Trigari, Antonella

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EQUILIBRIUM UNEMPLOYMENT, JOB FLOWS AND INFLATION DYNAMICS

by Antonella Trigari
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by Antonella Trigari²

In 2004 all ECB publications will feature a motif taken from the €100 banknote.

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2 Correspondence: IGIER, Bocconi University, Via Solasco 5, 20136 Milan, Italy. E-mail: antonella.trigari@uni-bocconi.it. Home page: www.igier.uni-bocconi.it/trigari. Tel: +39-02-58363040. Fax: +39-02-58363302.

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Abstract

In order to explain the joint fluctuations of output, inflation and the labor market, this paper first develops a general equilibrium model that integrates a theory of equilibrium unemployment into a monetary model with nominal price rigidities. Then, it estimates a set of structural parameters characterizing the dynamics of the labor market using an application of the minimum distance estimation. The estimated model can explain the cyclical behavior of employment, hours per worker, job creation and job destruction conditional on a shock to monetary policy. Moreover, allowing for variation of the labor input at the extensive margin leads to a significantly lower elasticity of marginal costs with respect to output. This helps to explain the sluggishness of inflation and the persistence of output after a monetary policy shock. The ability of the model to account for the joint dynamics of output and inflation rely on its ability to explain the dynamics in the labor market.

Keywords: Business Cycles, Search and Matching Models, Monetary Policy, Inflation
JEL Classification: E32, J41, J64, E52, E31
Non-Technical Summary

A classic challenge that macroeconomists face is to explain the cyclical fluctuations of output, unemployment and inflation. Recently, a new generation of monetary general equilibrium models with staggered price setting, often referred to as New Keynesian, has made important advances in explaining the links between money, output and inflation over the business cycle. However, these models have a great difficulty in explaining the sluggishness in inflation and the persistence in output that are observed in the data. What is more, they cannot explain why demand shocks, such as monetary policy shocks, should cause significant fluctuations in equilibrium unemployment. They fail to deal with unemployment as they assume a frictionless perfectly competitive labor market in which individuals vary the hours that they work, but the number of people working never changes.

In this paper I integrate a new Keynesian theory of money and inflation with a theory of equilibrium unemployment along the lines of the work by Mortensen and Pissarides (1994). The model is characterized by two main building blocks: nominal rigidities in price setting and search frictions in the labor market. To introduce nominal price rigidities, I assume that at least some firms are monopolistic competitive and face constraints on the frequency with which they can adjust the price of the good they produce, as in Calvo (1983). This leads to a theory of inflation that emphasizes its forward-looking nature as well as the role played by real marginal cost fluctuations in shaping inflation dynamics. To introduce equilibrium unemployment, I assume that the labor market displays search and recruiting frictions and the need to reallocate workers from time to time across alternative productive activities. Job flows and worker flows between labor market states are explicitly modeled and can be influenced by aggregate events or individual decisions. In particular, unemployment is treated as the endogenous outcome of job creation and job separation decisions of firms and workers. Finally, monetary policy is conducted according to a Taylor-type rule for the nominal interest rate.

In the model, it turns out that most of the fluctuation in total hours takes the form of fluctuations in the number of workers, the extensive margin, rather than changes in the hours that each individual works, the intensive margin. Moreover, changes in employment allow for changes in output without increased marginal costs. As a consequence, allowing for variations of the labor input at the employment margin leads to a significantly lower elasticity of marginal costs with respect to output. In turn, smaller variations in marginal costs induce smaller adjustments in prices. This raises the sluggishness of the price level to changes in aggregate demand and reduces the volatility of inflation. Finally, the lower sensitivity of the price level to variations in aggregate demand raises the persistence of the response of aggregate demand and output to a monetary shock.

After developing the theoretical model, I estimate a set of structural parameters that characterize the dynamics of the labor market and on which there is few or no independent evidence. I follow the estimation strategy adopted in Rotemberg and Woodford (1997), which can be seen as an application of the minimum distance estimation. Specifically, the structural parameters are chosen so that the impulse responses to a monetary shock of a set of endogenous variables in the
model match as closely as possible the responses estimated using a Vector Autoregressive (VAR) methodology. While this estimation strategy is widely adopted in the literature on dynamic general equilibrium models with money, no other study, to the best of my knowledge, has used it to estimate at least a set of the parameters that describe a labor market with matching frictions and endogenous job destruction.

The main results of the paper can be summarized as follows. First, I obtain consistent estimates of a set of labor market parameters. When previous estimates are available, the estimates that I obtain are consistent with the previous ones. Second, when I compare the model with equilibrium unemployment to the baseline new Keynesian model, I show that the response of inflation to a monetary shock is significantly less volatile and more persistent. The response of output is also considerably more persistent. Third, the estimated model does a very good job in accounting quantitatively for the response of the US economy to a monetary policy shock. The model can reproduce the large hump-shaped response of output together with the sluggish response of inflation. It also accounts for the large, persistent decrease in employment (the extensive margin) together with the small, transitory fall in average hours per worker (the intensive margin) after a contractionary monetary shock. Finally, it explains the transitory fall in job creation and the larger and more persistent raise in job destruction that is observed in the data. It is important to point out that the ability of the model to account for the joint dynamics of output and inflation rely on its ability to explain the dynamics in the labor market.
1 Introduction

A classic challenge that macroeconomists face is to explain the cyclical fluctuations of output, unemployment and inflation. Recently, a new generation of monetary optimizing general equilibrium models, often referred to as new keynesian\(^1\), has made important advances in explaining the links between money and the business cycle. Building on the traditional keynesian theory of fluctuations, these studies assume that there are barriers to the instantaneous adjustment of nominal prices. The emphasis, then, is on the demand-side transmission mechanism of monetary policy. Although these models are widely used to explain the joint dynamics of output and inflation, they cannot explain why aggregate shocks, in particular monetary policy shocks, should cause significant and persistent fluctuations in equilibrium unemployment.

New keynesian models abstract from unemployment as they assume a frictionless perfectly competitive labor market in which individuals vary the hours that they work, but the number of people working never changes. Even if changes in total hours are interpreted as changes in the number of people working shifts of fixed length, as in the indivisible labor literature\(^2\), this process takes no time and no other resources. In addition, these models do not allow for any heterogeneity among jobs or workers. As a consequence, there is no reason why old jobs should be destroyed and new ones created or why workers should be reallocated from time to time across alternative jobs.

If we want to investigate the effects of monetary policy on unemployment, as well as on job creation and job destruction, we need a richer labor market structure. Such labor market is one where workers look for jobs, hold them and loose them and where existing jobs are continuously replaced by new ones. The search and matching approach to labor market equilibrium, along the lines of the work by Mortensen and Pissarides (1994) and Pissarides (2000), provides a theory of equilibrium unemployment that captures these features of the labor market. In this paper I integrate this approach to unemployment into an otherwise standard new keynesian model.

The second reason to study this integrated framework is that labor market search considerations may help to solve the problems that new keynesian models have in explaining the sluggish response of prices and inflation together with the large, persistent response of output to demand shocks, such as monetary policy shocks. With output being demand-determined, these models predict that the number of worked hours varies significantly as a consequence of a monetary policy shock. In the absence of an implausibly high labor supply elasticity, this leads to sizeable movements in wages and marginal costs. The large variation in marginal costs, then, induces firms setting their prices to make large price adjustments and causes inflation to respond substantially. The evidence, however, shows that the response of inflation to a monetary policy shock is relatively small.

With equilibrium unemployment, it turns out that most of the fluctuation in total hours takes the form of fluctuations in the number of workers, the extensive margin, rather than changes in the hours that each individual works, the intensive margin. Moreover, changes in employment allow for

\(^1\)See Galí (2000) for a survey.
changes in output without increased marginal costs. As a consequence, allowing for variations of the labor input at the employment margin leads to a significantly lower elasticity of marginal costs with respect to output. In turn, smaller variations in marginal costs induce smaller adjustments in prices. This raises the sluggishness of the price level to changes in aggregate demand and reduces the volatility of inflation. Finally, the lower sensitivity of the price level to variations in aggregate demand raises the persistence of the response of aggregate demand and output to a monetary shock.

A third benefit of this research strategy is that it permits to account for the joint response of output and inflation without assuming an implausibly high value of the intertemporal elasticity of substitution of leisure. Precisely, as I discuss in Section 5, I will assume a degree of intertemporal substitution that is consistent with the evidence from microeconomic studies.

The model that I develop in this paper is characterized by two main building blocks: nominal rigidities in price setting and search and matching frictions in the labor market. One complication is that when firms set prices on a staggered basis the job creation and destruction decisions become highly intractable. To avoid this problem I distinguish between two types of firms: retail firms and intermediate goods firms.\(^3\)\(^4\) Firms produce intermediate goods in competitive markets using labor as their only input, and then sell their output to retailers who are monopolistic competitive. Retailers, finally, sell final goods to the households. Then, I assume that price rigidities arise at the retail level, while search frictions occur in the intermediate goods sector.

After developing the theoretical model, I estimate a set of structural parameters that characterize the dynamics of the labor market and on which there is few or no independent evidence. I follow the estimation strategy adopted in Rotemberg and Woodford (1997) and other studies\(^5\), which can be seen as an application of the minimum distance estimation. Specifically, the structural parameters are chosen so that the impulse responses to a monetary shock of a set of endogenous variables in the model match as closely as possible the responses estimated using a Vector Autoregressive (VAR) methodology. While this estimation strategy is widely adopted in the literature on dynamic general equilibrium models with money, no other study, to the best of my knowledge, has used it to estimate at least a set of the parameters that describe a labor market with matching frictions and endogenous job destruction.

The main results of the paper can be summarized as follows. First, I obtain consistent estimates of a set of labor market parameters. When previous estimates are available, the estimates that I obtain are consistent with the previous ones. Second, when I compare the model with equilibrium unemployment to the baseline new keynesian model, I show that the response of inflation to a monetary shock is significantly less volatile and more persistent. The response of output is also considerably more persistent. Third, the estimated model does a very good job in accounting quan-

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3This modelling device has first been introduced by Bernanke, Gertler and Gilchrist (1999) in their study of the financial accelerator mechanism.

4For simplicity, I will often refer to retail firms as retailers and to intermediate goods firms as simply firms.

titatively for the response of the US economy to a monetary policy shock. The model can reproduce the large hump-shaped response of output together with the sluggish response of inflation. It also accounts for the large, persistent decrease in employment (the extensive margin) together with the small, transitory fall in average hours per worker (the intensive margin) after a contractionary monetary shock. Finally, it explains the transitory fall in job creation and the larger and more persistent raise in job destruction that is observed in the data. It is important to point out that the ability of the model to account for the joint dynamics of output and inflation rely on its ability to explain the dynamics in the labor market.

Several recent papers have considered search and matching in a real business cycle model and showed that this new framework improves the empirical performance of the standard model in several directions (Merz, 1995, Andolfatto, 1996, and den Haan, Ramey and Watson, 2000). These non-monetary models, however, are not suitable to study how search and matching shape the response of the economy to monetary policy shocks. Cooley and Quadrini (1999) integrate a model of equilibrium unemployment with a limited participation model of money. Their model is consistent with evidence about the impact of monetary policy shocks on the economy and can produce labor market dynamics that fit the data. However, their analysis focuses on the cost channel, or supply-side channel, of monetary transmission and ignores the demand-type channel due to nominal price rigidities. A recent paper by Walsh (2003), written independently from this paper, also studies the interaction between price rigidities and labor-market search. This paper, however, considers only the extensive margin, while I consider the intensive as well as the extensive margin. This allows me to explain the dynamics of hours per worker over the cycle as well as the dynamics of employment. The two papers also differ in other modeling aspects. Moreover, differently from Walsh, I evaluate the empirical performance of the model based on its ability to match conditional second moments, i.e., second moment conditional on a particular source of fluctuations. The advantages of this evaluation criterion are clearly presented in Gali (1999).

Finally, using an application of minimum distance estimation, I also provide estimates of a set of the structural parameters that characterize a labor market with search and matching frictions and on which there is few or no independent evidence. Dotsey and King (2001) show that modifying a benchmark new keynesian model to allow for a number of “supply side” features helps to account for the large and persistent response of output to monetary shocks. In particular, among these features, they allow for changes of the labor input along the extensive margin by introducing a labor force participation decision in addition to the hours of work decision. Then, making the

6Moreover, as I discuss later, allowing for variation at both margins has the implication that the model developed in this paper nests a baseline new keynesian model with a frictionless perfectly competitive labor market. It is this property that makes the two models easily comparable. Specifically, any difference in the dynamics of the two models must be associated with the dynamics of employment, which are in turn determined by the dynamics of job creation and job destruction.

7More precisely, I evaluate the empirical performance of the model in terms of its ability to match the estimated responses of output, inflation and the labor market to a monetary policy shock.
supply elasticity of employment much larger than the supply elasticity of hours per worker, they assume that most of the variation of the labor input over the business cycle occurs at the extensive margin, as it is in the data. In this paper, instead, I investigate whether a fully microfounded specification of the labor market with involuntarily equilibrium unemployment can account for this feature of the data without appealing to high labor supply elasticities. Finally, in Trigari (2003), I develop a model similar in the spirit to the one presented in this paper. However, in that paper I focus on explaining the dynamics of the real wage and its implications for inflation. In order to do this, besides studying a Nash bargaining process, I also develop an alternative bargaining model.

The remainder of the paper is organized as follows: Section 2 presents the evidence related to the response of output, inflation and the labor market to a monetary shock, Section 3 describes the model, Section 4 presents the dynamics of the model around the steady state, Section 5 brings the model to the data and discusses the estimation, Section 6 presents the results and Section 7 concludes.

2 Evidence: output, inflation and the labor market

In this Section I describe a set of stylized facts related to the behavior of output, inflation and a set of labor market variables in face of a monetary shock. More specifically, I use a VAR methodology to estimate the dynamic response of the variables of interest to an identified exogenous monetary policy shock. The short-term nominal interest rate is taken to be the instrument of monetary policy and the identification strategy is described in the Appendix.

The variables included in the analysis are measures of output, inflation and the nominal interest rate, to which I add four labor market variables. The labor market variables that I include are measures of employment, average hours per worker, the job creation rate and the job destruction rate. I include four lagged values of all variables in the VAR. Estimates are based on quarterly US data from 1972:2 to 1993:4.

The series for the nominal interest rate is the Federal Funds rate, annualized and averaged over the quarter. The series for output is the log of quarterly real GDP and the series for inflation is the annualized rate of change of the GDP deflator between two consecutive quarters. The series for employment is the log of total employees in nonfarm establishments. The series for average hours per worker is constructed by subtracting the previous variable from the log of total employee-hours in nonagricultural establishments. The series for job creation and job destruction are taken from Davis, Haltiwanger, and Schuh “Job Creation and Destruction” database. They are, respectively, the log of the quarterly job creation rate for both startups and continuing establishments in the manufacturing sector and the log of the quarterly job destruction rate for both shutdowns and continuing establishments in the manufacturing sector.

Figure 1 reports the responses over time of output, inflation and the Federal funds rate to a

\[ \text{The choice of the sample period is explained by the availability of the data on job creation and job destruction.} \]
one percent increase in the Federal funds rate and Figure 2 the responses of employment, average hours per worker, the job creation rate and the job destruction rate to the same shock. The solid lines display the point estimates of the coefficients. The dashed lines are two standard deviation confidence intervals. The impulse response functions of inflation and the Federal funds rate are reported in percentage points. The other impulse responses are reported in percentage deviations from each variable’s unconditional mean. The horizontal axis indicate quarters.

The results suggested by Figure 1 are standard in the VAR literature on monetary policy. After a contractionary monetary shock there is a large hump-shaped fall in output accompanied by a sluggish persistent decrease in inflation. The peak fall in output is about 0.4 percent and that of annualized inflation about 0.3 percent. Existing optimizing monetary general equilibrium models have shown a great difficulty in explaining this joint dynamic behavior of output and inflation. In general, they predict a much larger response of inflation.

Figure 1: Estimated impulse responses to a monetary shock

Figure 2, instead, presents some new results about the response of the labor market to a monetary shock. First, as we can see from the plots, the labor input adjusts along both the extensive and the intensive margin. As a consequence of the tightening in monetary policy, both employment and hours per worker fall. However, while the fall in employment is large and persistent,
there is only a small transitory decrease in hours per worker. Therefore, the labor input shows a significantly different cyclical behavior at the extensive and the intensive margin. Second, the response of employment is explained by variations at both the job creation and the job destruction margin. The monetary contraction causes a fall in job creation and a raise in job destruction. The decrease in job creation is transitory with a peak response of about 3.4 percent, while the increase in job destruction is larger and more persistent with a peak response of about 4.5 percent.

![Figure 2: Estimated impulse responses to a monetary shock](image)

### 3 The model

The proposed model with nominal price rigidities and search and matching in the labor market has four sectors. The sectors include the households, the (intermediate goods) firms, the retailers and a monetary authority. Each sector’s environment is discussed in detail below.
3.1 Households

Each household is thought of as a very large extended family which contains a continuum of members with names on the unit interval. In equilibrium, some members will be unemployed while some others will be working for firms. Each member has the following period utility function:

\[ u(c_t, c_{t-1}) - g(h_t, a_t), \]

where

\[ u(c_t, c_{t-1}) = \log(c_t - ec_{t-1}) \]

and

\[ g(h_t, a_t) = \kappa h_t^{1+\phi} + \chi_t a_t. \]

The variable \( c_t \) is consumption of a final good, \( h_t \) is the hours of work, \( a_t \) is a shock to the disutility from working and \( \chi_t \) is an indicator function taking the value of one if the individual is employed and zero if unemployed. When \( e > 0 \), the model allows for habit formation in consumption.\(^9\) The preference shock \( a_t \) is idiosyncratic to the individual and is assumed to be independently and identically distributed across individuals and times with cumulative distribution function \( F(a_t) \). The cumulative distribution function \( F(a_t) \) is assumed to be lognormal with parameters \( \mu_a \) and \( \sigma_a \).\(^{10}\) A high preference shock \( a_t \) causes a high disutility from working.\(^{11}\)

The presence of equilibrium unemployment introduces heterogeneity in the model. In the absence of perfect income insurance, each individual’s labor income differs based on his employment status. In this case, the individuals’ saving decision would become dependent on their entire employment history. To the purpose of this paper, I avoid these distributional issues by assuming that family members pool their incomes and chose per capita consumption and asset holdings to

\(^9\) McCallum and Nelson (1999), Fuhrer (2000) and Christiano et al. (2001) show that habit formation in consumption preferences is important to understand the transmission mechanism of monetary shocks. In particular, it helps to account for the hump-shaped decrease in consumption together with the rise in the real interest rate after a contractionary monetary shock. In this paper, habit persistence in consumption is also important to account for the response of the labor market. Without habit persistence, the larger change in consumption and output (since output is demand-determined) would occur in the first period following the monetary shock. Since employment, as it will be clear below, moves gradually, hours per worker would fluctuate significantly in the first period in order to accommodate the initial change in output. In the data, however, the initial response of hours per worker is relatively small.

\(^{10}\) Note that these parameters do not coincide with the mean and the variance of \( a_t \).

\(^{11}\) Assuming that the idiosyncratic shock enters additively avoids the problem of excessive variation in hours worked across individuals. In particular, since individuals are identical in all aspects other than the preference shock, it will be the case that they all work the same number of hours.
maximize the expected lifetime utility of the representative household:\footnote{The same result could be obtained with a more sophisticated variant of the income-pooling hypothesis if the individuals insure one another against the risk of being unemployed. See as an example Andolfatto (1996).}

\[
E_t \sum_{s=0}^{\infty} \beta^s [u(c_{t+s}, c_{t+s-1}) - G_{t+s}],
\]

where \( \beta \in (0,1) \) is the intertemporal discount factor and \( c_t \) is per capita consumption of each family member at date \( t \). The variable \( G_t \) denotes the family’s disutility from supplying hours of work at date \( t \), i.e., the sum of the disutilities of the members who are employed and supply hours of work. The representative household does not choose hours of work. These are determined through decentralized bargaining between firms and workers. Therefore, for simplicity, I do not make explicit the family’ disutility term at this point.\footnote{This term is nevertheless important to derive the value of employment and unemployment for a worker from the family problem. See the Appendix for details.}

Households own all firms in the economy and face, in each period, the following budget constraint:

\[
c_t + \frac{B_t}{p_t r^n_t} = d_t + \frac{B_{t-1}}{p_t},
\]

where \( p_t \) is the aggregate price level, \( B_t \) is per capita holdings of a nominal one-period bond and \( r^n_t \) is the gross nominal interest rate on this bond, which is certain at the issuing date. The variable \( d_t \) is the per capita family income in period \( t \).\footnote{The family income is the sum of the wage income earned by employed family members, the non-tradable output of final good produced at home by unemployed family members and the family share of aggregate profits from retailers and matched firms.}

The representative household chooses consumption and asset holdings to maximize (4) subject to (5). Furthermore, as in Rotemberg and Woodford (1997), I assume that households must choose their consumption level at date \( t \) with the information set available at date \( t-2 \).\footnote{As Rotemberg and Woodford (1997) point out, this information lag could also be interpreted as a decision lag.} This assumption is consistent with the identifying restriction imposed in the VAR considered in Section 2, according to which all variables in the information set of the central bank are prevented from responding contemporaneously to a monetary shock. In addition, this assumption is necessary to match the initial delay in the observed response of output. As Figure 1 shows, the tightening in monetary policy has a significant effect on output only after two quarters. The household’s optimal choice of consumption, then, must satisfy:

\[
E_{t-2} \lambda_t = E_{t-2} u_{c,t},
\]
of additional income to the expected utility of additional consumption based on the knowledge of period \( t - 2 \). The variable \( u_{c,t} \) is the realized value of the marginal utility of consumption at date \( t \):

\[
    u_{c,t} = \frac{\partial u(c_t, c_{t-1})}{\partial c_t} + \beta E_t \frac{\partial u(c_{t+1}, c_t)}{\partial c_t} = \frac{1}{(c_t - ec_{t-1})} - \beta e E_t \frac{1}{(c_{t+1} - ec_t)}.
\]  

In addition the marginal utility of income satisfies:

\[
    \lambda_t = \beta E_t [r_t \lambda_{t+1}],
\]

where \( r_t \) is the gross real interest rate:

\[
    r_t = \frac{p_t}{p_{t+1}} r_t^p.
\]

### 3.2 Firms and the labor market

Firms producing intermediate goods sell their output in competitive markets and use labor as their only input. They meet workers on a matching market. That is, firms cannot hire workers instantaneously. Rather, workers must be hired from the unemployment pool through a costly and time-consuming job creation process. Workers’ wages and hours of work are determined through a decentralized bargaining process. Finally, matched firms and workers may decide to endogenously discontinue their employment relationship.

#### 3.2.1 Matching market and production

In order to match with a worker, firms must actively search for workers in the unemployment pool. This idea is formalized assuming that firms post vacancies. On the other hand, unemployed workers must look for firms. I assume that all unemployed workers search passively for jobs.

Each firm has a single job that can either be filled or vacant and searching for a worker. Workers can be either employed or unemployed and searching for a job.\(^{16}\) Denote with \( v_t \) the number of vacancies posted by firms at date \( t \) and with \( u_t \) the number of workers seeking for a job at date \( t \).

Vacancies are matched to searching workers at a rate that depends on the number of searchers on each side of the market, i.e., the number of workers seeking for a job and the number of posted vacancies. In particular, the flow of successful matches within a period, denoted with \( m_t \), is given by the following matching function:

\(^{16}\) All unmatched workers are assumed to be part of the unemployed pool, i.e., I abstract from workers’ labor force participation decisions.
\[ m_t = \sigma_m u_t^\sigma v_t^{1-\sigma}, \]  

where \( \sigma \in (0,1) \) and \( \sigma_m \) is a scale parameter reflecting the efficiency of the matching process. Notice that the matching function is increasing in its arguments and satisfies constant returns to scale. It is convenient to introduce the ratio \( v_t/u_t \) as a separate variable denoted with \( \theta_t \). This ratio is the relative number of searchers and measures the labor-market tightness.

The probability that any open vacancy is matched with a searching worker at date \( t \) is denoted with \( q_t \) and is given by:

\[ q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma}. \]  

This implies that firms with vacancies find workers more easily the lower is the market tightness, that is, the higher is the number of searching workers relative to the available jobs. Similarly, the probability that any worker looking for a job is matched with an open vacancy at time \( t \) is denoted with \( s_t \) and is given by:

\[ s_t = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma}. \]  

Analogously, searching workers find jobs more easily the higher is the market tightness, that is, the higher is the number of vacant jobs relative to the number of available workers.

If the search process is successful, the firm operates a production function \( f(h_t) = h_t \), where \( h_t \) is the time spent working at date \( t \). Employment relationships might be severed for exogenous reasons at the beginning of any given period. I denote with \( \rho^x \) the probability of exogenous separation. Furthermore, a matched pair may choose to separate endogenously. If the realization of the match-specific preference disturbance \( a_t \) is above a certain threshold, which I denote \( a^*_t \), a firm and a worker discontinue their relationship. The probability of endogenous separation is \( \rho^e_t = \Pr(a_t > a^*_t) = 1 - F(a^*_t) \) and the overall separation rate is \( \rho_t = \rho^x + (1 - \rho^x)\rho^e_t \). If either exogenous or endogenous separation occurs, production does not take place.

Let us now characterize the employment dynamics. First, because job searching and matching is a time-consuming process, matches formed in \( t-1 \) only start producing in \( t \). Second, employment relationships might be severed for both exogenous and endogenous reasons in any given period, so that the stock of active jobs is subject to continual depletion. Hence, employment \( n_t \) evolves according to the following dynamic equation:

\[ n_t = (1 - \rho_{t-1}) n_{t-1} + m_{t-1}, \]  

which simply says that the number of matched workers at the beginning of period \( t \), \( n_t \), is given by the fraction of matches in \( t-1 \) that survives to the next period, \( (1 - \rho_{t-1}) n_{t-1} \), plus the newly-formed matches, \( m_{t-1} \).
The labor force being normalized to one, the number of unemployed workers at the beginning of any given period is \(1 - n_t\). This is different from the number of searching workers in period \(t\), \(u_t\), which is given by:

\[ u_t = 1 - (1 - \rho_t) n_t \]  

since some of the employed workers discontinue their match and search for a new job in the same period.

### 3.2.2 Bellman equations

To make the exposition of the following sections easier, I describe here the Bellman equations that characterize the problem of firms and workers.

Denote with \(J_t\) the value of a job for a firm at date \(t\) measured in terms of current consumption of the final good. This is given by:

\[
J_t(a_t) = x_t f(h_t) - w_t(a_t) h_t + \mathbb{E}_t \beta_{t+1} (1 - \rho_{t+1}) \int_0^{a_{t+1}} J_{t+1}(a_{t+1}) \frac{dF(a_{t+1})}{F(a_{t+1})},
\]  

(15)

where \(x_t\) and \(w_t\) denote, respectively, the relative price of the intermediate good and the hourly wage rate at date \(t\). Note that the hourly wage rate depends on the idiosyncratic realization of the preference shock. The current value of the job is simply equal to the profits: \(x_t f(h_t) - w_t(a_t) h_t\). The future expected present value of the job, instead, can be explained as follows. Next period, with probability \(1 - \rho_{t+1}\) the match is not severed. In this event the firm obtains the future expected value of a job, where the expected value is conditional on having the preference shock \(a_{t+1}\) below the separation threshold \(\underline{a}_{t+1}\). With probability \(\rho_{t+1}\), instead, the match is discontinued in \(t+1\) and the firm obtains a future value equal to zero. Finally, the expected future value of the job is discounted according to the factor \(\beta_{t+1}\), where \(\beta_{t+s} = \beta_{t+s}^\lambda \frac{\lambda_{t+s}}{\lambda_t} \).  

Denote with \(V_t\) the value of an open vacancy for a firm at date \(t\) expressed in terms of current consumption. With probability \(q_t (1 - \rho_{t+1})\) the vacancy is filled in \(t\) and it is not discontinued in \(t+1\). In this case the vacancy obtains the future expected value of a job. With probability \(1 - q_t\) the vacancy remains open with future value \(V_{t+1}\). Finally, with probability \(q_t \rho_{t+1}\) the vacancy is filled in \(t\) but the new match is discontinued in \(t+1\). In this case the future value is zero. Denoting with \(\kappa\) the utility cost of keeping a vacancy open, \(V_t\) can be written as:

\[
V_t = -\kappa + \mathbb{E}_t \beta_{t+1} \left[ q_t (1 - \rho_{t+1}) \int_0^{a_{t+1}} J_{t+1}(a_{t+1}) \frac{dF(a_{t+1})}{F(a_{t+1})} + (1 - q_t) V_{t+1} \right],
\]  

(16)

17The use of this discount factor effectively evaluates profits in terms of the values attached to them by the households, who ultimately own firms.
where $\kappa t$ is the utility cost expressed in terms of current consumption.

Denote now with $Wt$ and $Ut$, respectively, the employment and the unemployment value for a worker at date $t$ expressed in terms of current consumption. Consider first the situation of an employed worker. The current value of employment is the labor income net of the labor disutility. Next period, with probability $1 - rt+1$ the match is continued and the worker obtains the future expected value of employment. In contrast, with probability $rt+1$ the match is severed and the worker becomes unemployed with future value $Ut+1$. Therefore, $Wt$ can be written as:

$$Wt(a_t) = wt(a_t) h_t - \frac{g(h_t, a_t)}{\lambda_t} + E_t \beta_{t+1} \left[ (1 - rt+1) \int_0^{\omega_{t+1}} (W_{t+1}(a_{t+1}) - Ut+1) \frac{dF(a_{t+1})}{F(\omega_{t+1})} + Ut+1 \right],$$

where $g(h_t, a_t)$ is the disutility from supplying hours of work expressed in terms of current consumption.

Finally, consider the situation of an unemployed worker. His current value is equal to the benefit $b$ from being unemployed. I assume that each unemployed individual produces at home a non-tradable output $b$ of the final good. Then, with probability $st(1 - rt+1)$ the unemployed worker is matched with a firm in period $t$ and continues in the match in $t+1$. In this case he obtains the future expected value of being employed. With probability $1 - st + st rt+1$, instead, the worker remains in the unemployment pool. Therefore, $Ut$ is given by:

$$Ut = b + E_t \beta_{t+1} \left[ st(1 - rt+1) \int_0^{\omega_{t+1}} (W_{t+1}(a_{t+1}) - Ut+1) \frac{dF(a_{t+1})}{F(\omega_{t+1})} + Ut+1 \right].$$

### 3.2.3 Vacancy posting

In this Section I study the opening of new vacancies. Note that opening a new vacancy is not job creation. Job creation takes place when a firm with a vacant job and an unemployed worker meet and agree to form a match.

As long as the value of a vacancy $V_t$ is greater than zero, firms will open new vacancies. In this case, however, as the number of vacancies increases, the probability $q_t$ that any open vacancy finds a suitable worker decreases. A lower probability of filling a vacancy reduces the attractiveness of recruitment activities, thus decreasing the value of an open vacancy. In equilibrium, free entry ensures that $V_t = 0$ at any time $t$. Furthermore, I make a similar timing assumption as for the choice of consumption. I assume that firms must choose the vacancies $v$ at date $t$ on the basis of the information available at date $t - 2$. Hence, from (16) the condition for the posting of new vacancies is:

18Because there is perfect income insurance it is not straightforward to define these values. In the Appendix $W_t$ and $U_t$ are derived from the family problem.
\[ E_{t-2} \kappa \lambda q_t = E_{t-2} \beta_{t+1} (1 - \rho_{t+1}) \int J_{t+1} (a_{t+1}) \frac{dF (a_{t+1})}{F (a_{t+1})}. \]  

Noting that \( 1/q_t \) is the expected duration of an open vacancy, equation (19) simply says that in equilibrium the expected cost of hiring a worker is equal to the expected value of a match.

Substituting recursively equation (15) into (19) and using the law of iterated expectations I obtain:

\[ E_{t-2} \kappa \lambda q_t = E_{t-2} \sum_{s=1}^{\infty} \beta_{t+s} \left( \prod_{k=1}^{s} (1 - \rho_{t+k}) \right) \int \pi_{t+s} (a_{t+s}) \frac{dF (a_{t+s})}{F (a_{t+s})}, \]

where the variable \( \pi_t (a_t) \) is the profits of the firm at date \( t \).

For simplicity, assume for a moment that vacancies at time \( t \) are chosen on the basis of the information available at time \( t \). Then, equation (20) implies that, holding constant \( \lambda_t \), a decrease in the sum of expected future profits must be associated with an increase in \( q_t \). Given the specification of the matching function, this requires either a decrease in the number of vacancies posted, \( v_t \), or an increase in the number of searching workers, \( u_t \). If job destruction was exogenous, the number of searching workers would not change together with the number of vacancies, but only the following period. In this case, the increase in \( q_t \) would be unambiguously associated with a fall in \( v_t \). The decrease in the number of posted vacancies, in turn, would cause a decrease in next period employment, \( n_{t+1} \). With endogenous job destruction, instead, the number of searching workers changes together with the number of vacancies. In particular, if the decrease in profits is caused by a persistent contractionary aggregate shock, as I discuss below, the job destruction rate \( \rho_t \) is likely to increase and so is the number of workers searching for a job, \( u_t \). However, unless the increase in the number of searching workers is extremely large, the increase in \( q_t \) will be associated with a fall in \( v_t \). Monetary policy shocks will affect the rate at which vacancies are posted and, consequently, employment through the above mechanism. A persistent raise in the nominal interest rate, which results in an increase in the real interest rate due to price rigidities, modifies the aggregate consumption behavior of the households and diminishes current and future aggregate demand. Since monopolistic competitive retailers produce to meet demand, this reduces their current and future demand for intermediate goods, which they use as inputs. The resulting persistent decrease in the relative price of intermediate goods, \( x_t \), leads to a fall in firms’ expected future profits. The fall in profits, finally, decreases the number of posted vacancies and reduces employment next period.

---

19 By assuming away the timing assumption, equation (20) becomes:

\[ \frac{\kappa}{\lambda_t q_t} = E_t \sum_{s=1}^{\infty} \beta_{t+s} \left( \prod_{k=1}^{s} (1 - \rho_{t+k}) \right) \int \pi_{t+s} (a_{t+s}) \frac{dF (a_{t+s})}{F (a_{t+s})}. \]
Now, the consideration of the timing assumption has the only implication that a monetary shock at date \( t \) will affect the probability of filling a vacancy and the number of posted vacancies at time \( t + 2 \), rather than at time \( t \). The above transmission mechanism is unchanged.

Finally, note that equation (20) can be rearranged to a first-order difference equation in \( q_t \):

\[
E_{t-2} \frac{\kappa}{\lambda_t} q_t = E_{t-2} \beta_{t+1} (1 - \rho_{t+1}) \int_0^{\tilde{\omega}_{t+1}} \frac{\tilde{\pi}_{t+1}(a_{t+1}) dF(a_{t+1})}{F(a_{t+1})} + E_{t-2} \beta_{t+1} (1 - \rho_{t+1}) \frac{\kappa}{\lambda_{t+1} q_{t+1}}. \tag{21}
\]

### 3.2.4 Bargaining

In equilibrium, matched firms and workers obtain from the match a total return that is strictly higher than the expected return of unmatched firms and workers. The reason is that if the firm and the worker separate, each will have to go through an expensive and time-consuming process of search before meeting another partner. Hence a realized job match needs to share this pure economic rent which is equal to the sum of expected search costs for the firm and the worker. The most natural way to do this is through bargaining.

Bargaining takes place along two dimensions, the real wage and the hours of work. I assume Nash bargaining. That is, the outcome of the bargaining process maximizes the weighted product of the parties’ surpluses from employment:

\[
(W_t(a_t) - U_t)^\eta (J_t(a_t) - V_t)^{1-\eta}, \tag{22}
\]

where the first term in brackets is the worker’s surplus, the second is the firm’s surplus, and \( \eta \) reflects the parties’ relative bargaining power, other than the one implied by the “threat points” \( U_t \) and \( V_t \).

Because the firm and the worker bargain simultaneously about wages and hours, the outcome is (privately) efficient and the wage plays only a distributive role. The Nash bargaining model, in effect, is equivalent to one where hours are chosen to maximize the joint surplus of the match, while the wage is set to split that surplus according to the parameter \( \eta \).

Together the firm and the worker choose the wage \( w_t \) and the hours of work \( h_t \) to maximize (22), taking as given the relative price \( x_t \).

The wage \( w_t \) chosen by the match satisfies the optimality condition:

\[
\eta J_t(a_t) = (1 - \eta) (W_t(a_t) - U_t). \tag{23}
\]

\(^{20}\) I will treat \( \eta \) as a constant parameter strictly between 0 and 1.

\(^{21}\) It must be emphasized that the outcome predicted by the Nash bargaining model is generally not efficient from the viewpoint of society as a whole.
As mentioned above, this condition implies that the total surplus that a job match creates is shared according to the parameter $\eta$. To see why, let $S_t (a_t) = W_t (a_t) - U_t + J_t (a_t)$ denote the total surplus from a match. Finally, from (23) we obtain $W_t (a_t) - U_t = \eta S_t (a_t)$ and $J_t (a_t) = (1 - \eta) S_t (a_t)$.

Although (23) explicitly takes into account the dynamic implications of the match, it can be rewritten as a wage equation that only includes contemporaneous variables. To this purpose, substitute (15), (17) and (18) into (23), using also (19) and (24). This gives the following wage equation:

$$w_t (a_t) h_t = \eta \left( x_t f(h_t) + \frac{\kappa}{\lambda_t} q_t \right) + (1 - \eta) \left( \frac{g(h_t, a_t)}{\lambda_t} + b \right),$$  \hspace{1cm} (24)

Finally, replacing the expressions for $f(h_t)$ and $g(h_t, a_t)$ and using the fact that $\frac{\kappa}{\lambda_t} = \theta_t$ from (11) and (12), I obtain:

$$w_t (a_t) h_t = \eta \left( x_t h_t + \frac{\kappa}{\lambda_t} h_t \right) + (1 - \eta) \left( \frac{\kappa h_t^{1+\phi}}{1+\phi} + a_t \right),$$  \hspace{1cm} (25)

which can be interpreted as follows. The wage shares costs and benefits from the activity of the match according to the parameter $\eta$. In particular, the first term on the right-hand side indicates that the worker is rewarded for a fraction $\eta$ of both the firm’s revenues and the saving of hiring costs that the firm enjoys when a job is created$^{22}$. The second term indicates that the worker is compensated for a fraction $1 - \eta$ of both the disutility he suffers from supplying hours of work and the foregone benefit from unemployment. Note that a high preference shock $a_t$ causes a high wage.

In a frictionless perfectly competitive labor market, the wage would equal the marginal rate of substitution between consumption and leisure. With bargaining and equilibrium unemployment the wage does not equal (although is related to) the marginal rate of substitution. In particular, from (25) the wage also depends on the state of the labor market as it is measured by the exit rate from unemployment or the labor market tightness, $\theta_t$. In a tight labor market, knowing that finding another job is likely to be easy, workers will only accept a higher wage. Conversely, in a depressed labor market they will be willing to settle for a lower wage. The level of the benefit from unemployment affects the equilibrium wage through a similar channel: the higher the benefit, the lower the cost of being unemployed and the higher the bargained wage. The bargained wage, then, will behave quite differently from the competitive wage.

Let us now turn to the determination of hours. The hours of work, $h_t$, chosen by the match satisfy the following optimality condition:

$$\eta J_t (a_t) \left( \frac{g(h_t, a_t)}{\lambda_t} - w_t (a_t) \right) = (1 - \eta) \left( W_t (a_t) - U_t \right) \left( x_t h_t - w_t (a_t) \right),$$  \hspace{1cm} (26)

$^{22}$The term $\frac{\kappa}{\lambda_t}$ reflects the total hiring cost in the economy. Then, $\frac{\kappa}{\lambda_t} \theta_t = \frac{\kappa}{\lambda_t} q_t$ is the hiring cost per unemployed worker.
which can be simplified, using (23), to:

\[
x_t f(h_t) = \frac{g_h(h_t, a_t)}{\lambda_t},
\]

where the value of the marginal product of labor is equated to the marginal rate of substitution between consumption and leisure. Thus, the first order condition determining the hours worked is exactly the same as in a competitive labor market. This happens because the correct measure of labor costs to the firm is the marginal rate of substitution, rather than the wage. In other words, the wage only plays a distributive role.

Finally, using the expressions for \( f(h_t) \) and \( g(h_t, a_t) \), the optimal hours condition is:

\[
x_t = \kappa h \phi t \lambda t,
\]

where optimal hours do not depend on the realization of the preference shock. Note also that, as previously mentioned, the choice of hours that solves the bargaining problem also maximizes the joint surplus.

### 3.2.5 Endogenous separation

In this Section I study the separation decision of a firm-worker pair. A successful match is endogenously discontinued whenever the realization of the preference shock makes the value of the joint surplus of the match equal to zero or negative. The condition that implicitly defines the threshold value \( a_t \) is \( S_t(a_t) = 0 \). Because the firm and the worker share the joint surplus according to the bargaining power \( \eta \), \( S_t(a_t) = 0 \) if and only if \( J_t(a_t) = W_t(a_t) - U_t = 0 \). Thus, the job destruction condition can be written as \( J_t(a_t) = 0 \). In addition, I assume that firms and workers must decide whether to separate in \( t \) on the basis of the information available at time \( t - 2 \). Using (15) and (19) this condition becomes:

\[
E_{t-2} \left[ \tilde{\pi}_t(a_t) + \frac{\kappa}{\lambda_t q_t} \right] = 0.
\]

For simplicity, assume for a moment that firms and workers decide whether to separate in \( t \) on the basis of the information available at time \( t \). Then, equation (29) implies that a fall in the expected future profits, i.e., a decrease in \( \frac{\kappa}{\lambda_t q_t} \), must be associated with an increase in expected profits at \( t \) evaluated at \( a_t \). If the decrease in expected future profits is caused by a persistent contractionary aggregate shock, current profits at any given realization of the preference shock are likely to fall as well. In this case, the increase in \( \tilde{\pi}_t(a_t) \) requires a decrease in \( a_t \). Monetary policy shocks will affect the separation decision of firms and workers and, consequently, employment through the above mechanism. As previously discussed, a persistent increase in the nominal interest rate reduces current and future expected profits at any given level of \( a_t \). This, in turn, decreases the value of \( a_t \) above which the firm and the worker decide to separate. A lower threshold \( a_t \) raises the
current separation rate $\rho_t$ on impact and decreases the number of people actually working within the same period. Finally, taking into account the timing assumption has the only implication that a monetary shock at date $t$ will only affect the threshold value of the idiosyncratic shock and the separation rate at time $t+2$. The transmission mechanism is again unchanged.

### 3.2.6 Job creation, job destruction and employment

I define labor market flows following den Haan, Ramey and Watson (2000). They begin with the observation that flows of workers out of employment relationships are larger than flows of jobs out of firms. This implies that a fraction of the firms experiencing separations from workers must attempt to refill the jobs left vacant and be successful at doing it within the same period. To take this observation into account, they assume that firms experiencing exogenous separations immediately repost the resulting vacancies, while firms experiencing endogenous separations do not. This implies that $\rho^x n_t$ separations are reposted and $q_t \rho^x n_t$ separations are refilled within the same period. Finally, they assume that a job is neither created or destroyed by a firm that both loses and gains a worker in the same period.

Job creation, then, is defined to be equal to the number of newly-created matches net of the number of matches serving to refill the reposted vacancies. The job creation rate is given by:

$$jc_t = \frac{m_t}{n_t} - q_t \rho^x$$

(30)

Job destruction, in turn, is defined as the total number of separations net of the number of separations that are reposted and successfully refilled. The job destruction rate is given by:

$$jd_t = \rho_t - q_t \rho^x$$

(31)

Employment variation, finally, is the outcome of job creation and job separation decisions of firms and workers. Substituting (30) and (31) into (13) and rearranging, I obtain:

$$\frac{n_{t+1} - n_t}{n_t} = jc_t - jd_t.$$  

(32)

### 3.3 Retailers and price setting

There is a continuum of monopolistic competitive retailers indexed by $i$ on the unit interval. Retailers do nothing other than buy intermediate goods from firms, differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, then re-sell them to the households.

Let $y_{it}$ be the quantity of output sold by retailer $i$ and let $p_{it}$ be the nominal sale price. Final goods, denoted with $y_t$, are the following composite of individual retail goods:
\[
y_t = \left[ \int_0^1 y_{it}^{-1} \, dt \right]^{\frac{1}{1-\varepsilon}},
\]  
(33)

where \( \varepsilon \), which is assumed to be greater than one, is the elasticity of substitution across the differentiated retail goods.

Given the index (33) that aggregates individual retail goods into final goods, the demand curve facing each retailer is given by:

\[
y_{it} = \left( \frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t.
\]  
(34)

The aggregate price index, which is defined as the minimum expenditure required to purchase retail goods resulting in one unit of the final good, is:

\[
p_t = \left[ \int_0^1 p_{it}^{1-\varepsilon} \, dt \right]^{\frac{1}{1-\varepsilon}}.
\]  
(35)

As in Calvo (1983), I assume that in any given period each retailer can reset its price with a fixed probability \( 1 - \varphi \) that is independent of the time elapsed since the last price adjustment. This assumption implies that prices are fixed on average for \( \frac{1}{1-\varphi} \) periods.\(^{23}\) Moreover, consistently with the identification assumption made in the VAR analysis, I assume that the retailers who get to change their prices at date \( t \) must decide on the basis of the information available at date \( t - 2 \). Finally, I follow Gali and Gertler (1999) and Amato and Laubach (2000) by assuming that there are two types of retailers that differ in the way they reset prices. A fraction \( 1 - \omega \) of the retailers, which are referred to as “forward-looking”, set prices optimally, given the restriction on the frequency with which they can adjust their price. The remaining fraction \( \omega \) of the retailers, which are referred to as “backward-looking”, instead follow a simple rule of thumb.

The average price of the retailers that do not adjust their price can be shown to be simply \( p_{t-1} \). Thus, given (35), the aggregate price level evolves according to the following equation:

\[
p_t = \left[ \varphi p_{t-1}^{1-\varepsilon} + (1 - \varphi) p_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},
\]  
(36)

where \( p_t \) is the average of the newly reset prices at date \( t \). Let \( p_t^f \) be the price set by the forward-looking retailers and \( p_t^b \) the price set by the backward-looking retailers. The average price \( p_t \) may then be expressed as follows:

\[
p_t = \left[ (1 - \omega) p_t^{f1-\varepsilon} + \omega p_t^{b1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.
\]  
(37)

\(^{23}\) The Calvo’s model avoids keeping track of every agent’s pricing decision when prices are fixed for a certain number of periods.
Forward-looking retailers choose their price to maximize expected future discounted profits given the demand for the good they produce and under the hypothesis that the price they set at date \( t \) applies at date \( t + s \) with probability \( \varphi^s \). Retailers, then, maximize

\[
E_{t-2} \sum_{s=0}^{\infty} \varphi^s \beta_{t+s} \left[ \frac{p_t}{p_{t+s}} - x_{t+s} \right] y_{it,t+s},
\]

where \( y_{it,t+s} \) denotes the demand for good \( i \) at date \( t + s \) conditional on the price set at date \( t \). Note that the relative price of intermediate goods, \( x_t \), coincides with the real marginal cost faced by retailers.

The solution to this problem gives the following expression for the optimal reset price, \( p_f^t \):

\[
p_f^t = \mu E_{t-2} \sum_{s=0}^{\infty} \omega_{t,t+s} x^n_{t+s},
\]

where \( \mu = \frac{\varphi}{\varepsilon - 1} \) is the flexible-price markup and \( x^n_t = p_t x_t \) is the nominal marginal cost at date \( t \). The weights \( \omega_{t,t+s} \) are given by

\[
\omega_{t,t+s} = \frac{\varphi^s \beta_{t+s} R_{it,t+s}}{E_{t-2} \sum_{k=0}^{\infty} \varphi^k \beta_{t+k} R_{it,t+k}},
\]

where \( R_{it,t+s} \) denotes revenues from good \( i \) at time \( t + s \) conditional on the price set at date \( t \). Thus, a forward-looking retailer sets its price equal to a markup \( \mu \) over a weighted average of expected future marginal costs, where the weights represent the relative proportion of expected discounted revenues at each future date.\(^{24}\)

Backward-looking retailers are assumed to obey the following rule of thumb, as in Galí and Gertler (1999):

\[
p_b^t = (1 + \pi_{t-1}) p_{t-1},
\]

where \( \pi_t \) is the inflation rate at time \( t \). That is, they set their price equal to the average of the last period reset prices, \( \bar{p}_{t-1} \), after applying a correction for inflation. It can be shown that there are not persistent deviations of the rule of thumb from the optimal pricing behavior.

Finally, the model is closed by imposing the economy-wide resource constraint

\[
c_t = y_t,
\]

and the market clearing condition in the intermediate good sector

\[
y_t = n_t (1 - \rho_t) f(h_t),
\]

\(^{24}\)In the limiting case in which retailers are allowed to reset their price every period (\( \varphi = 0 \)), equation (39) reduces to the standard condition that the price is a constant markup over the nominal marginal cost.
where $y_t$ is aggregate demand, $n_t (1 - \rho_t)$ is the number of firms actually producing in $t$ and $f(h_t)$ is each firm’s production.

### 3.4 Monetary authority

The monetary authority conducts monetary policy using the short-term nominal interest rate as the policy instrument and lets the nominal amount of money adjusting accordingly. The gross nominal interest rate $r^n_t$ follows a Taylor-type rule of the following type:

$$r^n_t = (r^n_{t-1})^{\rho_m} E_t (p_{t+1}/p_t)^{\gamma_p (1-\rho_m)} y_t^{\gamma_y (1-\rho_m)} e^{\varepsilon_m}_t.$$  

(44)

The parameter $\rho_m$ measures the degree of interest rate smoothing and is included following the empirical evidence presented in Clarida, Galí and Gertler (2000). The parameters $\gamma_p$ and $\gamma_y$ are the response coefficients of inflation and output. Finally, $\varepsilon_m^t$ is an i.i.d. monetary policy shock.

### 4 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation of equations (6), (7), (8), (9), (10), (11), (12), (13), (14), (21), (25), (28), (29), (30), (31), (36), (37), (39), (40), (41), (42), (43), (44) around a deterministic steady state, with zero inflation. In what follows variables with a “hat” denote log-deviations from the steady state value, while variables without a time subscript denote steady state values.

**Taylor-type interest rate rule**

$$\hat{r}_t^n = \rho_m \hat{r}_{t-1}^n + (1 - \rho_m) \gamma_p \pi_t + (1 - \rho_m) \gamma_y y_t + \varepsilon_m^t$$  

(45)

**Euler equation**

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{r}_t$$  

(46)

**Marginal utility of consumption**

$$(1 - \beta e) \hat{\lambda}_t = \frac{e}{1 - e} \hat{\lambda}_{t-1} - \frac{1 + \beta e^2}{1 - e} \hat{\pi}_t + \frac{\beta e}{1 - e} E_t \hat{\pi}_{t+1}$$  

(47)

**Real interest rate**

$$\hat{r}_t = \hat{r}_t^n - E_t \pi_{t+1}$$  

(48)

**Hours per worker**
\[ \hat{x}_t = \phi \hat{h}_t - \hat{\lambda}_t \] (49)

**Phillips curve**

\[ \pi_t = \varphi_x \hat{x}_t + \varphi_f \pi_{t+1} + \varphi_b \pi_{t-1} \] (50)

where \( \varphi_x = \frac{(1-\beta \varphi)(1-\varphi)(1-\omega)}{\varphi_f} \), \( \varphi_f = \frac{\varphi}{\varphi_b} \), \( \varphi_b = \omega \) and \( \kappa = \varphi + \omega [1 - \varphi (1 - \beta)] \)

**Resource constraint**

\[ \hat{y}_t = \hat{c}_t \] (51)

**Market clearing**

\[ \hat{y}_t = \hat{h}_t + \hat{n}_t + \eta_{F,a} \hat{a}_t \] (52)

where \( \eta_{F,a} = \frac{\partial F(a)}{\partial a} \)

**Matching function**

\[ \hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t \] (53)

**Transition probabilities**

\[ \hat{q}_t = \hat{m}_t - \hat{v}_t \] (54)

\[ \hat{s}_t = \hat{m}_t - \hat{u}_t \] (55)

**Market tightness**

\[ \hat{\theta}_t = \hat{v}_t - \hat{u}_t \] (56)

**Employment**

\[ \hat{n}_t = (1 - \rho) \hat{n}_{t-1} + (1 - \rho) \eta_{F,a} \hat{a}_{t-1} + \rho \hat{m}_{t-1} \] (57)

**Searching workers**

\[ \hat{u}_t = -\frac{n}{\eta} (1 - \rho) \left( \hat{n}_t + \eta_{F,a} \hat{a}_t \right) \] (58)

**Vacancy posting condition**

\[ \hat{q}_t = -\nu_1 \left( \hat{x}_{t+1} + \hat{h}_{t+1} \right) + \beta (1 - \rho) \eta s \hat{t}_{t+1} + \beta (1 - \rho) \hat{g}_{t+1} - (1 + \nu_2) \hat{\lambda}_{t+1} + \hat{\lambda}_t \] (59)
where $\nu_1 = \frac{\phi}{1+\rho} x h \lambda \left( a - \frac{H(a)}{F(a)} \right)^{-1}$, $\nu_2 = \frac{H(a)}{F(a)} \left( a - \frac{H(a)}{F(a)} \right)^{-1}$ and $H(a) = f'_{ad} F'(a)$

Separation condition

$$\varsigma \left( \tilde{x}_t + \tilde{h}_t \right) - (1 - \eta) \frac{a}{\lambda} (\tilde{a}_t - \lambda_t) - \frac{\eta s k}{q} \tilde{q}_t - \frac{\kappa}{q} \tilde{q}_t = 0 \quad (60)$$

where $\varsigma = (1 - \eta) \frac{\phi}{1+\rho} x h$

Job creation rate

$$\hat{j}_c_t = \chi (\tilde{m}_t - \tilde{n}_t) + (1 - \chi) \tilde{q}_t \quad (61)$$

where $\chi = \frac{1}{1-aq}$ and $\alpha = \frac{\rho^e}{\rho}$

Job destruction rate

$$\hat{j}_d_t = -\chi \frac{1 - \rho}{\rho} \eta F, a \tilde{a}_t + (1 - \chi) \tilde{q}_t \quad (62)$$

The model presented in this paper nests a baseline new keynesian model with a frictionless and competitive labor market. The baseline model can be obtained by assuming that the rates of job creation and job destruction are constant at their steady state values. This implies that all labor market variables specific to the search and matching framework are also constant at their steady state values. The baseline sticky prices model, then, is described by equations (45), (46), (47), (48), (49), (50), (51) and (52), where in equation (52) $\tilde{n}_t$ and $\tilde{a}_t$ are both equal to zero.

This has the extremely convenient implication that the two models can be easily comparable. In particular, any difference in the dynamics of those variables that belong to both models must be associated with the dynamics of job creation and job destruction that, in turn, determine the dynamics of employment.

5  Bringing the model to the data

In this Section I describe the econometric methodology that I use to evaluate the model developed in Section 3. The model parameters can be divided in three groups. The first group is composed by the parameters that characterize the Taylor rule and is given by $\{\rho, \gamma \}$. The second group is given by the structural parameters that affect the dynamics of both the search model and the baseline new keynesian model. This group is given by $\{\beta, \phi, \kappa, \varepsilon, \varphi, \omega\}$. The third group includes the structural parameters that describe the labor market in the search model. This group does not affect the dynamics of the baseline model and is composed by $\{\rho, \alpha, \sigma, q, \eta, \mu, \sigma_{\alpha}\}$.\footnote{These variables are $n_t, u_t, m_t, s_t, \zeta_t, \eta, \mu, \sigma_{\alpha}$.}

\footnote{Recall that $\alpha = \frac{\rho^e}{\rho}$.}
First, I set the Taylor rule parameters as follows: the interest rate smoothing parameter $\rho_m$ is set to be equal to 0.85, and the parameters $\gamma_x$ and $\gamma_y$ to 1.5 and 0.5, respectively. These values are roughly consistent with the estimates presented in Clarida, Galí and Gertler (2000).

Second, I calibrate the parameters of the second group, with the exception of the habit persistence parameter $e$. Specifically, I set the quarterly discount factor $\beta$ to 0.99, which implies a quarterly real rate of interest of approximately 1 percent. In order to calibrate the parameter $\phi$, note first that $1/\phi$ is the intertemporal elasticity of substitution of leisure. The value of this elasticity has been a substantial source of controversy in the literature. Most microeconomic studies estimates this elasticity to be small, close to 0 and not higher than 0.5.\textsuperscript{27} Students of the business cycle, however, tend to work with elasticities that are much higher than microeconomic estimates, typically unity and above. In such a way they can approximate the absence of the extensive margin variation of the labor input. Since the model that I develop in this paper can account for both margins, I accordingly set $\phi$ equal to 10, which implies an elasticity of intertemporal substitution of 0.1. I then choose $\kappa_h$ so that the time spent working in the steady state, $h$, is equal to 1/3. Then, I set the probability that a firm does not change its price within a given period, $\varphi$, equal to 0.85, implying that the average period between price adjustments is around 6.5 quarters. The fraction $\omega$ of backward-looking retailers is set to 0.5. Both values are consistent with the estimates in Galí and Gertler (1999).\textsuperscript{28} Finally, I assume that the markup of prices on marginal costs is on average 10 percent. This amounts to setting $\varepsilon$ equal to 11.

Third, I estimate most of the structural parameters that characterize the labor market in the search model. Moreover, since the habit persistence parameter is important to explain the dynamics of the labor market, I include it in the group of parameters to be estimated. The following two sections describe the estimation procedure and results.

### 5.1 Minimum distance estimation

I follow the estimation strategy adopted in Rotemberg and Woodford (1999), Gilchrist and Williams (2000), Christiano, Eichenbaum and Evans (2001), Amato and Laubach (2003) and Boivin and Giannoni (2003). This strategy can be seen as an application of the minimum distance estimation. Specifically, the structural parameters are chosen so that the impulse responses to the monetary shock of the endogenous variables in the model match as closely as possible the responses estimated from the VAR.

More formally, denote with $\Psi$ the vector of structural labor-market parameters to be estimated

\textsuperscript{27}For a survey of the literature see Card (1994).

\textsuperscript{28}It is important to point out that it is not necessary to rely on such high values of the parameters $\varphi$ and $\omega$ to explain inflation dynamics in the data. In particular, I could allow for heterogeneous labor services as in Rotemberg and Woodford (1999) and Boivin and Giannoni (2003) and, everything else equal, significantly reduce the value of both parameters. However, for clarity of presentation and analogously to Galí and Gertler (1999), I do not include in the model this additional feature. Moreover, as I discuss below, the important result is that for given values of $\varphi$ and $\omega$, whichever values I assume, the response of inflation is much smaller than in the baseline new keynesian model.
and with \( g_M(\Psi) \) the vector-valued function containing the model-based impulse response functions. Then, denote with \( \Phi \) the vector of the estimated VAR coefficients and with \( g_V(\Phi) \) the vector-valued function containing the VAR-based impulse response functions. The minimum distance estimator, \( \hat{\Psi} \), can be obtained by minimizing the objective function

\[
L(\Psi) = [g_M(\Psi) - g_V(\Phi)]' \Lambda [g_M(\Psi) - g_V(\Phi)],
\]

with respect to \( \Psi \) and subject to the theoretical constraints on the values of the parameters. In the objective function, \( \Lambda \) denotes a diagonal weighting matrix with the inverse of each impulse response’s variance along the diagonal. The choice of this weighting matrix effectively takes into account that some of the points estimates of the impulse responses are less accurate than others. Finally, I consider in the estimation the impulse responses of the variables \( r_t, y_t, \pi_t, n_t, h_t, jct \) and \( jd_t \) over the first twenty periods after the monetary policy shock.

As Dridi, Guay and Renault (2003) and Boivin and Giannoni (2003) point out, although this estimation strategy is similar in the spirit to a calibration exercise, it produces consistent estimates of the structural parameters on which it is possible to perform statistical inference. These authors also argue that, since the structural model cannot explain all features of the data, it should be estimated only on the basis of some well-chosen moments of the data, which are consistent with the main purpose of the model. Given that the main goal of this study is to explain the response of the economy to a monetary policy shock, the estimation based on the impulse responses permits to focus on the moments of the data that the model seeks to explain.

Among the labor market parameters, three of them can be easily calibrated from the data. In particular, the empirical literature provides us with several measures of the US worker separation rate. Davis, Haltiwanger and Schuh (1996) compute a quarterly worker separation rate of about 8 percent, while Hall (1995) reports this rate to be between 8 and 10 percent. Accordingly, I set the overall separation rate \( \rho \) to 0.08. In order to calibrate \( \sigma \), I follow den Haan, Ramey and Watson (2000). First, as previously discussed, they assume that only exogenous separations are reposted. Then, based on evidence reported by Davis, Haltiwanger and Schuh, they calculate that the rate at which separations are reposted by firms is equal to 0.68. This implies that \( \alpha = 0.68 \) and \( \rho^x = 0.054 \). Then, I set the steady state probability that a firm fills a vacancy, \( q \), to be equal to 0.7, as in Cooley and Quadrini (1999) and den Haan, Ramey and Watson (2000). This value imply that the average time a vacancy is filled is 1.4 quarters. The vector of parameters to be estimated, then, is given by \( \Psi = [\sigma, \eta, n, e, \mu_a, \sigma_a] \). Finally, it may seem reasonable to calibrate from the data also \( n \), the steady state employment rate. Below I discuss why I choose to estimate this steady state value.

5.2 Estimation results

The estimates of the parameters \( \sigma, \eta, n, e, \mu_a \) and \( \sigma_a \) are reported in Table 1, along with the corresponding standard errors. I perform the estimation in three stages. In the first stage I estimate all six parameters. The results are reported in the second column of Table 1. The elasticity of new
matches with respect to the number of searching workers, \( \sigma \), is estimated to be 0.56. This value is higher but not too far from the estimate of 0.4 obtained by Blanchard and Diamond (1989) and it is consistent with the evidence summarized by Petrongolo and Pissarides (2001).

The estimate of the habit persistence parameter, \( e \), is 0.55. This is close to the estimate of 0.63 reported in Christiano, Eichenbaum and Evans (2001). As previously mentioned, besides helping the model to reproduce the hump-shaped responses of output and consumption, the presence of habit formation in preferences also enhances the ability of the model to account for the joint response of the extensive and intensive margins of variation of the labor input. Without habit persistence, in particular, the initial response of hours per worker would be significantly higher, although still as transitory as in the data.

Table 1: Estimates of structural labor-market parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates I</th>
<th>Estimates II</th>
<th>Estimates III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>0.558</td>
<td>0.558</td>
<td>0.545</td>
</tr>
<tr>
<td>(0.0843)</td>
<td>(0.0448)</td>
<td>(0.0311)</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.1</td>
<td>0.102</td>
<td>0.5</td>
</tr>
<tr>
<td>(1.0098)</td>
<td>(0.8568)</td>
<td>(--)</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>0.753</td>
<td>0.753</td>
<td>0.747</td>
</tr>
<tr>
<td>(0.0396)</td>
<td>(0.0107)</td>
<td>(0.0112)</td>
<td></td>
</tr>
<tr>
<td>( e )</td>
<td>0.549</td>
<td>0.549</td>
<td>0.55</td>
</tr>
<tr>
<td>(0.0051)</td>
<td>(0.0051)</td>
<td>(0.0062)</td>
<td></td>
</tr>
<tr>
<td>( \mu_a )</td>
<td>2.86e-009</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(0.4723)</td>
<td>(--)</td>
<td>(--)</td>
<td></td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>0.410</td>
<td>0.410</td>
<td>0.382</td>
</tr>
<tr>
<td>(0.1170)</td>
<td>(0.0769)</td>
<td>(0.0095)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are in parenthesis. (--) denotes that the standard error is not available because the parameter is calibrated.

The reason why I choose to estimate the steady state employment ratio \( n \) is that on one hand it may have considerable effects on the dynamics of the labor market, on the other there is no unambiguous way to calibrate it from the data. More precisely, as an example, Andolfatto (1996) sets the employment rate \( n \) to 0.54, while den Haan, Ramey and Watson (2000) set it to 0.89. These values, which are obviously larger than in the data, can be justified by interpreting the unmatched workers in the model as being both unemployed and partly out of the labor force. This interpretation is consistent with the abstraction in the model from labor force participation decisions. Another way to rationalize a lower value for \( n \) is the following. It is assumed in order to capture labor force participation changes. When the steady state fraction of searchers is low, the
model implies that a small percentage decrease in the number of employed workers causes a large percentage increase in the numbers of workers looking for a job. In reality, however, a lower probability of finding a job reduces the labor force participation. In that case, a decrease in the number of employed people does not necessarily translates in a one-to-one increase in the number of people searching for a job. As a result, the probability of filling a vacancy may increase by a lower amount. A possible way to take this labor force participation effect into account is to assume a higher steady state value for the fraction of searching workers. The estimate of \( n \) that I obtain is 0.75. This estimate lies between the value used by Andolfatto (1996) and that used by den Haan, Ramey and Watson (2000).

The relative bargaining power, \( \eta \), is estimated to be 0.1. However, this parameter is not very precisely estimated. This may suggest that \( \eta \) does not have a large effect on the dynamics of the model. I return on this point below. Finally, the estimate of the parameter \( \mu_a \) of the lognormal is driven to 0 and the estimate of the parameter \( \sigma_a \) is 0.41.\(^{29}\) These values, in turn, determine the steady state value of the threshold, \( a \), and the elasticity of the survival rate to changes in the threshold, \( \eta_{F,a} \), from the steady state relationships. The implied values for \( a \) and \( \eta_{F,a} \) are, respectively, 2.2 and 0.17.

In the second stage of the estimation, I set the value of \( \mu_a \) to 0 and estimate \( \sigma, \eta, n, e \) and \( \sigma_a \). The results are reported in the third column of Table 1. As can be seen from the table, the new estimates are the same as the estimates in the first stage, only the standard errors are lower. However, the bargaining power \( \eta \) remains imprecisely estimated. For this reason, in the third stage I simply set \( \eta \) to 0.5 - a value that assigns equal bargaining power to the worker and the firm - and estimate \( \sigma, n, e \) and \( \sigma_a \). The fourth column of Table 1 reports the estimation results and shows that the estimates of all parameters are almost unaffected by setting \( \eta \) to 0.5. This confirms the above suggestion that the bargaining power has a negligible impact on the dynamic behavior of the model. The reason why it is so is that with Nash bargaining the real wage plays only a distributive role. In other words, although the dynamic behavior of the real wage is affected by \( \eta \), the model dynamics are not significantly affected by the behavior of the real wage.\(^{30}\)

Finally, given the above estimates, the steady state probability that a worker finds a job, \( s \), is calculated from the steady state relationships to be 0.2. This value imply that the average time a worker finds a job is 5 quarters. The parameters \( \kappa \) and \( b \) are also derived from the steady state calculation and are equal to 0.4 and 0.03, respectively.

\(^{29}\)The values of the mean and the variance of \( a_t \) can then be calculated to be 1.1 and 0.2, respectively.

\(^{30}\)In Trigari (2003), within a similar model to the one developed here, I study an alternative bargaining model to the Nash bargaining and show that in this case the wage is allocative. As a consequence, the bargaining power \( \eta \) becomes important to explain the model dynamics.
6 Findings

First, I compare the predictions of the model developed in this paper - which I will refer to, for simplicity, as the search model - with those of the baseline new keynesian model.

Figure 3 shows the response of several variables to a monetary shock. The monetary shock is a one percent increase in the nominal interest rate. For each variable I plot the response in the search model and the baseline model. As can be seen from the figure, output, inflation, marginal costs and hours have a similar qualitative response in the two models. Note that, for comparison reasons, in the search model I plot hours per worker rather than total hours. In both models, a raise in the nominal interest rate causes an increase in the real interest rate because there are price rigidities. As a consequence of the raise in the real interest rate, aggregate demand, output of final goods and hours worked decrease. The fall in output and hours can only occur at decreased marginal costs. Finally, because prices are set based on expected future marginal costs, inflation decreases. Therefore, the two models are observationally equivalent. That is, the introduction of search frictions does not change the nature of the baseline model dynamics.

Figure 3: Search versus new keynesian model

Note that, although the equations in the model involve a quarterly inflation rate, for clarity reasons I plot the annualized inflation rate.
From a quantitative point of view, however, the search and the baseline model behave extremely differently. In the search model the response of inflation is significantly less volatile. The response of output is larger and more persistent. This happens because the search model implies a substantially lower elasticity of marginal costs with respect to output. The figure shows that a given fall in output is associated with a much lower decrease in the level of marginal costs than in the baseline model. In turn, smaller variations in marginal costs induce firms setting their prices to make smaller adjustments in prices. This increases the sluggishness of the aggregate price level to changes in aggregate demand and reduces the volatility of inflation. In particular, while in the baseline model a peak decrease in output of about 0.23 percent is associated with a peak fall in inflation of around 0.63 percent, in the search model output falls by about 0.33 percent and inflation by only 0.27 percent. Finally, the lower sensitivity of the price level to variations in aggregate demand raises the persistence of the response of aggregate demand and output to a monetary shock. In the baseline model output goes back to its steady state value after 9 quarters, while in the search model it takes around 18 quarters.

![Figure 4: Extensive and intensive margin](image)

The elasticity of marginal costs with respect to output is lower in the search model for two
reasons. First, changes in the labor input at the extensive margin allow for adjustments in output without changed marginal costs. To see this, write the log-linearized real marginal cost as
\[ b_x^t = \phi b_h^t - b_\lambda^t, \]
from equation (49). This implies that changes at the intensive margin cause changes in marginal costs according to the parameter \( \phi \), while changes at the extensive margin do not affect marginal costs.\(^{32}\) This happens because variations in hours per worker involve changes in the disutility cost from supplying labor, while changes in employment only represent changes in the economy’s capacity level. In the baseline model, instead, all changes in the labor input occur at the intensive margin and affect marginal cost as above, proportionally to \( \phi \). Now note that final output is given by \( \hat{y}_t = \hat{h}_t \) in the baseline model and \( \hat{y}_t = \hat{h}_t + \hat{n}_a^t \) in the search model, with \( \hat{n}_a^t \) denoting active employment in \( t \).\(^{33}\) Substituting, then, hours for final output in the expression for marginal cost gives \( \hat{x}_t = \phi \hat{y}_t - \hat{\lambda}_t \) and \( \hat{x}_t = \phi (\hat{y}_t - \hat{n}_a^t) - \hat{\lambda}_t \), respectively. These expressions imply that a given change in output causes a lower change in marginal cost in the search model. Marginal costs are lower by exactly the change in active employment, weighted by \( \phi \). Secondly, the elasticity of marginal cost with respect to output in the search model will be lower the larger is the share of the fluctuation in total hours that takes the form of fluctuations in the number of people working rather than changes in the hours by employed workers. Figure 4 plots the responses of total hours, active employment and hours per worker in the search model. The percent change in total hours is the sum of percent changes in employment and hours per worker. The figure shows that the decrease in the number of people working is significantly larger and more persistent than the fall in the hours per worker. Initially, the fall in the demand for intermediate goods reduces its relative price and reduces hours per worker. At the same time, the lower profitability of firms induces less firms to post vacancies and more firms to separate from their workers. As the number of intermediate goods firms producing gradually decreases, the demand of intermediate goods per firm gradually increases. As a consequence, the responses of output per firm and hours of work in the intermediate goods sector are reverted fairly quickly.

It must be emphasized that I have assumed a degree of intertemporal substitution in the supply of hours that is consistent with microeconomic estimates. Instead, general equilibrium models of the business cycle, among which sticky prices models, tend to assume much higher values of this elasticity, typically unit and above. By doing so, they can approximate some implications of the model with both margins of adjustment. Of course, such model cannot explain what drives fluctuations in employment as opposed to hours per worker, why there is unemployment in equilibrium or, more generally, the behavior of the labor market over the business cycle.

Figure 5 presents the dynamics of the labor market in the search model after a monetary policy shock. The response of employment is explained by the dynamics of job creation and job

\(^{32}\)Of course, changes at both margins have a second-order general equilibrium effect on real marginal costs \( \hat{x}_t \) through \( \hat{\lambda}_t \).

\(^{33}\)Active employment, \( n_a^t \), is the number of employed people actually working in period \( t \) and it is different from \( n_t \), the number of employed people at the beginning of period \( t \). In particular, \( n_a^t \) is given by: \( n_a^t = (1 - \rho^t) n_t = (1 - \rho^t) F(a^t) n_t \). Log-linearizing, we obtain: \( \hat{n}_a^t = \hat{n}_t + \eta F(a^t) \hat{a} \).
destruction. Since the labor force is assumed to be constant, the response of unemployment is just
the mirror of the response of employment. Recall, from equation (32), that employment growth is
given by $\frac{n_{t+1} - n_t}{n_t} = j_c_t - j_d_t$. Thus, employment falls if job creation is lower than job destruction.
As can be seen from the figure, a contractionary monetary shock decreases job creation and raises
job destruction. The raise in job destruction is slightly greater and significantly more persistent
than the decrease in job creation. Thus, most of the decrease in employment is due to the response
of job destruction, rather than job creation. In particular, while the reduction in job destruction
persists for nine periods, job creation raises above the steady state in the fourth period and above
the job destruction rate in the fifth period. This implies that from the sixth period on employment
begins to raise and unemployment to decline.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Labor-market dynamics}
\end{figure}

The responses of job creation and destruction, in turn, can be explained as follows. A persistent
raise in the nominal interest rate causes a decrease in current and expected future aggregate demand.
The fall in aggregate demand, in turn, decreases the demand for intermediate goods and the profits
of firms producing them. This diminishes the value of the idiosyncratic shock above which the
firm and the worker decide to separate and raises the separation rate. Because of the timing assumption,
the monetary shock only affects the threshold value of the idiosyncratic shock and the separation
rate after two periods. The decrease in profits also reduces the value of opening a vacancy and induces firms to post less vacancies. The decrease in the number of posted vacancies diminishes both the number of new matches and the job creation rate. Again, the number of vacancies and the job creation rate respond to the monetary shock with a two-period delay.

The decrease in the number of posted vacancies and the increase in the number of searching workers cause the labor market tightness to decrease. Thus, the probability of filling a vacancy raises while the probability of finding a job drops. The higher probability of hiring a worker increases the attractiveness of hiring activities and the expected future value of a match. Therefore, job creation starts to increase and job destruction to fall.

Figure 6: Estimated versus model responses

Figure 6 plots the model impulse responses of output, inflation and the nominal interest rate to the monetary shock against the estimated impulse responses in the US economy. Figure 7 plots the model responses of employment, hours per worker, the job creation rate and the job destruction rate against the estimated responses in the US economy. The solid and dashed lines denote, respectively, the estimated impulse responses and the two standard deviations confidence intervals, while the lines with circles denote the simulated responses in the model.\footnote{As Figure 6 and 7 show, the model}

\footnote{Again, even if the equations in the model involve quarterly inflation and nominal interest rates, for clarity and}
does a good job in accounting for the dynamic response of the US economy to a monetary policy shock.

The first dimension in which the model can reproduce the data is the joint dynamic behavior of output and inflation. Basically, the simulated responses of output and inflation are everywhere within the respective confidence intervals. However, while the model generates significantly more persistence in output than the baseline new-Keynesian model, Figure 6 suggests that output is not yet as persistent as in the data. Second, the model is able to reproduce the quantitative behavior of the variation of the labor input at both margins of adjustment. It generates a small, transitory fall in hours per worker together with a larger, more persistent fall in employment. Likewise the response of output, however, the response of employment is less persistent than in the data. Third, the model explains the joint behavior of job creation and job destruction. In particular, it can account for the larger response of job destruction than job creation and for the observed upturn in job creation. This upturn occurs because the larger pool of unemployed workers looking for a

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Footnote:

35 Note that in Figure 7, differently from Figure 5, I plot the model response of active employment. Conceptually, this is the right measure of employment to be compared with the data.
job stimulates firms to post new vacancies. The model can also account for the higher degree of persistence in job destruction with respect to job creation that is observed in the data. Note, finally, that the simulated impulse responses of all four labor market variables are everywhere within the respective confidence intervals.

7 Conclusions

This paper builds on the new keynesian theory of money and inflation and the modern theory of equilibrium unemployment. Both theories have been introduced previously in the macroeconomic literature and extensively used for both normative and positive analysis. But the combination of these theories into a single dynamic general equilibrium model provides new insights on the linkages between money, business cycle fluctuations and the dynamics of the labor market.

There are three basic findings. The first concern the estimation results. I obtain consistent estimates of a set of structural parameters that characterize the labor market, on which there is few or no independent evidence. When previous estimates are available, the estimates that I obtain are consistent with the previous ones. The second finding concerns the cyclical behavior of the labor market when money is the driving force behind aggregate fluctuations. The paper shows that the demand-side channel of monetary transmission seems to be a good candidate to explain the fluctuations in employment and job flows over the business cycle. The third finding concerns the role of labor market dynamics in shaping the joint dynamics of output and inflation. These variables are the focus of the recent literature that analyzes the effects of monetary policy shocks in the presence of nominal price rigidities. The results indicate that, when labor market search is incorporated into a standard new keynesian model, the ability of the model to explain the response of output and inflation improves along a number of dimensions. In general, the estimated model does a very good job in accounting quantitatively for the response of the US economy to a monetary shock.

The ultimate objective of developing quantitative monetary general equilibrium models of the business cycle is to design an optimal, or at least desirable, monetary policy. The model developed in this paper could then be used to perform a welfare analysis of the consequences of alternative monetary policies. In particular, the model provides the basis for thinking about the implications of different labor market policy regimes for the optimal monetary policy. I plan to explore these issues in future research.
8 Appendix

Derivation of the surplus from employment for a worker

This section of the Appendix shows how the surplus from employment for a worker - the difference between the employment and unemployment values - can be obtained from the family’s problem. In this way, it is possible to rationalize the existence of bargaining between workers and firms when workers are perfectly insured against the risk of being unemployed, as it is assumed in the paper. The argument is based on the assumption that workers value their actions in terms of the contribution these actions give to the utility of the family to which they belong. This implies that the surplus from employment for a worker can be defined as the change in the family’s utility from having one additional member employed.

Suppose that there is a continuum of identical families indexed on the unit interval. Each of these families has a continuum of members indexed by \( i \in [0, 1] \). A fraction \( n^a_t \) of these members is employed, while the remaining fraction \( 1 - n^a_t \) is unemployed. Recall that \( n^a_t \) denotes the number of individuals that are actually working in period \( t \). This is different from \( n_t \), the number of individuals that are employed at the beginning of period \( t \), previously to the realization of the idiosyncratic shock. The representative family’s optimal value function, denoted with \( \Omega_t \), can be written as:

\[
\Omega_t (n^a_t) = u(c_t, c_{t-1}) - \int_0^{n^a_t} g(h_t, a_{it}) \, di + \beta E_t \left[ \Omega_{t+1} (n^a_{t+1}) \mid a_{it+1} \leq a_{it+1} \right] \quad (63)
\]

Note that the family’s disutility from having a fraction \( n^a_t \) of its members supplying hours of work, previously denoted with \( G_t \), is made explicit in (63) and is equal to \( \int_0^{n^a_t} g(h_t, a_{it}) \, di \). The symbol \( a_{it} \) denotes the idiosyncratic shocks to the individual \( i \)’s disutility from working.

Each family faces the following budget constraint:

\[
c_t + \frac{B_t}{p_t r_t} = \int_0^{n^a_t} w_t(a_{it}) \, h_t \, di + (1 - n^a_t) b + \delta_t + \frac{B_{t-1}}{p_t} \quad (64)
\]

where the per capita family’s income, previously denoted with \( d_t \), is the sum of the first three terms on the right-hand side of the budget constraint. More precisely, the family obtains income from having a fraction \( n^a_t \) of its members working at the hourly wage \( w_t(a_{it}) \) and a fraction \( 1 - n^a_t \) producing at home a non-tradable output \( b \) of final goods. Finally, \( \delta_t \) denotes the family’s per capita share of aggregate profits from retailers and intermediate goods firms, net of the vacancy posting costs.

The fraction of employed members evolves accordingly to the following dynamic equation:

\[
n^a_{t+1} = (1 - \rho_{t+1}) n^a_t + s_t (1 - \rho_{t+1}) (1 - n^a_t) \quad (65)
\]

where the representative family takes as given the probability \( s_t \) at which the search activity by the unemployed members leads to a job match.
Denote now with $\tilde{S}_t^W(a_{it})$ the surplus from employment for a worker. As previously said, this is defined as the change in the family’s optimal utility from having an additional member employed, that is,

$$\tilde{S}_t^W(a_{it}) \equiv \frac{\partial U_t(n_t^a)}{\partial n_t^a} \tag{66}$$

Taking the derivative of $\Omega_t$ in (63) with respect to $n_t^a$ subject to equations (64) and (65) gives:

$$\frac{\partial \Omega_t(n_t^a)}{\partial n_t^a} = \lambda_t w_t(a_{it}) h_t - \lambda_t b - g(h_t, a_{it}) + \beta E_t \left[ (1 - s_t) (1 - \rho_{t+1}) \frac{\partial \Omega_{t+1}(n_{t+1}^a)}{\partial n_{t+1}^a} \right] \eta_{t+1}^a \leq \eta_{t+1}$$ \tag{67}

The surplus from employment, then, is given by the following expression:

$$\tilde{S}_t^W(a_{it}) = \lambda_t w_t(a_{it}) h_t - \lambda_t b - g(h_t, a_{it}) + \beta E_t \left[ (1 - s_t) (1 - \rho_{t+1}) \int_0^{\tilde{S}_{t+1}^W(a_{t+1})} \frac{dF(a_{t+1})}{F(a_{t+1})} \right] \tag{68}$$

where the index $i$ is omitted for simplicity.

Finally, denote with $S_t^W(a_{it})$ the value of the surplus from employment in terms of current consumption of final goods, i.e.,

$$S_t^W(a_{it}) \equiv \frac{\tilde{S}_t^W(a_{it})}{\lambda_t} \tag{69}$$

After substituting into the above identity the expression for $\tilde{S}_t^W(a_{it})$ and rearranging, the value of the surplus in terms of current consumption can be written as:

$$S_t^W(a_{it}) = w_t(a_{it}) h_t - b - \frac{g(h_t, a_{it})}{\lambda_t} + E_t \beta_{t+1} \left[ (1 - s_t) (1 - \rho_{t+1}) \int_0^{\tilde{S}_{t+1}^W(a_{t+1})} \frac{dF(a_{t+1})}{F(a_{t+1})} \right] \tag{70}$$

This equation corresponds to the difference between the value of employment (17) and the value of unemployment (18) that are reported in the paper.

### Identifying monetary policy shocks

In this section of the Appendix I briefly describe the identification strategy of the monetary policy shock. Following Christiano et al. (2000), and others, I assume that the central bank conducts its monetary policy following a simple reaction function. More precisely, in each period $t$, the policymaker sets its instrument - the short-term nominal rate $r_t^n$ - in a systematic way using a simple rule that exploits the available information at time $t$, $I_t$. The monetary policy rule can be written as:

$$r_t^n = F(I_t) + \varepsilon_t^m,$$ \tag{71}
where \( f \) is a linear function and \( \varepsilon_t^m \) is the monetary policy shock. The identification scheme is based on the recursiveness assumption, according to which monetary policy shocks are orthogonal to the information set of the monetary authority, \( I_t \).

Let \( y_t \) denote the \((n \times 1)\) vector of the variables included in the analysis, i.e., the instrument and the variables in the information set of the monetary authority. The vector \( y_t \) is partitioned so that the monetary policy instrument is ordered last, in the \( n^{th} \) position. Then, the dynamic behavior of \( y_t \) is assumed to be represented by the following VAR of order \( p \):

\[
y_t = c + A_1 y_{t-1} + \ldots + A_p y_{t-p} + B \varepsilon_t,
\]

where \( c \) is a \((n \times 1)\) vector of constants, the \( A_i \)'s are \((n \times n)\) matrices of coefficients, \( B \) is a \((n \times n)\) lower triangular matrix with unit diagonal elements and \( \varepsilon_t \) is a \((n \times 1)\) vector of mutually and serially uncorrelated structural shocks with zero mean and constant variance. The \( n^{th} \) element of \( \varepsilon_t \) is the monetary policy shock, \( \varepsilon_t^m \). The lower-triangularity of \( B \) implies that all variables in the information set are assumed to be predetermined with respect to the monetary policy shock.

Equivalently, we can write:

\[
A(L) y_t = c + B \varepsilon_t,
\]

where \( A(L) = [I_n - A_1 L - \ldots - A_p L^p] \) and \( L \) is the lag operator. Using OLS, we can estimate the coefficient matrices \( A(L) \), \( c \), \( B \) and the variance-covariance matrix of \( \varepsilon_t \).

Given these estimates, the impulse responses functions to a monetary shock of the variables belonging to \( y_t \) can be obtained from the infinite Moving Average (MA) representation of the structural VAR. This is given by:

\[
y_t - y = H(L) \varepsilon_t,
\]

where \( y = [A(L)]^{-1} c \) is the unconditional mean of \( y_t \) and \( H(L) = [A(L)]^{-1} B \) embeds the impulse response coefficients.

Equivalently, we have:

\[
\hat{y}_t = \varepsilon_t + H_1 \varepsilon_{t-1} + H_2 \varepsilon_{t-2} + \ldots + H_s \varepsilon_{t-s} + \ldots,
\]

where \( H(L) = [I_n + H_1 L + \ldots + H_p L^p + \ldots] \) and \( \hat{y}_t = y_t - y \) is the deviation of \( y_t \) from its unconditional mean. In particular, a plot of the \((i,n)^{th}\) element of \( H_s \) as a function of \( s \) is the estimated impulse response function of \( \hat{y}_t \) to a monetary shock, for any variable \( i \) in \( y_t \).\(^{36}\) This dynamic path is invariant to the ordering of the variables contained in \( I_t \).

\(^{36}\) In practice, the sum in (75) is truncated at a large but finite lag.
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