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VOLATILITY OF INTEREST RATES IN THE EURO AREA: EVIDENCE FROM HIGH FREQUENCY DATA

BY NUNO CASSOLA AND CLAUDIO MORANA

June 2003
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BY NUNO CASSOLA² AND
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June 2003

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Abstract

This paper estimates the factors underlying the volatility of the euro overnight interest rate and its transmission along the euro area money market yield curve. A new multivariate unobserved components model is proposed allowing for both long-memory and stationary cyclical dynamics. Using hourly data the estimates show repetitive intraday and monthly patterns that can be explained by the microstructure of the money market and the institutional features of the Eurosystem’s operational framework for monetary policy implementation. Strong persistence is detected in all log-volatility processes and two common long-memory factors are extracted. The first factor explains the long-memory dynamics of the shortest maturity. The second factor explains the transmission of volatility along the money market yield curve. We find evidence that most liquidity effects are cyclical, confined to the end of reserve maintenance periods, and are not transmitted along the money market yield curve.

Keywords: Money market microstructure; money market interest rates; liquidity effect; stochastic volatility; fractional integration and cointegration.

JEL classification: C32; E43; F30; G10
Non-technical summary

An often asked question is whether the specific characteristics of the operational framework of the Eurosystem have any impact on the volatility of money market interest rates and on its transmission to longer term rates. Since the volatility of money market interest rates is generally believed to be the main factor underlying time-varying term premia, this question is relevant for the European Central Bank (ECB) given the current focus on steering money market interest rates in the implementation of the monetary policy stance. In fact, the monetary policy signalling could be blurred by the volatility of short-term interest rates, term premia could distort the information content of forward interest rates and the ex-post assessment of the predictability of the actions of the ECB would become more difficult. Thus, it seems warranted to empirically identify the sources of the volatility of short-term interest rates and assess the extent to which it is transmitted to longer-term rates. To address these issues we use a novel data set composed of hourly observations for the overnight rate, and the two-week, one-month, three-month, and twelve-month EONIA swap rates. The sample is from 4/12/2000 through 31/05/2002, for a total of 3510 hourly observations, or 390 working days. A new multivariate unobserved components model is used allowing for both long-memory and stationary cyclical dynamics. As common in time series analysis, interest rate volatility is decomposed into a cyclical component and a persistent (trend) component. The cyclical component is further decomposed into a deterministic part and a stochastic part, the latter capturing the possibly time-irregular seasonal patterns of
volatility. The persistent component is assumed to capture the long-memory behaviour of interest rate volatility. We interpret the cyclical components as related (mainly) to the characteristics of the operational framework, payment systems and accounting conventions - they should have either no impact on longer-term interest rates or be idiosyncratic. Thus, the trend component should capture the underlying economic determinants of volatility and should explain its transmission along the term structure. The cyclical components are modelled using a functional form combining sine and cosine terms. Additionally, impulse dummies are included to account for institutional features of the operational framework, including the beginning and the end of the reserve maintenance period and the open market operations allotment and settlement days. Other impulse dummies account for special calendar effects, like holidays, and also accounting practices and payment system related issues, like end-of-month and end-of-year effects. Dummies are also included for identifying the impact of the press conferences following the ECB’s Governing Council (GC) meetings distinguishing between days with and without interest rate change decisions. One break is considered in the volatility of interest rates, at the end of November 2001, which is associated with the reduction in the frequency of the monetary policy decision making GC meetings. The estimates show that a number of factors significantly affect the pattern of volatility. There is a significant increase in the volatility of the overnight interest rate at the beginning and towards the end of the maintenance period and on allotment days. Furthermore, end-of-month and end-of-year effects are also significant for the overnight interest rate. By contrast, the settlement of the weekly open market operations does not seem to impact
on the volatility of the overnight interest rate. Overall the results suggest that the impact of the institutional features of the operational framework are well captured by the cyclical deterministic components of volatility. These are concentrated at the shortest end of the curve, and do not affect volatility along the money market yield curve. It is also interesting to note that the impact on interest rate volatility of accounting practices and payment system factors (end of month and end of year effects) are also confined to the shortest end of the curve. Finally, the “pure” monetary policy factors (i.e. press conference following the GC meeting and interest rate decisions) are associated with higher volatility along the yield curve. The “trend” components capture the underlying economic determinants of volatility and help explaining transmission along the term structure. We found evidence of three fractional cointegrating relationships between the five volatility processes. Therefore, two common long-memory factors drive the long-run evolution of the series. The first factor explains the long-memory dynamics of the volatility at the shortest end of the curve (overnight rate, two-week rate and 1-month rate). More importantly, shocks to this factor are not transmitted further along the term structure. The forward propagation of shocks along the term structure is explained by the second long-memory factor, which affects all maturities beyond the two-week rate. Thus, we may interpret the first factor as measuring the intensity of the shock to the short-term interest rates and the second factor as measuring the intensity of its transmission along the term structure. We are also able to quantify persistent effects of some liquidity shortages, which were transmitted along the money market yield curve. However, these effects were not the rule and could be explained by exceptional
circumstances. In general, liquidity effects were found to be cyclical, confined to the end of reserve maintenance periods, and not transmitted along the money market yield curve.
1 Introduction

Understanding the behaviour of the short-term interest rate and its volatility is at the center of two strands in the professional literature. On the one hand, the degree of persistence and the volatility of the short-term interest rate are the critical ingredients in most models of the term structure of interest rates, for their role in the determination of term premia, forward interest rates and the yields and volatilities of longer maturity interest rates. On the other hand, the literature with a market microstructure focus explains regular volatility patterns in the short-term interest rate either by institutional details associated with the operational framework of monetary policy implementation or by the impact of scheduled macroeconomic news announcements.

The main goal of the paper is to empirically study the euro area money market from a microstructure perspective. In particular, we focus on the factors underlying the volatility of the overnight interest rate and its transmission along the money market yield curve (up to the one-year maturity). We aim at empirically separating out the two sources of volatility referred in the microstructure literature, one related to the institutional features of the operational framework and payments system, and the other, related to the impact of policy decisions. The latter can be seen as reflecting the assessment by the money and financial markets, of the monetary policy stance and central bank actions.

The empirical analysis is implemented without reference to any specific structural model of the money market. Rather, we build an econometric model with deterministic and stochastic components. Deterministic and stochastic cyclical components are used to identify the institutional characteristics and the response of the market to monetary policy news, while not cyclical stochastic components are employed to model the long-run volatility dynamics. A residual stochastic component is also included which should capture other (less relevant) factors affecting the decisions of market participants.

Why is this identification useful? A natural and often analyzed question has to do with the impact of central bank policy choices on term premia and on the transmission of volatility along the term structure of interest rates. This question is relevant for central banks that focus on steering money market interest rates in their implementation of the monetary policy stance as its signalling may be blurred by the volatility of short-term interest rates. Our econometric model may therefore be useful to study the reaction of market interest rates to the actions of the central bank. Moreover, the empirical framework proposed in the paper is very flexible and is well suited
for testing the theoretical predictions of specific models.

In order to achieve our target we use high frequency data on the level of the euro area\(^1\) interest rates along several points of the money market yield curve. More precisely, we use a novel data set composed of hourly observations for the overnight rate, and the two-week, one-month, three-month, and twelve-month EONIA swap rates. Hourly observations have been computed as averages of bid-ask quotes taken from REUTERS screens. The sample is from 4/12/2000 through 31/05/2002, for a total of 3510 hourly observations, or 390 working days. These data are used to estimate a multivariate model with deterministic and stochastic components of volatility. The econometric framework implemented is novel and extends existing methodologies in jointly modelling common long-memory volatility dynamics and stochastic intraday cyclical effects. The extension implements, for the purposes of estimation, a frequency domain methodology and the Wiener-Kolmogorov theory of signal extraction.

In the paper, we show how the model can be used to analyse the impact of «events» on the intraday volatility dynamics of the overnight interest rate and other money market interest rates. We explain how to assess the magnitude of the different impacts, relative to non-intervention and non-announcement days, and the time pattern of decay of the impact of the various shocks on volatility. The high frequency data allows to accurately investigate both features, since the timing of interest rate setting is known and the pattern of decay can be examined at a high frequency. The outcome of this exercise is an evaluation of market participants’ reactions to shocks, and a test as to whether different time patterns can be detected for different maturities.

The remainder of the paper is organized as follows. Section 2 discusses the issue of intra-day and intra-week volatility dynamics with a special focus on the European money market. In this section we review the relevant theoretical literature and discuss the institutional factors which may account for repetitive volatility patterns. Section 3 describes the econometric methodology. In this section the interest rate process is written in such a way that its conditional volatility can be emphasized, which is assumed to depend on both

\(^1\)Euro area refers to those countries of the European Union (EU) participating in Stage III of Economic and Monetary Union (EMU), which started in 1 January 1999. The 12 countries that share a common currency, the euro, and a single monetary policy are, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. The network constituted by the ECB and the National Central Banks (NCBs) of the 12 participating member countries is referred to as the Eurosystem. The other members of the European Union not participating in the single currency are Denmark, Sweden and the United Kingdom.
deterministic and stochastic components. Section 4 describes the data set, presents some descriptive statistics, the estimation of deterministic intraday patterns and the persistence analysis. Section 5 presents the results of the multivariate analysis, building on the stochastic intradaily cyclical patterns and the fractional cointegration properties of the persistent components of volatility. The latter are determined by two factors only and it is shown how these react to monetary policy decisions, and exceptional allotment decisions by the ECB. The main conclusions are presented at the end of the paper.

2 Intraday volatility dynamics in the money market

A well documented finding in the literature is the existence of intraday repetitive patterns in the volatility of financial assets (see for instance Dacorogna et al. [19], Andersen and Bollerslev [1], Payne [53], Beltratti and Morana [10], [11]). Evidence provided for stock market data and exchange rates points to complex intraday volatility dynamics, reflecting the intensity of trading activity in the market. Can we expect to find similar issues for the money market? Our answer is positive. In this section we briefly review the market microstructure literature that explains such patterns.

Spindt and Hoffmeister [64] show that in a continuous market, asynchronous trading, regulatory constraints, and accounting conventions that focus agents’ attention on discrete time instants, have important impacts on the dynamics of trading activity and realized market prices. Applied to the US Fed funds market the model predicts that the variability of the US federal funds is higher towards the end of each business day and highest near the end of maintenance periods. The model also predicts higher variance of the federal funds on days just prior to holidays and on Fridays. Griffiths and Winters [32] extended Spindt and Hoffmeister [64] model by including additional regulatory constraints to show strong inter- and intra-period incentives to borrow and lend in the federal funds market at predictable points in the 10 trading day maintenance period. These features lead to predictable changes in the variance of Fed funds on a daily and intraday basis which are supported by empirical evidence. Clouse and Dow [18] model the federal funds market allowing for a fixed cost facing banks that borrow at the discount window. The existence of the fixed cost is critical for explaining occasional instances of extremely high funds rates and also provides an explanation for heterogeneous behavior across banks towards the discount window and for higher average funds rates at the end of maintenance periods. Hamilton
[33] argues that line limits, transaction costs, and weekend accounting conventions explain why the Fed funds rate tends to fall during the two-week reserve maintenance period, with sharper than usual drops on Fridays and before holidays but an abrupt upsurge at the end of the period.

A common feature of these studies is that they omit the "supply" side, e.g. central bank liquidity and/or interest rate policy. Hamilton [34] models the supply and demand for Federal Reserve deposits and finds empirical evidence that a shock in the supply of reserves will only induce banks to borrow at the discount window if it occurs on settlement Wednesday or the last day of the quarter. Bartolini et al. [9] model banks’ liquidity management and official intervention policies jointly, in a setting which accounts explicitly for the main institutional features of the US federal funds market. The model predicts biweekly patterns in the volatility of the Fed funds and on its response to changes in target rates and in intervention procedures, such as those implemented by the Fed in 1994. The empirical evidence tends to support the models’ predictions. Prati et al. [54] show in a cross-country study that the high-frequency behavior of both levels and volatility of short-term interest rates can be related to differences in the role of reserve requirements, standing facilities, style of monetary intervention, and choice of nominal anchor.

The process of EMU led to the creation of the ECB and to the introduction, in January 1999, of a new operational framework for the implementation of monetary policy in twelve European countries. The Eurosystem provides the bulk of liquidity to the banking system through regular open market operations (see ECB [25] for details on the liquidity management of the ECB). The main refinancing operations (MROs) are liquidity-providing reverse transactions (against eligible collateral) with a weekly frequency and normally with a maturity of two weeks. The MROs are executed through standard tenders. From January 1999 until June 2000, fixed rate tenders were the norm replaced, after June 2000, by variable rate tenders with a pre-announced minimum bid rate.2

2Credit institutions in the euro area must hold compulsory deposits on accounts with the Eurosystem (required reserves). Reserve requirements, which are known before the start of the maintenance period, must be held on average over a one-month reserve maintenance period (averaging mechanism). In fact, compliance with reserve requirements is determined on the basis of the average of daily balances on credit institution’s reserve accounts with the Eurosystem. Maintenance periods start on the 24th calendar day of each month and end on the 23rd calendar day of the following month. Reserves are renumerated at a rate that corresponds to the average over the maintenance period of the marginal rate of the MROs. One of the key functions of the minimum reserve system is to stabilise money market interest rates. This function is performed by the averaging provision. The averaging provision allows credit institutions to smooth out daily liquidity
Several models of the overnight market for funds are available embodying the stylized features of the new euro area environment. Quirós and Mendiázabal [55] model reserve management by banks in an environment with an averaging provision for the reserve requirement and where banks have two standing facilities at their disposal, a lending facility and a deposit facility. The model predicts a process for the interest rate that tends to be higher on average and more volatile as the end of the maintenance period approaches, and provide evidence showing that the introduction of the standing facilities stabilized overnight interest rates in Germany after EMU.

Other authors focused on modelling the tenders (auctions) conducted by the ECB to provide refinancing to the banking system (Ayuso and Repullo [6], Bindseil [13], Catalão-Lopes [17], Ewerhart [27], Välikimäki [65], [66], and [67]). This literature shows that tender procedures, banks bidding behavior and central bank allotment policy, interact in a complex manner leading to different predictions about the dynamic and volatility of the overnight interest rate. As detailed in section 4, during the period covered by our study the ECB decreased its interest rates four times. Thus, the sample period is marked by recurrent market expectations of interest rate decreases. Whenever banks expect a reduction in the official interest rates of the ECB within the current maintenance period, the fulfillment of the reserve requirement is postponed and the demand for current liquidity is shifted downwards. This pushes short-term interest rates in the interbank market temporarily below the minimum bid rate and makes central bank refinancing non-competitive. Consequently, the bids at the MRO that settles before the GC meeting where the interest rate decrease is expected, will tend to be below the so-called

\footnote{For the purpose of controlling short-term interest rates in the money market and, in particular, restricting their volatility, the Eurosystem offers standing facilities to its counterparts (available on their own initiative). One, the marginal lending facility, provides overnight loans against collateral at a penalty rate. Thus, banks only use the facility to obtain funds as a last resort. Another, the deposit facility allows banks to make overnight deposits with the Eurosystem. As the rate offered by the Eurosystem is in general below market rates, banks use the facility only if they cannot use their funds in any other way. As access to the standing facilities is unlimited the rates on the standing facilities determine the corridor within which the overnight money market rate can fluctuate.}
neutral level, an incident that has been labeled underbidding (see Bindseil [13], Everhart [27] and Välimäki [67]).\textsuperscript{4} The resulting liquidity imbalance following an underbidding episode may pull short-term interest rates up, irrespective of whether the expected policy decision materializes or not. In fact, if the ECB does not provide in the subsequent MROs the extra liquidity needed to rebalance the market, the perceived probability of ending the maintenance period in the marginal lending facility increases. Thus, both the current overnight interest rate and its future expected value increase. The volatility of interest rates is also likely to increase, independently of the source of shocks, as the elasticity of demand is lower at the low level of reserve holdings.

In an empirical paper closer to ours, Hartmann et al. [36] consider how four institutional factors influence the evolution of prices and quantities in the euro area money market.\textsuperscript{5} Their main findings were: a hump-shaped intraday volatility pattern; an increase in volatility at the end of the maintenance period; an increase in volatility at the end of the year, and due to the Y2K effect. These patterns can be related to the instruments for monetary policy operations and liquidity management, the private market financial instruments and trading mechanisms for funds, and the payment and settlement infrastructure for the transfer of those funds. Other findings that can be explained by the monetary policy strategy of the central bank and its policy decision making process were an increase in volatility on meeting days of the Governing Council (GC) of the European Central Bank (ECB), and after the release of new M3 data.\textsuperscript{6}

3 Econometric methodology

Our empirical application uses high frequency observations in order to estimate the various components of the volatility of money market interest rates. There are several approaches to dealing with intraday seasonals. Andersen and Bollerslev [1], [2], [3], [4] and Payne [53] propose a deterministic model. Dacorogna et al. [19] use a time deformation method to obtain a well-behaved volatility process when measured in a new scale, called theta-time. Beltratti

\textsuperscript{4}In the neutral or benchmark allotment the refinancing needs of the banking system are taken into account so that reserve requirements are fulfilled, on aggregate, smoothly (i.e. proportionally over time) over the maintenance period.

\textsuperscript{5}Their paper is mainly descriptive, as no econometric model is presented, and focus only on the volatility of the overnight interest rate, which is sampled at a 3 hour frequency. The period covered in their study is from November 1999 until March 2000. However, they look also at bid-ask spreads and quoting frequency.

\textsuperscript{6}The finding of increasing volatility around GC meetings is confirmed by our results.
and Morana [10], [11] and Morana and Beltratti [51] propose a stochastic volatility model which generalises the previous methodologies, allowing for stochastic seasonal volatility dynamics, and analyze its properties in terms of both consistency with the theoretical aggregation properties of GARCH and flexibility to describe the reactions to announcements and central bank interventions.

The latter approach is motivated by the idea that the intraday seasonal patterns in volatility of financial asset prices are not exactly repeated over time. For example, the geographical model of Dacorogna et al. [19] interprets volatility in terms of the level of activity in the market which may be proxied by the number of traders who are active in the market. Market activity is highest when there is an overlap between two important markets belonging to different time zones. The level of activity is presumably stochastic, depending on how many traders actually participate in the market, a factor which may well change over time, especially if traders react to the trading of other investors. Thus, according to this interpretation it cannot be ruled out that the intraday seasonal component is stochastic. In summary, the empirical findings referred provide statistical support for a more general modelling of intra-day dynamics.

A weakness in the volatility model of Beltratti and Morana [10] is the assumption of an autoregressive model for the true unobserved volatility and the finding that the coefficient of the autoregressive components implies a unit root, so that volatility is an integrated process. Recent contributions to the literature have cast doubts on the IGARCH hypothesis (see Dacorogna et al. [19], Ding et al. [20], Ding and Granger [21], [22]. Lobato and Savin [46], Baillie et al. [7], Bollerslev and Mikkelsen [14], Henry and Payne [41], Andersen and Bollerslev [2], Morana and Beltratti [50], Andersen et al. [5], Bollerslev and Wright [15], Eibens [24]), providing evidence of long-memory in financial asset volatility. A general finding is that volatility is a strongly persistent but mean reverting process.

The model used in this paper generalizes the long-memory stochastic volatility model of Harvey [38] and Breidt et al. [16] by including a stochastic cyclical component; it generalises Beltratti and Morana [10] by allowing for fractional integration in the volatility process. Moreover, it generalises both the above mentioned approaches since the model is multivariate, extending the common long-memory factor model of Morana [49] by the inclusion of a stochastic cyclical component. The model is set in the tradition of SUTSE model of Harvey [39].
3.1 Volatility model

The volatility model for the interest rate measured at intraday frequencies can be written as follows:

\[ i_{t,m} = \bar{i}_{t,m} + \sigma_{t,m} \varepsilon_{t,m} = \bar{i}_{t,m} + \sigma \varepsilon_{t,m} \exp \left( \frac{\mu_{t,m} + \zeta_{t,m}}{2} \right), \tag{1} \]

for \( t = 1, \ldots, T \), and \( m = 1, \ldots, M \), where \( i_{t,m} \) is the interest rate on a bond or a contract with an unspecified maturity, the index \( t \) refers to days and the index \( m \) to the intraday periods, \( \sigma \) is a scale factor, \( \varepsilon_{t,m} \sim NID(0, 1) \), \( \mu_{t,m} \) is the long memory volatility component:

\[ (1 - L)^d \mu_{t,m} = \eta_{t,m} \quad \eta_{t,m} \sim NID(0, \sigma^2 \eta) \quad 0 < d < 0.5, \tag{2} \]

\( \zeta_{t,m} \) is the cyclical volatility component and \( \bar{i}_{t,m} = E[i_{t,m}|I_{t,m}] \), where the information set \( I_{t,m} \) is composed of lagged values of the interest rate and contemporaneous values of the deterministic variables. Squaring both sides, and taking logs, the model may be rewritten as:

\[ \ln \left( \frac{i_{t,m} - \bar{i}_{t,m}}{\bar{i}_{t,m}} \right)^2 = \ln \left[ \sigma \varepsilon_{t,m} \exp \left( \frac{\mu_{t,m} + \zeta_{t,m}}{2} \right) \right]^2, \tag{3} \]

that is:

\[ 2 \ln \left| \frac{i_{t,m} - \bar{i}_{t,m}}{\bar{i}_{t,m}} \right| = \varphi + \mu_{t,m} + \zeta_{t,m} + w_{t,m} \tag{4} \]

where \( \varphi = \ln \sigma^2 + E[\ln \varepsilon_{t,m}^2] \) and \( w_{t,m} = \ln \varepsilon_{t,m}^2 - E[\ln \varepsilon_{t,m}^2] \).

By writing the cyclical component as a linear combination of sine and cosine terms, that is

\[ \psi_{t,m} = \delta_c \cos \lambda_c k_{t,m} + \delta_s \sin \lambda_c k_{t,m} \tag{5} \]

where \( k_{t,m} = 1, \ldots, TM \) indicates the observations in the sample, \( \lambda_c \) is the frequency of the cycle measured in radians, \( (\delta_c^2 + \delta_s^2) \) is the amplitude and \( \tan^{-1}(\delta_s/\delta_c) \) is the phase. The cycle at the fundamental frequency is stochastic by means of time-varying parameters \( \delta_c \) and \( \delta_s \). This allows the cyclical component, while maintaining constant its period, to evolve over time with respect to both the vertical (amplitude) and horizontal axes (phase). Following Harvey [39], the stochastic cyclical component can be rewritten in state space form as

\[ \begin{bmatrix} \psi_{t,m} \\ \psi^*_{t,m} \end{bmatrix} = \begin{bmatrix} \cos \lambda_c & \sin \lambda_c \\ -\sin \lambda_c & \cos \lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t,m-1} \\ \psi^*_{t,m-1} \end{bmatrix} + \begin{bmatrix} \kappa_{t,m} \\ \kappa^*_{t,m} \end{bmatrix}, \tag{6} \]
where $0 \leq \rho \leq 1$ is a damping factor, $\psi_{1,0} = \delta_c$, $\psi_{1,0} = \delta_s$, $\kappa_{t,m} \sim NID(0, \sigma_{\kappa}^2)$ and $\kappa_{t,m}^{*} \sim NID(0, \sigma_{\kappa}^{*2})$, are white noise disturbances with $Cov(\kappa_{t,m}, \kappa_{t,m}^{*}) = 0$. Equivalently, the stochastic cyclical component can be rewritten in univariate form as follows

$$c_{t,m} = \frac{(1 - \rho \cos \lambda_c L) \kappa_t + (\rho \sin \lambda_c L) \kappa_t^{*}}{(1 - 2\rho \cos \lambda_c L) + \rho^2 L^2}. \quad (7)$$

A deterministic cyclical component can be added to model the signal at the harmonics of the fundamental frequency as in Beltratti and Morana [10]. Following Andersen and Bollerslev [4], the deterministic cyclical components can be modelled in terms of the Fourier flexible functional form originally proposed by Gallant [29]:

$$c_{d,m} = c_0 + c_1 m + c_2 m^2 + \sum_{p=2}^{P} (\delta_{cp} \cos \frac{2\pi}{M} m + \delta_{sp} \sin \frac{2\pi}{M} m). \quad (8)$$

The volatility model can be written in multivariate form as follows

$$h_{t,m} = c_{d,t,m} + \Theta \mu_{t,m} + \Phi c_{t,m} + w_{t,m}, \quad (9)$$

where: $h_{t,m}$ is a $N \times 1$ vector of log variance processes ($N = 5$) at time $t$, $m$, $c_{d,t,m}$ is a $N \times 1$ vector of deterministic cyclical components, $\mu_{t,m}$ is a $q \times 1$ vector of long memory factors $(1 - L)^q \mu_{t,m} = \eta_{t,m}, \eta_{t,m} \sim NID(0, \Sigma_\eta)$, with $q \leq N$, $c_{t,m}$ is a $s \times 1$ vector of stochastic cyclical components, with $\kappa_{t,m} \sim NID(0, \Sigma_\kappa)$, $\kappa_{t,m}^{*} \sim NID(0, \Sigma_\kappa^{*})$ with $s \leq N$, $\Theta$ and $\Phi$ are factor loading matrices of dimension $N \times k$, and $N \times s$, respectively, $w_{t,m}$ is a $N \times 1$ vector of i.i.d. innovations, $w_{t,m} \sim IID(0, \Sigma_w)$. The innovation vectors $\eta_{t,m}, \kappa_{t,m}, \kappa_{t,m}^{*}$ and $w_{t,m}$ are assumed to be pairwise orthogonal. When the loading matrix $\Theta$ shows reduced rank $q < N$ then there exist $q$ common long memory factors. This implies the existence of $N - q$ fractional cointegration relationships relating the $N$ log variance processes. On the other hand, a reduced rank loading matrix $\Phi$, $s < N$, implies the existence of $s$ common stochastic cycles among the $N$ log variance processes.

### 3.2 Estimation of the cyclical volatility model

Because a long memory component is included in the specification, the full model cannot be written in state space form and estimated by the Kalman filter. On the other hand, the model may be easily estimated in the frequency domain. However, since deterministic components cannot be handled in the frequency domain, a two step procedure is required. In the first step the
deterministic cyclical component is fitted to the data by OLS; then in the second step ML estimation in the frequency domain yields estimates of the stochastic components of the model. Given the assumption of orthogonality between the various components, the spectral generating function of the full model is additive in the spectral generating functions of the different components. We have therefore

$$G(\lambda_j) = G(e^{i\lambda_j}) = G_\mu(\lambda_j) + G_c(\lambda_j) + G_w(\lambda_j), \tag{10}$$

with $\lambda_j = 2\pi j/TM$ denoting the frequency in radians,

$$G_\mu(\lambda_j) = \Theta [D(\lambda_j)^* \odot \Sigma_q] \Theta',$n

and the elements of the $q \times 1$ vector $D(\lambda_j)^*$ are given by $[2(1 - \cos \lambda_j)]^{-d_i}$

$i = 1, \ldots, q$ and $\odot$ denotes the Adamar product,

$$G_c(\lambda_j) = \Phi [C(\lambda_j)^* \odot \Sigma_k] \Phi',$n

and the elements of the $s \times 1$ vector $C(\lambda_j)^*$ are given by

$$\frac{|1 - \rho_l \cos \lambda_{c,l} e^{i\lambda_j}|^2 + |\rho_l \sin \lambda_{c,l} e^{i\lambda_j}|^2}{|1 - 2\rho_l \cos \lambda_{c,l} e^{i\lambda_j} + \rho_l^2 e^{2i\lambda_j}|^2} = \frac{1 + \rho_l^2 - 2\rho_l \cos \lambda_{c,l} \cos \lambda_j}{1 + \rho_l^4 + 4\rho_l^2 \cos^2 \lambda_{c,l} - 4\rho_l(1 + \rho_l^2) \cos \lambda_{c,l} \cos \lambda_j + 2\rho_l^2 \cos 2\lambda_j},$$

$l = 1, \ldots, s$, and

$$G_w(\lambda_j) = \Sigma_w.$n

Estimation can be carried out by maximum likelihood estimation in the frequency domain. Let $\omega_{h,f}(\lambda_j) = \frac{1}{\sqrt{2\pi/TM}} \sum_{n=1}^{TM} h_{n,f}^* e^{in\lambda}$ denote the discrete Fourier transform of $h_{n,f}^* = 2\ln |f_{n} - \bar{f}_{n} - \bar{c}_{f,n}|, n = 1, \ldots, TM, f = 1, \ldots, N$ and $I(\lambda_j)_{f,r} = \omega_{h,f}(\lambda_j) \bar{\omega}_{h,r}(\lambda_j) f, r = 1, \ldots, N$ a generic element of the cross-periodogram, where $\bar{\cdot}$ denotes complex conjugation and transposition.

The log-likelihood function can be written as

$$\ln L = -\frac{NTM}{2} \ln 2\pi - \frac{1}{2} \sum_{j=1}^{TM-1} \ln |G(\lambda_j)| - \pi tr \left( \sum_{j=1}^{TM-1} (G(\lambda_j)^{-1})^T \text{Re} I(\lambda_j) \right),$$

where $I(\lambda_j)$ is the cross periodogram matrix.
Asymptotic properties  The asymptotic theory of the frequency domain ML estimator for long memory processes is still incomplete. As suggested by Giraitis and Robinson [30], a central limit theorem can be derived from the works of Hannan [35] and Robinson [56]. Under the assumptions stated in Robinson [56], we have

$$T^{1/2}(\hat{\chi} - \chi) \overset{d}{\to} N(0, 2\Omega^{-1} + \Omega^{-1}\Xi\Omega^{-1}),$$

where $\chi$ denotes the vector of parameters,

$$\Omega = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \left( \frac{\partial}{\partial \lambda} \ln g(\lambda, \chi) \right) \left( \frac{\partial}{\partial \lambda} \ln g(\lambda, \chi) \right)' d\lambda,$$

$$\Xi = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} \frac{f(\lambda, \zeta, -\zeta)}{g(\lambda, \chi)g(\mu, \chi)} \left( \frac{\partial}{\partial \lambda} \ln g(\lambda, \chi) \right) \left( \frac{\partial}{\partial \lambda} \ln g(\lambda, \chi) \right)' d\lambda du,$$

and $g(\lambda, \chi)$ is the spectral generating function of $h_t^\ast$, $f(\lambda, \zeta, -\zeta)$ is the fourth cumulant spectral density function of $h_t^\ast$, which vanishes under Gaussianity, and $\zeta$ is the mean of $h_t^\ast$. Therefore the estimator is in general consistent and asymptotically normal, but asymptotically efficient only under the assumption of normality of the innovation process.

Identification of the common long memory and cyclical factors model  Identification of the loading parameters requires $\Sigma_\eta = I_q$, $\Sigma_k = I_s$ and the imposition of $q(q-1)/2$ restrictions on the loading matrix $\Theta$ and $s(s-1)/2$ restrictions on the loading matrix $\Phi$. Alternatively, identification requires standardised factor loading matrices ($\theta_{i,i} = 1$ for $i = 1, ..., q$; $\phi_{i,i} = 1$ for $i = 1, ..., s$), diagonal $\Sigma_\eta$ and $\Sigma_k$ matrices, and the imposition of $k(k-1)/2$ restrictions on the factor loading matrix $\Theta$ and $s(s-1)/2$ restrictions on the loading matrix $\Phi$. The $q(q-1)/2$ restrictions on the loading matrix $\Theta$ and $s(s-1)/2$ restrictions on the loading matrix $\Phi$ can be imposed by setting $\theta_{i,j} = 0$ for $j > i$, $i = 1, ..., N$, $j = 1, ..., q$ and $\phi_{i,j} = 0$ for $j > i$, $i = 1, ..., N$, $j = 1, ..., s$. After estimation the loading matrix can be rotated to give economic interpretation to the factors. Moreover assuming that $\rho > 0$, $0 \leq \lambda_c \leq \pi$, $\kappa_{t,m,j}$ and $\kappa_{t,m,j}^\ast$ are not correlated is sufficient to achieve identification of the two components $\psi_{t,m,j}$ and $\psi_{t,m,j}^\ast$ of each cycle. In addition, the restriction of equal variance-covariance matrices $\Sigma_\kappa = \Sigma_\kappa^\ast$, is imposed for parsimony reasons. See Harvey [39] for further details on identification.

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7See Giraitis and Robinson [30] for a recent account of the available results.
**Optimal smoothing** Optimal smoothing can be obtained via a generalisation of the Wiener-Kolmogorov classical formulas. Given the model

\[ h_{t,m} - \hat{c}_{d,t,m} = \Theta \mu_{t,m} + \Phi c_{t,m} + w_{t,m}, \]  

(14)

the optimal estimators of \( \mu_{t,m} \) and \( c_{t,m} \) in a doubly infinite sample are

\[ \hat{\mu}_{t,m\mid\infty} = \sum_{j=-\infty}^{+\infty} \Pi_{\mu,j} L_{t,m}^j (h_{t,m} - \hat{c}_{d,t,m}), \]  

(15)

\[ \hat{c}_{t,m\mid\infty} = \sum_{j=-\infty}^{+\infty} \Pi_{c,j} L_{t,m}^j (h_{t,m} - \hat{c}_{d,t,m}), \]

where the weights matrices \( \Pi_{\mu,j} \) and \( \Pi_{c,j} \) are easily computed by a frequency domain approach. We have

\[ \Pi_{\mu}(\exp(i\lambda_j)) = \]  

\[ [\Theta [D(\lambda_j) \ast \Sigma_{\eta}] \Theta'] [\Theta [D(\lambda_j) \ast \Sigma_{\eta}] \Theta' + \Phi [C(\lambda_j) \ast \Sigma_{\kappa}] \Phi' + \Sigma_c]^{-1}, \]

\[ \Pi_{c}(\exp(i\lambda_j)) = \]  

\[ [\Phi [C(\lambda_j) \ast \Sigma_{\kappa}] \Phi'] [\Theta [D(\lambda_j) \ast \Sigma_{\eta}] \Theta' + \Phi [C(\lambda_j) \ast \Sigma_{\kappa}] \Phi' + \Sigma_c]^{-1}, \]

and the weights matrices can then been obtained by means of inverse Fourier transforms. Under Gaussianity this yields the minimum mean square error estimator of the persistent and cyclical components, and the minimum mean square error estimator within the class of linear estimators when Gaussianity does not hold.

Recently Harvey and Trimbure [37] have shown the close relation that exists between low-pass and band-pass Butterworth filters (BF) and the model consistent filters which can be obtained from structural time series models, or Generalised Butterworth filters (GBF), through the Wiener-Kolmogorov formulas. In addition to be more general than the BF filters or the standard band-pass filter (Baxter and King [8]), since they allow, for instance, for stationary cycles \( (\rho < 1) \), the GBF filters allows to extract multiple unobserved components in a fully consistent way, avoiding the usual problem associated with the consecutive extraction of components from adjusted observations, namely the dependence of the extracted component on the previous applications of the filter. In addition the GBF filter is the only filter which can handle long memory trend processes. In fact the \((1 - L)^d\) term is included in its specification, while standard low-pass filters are suited only for processes with integer order of integration. As noted by Murray [52] and Benati [12], the application of standard band-pass filters on fractionally integrated processes may lead to serious distortion on the estimated components.
4 The data

The data employed in this study are hourly observations for the overnight rate, and the two-week, one-month, three-month, and twelve-month EONIA swap rates. Hourly observations have been computed as averages of bid-ask quotes taken from Reuters screens. We are aware that these quotes are only indicative. Yet, like Hartmann et al. [36] we assume that there is a close link between market activity and quotes updating.8 The sample is from 4/12/2000 through 31/05/2002, for a total of 3510 hourly observations, or 390 working days. Weekends were removed, and each working day consists of nine intradaily observations (from 9 a.m. to 5 p.m.).

Except the overnight interest rate the other interest rates used in our study are EONIA swap rates. Whereas the EONIA is solely an overnight rate, the EONIA swap rates are traded for maturities from one to three weeks and one to twelve months. Given that the swaps are linked to the EONIA rate they have some advantages compared to, say EURIBOR rates. In an EONIA swap two parties agree to exchange the difference between the interest accrued, at an agreed fixed interest rate for a given period, on an agreed notional amount and the interest accrued on the same amount by compounding EONIA daily over the term of the swap. The ‘fixed leg’ of this agreement is referred to as the EONIA swap rate. The nature of the swap arrangement limits its credit risk since no principal amounts are exchanged.9

Ideally, repo rates should have been used in the study because, referring to collateralized transactions, these are credit risk free. However, the integration of the national repo markets across the euro area is lagging behind the level of integration reached in other markets. Consequently, a single repo yield curve, covering different maturities, has not yet emerged for the euro area (on the repo market in the euro area see ECB [26]). On the other hand, since early 1999 swaps linked to the EONIA have replaced swaps linked to the EURIBOR as the main reference swap rate in the euro money market. In spite of not being credit risk free, the euro area swap market has the advantage over the repo market that it is deeper and more (highly) liquid and that a single yield curve has emerged for the whole euro area.10

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8 One also could argue that when market activity is high traders do not have time to update quotes implying a negative association between trading intensity and price changes. This issue cannot be settled until actual transactions data is available. Nevertheless, evidence from the panel data underlying the calculation of the EONIA suggests that there is a close association between the time series (and the cross sectional) dispersion of prices (measured by the daily range of quoted prices) and the daily traded volumes.

9 We are grateful to Snorre Evjen for helpful discussions on this topic.

10 For further information on the development and characteristics of the EONIA swap market see Santillan et al. [63]
4.1 Descriptive statistics

The hourly volatility process has been computed by squaring the hourly conditional mean innovations. The conditional mean has been modelled by an autoregression, after having accounted for the break process generated by monetary policy decisions concerning the levels of the ECB interest rates (minimum bid rate and the rates on the standing facilities).

Over the sample considered in the study the minimum bid rate was cut by 150 bp, from 4.75% to 3.25%. The interest rate cut was progressive and carried out in four steps (10/05/01: -25bp; 30/08/01: -25bp; 17/09/01: -50bp; 8/11/01: -50bp), generating five regimes in interest rates data. Since the timing of the breaks is known, dummy variables can be constructed and their statistical significance can be tested by means of ADF regressions augmented for the break process. In this framework it is possible to assess the stationarity of the series, controlling for structural change. Table 1 (Panel A) reports the ADF tests for the two competing specifications of the conditional mean. The unit root hypothesis is always rejected at the 5% level, and the AIC criterion and the F-test for the joint significance of the dummy variables support the model augmented for the break process. Yet, shock persistence is still strong, particularly for the twelve-month rate.\footnote{The estimated long-run multipliers are 0.96 for the overnight rate, 0.86 for the two-week rate, 0.95 for the one-month rate, 0.99 for the three-month and twelve-month rate. A full set of results for the unit root tests is available from the authors upon request.}

Table 1 also reports summary statistics on the distributional properties of interest rates, and the conditional variance and log-variance processes in the five regimes.\footnote{The regimes are as follows: 4/11/00-10/05/00, 11/05/01-30/08/01, 31/08/01-17/09/01, 18/09/01-8/11/01, and 9/11/01-31/05/02.} The dispersion of interest rates over the full sample tends to decrease as the maturity increases: the standard deviations of the overnight rate is in fact about 50% larger than that of the one year rate. On the other hand, no clear pattern can be detected for the other moments. Also the relationship between the level of the interest rate and volatility does not seem to be univocal, with volatility increasing or falling with the level of the interest rate. The evidence suggests that the first regime (4/12/00-10/05/01) and the second regime (11/05/01-30/08/01) were periods of low interest rate volatility, whilst volatility increased in the third (31/08/01-17/09/01) and fourth regimes (18/09/01-8/11/01), to fall again in the most recent period (9/11/01-31/05/02), except for the twelve month rate. However, the Kokoszka-Leipus [44] test for structural change detects only one significant break point in all the log variance processes, except for the twelve-month rate. The location of the break point varies slightly across maturities, pointing to November 2001.
for the overnight and three-month rates and to December 2001 for the two-week rate and the three-month rate.\footnote{At its meeting on 8 November 2001 the Governing Council of the European Central Bank decided that under normal circumstances, only the first of its bi-monthly meetings would be an interest rate decision-making meeting. Before that date interest rate decisions could (and have in one occasion) take place at any of the two meetings. Thus, the reduction in the frequency of potential changes in ECB’s interest rates may have contributed to reducing volatility at the shortest end of the money market yield curve. However, the reduction in volatility may be attributed also to the absence of expectations of interest rate changes in the last part of the sample period due to business cycle reasons.} Therefore, the reduction in volatility is statistically significant only in the most recent regime. The dependence of the dispersion on the maturity can also be noted from the variance and log variance series (Panel B and Panel C). The mean log variance displays a U-shape pattern with its minimum at the three-month rate, with log variance dispersion steadily decreasing as the maturity horizon increases. This finding is however not robust across regimes, with the three-month rate showing the lowest dispersion only in two out of five cases. In fact, the volatility curve shows in most regimes a more complex J-pattern. On the other hand, the conditional variance process tends to become less dispersed as the maturity horizon increases. This finding is also robust across subsamples, with the three-month rate showing the lowest dispersion in three out of five cases. Finally, the null of normality for the log variance processes is generally rejected.

Table 2 reports the correlation matrix for the interest rate series. All interest rates are highly correlated, with noticeably higher correlations for consecutive maturities, namely for the one- and three-month rates (0.99), and the three- and twelve-month rates (0.91). The variance and log variance series show a similar pattern. As far as the log variance series are concerned, the largest correlation is between the three- and one-month rates (0.30), followed by the correlation between the one-month and two-week rates (0.24), and between the overnight and the two-week rates (0.20).

Evidence of cyclical patterns in the volatility of interest rates can be gauged from the autocorrelation function for the log variance processes (Figure 1). Two features can be noted. Firstly, there is evidence of a slow decay following a hyperbolic law, rather than an exponential law. Secondly, the decay is not monotonic, showing repetitive patterns with a daily period (nine observations). The former finding points to the presence of long memory in volatility, while the latter finding points to the existence of a daily cycle in volatility.\footnote{Unaccounted structural breaks can also explain the finding of a hyperbolic decay of the autocorrelation function. We further analyse this issue in the following section.} Interestingly, the correlogram shows cyclical dynamics occurring in the two week rate at a frequency higher than the daily one, showing a period of approximately 4.5 hours (1/2 day).
4.2 Deterministic cyclical volatility

Given the rich volatility dynamics the modelling of the log variance processes is carried out in two steps, as explained in the methodological section. The intraday frequencies are assumed to be generated by a deterministic process which is modelled by a flexible Fourier functional form. Then, in order to allow the cycle to change in phase and amplitude, the daily pattern is modelled by a stochastic cycle. The procedure allows the estimation of repetitive patterns which randomly fluctuate around the average deterministic pattern, and therefore yields more realistic and flexible estimates of the cyclical volatility dynamics. If the repetitive patterns reflect trading intensity in the market, there is no reason to assume that the latter may be constant over time.

The estimated flexible Fourier regression is as follows:

\[ c_{d,m} = c_0 + \sum_{p=1}^{2} d_p m^p + \sum_{p=2}^{4} (\delta_{cp} \cos \frac{p2\pi}{M} m + \delta_{sp} \sin \frac{p2\pi}{M} m), \]

where the index \( m \) refers to the daily period \((M = 9)\).

Impulse dummies were also included in the specification in order to account for: (i) special calendar effects, like holidays (Christmas, New Year, Easter and Easter Monday, Labour day); (ii) institutional features of the operational framework (beginning of the maintenance period, number of days before the end of the maintenance period, settlement of the MRO); (iii) accounting practices and payment system related issues, like end of the month and end of the year effects; and (iv) monetary policy related effects (press conference following the Governing Council meeting, interest rate changes).\(^{15}\)

\(^{15}\)The announcement of the MRO tender takes place on Monday at 3.30 p.m., a maximum 24 hours before the submission deadline (Tuesday at 9.30 a.m.) and does not contain “news”. The announcement of the allotment result follows within two hours of the submission of bids (Tuesday, at 11.15 a.m.). The announcement of the allotment result contains, among other, information on the total number of bidders, the total amount bid, the total amount allotted (allotment ratio) and the minimum accepted interest rate (so-called marginal rate).

\(^{16}\)The GC of the ECB is the monetary policy decision body for the euro area. The GC meets every two weeks usually on Thursdays. Unless exceptional circumstances arise interest rate decisions by the GC are taken only at the first meeting of the month. The decision is first announced to the markets at 1.45 pm on a Council day. The first CG meeting of the month is followed by a press conference starting at 2.30 pm, in which the President of the ECB makes an introductory statement summarising the meeting and answers questions by the press. Thus, some Thursdays and/or GC days might be "news" days which convey information to the money market, in so far as the interest rate decisions (timing and magnitude of changes) and the conjunctural assessment (President’s statement) are not fully anticipated by market participants.
Finally, (v) a dummy variable was also included to account for the fall in volatility, after November 2001, associated with the most recent regime.\textsuperscript{17}

The estimated Fourier regressions are reported in Table 3. Most cyclical terms are strongly significant for all log variance processes, coherent with the correlogram (see Panel A). The holidays dummy is strongly significant for all the log variance processes, except for the two-week rate, pointing, as expected, to a reduction in volatility (see Panel B in Table 3). The impact of the institutional features of the operational framework on volatility is documented in Panel B (Table 3). It is noticeable the sharp increase in volatility of the overnight interest rate towards the end of the maintenance period.\textsuperscript{18} Some extra volatility in the overnight interest rate is also noticeable at the beginning of the maintenance period, albeit smaller in magnitude than the end of period effect.\textsuperscript{19} However, the settlement of the MROs does not impact on volatility, while the allotment dummy has a positive impact on the volatility of the overnight and two-week rate. These results illustrate the fact that the allotment on Tuesday may convey news to the market. In fact, in order to facilitate the preparation of the bids the ECB makes available via wire services key liquidity figures on Monday, the announcement day (e.g. the forecast of the aggregated reserve requirements of the banking system and of the average of the autonomous factors). With this information on "quantities" together with some technical assumptions about excess reserves and reserve requirements banks may calculate the so-called neutral (benchmark) allotment. Given the availability of market "prices" of close substitutes to (two week) central bank refinancing (two-week EONIA swap, repo and deposit rates), banks can prepare their bids with "full" information on quantities and prices. The result of the allotment may contain "news" on the current and prospective refinancing needs, in so far as it deviates from the neutral benchmark. For example, in its allotment decisions the ECB sometimes took into consideration counterpart’s bidding behaviour in previous MROs.

The increase in volatility at the end of the maintenance period has been previously documented, both for the US Fed funds rate and for the euro

\textsuperscript{17} We have also allowed sine and cosine terms involving the weekly frequency, in order to capture day-of-the-week effects. These resulted not to be statistically significant, once allowed for the dummies under ii), iii) and iv).

\textsuperscript{18} End of maintenance period and end of year effects are also significant for the two-week rate, although their impact on volatility is much smaller than for the overnight rate.

\textsuperscript{19} The liquidity situation at the beginning of each maintenance period is somewhat more volatile than during the remainder of the maintenance period (except after the last MRO). This is due to the fact that there is a liquidity "hangover" from one maintenance period to the next, given that the maturity of the operations is two weeks and its frequency is weekly.
area. The important point to add, which had not yet been documented, at least for the euro area, is that the impact of the institutional features of the operational framework on volatility seems to be concentrated at the shortest end of the curve, thus not affecting volatility along the money market yield curve. It is also interesting to note that the impact on interest rate volatility, of accounting practices and payment system factors (end of month and end of year effects) are also confined to the shortest end of the curve. Nevertheless, there seems to be an end-of-month effect significantly affecting (only) the three-month rate. Finally, the “pure” monetary policy factors (press conferences following the GC meetings and interest rate changes) are associated with higher volatility for all maturities. The impact of press conferences on volatility is further explored in the following sections.

In Figure 2 the estimated deterministic volatility cycle is plotted for all series over the first week of the sample. The estimated weekly pattern is similar across maturities, with correlation coefficients being higher for longer maturities than for shorter maturities. For instance, the one-month and three-month rate cycles show a correlation coefficient equal to 0.86, while the correlation coefficient between the one-month and the overnight rate cycles is equal to 0.70. On average volatility is higher on Tuesday than in any other day of the week for the overnight and two-week rates.

The deterministic average daily pattern estimated by the filter is also interesting (Figure 3). Volatility is high at the beginning of the day, falls not monotonically until 2 p.m., and increases again until 5 p.m. There are however some noteworthy differences across maturities, which explain the moderate correlation coefficients.

For instance, while the U-shaped pattern seems to be an appropriate description of the average intraday dynamics, it is not homogeneous across maturities, particularly at the end of the working day. While for the overnight rate volatility increases, albeit not monotonically, after the lunch through, for the other maturities the “after lunch” volatility peak is reached between 3 p.m. and 4 p.m., with volatility falling thereafter. This seasonal pattern is clearly associated with trading occurring over the day, and consistent with previous evidence provided for the exchange rates and stock market data.

Finally, in Figure 4 the deterministic filter for the overnight interest rate is plotted over a generic maintenance period. The pattern detected in the data is similar to the one reported by Prati et al. [54] for the euro area, and also confirmed by Hartmann et al. [36], with volatility being high at the beginning of the maintenance period, falling over time, to increase again towards the end of the maintenance period. Such pattern though is not

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20 As explained the result of the allotment on Tuesday may convey news to the market.
present in the volatility of other interest rates (see Figure 5). These patterns are fully in line with the predictions of the theoretical models surveyed in section 2.

4.3 Persistence analysis

Table 4 reports the results of the persistence analysis for the break/cycle-free log variance series. By controlling for both deterministic cyclical components and structural change, the persistence analysis should provide reliable results concerning the causes of volatility persistence. In fact unaccounted structural breaks may lead to upward biased estimates of the persistence parameter, while unaccounted cyclical features may lead to downward biased estimates of the persistence parameter\textsuperscript{21}. In particular, in the analysis we have employed the local Whittle estimator suggested by Künsch [45] and Robinson [60] and, to assess the robustness of the results, we compared the estimates with those obtained using the averaged periodogram estimator (Robinson [57], Lobato and Robinson [47]), the log periodogram regression (Geweke and Porter-Hudak [31]; Robinson [59]) and a new estimator proposed by Robinson [61].

Robinson [60] has shown that the local Whittle estimator dominates the averaged periodogram estimator and the log periodogram estimator under several respects. Firstly, it is asymptotically efficient, secondly it does not require the trimming of the lowest frequency estimates, thirdly it does not assume Gaussianity, and fourthly it does not require the selection of a numerical value for any additional parameter. On the other hand, the estimator proposed by Robinson [61], which we denote as the LM estimator, is consistent. In empirical applications it has been found that the LM estimator is in general more robust to the bandwidth than the other available estimators. Optimal bandwidth theory (Robinson [58], Henry and Robinson [40], Hurvich et al. [43], Delgado and Robinson [23], Henry [42]) has been employed to select the number of periodogram ordinates to be used in the estimation of the Hurst exponent. In addition we have implemented the LM, LR and Wald stationarity tests (Lobato and Robinson [48], Robinson [61]). The latter statistics assume weak dependence under the null (see the Appendix for a description of the statistics).\textsuperscript{22}

As shown in Table 4, there is clear evidence of a strong degree of long memory for all the log variance series. The estimated Hurst exponents are in fact statistically different and larger than the I(0) threshold (0.5), and fall

\textsuperscript{21} This latter result was firstly documented for the persistence parameter estimated from GARCH models. See for instance Andersen and Bollerslev [1].

\textsuperscript{22} Optimal bandwidth estimation was carried out by means of a GAUSS routine written by Mark Henry, to whom the authors are grateful.
in the long memory interval \((0.5 < H < 1)\), pointing to both strong persistence and covariance stationarity. Similar estimates have been obtained from the different estimators for the various log variance processes, with the LM estimator in general providing the more conservative estimates, and the twelve-month rate log variance series showing the lowest degree of persistence.\(^{23}\) For the overnight rate and the one-month rate the estimated Hurst exponent ranges between 0.71 and 0.77, and 0.72 and 0.76, respectively, while for the two-week rate and the three-month rate ranges between 0.75 and 0.85, and 0.73 and 0.84, respectively. On the other hand, for the twelve-month rate the estimates range between 0.63 and 0.66. Coherent with the above results, the stationarity tests clearly point to the rejection of the null of weak dependence, against the alternative of long memory.

Despite the range of variability, the null of equal fractional differencing parameters cannot be rejected for all the log variance processes. In Figure 6 we report the p-value of the test (Robinson [59]) for all possible pairs of log variance processes and the estimated fractional differencing parameter for bandwidths between 40 and 200 periodogram ordinates, obtained by means of the bivariate log-periodogram estimator (Robinson [59]). As shown in the plot, the null of equal fractional differencing parameters is never rejected at the 1% level. However, the estimates tend to show some variability, with most of them falling in the interval 0.25 and 0.35 in the (relatively) stable region (140-200 ordinates), with the average estimate ranging between 0.28 and 0.30.\(^{24}\) For bandwidths larger than 200 ordinates the evidence of a common fractional differencing parameter is still strong for the overnight rate, the two-week rate and the one-month rate, while rejection is found for the three-month and twelve-month rates relatively to the shortest maturities, but not relatively to each other.\(^{25}\) The overall conclusion favours the hypothesis of a common degree of fractional integration, inviting cointegration analysis.

To test for fractional cointegration we employed the Robinson and Yajima [62] fractional cointegrating rank test (see the Appendix for a description of the methodology). In order to assess the robustness of the findings, we report in Table 5 the results assuming three values for the fractional differencing parameter, namely \(d = 0.25, 0.30, 0.35\). In addition, since the test may be sensitive to the number of periodogram ordinates employed, we computed the test for bandwidths ranging between one and ten ordinates. A first interesting finding is that the test seems to be robust to the value of the fractional differencing parameter. For bandwidths smaller than four ordinates

\(^{23}\)The Hurst exponent is related to the fractional differencing parameter through the relationship \(d = H - 0.5\).

\(^{24}\)In the plot we have reported the estimated Hurst exponent: \(H = d + 0.5\).

\(^{25}\)This latter results are available upon request to the authors.
the results suggest that the two largest eigenvalues are sufficient to account for more than 90% of total variance, while for larger bandwidths the proportion of explained variance exceeds 80%. Therefore, two common long memory factors seem to be sufficient to explain the variability of the series, pointing to the existence of three fractional cointegration relationships among the five log variance processes. In correspondence of the selected bandwidth (three ordinates)\(^2\), the Robinson and Yajima [62] test points to the existence of two cointegration relationships (10% significance level). However, since the selection of the threshold for the critical value (0.01/N = 0.002) is arbitrary, on the basis of the proportion of explained variance we have concluded in favour of three cointegrating vectors, and therefore two common long memory factors. From Table 5 it can also be noted that there is strong evidence against the existence of four cointegrating vectors, i.e. a single long memory factor driving the five log variance processes.

5 Common factor analysis

The estimation of the common long memory factors has been carried out by means of the common long memory factor model plus stochastic cycle discussed in the methodological section. On the basis of the fractional cointegration analysis, a two common long memory factor model was selected, while a single cycle was selected on the basis of the evidence provided by the correlogram and the intradaily deterministic cycle analyses. Therefore, the estimated model can be written as

\[
\mathbf{h}_{t,m}^{bcf} = \Theta \mathbf{\mu}_{t,m} + \Phi \mathbf{c}_{t,m} + \mathbf{w}_{t,m},
\]

where: \(\mathbf{h}_{t,m}^{bcf}\) is a 5 \times 1 vector of break/(deterministic) cycle free log variance processes at time \(t, m\), \(\mathbf{\mu}_{t,m}\) is a 2 \times 1 vector of long memory factors 

\((1 - L)^d \mathbf{\mu}_{t,m} = \eta_{t,m}, \eta_{t,m} \sim NID(0, \Sigma_{\eta})\), 

\(\mathbf{c}_{t,m}\) is a 5 \times 1 vector of stochastic cyclical components, with 

\(\kappa_{t,m}^{c} \sim NID(0, \Sigma_{\kappa}), \kappa_{t,m}^{s} \sim NID(0, \Sigma_{\kappa^{s}}). \Theta \) and \(\Phi \) are factor loading matrices of dimension 5 \times 2, and 5 \times 5, respectively, with the former written in such a way to show unitary diagonal elements in the upper 2 \times 2 submatrix, and the latter set equal to the identity matrix.

Finally, \(\mathbf{w}_{t,m}\) is a 5 \times 1 vector of i.i.d. innovations, \(\mathbf{w}_{t,m} \sim IID(0, \Sigma_{w})\). The innovation vectors \(\eta_{t,m}, \kappa_{t,m}, \kappa_{t,m}^{s}\) and \(\mathbf{w}_{t,m}\) are assumed to be pairwise orthogonal, but no restrictions have been imposed on the variance-covariance matrices \(\Sigma_{\eta}, \) and \(\Sigma_{\kappa}, \) while the matrix \(\Sigma_{\eta}\) has zero off-diagonal elements. For parsimony reasons we have also assumed \(\Sigma_{\kappa} = \Sigma_{\kappa^{s}}. \) Coherent with the

\(^{2}\) The results for bandwidth equal to four and five ordinates are similar.
properties of the autocorrelation functions, the period of the stochastic cycle has been set to nine hours for all the maturities, apart from the two week rate for which the period was set to 4.5 hours.

**Stochastic cyclical components** As shown in Table 6 (Panel B), a stochastic cycle can be detected in all the log variance series, although the estimated variance of the innovation tends to differ noticeably across maturities. Interestingly, the stochastic cyclical component seems to matter more for the overnight, the three-month and twelve-month rates, than for the two-week rate. On the other hand, the estimated damping parameters is similar across maturities (0.30-0.39), apart from the overnight rate, for which a higher value has been found (0.55). The information conveyed by the stochastic cyclical component tends to be idiosyncratic, since the processes are weakly correlated, with the two week and overnight rates being the only exception (the correlation coefficient is about -0.85). The flexibility allowed by the stochastic specification is evident from Figure 7, where the stochastic and deterministic weekly cycles, for the overnight and the twelve-month rates, are contrasted over two weeks in the sample. Contrary to the deterministic model, the stochastic specification allows for significant changes in amplitude and phase, being however close to the average behaviour described by the deterministic cycle.

The stochastic specification also allows investigating the reaction of volatility to specific events. In Figure 8 we contrast the deterministic and stochastic cycles for two days with GC meetings followed by press conferences, over the 2 p.m. - 4 p.m. time interval. The reaction of the market to the press conferences was different. In one case (6/12/2001) the reaction suggests resolution of market uncertainty - volatility started above average and declined sharply within two hours. By contrast, in 4/4/2002, the reaction suggests enhanced market uncertainty - volatility started below average and rose sharply within two hours.

**Long memory components** As shown in Table 6 (Panel A), the estimated fractional differencing parameter is coherent with the results of the semiparametric analysis, pointing to a moderate degree of long memory in all the log variance processes \((d = 0.28)\). The estimated factor loading matrix suggests an interpretation of the common factors. The first factor seems to explain the long-memory dynamics of the shorter-term rates. Shocks to this factor do not seem to transmit further along the term structure. On the other hand, the forward propagation of shocks along the term structure seems to be explained by the second long-memory factor, which affects all other ma-
turities. The two factors also differ in terms of variability, with the first factor showing an innovation variance five times larger than the one of the second factor. As expected, the two factors are orthogonal (the correlation coefficient is about 0.28), and therefore are related to different persistent dynamics. However, as it is apparent from Figure 9, some of the peaks in the conditional volatility factors are highly correlated, which is an intriguing feature of the data given that the factors are orthogonal and all cyclical components have been filtered out.

In order to understand the underlying causes of these patterns Figures 9 and 10 plot the two conditional volatility factors (smoothed; middle and lower pannels). In Figure 9, in the upper panel, a dummy variable is plotted identifying the press conferences following the GC of the ECB and also the interest rate decreases that were decided (at the moment they were announced). In Figure 10, a dummy variable is plotted identifying the end of maintenance periods.

Two facts are noteworthy. Firstly, both factors show volatility peaks around GC meetings in which interest rate decisions were taken, suggesting that these decisions were informative (Figure 9). There is an additional volatility peak (after the 200th day in the sample) which is associated with the ECB interest rate decrease in the aftermath of the terrorist attacks of the 11th September 2001. Secondly, we are left with a few volatility peaks that coincide with specific maintenance periods (Figure 10). All cases (four: around days 60, 100, 240 and 260) correspond to maintenance periods that were marked by underbidding incidents (February, April, October, and November all in 2001). These episodes, in turn, were all related to expectations of interest rate decreases, as explained in section 2. Figure 11 complements the analysis by showing the estimated persistent and cyclical (stochastic) components of the log variance process for the 12-month EONIA swap rate. The persistent component shows peaks corresponding to maintenance periods that were marked by underbidding incidents. The episodes in April and November 2001, also seem to have affected the stochastic cyclical component.

Thus, we are able to document, for the first time with euro area data, the effects of liquidity shortages created, or expected in the aftermath of the underbidding episodes. However, these effects are not the rule and can be explained by exceptional circumstances. In general, as documented in section 4, the temporary liquidity effects noticed at the end of each maintenance period, matter only for the shortest maturities and are not transmitted along the money market yield curve.
Residual components  Finally, the residual components show a non negligible negative correlation for consecutive maturities, apart from the twelve month rate, with correlation decreasing as the distance between maturities increases. The specification tests, are shown in Figure 12. There is little evidence of misspecification and evidence of short-term serial correlation is not compelling, even for the shortest maturities.

6 Conclusions

In the paper we investigated the properties of the volatility processes of money market interest rates in the euro area. For that purpose, we developed a new multivariate unobserved components model, that allows for both long-memory and stationary cyclical dynamics. The empirical analysis was conducted using high frequency data. The estimates showed the presence of repetitive intradaily and monthly patterns that can be explained by the microstructure of the money market. For example, a U-shaped volatility pattern could be detected, with higher volatility at the beginning and end of the day, with a minimum during the lunch break. Such pattern is also a feature of volatility within the reserve maintenance period, but only for interest rates at the shortest end of the maturity spectrum. Strong persistence was detected in the volatility processes, pointing to a moderate degree of long-memory, also after accounting for structural breaks, which is common across maturities (about 0.28). We found that two common long-memory factors drove the volatility processes. The first explained the long-memory dynamics of the shortest maturity. The other explained the transmission of volatility to other maturities. We showed that the announcement of interest rate changes exercised the strongest impact on the volatility of the shortest maturities. We were also able to document persistent effects of liquidity shortages, which were transmitted along the money market yield curve. However, these effects were not the rule and could be explained by exceptional circumstances. In general, liquidity effects were found to be cyclical and thus, short-lived, confined to the end of maintenance periods, and not transmitted along the money market yield curve.
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7 Appendix: Semiparametric methodologies and stationarity test

Local Whittle estimator (Kunsch [45]; Robinson [60]) It requires the minimization of the following objective function

\[ Q(C, H_{LW}) = \frac{1}{m} \sum_{j=1}^{m} \left( \log C \lambda_j^{-2H} + \frac{\lambda_j^{2H-1}}{C} I(\lambda_j) \right) \]

where \( I(\lambda_j) \) is the periodogram at frequency \( \lambda_j = 2\pi j / T \), \( j = 1, \ldots, m \), \( m \) is the bandwidth parameter, \( C \) is a positive constant. For \( H < 0.5 \) the process is antipersistent, for \( H > 0.5 \) it is long memory, and for \( H = 0.5 \) it is weakly dependent. It is shown that

\[ \sqrt{m} \left( \hat{H}_{LW} - H \right) \overset{d}{\rightarrow} N\left( 0, \frac{1}{4} \right) . \]

LM estimator (Robinson [61]) An alternative consistent estimator for \( H \) is

\[ \hat{H}_{LM} = \frac{\sum_{j=1}^{m} (1 - 2v_j) I(\lambda_j)}{\sum_{j=1}^{m} (2 - 2v_j) I(\lambda_j)} \]

where \( v_j = \log j - \frac{1}{m} \sum_{j=1}^{m} \log j \). We denote this estimator as \( H_{LM} \) since it can be derived from the LM test of Lobato and Robinson [48]. The LM estimator has the same limiting distribution of the Local Whittle estimator under weak dependence, but its asymptotic distribution under long memory is still unknown.

Averaged periodogram estimator (Robinson [57]; Lobato and Robinson [47]) Another estimator widely used in empirical work is obtained from the averaged periodogram

\[ \hat{H}_{AP,q} = 1 - \frac{1}{2 \ln q} \ln \left( \frac{\hat{F}(qm)}{\hat{F}(\lambda m)} \right) \]
where \( \hat{F}(\lambda) = \frac{2\pi}{n} \sum_{j=1}^{[\lambda n/2\pi]} I(\lambda_j) \). The limiting distribution of \( H_{AP,q} \) is

\[
m^{1/2} \left( \hat{H}_{AP,q} - H \right) \xrightarrow{d} N \left( 0, \frac{(1 + q^{-1} - 2q^{1-2H})(1-H)^2}{(\ln q)^2} \frac{1}{(3 - 4H)} \right)
\]

for \( \frac{1}{2} \leq H \leq \frac{3}{4} \) and

\[
m^{2-2H} \left( \hat{H}_{AP,q} - H \right) \xrightarrow{d} N \left( 0, \frac{(1 - q^{2H-2})(1-H)}{\ln q} \frac{\Gamma(2(1-H)) \cos((1-H)\pi)}{(2\pi)^{2-2H}} P \right)
\]

as \( T \to \infty \), where \( P \) is a random variable with unknown distribution, for \( \frac{3}{4} < H < 1 \).

**Log periodogram estimator (Geweke and Porter-Haudak [31]; Robinson [59])** A consistent but less efficient estimate of the fractional differencing parameter can be obtained by the log periodogram regression

\[
\ln I(\lambda_j) = c + d(-2 \log \lambda_j) + \mu_j \quad j = 1, \ldots, m,
\]

where \( l \) is a trimming parameter. It has been shown that

\[
\sqrt{m} \left( \hat{d}_{LP} - d \right) \xrightarrow{d} N \left( 0, \frac{\pi^2}{24} \right).
\]

A test for the equality of the fractional differencing parameter \( H_0 : \mathbf{P}d = 0 \) for two processes can be computed in the following way

\[
T = \hat{d} \mathbf{P}' \left[ \mathbf{0}, \mathbf{P} \right] \left\{ (\mathbf{Z}' \mathbf{Z})^{-1} \otimes \hat{\Omega} \right\} \left[ \mathbf{0}, \mathbf{P} \right]'^{-1} \mathbf{P} \hat{d}' \sim \chi^2_1,
\]

where \( \mathbf{Z} = \left( \mathbf{Z}_{d+1} \ldots \mathbf{Z}_m \right)' \), \( \mathbf{Z}_j = \left( \begin{array}{c} 1 \\ -2 \log \lambda_j \end{array} \right) \), \( \mathbf{P} = \left( \begin{array}{cc} 1 & -1 \end{array} \right) \), and \( \hat{\Omega} \) is the sample variance covariance matrix of the error terms. A constrained estimate of the fractional differencing parameter, under the constraint \( d = \mathbf{Q} \theta \), can then be computed as

\[
\left[ \begin{array}{c} \hat{\theta} \\
\end{array} \right] = \left\{ \mathbf{Q}' \left( (\mathbf{Z}' \mathbf{Z})^{-1} \otimes \hat{\Omega}^{-1} \right) \mathbf{Q}_1 \right\}^{-1} \mathbf{Q}' \text{vec} \left( \hat{\Omega}^{-1} \mathbf{Y}' \mathbf{Z} \right)
\]

where \( \mathbf{Q}_1 = \left[ \begin{array}{cc} \mathbf{I}_2 & 0 \\
0 & \mathbf{Q} \end{array} \right] \), \( \mathbf{Q} = \mathbf{e}_2 \), and \( \mathbf{Y} = \left( \begin{array}{cc} \ln I(\lambda_j)_1 & \ln I(\lambda_j)_2 \end{array} \right) \) .
**Stationarity test** A test for the null $H_0 : H = 0.5$ against a two-sided alternative can be computed as follows

$$LM = m \left( \sum_{j=1}^{m} v_j I(\lambda_j T) \right)^2 \frac{d}{\chi^2_1},$$

$$W = 4m \left( \hat{H}_{LM} - H \right)^2 \frac{d}{\chi^2_1},$$

$$LR = 2m \left\{ \frac{m}{\sum_{j=1}^{m} \lambda_j^{2H-1} I(\lambda_j T)} \log \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j + \left(2\hat{H}_{LM} - 1\right) \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right\} \frac{d}{\chi^2_1}.$$

**Bandwidth selection** A theory of optimal bandwidth selection, based on the minimization of the asymptotic MSE, has been proposed for various semiparametric estimators (Robinson [58], Henry and Robinson [40], Hurvich et al. [43]) and efforts have been made to make it implementable (Delgado and Robinson [23], Henry [42]). Following Henry [42] the optimal bandwidths can be stated as follows:

$$m_{LW}^* = \left( \frac{3}{4\pi} \right)^{4/5} \left| \frac{d^*}{12} \right|^{-2/5} T^{4/5}$$

for the local Whittle estimator and $-\frac{1}{2} < d < \frac{1}{2}$;

$$m_{LP} = \left( \frac{27}{512\pi^2} \right)^{1/5} \left| \frac{d^*}{12} \right|^{-2/5} T^{4/5}$$

for the log periodogram estimator, $-\frac{1}{2} < d < \frac{1}{2}$ and $d^* \neq 0$;

$$m_{AP} = \left( \frac{3 - 2d^*}{4(2\pi)^4(1-4d^*)} \right)^{1/5} \left| \frac{d^*}{12} \right|^{-2/5} T^{4/5}$$

for the averaged periodogram estimator with $0 < d < \frac{1}{4}$, while for $\frac{1}{4} < d < \frac{1}{2}$
\[
m_{AP_2} = \frac{T^{2\pi d_x}}{2\pi} \left\{ \frac{2\Gamma(1 - 2d_x) \cos \left( \left( \frac{1}{2} - d_x \right) \pi \right)}{8} \left( \frac{2d_x - 3}{2d_x (\tau^* + d_x/12)} \right) \right. \\
+ \frac{1}{\tau^* + d_x/12} \left[ \frac{(3/2d_x)^2}{(2d_x)^2} \frac{(1/2 - d_x)}{2d_x (4d - 1)} \right] - \frac{1}{(4d_x + 1) (d_x + 1/2)^2} \left\{ \frac{1}{\Gamma(4d_x + 2)} \right\} \frac{1}{\tau^*} \\
- \left. \frac{1}{\Gamma(4d_x + 2)} \right\} \frac{1}{\tau^*} \right\}^{\frac{1}{4\pi}} \\
\]

finally, for the LM estimator proposed by Robinson [61] we used

\[
m_{LM} = \left( \frac{3}{4\pi} \right)^{4/5} |\tau^*|^{-2/5} T^{4/5}.
\]

Optimal bandwidths and the corresponding estimates of \(d\) are then derived together using the recursion

\[
\hat{d}_x^{(k)} = d(\hat{\lambda}^{(k)}), \\
\hat{\lambda}^{(k+1)} = \lambda^{(k)}(\hat{d}_x^{(k)}, \tau^*)
\]

\(i = LW, AW\), for the Gaussian semiparametric estimator and the averaged periodogram estimator, while for the log periodogram estimator and the LM estimator the formula simply is \(\hat{d}_x = d(\hat{\lambda}_j)\) \(j = LP, LM\). The recursions were started setting \(\hat{\lambda}^{(0)} = [T^{1/5}]\) and an estimate of the smoothness parameter \(\tau^* = \frac{f^{*\prime\prime}(0)}{2f^* (0)}\), where the function \(f^*(\cdot)\) describes a short-term correlation structure in the spectral density of the process \(f(\lambda) = |1 - \exp(i\lambda)|^{1-2\hat{\mu}} f^*(\lambda)\), was obtained using the least square regression suggested by Delgado and Robinson [23]

\[
I(\lambda_j) = \sum_{k=0}^{2} Z_{jk} \left( \hat{H} \right) \hat{\beta}_k + \hat{\varepsilon}_j \quad j = 1, ..., \hat{\lambda}^{(0)}
\]

where \(Z_{jk} \left( \hat{H} \right) = |1 - \exp(i\lambda_j)|^{1-2\hat{\mu}} \lambda_j^k/k!\). \(\tau^*\) is then estimated by \(\hat{\beta}_2/2\hat{\beta}_0\).

**Fractional cointegration test**  Let's consider the \(n \times 1\) vector \(w_t\) of long memory processes with continuous spectral distribution function satisfying the condition \(f_w(\lambda) \sim A E \tilde{A}\) as \(\lambda \rightarrow 0^+\), where \(\tilde{A}\) denotes the complex conjugate of \(A\), \(E\) is a real symmetric matrix of dimension \(n \times n\), and
$$\Lambda = \text{diag} \left\{ e^{i d_i / 2} / \lambda^{-d_i} \right\}_{i=1}^{n}, \quad 0 < d_i < 1/2, \ i = 1, \ldots, n.$$ Fractional cointegration implies $\beta' \mathbf{E} \beta = 0$, so that $\mathbf{E}$ is of reduced rank $k = n - r$, where $r$ is the number of cointegration relations. Therefore, the number of cointegration relations is given by the number of non-zero eigenvalues of the $\mathbf{E}$ matrix ($r$), and the number of common long memory factors is given by $k = n - r$.

Robinson and Yajima [62] have proposed a cointegrating rank test based on the significance of the eigenvalues of the $\mathbf{E}$ matrix. Assuming that all the processes are characterised by the same order of fractional integration, or that the series have been partitioned in groups according to the order of fractional integration, the estimator proposed by Robinson and Yajima [62] is

$$\hat{\mathbf{E}}_m = \frac{1}{m} \sum_{j=1}^{m} \lambda_j^{2 \hat{d}_j} \Re \{ I (\lambda_j) \},$$

where $\hat{d}_s = \frac{1}{n} \sum_{i=1}^{n} \hat{d}_i$, where $\hat{d}_i$ are the estimated fractional differencing parameters of each individual series.

Under the assumptions detailed in Robinson and Yajima [62], the estimator is consistent and asymptotically normally distributed

$$m^{1/2} \text{vec}(\mathbf{E}(\hat{d}_s) - \mathbf{E}) \rightarrow N(0, \frac{1}{2} (\mathbf{E} \otimes \mathbf{E} + (\mathbf{E} \otimes \mathbf{E}_1, \ldots, \mathbf{E} \otimes \mathbf{E}_p))),$$

where $\mathbf{E}_i$ denotes the $i$-th column of $\mathbf{E}$. By denoting the eigenvalues of the estimated $\mathbf{E}(\hat{d}_s)$ matrix as $\hat{\delta}_i$, $i = 1, \ldots, n$, ordered as $\hat{\delta}_1 > \ldots > \hat{\delta}_{n-r} > 0$, with $\hat{\delta}_{n-r+1} = \ldots = \hat{\delta}_n = 0$ for $r \geq 1$, it is shown that

$$m^{1/2} \left( \hat{\delta}_i - \delta_i \right) \sim N \left( 0, \delta_i^2 \right).$$

By defining

$$\hat{\pi}_j = \frac{\hat{\delta}_{n-j+1,n}^{(1)}}{\hat{\delta}_{1,n}^{(1)}}, \quad j = 1, \ldots, n - 1,$$

where $\hat{\delta}_{k,l}^{(i)} = \sum_{z=k}^{l} \hat{\delta}_z^{(i)}$, and

$$s_j = \frac{\hat{\delta}_{n-j+1,n}^{(1)} \hat{\delta}_{1,n-j}^{(1)} + \hat{\delta}_{1,n-j}^{(1)} \hat{\delta}_{n-j+1,n}^{(2)}}{\hat{\delta}_{1,n}^{(1)^2}},$$

it is shown that

$$m^{1/2} \left( \hat{\pi}_j - \pi_j \right) / s_j \overset{d}{\rightarrow} N(0, 1), \quad j = 1, \ldots, n - 1, \ r = 0.$$
In practice, since the asymptotic distribution of $\hat{\pi}_r$ is standard normal only when $r = 0$, a test for a non zero cointegration rank can be carried out by considering the 100(1-$\alpha$)% upper confidence interval

$$\hat{\pi}_r + s_\tau z_{\alpha}/m^{1/2},$$

not rejecting the null of rank = $r$ if $\hat{\pi}_r + s_\tau z_{\alpha}/m^{1/2} < 0.1/n$, against the alternative hypothesis of rank > $r$. A similar tests can be constructed by evaluating the number of zero eigenvalues of the zero frequency spectral matrix of the fractionally differenced processes.

Kokoszka and Leipus [44] estimator of the break points

Consider the following process

$$U_N(k) = \left(1/\sqrt{N} \sum_{j=1}^{k} r_j^2 - k \left(N \sqrt{N} \right) \sum_{j=1}^{N} r_j^2 \right)$$

for $0 < k < N$, where $r_t^2$ is the squared return at time $t$. The proposed estimator of the break point is

$$\hat{k} = \min \left\{ k : |U_N(k)| = \max_{1 \leq j \leq N} |U_N(j)| \right\},$$

i.e. the point at which there is the maximal evidence of a break point. The statistical significance of the break point can be evaluated using the results

$$\sup \{ |U_N(k)| \} / \hat{\sigma} \rightarrow_{DP,1} \sup \{ B(k) : k \in [0,1] \},$$

where $B(k)$ is a Brownian bridge and $\sigma^2 = \sum_{j=\infty}^{\infty} \text{cov}(r_j^2, r_0^2)$. The 90%, 95%, and 99% critical values (two sided test) are 1.22, 1.36 and 1.63, respectively.
Table 1, Panel A: Summary statistics for hourly interest rates

<table>
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<th>Mean</th>
<th>i0_R1</th>
<th>i0_R2</th>
<th>i0_R3</th>
<th>i0_R4</th>
<th>i0_R5</th>
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<th>i0.5_R4</th>
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<td>0.095</td>
<td>0.056</td>
<td>0.324</td>
<td>0.136</td>
<td>0.082</td>
<td>0.042</td>
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<td>0.082</td>
<td>0.044</td>
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<tr>
<td>Sk</td>
<td>1.764</td>
<td>5.111</td>
<td>2.253</td>
<td>0.855</td>
<td>0.838</td>
<td>-3.914</td>
<td>-1.778</td>
<td>-1.205</td>
<td>-0.693</td>
<td>2.813</td>
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<tr>
<td>Ex-k</td>
<td>3.783</td>
<td>44.21</td>
<td>13.26</td>
<td>2.112</td>
<td>13.80</td>
<td>27.5</td>
<td>6.017</td>
<td>1.220</td>
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<tr>
<td>Sk</td>
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| t_ADF_bp | -7.605 | -8.713 | -8.937 | -4.203 | 3.003 |
| t_ADF    | -2.021 | -0.992 | -0.914 | -1.210 | -1.949 |
| F        | 0.000  | 0.000  | 0.000  | 0.000  | 0.005 |
Table 1, Panel B: Summary statistics for hourly variances

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<td>0.030</td>
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<td>0.012</td>
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<td>12.91</td>
<td>16.66</td>
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<td>9.737</td>
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<td>0.001</td>
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Table 1, Panel C: Summary statistics for hourly log variances

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<th>i_{3,R3}</th>
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<td>2.779</td>
<td>2.962</td>
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<td>2.617</td>
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<td>-0.134</td>
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<td>-0.854</td>
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<td>-0.659</td>
<td>-0.359</td>
<td>-0.210</td>
<td>-0.649</td>
<td>-0.222</td>
<td>-0.852</td>
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<td>1.532</td>
<td>1.111</td>
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</tbody>
</table>

The table reports summary statistics for daily interest rates (Panel A), squared innovations (Panel B) and log squared innovations (Panel C). \( i_j \) denotes the overnight rate, the one-week rate, the one-month rate, the three-month rate, and the twelve-month rate, respectively. \( R_t \) \( i = 1, \ldots, 5 \) denotes the monetary policy regimes. The last table reports figures for the whole sample. \( N_{BJ} \) is the Bera Jarque normality test. \( t_{ADF,bp} \) is the ADF statistics for the model augmented for the break process. \( t_{ADF} \) is the standard ADF statistics, \( AIC_{bp} \) is the Akaike Information Criterion for the model augmented for the break process, AIC is the Akaike Information Criterion for the standard ADF regression, and \( F \) is the p-value of the F statistic for the significance of the break process. KL denotes the Kokoszka and Leipus \( [44] \) test for breaks in variance (5% critical value: 1.36; 10% critical value: 1.22). The time span analysed is 04/12/2000:31/05/02, for a total of 3510 observations.
Table 2, Panel A: Correlation matrix

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<th>(i_3)</th>
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<tr>
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<td>(i_3)</td>
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<table>
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<tr>
<th></th>
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</table>

The table reports the correlation coefficients for daily interest rates (IR), squared innovations (V) and log squared innovations (LV) over the full sample.

\(i_j\), \(j = 0, 0.5, 1, 3, 12\), denotes the overnight rate, the two-week rate, the one-month rate, the three-month rate, and the twelve-month rate, respectively. The time span analysed is 04/12/2000:31/05/02, for a total of 3510 observations.
Table 3, Panel A: Deterministic cyclical model, cyclical terms, OLS estimates

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<tr>
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<td>$\delta_{c,4}$</td>
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Table 3, Panel B: Deterministic cyclical model, intervention dummies, OLS estimates

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<th></th>
<th>$i_0$</th>
<th>$i_{0.5}$</th>
<th>$i_1$</th>
<th>$i_3$</th>
<th>$i_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOL</td>
<td>-2.575</td>
<td>-0.115</td>
<td>-1.721</td>
<td>-1.578</td>
<td>-1.359</td>
</tr>
<tr>
<td></td>
<td>(1.160)</td>
<td>(0.569)</td>
<td>(0.519)</td>
<td>(0.393)</td>
<td>(0.401)</td>
</tr>
<tr>
<td>SMP</td>
<td>0.823</td>
<td>-0.155</td>
<td>-0.510</td>
<td>0.170</td>
<td>-0.211</td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
<td>(0.429)</td>
<td>(0.370)</td>
<td>(0.301)</td>
<td>(0.205)</td>
</tr>
<tr>
<td>EMP</td>
<td>2.312</td>
<td>0.602</td>
<td>0.365</td>
<td>0.296</td>
<td>-0.073</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.275)</td>
<td>(0.247)</td>
<td>(0.203)</td>
<td>(0.169)</td>
</tr>
<tr>
<td>EY</td>
<td>1.667</td>
<td>0.642</td>
<td>-0.033</td>
<td>0.406</td>
<td>-0.271</td>
</tr>
<tr>
<td></td>
<td>(0.656)</td>
<td>(0.377)</td>
<td>(0.364)</td>
<td>(0.301)</td>
<td>(0.382)</td>
</tr>
<tr>
<td>EM</td>
<td>0.590</td>
<td>-0.125</td>
<td>0.057</td>
<td>0.634</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>(0.472)</td>
<td>(0.345)</td>
<td>(0.282)</td>
<td>(0.199)</td>
<td>(0.235)</td>
</tr>
<tr>
<td>SMRO</td>
<td>-0.176</td>
<td>0.190</td>
<td>-0.124</td>
<td>-0.146</td>
<td>-0.053</td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
<td>(0.210)</td>
<td>(0.171)</td>
<td>(0.125)</td>
<td>(0.129)</td>
</tr>
<tr>
<td>AMRO</td>
<td>0.606</td>
<td>0.561</td>
<td>0.102</td>
<td>0.104</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.240)</td>
<td>(0.267)</td>
<td>(0.282)</td>
<td>(0.215)</td>
<td>(0.211)</td>
</tr>
<tr>
<td>PR</td>
<td>0.900</td>
<td>1.339</td>
<td>2.099</td>
<td>1.270</td>
<td>1.592</td>
</tr>
<tr>
<td></td>
<td>(0.563)</td>
<td>(0.597)</td>
<td>(0.562)</td>
<td>(0.370)</td>
<td>(0.359)</td>
</tr>
<tr>
<td>IRCtime</td>
<td>1.194</td>
<td>0.837</td>
<td>-0.247</td>
<td>0.507</td>
<td>-0.637</td>
</tr>
<tr>
<td></td>
<td>(0.538)</td>
<td>(0.787)</td>
<td>(0.842)</td>
<td>(0.503)</td>
<td>(0.691)</td>
</tr>
<tr>
<td>IRC</td>
<td>-0.884</td>
<td>-6.134</td>
<td>-5.090</td>
<td>-4.359</td>
<td>-1.272</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(1.990)</td>
<td>(1.100)</td>
<td>(1.170)</td>
<td>(0.870)</td>
</tr>
</tbody>
</table>

$R^2$  | 0.167  | 0.091    | 0.119  | 0.106  | 0.121    |

Table 3, Panel C: Deterministic cyclical model, correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>DC</th>
<th>$i_0$</th>
<th>$i_{0.5}$</th>
<th>$i_1$</th>
<th>$i_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.747</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_{0.5}$</td>
<td>0.716</td>
<td>0.863</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_1$</td>
<td>0.646</td>
<td>0.663</td>
<td>0.706</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$i_3$</td>
<td>0.525</td>
<td>0.518</td>
<td>0.551</td>
<td>0.834</td>
<td></td>
</tr>
</tbody>
</table>

The table reports OLS estimates of the deterministic cyclical model with Newey-West standard errors in brackets (Panel A and B). Panel C reports the correlation matrix of the estimated deterministic cyclical components.

The time span analysed is 04/12/2000-31/05/2002, for a total of 3510 observations.
Table 4: Semiparametric analysis and stationarity tests: hourly log variances

<table>
<thead>
<tr>
<th></th>
<th>$i_0$</th>
<th>$i_{0.5}$</th>
<th>$i_1$</th>
<th>$i_3$</th>
<th>$i_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{LW}$</td>
<td>0.769</td>
<td>0.845</td>
<td>0.762</td>
<td>0.841</td>
<td>0.651</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.039)</td>
<td>(0.032)</td>
</tr>
<tr>
<td></td>
<td>[353]</td>
<td>[389]</td>
<td>[391]</td>
<td>[161]</td>
<td>[245]</td>
</tr>
<tr>
<td>$H_{LP}$</td>
<td>0.773</td>
<td>0.837</td>
<td>0.752</td>
<td>0.730</td>
<td>0.626</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
<tr>
<td></td>
<td>[538]</td>
<td>[524]</td>
<td>[963]</td>
<td>[322]</td>
<td>[448]</td>
</tr>
<tr>
<td>$H_{AP}$</td>
<td>0.735</td>
<td>0.817</td>
<td>0.744</td>
<td>0.811</td>
<td>0.656</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.048)</td>
<td>(0.012)</td>
<td>(0.038)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[729]</td>
<td>[261]</td>
<td>[464]</td>
<td>[168]</td>
<td>[234]</td>
</tr>
<tr>
<td>$H_{LM}$</td>
<td>0.712</td>
<td>0.746</td>
<td>0.720</td>
<td>0.798</td>
<td>0.643</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.034)</td>
<td>(0.031)</td>
<td>(0.040)</td>
<td>(0.032)</td>
</tr>
<tr>
<td></td>
<td>[335]</td>
<td>[223]</td>
<td>[262]</td>
<td>[155]</td>
<td>[247]</td>
</tr>
<tr>
<td>$W$</td>
<td>59.99</td>
<td>53.90</td>
<td>50.72</td>
<td>55.08</td>
<td>20.06</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$LM$</td>
<td>180.3</td>
<td>208.5</td>
<td>161.7</td>
<td>337.6</td>
<td>39.24</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$LR$</td>
<td>106.9</td>
<td>119.2</td>
<td>96.95</td>
<td>136.9</td>
<td>27.03</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

The table reports semiparametric estimates of the Hurst exponent and stationarity tests, with standard errors or p-values in brackets and selected bandwidths in square brackets. $H_i$ $i = LW$, $LP$, $AP$, $LM$ is the estimate of the Hurst exponent using the Gaussian semiparametric estimator, the log periodogram estimator, the averaged periodogram estimator, and the LM estimator. $LM$ is the LM test; $W$ is the Wald test; $LR$ is the Likelihood Ratio test. In the implementation we used $q = 0.5$ for the averaged periodogram estimator and $l = 0$ for the log periodogram estimator. $i_j$ $j = 0, 0.5, 1, 3, 12$ denotes the overnight rate, the two-week rate, the one-month rate, the three-month rate and the twelve month-rate, respectively. The time span analysed is 04/12/2000-31/05/2002, for a total of 3510 observations.
Table 5: Fractional cointegration analysis

\[ d = 0.25 \]

<table>
<thead>
<tr>
<th></th>
<th>( eig )</th>
<th>( pv )</th>
<th>( r = 1 )</th>
<th>( r = 2 )</th>
<th>( r = 3 )</th>
<th>( r = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 2.054 )</td>
<td>( 1.796 )</td>
<td>( 0.085 )</td>
<td>( 0.052 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.515 )</td>
<td>( 0.451 )</td>
<td>( 0.021 )</td>
<td>( 0.013 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.025 )</td>
<td>( 0.022 )</td>
<td>( 0.020 )</td>
</tr>
<tr>
<td></td>
<td>( 0.066 )</td>
<td>( 0.057 )</td>
<td>( 0.052 )</td>
<td>( 1.139 )</td>
<td>( 0.947 )</td>
<td>( 0.845 )</td>
</tr>
</tbody>
</table>

\[ d = 0.30 \]

<table>
<thead>
<tr>
<th></th>
<th>( eig )</th>
<th>( pv )</th>
<th>( r = 1 )</th>
<th>( r = 2 )</th>
<th>( r = 3 )</th>
<th>( r = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 1.212 )</td>
<td>( 1.029 )</td>
<td>( 0.049 )</td>
<td>( 0.030 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.522 )</td>
<td>( 0.444 )</td>
<td>( 0.021 )</td>
<td>( 0.013 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.025 )</td>
<td>( 0.022 )</td>
<td>( 0.020 )</td>
</tr>
<tr>
<td></td>
<td>( 0.066 )</td>
<td>( 0.060 )</td>
<td>( 0.052 )</td>
<td>( 0.942 )</td>
<td>( 0.806 )</td>
<td>( 0.734 )</td>
</tr>
</tbody>
</table>

\[ d = 0.35 \]

<table>
<thead>
<tr>
<th></th>
<th>( eig )</th>
<th>( pv )</th>
<th>( r = 1 )</th>
<th>( r = 2 )</th>
<th>( r = 3 )</th>
<th>( r = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 0.715 )</td>
<td>( 0.590 )</td>
<td>( 0.028 )</td>
<td>( 0.018 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.529 )</td>
<td>( 0.437 )</td>
<td>( 0.021 )</td>
<td>( 0.013 )</td>
<td>( 0.000 )</td>
<td>( 1% ) 5% 10%</td>
</tr>
<tr>
<td></td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.000 )</td>
<td>( 0.025 )</td>
<td>( 0.022 )</td>
<td>( 0.020 )</td>
</tr>
<tr>
<td></td>
<td>( 0.066 )</td>
<td>( 0.056 )</td>
<td>( 0.051 )</td>
<td>( 0.852 )</td>
<td>( 0.740 )</td>
<td>( 0.681 )</td>
</tr>
</tbody>
</table>

The Table reports the Robinson and Yajima [62] fractional cointegrating rank test. \( eig \) denotes the estimated eigenvalues, \( pv \) the proportion of explained variance, and \( rank = i, i = 1, ... , 3 \), denotes the corresponding test at the given significance level (1\%, 5\%, 10\%). The bandwidth was set to three ordinates.
Table 6, Panel A: Common long memory factor model: $\Theta$ matrix and $\Sigma_\eta$ matrix

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_0$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$i_{0.5}$</td>
<td>1.309</td>
<td>(0.047)</td>
</tr>
<tr>
<td>$i_1$</td>
<td>0.332</td>
<td>2.730</td>
</tr>
<tr>
<td>$i_3$</td>
<td>0.026</td>
<td>(0.095)</td>
</tr>
<tr>
<td>$i_{12}$</td>
<td>0.207</td>
<td>0.453</td>
</tr>
</tbody>
</table>

$d$ | 0.279 | 0.279 |

$\sigma^2_{\eta_i}$ | 2.161 | 0.399 |

Table 6, Panel B: Common long memory factor model: $\Sigma_\kappa$ matrix and damping factors

<table>
<thead>
<tr>
<th>$\Sigma_\kappa$</th>
<th>$i_0$</th>
<th>$i_{0.5}$</th>
<th>$i_1$</th>
<th>$i_3$</th>
<th>$i_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_0$</td>
<td>1.875</td>
<td>-0.876</td>
<td>0.137</td>
<td>0.169</td>
<td>0.083</td>
</tr>
<tr>
<td>$i_{0.5}$</td>
<td>-1.081</td>
<td>0.812</td>
<td>-0.183</td>
<td>0.233</td>
<td>-0.275</td>
</tr>
<tr>
<td>$i_1$</td>
<td>0.218</td>
<td>-0.192</td>
<td>1.351</td>
<td>-0.002</td>
<td>0.173</td>
</tr>
<tr>
<td>$i_3$</td>
<td>0.294</td>
<td>-0.267</td>
<td>-0.003</td>
<td>1.615</td>
<td>0.348</td>
</tr>
<tr>
<td>$i_{12}$</td>
<td>0.175</td>
<td>-0.381</td>
<td>0.310</td>
<td>0.680</td>
<td>2.365</td>
</tr>
</tbody>
</table>

$\rho$ | 0.550 | 0.296 | 0.393 | 0.330 | 0.322 |

(0.011) | (0.050) | (0.000) | (0.000) | (0.000) |

The table reports FDML estimates of the common long memory factor model. Panel A reports the estimated loading matrix for the common long memory factors ($\mu_1, \mu_2$), the estimated fractional differencing parameter ($d$), the estimated variance-covariance matrix of the cycles innovations ($\Sigma_\kappa$ matrix) (lower triangular matrix), the correlation matrix of the cycles innovations (upper triangular matrix), and the estimated damping parameters ($\rho$). $i_{ij}$, $j = 0, 0.5, 1, 3, 12$ denotes the overnight rate, the two-week rate, the one-month rate, the three-month rate and the twelve-month rate, respectively. The time span analysed is 04/12/2000-31/05/2002, for a total of 3510 observations.
Table 6, Panel C: Common long memory factor model: $\Sigma_w$ matrix

\[
\begin{array}{cccccc}
\Sigma & i_0 & i_{0.5} & i_1 & i_3 & i_{12} \\
i_0 & 2.538 & -0.955 & -0.219 & -0.115 & -0.127 \\
i_{0.5} & -1.475 & 0.940 & -1 & -0.080 & -0.058 \\
i_1 & -0.440 & -1.292 & 1.593 & -0.717 & -0.155 \\
i_3 & -0.273 & -0.116 & -1.348 & 2.222 & 0.000 \\
i_{12} & -0.351 & -0.098 & -0.339 & 0.001 & 2.990 \\
\end{array}
\]

Table 6, Panel D: Common long memory factor model: specification tests

\[
\begin{array}{cccccc}
& i_0 & i_{0.5} & i_1 & i_3 & i_{12} \\
N_{BJ} & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
AR(5) & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
AR(20) & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
ARCH(5) & 0.000 & 0.000 & 0.029 & 0.780 & 0.502 \\
ARCH(20) & 0.000 & 0.000 & 0.052 & 0.781 & 0.848 \\
\end{array}
\]

The table reports FDML estimates of the common long memory factor model. Panel C reports the estimated variance-covariance matrix of the residual components ($\Sigma_w$ matrix) (lower triangular matrix), and the correlation matrix of the residual components (upper triangular matrix). Panel D reports the specification tests. $N_{BJ}$ is the Bera Jarque normality test, $AR(k)$ is the LM test for serial correlation up to the $k$th order and $ARCH(k)$ is the test for the ARCH effects. $i_j \ j = 0, 0.5, 1, 3, 12$ denotes the overnight rate, the two-week rate, the one-month rate, the three-month rate and the twelve-month rate, respectively. The time span analysed is 04/12/2000-31/05/2002, for a total of 3510 observations.
Figure 1: Log variance processes, autocorrelation functions (overnight rate (LV0), two-week rate (LV0.5), one-month rate (LV1), three-month rate (LV3), twelve-month rate (LV12)).
Figure 2: Deterministic weekly cycle, log variance processes (overnight rate (LV0), two-week rate (LV0.5), one-month rate (LV1), three-month rate (LV3), twelve-month rate (LV12)).
Figure 3: Average daily cycle (overnight rate (LV0), two-week rate (LV05), one month rate (LV1), three month rate (LV3), twelve month rate (LV12)).
Figure 4: Overnight rate, deterministic cycle over the maintenance period, with spline interpolator.
Figure 5: Maintenance period (two-week rate (LV0.5), one-month rate (LV1), three-month rate (LV3), twelve-month rate (LV12)).
Figure 6: Order of integration equality test and estimated Hurst exponent (overnight rate (0), two-week rate (0.5), one month rate (1), three-month rate (3), twelve-month rate (12)).
Figure 7: Deterministic (dc) and total (stochastic plus deterministic) (sc) cycles (overnight rate (LV0), two-week rate (LV05), one-month rate (LV1), three-month rate (LV3), twelve-month rate (LV12)).
Figure 8: Impact of Press Conferences after GC meetings on the volatility of the overnight rate (d: deterministic cycle; s: total - deterministic and stochastic - cycle).
Figure 9: Conditional variance factors and the timing of GC meetings
Figure 10: Conditional variance factors and the end of maintenance periods
Figure 11: Persistent and cyclical stochastic components of the volatility of the 12 month EONIA swap rate
Figure 12: Specification tests: Normality (N), correlogram for residuals (ACF), correlogram for squared residuals (ACF2) (overnight rate (LV0), two-week rate (LV0.5), one-month rate (LV1), three-month rate (LV3), twelve-month rate (LV12)).
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