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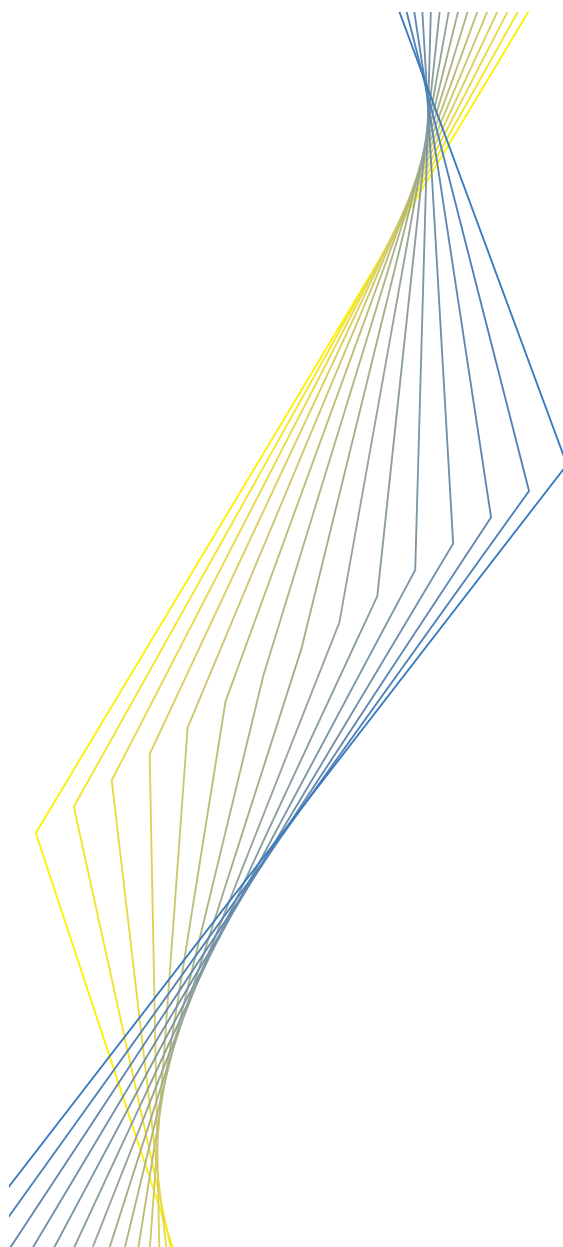
**MACROECONOMIC MODELLING  
OF MONETARY POLICY**

**BY MATT KLAEFFLING**

**SEPTEMBER 2003**

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**MACROECONOMIC MODELLING  
OF MONETARY POLICY<sup>1</sup>**

**BY MATT KLAEFFLING<sup>2</sup>**

**SEPTEMBER 2003**

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## Abstract

This paper proposes a new paradigm for the analysis of monetary policy. From an econometric point of view this new approach is just as easy to implement as reduced form analysis, but is robust to the Lucas critique. It requires no explicit prior theory and yet it encompasses all standard DSGE models.

After introducing this new paradigm I study US monetary policy and look at the nature and the effect of monetary policy, discuss the transmission mechanism and the policy rule implied by the data, and perform counterfactual policy analysis.

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## Non-technical Summary

Ever since the rational expectations revolution transformed theoretical macroeconomics in the late 1970s empirical macroeconomics had to face the following problem: How can one estimate relationships between macroeconomic variables such as inflation and growth and at the same time take account for the fact that both today's spending as well as price decisions depend on the likely future evolution of the economy, which as of today is not known? In other words, if economic actions depend on expectations and these expectations are not observable, how can we incorporate them into empirical models? Further, since much of empirical macroeconomics focuses on the implications of alternative economic policies for growth and inflation, how can one incorporate the Lucas critique, i.e. the notion that expectations about the future and hence today's actions are functions of the policy regime? And how can we take into account the fact that future policies might differ from those observed in the past?

This paper proposes a new technique to estimate the effect of changes in economic policy, and monetary policy in particular, on the macroeconomy. What is novel about this technique is that it estimates a forward-looking representation from the data and then calculates the implied rational expectations that correspond to this representation. These techniques are straightforward generalizations of techniques that are often used in theoretical macroeconomics, but that have not been applied in the empirical macroeconomics literature until now.

The paradigm of this paper defines a conditional model for the macroeconomy as a (forward-looking) function of super-exogenous<sup>1</sup> policy. It then adds a "marginal" model for policy. As a first application we estimate both components from past data. We then calculate the rational expectations solution for this economy and analyze the effects of an unexpected monetary policy easing. Further, since policy is super-exogenous, we can also calculate the responses to any given hypothetical alternative policy rule given the rational expectations solution that corresponds to this particular alternative policy.

The primary contribution of this paper lies in presenting a novel approach for empirical macroeconomics. It is shown how we can combine the insights of the Lucas critique with rational expectations solution techniques to analyze the effects of policy shocks for the alternative cases of either estimated or counterfactual policy rules. As a side-effect we obtain an identification technique that solves a problem familiar from the VAR literature on shock identification. Finally we relate the techniques to optimal policy and asset pricing.

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<sup>1</sup>The concept of super-exogeneity is reviewed in detail in the paper.

# 1 Introduction

While the question of the effect of monetary policy on the macroeconomy is probably the most studied single question in macroeconomics, the answers to this question still widely diverge, depending on the modeling paradigm used in the analysis. This paper argues that the two standard paradigms - structural and reduced form analysis - are intrinsically ill-suited for empirical policy analysis. We propose a hybrid approach that avoids some of the difficulties of both.

The first approach is structural modeling in the RBC/DSGE tradition. This approach focuses on modeling the macroeconomy from microeconomic first principles. The problem here is that the assumptions about the structure of the economy that have to be made to render this approach feasible, are heroic - at best. Absent general agreement as to how relevant the first principles of neo-classical microeconomics are for a world characterized by incomplete markets, asymmetric information and heterogeneous agents, one may ask what informative value micro-structural models have for empirical macroeconomics. The most mechanical alternative is to throw all prior theory overboard and to focus on simple regression-based correlations between key variables. This is done in the VAR literature. Unfortunately, reduced-form models, such as VARs are subject to the Lucas critique and their usefulness for policy analysis is limited.<sup>1</sup>

This paper proposes a novel approach: to do data-driven general equilibrium policy analysis that is robust to the Lucas critique, not model-based, and consistently estimates a reduced-form representation of a dynamic, forward-looking, stochastic general equilibrium economy under the simple assumption that the data is generated by a model of this class and that the equilibrium is stable. The model is trivial to estimate from an econometric point of view, in that it is based on 2 sets of regressions that can be run by instrumental variable techniques. While the details of this new 'semi-structural' paradigm are outlaid in detail in section 3 I can sum up its main features in that I can analyze the effect of monetary policy by putting together two elements:

1. A conditional macroeconomic model, i.e. a law of motion for the state of the economy given a policy rule, and
2. A monetary policy rule, i.e. an equation that relates the monetary authority's control variable, the federal funds rate, to the state of the economy

Under the assumptions, standard in DSGE analysis, that the policy rule is taken as given by all private agents, the resulting model is robust to the Lucas critique as well as to the generic criticism of imposing 'ad hoc' economic structure and can be used for empirical and counterfactual policy analysis.

The rest of this paper is organized as follows. Section 2 briefly looks at the two competing paradigms of the macroeconomic literature - structural and

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<sup>1</sup>The obvious exception here is when the economy is backward-looking as is the case in the traditional Keynesian models of the Cowles Commission.



reduced-form analysis - and proposes the novel hybrid method, the so-called semi-structural model paradigm. Section 3 lays out the complete model for the macroeconomy and shows how to map very basic low-level structural assumptions into a full model for the economy. Section 4 reports empirical results for the US economy. Section 5 discusses how this new paradigm can be related to a number of other issues in monetary and financial economics. Section 6 concludes.

## 2 Empirical Macroeconomics - How much structure do we need?

Dynamic stochastic general equilibrium models (DSGE) start by formulating control problems for all the agents in the economy: consumers, producers, the government,... . These control problems take the form of maximizing some objective function subject to exogenous laws of motion for the state variables and given informational, technological and behavioral constraints. The resulting control choices, along with the laws of motion for the states are then aggregated to economic variables that map into the theoretical analogs of empirically observable variables. Forward-looking nonlinear structural models can be solved by focusing on the first-order necessary conditions, linearizing around the steady state and applying system-reduction techniques to obtain a backward-looking reduced form.<sup>2</sup> Denote the vector of observables by  $Y_t$ , the vector of (augmented) states by  $X_t$  and the vector of structural shocks by  $u_t$ . The resulting system can then be written in reduced-form as a state-space system as

state:

$$X_t = A(\theta)X_{t-1} + B(\theta)u_t \quad (1)$$

observation:

$$Y_t = C(\theta)X_t \quad (2)$$

where  $\theta$  is a vector of structural parameters of economic meaning and  $A(\theta)$ ,  $B(\theta)$  and  $C(\theta)$  etc. nonlinearly map the structural parameters,  $\theta$ , into reduced-form parameters. The system (1)-(2) can then be used to estimate the vector  $\theta$ . Note that unrestricted estimation of the same system would violate the cross-equational restrictions imposed by theory.<sup>3</sup>

The problem with such models is that while they provide an interesting benchmark they are almost certainly misspecified along a number of important dimensions. They usually rely on the existence of an aggregate production function, a representative consumer, incredible cognitive and computational abilities on the part of all agents, dubious ways of introducing money, etc.. In particular, one standard critique of structural models is that they show monetary shocks to not have noteworthy real effects unless strong forms of price and/or

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<sup>2</sup>See McGrattan (1994) and McGrattan, Rogerson and Wright (1996) for examples of log-linearized models. For linear-quadratic models see the book by Hansen and Sargent (1997)

<sup>3</sup>See Geweke (1999), Schorfheide (2001) and Fernandez-Villaverde and Rubio (2001) for Bayesian approaches to estimating potentially misspecified structural models.

wage stickiness are assumed (see Yun (1999), Kim (1998) and Chari, Kehoe and McGrattan (1998, 2000)). This observation has resulted in two contrasting conclusions. Proponents of structural models stress the finding that there does not seem to be *any* (standard) structural model that is capable of producing important effects of money on output to indicate that money *does not* have such an effect. This point of view is based on the a priori that, in principle, there is some structural, micro-founded model that can capture the essential features of the real world economy. It is then merely a question of perturbing existing models in new dimensions such as heterogeneity (Krusell and Smith, 1998), habit formation (Fuhrer (2000), Ljungqvist and Uhlig (2000), Boldrin, Christiano and Fisher (2001)), etc. in order to eventually converge to an ideal model.<sup>4</sup>

The alternative point of view suggests that the number of dimensions along which the current generation of DSGE models is misspecified is simply too large to make structural analysis a fruitful way for reading the data. Instead, simple reduced-form models are used as representations of the data. In his seminal paper 'Macroeconomics and Reality' Chris Sims (1980) proposed to simply project the vector of variables of interest on lagged values of itself. Such vector autoregressions (VARs) are by now the most standard way of looking at empirical phenomena in all fields of macroeconomics and have proven to be useful for forecasting as well (Doan, Litterman, and Sims (1983)). From the perspective of policy analysis the crucial deficiency of this paradigm lies in the Lucas critique.

Lucas (1976) building on Muth (1960, 1961) showed how the reduced-form solutions of forward-looking rational expectation models are functions of the law of motion of the exogenous driving process. For the analysis of monetary policy this means that for any given policy rule the dynamics of the economy, and hence the transmission mechanism, will be different. In terms of the notation of (1)-(2) this means that the reduced-form matrices  $A$ ,  $B$ , and  $C$  are functions of the policy rule. Since a structural model has the matrices  $A$ ,  $B$ , and  $C$  determined as explicit functions of the structural parameters,  $\theta$ , which include the parameters of the policy rule, we can explicitly recalculate these matrices for alternative policy rules. Structural models in this sense are therefore robust to the Lucas critique. Reduced form models on the other hand estimate the analogs of matrices  $A$ ,  $B$ , and  $C$  from a given sample with an associated policy regime. If this regime were to change these matrices would a priori change as well, thus exposing reduced form analysis to the Lucas critique.

Relative to these two approaches this paper proposes a hybrid approach. Within the paradigm of structural dynamic general equilibrium we limit ourselves to the basic pillars: expectations are rational, policy is exogenous, and there exists a stable rational expectations equilibrium. As to the exact structure of the economy, i.e. preference patterns, production technologies, etc., we remain agnostic and, instead, take our modeling framework directly to the data. The semi-structural model of this paper is then a parsimonious time-series rep-

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<sup>4</sup>This implicitly assumes that all models are nested. In order to compare non-nested sets of models one could follow the Bayesian approach laid out in Schorfheide (2001) or Fernandez-Villaverde and Rubio (2001).

resentation that encompasses a large set of DSGE models in the sense that if the data is generated by an economy with rational agents that take policy as exogenous and that has a stable equilibrium, then we can estimate the data-generating process and subsequently perform counterfactual policy analysis - despite having imposed no explicit prior theory (!). To be specific, the class of models I consider includes all models that are linear with a dimensionality of observable variables at least equal to the number of structural shocks affecting the economy.

I start by assuming that there exists *some* structural (factor) model that generates the economy. This means that a given economic environment (set of endowments, information sets, technical and behavioral structure...) gives rise to a set of relationships mapping the respective relevant state variables into the controls of the different agents (consumers, producers, etc.)

$$c_t^j = g(\theta, s_t^j), \quad \forall j \in J$$

where  $c_t^j$  and  $s_t^j$  denote the control and state vector for agents  $j$ , respectively and  $\theta$  is the vector of structural parameters. Since the sets of controls and states are not mutually exclusive at the aggregate level, let me denote all variables of economic meaning by an augmented vector  $X_t$ . Policy variables are captured by a vector  $P_t$  that is super-exogenous (in the sense of Engle, Hendry and Richards, 1983, see below).

Aggregating and imposing market clearing conditions then leads to a system of equations describing the dynamics of the economy

$$\begin{aligned} X_t = & A^1(\theta)E[X_{t+1}] + A^2(\theta)X_{t-1} \\ & + A^{1P}(\theta)E[P_{t+1}] + A^{2P}(\theta)P_t + A^{3P}(\theta)P_{t-1} + B(\theta)u_t \end{aligned} \quad (3)$$

The notation in (4) explicitly shows how today's actions are functions of expected future actions, past actions, and current, past, and expected future policy. Augmenting vectors  $X_t$  and  $P_t$  I can rewrite the law of motion for the economy more compactly as

$$\Psi^1 E[X_t] = \Psi^0 X_{t-1} + \Psi^P(L)E[P_t] + \Sigma u_t, \quad (4)$$

where  $\Psi^0$ ,  $\Psi^1$ ,  $\Psi^P$ , and  $\Sigma$  are matrices whose elements are nonlinear functions of the (unknown) structure of the economy. Given that this structure is here considered unknown, I will here oppress the dependence on  $\theta$ . (4) is a system of equations that I can estimate (!).

Let me denote the econometric model by hatted matrices

$$\widehat{\Psi}^1 E[X_t] = \widehat{\Psi}^0 X_{t-1} + \widehat{\Psi}^P(L)E[P_t] + \widehat{\Sigma} u_t, \quad (5)$$

I call (5) a conditional model for the macroeconomy given a super-exogenous policy rule. If I then merely add any arbitrary - estimated or calibrated - policy

rule I can then solve the combined general equilibrium system for a reduced-form law of motion. (See section 3 or appendix 1)<sup>5</sup>

$$\begin{bmatrix} X_t \\ P_t \end{bmatrix} = Y_t = \Omega Y_{t-1} + B^\Omega u_t \quad (6)$$

What this means is that if the underlying data generating process is of the assumed linear form, then I can then consistently estimate the impact of the super-exogenous subvector  $P_t$  on the entire economy without having to make any assumptions about the economic structure of the economy under the maintained hypothesis that the equations determining the super-exogenous subvector  $P_t$  are correctly specified. For consistency all lag- and lead-orders are chosen by Schwarz's Bayesian Information Criterion (BIC).

What distinguishes the semi-structural model of this paper from the fully structural models of the literature is that fully structural models require a *complete* specification of the structure of the economy, whereas the semi-structural model merely requires specification of the policy rule. If all other agents of the economy take the policy rule as given, then this semi-structural approach is equivalent to data-driven general equilibrium modeling and as far as policy analysis is concerned the Lucas critique *does not apply*.

To see this key point recall we can always write the density of the Markov process  $X_t = [Y_t, P_t]$  as  $f(X_t|X_{t-1}, \phi) = f(Y_t|P_t, X_{t-1}, \phi_2)f(P_t|X_{t-1}, \phi_1)$ . Weak exogeneity of  $P_t$  with respect to a parameter vector of interest  $\gamma = g(\phi_2)$  then implies that  $\phi_1$  and  $\phi_2$  must be variation free, i.e.  $\phi_1$  and  $\phi_2$  should not be subject to any restrictions. This implies that the marginal model for  $P_t$  is not informative on  $g(\phi_2)$ . If in addition  $\phi_2$  is invariant to the set of possible variations in  $\phi_1$  that is defined by  $\phi_{1,i}^g = \{h_i : \phi_{1,i} = h(\phi_i, g), \forall i\}$ , then  $P_t$  is termed super-exogenous with respect to  $g(\phi_2)$ . In the current context  $P_t$  denotes policy,  $Y_t$  the remainder of the economy's variables, and  $\phi_2$  parameterizes the economy's law of motion, i.e.  $\phi_2 = \{\Psi^1, \Psi^0, \Psi^P(\cdot)\}$ . Super-exogeneity of policy then means that I can write the general equilibrium law of motion as a conditional model for the economy - ' $f(Y_t|P_t, X_{t-1}, \phi_2)$ '- given some marginal model for policy- ' $f(P_t|X_{t-1}, \phi_1)$ '-, and that the parameter vector  $\phi_2$  that parameterizes the (forward-looking) model dynamics is independent of changes of the marginal model, i.e. the policy rule,  $\phi_1$ . All the Lucas critique says is that the general equilibrium reduced-form coefficients  $\phi^* = \{\Omega, B^\Omega\}$  are functions of the parameters that describe the density of the driving process, here policy, i.e.  $\phi_1$ . Clearly, the solution of the general equilibrium-model defined by the mapping  $\{\phi_1, \phi_2\} \mapsto \phi^*$  depends on the policy rule,  $\phi_1$ . Since the present paradigm explicitly considers the forward-looking behavior of the agents through the matrices  $\Psi^1$ ,  $\Psi^0$ , and the matrix polynomial  $\Psi^P(\cdot)$  it is not only that the present model is robust to the Lucas critique<sup>6</sup>, but it constructively applies its tenants at a practical level. Thus, it is possible to do counterfactual policy analysis just

<sup>5</sup>The techniques laid out in the appendix directly follow from Sims (2002).

<sup>6</sup>The present modeling approach is robust to the Lucas critique under the assumptions that the data-generating process is linear and that policy is super-exogenous. The Lucas critique in a more general sense applies to the present model for it may be argued that there have

like with fully-specified structural models.<sup>7</sup> appendix 2 illustrates the statement that any DSGE model is an element of the class of models here considered by means of a simple but representative model example.

The empirical model of this paper, which will be described in detail in the next section, models the economy as a reduced form model with a super-exogenous process for monetary policy. Monetary policy then is modeled with an estimated forward-looking policy rule.

## 2.1 Empirical Evidence on the Macroeconomic Effects of Money

The body of empirical literature that looks at the effect of monetary policy on the macroeconomy is impressive. (See Christiano, Eichenbaum and Evans (1999), CEE) for an excellent literature review). The main findings can be summed up as follows.

While monetary shocks are correlated with important movements in real variables, the bulk of the variability is due to systematic policy, i.e. to reactions to other variables/structural shocks in the same period. The potential effects of monetary shocks being measured by orthogonalized impulse responses show that a monetary innovation leads to a hump shaped and drawn out response in output. While this effect could be of quantitative importance, some assert that the historic variance of policy shocks has been so low during the past decades that the variance share of real variables due to monetary policy shocks in the sample is small at all but the very short horizon. This last argument depends crucially on the exact specification of the VAR. In fact, while the short-run effect is fairly robust across specifications, the path from the hump-shaped maximum effect to the long-term effect varies greatly across specifications.

Many other interesting questions about empirical monetary macroeconomics arise. The problem with these questions is that they cannot be easily mapped into both frameworks and sometimes neither one of the two. How shall one disentangle the transmission mechanism from the policy rule in a reduced-form model? What is the value to a policy maker to know optimal monetary policy in a calibrated and stylized model if he knows that the model is wrong<sup>8</sup>? In

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been regime changes during the sample period, which would bias the entire model. Such a disclaimer, in fact, applies to all models, theoretical or empirical, for any structural model might just as well ignore some structural change in the economy such as a policy regime change. As always, the model here is consistently estimated under the Null that it is correctly specified, which here means in particular that there is no policy regime change during the sample period.

<sup>7</sup>On the other hand, since reduced form analysis ignores the presence of the polynomial  $\Psi^P(\cdot)$  it can only be useful for policy analysis if  $\Psi^P(\cdot)$  is not present in the data-generating process, i.e. if the entire economy is backward-looking

<sup>8</sup>Recall that the RBC school objects to estimation in favor of calibration because the model is known to be wrong. Here I go one step further. I build on the methodological paradigm of the RBC school as far as the assumption of a unique and stable rational expectations equilibrium goes. But as far as policy analysis is concerned I can then drop the entire theoretical underpinning - which is known to be wrong to start with - and instead estimate a simple forward looking equations for the macroeconomic variables of interest directly from the data.

fact, the traditional RBC/DSGE and VAR approaches can be compared directly only as far as the usual impulse-response analysis and variance decomposition accounting. On the other hand, the new paradigm of this paper allows analysis of a variety of additional questions. Thus, we can not only compare its implications to those of the two alternative modeling approaches, but can show where the other approaches go wrong. This is because both the reduced form as well as the fully structural approach are restricted subcases of the new 'semi-structural' paradigm.<sup>9</sup>

The new paradigm of this paper allows full empirical and counter-factual policy analysis in a simple estimated model. For example, one can disentangle the nature of the policy rule from the monetary transmission mechanism. Also, we can estimate policy rules for different time periods and compare the monetary transmission mechanism through time by looking at the response function of output to monetary innovations to the estimated as well as hypothetical policy rules. This distinction is important in the cross-sectional comparison of monetary policy rules, for example. Asymptotically free of the restrictions of a potentially misspecified economic structure due to its estimated form, and robust to the Lucas critique under the maintained assumption of super-exogeneity of the estimated policy rule, the present model is subjected to a number of policy exercises in Section 4, where the relevant literature is discussed.

## 2.2 Monetary Policy Rules

The literature on monetary policy rules is enormous<sup>10</sup> and I will limit myself to some recent papers that try to combine a conditional macroeconomic model and a policy rule in a vain similar to the one of this paper.

Clarida, Gali, and Gertler (2000, CGG) is probably the most quoted piece of recent work in this field. CGG estimate Taylor-type policy rules for the US, allowing for smoothing, and discuss the issues of structural breaks (pre-Volker vs. Volker-Greenspan) as well as the relevance of sunspots in the context of a simple sticky price model for the macroeconomy. I will get back to both issues below.

Boivin and Giannoni (2001) estimate VARs for different time periods and calibrate a 'structural' model - with backward-looking price setters and habit formation - to the impulse-responses of a VAR to perform counterfactual policy experiments. They fail to realize that strictly speaking there is no need to calibrate a 'structural' model to do counterfactual policy analysis and that their doing so unnecessarily limits the practical relevance of their results.

Finally, Soderlind (2001) estimates a simple DSGE model by maximum likelihood and then performs counterfactual policy analysis. He asks "*What if the Fed had been an inflation Nutter?*" and compares the time path of macroeco-

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<sup>9</sup>This is certainly true for the conditional macroeconomic model, i.e. for the model of the macroeconomy given a policy rule. If the policy rule is correctly specified then this naturally applies to the complete general-equilibrium model.

<sup>10</sup>For a comprehensive overview over this huge literature, see the homepage on monetary policy rules by John Taylor at <http://www.stanford.edu/~johntayl/PolRulLink.htm>

conomic variables had the Federal Reserve acted much more vigorously to counteract inflation. Again, the author fails to realize that no structural model is necessary and that the imposition of one conditions his results on a particular economy that may or may not be a good empirical model.

Relative to the literature this paper proposes an empirical model for the study of monetary policy that has two characterizing new features.

First, it proposes to model the macroeconomy not with a stylized structural model, but with an estimated forward-looking linear model.

Second, money, or more precisely, the federal funds rate, is modeled with a structural equation. Rather than simply regressing the federal funds rate on a vector of observable variables as is done in the pure reduced form approach, the federal funds rate is here modeled directly with a forward-looking policy rule, i.e. a functional relationship between the control variable, the federal funds rate, and expectations of future variables that describe the state of the economy. Here it will be assumed that the monetary authority determines the federal funds rate as a function of the path of inflation and output growth only. The advantage of this explicit policy rule specification is that it yields an economically interpretable policy rule rather than reduced form coefficients devoid of any direct meaning. Estimates of such policy rules were used in Clarida, Gali and Gertler (1998a,b, 2000) and Batini and Haldane (1999) among others, but to the knowledge of the author such a rule has not been used in combination with an estimated transmission mechanism that would allow the quantitative analysis of the economic effects of such a rule. The estimated model of this paper will allow me to obtain a numerical estimate of the policy reaction to expected inflation and output growth as well as an estimate of how the central bank endogenously reacts to macroeconomic shocks via their dynamic effect on inflation and growth. The technical details of the model will be presented in the following section.

### 3 The Model

The model has two main ingredients, the reduced form model for the macroeconomy and the structural policy rule, each which will be presented now.

#### 3.1 The macroeconomy

The observed economy is denoted by a vector  $X_t$ , and is potentially observed with noise.

$$(1 - L)X_t = \mu_X + HF_t + \eta_t, \quad (7)$$

where  $E[\eta_t] = 0, E[\eta_t F_t^T] = 0$

The dynamics of the economy are driven by a vector of mutually orthogonal factors,  $F_t$ , with dynamics that are given the following linear difference equation

$$F_t = A(L)F_{t-1} + C(L)E[P_t] + Bs_t \quad (8)$$

Denote the dimensionality of the vector of economic variables,  $X_t$ , and the vector of factors,  $F_t$ , by  $n$  and  $q \leq n$  respectively.<sup>11</sup>  $P_t$  denotes a vector of exogenously determined variables, here limited to the monetary policy's control variable, the federal funds rate. Note that since this is a dynamic forward-looking rational expectations model, both  $A(L)$  and  $C(L)$  are a lead and lag polynomial in  $L$ .

At this point I would like to consider two sub-classes of models for the economy that have both been empirically applied in related contexts and which have different advantages in the current setting. First, I could model the factors  $F_t$  by principal component analysis (PCA). In this case I can extract  $F_t$  as the first  $q$  principal components of the variance matrix of  $(1 - L)X_t$ .  $H$  is the matrix obtained from the eigenvectors associated with the first  $q$  eigenvalues and  $\eta_t$  is residually defined. The intuition for modeling the economy through a small number of factors is as follows. Macroeconomic theory at a very broad level can be cast in terms of a small number of aggregate shocks. In addition, an unappealing aspect of the shocks recovered from a VAR is that it is hard to come up with economic classification and identification schemes for  $n$  shocks for  $n$  larger than 3. By assuming that the structural shock vector is of reduced dimension, the identification problem is significantly reduced. Obviously, this assumption could be false, but even then, as long as I have correctly identified the largest  $q$  shocks, the orthogonality of the factors assures that the resulting system that describes the monetary transmission mechanism has maximal explanatory power relative to any other linear system of that order.

Alternatively, I could estimate a model of the type (8), where the factors are rotations of the observable variables  $X_t$  themselves (i.e.  $q = n$ ). This would amount essentially to estimating a VAR with  $P_t$  as an exogenous regressor.

In the empirical application below I will focus on the case where the macro-model is written in terms of two observables, output growth, and inflation<sup>12</sup>.

It is important to stress that since  $ff_t$  is modeled as super-exogenous, the specification (8) is the most general time series representation of any economic system in the following sense. If all variables that affect the economy were observable and the data-generating process where a structural economy is defined in the sense of section 2, then estimation of (8) would recover the true model in the limit. See appendix 1 for details.

**correlated estimates of structural shocks** The DSGE literature usually considers structural shocks to be orthogonal, i.e. one may label an in-

<sup>11</sup>This means that the spectral frequency matrix is of rank  $q$  at all frequencies. At an intuitive level, this is related to the hypothesis of a reduced rank matrix at *frequency zero* implicit in co-integration analysis. Obviously, the hypothesis here is much stronger in that it applies at *all frequencies*.

<sup>12</sup>While I did apply the standard battery of unit root tests to check whether the assumption of integration of order one,  $I(1)$ , for the levels of output and growth is justified - in fact, I cannot reject the Null of  $I(1)$  for both series even at the 10 per cent significance level - it is well-known that the power of these tests relative to local alternatives goes to zero, in particular given the sample size here at hand.



novation to monetary growth or the fed funds rate as a monetary shock, or an innovation to the level of productivity as a productivity shock. Since the DSGE literature focuses on calibrated models, the issue of uncorrelated structural shocks is a convenient structural assumption. On the other hand, the VAR literature estimates linear models and finds the residuals of the various equations to be highly correlated, which leads to the uncomfortable problem of identification of an impact matrix. Usually, one therefore imposes some type of more-or-less convincing set of restrictions on the impact matrix in order to be able to discuss the effects of 'structural', shocks that are here again assumed to be orthogonal, but affecting more than one variable on impact.<sup>13</sup> In the present context, we have already solved for the simultaneous feedback of the entire economic system when solving the semi-structural forward-looking form representation for its reduced-form analog. As such, it doesn't make a lot of sense to talk about an implicit Wold ordering or impact matrix restrictions. Instead, in calculating the impulse-responses to a structural shock to, say, inflation, we have actually already calculated the simultaneous effect on all other variables, on expectations of future policy, and the feedback effect of all these changes on the shocked variable itself. Hence, if we find the estimated variance-covariance matrix of the shocks to the economy to be non-diagonal, then it is because structural shocks are, in the data, not uncorrelated. As a result, a simulation of the theoretical model that assumes an orthogonal shock structure will fail to generate a theoretical variance-covariance matrix that is equivalent to that estimated from the data. Yet, for the exercise of policy analysis it is important to do counterfactual analysis. Counterfactual analysis here then means that I abstract from the correlation of the estimated structural shocks in the data and look at the effects of a hypothetical structural shock to a particular variable *ceteris paribus*, i.e. all other variables unshocked. Since I solve for the endogenous simultaneous effect on all other variables a shock to this one particular variable will in general equilibrium generally affect other variables simultaneously, but this effect is independent of any assumed Wold ordering or statistical exclusion restrictions.

### 3.2 The Central Bank

The objective function of the central bank is simple. I here do not assume any particular parametric form for its objective function, but rather assume that its choice is given by setting its target interest rate equal to a linear combination of current, past, and expected future deviations of inflation and output growth from their targets.

$$ff_t^* = E[a(L^{-1})(\pi_t - \mu_\pi) + b(L^{-1})(y_t - \mu_y)|F_t] + u_t^m, \quad (9)$$

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<sup>13</sup>The VAR literature in fact confounds two issues: correlation of shocks and instantaneous effect. In a VAR setting we could not distinguish between variable A at time t reacting to the level of variable B at time and a correlation between the shocks to A and B, which could mean that variable A does not react to variable B at time t. A projection residual would not allow us to distinguish between the two cases.

where  $F_t = \left\{ \{\pi_s\}_{s \leq t}, \{y_s\}_{s \leq t}, \{ff_s\}_{s < t} \right\}$ .

The realized federal funds rate is then modeled as a linear combination of this target rate and past periods' realized rates, i.e. the central bank smooths the interest rate path across time.

$$ff_t = \rho(L)ff_{t-1} + (1 - \rho(1))ff_t^* \quad (10)$$

The reduced form of the resulting joint economic system can then be written in the following augmented state-space form

- forward-looking state equation:

$$\Psi^1 E_t \begin{bmatrix} F_t \\ ff_t \end{bmatrix} = \mu_\Phi + \Psi^0 \begin{bmatrix} F_{t-1} \\ ff_{t-1} \end{bmatrix} + \tilde{B} \begin{bmatrix} s_t \\ u_t^m \end{bmatrix} \quad (11)$$

- observation equation:

$$Y_t = \begin{bmatrix} X_t \\ ff_t \end{bmatrix} = \mu_\Theta + \Theta \begin{bmatrix} F_t \\ ff_t \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \eta_t, \quad (12)$$

where  $s_t$ ,  $u_t^m$  and  $\eta_t$  denote macro shocks, the policy shock, and observation noise respectively and  $F_t$  and  $ff_t$  are appropriately augmented.

While this is a system that I can estimate by GMM, I cannot use it for simulation or analysis just yet. The problem is that it is a matrix difference equation with nontrivial leads and lags that involves expectations that I first have to solve for. In order to do so I have to check the stability and uniqueness conditions of Blanchard and Kahn (1980). Intuitively, these conditions check if there is a unique rational expectations equilibrium that can be associated with the system (11)-(12) (see Sims (2002) or Klein (2000)). If both conditions are met then I can uniquely project all expectations of future variables on variables known at time  $t$  and obtain a backward-looking representation that I can make use of for policy analysis.

The resulting reduced-form representation can then be written as

$$\begin{bmatrix} F_t \\ ff_t \end{bmatrix} = \mu_\Omega + \Omega \begin{bmatrix} F_{t-1} \\ ff_{t-1} \end{bmatrix} + B^\Omega \begin{bmatrix} s_t \\ u_t^m \end{bmatrix} \quad (13)$$

The impact matrix  $B^\Omega$  in column  $j$  then shows the impact effect of a structural shock to variable  $j$ , where structural means that this shock shocks only variable  $j$  and no other variable either directly or through correlated shocks. However, given that the model is forward-looking, a shock to variable  $j$  implies a changed future time path for this variable and all other variables that are affected by it. These changes in expectations of the future then affect all of today's variables that are forward-looking. As a result, the impact matrix  $B^\Omega$

in (13) has a zero in position  $B_{i,j}^\Omega$ ; only if the respective variable  $i$  is backward-looking. See appendix 2 for details. For convenience we put state the core of the present paradigm in the form of a proposition

**Proposition 1** *Suppose the dynamics of an economy can be described by the following forward-looking state equation*

$$\Psi^1 E_t \begin{bmatrix} F_t \\ f f_t \end{bmatrix} = \mu_\Phi + \Psi^0 \begin{bmatrix} F_{t-1} \\ f f_{t-1} \end{bmatrix} + \tilde{B} \begin{bmatrix} s_t \\ u_t^m \end{bmatrix} \quad (14)$$

*Then one can check whether it is stable and if so associate with it a backward-looking state equation of the form*

$$\begin{bmatrix} F_t \\ f f_t \end{bmatrix} = \mu_\Omega + \Omega \begin{bmatrix} F_{t-1} \\ f f_{t-1} \end{bmatrix} + B^\Omega \begin{bmatrix} s_t \\ u_t^m \end{bmatrix} \quad (15)$$

*The solution (15) might be unique or not, a condition we can check by looking at the eigenvalues of the canonical representation associated with (14). (See Appendix 2 for details.) If the system is stable and unique, then the impact matrix is  $B^\Omega$  identifiable. If the system is stable and not unique then it allows for sunspot dynamics. In the sunspot case usually it is neither the impact effect, the size of the shock, nor its dynamic effect on the system that are identifiable.*

While this paper focuses on the analysis of a semi-structural model for monetary policy the proposition has direct implications for the popular empirical tool that are vector autoregressions.

**Corollary 2** *If we assume that all structural shocks are uncorrelated, then we can apply the same techniques to a (potentially forward-looking) vector autoregression*

$$(\Delta - \Phi_{B1}L^{-1} - \Phi_{B2}L^{-2} - \Phi_{F1}L - \Phi_{F2}L^2)E_t [Y_t|F_t] = u_t, \quad (16)$$

*where  $Y_t$  denotes the vector of variables,  $\Delta$  a matrix with unit diagonal elements,  $u_t$ , the vector of shocks, and the information set  $F_t$  includes all past realizations as well as for each equation  $i$  all current observations for all variables  $j \neq i$ . This way we can condition on the time  $t$  realizations of all other variables in a given period before analyzing the effects of an innovation to variable  $i$ .*

The corollary implies that we can use the present techniques to analyze standard VARs without making assumptions about the impact matrix as is usually done (imposing Wold orderings, long-run effects, zero restrictions, etc.).

There are some similarities worth noting between the present approach and those proposed by Rigobon (2003) and Beyer and Farmer (2003). All three approaches attempt to get around the usual identification problem of the impact matrix in multivariate systems by exploiting particular knowledge about the system under study. In the case of Rigobon and Beyer and Farmer (2003) this is

done by testing for constancy in the identified parameters of an equation while reestimating the same equation for different sample periods. If the identified parameters are stable the different estimated variance-covariance matrices for the different samples then allow identification of the otherwise unidentified parameters (of the impact matrix). In these papers the method of estimation is ML/OLS, whereas in the present paper we move to GMM/IV. Our method of estimation allows us to apply our techniques also to setting where the underlying process is of homoscedastic through time. On the other hand we slightly loose efficiency, which is a bigger problem in applications where the data has low autocorrelation and when there are no alternative good instruments available. Which of these alternative estimation techniques for otherwise unidentifiable parameters is more efficient or robust is clearly dependent on the application at hand.

The following section will present empirical results for postwar US time series.

## 4 Empirical Results

This section reports the estimation results of the model outlined in the previous section for an application to quarterly U.S. data (the sample is 1984:01 to 2000:04).<sup>14</sup> The lag and lead orders of the macroeconomic factor model as well as the policy rule were chosen by Schwarz's Bayesian Information Criterion (BIC). Since all lead and lag orders are chosen simultaneously I have to evaluate a total number of models that is given by the product of the all the possible lead and lag orders I consider for the various polynomials. Specifically, I consider lags and leads in the policy rule from 1 to 8 and a smoothing polynomial of lag order 0 to 8. Likewise, for the conditional model I consider leads and lags in macro factors up to 8 periods, and leads and lags in the short rate of up to 8 periods. I also allow for contemporaneous feedback in both directions.<sup>15</sup> All estimates are based on exactly-identified linear IV estimation using lagged observables, the robust variance-covariance estimates are constructed following Newey and West (1987).

### 4.1 The macroeconomic model

For the conditional model of the macroeconomy or, for purposes of the discussion of monetary policy, the transmission mechanism, the BIC chooses the minimal orders of zero lead and zero lags for the polynomial associated with

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<sup>14</sup>All data are from FRED, the economic web database at the Federal Reserve Bank of St. Louis. The federal funds rateseries refers to the quarterly averages obtained from monthly data (source: H.15 release of the Federal Reserve Board of Governors) . The output growth series refers to changes in logs of total seasonally adjusted real output in chained 1996 Dollars (source: U.S. Department of Commerce, Bureau of Economic Analysis). The inflation rate refers to the *consumer price index for all urban consumers: all items, seasonally adjusted* (source: U.S. Department of Labor, Bureau of Labor Statistics)

<sup>15</sup>I thus estimate and compare a total of 419904 model specifications.

the policy variable. The order of the autoregressive polynomial is chosen to be one lag and zero leads. The numerical point estimate is as follows (t-statistics in parentheses)

$$\begin{aligned} \begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} \mu_y \\ \mu_\pi \end{bmatrix} + \begin{bmatrix} 0.81 & 0.14 \\ (1.68) & (2.26) \\ 0.06 & 0.80 \\ (0.64) & (5.48) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \pi_{t-1} \end{bmatrix} \\ &+ \begin{bmatrix} -0.35 \\ (-4.38) \\ -0.13 \\ (0.67) \end{bmatrix} ff_t + Be_t \end{aligned} \quad (17)$$

## 4.2 The monetary policy rule

The policy rule has a minimum lead order of 0, i.e. only contemporaneous macro variables enter, and no lags. A marginal effect of the contemporaneous state of the economy given expectations about the future as well as a marginal effect of the past is decided against based on the BIC. The smoothing polynomial is of order 2. The estimated policy rule looks as follows (t-statistics in parentheses)

$$\begin{aligned} ff_t &= \mu_{ff} + \underset{(8.85)}{1.32} ff_{t-1} - \underset{(-2.77)}{0.39} ff_{t-2} \\ &\quad - \underset{(-1.18)}{0.70} (\Delta y_t - \mu_y) + \underset{(2.15)}{2.42} (\pi_t - \mu_\pi) + e_t^m \end{aligned} \quad (18)$$

Three comments are in order. First, the response to deviations from the (assumed constant) target level of inflation is strong. To those familiar with the literature on determinacy in calibrated sticky price models (see Kerr and King (1996), Clarida, Gali and Gertler (1998a), or Woodford (2002)), this may be comforting. In standard calibrated models, the policy rule generally has to react with a coefficient of at least unity to deviations in future inflation in order to assure determinacy of the equilibrium dynamics. Otherwise, the model's dynamics are indeterminate in the sense that nonfundamental shocks to expectations - so-called sunspot shocks - can generate self-fulfilling dynamics that qualify as dynamic rational expectations equilibria. As will be shown below, the standard determinacy frontier is very much model-dependent, however, and the present model would actually still be determinate even if we calibrated this response to be nil.

Secondly, the importance of the smoothing polynomial, which has been shown in other studies (see, for example, Clarida, Gali, and Gertler (1998a, 2000)), seems noteworthy. In effect, the almost unit root behavior means that it is not so much the interest rate that reacts to changes in expectations about future policy, but more the change in the interest rate. It is oftentimes argued that smoothing is a policy means to promote financial stability (see Estrella (2001) for a literature review). Intuitively, the argument goes, higher variability in the short rate will lead, *ceteris paribus*, to higher long-term yields. Since the

short-rate is clearly only an intermediary tool in influencing medium- and long-term yields which are more important for investment decisions on the income side and real estate and equity wealth on the wealth side, lower interest rate variability at the short end of the yield curve has direct implications for long-term yields, and thus the level of wealth and production of an economy, and may for this reason be desirable. This is clearly a fallacy of decomposition, for due to Jensen's inequality term, higher short-term interest rate variability actually *lowers* longer term interest rates in a model where the local expectations hypothesis holds. As a result, the apparent presence of the smoothing character of empirical policy might not be welfare-improving at all. On the other hand Sack and Wieland (2000) present theoretical considerations for why smoothing might be optimal for reasons such as measurement errors in target variables or model uncertainty. In any event, we do retain the smoothing specification because it clearly improves the statistical fit of the model and allows better comparison with other studies.

Finally, the interest rate response with respect to expected future growth is negative. Here it is important to note that this is the marginal effect of expected future growth, i.e. the effect of expected growth given expected future inflation expectations. Since a shock to growth that does not move inflation expectations does not put the monetary authority's goal of inflation stability at risk, this does not seem surprising. On the other hand, the conditional correlation between growth and inflation is positive. Also note that the interest rate reaction with respect to inflation relative to the reaction to growth is stronger than the ratio of the relative multipliers requires. Consequently, an expansion caused by a monetary ease will be stabilized through future rises in interest rates and the model is dynamically stable.

### 4.3 Dynamic Effects of Monetary and Macroeconomic Shocks

Figures (1)-(3) shows the impulse responses of output growth and inflation along with bootstrapped 95% confidence bands<sup>16</sup> to a one-time one percent deviation from the policy rule for the system (17)-(18).

Note that since the monetary shock is defined relative to a policy rule, there is an automatic feedback effect to the degree that the monetary shock moves the entire future path of growth and inflation to which current policy in turn reacts. Hence, the effect of a policy-induced boom in output and the associated rise in inflation leads to an automatic stabilization of realized interest rates. This effect is fairly weak, however, and a one percent shock to the policy rule

<sup>16</sup>The error bands are obtained from the following simple bootstrap procedure. Given the empirical distribution of the system's shock vector  $\{u_s\}_{s \in T}$  I draw  $N = 500$  time series of length  $T$  from this empirical distribution (with replacement), which I denote as  $\{\hat{u}_s^i\}_{s \in T}$  for  $i = 1, \dots, N$ . I then generate  $N$  artificial samples  $\{\hat{Y}_s^i\}_{s \in T}$  by feeding the system the shock series  $\{\hat{u}_s^i\}_{s \in T}$  and reestimate my model from each artificial sample. Finally I compute my statistics of interest  $g(\hat{Y}_s^i)$  and calculate the cut-off  $g_{\frac{\alpha}{2}}(\hat{Y}_s^i)$  and  $g_{1-\frac{\alpha}{2}}(\hat{Y}_s^i)$  values that define a confidence interval with coverage of  $(1-\alpha)\%$ . My point estimate will be the bootstrap-bias-corrected mean  $\bar{g} = \sum_{i=1}^N g(\hat{Y}_s^i)$ .

in general equilibrium results in a reduction in realized interest rates of 99 basis points.

Let me make four comments about this impulse-response. First, the response of output growth is stronger than that of inflation, a result that may be surprising to those who think in monetarist terms, but a result that is often encountered in other empirical estimates of the effect of monetary shocks (see, for example, Christiano, Eichenbaum, and Evans (1999)).

Second, the presence of the estimated smoothing behavior in a forward-looking model of rational expectations determines the reaction of the macroeconomy to a policy shock. Consequently, the reaction is delayed and hump-shaped, maximal only more than two years after the impact of the shock, and generally following the pattern of interest rates closely with a lag of 7 and 8 quarters year and multipliers of -1.2 and -0.6 for output growth and inflation, respectively. Figure (4) plots the conditional expectations of inflation and output growth divided by their respective multipliers at time  $t + s + 7(8)$  along with the conditional expectations of the interest rates at time  $t + s$  conditional on a monetary shock at time  $t$ .

Third, the above specification is extremely parsimonious. Since it is well-known that the BIC tends to choose rather underparameterized models in finite samples I have repeated the order selection procedure with Akaike's information criterion (AIC). The AIC-specification chooses a lag order in macro variables of 2 for the macroeconomic model and a smoothing polynomial of order 3. All of the results reported below are very much the same for this alternative specification.

Finally, note that the above responses are all with respect to a 1 percent shock to the policy rule, i.e. a deviation of realized interest rates from their expected value by 100 basis points. Naturally this is just a convenient normalization.

Overall we observe a clear qualitative finding: both output growth and inflation rise temporarily after an interest rate shock. The response seems to be hump-shaped with a maximal effect after approximately two years.

## 5 Policy Analysis

### 5.1 Comparing Historical Policy Rules

The key insight in rational expectations macroeconomics (Muth (1960, 1961), Lucas (1976)) is that the optimal decision rules in dynamic and stochastic control problems, such as consumption or investment demand, are function of the moments of the law of motion of the states that affect the control problem. For our analysis this means that any change in the policy rule could potentially cause a change in the law of motion of the economy, thus rendering exercises of comparative dynamics useless. However, as shown in section 2, the present model has the forward-looking nature of economic actions incorporated, thus rendering it immune to the Lucas critique as far as policy experiments are concerned. Certainly, the reduced form of the model changes as a function of monetary policy,

but given the maintained assumption that policy is determined exogenously, we can manipulate the policy rule and perform counterfactual experiments with this estimated model.

As an example, I have done an exercise analog to the one presented in Clarida, Gali and Gertler (1998b, CGG). CGG compare the implications of different estimated policy rules, notably those estimated for the pre Volker vs. the Volker-Greenspan era. What distinguishes the present exercise from the one performed in CGG is that CGG enter their estimated policy rule in a small-scale sticky-price model, whereas I use the estimated policy rule in combination with the transmission mechanism, the conditional model of the macroeconomy, estimated from post-1984 data. I am thus looking at what would have happened if the Federal Reserve System had continued to use the same policy rule it had used from 1964 to 1982.

The primary difference is the smaller reaction of the interest rate to expected future growth and inflation. The estimated rule is as follows

$$ff_t = \mu_{ff} + \underset{(6.54)}{0.91} ff_{t-1} + \underset{(0.18)}{0.02} ff_{t-2} + \underset{(-0.37)}{0.32} (\Delta y_t - \mu_y) + \underset{(0.11)}{0.25} (\pi_t - \mu_\pi) + e_t^m \quad (19)$$

The policy rule has exactly the same orders as the one estimated for the Volker-Greenspan era. Note the clear lack of significance of the forward-looking terms in the policy rule. According to (19) policy in the period 1964-1982 could then be described as largely passive. Still the effect of interest rate shocks on the macroeconomy is largely the same because of the strong direct effects on the macroeconomy that are not offset through countercyclical policy. Figure (5) plots the impulse responses. Also note that while the response to a normalized interest rate shock of 100 basis points is actually smaller with the pre-1982 rule than with respect to the post-1984 rule, the standard deviation in the former case is much larger in the earlier period (120 (!) basis points) than in the latter (35 basis points).

## 5.2 Sunspots and Indeterminacy in Linear Rational Expectation Models

In section 3 the forward-looking semi-structural model had to be solved for a unique rational expectations solution by projecting all variables of future time periods on the current information set in a way that stability of the system was assured. While stability requires a certain root condition to hold, it does not generally reduce the set of stable solutions to a singleton. An additional root condition can be checked to see if uniqueness holds - and it does in the case of the point estimates of this paper's model -, but it is straightforward to calibrate - or, for that matter, estimate - models that fail the uniqueness condition. In this case there exists an  $r$ -dimensional space of so-called 'sunspot' equilibria, where  $r$  denotes the number of *excess stable roots*. (see Sims (2002), Klein (2000), Lubik and Schorfheide (2001), and appendix 2 for details)

Kerr and King (1996) were probably the first to note that simple policy rules a la Taylor can result in indeterminacy in calibrated sticky-price models. As



already mentioned, Clarida, Gali and Gertler (1998b, CGG) estimate a simple Taylor-type policy rule and plug it into a calibrated model. At their point estimates for U.S. data covering the pre-Volker era, they find indeterminacy to be a problem and argue that passiveness of monetary policy could generically be an empirically relevant cause for nonuniqueness. Woodford (2002) presents a textbook treatment of standard sticky-price models and discusses limits on policy rules that assure determinacy. In these standard sticky-price model uniqueness is usually guaranteed if interest rates respond to inflation by a more-than-proportional increase in nominal interest rates, i.e. real rates have to rise. As the estimates of this paper show (see (19)), this result is a by-product of the calibrated transmission mechanism of inflation and interest rates in these models.

To the knowledge of the author the only published econometric study of determinacy in linear rational expectations models seems to be Farmer and Guo (1995, FG). FG estimate an RBC model from postwar US data and find evidence for indeterminacy.<sup>17</sup> On the other hand, the point estimates of this paper suggest for the period under study U.S. data does not seem to be characterized by indeterminacy in the sense that models with indeterminacy simply imply higher BIC values. Such representations are usually very far forward-looking and imply extremely erratic impulse responses. As such it appears that they are mere artifacts of overfitting the data. Intuitively, the relative unimportance of monetary policy for the question of indeterminacy is due to its small effect on the macroeconomy. While in calibrated sticky price models monetary variability is the key factor driving macroeconomic variables, the present estimated model shows that at least for the sample period and for the U.S. monetary variability seems to a minor factor and given the inherent stability of the economic system (unknown to us for we have only estimated a reduced form representation of the data), monetary policy does not seem to induce macroeconomic indeterminacy.

### 5.3 Optimal Policy

To many, the quest for optimal policy is the *raison d'être* of monetary economics and the literature of how to define, qualify, and conduct optimal policy is enormous and still rapidly evolving. At the same time, two basic assumptions drive most of the results. The first assumption is about how to

model the central bank's loss function. The second is about how to model the central bank's policy rule.

As to the central bank's loss function, many studies in the general equilibrium literature use the utility function of a presumed representative agent (see Rotemberg and Woodford (1997)). Alternatively, the official mandate of central

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<sup>17</sup>The relative lack of impact of this study on the profession may be in part due to the brilliant discussion by Rao Aiyagari (1995), who analytically shows why indeterminacy is unlikely to be a credible factor in standard RBC models. Yet, Aiyagari's arguments are very model-specific and, as he notes, do in no way limit the relevance of asking whether indeterminacy could be an issue in other models and, more to the point, whether indeterminacy could be a real-world phenomenon.

banks to control inflation and to promote economic stability may suggest some other weighing of inflation and output variability with weights determined by the political mandate specific to the central bank of the economy under study. Given an objective function optimal policy is then straightforward to calculate both for the discretion as well as for the commitment case given that we are equipped with a Lucas-robust estimate of the transmission mechanism. For computational details, see Soderlind (1999), for example.

#### 5.4 Asset-Pricing Implications

Appendix 3 shows how the present model also is a general equilibrium asset pricing model. In fact, following the DSGE motivation, if the observed data is generated by a unique and stable rational-expectations equilibrium, then the present model can be augmented to consistently estimate the pricing kernel of the economy as long as the latter is affine without *any necessity to specify a utility function*. Intuitively, the pricing kernel has to be measurable with respect to the relevant filtration. But in the Markov case the relevant filtration is given by current period state variables. Thus, making an assumption about a flexible functional form for the pricing kernel renders calculation of the interest rate term structure a straightforward exercise. For details, see appendix 3.

## 6 Conclusion

This paper proposes a data-driven alternative to structural DSGE analysis. The proposed 'semi-structural' model is an econometric model that nests all standard structural DSGE models as special subcases.

Summing up, the present state of the literature on macroeconomic general equilibrium modeling offers two alternative routes: Either one makes very restrictive - ad hoc (?) - assumptions about the (unobservable) structure of the economy or one models the economy in reduced form and tries to give some structural interpretation to the reduced-form representation. Given the Lucas critique as well as the (structural) VAR critiques of Canova and Pina (2000), Sartre (1987), Rudebusch (1999), or Klaeffering (2003) it is not clear how useful this approach is for empirical policy analysis. In addition, certain questions cannot be asked in a reduced-form setting. The policy rule of the monetary policy maker and the monetary transmission mechanism, for example, are convoluted and cannot be studied separated. Relative to these two alternatives, the approach of this paper instead focuses on the pure logical implications of the structural general equilibrium paradigm: agents are forward-looking, adjustment lags are finite, policy is exogenous, and the equilibrium is stable.

Building on these basic pillars I propose a time-series approach to macroeconomic general equilibrium modeling. It combines an estimated, forward-looking, structural equation for a policy variable with an estimated, forward-looking, reduced-form system for a vector of key economic variables to obtain the empirically estimated analog to a DSGE model solution.

This procedure allows me to make structural inference on all of the effects of the modeled policy variable on the economy including: impulse-response analysis, comparative and separate study of the policy rule and the transmission mechanism across time, the stabilizing or destabilizing role of monetary policy, optimal policy, and the effect of policy shocks on the term structure of interest rates.

## A Appendix

### A.1 Solving a dynamic forward-looking general equilibrium model for a backward-looking reduced form

The following algorithm is a conceptually straightforward application of the techniques of Klein (2000), and, in particular, Sims (2002).<sup>18</sup>

I start with an economic system that is comprised of two building blocks: a conditional model for the macroeconomy, (20), given super-exogenous policy rule and a marginal model for policy, (21), that closes the system.

$$X_t = A(L)X_{t-1} + C(L)E[P_t] + Bu_t^{macro} \quad (20)$$

$$P_t = D(L)P_{t-1} + F(L)E[X_t] + u_t^p, \quad (21)$$

where  $A(L)$ ,  $C(L)$  and  $F(L)$  are forward- and backward-looking polynomials,  $D(L)$  is backward-looking.

More compactly, I can rewrite the general equilibrium representation as (22)

$$\Psi^1 Y_t = \Psi^0 Y_{t-1} + \Sigma u_t + \Xi \varsigma_t, \quad (22)$$

where  $Y_t = \begin{bmatrix} X_t \\ P_t \\ E_t \end{bmatrix}$  is a stacked vector of economic variables and expectations respectively,  $u_t$  denotes the vector of i.i.d. (w.l.o.g.) structural economic shocks and  $\varsigma_t$  denotes endogenous expectational errors. The subvector  $E_t$  is obtained by stacking expectations. For example  $e_t^{ij}$  denotes the time  $t$  expectation of variable  $j$  for period  $t+i$ , i.e.  $e_t^{ij} = E[w_{j,t+i}]$ , where  $w_j$  is the position  $j$  element of the subvector  $W_t = \begin{bmatrix} X_t \\ P_t \end{bmatrix}$ .

Following Sims (2002) I will solve for an equilibrium law of motion of  $Y_t$  in autoregressive form

$$Y_t = \Omega Y_{t-1} + B^\Omega u_t \quad (23)$$

The algorithm is presented in 5 steps.

- Step 1: Apply the QZ transform to the matrix pencil in (22) and rewrite as

$$Q^T S Z^T Y_t = Q^T T Z^T Y_{t-1} + \Sigma u_t + \Xi \varsigma_t, \quad (24)$$

where  $Q$  and  $Z$  are orthonormal matrices, i.e.  $QQ^T = ZZ^T = I$ , and  $S$  and  $T$  are upper triangular.

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<sup>18</sup>We present the techniques from the point of view of the model as we present it in the main text of the paper. However, it is noteworthy that (22) could equally well capture a forward-looking VAR (potentially only including present and past values). In this sense we obtain an alternative identification technique for VARs that make the oxymoron Structural VAR obsolete, albeit at a (usually small) efficiency cost due to the necessity to move from ordinary least squares to instrumental variable techniques.

- Step 2: Premultiply (24) with  $Q$  and reorder the system's equations such that the ratios of the respective diagonal elements of  $S$  and  $T$  are increasing in absolute value. Recycling notation, the resulting reordered equation system can be written as

$$SZ^T Y_t = TZ^T Y_{t-1} + Q(\Sigma u_t + \Xi \varsigma_t). \quad (25)$$

- Step 3: Define the vector of canonical variables

$$Z^T Y_t = \begin{bmatrix} \theta_t \\ \delta_t \end{bmatrix}, \quad (26)$$

where  $\delta_t$  denotes the subvector associated with unstable roots. I will denote  $\theta_t$  as canonical states. Partitioning  $Z, T, S$ , and  $Q$  as  $S = \begin{bmatrix} S_{\theta\theta} & S_{\theta\delta} \\ 0 & S_{\delta\delta} \end{bmatrix}$ ,  $T = \begin{bmatrix} T_{\theta\theta} & T_{\theta\delta} \\ 0 & T_{\delta\delta} \end{bmatrix}$ , and  $Q = \begin{bmatrix} Q^\theta \\ Q^\delta \end{bmatrix}$  I can then impose the stability of the economic system by setting  $\delta_s = 0, \forall s$ .<sup>19</sup> This amounts to imposing that

$$\text{span}(Q^\delta \Sigma) \subseteq \text{span}(Q^\delta \Xi) \quad (27)$$

In order to allow for the case that the number of explosive roots generates fewer than maximal restrictions I calculate the singular value decomposition of  $Q^\delta \Xi$ <sup>20</sup>

$$Q^\delta \Xi = [ U_1 \quad U_2 ] \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 D_{11} V_1^T \quad (28)$$

Further, let me

Then

$$\varsigma_t = U_1^T D_{11}^{-1} V_1 Q^\delta \Sigma u_t \quad (29)$$

These are the restrictions on the endogenous expectational errors given by the stability condition. (27) is the condition that allows me to solve the model. It is a necessary and sufficient condition. In order to obtain a unique solution however I have to take a look at how the shock process enters in the law of motion of the canonical state variables, which is obtained from (25).

$$\theta_t = S_{\theta\theta}^{-1} T_{\theta\theta} \theta_{t-1} + S_{\theta\theta}^{-1} Q^\theta (\Sigma u_t + \Xi \varsigma_t) \quad (30)$$

<sup>19</sup>This is true as long as the growth rate of  $\{u_s\}$  is bounded by the smallest explosive root of the process. This condition is trivially satisfied for a martingale shock process.

<sup>20</sup>See also Lubik and Schorfheide (2001).

## A.2 A simple example of a Neo-Keynesian model that fits the present framework

The present example builds around the paradigm of the neoclassical synthesis (Goodfried and King (1997)) and is a nutshell version of a class of models that includes Jeanne (1997), Yun (1999), Kim (1998), Gali (2001) or Christiano, Eichenbaum, and Evans (2001) among many others.

The nutshell model excludes capital and involves three types of agents: a continuum of consumers of measure unity, a continuum of producers of measure unity, and a central bank determining policy.

**consumers** The consumers choose sequences  $\left\{ C_{t+i}, C_{t+i}(z), N_{t+i}, \frac{M_{t+i}}{P_{t+i}}, \frac{B_{t+i}}{P_{t+i}} \right\}_{i=0}^{\infty}$  to maximize lifetime welfare, defined as

$$\max E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1-\gamma} C_{t+i}^{1-\gamma} + \frac{a}{1-\varepsilon} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\varepsilon} - \frac{k}{\mu_n} N_{t+i}^{\mu_n} \right] \right\} \quad (31)$$

subject to

$$C_t = \frac{W_t}{P_t} N_t + \Pi_t + TR_t - \frac{M_t - M_{t-1}}{P_t} - \frac{\frac{1}{1+i_t} B_t - B_{t-1}}{P_t}$$

and to

$$C_t = \left[ \int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$C_t$  denotes the basket of differentiated goods with constant elasticity of substitution  $\theta$ . Households choose not only the total level of consumption per period,  $C_t$ , but also the composition  $\{C_t(z)\}_{z \in [0,1]}$  given prices  $\{P_t(z)\}_{z \in [0,1]}$ .

It is straightforward to show<sup>21</sup> that given constant elasticity of substitution the demand for each good  $C(z)$  is given by

$$C(z) = \left[ \frac{P_t(z)}{P_t} \right]^{-\theta} C_t \quad (32)$$

The price index as the minimum expenditure that yields one unit of the composite consumption good is then

$$P_t = \left[ \int_0^1 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \quad (33)$$

Further, the first-order necessary condition for utility maximization are given by

$$\frac{P_t}{P_{t+1}} \beta E_t C_{t+1}^{-\gamma} = \frac{1}{1+i_t} C_t^{-\gamma} \quad (34)$$

<sup>21</sup>See, for example, Obstfeld and Rogoff (1996).

$$C_t^{-\gamma} \frac{W_t}{P_t} = k N_t^{\mu_n - 1} \quad (35)$$

$$\frac{i_t}{1 + i_t} C_t^{-\gamma} = a \left( \frac{M_t}{P_t} \right)^{-\varepsilon} \quad (36)$$

(34) is the standard Euler equation for consumption, whereas (35) is the intratemporal leisure-labor trade-off condition, and (36) gives money demand as a function of the interest rate and marginal utility of consumption.

**producers** The production side is given by a continuum  $z \in [0, 1]$  of producers that set prices in advance given demand functions (32) for their products. For simplicity we suppose that their production technology is given by  $Y_t(z) = N_t(z)$ . We do, however, superimpose exogenous price-stickiness a la Calvo (1983). The firms' problem is then

$$\max_{\{P_t\}_{t=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \rho^i \Pi_{t+i} \right\} \quad (37)$$

subject to

$$\Pi_{t+i} = \frac{P_t(z)}{P_t} Y_t(z) - \frac{W_t}{P_t} N_t(z) \quad (38)$$

$$Y_t(z) = D_t(z) = N_t(z) \quad (39)$$

$$D(z) = \left[ \frac{P_t(z)}{P_t} \right]^{-\theta} C_t^{22} \quad (40)$$

and only a fraction  $1 - \varphi$  of randomly-chosen firms get to readjust in any given period. The pricing kernel for profits is given by discounted relative marginal utility  $\rho_t^i = \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\gamma}$ .

The first-order necessary condition for firms setting their prices in period  $t$  is then given by

$$P_t^*(z) = \frac{\theta}{\theta - 1} E_t \sum_{i=0}^{\infty} \varpi_{t+i} W_{t+i} \quad (41)$$

where  $\varpi_{t+i} = \frac{\varphi^i \rho_t^i D_{t+i}}{E_t \sum_{i=0}^{\infty} \varphi^i \rho_t^i D_{t+i}}$  and  $\rho_t^i = \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\gamma}$

(41) shows how in the case of exogenously-imposed price-stickiness a la Calvo today's optimal price is a weighted average of the expected future marginal costs of production. The price level is then given by

$$P_t = \left[ \varphi P_{t-1}^{1-\theta} + (1 - \varphi) P_t^{*1-\theta} \right]^{\frac{1}{1-\theta}} \quad (42)$$

Given some form of monetary policy to be discussed further below we then obtain the general equilibrium for this economy by assuming that it is symmetric, i.e.

$$Y_t(z) = Y_t \forall z \quad (43)$$

<sup>22</sup>Since we assume away capital  $C_t(z) = Y_t(z) \forall z \in [0, 1]$

$$N_t(z) = N_t \forall z \quad (44)$$

and imposing market clearing, i.e.

$$C_t = Y_t \quad (45)$$

and

$$B_t = 0 \quad (46)$$

In log-linearized form I can then define the dynamics of the equilibrium implicitly by the following system of equations, (47)-(50).

- The IS curve

$$y_t = -\frac{1}{\gamma}(i_t - \pi_t) + E_t y_{t+1}, \quad (47)$$

where  $\pi_t = p_{t+1} - p_t$

- The forward-looking Phillips-curve<sup>23</sup>

$$\pi_t = \lambda y_t + \beta E_t \pi, \quad (48)$$

where  $\lambda = \frac{(1-\varphi)(1-\beta\varphi)}{\varphi}(\gamma + \mu_n - 1)$

- The LM curve

$$m_t - p_t = a_1 y_t - a_2 i_t, \quad (49)$$

where  $a_1 = \frac{\gamma}{\varepsilon}$ ,  $a_2 = \frac{1}{\varepsilon i}$

- The log-linearized equation for wages

$$w_t - p_t = b y_t, \quad (50)$$

where  $b = (\gamma + \mu_n - 1)$ ,

- and finally, some yet to be specified monetary policy rule for the policy control variable, here the nominal interest rate.

Given that (49) and (50) merely define demand for real balances and real wages, respectively, let us drop them for they are inconsequential at this point.

The equilibrium dynamics will then be defined by two equations, the IS curve, (47), the Phillips curve, (48), and an exogenous process for policy, i.e. the nominal interest rate  $i_t$ .

In vector form, we can describe the equilibrium dynamics as

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \frac{\gamma}{\gamma+\lambda} & 0 \\ \frac{\gamma\lambda}{\gamma+\lambda} & \frac{\beta\gamma\lambda}{\gamma+\lambda} \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} -\frac{1}{\gamma+\lambda} \\ 0 \end{bmatrix} i_t \quad (51)$$

<sup>23</sup>The Phillips curve is obtained by combining the log-linearized version of the first-order condition for optimal pricing, (41), with the log-linearized version of the equation for wages, (35).



or

$$X_t = A^1 E[X_{t+1}] + A^2 X_{t-1} + A^{1P} E[P_{t+1}] + A^{2P} P_t + A^{3P} P_{t-1} + B u_t, \quad (52)$$

a special case of (48), with  $X_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$ ,  $P_t = i_t$ ,  $A^1 = \begin{bmatrix} \frac{\gamma}{\gamma+\lambda} & 0 \\ \frac{\gamma\lambda}{\gamma+\lambda} & \frac{\beta\gamma\lambda}{\gamma+\lambda} \end{bmatrix}$ ,  $A^{2P} = \begin{bmatrix} -\frac{1}{\gamma+\lambda} \\ 0 \end{bmatrix}$ ,  $A^2 = 0_{2 \times 2}$ ,  $A^{1P} = A^{3P} = 0_{2 \times 1}$ , and structural economic shocks  $u_t$  that can be interpreted as shocks to the IS equation and the Phillips Curve, respectively.

Clearly, in this simple DSGE model we can analyze the economic implications of arbitrary policy rules for the policy variable  $i_t$ . The matrices  $A^1$ ,  $A^2$ ,  $A^{1P}$ ,  $A^{2P}$ ,  $A^{3P}$  are unaffected by the particular policy rule under consideration, for they are functions of the deep parameters of the economy,  $\{\gamma, \lambda, \beta\}$ .

Estimation of the semi-structural form (52) would clearly be inefficient in the case that we know the model (in which case we could simply estimate  $\{\gamma, \lambda, \beta\}$  by efficient maximum likelihood). On the other hand, estimation of (52) would be robust to model uncertainty in the sense that if there is some unknown stable DSGE model that generates the data, then the data-generating process has an estimable representation of the form (52), and I can do empirical and counterfactual policy analysis with the estimated model without having to write down a model such as the present one that - in the spirit of the RBC school - is known to be wrong to start with.

### A.3 Computing a macroeconomic term structure model

The bond market is modeled with two key assumptions. I will introduce and discuss them in turn.

**Condition C1: Absence of Arbitrage (A $\circ$ A)**

According to Harrison and Kreps (1979) absence of arbitrage implies the existence of a measure  $Q$  under which all discounted asset prices are martingales, i.e.

$$E_Q[e^{-r_t} p_{i,t+1} | F_t] = p_{i,t}, \quad (53)$$

where  $r_t$  denotes the risk-free rate (here the short rate), subindex  $i$  denotes an asset in the asset span, and  $E_Q$  denotes the expectation under the Martingale equivalent measure  $Q$  given some filtration  $F_t$ .<sup>24</sup>

Further, I can relate the expectation under the induced measure, denote it by  $P$ , and the expectation under the Martingale equivalent measure,  $Q$ , by Girsanov's theorem

$$E_P\left[\frac{\frac{dP_{t+1}}{dQ_{t+1}} p_{i,t+1}}{\frac{dP_t}{dQ_t}} \middle| F_t\right] = E_Q[p_{i,t+1} | F_t] \quad (54)$$

Let us denote the discounted ratio of the Radon-Nikodym derivatives as a pricing kernel,  $m_{t+1}$ ,

$$m_{t+1} = e^{-r_t} \frac{\frac{dP_{t+1}}{dQ_{t+1}}}{\frac{dP_t}{dQ_t}} \quad (55)$$

Thus

$$p_{i,t} = E_P[m_{t+1} p_{i,t+1} | F_t], \quad (56)$$

where  $m_t$  is  $F_t$ -measurable.

Note that this expectation is with respect to the induced measure  $P$ , the density that can be estimated by maximum likelihood. I will make use of relationship (56) to estimate the pricing kernel,  $m_t$ , from the sample observations I have on the asset span across time. Here I will follow much of the literature on empirical asset pricing (Constantinides, 1992, Ang and Piazzesi, 2001) and assume a particular functional form for the Radon-Nikodym derivative, which in turn will allow me to retrieve the pricing kernel given its definition in 56.

**Condition C2: Known Functional Form for Radon-Nikodym derivative**

$$\frac{dP_{t+1}}{dQ_{t+1}} = \frac{dP_t}{dQ_t} \exp\left(-0.5\lambda_t^T \lambda_t - \lambda_t u_t\right), \quad (57)$$

<sup>24</sup>The martingale-equivalent measure  $Q$  could be dependent on the filtration, i.e. if there was heterogeneity in information sets each information set would have its associated measure. If markets are incomplete the measure  $Q$  is not defined uniquely, but I will here implicitly assume that it is unique through time.

The measure  $Q$  that I will be looking for will also have to be adapted to the information set of the econometrician. All moments of the model, such as impulse-responses or variance decompositions, will then be conditional on this filtration and measure.

where

$$\lambda_t = \lambda_0 + \lambda_1 X_t \quad (58)$$

(58) says that the price of risk,  $\lambda_t$ , is an affine function in a vector of states which includes the macroeconomic factors,  $F_t$  and the short rate,  $f_t$ . Endowed with a model for the short rate and a pricing kernel I can then use the chain law that links the short rate to zero bonds and iteratively build up bond prices for zero coupons for any maturity.

$$b_t^1 = E_P[m_{t+1}b_{t+1}^0|F_t] = E_P[m_{t+1}|F_t] = e^{-ff_t} \quad (59)$$

and

$$b_t^s = E_P[m_{t+1}b_{t+1}^s|F_t] \quad (60)$$

In order to get an explicit solution for yields as a function of the state vector I simply have to write on the iterative definitions of bond prices as conditional expectations implicit in (59-60). To do so I start by solving for the price of a 1-period bond as follows

$$\begin{aligned} b_t^1 &= E_P[m_{t+1}b_{t+1}^0|F_t] \\ &= E_P \left[ \exp -\frac{1}{2}\lambda_t^T \lambda_t - \lambda_t u_t | F_t \right] \end{aligned} \quad (61)$$

and iterate from there on.

Finally, I can obtain the continuously compounded nominal yield  $i_t^s$  from the price of the zero coupon bonds.

$$i_t^s = -\log \left( \frac{b_t^s}{s} \right) \quad (62)$$

I have the yield curve.

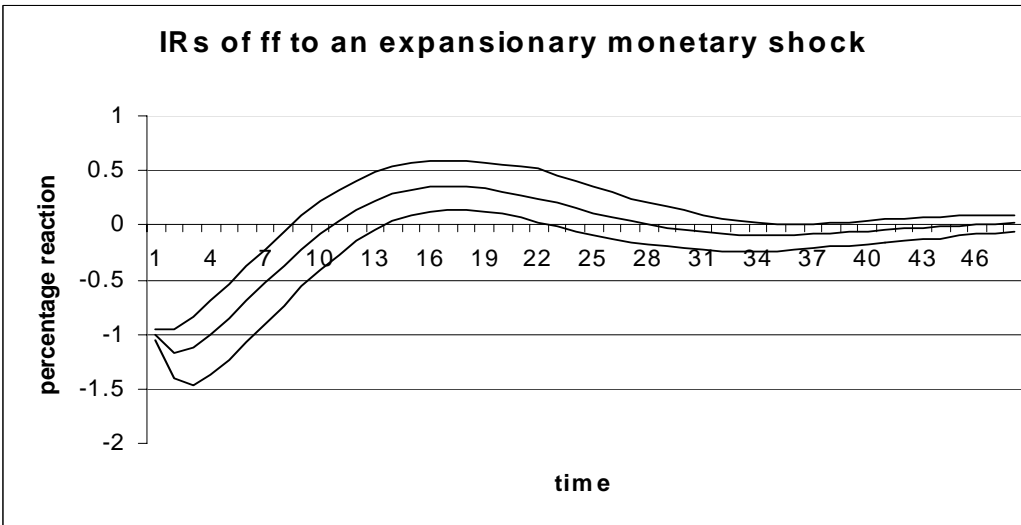


Figure 1:

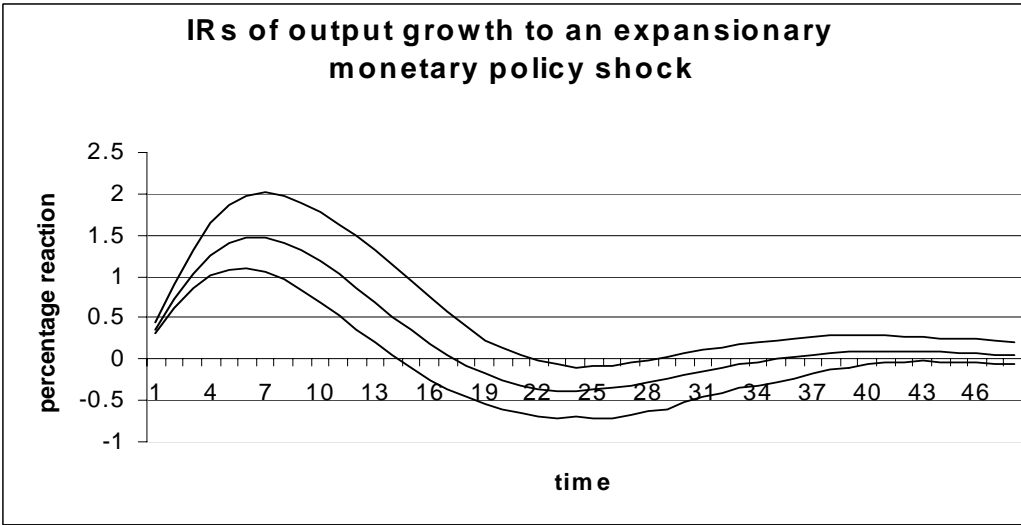


Figure 2:

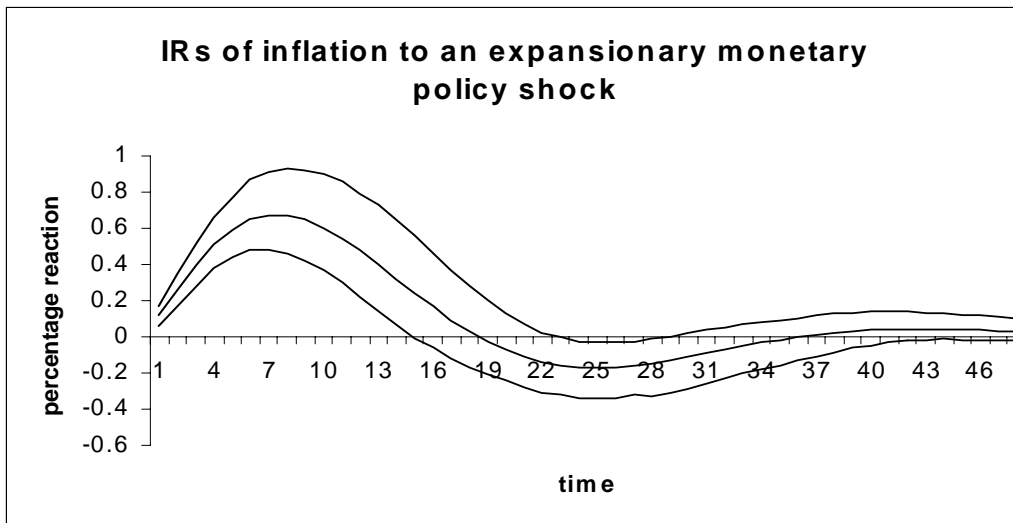


Figure 3:

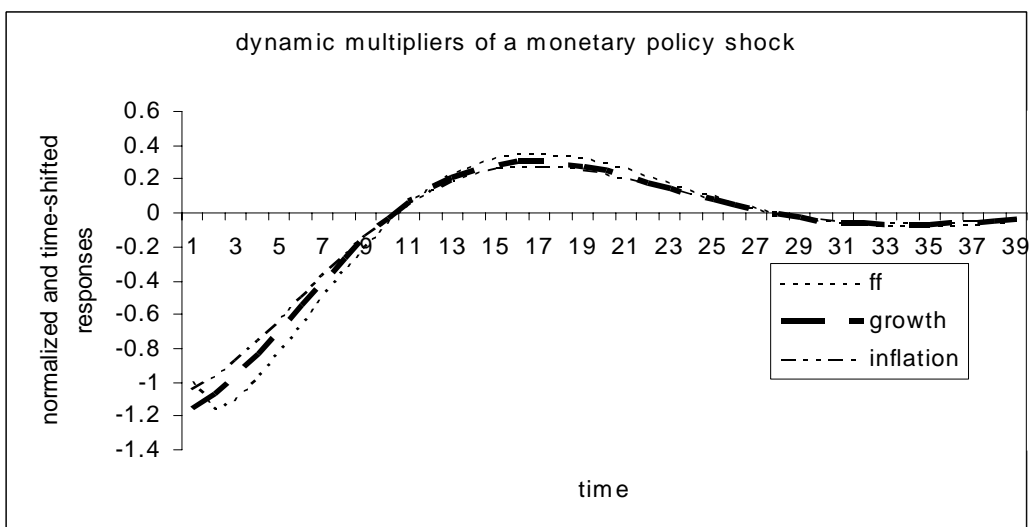


Figure 4:

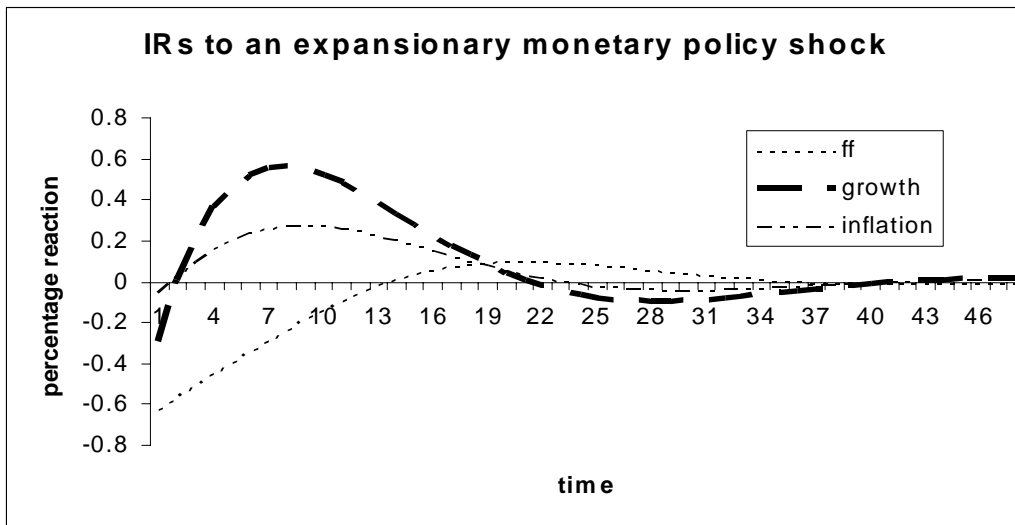


Figure 5:

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