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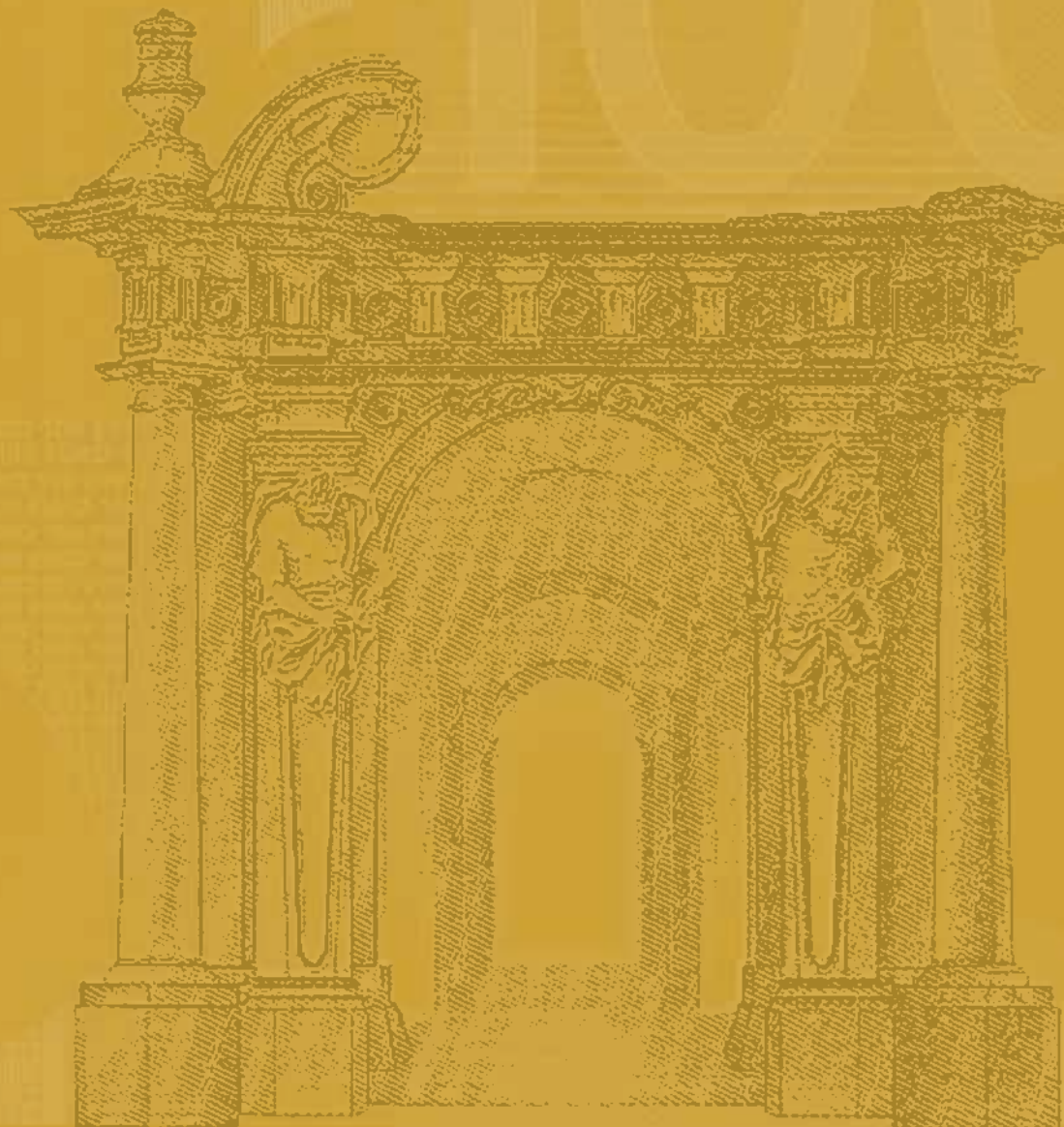
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WORKING PAPER SERIES

NO. 305 / FEBRUARY 2004

**A STRUCTURAL
COMMON FACTOR
APPROACH TO
CORE INFLATION
ESTIMATION AND
FORECASTING**

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In 2004 all ECB publications will feature a motif taken from the €100 banknote.



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A STRUCTURAL COMMON FACTOR APPROACH TO CORE INFLATION ESTIMATION AND FORECASTING¹

by Claudio Morana²

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Abstract

In the paper we propose a new methodological approach to core inflation estimation, based on a frequency domain principal components estimator, suited to estimate systems of fractionally cointegrated processes. The proposed core inflation measure is the scaled common persistent factor in inflation and excess nominal money growth and bears the interpretation of monetary inflation. The proposed measure is characterised by all the properties that an “ideal” core inflation process should show, providing also a superior forecasting performance relative to other available measures.

Keywords: long memory, common factors, fractional cointegration, Markov switching, core inflation, euro area.

JEL classification: C22, E31, E52.

Non-technical summary

While the available core inflation measures differ in terms of the statistical or econometric tools employed for estimation, there is substantial agreement in the literature concerning the theoretical framework of reference and the properties that a core inflation process should show. According to Brian and Cecchetti (1994), a core inflation process should be highly persistent, forward looking and tied to monetary dynamics. Coherent with the quantity theory of money, this latter property implies that core inflation should measure the inflation rate determined by the monetary authority. However, although most of the core inflation processes proposed in the literature make reference to the quantity theory framework (Brian and Cecchetti, 1994; Quah and Vahey, 1995; Bagliano and Morana, 1999, 2003a,b; Bagliano et al., 2002, 2003c; Cogley, 2002), the linkage between inflation and excess nominal money growth is only indirect, since at most either monetary aggregates have been considered in the information set, and a Cambridge real money demand has been estimated in levels (Bagliano et al., 2002, 2003c, Cassola and Morana, 2002), or a long-run relationship linking inflation and nominal money growth has been estimated (Bagliano and Morana 1999, 2003a,b), the latter leading to a core inflation process bearing the interpretation of the common permanent component in inflation and nominal money growth. Morana (2002) has recently made some progress on this issue, estimating the core inflation process as the scaled common persistent factor in inflation and excess nominal money growth, annihilated by the quantity theory long-run relationship. Coherent with recent contributions in the literature, which point to the presence of long memory and structural change in inflation (see for instance Hassler and Wolters, 1995; Baillie et al., 1996; Delgado and Robinson, 1994; Bos et al., 1999, 2001; Ooms and Doornik, 1999; Morana, 2000, 2002; Hyung and Franses, 2001; Baum et al., 2001), a more accurate modelling of the persistence properties of core inflation is also allowed in this framework.

In the paper we propose a new methodological approach to core inflation estimation, grounded on recent results of Morana (2003). Our definition of core inflation is the same as the one proposed by Morana (2002). However, differently from Morana (2002), estimation is carried out by means of a principal components frequency domain approach, suited to estimate systems of fractionally cointegrated processes. The approach improves upon the common long memory factor model of Morana (2002)

under several respects. Firstly, the estimation procedure is simplified, since the maximisation of the spectral likelihood function is not required, making the approach suitable to handle large systems and sample sizes. Secondly, the extraction of the persistent component is carried out through the Kasa (1992) decomposition, avoiding end-sample problems and arbitrary in the selection of the length of leads and lags, which is a drawback of double-sided filters. This also grants computability in real time of the core inflation measure. The main results of the paper are as follows. Firstly, by using an extended data set for the period 1980:1-2003:3, comprising data for Greece since the 1980s, and a different methodological approach, we confirm previous results of Morana (2000, 2002), concerning the presence of regime shifts and long memory in euro area HICP inflation. We also confirm the existence of a long-run linkage between inflation and excess nominal money growth. In fact, over the period 1980-2003 inflation and excess nominal money growth in the euro area share a common break process, with a near homogeneous cofeature (cobreaking) vector, which can be related to monetary policy regimes, i.e. to the break process in nominal money growth. We also find that break-free inflation and excess nominal money growth are fractionally cointegrated long memory processes. Therefore, inflation persistence can be accounted by both regime shifts and long memory dynamics. Coherent with the persistence properties of inflation and the structural linkages with the excess nominal money growth process, we compute the core inflation process as the scaled common persistent factor in inflation and excess nominal money growth. We find that the proposed core inflation measure is characterised by all the properties that a core inflation measure should show, namely forecasting ability, smoothness, robustness, theoretical foundation, computability in real time. In addition, by construction, the proposed core inflation process is tied to monetary aggregates, bearing the interpretation of monetary inflation rate. An out of sample forecasting exercise also shows that the proposed core inflation measure outperforms other available approaches both in the short and medium term.

1 Introduction

While the available core inflation measures differ in terms of the statistical or econometric tools employed for estimation, there is substantial agreement in the literature concerning the theoretical framework of reference and the properties that a core inflation process should show. According to Brian and Cecchetti (1994), a core inflation process should be highly persistent, forward looking and tied to monetary dynamics. Coherent with the quantity theory of money, this latter property implies that core inflation should measure the inflation rate determined by the monetary authority. However, although most of the core inflation processes proposed in the literature make reference to the quantity theory framework (Brian and Cecchetti, 1994; Quah and Vahey, 1995; Bagliano and Morana, 1999, 2003a,b; Bagliano et al., 2002, 2003c; Cogley, 2002), the linkage between inflation and excess nominal money growth is only indirect, since at most either monetary aggregates have been considered in the information set, and a Cambridge real money demand has been estimated in levels (Bagliano et al., 2002, 2003c, Cassola and Morana, 2002), or a long-run relationship linking inflation and nominal money growth has been estimated (Bagliano and Morana 1999, 2003a,b), the latter leading to a core inflation process bearing the interpretation of the common permanent component in inflation and nominal money growth. Morana (2002) has recently made some progress on this issue, estimating the core inflation process as the scaled common persistent factor in inflation and excess nominal money growth, annihilated by the quantity theory long-run relationship.¹ The core inflation process proposed by Morana (2002), therefore, is derived from the estimation of a structural model for inflation, granting a theoretical definition to the core inflation process in terms of monetary inflation rate.² Coherent with recent contributions in the literature, which point to the presence of long memory and structural change in inflation (see for instance Hassler and Wolters, 1995; Baillie et al., 1996; Delgado and Robinson, 1994; Bos et al., 1999, 2001; Ooms and Doornik, 1999; Morana, 2000, 2002; Hyung and Franses, 2001; Baum et al., 2001), a more accurate modelling of the persistence properties of core inflation is also allowed in this framework.

¹In Bagliano and Morana (2003a,b) nominal money growth and inflation are modelled as $I(1)$ processes and the quantity theory relationship involves only the nominal money rate of growth and the inflation rate, being output growth assumed to be $I(0)$. In Morana (2002) the quantity theory relationship involves excess nominal money growth and inflation, since both variables are modelled as long memory processes.

²Yet, the concept of core inflation is not univocally defined. For instance, Mankiw and Reiss (2002) is another contribution to the theoretical foundation of the core inflation process, but the proposed core inflation concept is unrelated to monetary dynamics. See also Winne (1999).

In the paper we propose a new methodological approach to core inflation estimation, grounded on recent results of Morana (2003). Our definition of core inflation is the same as the one proposed by Morana (2002). However, differently from Morana (2002), estimation is carried out by means of a principal components frequency domain approach, suited to estimate systems of fractionally cointegrated processes. The approach improves upon the common long memory factor model of Morana (2002) under several respects. Firstly, the estimation procedure is simplified, since the maximisation of the spectral likelihood function is not required, making the approach suitable to handle large systems and sample sizes. Secondly, the extraction of the persistent component is carried out through the Kasa (1992) decomposition, avoiding end-sample problems and arbitrary in the selection of the length of leads and lags, which is a drawback of double-sided filters. This also grants computability in real time of the core inflation measure.

The main results of the paper are as follows. Firstly, by using an extended data set for the period 1980:1-2003:3, comprising data for Greece since the 1980s, and a different methodological approach, we confirm previous results of Morana (2000, 2002), concerning the presence of regime shifts and long memory in euro area HICP inflation. We also confirm the existence of a long-run linkage between inflation and excess nominal money growth. In fact, over the period 1980-2003 inflation and excess nominal money growth in the euro area share a common break process, with a near homogeneous cofeature (cobreaking) vector, which can be related to monetary policy regimes, i.e. to the break process in nominal money growth. We also find that break-free inflation and excess nominal money growth are fractionally cointegrated long memory processes. Therefore, inflation persistence can be accounted by both regime shifts and long memory dynamics. Coherent with the persistence properties of inflation and the structural linkages with the excess nominal money growth process, we compute the core inflation process as the scaled common persistent factor in inflation and excess nominal money growth. We find that the proposed core inflation measure is characterised by all the properties that a core inflation measure should show, namely forecasting ability, smoothness, robustness, theoretical foundation, computability in real time. In addition, by construction, the proposed core inflation process is tied to monetary aggregates, bearing the interpretation of monetary inflation rate. An out of sample forecasting exercise also shows that the proposed core inflation measure outperforms other available approaches both in the short and medium term.

The rest of the paper is organised as follows. In section two we discuss the economics of core inflation, showing that a common theoretical framework can be found for the various core inflation measures proposed in the literature.

In section three we show how quantity theory is employed for the computation of the proposed core inflation measure. In sections four and five we introduce the econometric methodology and present the results. Finally, in section six we conclude.

2 The economics of core inflation

Despite the differences in the statistical approach used for estimation, it is possible to find in the quantity theory of money a common theoretical framework underlying the various measure of core inflation recently proposed in the literature. The quantity theory of money predicts that inflation is a monetary phenomenon. The relationship between the money supply and the price level can be stated through the Fisher's transaction equation

$$MV = PY,$$

where M is the nominal money supply, V is velocity, P is the price level, and Y is real output. By taking relative changes, we then have

$$\pi = m + v - y,$$

stating that the inflation rate (π) is equal to the excess nominal money rate of growth ($m - y$), corrected for the drift in velocity (v). In the standard version, therefore, the theory predicts that inflation in the steady state is determined by the nominal money rate of growth, being velocity constant and the output growth rate equal to zero.

This framework is clearly consistent with the general definition of core inflation as the (long-run) persistent inflation process, tied to monetary dynamics (Brian and Cecchetti, 1994). In general, however, the theoretical long-run linkage between inflation and nominal money growth has been only indirectly exploited in the computation of the core inflation process, being the core inflation measure proposed by Morana (2002) the only exception in the literature.

For instance, Bryan and Cecchetti (1994) justify the use of limited influence estimators through an implicit reference to the quantity theory of money and an explicit reference to the price setting model of Ball and Mankiw (1992). Ball and Mankiw (1992) assume that firms at each point in time decide to change prices according to the core inflation rate, which is defined as the nominal money rate of growth, under the assumption of constant velocity and zero trend output growth ($\pi^c = m$). Then, after having set prices

they observe a realisation from a zero mean price shock distribution. In order to change prices immediately after having observed the shock, firms have to pay a menu cost. Hence, they will choose to reset prices only if the observed shock is large enough. If the price shock distribution is skewed, also the actual inflation distribution will be skewed. It follows that computing the expectation of the cross sectional inflation distribution at each point in time will fail to deliver an estimate of the core inflation rate ($E\pi_{s,t} \neq \pi_t^c = m_t$). On the other hand, an accurate measure of the central tendency of the inflation distribution can be computed from the central part of the distribution, neglecting the tails. Limited influence estimators, such as the trimmed mean or the weighted median, can then successfully deliver the required core inflation estimate.³

The reference to the quantity theory of money is also implicit in the framework suggested by Quah and Vahey (1995), Blix (1995), Bagliano et al. (1999, 2002, 2003a,b,c)⁴. The common element in these approaches is the exploitation of a long-run neutrality restriction to identify the nominal shock underlying the core inflation process. As predicted by quantity theory, changes in the money supply will only affect the price level in the long-run, being output determined by supply side factors. The challenge is then to identify the monetary shock, which does not have a long-run impact on real activity. Given the assumption made on the persistence properties of the series, Bagliano and Morana (1999, 2003ab) have established a linkage between inflation and nominal money growth, deriving a core inflation process which bears the interpretation of the common permanent component in inflation and nominal money growth. However, Cassola and Morana (2002) have shown that the core inflation process obtained as the long-run inflation forecast from a common trends model (the Beveridge-Nelson-Stock-Watson inflation trend), under a suitable specification of the cointegration space and

³Note that the rationale underlying the computation of variance weighted core inflation measures (Dow, 1994; Diewert, 1995) is different. In these latter measures the weighting of the different categories of goods is inversely proportional to their price changes variance. The approach can find some theoretical grounding in the recent work of Mankiw and Reis (2002), which establishes that in a stability price index, i.e. a price index that if targeted would lead to the lowest variability in economic activity, the weights would be larger for sectors that are sensitive to the state of the economy, experience few sectoral shocks, have sluggish prices, and are relatively small in the aggregate price index.

⁴Despite the fact that the SVAR approach and the common trends approach are strictly related, the core inflation process obtained from the former does not bear the interpretation of long-run inflation forecast (see Evans and Reichlin, 1994). This is an important drawback of the Quah and Vahey (1995) approach, since core inflation is an expectational variable (Eckstein, 1981), and an appropriate estimator should deliver a forward looking measure.

the hypothesis of long-run separation between the nominal and real side of the economy, i.e. $\gamma_3 = 0$, can be interpreted as the long-run excess nominal money growth process

$$\pi_t^* = m_t^* - \eta k_\theta = \gamma_2 \beta_t + \gamma_3 \theta_t,$$

where β_t is the nominal trend determined by the shocks which are output neutral in the long-run, θ_t is the real trend determined by productivity shocks, and k_θ is the drift in the real trend. The same interpretation also holds for the core inflation process obtained in Bagliano et al. (2002, 2003c).

On the other hand, in Morana (2002) the exploitation of quantity theory for the estimation of the core inflation process is direct, since the core inflation measure is given by the (scaled) common persistent factor in inflation and excess nominal money growth

$$\pi_t^c = \hat{\mu}_{cbp,t} + \hat{\mu}_{clm,t},$$

where $\hat{\mu}_{cbp,t}$ is the common break process in inflation and excess nominal money growth, bearing the interpretation of long-run inflation forecast, and being determined by the break process in the nominal money growth rate, i.e. by changes in monetary policy regimes; $\hat{\mu}_{clm,t}$ is the (scaled) common long memory factor in break-free inflation and excess nominal money growth, determined by output dynamics. An important novelty of the approach of Morana (2002) is that the quantity theory equation is directly estimated and interpretable as a cointegration and fractional cointegration relationship. In addition, differently from the other approaches, the core inflation process is assumed to be covariance stationary, but strongly persistent.⁵

While the core inflation measure proposed by Cogley (2002) is still coherent with a monetary interpretation of long-run inflation, its theoretical justification is different and related to the work of Sargent (1999). In this framework inflation persistence is explained by changes in monetary policy regimes, which alter the mean towards which actual inflation converges, i.e. by the inflation break process. However, in practice, the estimated core inflation process is fully unrelated to monetary aggregates, since the proposed constant gain filter is applied to the inflation process only.⁶

⁵By controlling for long memory and structural change the approach should allow a more accurate modelling of the persistence properties of inflation. Relatively to the core inflation measure proposed by Morana (2000), the approach allows to relate also the break-free persistent dynamics to the excess nominal money growth process. The theoretical framework is presented in detail in the following section.

⁶Both in Morana (2000, 2002) and Cogley (2002) there is the attempt to relate (core) inflation persistence to a break process determined by monetary policy regimes. However, the approach of Cogley (2002) seems to be suboptimal, relative to the direct estimation of the break process, for the purpose of tracking policy changes and estimating core inflation. The updating of the mean is in fact more timely when the break process is directly estimated, possibly exploiting information on monetary aggregates. Moreover, not allowing for long memory, the constant gain filter is not suited to extract the persistent inflation component.



Finally, no direct or indirect reference to a theoretical framework is made in the other approaches available in the literature (Bryan and Cecchetti, 1993; Arrazola and de Hevia, 2002, Angelini et al., 2001a,b; Cristadoro et al., 2001). Despite the different statistical framework employed, i.e. the Dynamic Factor Index (Stock and Watson, 1991), the Independent Inflation Rate, the Diffusion Index approach (Stock and Watson, 1998), and the Generalised Factor Model (Forni et al., 2000), respectively, the common element of these studies is the attempt to extract a noise-free measure of inflation, common to the various categories of goods (Brian and Cecchetti, 1993; Arrazola and de Hevia, 2002), i.e. a measure of monetary inflation unaffected by changes in relative prices, or which is determined by common real or nominal factors (Angelini et al., 2001a,b; Cristadoro et al., 2002), computed from a large information set comprised of variables which are believed to be related to inflation. These approaches are therefore purely statistical, and it is not granted that the factors determining the estimated core inflation process are suitable of economic interpretation, i.e. for instance whether they are related to the excess nominal money growth process.

A conclusion which can be drawn at this stage is that, if any, the available core inflation measures make reference to a common theoretical framework, i.e. the quantity theory of money. However, the linkage between inflation and excess nominal money growth has not been directly exploited in estimation in none of the approaches, apart from Morana (2002). As noted above, the general definition of core inflation requires the core inflation process to be forward looking, strongly persistent, not affected by idiosyncratic shocks associated with relative price changes, and tied to nominal money growth. The definition of core inflation provided in Morana (2002), i.e. the scaled common persistent factor driving inflation and excess nominal money growth, annihilated by the quantity theory long-run relationship, is coherent with all the elements of the above definition, in addition to be obtained directly from a structural model. Moreover, this measure of core inflation is the only measure available in the literature which accurately models the persistence properties of inflation.⁷

⁷It should be noted that, if the causes of inflation persistence are acknowledged, all the core inflation estimation approaches proposed in the literature, apart from Morana (2000, 2002), not allowing for long memory and structural change, are not suited to extract the persistent inflation component.

3 The theoretical framework

The inflation equilibrium relationship can be described by the following equation

$$\pi_t = m_t - \eta g_t + \varepsilon_{\pi,t}, \quad (1)$$

where π_t is the inflation rate, m_t is the nominal money rate of growth, g_t is the real output rate of growth, $\varepsilon_{\pi,t} \sim i.i.d. (0, \sigma_\pi^2)$ or follows a stationary ARMA process. Therefore, equation [1], as predicted by quantity theory, relates the long-run inflation rate to the long-run excess nominal money growth rate. The persistence properties of the excess nominal money growth process are therefore inherited by the inflation rate through the quantity theory relationship. Hence, when the excess nominal money growth process is a stationary process, it follows that also inflation should show the same property. Under stationarity, persistence can be explained by unaccounted structural breaks, long memory, or both. Depending on the cause of persistence, the common feature shared by inflation and excess nominal money growth may be described by a common break process, a common long memory component, or both. Morana (2002) has considered several cases, showing how the common persistent feature may be related to either nominal or real factors or both. In what follows we only sketch the case which has been found of empirical relevance for the euro area.

Persistent dynamics In our framework the following assumptions will be made. Firstly, the excess nominal money growth process is a perturbed long memory process (Granger and Marmol, 1997) subject to structural change. Both long memory and structural change explain the persistence of nominal money growth. The break process may be related to the working of monetary policy, in particular to disinflation policies, given the sample considered. On the other hand, long memory explains the persistence of real output growth. Hence, the excess nominal money rate of growth process is a long memory process subject to structural change. From these assumptions, through the equilibrium relationship [1], it also follows that inflation is a perturbed long memory process subject to structural change, and that

excess nominal money growth and inflation are cobreaking and fractionally cointegrated processes.

The model therefore can be set up as follows.

$$m_t = \mu_{clm,t}^n + \mu_{cbp,t} + \varepsilon_{m,t}, \quad (2)$$

$$g_t = - (1/\eta) \mu_{clm,t}^r + \varepsilon_{g,t}, \quad (3)$$

where $\mu_{cbp,t}$ is the break process, $\mu_{clm,t}^n \sim I(d)$ $0 < d < 0.5$ is the nominal long memory component, $\mu_{clm,t}^r \sim I(d)$ $0 < d < 0.5$ is the real long memory component, and $\varepsilon_{i,t} \sim i.i.d. (0, \sigma_i^2)$ $i = m, g$ or follows a stationary zero mean ARMA process. This implies that the excess nominal money growth process

$$em_t = \mu_{cbp,t} + \mu_{clm,t}^n + \mu_{clm,t}^r + \varepsilon_{m,t} - \eta\varepsilon_{g,t}, \quad (4)$$

is a perturbed long memory process subject to structural change. We also have

$$\pi_t = \mu_{cbp,t} + \mu_{clm,t}^n + \mu_{clm,t}^r + \varepsilon_{m,t} - \eta\varepsilon_{g,t} + \varepsilon_{\pi,t}, \quad (5)$$

i.e., inflation is a perturbed long memory process subject to structural change. Then the equilibrium relationship [1] can be interpreted as a cobreaking and fractional cointegration relationship, since

$$\pi_t - em_t = \varepsilon_{\pi,t}. \quad (6)$$

is a stable and weakly dependent process.⁸

As shown by Morana (2002), the core inflation process can then be constructed by adding the estimated persistent factors, i.e.

$$\begin{aligned} \pi_t^c &= \hat{\mu}_{cbp,t} + \hat{\mu}_{clm,t}^n + \hat{\mu}_{clm,t}^r \\ &= \hat{\mu}_{cbp,t} + \hat{\mu}_{clm,t}. \end{aligned}$$

Differently from Morana (2002), the long memory inflation component ($\hat{\mu}_{clm,t}$) is given by both nominal ($\hat{\mu}_{clm,t}^n$) and real ($\hat{\mu}_{clm,t}^r$) forces. This specification is justified by the empirical results of the paper, which suggests that also nominal money growth is a perturbed long memory process (subject to structural change).

⁸An implication of this result is that real money growth and output growth should be pure long memory processes and fractionally cointegrated. In fact, the above long-run relationship can be rewritten as $rm_t - \eta g_t = -\varepsilon_{\pi,t}$, where $rm_t = m_t - \pi_t$ is the real money growth process.

4 Econometric methodology

Lets assume the following common long memory factor model

$$\begin{aligned}\mathbf{x}_t &= \Theta \boldsymbol{\mu}_t + \mathbf{u}_t \\ \Delta^d \boldsymbol{\mu}_t &= \boldsymbol{\varepsilon}_t,\end{aligned}\tag{7}$$

where \mathbf{x}_t is a $p \times 1$ vector of observations on the p fractionally cointegrated processes, Θ is the $p \times k$ factor loading matrix with $k < p$, $\boldsymbol{\mu}_t$ is a $k \times 1$ vector of observations on the long memory factors ($I(d)$ $0 < d < 0.5$), $\boldsymbol{\varepsilon}_t \sim i.i.d.(\mathbf{0}, \Sigma_\varepsilon)$ with dimension $k \times 1$, \mathbf{u}_t is a $p \times 1$ vector of observations on the weakly dependent components ($I(0)$), with $\Phi(L)\mathbf{u}_t = \Omega(L)\mathbf{v}_t$, all the roots of the polynomial matrices in the lag operator $\Phi(L)$ and $\Omega(L)$ are outside the unit circle, and $\mathbf{v}_t \sim i.i.d.(\mathbf{0}, \Sigma_u)$ with dimension $p \times 1$.

Applying fractional differencing to (7), yields

$$\Delta^d \mathbf{x}_t = \Theta \boldsymbol{\varepsilon}_t + \Delta^d \mathbf{u}_t\tag{8}$$

and the associated spectral matrix

$$\mathbf{f}(\omega) = \Theta \mathbf{f}_\varepsilon(\omega) \Theta' + \Theta \mathbf{f}_{\varepsilon, \Delta^d u'}(\omega) + \mathbf{f}_{\Delta^d u, \varepsilon'}(\omega) \Theta' + \mathbf{f}_{\Delta^d u}(\omega),\tag{9}$$

where the $\mathbf{f}_i(\omega)$ matrices contain the spectral and cross spectral functions for the given vectors, evaluated at the frequency ω . Evaluation at the zero frequency yields

$$\mathbf{f}(0) = \frac{1}{2\pi} \Theta \Theta',\tag{10}$$

since $\mathbf{f}_{\varepsilon, \Delta^d u'}(0) = \mathbf{0}$, $\mathbf{f}_{\Delta^d u, \varepsilon'}(0) = \mathbf{0}$, $\mathbf{f}_{\Delta^d u}(0) = \mathbf{0}$.⁹

Hence $\mathbf{f}(0)$ inherits the properties of the matrix $\Theta \Theta'$, namely $\mathbf{f}(0)$ is symmetric, it is of reduced rank equal to $k < p$, and it is positive semidefinite.¹⁰

⁹ $\mathbf{f}_{\Delta^d u}(0) = \mathbf{0}$ follows from the fact that the \mathbf{u}_t vector is $I(0)$, so that applying the fractional differencing filter leads to an overdifferenced vector process with null spectral matrix at the zero frequency. $\mathbf{f}_{\varepsilon, \Delta^d u'}(0) = \mathbf{0}$, $\mathbf{f}_{\Delta^d u, \varepsilon'}(0) = \mathbf{0}$ follows from the above argument and the $\boldsymbol{\varepsilon}_t$ having a finite spectrum at the zero frequency. Moreover, the i.i.d. assumption implies $\mathbf{f}_\varepsilon(\omega) = \frac{1}{2\pi} \Sigma_\varepsilon$ at all frequencies. Since Σ_ε is orthonormal we then have $\mathbf{f}_\varepsilon(\omega) = \frac{1}{2\pi} \mathbf{I}_k$. See properties 1-3 in section 3 in Morana [59].

¹⁰Note that the same results hold for the case in which the \mathbf{u} vector is $I(b)$ $b > 0$ $d-b > 0$, since $\Delta^d \mathbf{u} \sim I(b-d)$.

Estimation and identification of the factor loading matrix From the symmetry property, it follows that the spectral matrix can be factorised as

$$2\pi\mathbf{f}(0) = \mathbf{Q}\Lambda\mathbf{Q}', \quad (11)$$

where Λ is the $p \times p$ diagonal matrix of (real) eigenvalues and the matrix \mathbf{Q} is the $p \times p$ matrix of its associated orthogonal eigenvectors. Since $\mathbf{f}(0)$ is of reduced rank k , only k eigenvalues are greater than zero. Hence, $\mathbf{Q}\Lambda^{\frac{1}{2}}$ contains k non zero columns and $\Lambda^{\frac{1}{2}}\mathbf{Q}'$ k non zero rows. Without lack of generality, by assuming that the eigenvalues are ordered in descending order, the matrix $\mathbf{Q}\Lambda^{\frac{1}{2}}$ can be partitioned as $\begin{bmatrix} (\mathbf{Q}\Lambda^{\frac{1}{2}})^* & \mathbf{0} \\ p \times k & p \times (p-k) \end{bmatrix}$, so that by the rule of the product of partitioned matrices we have

$$\begin{pmatrix} \mathbf{Q}\Lambda^{\frac{1}{2}} \\ p \times p \end{pmatrix} \begin{pmatrix} \Lambda^{\frac{1}{2}}\mathbf{Q}' \\ p \times p \end{pmatrix} = \begin{pmatrix} \mathbf{Q}\Lambda^{\frac{1}{2}} \\ p \times k \end{pmatrix}^* \begin{pmatrix} \Lambda^{\frac{1}{2}}\mathbf{Q}' \\ k \times p \end{pmatrix}^* + \mathbf{0} = \Theta\Theta'. \quad (12)$$

The matrix $(\mathbf{Q}\Lambda^{\frac{1}{2}})^*$ is therefore our estimator of the factor loading matrix Θ .

Lets write the factor loading matrix as

$$\Theta = \mathbf{Q}_0^* \boldsymbol{\rho}, \quad (13)$$

where $\boldsymbol{\rho}$ is a $k \times k$ matrix collecting the free parameters in Θ and \mathbf{Q}_0^* is the matrix $(\mathbf{Q}\Lambda^{\frac{1}{2}})^*$ with the identification conditions imposed in such a way that the upper square submatrix of order k is the identity matrix. This will yield $k(k-1)/2$ zero restrictions in Θ . From the relationship $\Theta\Theta' = 2\pi\mathbf{f}(0)$ we then have

$$\boldsymbol{\rho}\boldsymbol{\rho}' = (\mathbf{Q}_0'^* \mathbf{Q}_0^*)^{-1} \mathbf{Q}_0^* (2\pi\mathbf{f}(0)) \mathbf{Q}_0^* (\mathbf{Q}_0'^* \mathbf{Q}_0^*)^{-1}. \quad (14)$$

The matrix $\boldsymbol{\rho}\boldsymbol{\rho}'$ is positive definite and symmetric, containing $k(k+1)/2$ distinct parameters which can be estimated through its Choleski decomposition, leading to a lower triangular $\boldsymbol{\rho}$ matrix and to $k(k+1)/2$ independent equations. A total of k^2 over pk parameters in Θ will result to be identified using the above discussed procedure. Finally, the remaining $(p-k)k$ parameters will result to be identified by using the condition $\mathbf{Q}'_{k+1,\dots,p} \mathbf{Q}_0^* = \mathbf{0}$. After

estimation the matrix Θ may be rotated to add further interpretability to the results.¹¹

Estimation of the cointegration space Given the orthogonality property of the eigenvectors, it follows that

$$\mathbf{Q}'_{1,\dots,k} \mathbf{Q}_{k+1,\dots,p} = \mathbf{0}_{k \times (p-k)}, \quad (15)$$

where $\mathbf{Q}'_{1,\dots,k}$ and $\mathbf{Q}_{k+1,\dots,p}$ denote the submatrices composed of the k eigenvectors associated with the first k largest roots, and the last $r = p - k$ eigenvectors associated with the zero roots, respectively. Hence $\mathbf{Q}_{k+1,\dots,p}$ is a right null space basis of the transposed factor loading matrix, which is the definition of the cointegration space, since the cointegration relationships are the linear combinations of the variables which remove the persistent (I(d)) or permanent (I(1)) component from them. We can write therefore $\boldsymbol{\beta} = \mathbf{Q}_{k+1,\dots,p}$, where $\boldsymbol{\beta}$ denote the $p \times r$ cointegration matrix, obtaining

$$\boldsymbol{\beta}' \left(\mathbf{Q} \Lambda^{\frac{1}{2}} \right)^* = \boldsymbol{\beta}' \Theta = \mathbf{0}_{r \times k}.$$

Estimation of the common long memory factors and persistent-non persistent decomposition A persistent-non persistent decomposition (P-NP decomposition) of the observed variables can be performed through the decomposition of Kasa (1992), which can be written as

$$\begin{aligned} \mathbf{x}_t &= \Theta \boldsymbol{\mu}_t + \mathbf{u}_t \\ \boldsymbol{\mu}_t &= (\Theta' \Theta)^{-1} \Theta' \mathbf{x}_t \\ \mathbf{u}_t &= \boldsymbol{\beta} (\boldsymbol{\beta}' \boldsymbol{\beta})^{-1} \boldsymbol{\beta}' \mathbf{x}_t \end{aligned} \quad (16)$$

where $\Theta (\Theta' \Theta)^{-1} \Theta' \mathbf{x}_t$ is the persistent (long memory component) and $\boldsymbol{\beta} (\boldsymbol{\beta}' \boldsymbol{\beta})^{-1} \boldsymbol{\beta}' \mathbf{x}_t$ is the non persistent (I(0)) component or the less persistent I(b) component $b > 0$, $d - b > 0$, when $\mathbf{u}_t \sim I(b)$.¹² This decomposition has the important advantage of being implemented using the observed series and is suitable also

¹¹A similar approach has been proposed by King et al. (1991) and Warne (1993) for the identification of the common trends model in time domain.

¹²The \mathbf{u}_t vector is I(b) when the cointegrating residuals are I(b) or when the largest order of fractional integration of the cointegrating residuals is I(b). Note in fact that the \mathbf{u}_t vector is computed as a linear combination of the cointegrating residuals.

for the case of fractionally cointegrated $I(d)$ processes. In fact, the decomposition follows from the projection theorem, and it is always valid provided that, given a vector of \mathbf{x}_t in R^p , a closed subspace Θ of R^p is available. Then the vector \mathbf{x}_t can be decomposed in the sum of its projections on Θ and Θ_{\perp} , where the projection operators are $\Theta(\Theta'\Theta)^{-1}\Theta'$ and $\beta(\beta'\beta)^{-1}\beta'$ (see Kasa, 1992). See Morana (2003) for further details on the methodology and for a proof of consistency.

5 Results

Since a strong degree of long memory may be a spurious finding due to neglected structural change (Mikosch and Starica, 1998; Granger and Hyung, 1999), it is important to control for structural change when testing for long memory. However, with the available methodologies distinguishing between long memory and structural change is far from being clear-cut.¹³ Two possible strategies indicated in the literature so far are to allow for long memory and structural change when assessing the persistence properties of a time series (Hidalgo and Robinson, 1996; Kuan and Hsu, 2000; Kokoszka and Leipus, 2000), or, as suggested by Granger and Hyung (1999), testing for a spurious break process by checking whether the break-free series is characterised by antipersistence, while the actual series shows long memory. Morana (2002) has proposed an approach that unifies the above mentioned strategies, based on an augmented Engle and Kozicki (1993) feature test. The test amounts to checking the statistical significance of a candidate break process in an ARFIMA model. By controlling for both long memory and structural change, this test is expected to provide reliable results concerning the causes of persistence. The approach improves upon the first strategy since the available methodologies are suited to test for just one break point. Moreover, coherent with the second strategy, the evaluation of the actual presence of a break process can also be assisted by considering the implications of selecting the wrong model, i.e. the antipersistence induced by the removal of a spurious break process.

¹³For instance Hsu (2001) and Kuan and Hsu (1998) have shown that the Bai (1994) test rejects the null of no structural change with probability one and is biased to select a break point in the middle of the sample when the process is actually characterised by long memory and no structural change. In addition, Granger and Hyung (1999) have shown that the number of spurious breaks detected increases with the magnitude of the Hurst exponent and is zero only when the process is $I(0)$.

In the paper the break process has been estimated by means of a Markov switching model (Hamilton, 1990), which, as shown by Ang and Bekaert (1998), allows for consistent estimation of the break process, provided the omitted variables are not regime dependent. This approach has been successfully employed to this purpose in several papers (Morana 2000, 2002; Morana and Beltratti, 2002; Timmerman, 2001). See Morana (2002) for a description of the methodology followed to estimate the break process and test for the existence of a common break process.

5.1 Computing the excess nominal money growth process

As already noted in the theoretical section, the existence of a common break process and long memory factor driving inflation and excess nominal money growth implies that real money growth and output growth should be pure long memory processes and fractionally cointegrated. Evidence in favour of such properties have been provided by Morana (2002), using euro-11 area data for the period 1980-2000. In our study we have extended the previous analysis by considering an updated dataset (euro-12 area), comprising data also for Greece since 1980, spanning over the period 1980:1-2003:3.¹⁴

In the analysis we have proceeded as follows. Firstly, we have established the causes of persistence of the real money growth and output growth processes by testing for structural change and long memory. Secondly, we have estimated the long-run relationship linking the two processes. In fact, in order to compute the excess nominal money growth process we need an estimate of the output elasticity of real money balances (η).

5.1.1 Persistence analysis

Coherent with previous results of Morana (2002), the Schwarz-Bayes information criterion and the Likelihood ratio test (with p-value computed as in

¹⁴Figures for inflation are monthly rates of growth of euro-12 area seasonally adjusted HICP. The national HICP series have been extended backwards using growth rates in CPI (national definition), using consumer spending weights converted by irrevocable fixed exchange rates for EMU. Figures for M3 are monthly rates of growth of the month-end stocks of M3 (ECB database, in millions of euro, seasonally adjusted). Until 1997Q4 M3 data are based on stocks; from 1997Q4 on flow statistics. Monthly rates of growth of real M3 are computed by subtracting HICP inflation from the monthly rates of growth of nominal M3. Figures for GDP are national series on seasonally adjusted real GDP at market prices from BIS and AMECO. They are converted to euro via the irrevocable fixed conversion rates of 31 December 1998. Monthly figures are derived via interpolation.

Davies (1987)) suggest that a yearly constant mean model may be preferred to the Markov-switching model¹⁵. Moreover the Kokoszka and Leipus (2000) test does not allow to reject the null of no structural change, even at the 10% significance level for both processes.

Semiparametric estimators have been employed to assess the degree of persistence of the series (Table 1). According to the results of the Monte Carlo analysis reported in the Appendix, the estimator of Robinson (1998)¹⁶ has been employed in the analysis. In addition to be unbiased, the estimator is the one characterised by minimum RMSE, relative to the Local Whittle estimator (Kunsch, 1987; Robinson, 1995b), the log periodogram estimator (Geweke and Porter Hudak, 1983; Robinson, 1995), and the averaged periodogram estimator (Robinson, 1994; Lobato and Robinson, 1996)).¹⁷ Since when observational noise characterises the data all the above mentioned estimators may be affected by downward bias, the non linear log periodogram estimator (Sun and Phillips, 2003) has also been employed in the analysis. As shown in the Table, both the LM and the non linear log periodogram estimator point to a moderate degree of long memory for both real money growth and output growth. Over the investigated interval (20-138 periodogram ordinates), the estimates provided by the LM estimator range between a minimum of 0.30 and a maximum of 0.34 for real money growth, and between a minimum of 0.28 and a maximum of 0.32 for real output growth. On the other hand, the estimates provided by the non linear log periodogram estimator tend to be more unstable. For real money growth the non linear log periodogram estimator points to estimates in the range 0.35-0.37 in the stable region detected between 68 and 86 ordinates, with an average value equal to

¹⁵Estimation has been performed using the Ox code "MSVAR" written by H.M. Krolzig. A full set of results on the estimated break processes is available from the author upon request. Estimating the break process on low frequency data helps the detection of break points, since only infrequent changes are detected by the Markov switching model. See Morana and Beltratti (2002) and Morana (2002) for details on how to recover the implied high frequency break process.

¹⁶The estimator proposed by Robinson (1998), which we denote the LM estimator since it can be derived from the LM I(0) stationarity test of Lobato and Robinson (1998), has the same asymptotic properties of the local Whittle estimator suggested by Kunsch (1987), i.e. asymptotic normality, consistency, and efficiency, under the assumption of weak dependence. Under the assumption of long memory the estimator still retains the consistency property, although its asymptotic distribution is unknown. In the empirical application we have computed approximate standard errors assuming the same asymptotic distribution holding for the weak dependence case, i.e. $\sqrt{m}(\hat{H} - H) \xrightarrow{d} N(0, \frac{1}{4})$.

¹⁷Empirical applications have also shown that the LM estimator tends to provide more stable estimates than the other available estimators, particularly when the sample size is small.

0.36. The estimated inverse long-run signal to noise ratio in the same region ranges between 0.81 and 0.95, with an average value equal to 0.87. For output growth the estimator does not seem to be appropriate given that, over the interval analysed, the estimated inverse long-run signal to noise ratio is equal to zero, apart from the interval 22-46 ordinates, where is ranging between a minimum of 0.19 and a maximum of 0.298). Moreover, the estimated fractional differencing parameter tends to be unstable. In the interval 38-54 ordinates the estimates range between a minimum of 0.26 and a maximum of 0.36, with an average value equal to 0.30. A test for the equality of the fractional differencing parameter carried out in the framework of the multivariate non linear log periodogram estimator (Beltratti and Morana, 2003; see the Appendix) points to the non rejection of the null of equality at the 5% significance level over all the range investigated. Given the instability of the estimates provided by the non linear log periodogram estimator the joint estimate of the fractional differencing parameter has then been computed by averaging the upper bound estimates obtained from the LM estimator for the two processes, yielding a value equal to 0.33.

5.1.2 Fractional cointegration analysis

Given the evidence of observational noise in the real money growth process, the denoising approach of Beltratti and Morana (2003) has been implemented (see the Appendix for a description of the methodology). The trimmed centered filter (c_{70}^*) has been employed, with trimming bandwidth determined optimally through Monte Carlo simulation (70 ordinates). In Table 2, Panel A we report the results of the Monte Carlo simulation, while in Figure 1 we plot the actual and denoised real money growth process. As shown in the table, the optimally centered trimmed filter shows a RMSE which is close to the theoretical minimum achieved by the two sided parametric Wiener Kolmogorov filter, also showing the same U coefficient and RMSE decomposition. The superior performance of the centered trimmed filter relatively to the other semiparametric filter is also noticeable from the Table.

In Table 3 Panel A we report the results of the fractional cointegrating rank analysis, computed as in Robinson and Yajima (2002), and of the squared coherence analysis; in Panel B we report the estimated eigenvectors, with standard errors computed using the jack-knife.

A first important finding of the analysis is the strong evidence in favour of fractional cointegration between real money growth and output growth, given that the bulk of variance is explained by the largest eigenvalue. The proportion of variance associated with the largest eigenvalue tends to decrease as

the bandwidth increases: it is close to 95% for the selected bandwidth (two ordinates, Table 2, Panel A), and falls to about 84% for a bandwidth equal to five ordinates, stabilising at a value close to 0.70 thereafter (Figure 2). Given the potential downward bias affecting the estimated proportion of explained variance pointed out by the Monte Carlo analysis of Morana (2003), it is possible to safely conclude in favour of a single persistent factor driving the two processes, despite the Robinson and Yajima (2002) test points to rejection of the null of cointegration between the two processes. Further evidence in favour of cointegration is also provided by the squared coherence analysis tests. As shown by Morana (2003), fractional cointegration implies and it is implied by a unitary squared coherence at the zero frequency for the fractionally differenced processes in the bivariate framework, while the lack of fractional cointegration is implied by a zero squared coherence at the zero frequency.¹⁸ In fact, the point estimate of the squared coherence at the zero frequency is about 0.73 (0.20), and the null of no cointegration (orthogonality) can be rejected at the 5% significance level, while the null of cointegration cannot be rejected.¹⁹ The bias corrected estimates (see Morana, 2003), tend to be stable, ranging between 0.90 and 1 over the range selected for the stability analysis (1-30 ordinates), being equal to 0.95 in correspondence of the selected bandwidth (two ordinates). Following economic theory, we have then set to one the estimated parameter. Support for the imposed restriction is provided by the correlation between the estimated factors obtained from the unrestricted and restricted models, which is equal to 0.975. The importance of denoising the real money growth process before estimation can be gauged by comparing our estimate of the output elasticity of real money balances with the one obtained by Morana (2002), which was close to 1.39.

5.2 Common persistent features: the nominal side

According to the results of Morana (2002), a common break process, originating from nominal money growth, can be detected in euro-11 area excess

¹⁸As shown by Granger and Weiss (1983) and Levy (2002), the existence of cointegration between I(1) bivariate processes implies that the squared coherence at the zero frequency of the series in differences is equal to one, while when more than two processes are involved it is the multiple squared coherence to assume a unitary value. As argued by Granger and Weiss (1983) the same results hold also for the $CI(d, d)$ $0 < d < 1$ case. Morana (2003) has generalised the above findings, considering the case of vector long memory cointegrated processes, for the case $CI(d, b)$ $0 < d < 0.5$, $d > b$, for the series in differences ($\omega = 0$) and levels ($\omega \rightarrow 0^+$). Morana (2003) has also provided new results concerning the number of unitary and zero squared coherences at the zero frequency.

¹⁹The p-value of the zero squared coherence at the zero frequency test is 0.047, while the p-value of the unitary squared coherence at the zero frequency test is 0.283.

nominal money growth and inflation, while a common long memory factor, originating from output growth, explains the break-free persistent dynamics of the two series. In the analysis that follows we have assessed whether these features hold also for euro-12 area data. We have proceeded as follows. Firstly, we have tested for the existence of a common break process in nominal money growth, inflation and excess nominal money growth. Then, we have tested for fractional cointegration between the break-free inflation and excess nominal money growth processes. Coherent with our theoretical framework, we expect a long-run relationship relating the above mentioned variables, i.e a single common long memory factor driving the two series. As it will be shown below, differently from Morana (2002), the common long memory component in inflation and excess nominal money growth is determined by both real and nominal forces.

5.2.1 Structural break analysis: determining monetary policy regimes

A simpler strategy than the one suggested by Morana (2002) to test for a common break process in inflation and excess nominal money growth has been followed, since the previous section has already provided evidence of no breaks in the real variables. Hence, if a break process is found in the nominal variables, then it has to be common. Namely, a candidate common break process has been estimated from a multivariate Markov switching model assuming perfect correlation of the states across processes (see Krolzig, 1997). Then, the augmented Engle and Kozicki test has been employed to assess whether the estimated break process may be regarded as a real feature of the variables, while controlling for long memory. If the null of no feature can be rejected, then there is evidence in favour of the existence of a common break process for the variables analysed.

The results of the persistence analysis are reported in Table 4. The following findings are noteworthy. Firstly, the Schwarz-Bayes information criterion and the Likelihood ratio test support the constrained three regimes model, estimated on annual data, for all the processes considered. The estimated common break process bears the interpretation suggested by Morana (2002) also for the extended data set, pointing to a high inflation period (1980-1982), an average inflation period (1983-1993) during which disinflation policies were undertaken in many euro area countries (see also Soderstrom and Vredin, 2000), and a low inflation period (1994-2000) consistent with the price stability objective of the ECB (see also Cassola, 1999). In fact, in the low inflation regime the mean annual HICP inflation rate is not statistically different from the ECB reference value (2%) (Table 3, Panel B). In addition,

also the annual nominal money growth rate is close to its reference value (4.5%). Finally, the mean annual excess nominal money growth rate is not statistically different from the mean inflation rate, and numerically very close to it in the low inflation regime. This finding is consistent with the long-run relationship between inflation and excess nominal money growth postulated by quantity theory, pointing to a homogeneous cobreaking vector.²⁰ According to the estimated transition matrix (Table 3, Panel A), the estimated regimes are very persistent, with a low probability to switch from the low inflation regime to the average inflation regime, suggesting the credibility of the current monetary policy framework. However, since 2001 the Markov switching model detects a reversion to the average inflation regime. One problem with the estimation of the common break process using annual data is that, by construction, when moving from the annual model to the monthly or quarterly models, regime changes will be estimated in correspondence of the first month or quarter of the selected year. For robustness we have checked the dating of the regime shifts by estimating a quarterly common break process directly²¹. Interestingly, the quarterly model does not suggest a reversion to the average inflation regime since 2001. We regard this finding, therefore, as pointing to spurious evidence of a switch to the average regime for the series considered. The Kokoszka and Leipus (2000) test supports this conclusion, since this latter switch is only significant at the 5% level for the excess nominal money growth process, but not for money growth and inflation. Therefore, we did not estimate a break point occurring in 2001. In addition, the overlapping of the implied (obtained from the annual model) and actual quarterly common break processes is not perfect. The quarterly model suggests that the high inflation regime ended in 1983:3, while the average inflation regime in 1992:1. Since the quarterly model allows for more flexibility in determining break points, proving to be as successful as the annual model in uncovering the low frequency shifts with the data at hand, we have selected 1992:2 as the starting quarter for the low inflation regimes. The dating of the monthly common break process is therefore: 1980:1-1983:3(9) (high inflation regime), 1983:4(10)-1992:1(3) (average inflation regime), 1992:2(4)-2003:1(3) (low inflation regime). Then, the estimation of the monthly break process has been carried out by regressing the actual variables on the three step dummies corresponding to the identified regimes. Statistical tests provide support for our modelling strategy. In fact, the augmented Engle and

²⁰A formal test partially supports this conclusion. The estimated cobreaking vector for inflation and excess nominal money growth is $[1 - 0.807(0.040)]$, the p-value of the cobreaking test ($\chi^2_{(1)}$) is equal to 0.365. On the other hand, the null of homogeneity is rejected at the 1% significance level.

²¹The results are available upon request to the author.

Kozicki (1993) test unambiguously shows that the estimated break process is a common feature for all the series, since the null of no feature is rejected at the 1% level for all the processes.²² Moreover, coherent with the findings of Morana (2002), the break process seems to explain all the persistence in nominal money growth, since the estimated fractional differencing parameter is not statistically different from zero.²³ On the other hand, for the other processes still a significant degree of persistence can be detected, with the estimated fractional differencing parameter being close to 0.30 for both processes. The estimates of the fractional differencing parameter, obtained from the LM estimator, for the break-free processes are also coherent with these conclusions, allowing to exclude the presence of a spurious break process, and pointing to both long memory and structural change in excess nominal money growth and inflation. For inflation and nominal money growth the estimates of the fractional differencing parameter are stable, yielding an average estimate equal to 0.19 and 0.11, respectively, over the interval 40-138 ordinates. For excess nominal money growth the estimated fractional differencing parameter tends to increase with the bandwidth, yielding an average value equal to 0.14 over the same interval. The estimates provided by the non linear log periodogram estimator suggests that the LM estimator may be affected by downward bias. In fact, for all the processes the NLP estimator points to a larger degree of persistence over all the investigated range (20-138 ordinates), and to a significant inverse long-run signal to noise ratio for inflation and nominal money growth over most of the bandwidth analysed. A test for the equality of the fractional differencing parameter, carried out in the framework of the multivariate non linear log periodogram estimator does not allow to reject the null, at the 5% significance level, that the three process are characterised by the same degree of persistence for any of the bandwidths investigated. Contrary to the univariate estimates, the constrained model yields stable estimates for bandwidth larger than 68 ordinates, ranging between 0.28 and 0.31, with an average value equal to 0.30. Interestingly, nominal money growth shows a larger inverse long-run signal to noise ratio than inflation. In fact the average values over the stable region are close to 0.89 and 0.44, respectively. The zero value of the inverse signal to noise ratio for the excess nominal money growth process suggests that the output component dominates the nominal component, since for the former process we did not find any evidence of observational noise. Therefore, the

²²Estimation was performed using the ARFIMA Ox code written by J.A. Doornick and M. Ooms. See Sowell (1992) for details on ML estimation of ARFIMA models.

²³However, as it will be shown below, this result is spurious and due to the presence of observational noise, which downward biases the estimated fractional differencing parameter.

evidence suggests that the constrained estimate of the fractional differencing parameter is appropriate for the three processes, which, given the different degree of noisiness, is also consistent with the results of the augmented Engle and Kozicki (1993) test.

5.2.2 Fractional cointegration analysis

Noise-free nominal money growth and inflation series have been obtained by means of the denoising approach of Beltratti and Morana (2003). The results of the Monte Carlo analysis for the selection of the optimal bandwidth for the centered trimmed semiparametric filter are reported in Table 2, Panel B. The Monte Carlo simulation has been calibrated using the estimates for the inflation process, since for the money growth process the optimal trimming bandwidth is the same as the one determined for the real money growth process, given the estimated fractional differencing parameter and inverse long-run signal to noise ratio. As shown in the table, when the inverse long-run signal to noise ratio is equal to 0.45 the optimal trimming bandwidth for the centered semiparametric filter is equal to 60 ordinates (c_{60}^*). Again the optimally centered trimmed filter shows a superior performance relatively to the other semiparametric filters, and RMSE, U and RMSE decomposition which are close to the theoretical optimal values achieved by the two sided parametric Wiener Kolmogorov filter.

In Figure 1 we plot the actual and denoised nominal money growth and inflation processes. We also plot the denoised excess nominal money growth process. The latter has been computed by subtracting the output growth process from the denoised nominal money growth process. We have also applied a compression factor to the excess nominal money growth process equal to the ratio of the standard deviations of the break-free denoised inflation and excess nominal money growth processes.²⁴

In Table 6 Panel A we report the results of the fractional cointegrating rank test and the squared coherence analysis, while the estimated eigenvectors are reported in Panel B. As is shown in the table, the evidence in favour of fractional cointegration between break-free excess nominal money growth and inflation is strong. At the selected bandwidth (two ordinates) the Robinson and Yajima (2002) test does not allow to reject the null of cointegration at the 1% significance level, the largest eigenvalue explains all the variance and the squared coherence tests do not allow to reject the null

²⁴This correction ensures that the compressed excess nominal money growth process has the same standard deviation of the inflation process, and, as it will be shown later in the paper, is justified on the basis of compliance with economic theory and forecasting performance of the core inflation process.

of unitary squared coherence at the zero frequency, while the null of zero squared coherence is strongly rejected.²⁵ As is shown in Figure 2, the percentage of explained variance by the largest eigenvalue tends to decrease as the bandwidth increases, being larger than 80% up to a bandwidth equal to five ordinates, stabilising at a value close to 65% for bandwidths larger than fifteen ordinates. In correspondence of the selected bandwidth, the estimated bias corrected cointegrating parameter is equal to 0.95, value which is close to the unitary value predicted by economic theory, and consistent with previous findings of Morana (2002). The estimated cointegrating parameter is still close to one for bandwidths up to four ordinates. Following economic theory, we have imposed the homogeneity restriction. Support for the imposed restriction is provided by the correlation between the estimated factors obtained from the unrestricted and restricted models, which is equal to 0.997.

The estimated break-free core inflation processes obtained with (FDPC) and without (FDPC*) applying the compression factor are reported in Figure 3. For comparison also the estimated break-free core inflation process obtained using the approach of Morana (2002) (WK) is shown in the same plot. The estimated components are strongly correlated (the correlation coefficient is equal to 0.975 for FDPC and WK and 0.911 for FDPC* and WK), with the series obtained from the Kasa decomposition showing higher variability than the series obtained by means of the Wiener Kolmogorov filter only when the compression factor is not applied (the estimated standard deviations are 0.08 (FDPC), 0.15 (FDPC*) and 0.11(WK)). The close similarity between the estimated persistent components should be expected, given that both methods are suited to extract the long memory signal from the data. Moreover, given the different procedure followed to compute the weights used to derive the smoothed processes, a perfect overlapping of the estimated processes should not be expected. Theoretically, the Wiener-Kolmogorov filter yields optimal estimates of the persistent components (it is the minimum MSE estimator under the assumption of Gaussianity, and the minimum MSE estimator within the class of linear estimator when Gaussianity does not hold). Similar optimal properties have not been demonstrated for the Kasa (1992) decomposition. Both approaches are however appropriate to effect a persistent-non persistent decomposition (P-NP). Moreover, the key advantage of using the Kasa (1992) decomposition is that it allows the core inflation process to be computed in real time, avoiding arbitrary in the selection of the leads and

²⁵The p-value of the zero squared coherence at the zero frequency test is 0.008, while the p-value of the unitary squared coherence at the zero frequency test is 0.315.

lags which affects a two-sided filter.²⁶ Finally, the factor approach employed in this paper has the advantage, relative to the approach of Morana (2002), of not requiring the maximisation of the spectral likelihood function, avoiding the well known problems of local maxima and convergence which arise in numerical optimisation. This is a clear asset, particularly when the number of processes involved is large.

5.2.3 Computing and evaluating the core inflation process

Following the definition of core inflation provided in the methodological section, our measure of core inflation is obtained from the common persistent signal in inflation and excess nominal money growth, i.e. by adding the common break process to the common long memory components. We have therefore

$$\pi_t^c = \hat{\mu}_{cbp,t} + \hat{\mu}_{clm,t}.$$

In the implementation we have used the conditional expectation of the estimated factor ($E[\hat{\mu}_{clm,t}|I_{t-i}]$), obtained by applying the ARFIMA(0,0.30,0) filter, scaled by the estimated inflation loading, while the common break process $\hat{\mu}_{cbp,t}$ is the common break process estimated for the inflation process. This allows to smooth the filtered signal obtained through the Kasa decomposition (FDPC process).

As is shown in Figure 4, the estimated core inflation process (Kasa decomposition) shows the expected feature of being smoother than actual inflation (the standard deviations of the year on year rates are 2.23 and 2.10). As also expected, core inflation was significantly below actual inflation in correspondence of the second oil price shock, and significantly above it in correspondence of the mid-eighties oil price counter shock, not being affected by the non persistent inflation dynamics induced by the oil shocks.

As shown by Morana (2002), in a multi regime framework the break process can be interpreted as the long-run forecast, so that price stability may be assessed by evaluating whether the actual inflation observations belong to a regime characterised by an unconditional mean coherent with the HICP inflation reference value. As already noted, according to this criterion inflation developments since the start of Stage Three would have been consistent with the price stability reference value. However, as shown in Figure 4, our core inflation measure suggests that since June 2001 there is evidence of

²⁶Computability in real time of the core inflation measure proposed by Morana (2002) can be allowed by means of a one-sided filter. This is however suboptimal relatively to the use of the two-sided filter. See the Monte Carlo results reported in Table 2.

deviations from the reference value, which appear to have stabilised since September 2002. The core inflation estimate for March 2003, the last observation of our sample, is 2.5%. It is interesting to note that the ex-food and energy inflation rate provides similar evidence up to December 2002, also suggesting that since February 2000 the deviation of HICP inflation from the 2% threshold is mainly explained by developments in oil prices. Differently from the proposed core inflation measure, the ex-food and energy inflation rate shows a sudden drop in January 2003, stabilising below the 2% reference value thereafter.

Robustness analysis As far as the robustness property is concerned, we have compared the estimated core inflation processes obtained using the 1980:1-2003:3 and 1980:1-2002:3 samples. Over the period 1981:1-2002:3 the mean and maximum absolute deviations between the two estimated year on year processes have been equal to 0.08% and 0.3%, suggesting that our measure is robust to sample updating.

Forecasting analysis The forward looking property of the proposed core inflation measure is displayed in Figure 4. From the cross-correlogram it can be noted that the core inflation measure leads actual inflation at horizons of interest for the policy maker, while actual inflation shows some leading properties only in the very short-run. The forecasting performance of the proposed core inflation measure has also been assessed by means of an out of sample forecasting exercise, considering both the year on year HICP inflation rate and the HICP inflation three-year centered moving average as benchmarks. Four forecasting horizons have been employed, namely six, twelve, eighteen, and twenty four months. The forecasting performance has been contrasted with that of other core inflation measures, namely the (MS-) Cogley (2002) model²⁷, the Cristadoro et al. (2002) core inflation process, and the MS-ARFIMA model (Morana, 2000), in addition to a random-walk model for the year on year HICP inflation rate. The out of sample forecasting horizon selected is 1999:1-2002:4, in order to allow for comparison with the results reported in Cristadoro et al. (2002, Table 2) for their proposed core inflation measure. We denote the forecasting models as follows: FDPC (the proposed core inflation process), A_{ms} (MS-ARFIMA model), C_{ms} (MS-

²⁷The MS-Cogley (2002) model has been constructed by applying the Cogley (2002) filter on the break-free inflation series, where the break process has been estimated by means of a Markov switching model. This allows to assess the improvement in forecasting provided by the use of a long memory filter.

Cogley model), random walk model (RW), and CFRV^a (the Cristadoro et al. (2002) model).²⁸

The main results of the forecasting exercise are as follows. Firstly, allowing for long memory leads to better forecasts. The forecasts generated by the A_{ms} model are always superior to those of the C_{ms} model independently of the benchmark and the forecasting horizon, outperforming also the CFRV^a model and the RW model. Secondly, as already found by Nicoletti Altimari (2002) and Morana (2002), allowing for the information contained in monetary aggregates leads to better forecasts in the medium term for the euro area. In fact, at the twelve, eighteen and twenty four months horizons the FDPC model is the best forecasting model for both benchmarks, followed by the A_{ms}^f models, with the C_{ms} model ranking third and the CFRV^a ranking fourth. Moreover, the FDPC model is also the best forecasting model for the three-year centered moving average also at the six-month horizon.²⁹ Extending the out of sample forecasting exercise up to 2003:3 yields the same ranking of the models, apart from the CFRV^a model, for which forecasts are not available.³⁰

It can be concluded that the proposed core inflation measure shows all the properties that should characterise the “ideal” core inflation measure, namely smoothness, robustness, forecasting ability, theoretical foundation, and computability in real time. In addition, by construction, our core inflation measure is directly related to monetary aggregates, bearing the interpretation of monetary inflation rate.

²⁸The Markov switching models (FDPC, A_{ms} , C_{ms}) have been forecasted following a two-step procedure. Firstly, the break process is forecasted using the Markov switching model; secondly, the relevant break-free process is forecasted using the appropriate time series filter. The two forecasts are then added. On the other end, the Cristadoro et al. (2002) model has been forecasted by means of a naive model (random walk model), where the forecasts for the year on year rate are generated by annualising the last estimated value in the sample.

²⁹However, according to the West-Cho test (Table 7, Panel B) the ranking of the models is not statistically significant at the six-month horizon, independently of the benchmark (5% significance level). At the twelve-month horizon the forecasts from the A_{ms} and C_{ms} models are statistically different from the one generated from the RW model for both benchmarks. At the eighteen-month and twenty-four month horizons the forecasts from the FDPC, A_{ms} and C_{ms} models are statistically different from the ones generated by the RW model when the year on year rate is considered as the benchmark, while for the case of the three-year centered moving average only the forecasts from the A_{ms} and C_{ms} models are statistically different from the ones of the RW model. For the case of the year on year rate the forecasts from the A_{ms} and C_{ms} models are statistically different at the eighteen-month horizon, while at the twenty four-month horizon the forecasts from the FDPC and C_{ms} models are statistically different. The statistical significance of the difference of the RMSE for the CFRV^a model and the other models has not been tested, since, for this latter model the figures reported in the paper are from Cristadoro et al. (2002).

³⁰The results are available upon request to the author.

6 Conclusions

In this paper we have introduced a new approach to core inflation estimation, based on recent theoretical developments in the estimation of fractionally cointegrated processes (Morana, 2003). Our definition of core inflation is the same as the one proposed by Morana (2002), i.e. the scaled common persistent component in inflation and excess nominal money growth. The common persistent component is measured by the common break process in inflation and excess nominal money growth, explained by the break process in nominal money growth, and by the scaled common long memory factor, which, differently from Morana (2002), can be related to both nominal and real forces. The proposed measure of core inflation shows all the properties that should characterise the “ideal” core inflation process, namely smoothness, forecasting ability, economic interpretation, computability in real time and robustness. In addition, by construction, our core inflation measure is directly related to monetary aggregates, bearing the interpretation of monetary inflation rate.

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Table 1: Persistence analysis

	<i>rm</i>	<i>g</i>
SC_{ms}	4.314	3.641
SC_m	3.919	3.453
LR	0.603	0.162
	0.322	0.303
d_{LM}	(0.081)	(0.088)
	[38]	[32]
	0.337	0.302
d_{NLP}	(0.153)	(0.236)
	[74]	[52]
	0.878	0.000
β_{NLP}	(0.754)	(-)
	[76]	[52]
T	0.819	
	0.331	
d_{joint}	(0.051)	-

The table reports the Schwarz-Bayes information criterion for the Markov switching (SC_{ms}) and the linear (constant mean) model (SC_m). The variables considered are real money growth (rm) and real output growth (g). LR is the p-value of the likelihood ratio test, computed as in Davies (1987). d_b denotes the fractional differencing operator estimated in the augmented Engle and Kozicki regression, with standard error in brackets. AEK is the p-value of the augmented Engle and Kozicki test. d_{LM} and d_{NLP} are the fractional differencing operators estimated using the LM estimator and the non linear log periodogram estimator. β_{NLP} is the estimated inverse long-run signal to noise ratio. Standard errors and selected bandwidths are reported in brackets and square brackets, respectively. T is the p-value of the test for the equality of the fractional differencing parameters.

Table 2, Panel A: Monte Carlo results ($d = 0.30$, $N = 300$, $l = 3$, $sn = 0.9$)

sp	1s	2s	c	c_{70}^*	p	1s	2s	c
ρ	0.78	0.78	0.77	0.77	ρ	0.79	0.80	0.77
<i>bias</i>	-0.01	0.00	-0.01	0.00	<i>bias</i>	0.00	0.00	-0.01
<i>RMSE</i>	0.75	0.71	1.03	0.72	<i>RMSE</i>	0.70	0.68	0.73
<i>U</i>	0.41	0.38	0.47	0.36	<i>U</i>	0.36	0.34	0.38
U_m	0.02	0.01	0.03	0.02	U_m	0.01	0.01	0.02
U_v	0.43	0.30	0.55	0.14	U_v	0.20	0.15	0.25
U_c	0.55	0.69	0.42	0.84	U_c	0.78	0.84	0.72

Table 2, Panel B: Monte Carlo results ($d = 0.30$, $N = 300$, $l = 3$, $sn = 0.45$)

sp	1s	2s	c	c_{60}^*	p	1s	2s	c
ρ	0.86	0.86	0.86	0.85	ρ	0.86	0.87	0.86
<i>bias</i>	-0.00	-0.00	-0.01	0.00	<i>bias</i>	-0.00	-0.00	-0.00
<i>RMSE</i>	0.57	0.56	0.59	0.57	<i>RMSE</i>	0.56	0.55	0.57
<i>U</i>	0.30	0.28	0.32	0.28	<i>U</i>	0.28	0.27	0.29
U_m	0.01	0.00	0.02	0.01	U_m	0.01	0.01	0.01
U_v	0.25	0.18	0.32	0.13	U_v	0.13	0.09	0.16
U_c	0.74	0.82	0.66	0.86	U_c	0.86	0.90	0.83

The table reports the results of the Monte Carlo simulation exercise for the semiparametric (sp) and parametric (p; Harvey, 1993) denoising approaches. ρ is the correlation coefficient between the simulated and filtered processes, bias is the mean deviation of the two processes, RMSE is the root mean square error, U is the Theil (1961)' U index, U_m , U_v , U_c , are the mean, variance and covariance components obtained from the RMSE decomposition. One sided (three lags, 1s), two sided (three leads and three lags, 2s) and contemporaneous filters (zero leads and zero lags, c) have been considered. c_i^* denotes the optimal trimmed contemporaneous semiparametric filter with trimming ordinate equal to i . The sample size (N) is equal to 300 observations. The inverse long-run signal to noise ratio (sn) is equal to 0.9 and 0.45. The fractional differencing parameter (d) is equal to 0.3. 500 Monte Carlo replications have been computed.

Table 3, Panel A, Fractional cointegration analysis

	RY	
<i>eig</i>	0.003	1e-4
<i>pv</i>	0.95	0.05
	1%	5%
$r = 1$	0.125	0.103
	T_0	T_1
	0.047	0.283

Table 3, Panel B, Unrestricted and restricted eigenvectors

	E_1	E_2	RE_1	RE_2
<i>rm</i>	0.882	0.472	1 (-)	0.208 (0.001)
<i>g</i>	-0.472	0.882	-1 (-)	0.208 (0.002)

Panel A reports the Robinson and Yajima (2002) fractional cointegrating

rank test. *eig* denotes the estimated eigenvalues, *pv* the proportion of explained variance, and $r = 1$ denotes the corresponding test at the given significance level (1%, 5%). The last two rows of Panel A report the p-values of the zero squared coherence tests (T_0) and the unitary squared coherence tests, computed according to the modified procedure suggested in Priestly (1981, p.705) (T_1). Panel B reports the unrestricted (first two columns) and restricted (second two columns) eigenvectors of the scaled spectral matrix. The first column refers to the cointegration space, while the second column is the factor loading matrix. Standard errors have been computed using the jack-knife.

Table 4, Panel A: Transition matrix of mean switching: 3-regimes annual model

	Regime 1	Regime 2	Regime 3
Regime 1	0.85	0.08	0
Regime 2	0.15	0.92	0.38
Regime 3	0	0	0.72
Number of observations	6	13	2
Duration	7	12	3

Table 4, Panel B: Coefficients: switching unconditional means

	m	π	em
μ_1	4.467	1.932	1.912
	(0.328)	(0.423)	(0.412)
	5.032	1.974	2.846
μ_2	(0.385)	(0.364)	(0.552)
	7.673	3.550	5.529
	(0.240)	(0.310)	(0.300)
μ_3	7.797	3.811	5.429
	0.348	(0.329)	(0.499)
	10.510	8.849	9.950
μ_3	(0.612)	(0.791)	(0.766)
	10.510	8.850	9.951
	(0.817)	(0.772)	(1.171)

Table 4, Panel C: Coefficients: dummy variables

	m	π	em
μ_1	0.453	0.184	0.311
	(0.022)	(0.0129)	(0.028)
	5.436	2.208	3.732
μ_2	0.660	0.307	0.409
	(0.025)	(0.0146)	(0.032)
	7.920	3.684	4.908
μ_3	0.814	0.700	0.774
	(0.039)	(0.023)	(0.050)
	9.768	8.400	9.288

The table reports the transition matrix (Panel A) and the switching unconditional means (Panel B) for the multivariate estimate of the break processes (nominal money growth (m), inflation (π), excess nominal money growth (em)) obtained from the annual model, with standard errors in brackets. The third row reports the estimates of the switching means obtained by

neglecting the transition to the average inflation regime in 2001. The element i, j of the transition matrix is the probability that at time t there is a switch to regime i , given that at period $t - 1$ the system was in regime j . Figures are annual percentages. Panel C reports the implied estimates of the break process for the monthly model, with standard errors in brackets, and annualised values in the third row.

Table 5: Persistence analysis

	m	π	em
SC_{ms}	2.687	3.776	2.191
SC_m	2.476	2.931	2.021
LR	0.000	0.000	0.000
d_b	0.062	0.266	0.294
AEK_f	(0.048)	(0.049)	(0.053)
	[0.0000]	[0.0000]	[0.0000]
d_{LM}	0.111	0.190	0.141
	(0.064)	(0.064)	(0.060)
	[82]	[82]	[92]
d_{NLP}	0.342	0.370	0.291
	(0.164)	(0.151)	(0.182)
	[92]	[100]	[92]
β_{NLP}	1.165	0.625	0.000
	(0.595)	(0.490)	(-)
	[92]	[100]	[92]
T	[0.905]	[0.825]	[0.893]
d_{joint}	0.298		
	(0.314)		
	[74]		

The table reports the Schwarz-Bayes information criterion for the Markov switching (SC_{ms}) and the linear (constant mean) model (SC_m). LR is the p-value of the likelihood ratio test, computed as in Davies (1987). The variables considered are nominal money growth (m), excess nominal money growth (em), and inflation (π). d_b denotes the fractional differencing operator estimated in the augmented Engle and Kozicki regression, with standard error in brackets. AEK is the p-value of the augmented Engle and Kozicki test. d_{LM} and d_{NLP} are the fractional differencing operators estimated using the LM estimator and the non linear log periodogram estimator. β_{NLP} is the estimated inverse long-run signal to noise ratio. Standard errors and selected bandwidths are reported in brackets and square brackets, respectively. T is the p-value of the test for the equality of the fractional differencing parameters, in the order (m, π) , (m, em) , and (π, em) .

Table 6, Panel A: Fractional cointegration analysis

	RY	
<i>eig</i>	5e-4	1e-6
<i>pv</i>	1	0
	1%	5%
$r = 1$	0.005	0.005
	T_0	T_1
	0.008	0.315

Table 6, Panel B: Unrestricted and restricted eigenvectors

	E_1	E_2	RE_1	RE_2
π	0.781	0.625	1 (-)	0.133 (0.001)
<i>em</i>	-0.625	0.781	-1 (-)	0.133 (0.001)

Panel A reports the Robinson and Yajima (2002) fractional cointegrating

rank test. *eig* denotes the estimated eigenvalues, *pv* the proportion of explained variance, and $r = 1$ denotes the corresponding test at the given significance level (1%, 5%). The last two rows of Panel A report the p-values of the zero squared coherence tests (T_0) and the unitary squared coherence tests, computed according to the modified procedure suggested in

Priestly (1981, p.705) (T_1). Panel B reports the unrestricted (first two columns) and restricted (second two columns) eigenvectors of the scaled spectral matrix. The first column refers to the cointegration space, while the second column is the factor loading matrix. Standard errors have been computed using the jack-knife.

Table 7, Panel A: Forecasting analysis 1999:1-2002:4

HICP	6	12	18	24	MA36	6	12	18	24
<i>FDPC</i>	0.46	0.40	0.40	0.41	<i>FDPC</i>	0.20	0.13	0.11	-
<i>A_{ms}</i>	0.33	0.41	0.44	0.46	<i>A_{ms}</i>	0.21	0.16	0.19	-
<i>C_{ms}</i>	0.41	0.46	0.49	0.50	<i>C_{ms}</i>	0.37	0.23	0.20	-
<i>RW</i>	0.47	0.77	1.00	1.21	<i>RW</i>	0.67	1.01	1.26	-
<i>CFRV^a</i>	0.50	0.66	0.82	0.90	<i>CFRV^a</i>	-	-	-	-

Table 7, Panel B: West-Cho test, p-values

HICP _{6,12}	<i>FDPC</i>	<i>A_{ms}</i>	<i>C_{ms}</i>	<i>RW</i>	HICP _{18,24}	<i>FDPC</i>	<i>A_{ms}</i>	<i>C_{ms}</i>	<i>RW</i>
<i>FDPC</i>		0.10	0.56	0.98	<i>FDPC</i>		0.21	0.06	0.00
<i>A_{ms}</i>	0.85		0.21	0.37	<i>A_{ms}</i>	0.13		0.02	0.00
<i>C_{ms}</i>	0.39	0.17		0.62	<i>C_{ms}</i>	0.00	0.06		0.00
<i>RW</i>	0.07	0.03	0.04		<i>RW</i>	0.00	0.00	0.00	

MA36 _{6,12}	<i>FDPC</i>	<i>A_{ms}</i>	<i>C_{ms}</i>	<i>RW</i>	MA36 _{18,24}	<i>FDPC</i>	<i>A_{ms}</i>	<i>C_{ms}</i>	<i>RW</i>
<i>FDPC</i>		0.97	0.55	0.20	<i>FDPC</i>		0.76	0.75	0.05
<i>A_{ms}</i>	0.93		0.21	0.05	<i>A_{ms}</i>	-		0.60	0.04
<i>C_{ms}</i>	0.78	0.51		0.06	<i>C_{ms}</i>	-	-		0.03
<i>RW</i>	0.07	0.03	0.03		<i>RW</i>	-	-	-	

Panel A reports the RMSE of the year on year forecasts generated by the core inflation model (*FDPC*), the MS-ARFIMA model (*A_{ms}*), the MS-Cogley model (*C_{ms}*), the random walk model for the HICP inflation rate (*RW*), and the Cristadoro et al. (2002) core inflation process (Cristadoro et al., 2002, Table 2) (*CFRV^a*). Panel B reports the p-values of the West-Cho test for the equality of the RMSE. Upper diagonal elements refer to the forecasts for the 6-month and 18-month horizons, respectively; lower diagonal elements refer to the forecasts for the 12-month and 24-month horizons, respectively. The year on year rate (HICP) and the three-year centered moving average (MA-36) are the two benchmarks employed to evaluate the forecasting performance of the models.

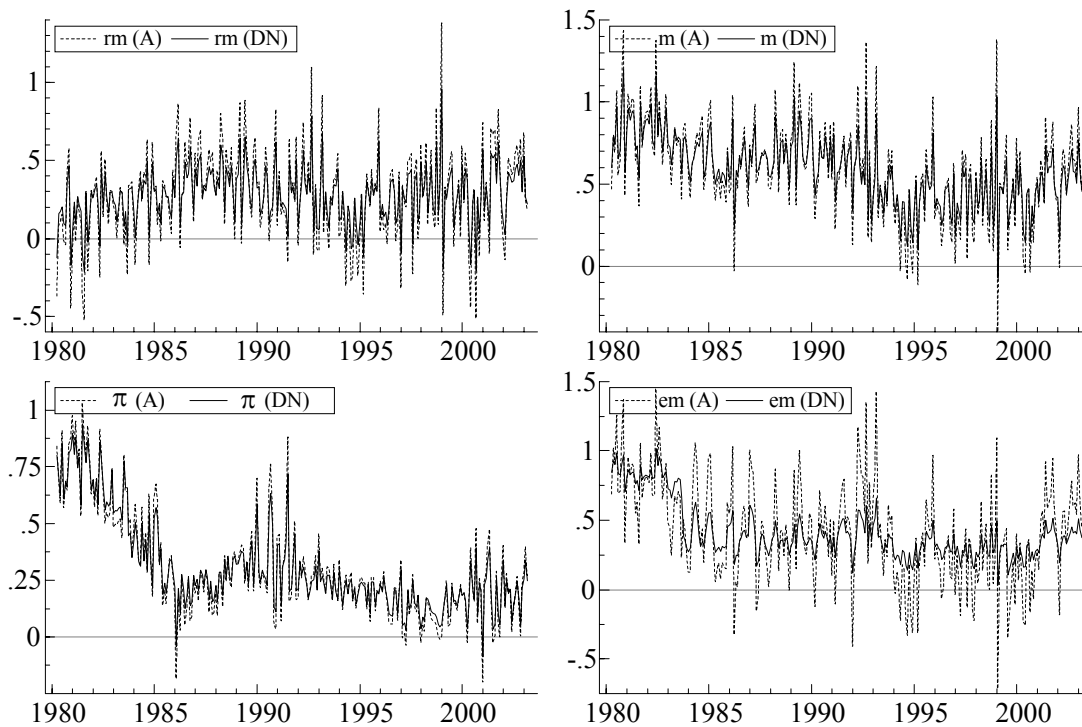


Figure 1: Actual (A) and denoised (DN) processes (real money growth (rm), nominal money growth (m), inflation (π), excess nominal money growth (em)).

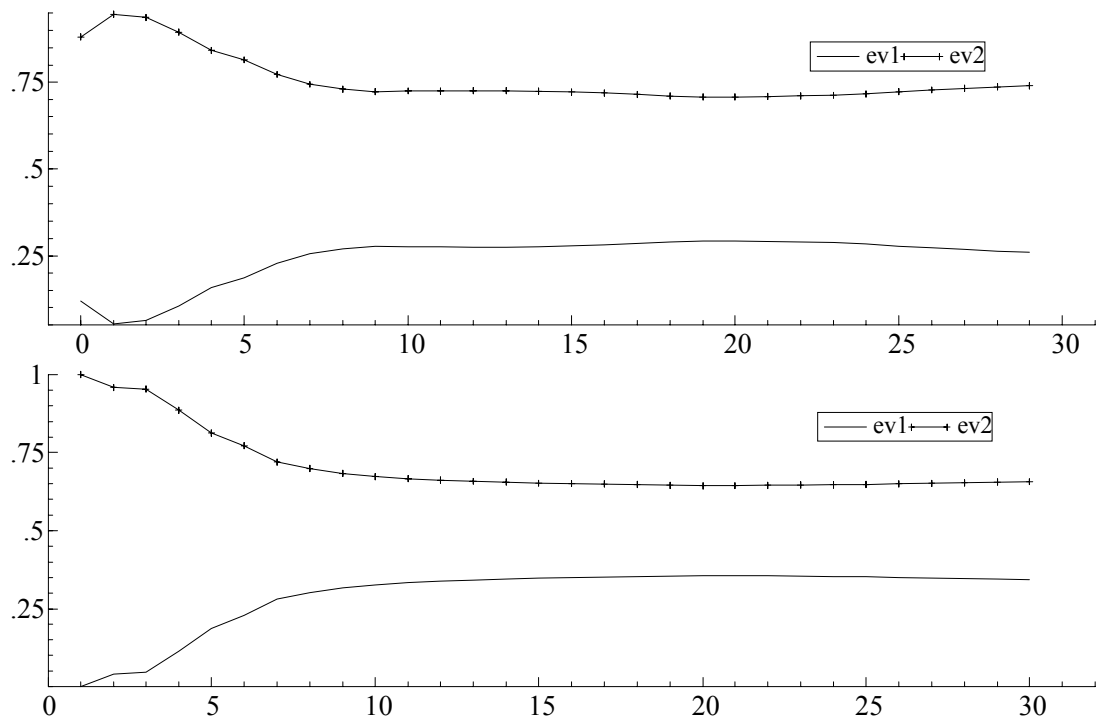


Figure 2: Estimated eigenvalues, proportion of explained variance (top plot: real money growth-output growth; bottom plot: inflation-excess nominal money growth).

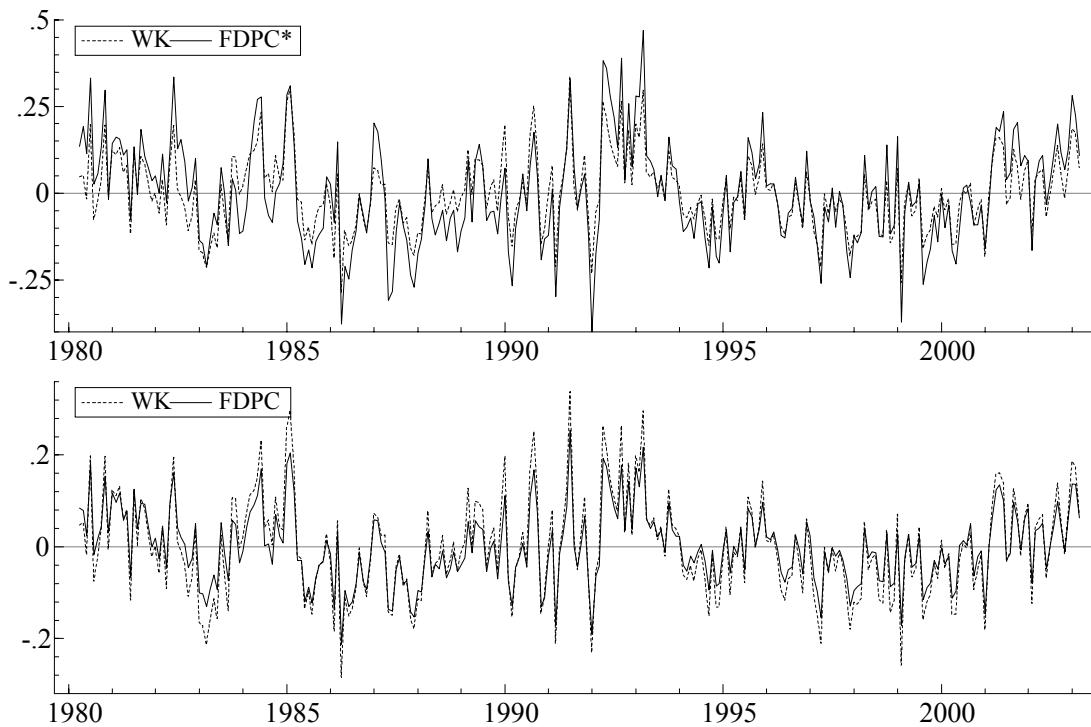


Figure 3: Estimated break-free core inflation process (Wiener Kolmogorov (WK) filter (Morana; 2002); Kasa decomposition (FDPC*; FDPC)).

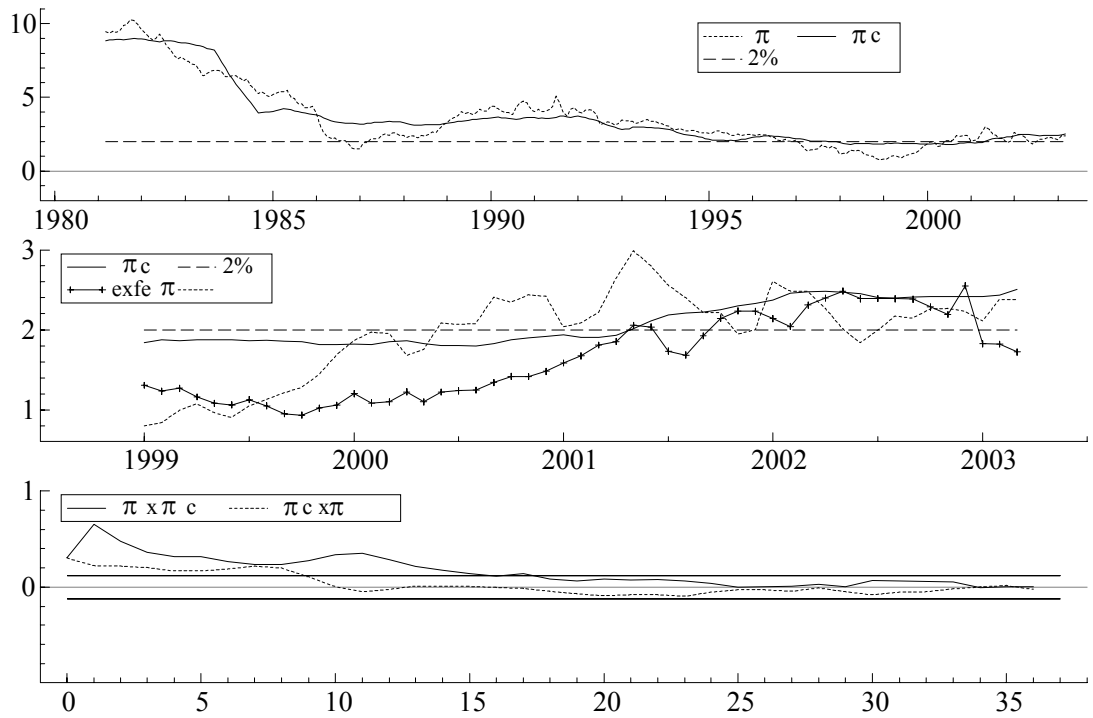


Figure 4: Actual inflation (HICP: π , ex-food and energy: $exfe$) and estimated core inflation process (π_c), MA-12 (top and center plots). Cross-correlation functions (bottom plot).

7 Appendix I: Monte Carlo Analysis

The performance of the semiparametric estimators (Local Whittle (LW), Averaged Periodogram (AP), Robinson, 1998 (LM), log periodogram (LP)) has been evaluated by means of a Monte Carlo experiment (see Appendix II for a description of the semiparametric estimators). The sample size has been set equal to 300 observations to match the sample size used in the paper. The simulated model is $(1 - \phi L)(1 - L)^d y_t = \varepsilon_t$, $\varepsilon_t \sim NID(0, 1)$. Four values have been employed for the autoregressive parameter $\phi = \{0, 0.3, 0.5, 0.7\}$ and the fractional differencing parameter $d = \{0.1, 0.2, 0.3, 0.4\}$. The number of Monte Carlo replications is 5000. Optimal bandwidth theory has been employed for the LW, LM and AP estimators (see Appendix II). For the LM estimator the optimal bandwidth has been determined through Monte Carlo simulation, since the analytical one has not yet been determined. The optimal bandwidth has been computed through bias minimisation, since this lead to a very little loss of efficiency. For reason of space we do not report the results for the case $\phi = 0$. Tables A1, A2 report the Monte Carlo bias (bias) and root mean square error (rmse) for the various estimators. The optimal bandwidth is reported in square brackets. As is shown in the tables, all the estimators tend to be unbiased, with the LM estimator always performing best in terms of efficiency. The LM estimator however shows some bias for the case $d = 0.40$ $\phi = 0.3, 0.5$, which is close in magnitude to the one shown by the average periodogram estimator.

Table A1: Monte Carlo results; sample length: 300 observations

	$\phi = 0.3$		$\phi = 0.5$		$\phi = 0.7$	
	<i>bias</i>	<i>rmse</i>	<i>bias</i>	<i>rmse</i>	<i>bias</i>	<i>rmse</i>
$d = 0.1$	-0.013	0.146	-0.017	0.193	-0.027	0.255
<i>LW</i>	[44]		[27]		[14]	
<i>LP</i>	0.040	0.107	0.056	0.142	0.038	0.236
	[80]		[52]		[24]	
<i>AP</i>	-0.033	0.178	-0.031	0.222	-0.042	0.251
	[45]		[32]		[22]	
<i>LM</i>	0.001	0.056	0.001	0.066	-0.001	0.076
	[74]		[45]		[29]	
$d = 0.2$	-0.027	0.168	-0.039	0.217	-0.031	0.258
<i>LW</i>	[39]		[23]		[16]	
<i>LP</i>	0.027	0.119	0.031	0.169	0.011	0.261
	[70]		[44]		[20]	
<i>AP</i>	-0.048	0.166	-0.056	0.207	-0.054	0.237
	[48]		[34]		[24]	
<i>LM</i>	0.000	0.037	0.000	0.043	0.000	0.046
	[129]		[81]		[48]	

The table reports the Monte Carlo bias and root mean square error for the semiparametric estimators (Local Whittle (LW), log periodogram (LP), averaged periodogram (AP), Robinson, 1998 (LM)). d is the fractional differencing parameter and ϕ is the autoregressive coefficient. The sample size is 300 observations and the number of replications is 5000.

Table A2: Monte Carlo results; sample length: 300 observations

	$\phi = 0.3$		$\phi = 0.5$		$\phi = 0.7$	
	<i>bias</i>	<i>rmse</i>	<i>bias</i>	<i>rmse</i>	<i>bias</i>	<i>rmse</i>
$d = 0.3$	-0.038	0.185	-0.053	0.230	-0.027	0.241
<i>LW</i>	[33]		[22]		[20]	
<i>LP</i>	0.018	0.141	0.012	0.204	0.018	0.267
	[59]		[37]		[18]	
<i>AP</i>	-0.066	0.152	-0.076	0.182	-0.066	0.205
	[46]		[35]		[26]	
<i>LM</i>	-0.041	0.033	-0.013	0.026	0.016	0.020
	[148]		[150]		[110]	
$d = 0.4$	-0.060	0.191	-0.061	0.208	-0.037	0.210
<i>LW</i>	[28]		[25]		[26]	
<i>LP</i>	0.060	0.168	-0.005	0.234	0.028	0.266
	[48]		[30]		[17]	
<i>AP</i>	-0.097	0.133	-0.104	0.159	-0.091	0.172
	[41]		[34]		[28]	
<i>LM</i>	-0.103	0.028	-0.086	0.024	-0.064	0.018
	[148]		[147]		[149]	

The table reports the Monte Carlo bias and root mean square error for the semiparametric estimators (Local Whittle (LW), log periodogram (LP), averaged periodogram (AP), Robinson, 1998 (LM)). d is the fractional differencing parameter and ϕ is the autoregressive coefficient. The sample size is 300 observations and the number of replications is 5000.

8 Appendix II: Econometric methodology

8.1 Break process estimation

Markov switching model Lets consider a k regime model for the unconditional mean of the return or the log variance series and let $\boldsymbol{\eta}$ be a vector consisting of the mean elements μ_1, μ_2 and the variance of the error process in the model $y_t = \mu_{s_t} + \varepsilon_t$, with $\varepsilon_t \sim N(0, \sigma^2)$. The transition between states is governed by a Markov chain whose realizations take on values in $\{1, \dots, k\}$, $p(s_t = j | s_{t-1} = i) = p_{ij}$, with $\sum_{j=1}^k p_{ij} = 1$. Let $\mathbf{p} = (p_{11}, p_{12}, \dots, p_{kk})'$ the $(k^2 \times 1)$ vector of transition probabilities. The econometrician is supposed to observe only the realizations of the variable y_t but not of the state s_t . The unknown parameters can be collected in the vector $\boldsymbol{\lambda} = (\mathbf{p}', \boldsymbol{\eta}')$ and maximum likelihood estimates of the parameters of the model can be obtained via the Expectation-Maximization algorithm. See Hamilton (1989) for further details.

The break process is then computed as

$$\hat{y}_t = \hat{\mu}_t = \sum_{s=1}^k \hat{p}_{t,s} \hat{\mu}_s$$

where $\hat{p}_{t,s,j}$ is the estimated probability that the observation t of process j belongs to state s and $\hat{\mu}_{s,j}$ is the estimated value of the mean in the s th state.

The break-free series can then be obtained as $y_{tbf} = y_t - \hat{y}_t$.

As suggested by Beltratti and Morana (2001), a more reliable estimate of the break process may be obtained from data sampled a low frequency.

Kokoszka and Leipus (2000) Consider the following process

$$U_N(k) = \left(\frac{1}{\sqrt{N}} \sum_{j=1}^k y_j - k \left(\frac{1}{N} \sum_{j=1}^N y_j \right) \right)$$

for $0 < k < N$, where y_t is the monthly process at time t . The proposed estimator of the break point is

$$\hat{k} = \min \left\{ k : |U_N(k)| = \max_{1 \leq j \leq N} |U_N(j)| \right\},$$

i.e. the point at which there is the maximal evidence of a break point. The statistical significance of the break point can be evaluated using the results

$$\sup \{|U_N(k)|\} / \hat{\sigma} \rightarrow_{D[0,1]} \sup \{B(k) : k \in [0, 1]\},$$

where $B(k)$ is a Brownian bridge and $\sigma^2 = \sum_{j=-\infty}^{\infty} \text{cov}(y_j^2, y_0^2)$. The 90%, 95%, and 99% critical values (two sided test) are 1.22, 1.36 and 1.63, respectively.

8.2 Semiparametric methodologies and stationarity test

Local Whittle estimator (Kunsch, 1987; Robinson, 1995b) It requires the minimization of the following objective function

$$Q(C, H_{LW}) = \frac{1}{m} \sum_{j=1}^m \left(\log C \lambda_j^{1-2H} + \frac{\lambda_j^{2H-1}}{C} I(\lambda_j) \right)$$

where $I(\lambda_j)$ is the periodogram at frequency $\lambda_j = 2\pi j/T$, $j = 1, \dots, m$, m is the bandwidth parameter, C is a positive constant, and H is the Hurst exponent, which is related to the fractional differencing parameter through the relation $H = d + 0.5$. For $H < 0.5$ the process is antipersistent, for $H > 0.5$ it is long memory, and for $H = 0.5$ it is weakly dependent. It is shown that

$$\sqrt{m} \left(\hat{H}_{LW} - H \right) \xrightarrow{d} N \left(0, \frac{1}{4} \right).$$

LM estimator (Robinson, 1998) An alternative estimator for H , with the same limiting distribution of the Local Whittle estimator under weak dependence, is

$$\hat{H}_{LM} = \frac{\sum_{j=1}^m (1 - 2v_j) I(\lambda_j)}{\sum_{j=1}^m (2 - 2v_j) I(\lambda_j)}$$

where $v_j = \log j - \frac{1}{m} \sum_{j=1}^m \log j$. We denote this estimator as H_{LM} since it can be derived from the LM test of Lobato and Robinson (1998).

Averaged periodogram estimator (Robinson, 1994; Lobato and Robinson, 1996) Another estimator is obtained from the averaged periodogram

$$\hat{H}_{AP,q} = 1 - \frac{1}{2 \ln q} \ln \left\{ \frac{\hat{F}(qm)}{\hat{F}(\lambda m)} \right\}$$

where $\hat{F}(\lambda) = \frac{2\pi}{n} \sum_{j=1}^{[\lambda n/2\pi]} I(\lambda_j)$. The limiting distribution of $H_{AP,q}$ is

$$m^{1/2} \left(\hat{H}_{AP,q} - H \right) \xrightarrow{d} N \left(0, \frac{(1 + q^{-1} - 2q^{1-2H}) (1 - H)^2}{(\ln q)^2 (3 - 4H)} \right)$$

for $\frac{1}{2} \leq H \leq \frac{3}{4}$, and

$$m^{2-2H} \left(\hat{H}_{AP,q} - H \right) \xrightarrow{d} N \left(0, \frac{(1 - q^{2H-2}) (1 - H) \Gamma(2(1 - H)) \cos((1 - H)\pi)}{(2\pi)^{2-2H}} P \right)$$

as $T \rightarrow \infty$, where P is a random variable with unknown distribution, for $\frac{3}{4} < H < 1$.

Log periodogram estimator (Geweke and Porter-Haudak, 1983; Robinson, 1995) A consistent but less efficient estimate of the fractional differencing parameter can be obtained by the log periodogram regression

$$\ln I(\lambda_j) = c + d(-2 \log \lambda_j) + \mu_j \quad j = l, \dots, m,$$

where l is a trimming parameter. It has been shown that

$$\sqrt{m} \left(\hat{d}_{LP} - d \right) \xrightarrow{d} N \left(0, \frac{\pi^2}{24} \right).$$

A test for the equality of the fractional differencing parameter $H_0 : \mathbf{P}\mathbf{d} = 0$ for two processes can be computed in the following way

$$T = \hat{\mathbf{d}}\mathbf{P}' \left[(\mathbf{0}, \mathbf{P}) \left\{ (\mathbf{Z}'\mathbf{Z})^{-1} \otimes \hat{\Omega} \right\} (\mathbf{0}, \mathbf{P})' \right]^{-1} \mathbf{P}\hat{\mathbf{d}}' \sim \chi_1^2,$$

where $\mathbf{Z} = (\mathbf{Z}_{l+1} \dots \mathbf{Z}_m)'$, $\mathbf{Z}_j = (1 \quad -2 \log \lambda_j)$, $\mathbf{P} = (1 \quad -1)$, and $\hat{\Omega}$ is the sample variance covariance matrix of the error terms. A constrained estimate of the fractional differencing parameter, under the constraint $d = \mathbf{Q}\boldsymbol{\theta}$, can then be computed as

$$\begin{bmatrix} \hat{c} \\ \hat{\theta} \end{bmatrix} = \left\{ \mathbf{Q}'_1 \left((\mathbf{Z}'\mathbf{Z})^{-1} \otimes \hat{\Omega}^{-1} \right) \mathbf{Q}_1 \right\}^{-1} \mathbf{Q}'_1 \text{vec} \left(\hat{\Omega}^{-1} \mathbf{Y}'\mathbf{Z} \right)$$

where $\mathbf{Q}_1 = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} \end{bmatrix}$, $\mathbf{Q} = \boldsymbol{\iota}_2$, and $\mathbf{Y} = (\ln I(\lambda_j)_1 \quad \ln I(\lambda_j)_2)$.

Multivariate non linear log periodogram estimator (Beltratti and Morana, 2003) Following Sun and Phillips (2003), consider the perturbed long memory process

$$\begin{aligned} x_t &= \mu_t + u_t \\ \Delta^d \mu_t &= \varepsilon_t, \end{aligned} \quad (17)$$

$0 < d < 0.5$, $\varepsilon_t \sim N.I.D.(0, \sigma_\varepsilon^2)$ and $u_t \sim N.I.D.(0, \sigma_u^2)$. The spectrum can then be written as

$$f_x(\omega_i) = \left(2 \sin \frac{\omega_i}{2} \right)^{-2d} f^*(\omega_i), \quad (18)$$

where $\omega_i = \frac{2\pi i}{T}$ denotes the frequency in radians and T is the sample size. By taking logs we have

$$\ln f_x(\omega_i) = -2d \ln \omega_i + \ln f^*(\omega_i) - 2d \ln \left(2\omega_i^{-1} \sin \left(\frac{\omega_i}{2} \right) \right). \quad (19)$$

By writing $\ln f^*(\omega_i) = \ln f_\varepsilon(\omega_i) + \frac{f_u(\omega_0)}{f_\varepsilon(\omega_0)} \omega_i^{2d} + O(\omega_i^{4d})$ and replacing $f_x(\omega_i)$ with the periodogram $I_x(\omega_i)$, we then have the non linear log periodogram regression

$$\ln I_x(\omega_i) = \alpha - 2d \ln \omega_i + \beta \omega_i^{2d} + w_x(\omega_i), \quad \omega_i \rightarrow 0^+ \quad (20)$$

where $w_x(\omega_i)$ is a disturbance term, and α , β , and d are the intercept, the inverse long-run signal to noise ratio, and the fractional differencing parameter, respectively. In particular, $\alpha = \ln f_\varepsilon(\omega_0) - c$ ($c = 0.577216\dots$ is the Euler constant), $\beta = f_u(\omega_0)/f_\varepsilon(\omega_0)$, where $f_\varepsilon(\omega_0)$ and $f_u(\omega_0)$ are the spectral matrix

at the zero frequency of the signal and noise components of the process x , respectively, and $w_x(\omega_i) = \ln f^*(\omega_i) - \ln f_\varepsilon(\omega_0) - \beta \omega_i^{2d} - 2d [\ln(2 \sin \frac{\omega_i}{2}) - \ln \omega_i]$.

The estimator proposed by Sun and Phillips (2003) is the minimizer of the averaged squared errors, requiring the minimization of the objective function

$$Q(d, \beta) = \frac{1}{m} \sum_{l=1}^m w_{\omega_l}^2, \quad (21)$$

$$w_{\omega_l} = \left(\ln I_{\omega_l} - \frac{1}{m} \sum_{k=1}^m \ln I_{\omega_k} \right) + 2d \left(\ln \omega_l - \frac{1}{m} \sum_{k=1}^m \ln \omega_k \right) - \beta \left(\omega_l^{2d} - \frac{1}{m} \sum_{k=1}^m \omega_k^{2d} \right).$$

Sun and Phillips (2003) have proved the consistency and asymptotic normality of the estimator.

When p perturbed long memory processes are available, a multivariate generalization can be implemented in a seemingly unrelated non linear log periodogram framework, similarly to the extension provided by Robinson (1995) for the linear log periodogram estimator. We would then have

$$\begin{aligned} \ln I_1(\omega_i) &= \alpha_1 - 2d_1 \ln \omega_i + \beta_1 \omega_i^{2d_1} + w(\omega_i)_1, \\ &\vdots \\ \ln I_p(\omega_i) &= \alpha_p - 2d_p \ln \omega_i + \beta_p \omega_i^{2d_p} + w(\omega_i)_p, \end{aligned} \quad (22)$$

The multivariate model can be estimated by means of a GLS approach, where the objective function to be minimized, concentrated with respect to the intercept vector, can be written as

$$Q(\mathbf{d}, \boldsymbol{\beta}) = \sum_{i=1}^p \sum_{j=1}^p \sigma^{ij} \mathbf{w}'_i \mathbf{w}_j, \quad (23)$$

where \mathbf{w}_s $s = i, j = 1, \dots, p$ is a $m \times 1$ vector of residuals, with generic element

$$\mathbf{w}_{s, \omega_l} = \left(\ln I_{s, \omega_l} - \frac{1}{m} \sum_{k=1}^m \ln I_{s, \omega_k} \right) + 2d_s \left(\ln \omega_l - \frac{1}{m} \sum_{k=1}^m \ln \omega_k \right) - \beta_s \left(\omega_l^{2d_s} - \frac{1}{m} \sum_{k=1}^m \omega_k^{2d_s} \right),$$

where m denotes the bandwidth employed for estimation. Finally, σ^{ij} denotes the i, j elements of the contemporaneous variance covariance matrix Σ , i.e. $\sigma^{ij} = E[w_{i,\omega_l} w'_{j,\omega_l}]$. Since the Σ matrix is not known, a two step procedure can be followed to obtain efficient estimates of the parameters. In the first step univariate estimation is performed on each equation separately by means of the estimator proposed by Sun and Phillips (2003), obtaining an estimate of the residuals vectors $\hat{\mathbf{w}}_s$ $s = 1, \dots, p$, which can be employed to compute a consistent estimate of the elements σ^{ij} $i, j = 1, \dots, p$ as $\hat{\sigma}^{ij} = \frac{\hat{\mathbf{w}}_i' \hat{\mathbf{w}}_j}{m}$. This yields the feasible GLS estimator, which requires the minimization of the function

$$Q(\mathbf{d}, \boldsymbol{\beta}) = \sum_{i=1}^p \sum_{j=1}^p \hat{\sigma}^{ij} \mathbf{w}'_i \mathbf{w}_j, \quad (24)$$

Asymptotic standard errors can be computed as the square root of the diagonal elements of the matrix

$$AsyVar \left[\hat{\mathbf{d}}, \hat{\boldsymbol{\beta}} \right] = \left[\sum_{i=1}^p \sum_{j=1}^p \hat{\sigma}^{ij} \mathbf{h}_i(\mathbf{d}, \boldsymbol{\beta})' \mathbf{h}_j(\mathbf{d}, \boldsymbol{\beta}) \right]^{-1}, \quad (25)$$

where $\mathbf{h}_s(\mathbf{d}, \boldsymbol{\beta})$ $s = i, j = 1, \dots, p$ is an $m \times 2p$ matrix of pseudoregressors obtained as the derivatives of the function $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ in the compact formulation of the model in deviations from the mean

$$\ln \tilde{\mathbf{I}}_s = \tilde{\mathbf{Z}}_s(\mathbf{d}, \boldsymbol{\beta}) + \mathbf{w}_s, \quad (26)$$

Since only the parameter d_s and β_s enter in the generic s th equation, the matrix \mathbf{h}_s will contain $2p - 2$ zero columns, corresponding to the omitted parameters. We then have

$$\mathbf{h}_{s,\omega_l,d_s} = -2 \left[\left(1 - \beta_s (\omega_l^{d_s})^2\right) \ln \omega_l - \frac{1}{m} \sum_{k=1}^m \left(1 - \beta_s (\omega_k^{d_s})^2\right) \ln \omega_k \right], \quad (27)$$

$$\mathbf{h}_{s,\omega_l,\beta_s} = \omega_l^{2d_s} - \frac{1}{m} \sum_{k=1}^m \omega_k^{2d_s}, \quad (28)$$

with $\mathbf{h}_{s,\omega_l,d_s}$ denoting the generic element (frequency ω_l) of the pseudoregressor vector obtained by differentiating the $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ function with respect to d_s , and $\mathbf{h}_{s,\omega_l,\beta_s}$ the generic element (frequency ω_l) of the pseudoregressor vector obtained by differentiating the $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ function with respect to β_s .

Linear restrictions can be easily tested in this framework. Of particular interest are restrictions which involve the equality of the fractional differencing parameter for two or more processes. Assuming the following ordering for the vector of parameters $(\mathbf{d}', \boldsymbol{\beta}')$, with $\mathbf{d} = (d_1, \dots, d_p)'$ and $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$, let's consider the case of homogeneous restrictions

$$H_0 : \mathbf{R}\mathbf{d} = \mathbf{0}, \quad (29)$$

where \mathbf{R} is an $h \times p$ matrix of rank equal to $h < p$. Following Robinson (1995), the test statistic is

$$\hat{\mathbf{d}}' \mathbf{R}' \left[(\mathbf{R}, \mathbf{0}) \left[\sum_{i=1}^p \sum_{j=1}^p \hat{\sigma}^{ij} \mathbf{h}_i(\mathbf{d}, \boldsymbol{\beta})' \mathbf{h}_j(\mathbf{d}, \boldsymbol{\beta}) \right]^{-1} (\mathbf{R}, \mathbf{0})' \right] \mathbf{R} \hat{\mathbf{d}} \sim \chi_{(h)}^2, \quad (30)$$

where $\mathbf{0}$ is a null matrix with dimension $h \times p$.

If the hypothesis under the null cannot be rejected, the restricted model can be estimated with gains in terms of efficiency. For instance, a model with equal fractional differencing parameter for the various processes can be easily estimated from the constrained model

$$\begin{aligned} \ln I_1(\omega_i) &= \alpha_1 - 2d \ln \omega_i + \beta_1 \omega_i^{2d} + w(\omega_i)_1, \\ &\vdots \\ \ln I_p(\omega_i) &= \alpha_p - 2d \ln \omega_i + \beta_p \omega_i^{2d} + w(\omega_i)_p, \end{aligned} \quad (31)$$

and minimizing the function in [24].

Denoising (Beltratti and Morana, 2003) Following Beltratti and Morana (2003), from the non linear log periodogram regression a semiparametric denoising approach can be easily implemented. By writing the noise corrected log periodogram for the generic j th process as

$$\ln I_j^c(\omega_i) = \ln I_j(\omega_i) - \hat{\beta}_j^{2d_j} \omega_i,$$

it is possible to recover an estimate of the periodogram for the unperturbed long memory process as

$$I_j^c(\omega_i) = \exp(\ln I_j^c(\omega_i)).$$

Similarly to the Wiener-Kolmogorov approach, two sided time domain weights to filter the long memory signal from the observed process, can be computed from the inverse Fourier transform of the (semiparametric) transfer function

$$h_\mu(\omega_i) = \frac{I_j^c(\omega_i)}{I_j(\omega_i)} \quad (32)$$

According to the results of the Monte Carlo simulation reported in Beltratti and Morana (2003), the performance of the semiparametric filter is very similar to the performance of the parametric filter of Harvey (1993). Both the approaches allow an accurate recovering of the simulated signal when the inverse long-run signal to noise ratio is low, with the performance worsening as the inverse of the signal to noise ratio increases. However, both filters are always unbiased, independently of the value of the long-run signal to noise ratio. According to the RMSE decomposition, the parametric model provides a superior performance than the semiparametric model as the long-run inverse signal to noise ratio increases. However, a modified version of the contemporaneous semiparametric filter in general outperforms also the two-sided parametric model. The modified semiparametric filter is computed as follows

$$\begin{aligned} \ln I_j^c(\omega_i) &= \ln I_j(\omega_i) - \hat{\gamma}_j \omega_i^{2d_j} \\ \hat{\gamma}_j &= 0 \quad 0 < \omega_i \leq \frac{2\pi m^*}{T} \\ \hat{\gamma}_j &= \hat{\beta}_j \quad \omega_i > \frac{2\pi m^*}{T}, \end{aligned}$$

i.e. by not filtering out the lowest frequencies in the computation of the term $\ln I_j^c(\omega)$. The optimal bandwidth m^* can be easily determined through Monte Carlo simulation by minimising the RMSE. The optimal bandwidth, obtained through the minimization of the RMSE, enable the semiparametric filter to achieve the same RMSE of the parametric model (which yields the minimum

mean square error under Gaussianity), yielding a superior performance in terms of RMSE decomposition. ³¹

Stationarity test A test for the null $H_0 : H = 0.5$ against a two-sided alternative can be computed as follows

$$LM = m \left(\frac{\sum_{j=1}^m v_j I(\lambda_{jT})}{\sum_{j=1}^m I(\lambda_{jT})} \right)^2 \xrightarrow{d} \chi_{(1)}^2,$$

$$W = 4m \left(\hat{H}_{LM} - H \right)^2 \xrightarrow{d} \chi_{(1)}^2,$$

$$LR = 2m \left\{ \log \frac{\sum_{j=1}^m I(\lambda_{jT})}{\sum_{j=1}^m \lambda_j^{2H-1} I(\lambda_{jT})} + \left(2\hat{H}_{LM} - 1 \right) \frac{1}{m} \sum_{j=1}^m \log \lambda_j \right\} \xrightarrow{d} \chi_{(1)}^2.$$

Bandwidth selection A theory of optimal bandwidth selection, based on the minimization of the asymptotic MSE, has been proposed for various semiparametric estimators (Robinson, 1994b; Henry and Robinson, 1996; Hurvich et al., 1998) and efforts have been made to make it implementable (Delgado and Robinson, 1996; Henry, 2001). Following Henry (2001) the optimal bandwidths can be stated as follows:

$$m_{LW}^* = \left(\frac{3}{4\pi} \right)^{4/5} \left| \tau^* + \frac{d_x}{12} \right|^{-2/5} T^{4/5}$$

for the local Whittle estimator and $-\frac{1}{2} < d < \frac{1}{2}$;

$$m_{LP} = \left(\frac{27}{512\pi^2} \right)^{1/5} |\tau^*|^{-2/5} T^{4/5}$$

³¹A GAUSS code implementing the optimal semiparametric filter is available upon request to the author.

for the log periodogram estimator, $-\frac{1}{2} < d < \frac{1}{2}$ and $\tau^* \neq 0$;

$$m_{AP_1} = \left(\frac{(3 - 2d_x)^2}{4(2\pi)^4(1 - 4d_x)} \right)^{1/5} \left| \tau^* + \frac{d_x}{12} \right|^{-2/5} T^{4/5}$$

for the averaged periodogram estimator with $0 < d < \frac{1}{4}$, while for $\frac{1}{4} < d < \frac{1}{2}$

$$m_{AP_2} = \frac{T^{\frac{2}{3-2d}}}{2\pi} \left\{ \frac{2\Gamma(1 - 2d_x) \cos\left(\left(\frac{1}{2} - d_x\right)\pi\right) (3 - 2d_x)}{8} \left(\frac{2d_x - 3}{2d_x(\tau^* + \frac{d}{12})} \right. \right.$$

$$+ \frac{1}{\left| \tau^* + \frac{d_x}{12} \right|} \left[\frac{(3/2d_x)^2}{(2d_x)^2(d_x + \frac{1}{2})^2} + 32\left(\frac{1}{2} - d_x\right) \left\{ \frac{1}{2d_x(4d - 1)} \right. \right.$$

$$\left. \left. - \frac{1}{(4d_x + 1)(d_x + \frac{1}{2})^2} - \frac{4\Gamma(2d_x)^2}{\Gamma(4d_x + 2)} \right\}^{\frac{1}{2}} \right\}^{\frac{1}{3-2d_x}};$$

finally, for the LM estimator proposed by Robinson (1998) we used

$$m_{LM} = \left(\frac{3}{4\pi} \right)^{4/5} |\tau^*|^{-2/5} T^{4/5}.$$

Optimal bandwidths and the corresponding estimates of d are then derived together using the recursion

$$\begin{aligned} \hat{d}_x^{(k)} &= d(\hat{m}^{(k)}), \\ \hat{m}^{(k+1)} &= m_i(\hat{d}_x^{(k)}, \tau^*) \end{aligned}$$

$i = LW, AW$, for the Gaussian semiparametric estimator and the averaged periodogram estimator, while for the log periodogram estimator and the LM estimator the formula simply is $\hat{d}_x = d(\hat{m}_j)$ $j = LP, LM$. The recursions were started setting $\hat{m}^{(0)} = [T^{4/5}]$ and an estimate of the smoothness parameter $\tau^* = \frac{f^{*''}(0)}{2f^*(0)}$, where the function $f^*(\cdot)$ describes a short-term correlation structure in the spectral density of the process $f(\lambda) = |1 - \exp(i\lambda)|^{1-2\hat{H}} f^*(\lambda)$, was obtained using the least square regression suggested by Delgado and Robinson (1996)

$$I(\lambda_j) = \sum_{k=0}^2 Z_{jk}(\hat{H}) \hat{\beta}_k + \hat{\varepsilon}_j \quad j = 1, \dots, \hat{m}^{(0)}$$

where $Z_{jk}(\hat{H}) = |1 - \exp(i\lambda_j)|^{1-2\hat{H}} \lambda_j^k / k!$. τ^* is then estimated by $\hat{\beta}_2 / 2\hat{\beta}_0$.³²

Fractional cointegration test (Robinson-Yajima, 2001) Lets consider the $n \times 1$ vector \mathbf{w}_t of long memory processes with continuous spectral distribution function satisfying the condition $f_w(\lambda) \sim \Lambda \mathbf{E} \bar{\Lambda}$ as $\lambda \rightarrow 0^+$, where $\bar{\Lambda}$ denotes the complex conjugate of Λ , \mathbf{E} is a real symmetric matrix of dimension $n \times n$, and $\Lambda = \text{diag} \{ e^{i\pi d_i/2} \lambda^{-d_i} \}_{i=1}^n$, $0 < d_i < 1/2$, $i = 1, \dots, n$. Fractional cointegration implies $\beta' \mathbf{E} \beta = \mathbf{0}$, so that \mathbf{E} is of reduced rank $k = n - r$, where r is the number of cointegration relations. Therefore, the number of cointegration relations is given by the number of zero eigenvalues of the \mathbf{E} matrix (r), and the number of common long memory factors is given by $k = n - r$.

Robinson and Yajima (2001) have proposed a cointegrating rank test based on the significance of the eigenvalues of the \mathbf{E} matrix. Assuming that all the processes are characterized by the same order of fractional integration, or that the series have been partitioned in groups according to the order of fractional integration, the estimator proposed by Robinson and Yajima (2001) is

$$\hat{\mathbf{E}}_m = \frac{1}{m} \sum_{j=1}^m \lambda_j^{2\bar{d}_*} \text{Re}(I(\lambda_j)),$$

where $\bar{d}_* = \frac{1}{n} \sum_{i=1}^n \hat{d}_i$, where \hat{d}_i are the estimated fractional differencing parameters of each individual series.

Under the assumptions detailed in Robinson and Yajima (2001), the estimator is consistent and asymptotically normally distributed

$$m^{1/2} \text{vec}(\mathbf{E}(\bar{d}_*) - \mathbf{E}) \rightarrow N(0, \frac{1}{2} (\mathbf{E} \otimes \mathbf{E} + (\mathbf{E} \otimes \mathbf{E}_1, \dots, \mathbf{E} \otimes \mathbf{E}_p))),$$

where \mathbf{E}_i denotes the i -th column of \mathbf{E} . By denoting the eigenvalues of the estimated $\mathbf{E}(\bar{d}_*)$ matrix as $\hat{\delta}_i$, $i = 1, \dots, n$, ordered as $\hat{\delta}_1 > \dots > \hat{\delta}_{n-r} > 0$, with $\hat{\delta}_{n-r+1} = \dots = \hat{\delta}_n = 0$ for $r \geq 1$, it is shown that

³²Optimal bandwidth estimation has been performed using a GAUSS routine written by Mark Henry, to whom the author is grateful.

$$m^{1/2} (\hat{\delta}_i - \delta_i) \sim N(0, \delta_i^2).$$

By defining

$$\hat{\pi}_j = \frac{\hat{\sigma}_{n-j+1,n}^{(1)}}{\hat{\sigma}_{1,n}^{(1)}} \quad j = 1, \dots, n-1,$$

where $\hat{\sigma}_{k,l}^{(i)} = \sum_{z=k}^l \hat{\delta}_z^i$, and

$$s_j = \frac{\hat{\sigma}_{n-j+1,n}^{(1)2} \hat{\sigma}_{1,n-j}^{(1)} + \hat{\sigma}_{1,n-j}^{(1)2} \hat{\sigma}_{n-j+1,n}^{(2)2}}{\hat{\sigma}_{1,n}^{(1)2}},$$

it is shown that

$$m^{1/2} (\hat{\pi}_j - \pi_j) / s_j \xrightarrow{d} N(0, 1) \quad j = 1, \dots, n-1, \quad r = 0.$$

In practice, since the asymptotic distribution of $\hat{\pi}_j$ is standard normal only when $r = 0$, a test for a non zero cointegration rank can be carried out by considering the $100(1-\alpha)\%$ upper confidence interval

$$\hat{\pi}_r + s_r z_\alpha / m^{1/2},$$

not rejecting the null of $rank = r$ if $\hat{\pi}_r + s_r z_\alpha / m^{1/2} < 0.1/n$.

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