

Klaeffling, Matt

Working Paper

Monetary policy shocks - a nonfundamental look at the data

ECB Working Paper, No. 228

Provided in Cooperation with:

European Central Bank (ECB)

Suggested Citation: Klaeffling, Matt (2003) : Monetary policy shocks - a nonfundamental look at the data, ECB Working Paper, No. 228, European Central Bank (ECB), Frankfurt a. M.

This Version is available at:

<https://hdl.handle.net/10419/152662>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

EUROPEAN CENTRAL BANK
WORKING PAPER SERIES

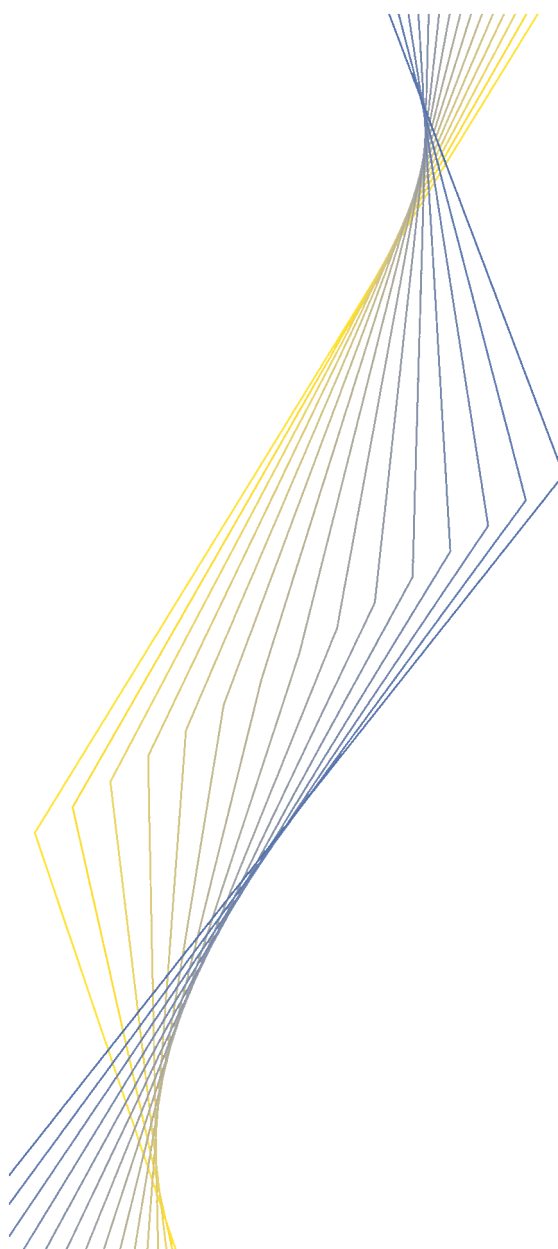


WORKING PAPER NO. 228
MONETARY POLICY SHOCKS -
A NONFUNDAMENTAL LOOK
AT THE DATA

BY MATT KLAEFFING

May 2003

EUROPEAN CENTRAL BANK
WORKING PAPER SERIES



WORKING PAPER NO. 228

**MONETARY POLICY SHOCKS -
A NONFUNDAMENTAL LOOK
AT THE DATA¹**

BY MATT KLAEFFING²

May 2003

¹ The author would like to thank Jim Bullard, Gary Chamberlain, Larry Christiano, Mike Golosov, Alexei Onatski, Ricardo Reis, Jim Stock, Oleg Tsyvinski, Juergen von Hagen and seminar participants at the Harvard macro and econometric lunches, the ZEI Summer School on Monetary Policy, the Federal Reserve Bank of Minnesota, the Federal Reserve Bank of St. Louis and the European Central Bank. This paper was written while the author was a Visiting Fellow at the Harvard Economics Department and during visits to the Institute for Empirical Macroeconomics at the Federal Reserve Bank of Minnesota and the Federal Reserve Bank of St. Louis, whose hospitality is thankfully acknowledged. The opinions expressed herein are those of the authors and do not necessarily represent those of the European Central Bank. This paper can be downloaded without charge from <http://www.ecb.int> or from the Social Science Research Network electronic library at: http://ssrn.com/abstract_id=xxxxxx

² Zentrum fuer Europaeische Integrationsforschung (ZEI), University of Bonn. E-mail: klaeffing@yahoo.com

© European Central Bank, 2003

Address	Kaiserstrasse 29 D-60311 Frankfurt am Main Germany
Postal address	Postfach 16 03 19 D-60066 Frankfurt am Main Germany
Telephone	+49 69 1344 0
Internet	http://www.ecb.int
Fax	+49 69 1344 6000
Telex	411 144 ecb d

All rights reserved by the author/s.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

The views expressed in this paper do not necessarily reflect those of the European Central Bank.

ISSN 1561-0810 (print)

ISSN 1725-2806 (online)

Contents

Abstract	4
Non-technical summary	5
1 Introduction	7
2 Conditional and unconditional moments	9
3 Vector autoregressions and nonfundamentalness	10
3.1 Traditional (fundamental) VAR analysis	10
3.2 Nonfundamental VAR analysis	12
4 The effect of money on output	15
5 Conclusion	20
A Invertibility, fundamentalness and Blaschke matrices	22
B References	25
Figures	28
European Central Bank working paper series	31

Abstract

VAR studies of the effects of monetary policy on output suggest that a contractionary impulse results in a drawn-out, hump-shaped response of output. Standard structural economic models are generally not able to reproduce such a response. In this paper I look at nonfundamental representations that are observationally equivalent to a VAR. I find that the quantitative effect of a monetary policy shock on output might be much smaller and much more short-lived than the VAR studies suggest. I conclude that the apparent discrepancy between the VAR findings and standard structural models may be spurious and that the general tendency to append non-structural, ad hoc features to structural models should be questioned.

JEL classification: C12,E52

Keywords: Nonfundamental Representations, Blaschke Matrices, VAR Models, Monetary Policy

Non-technical Summary

Questions of inference in dynamic contexts are difficult. Given that experiments are practically not feasible in social sciences we have to rely on statistical approximations to model interdependencies which may or not be causal. Social scientists model dynamic correspondences between variables of interest statistically and then try to identify exogenous, or "independent", movements in a given variable and trace its effects on other variables on impact and across time. Clearly, the validity of the estimated relationships relies on two things: the correct identification of the statistical relationship between the variables per se and the correct identification of what constitutes an "independent" movement of the "causal" variable under consideration. This paper focuses on the second issue and specifically does so for the question of "What are the effects of monetary policy shocks?".

The experiment under analysis is the key question of monetary policy analysis: What are the effects of an unexpected monetary tightening on output? This question relies on correctly identifying what an "unexpected tightening" represents. We here map this question into the framework of Vector Autoregressions (VARs) and investigate what the existence of so-called "nonfundamental" representations implies for the effect of monetary tightenings relative to the conclusions of the VAR literature on this issue. "Nonfundamentality" is a statistical feature where one statistical representation is observationally equivalent to another, while the two representations can imply wildly different implications for dynamic responses to shocks to the two systems. Intuitively, the difference lies in that only the variables themselves are observed, whereas the monetary "shocks" are not. It occurs that for a given VAR there exists an infinity of alternative "nonfundamental" representations that each imply different shock definitions (and hence different economic responses to these shocks). We can think of each representation as defined by a root to a system and just as there are always two roots to a standard square-root problem, these dynamic system also have several roots (infinitely many), which define a distinct representation each.

In this paper I reproduce one famous benchmark analysis of the effects of monetary shocks due to Christiano, Eichenbaum, and Evans (1999,CEE) and compare the effects of CEE's Vector Autoregression to alternative representations that are nonfundamental, i.e. that are observationally equivalent to CEE's VAR. I characterize a subset of the responses that can be constructed by defining certain bounds of minimum and maximum effects. I argue that the VAR essentially captures the maximal effect of monetary shocks on output and may therefore

overstate the actual effect. On the other hand, the minimum effects are much closer to the predictions of a large class of macroeconomic models. This is because one of the great 'puzzles' of monetary economics is that it is difficult to build macroeconomic models that are based on explicitly maximizing agents that show important effects of monetary policy shocks unless strong price (and/or) wage rigidities are imposed.

I conclude by more generally questioning the use of responses implied by Autoregressions to direct the specification of macroeconomic models. Specifically, the strong responses to monetary shocks, as typically indicated by fundamental representations have led to a strong focus on justifying price and wage stickiness. In view of the issue of nonfundamental representations that are observationally equivalent to the fundamental VAR, it might well be that the effects of monetary shocks are much smaller than found in the VAR literature, which would much weaken the case for the necessity of price and wage stickiness in macroeconomic models.

1 Introduction

Ever since Sims' (1980) seminal work identified, or structural, Vector Autoregressions (VARs) have been an extremely popular device to capture key empirical data patterns without the need for an explicit economic model. It occurs that applications in the various fields of macroeconomics—monetary and fiscal economics, financial economics or international economics—often encounter patterns in the data that theoretical models have difficulties reproducing. If these patterns are robust across time and in the cross-section they are termed 'puzzles'. As theory advances, models are constructed that aim at reproducing these new patterns, often at the cost of economic plausibility. This paper proposes to rethink this practice. The argument builds on the theory of nonfundamental representations for vector autoregressive time series, a theory that was introduced into macroeconomics by Hansen and Sargent (1991) and Lippi and Reichlin (1993, 1994)¹. By looking at the implications of nonfundamentality for VAR studies this paper follows a broader literature² in arguing that the findings of the identified VAR literature are not as robust as they may seem and that discrepancies between structural models and VAR readings of the data need not be taken as direct evidence against the structural approach.

As is well known (see Hansen and Sargent 1981a, 1991), there exists an infinity of observationally equivalent moving-average representations for any given time series process, only one of which is the structural moving-average representation, i.e., the data-generating process. As Lippi and Reichlin (1993, 1994) have pointed out the impulse responses and variance decompositions implied by these different representations can vary greatly from those implied by a conventionally identified VAR representation. In the conventional structural VAR literature the identification problem has been recognized as far as there structural disturbances are assumed to be identified up to an orthonormal matrix only. But limiting the identification problem to one of identifying an orthonormal matrix amounts to imposing a characteristic on the innovations that is called fundamentalness. Fundamentalness means that the structural shocks can be recovered from current and past observations. But there exist other moving-average representations that are observationally equivalent to the fundamental one. All of these representations are nonfundamental, i.e. the structural disturbances in these representations cannot be recovered from current and past observations. In these representations current and past shocks span a strictly

¹While Lippi and Reichlin (1994) lay out the theory behind Blaschke matrices, Hansen and Sargent (1991) and Lippi and Reichlin (1993) discuss implications of nonfundamentality for applied work. Hansen and Sargent discuss a case where the econometrician's information set is a strict subset of the relevant information set. They show in a theoretical example how the impulse responses from the estimated model could be different from the true responses (see also the appendix). Lippi and Reichlin (1993) redo the exercise of Blanchard and Quah (1989), i.e. they calculate the variance share of output due to demand shocks. Hansen and Sargent (1980, 1981a, 1981b) and Townsend (1983) also discuss Blaschke matrices in related contexts.

²See, for example, Sartre (1997), Faust and Leeper (1997), Canova and Pina (1998) and Rudebusch (1998).

larger space than current and past observations. In general, there seems to be no economic reason to exclude such cases a priori, yet in empirical time series modeling this is almost always done. Specifically, nonfundamentality of the structural moving-average representation is an issue that is likely to arise in forward-looking, rational-expectations models when the econometrician does not observe all relevant variables (see Hansen and Sargent, 1991 and the appendix to this paper). The novelty of the approach proposed in this paper is to use the apparent lack of identification constructively. Specifically, I take the interaction between empirical analysis and theory as a way to identify a data representation out of an equivalence class that has theoretical appeal.

In this paper I will focus on the effect of monetary policy shocks on output.³ This is a particularly interesting question in that it is one that has generated a huge literature⁴ and at the same time constitutes a prime example of a central question of macroeconomics on which the profession has still not converged to a unified answer. At this point, the dispute here is much less about how to read the evidence from a VAR as about how to reconcile the results of VAR studies with those of other types of macroeconomic models, notably structural models in the RBC/DSGE⁵ tradition. Reduced-form data analyses such as VARs show a fairly robust finding about the effect of money on output: an unanticipated contractionary impulse to money results in a long-lived, hump-shaped response of output. The difficulty with standard micro-founded structural models is that these models can only reproduce a strong and persistent output response to a monetary impulse if prices are sticky for a very long time.⁶ The reaction with respect to this apparent discrepancy has generally been to amend structural theoretical models with nominal rigidities thus enabling them to quantitatively

³See Klaeffling (2001) for a related application to the foreign exchange market. There monetary shocks are identified by assuming that structural monetary shocks should not lead to a conditional forward excess return.

⁴See, for example, Cochrane (1994, 1998), Bernanke and Mihov (1995), Sims and Zha (1996), Bagliano and Favero (1998) and Christiano, Eichenbaum and Evans (1999, 2001).

⁵RBC stands for Real Business Cycle and DSGE stands for Dynamic Stochastic General Equilibrium. The latter class of models are structural models that essentially build on the former class methodologically but are augmented to include new elements, notably money, that were absent from the traditional RBC models.

⁶At this point there are three standard channels through which 'micro-founded' models theoretically introduce a quantitatively important output response to a monetary shock. The first approach is followed in a series of papers by Chari, Kehoe and McGrattan (1998, 2000, CKM). In their models prices are set fix for 4 periods, where one period is normalized to represent one quarter. Alternatively, numerous authors (Jeanne (1997) or Kollmann (2001)) follow Yun (1999) and assume Calvo-type pricing (Calvo, 1983), setting the number of firms that are allowed to modify prices each periods to one in four. In an interesting novel calibration procedure Christiano, Eichenbaum and Evans (2001) calibrate a model with sticky prices and wages *a la* Calvo. They estimate their model using particular conditional moments of the data implicit in an identified VAR. In fact, their conditional moment conditions are the response functions of output and other variables to a monetary impulse. Their calibration implies optimal adjustment in prices every 2 quarters and in wages every 3 to 4 quarters. Thirdly, Kim (1998) and Ireland (2001) estimate models assuming quadratic adjustment costs in prices. Thus, all these models have to resort to a degree of stickiness that would be qualified as extreme given economic priors.

reproduce the VAR finding⁷. An alternative reaction has been put forth by Chari, Kehoe and McGrattan (CKM, 1998, 2000). CKM show that from a wide range of structural models there seems to be none that can replicate an important and persistent effect of monetary shocks on output unless extreme assumptions about stickiness are made. While this is not different from the findings of other authors, their interpretation of this is with a twist: CKM argue that if there is no structural model that can generate big effects of money on output, then money probably *does not* generate such an effect. Their argument implies that the VAR findings are entirely spurious. The findings of this paper are corroborative in the sense that within a VAR setting I find that the effects may in fact be much smaller and short-lived than previously found. While the qualitative response found in this paper is supportive of the VAR findings in that I find a short-lived hump-shaped response, the quantitative findings suggest that the focus of the latest generation on DSGE models on sticky-prices may be overemphasized.

The remainder of the paper is organized as follows. Section 2 briefly goes over the distinction between conditional and unconditional moments, a distinction that will be important for the arguments made in section 3, where nonfundamentality is discussed in the context of Vector Autoregressions. Section 4 briefly reviews the main findings of the VAR literature on the effect of money on output and reproduces a benchmark VAR model. I then investigate the range of the equivalence class of nonfundamental representations and compare the implied conditional moments. Section 5 concludes. The appendix discusses the issue of nonfundamentality in greater detail.

2 Conditional and Unconditional Moments

‘... Models need to be tested as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies or parts of economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions.’
Lucas (1980)

The Lucas program calls for the testing of a theoretical economy by looking at the model economy’s ability to reproduce observable real-world features of the data. Only a model that can satisfactorily reproduce observable characteristics of the data, Lucas argues, should be relied upon for the analysis of questions that cannot be answered by just looking at the data. The latter include notably the endogenous responses to exogenous shocks and counterfactual policy analysis. For the purpose of this paper the interesting question is how the endogenous variables of the economy react to an unforecastable shock to monetary policy.

⁷See the ‘neoclassical synthesis’ by Goodfriend and King (1997) and the references in the previous footnote. See also the papers by Dib and Phaneuf (2001), Christiano, Eichenbaum and Evans (2001) and Boivin and Giannoni (2001).

The endogenous reaction to an exogenous shock is an example of a conditional moment. Conditional moments differ from unconditional moments in that the later are observable in the limit. Unconditional moments are essentially the moments that describe the autocovariance-generating function of a process, and functions thereof. Examples of unconditional moments would be variances and covariances. Structural models can be compared to real world data by calculating the theoretical unconditional moments of a model economy and comparing them to their empirical counterparts. Arguably, structural models have been fairly successful in replicating the most salient unconditional moments of the data.⁸ At the same time some conditional moments of these same models are strikingly at odds with their empirical analogs in the VAR literature. Conditional moments can be defined as moments of a time series conditional on a change in another time series.⁹ In a theoretical model these values are straightforward to calculate. To see this suppose that the autoregressive representation of the theoretical economy is given by

$$Y_t = Y_{t-1}A + X_tB, \quad (1)$$

where Y_t and X_t denote the vector of endogenous and exogenous variables. The vector of exogenous variables, X_t , is composed of economic variables that are determined outside the model and exogenous shock processes. Thus a conditional moment would be, for example, the conditional expectation of the response in the endogenous variable Y_i at time $t + s$ to a change in an exogenous variable X_j at time t ,

$$E_t \left[\frac{\partial Y_{i,t+s}}{\partial X_{j,t}} \right] \quad (2)$$

Given a model of the form (1) all unconditional and conditional moments can be calculated analytically. For example, the moment in (2) can be seen to be $(I_jBA^{s-1})I_i^T$, where I_k is a row vector of zeros with unity in the $k - th$ position. While these moments can be calculated easily for model economies, their empirical counterparts are not as easy to obtain. In particular, in the VAR literature the only exogenous variables are shocks and these are not uniquely identified. Hence, conditional moments such as (2) are not uniquely identified either. The next section of this paper will deal with the issue of unidentified conditional moments in the context of VAR analysis.

3 Vector Autoregressions and Nonfundamentallness

3.1 Traditional (fundamental) VAR analysis

⁸See McGrattan, Rogerson and Wright (1997) and Kollmann (2001) for representative applications.

⁹Note that this definition is nonstandard for I am 'conditioning' on a known, i.e. deterministic change in a variable. Usually, conditioning is referred to integrating over the distribution of a random variable, i.e. taking an expectation.

Starting with Sims (1980) critique of the simultaneous equation macroeconomic models, VARs have been used extensively in an effort to impose as little structure on the data as possible in hope of gaining insights that are not model-dependent and could rightfully be called ‘stylized facts’. VAR analysis has to its advantage that all elements of the vector of observables are treated simultaneously as endogenous. Therefore, while being subject to the Lucas critique, it is not subject to the traditional critique of simultaneous equation models of implying ‘incredible identifying restrictions’ (Sims, 1980). Hence, this approach allows what would seem to be an agnostic - unbiased from priors - and efficient look at comovements in the data. From these comovements conditional movements can be inferred by putting structure on the impact matrix (see below).

Most stylized facts in the literature relating to conditional moments in macroeconomics are derived from some version of this VAR setup. As I will show now, these stylized facts are much more stylized than factual.

Suppose the econometrician estimated the following VAR model:

$$Y_t A(L) = e_t, \quad (3)$$

with statistical, or fundamental, innovations

$$e_t = Y_t - E[Y_t | F_t], \quad (4)$$

with $F_t = \{Y_\tau\}_{\tau < t}$. It is usually assumed that the statistical innovations, e_t , are linear combinations of the structural innovations of the model, s_t . To identify the later, the econometrician thus has to identify an impact matrix B_0 in

$$e_t = s_t B_0 \quad (5)$$

In order to identify the impact matrix, B_0 , a nonlinear system of equations must be solved, which is given by

$$\Sigma = B_0' \Omega B_0, \quad (6)$$

where $\Sigma = E[e_t^T e_t]$ and $\Omega = E[s_t^T s_t]$

Given an $(n * 1)$ vector of shocks and assuming diagonal Ω , this system involves n^2 unknowns and only $\frac{n(n+1)}{2}$ estimable coefficients. Therefore another $n(n-1)$ restrictions are needed.

Defining an appropriately augmented vector, \tilde{Y}_t , I can rewrite (3) as

$$\tilde{Y}_t = \tilde{Y}_{t-1} \tilde{A} + s_t \tilde{B}, \quad (7)$$

where \tilde{A} and \tilde{B} denote the companion matrices associated with $A(L)$ and B , and I can calculate the impulse-response function of Y_i to a shock in X_j as

$$E \left[\frac{\partial Y_{i,t+s}}{\partial X_{j,t}} \mid F_t \right] = I_j \tilde{B} \tilde{A}^s I_i^T \quad (8)$$

Traditionally the structural ordering implicit in the Choleski decomposition has been used since Sims (1980). In that case B_0 is simply the Choleski factor.

The Choleski factor is being constructed by iterative projections of the statistical residuals from the $i - th$ equation on all the residuals for equations 1 through $i - 1$. This amounts to treating variable $i - 1$ as predetermined in forming the expectation of variable i , for its structural residual is defined as

$$s_{it} = Y_{i,t} - E[Y_{i,t} | F_t], \quad (9)$$

where $F_t = (\{Y_\tau\}_{\tau < t}, \{e_{j,t}\}_{j < i})$.

Thus the Choleski decomposition implies a Wold causal ordering.

While other identification schemes have been put forward¹⁰ I will limit myself to the case where the (partial) Wold ordering implicit in the Choleski decomposition is *correct*. In particular, since I am only focusing on identifying the reactions to monetary policy shocks, I merely have to know what variables are ordered before and after the monetary policy variable, but do not need to know the exact ordering of all variables¹¹.

Most of the literature on the robustness of the stylized facts of the VAR literature, in particular with respect to the effect of money on output, has focused on alternative impact matrices, B_0 . What this impact matrix does is simple: it defines the statistical innovations as linearly weighted averages of the structural shocks. Thus, in general, there is a problem of identification in the VAR literature that relates to the weighing of the various structural shocks at a point in time. What is observed is only a weighted average of all the current-period shocks and the problem lies in identifying the weights of this reweighing scheme. The next section will take this problem to the next level and show that there is in fact a restriction in (5) that is rarely discussed in the literature. I will show that the statistical innovations, e_t , for the class of models of (7) are in general weighted averages of all *present and past* structural shocks. Simply stating that statistical shocks of current-period structural shocks only, as does (5) implies a restriction devoid of any theoretical foundation, whose implications for statistics such as the impulse responses in (8) need to be explored.

3.2 Nonfundamental VAR analysis

I will assume that the econometrician has what seems a well-specified model in the sense that the data-generating process has exactly the same autocovariance-generating function as the VAR model that the econometrician uses as a data representation.¹² The true data-generating process however differs from a standard VAR in that the statistical innovations, e_t , are weighted averages of current

¹⁰See Blanchard and Quah (1989) and Sims and Zha (1996), for example.

¹¹Christiano, Eichenbaum and Evans (1999) show that if one wants to identify the conditional moments relative to a shocked variable $y_{i,t}$ then one merely has to know the position in the recursive Wold ordering of that variable, i.e one only has to be able to partition the vector of economic variables as $y = [y_{1,t}, y_{i,t}, y_{2,t}]^T$ where the shocks to variables $j = 1 : i - 1$ occur prior to the shock to variable i and the shocks to variables $j = i + 1 : n$ occur later in the Wold ordering. The exact ordering of the variables in the subvectors $y_{1,t}$ and $y_{2,t}$ is irrelevant for identification of the effects of shocks to $y_{i,t}$.

¹²This means, in particular, that the fundamental shocks are white noise.

and past structural shocks and not simply weighted averages of current structural shocks. At the same time e_t are white noise, so that the econometrician would never suspect any time series misspecification. The fact that the statistical shocks are functions of current and past structural shocks means that I could rewrite the data-generating process as a special case of a vector autoregressive-moving-average process (VARMA).¹³ As far as the econometrician is concerned the space of potential structural models is thus extended. Even conditioning on a Wold ordering, there exists an infinite-dimensional class of equivalent representations. Formally VAR is only identified up to a matrix polynomial $G(L)$ called Blaschke matrix¹⁴ which has the following 2 properties

- (i) $\det(G(z))$ does not vanish on the complex unit circle
- (ii) $G(z)G^T(z^{-1}) = I$, where $G^T(\cdot)$ denote the matrix obtained by transposing and taking conjugate coefficients,
i.e. $G(z)^{-1} = G^T(z^{-1})$

Further, given a particular identification scheme for the impact matrix B_0 the full space can be generated by multiplying elementary Blaschke matrix, where an elementary Blaschke matrix is a diagonal matrix with typical element $\frac{\alpha^i - z}{1 - \bar{\alpha}^i z}$, where $\bar{\alpha}$ denotes the complex conjugate of α . (see Lippi and Reichlin, 1994) To generate a particular element of the class of equivalent representations take a Blaschke matrix $G(L)$ and postmultiply (1) with $G(L)^{-1}$ to obtain

$$\begin{aligned} Y_t A(L) G(L)^{-1} &= Y_t A^*(L) \\ &= e^* \\ &= s_t^* \tilde{B}, \end{aligned} \tag{10}$$

where $A^*(L) = A(L) G(L)^{-1}$, and $s_t = s_t^* G(L)$.

The resulting impulse-responses can then be calculated as

$$E \left[\frac{\partial y_{i,t+s}}{\partial s_{j,t}^*} \mid F_t \right] = I_j \tilde{B}^* \tilde{A}^{*s} I_i^T, \tag{11}$$

where \tilde{B}^* and \tilde{A}^* denote the companion matrices associated with B^* and $A^*(L)$ respectively. The IRs given by (11) can be strikingly different from those given by (8).

Now suppose that the data-generating process is given by (10). While the econometrician focusing on the VAR representation would be focusing on (3), and would believe that the structural innovations were given by $[B_0]^{-1} e_t$, the agents of the economy know the data-generating process and realize that the structural innovations are given $[B_0]^{-1} e_t^*$. Representation (3) is called the fundamental representation and all elements of the equivalence class (10) are called

¹³In fact, it can be shown that if the econometrician would explore a more general VARMA model by maximum likelihood estimation, using the VAR as starting values, he would conclude that the VAR is the maximum likelihood estimate even if it is not the data-generating process. This is due to the nonlinear likelihood surface and the (local) optimality of the VAR specification

¹⁴See the references in footnote 1 for further references.

nonfundamental for the structural shocks cannot be recovered through an autoregression of Y_t . For a more detailed discussion of the notion of nonfundamentality see the appendix.

Recall that the fundamental innovations are weighted averages of current and lagged structural innovations. Thus, the shocks recovered by the econometrician, e_t , are in part reactions to current news, in part to old news. To see this take the scalar case, where $G(z) = \frac{\alpha - z}{1 - \alpha z} = \frac{1 - \frac{1}{\alpha}z}{1 - \alpha z} \alpha$.

Then

$$s_t = s_t^* G(L) \quad (12)$$

$$\begin{aligned} &= \frac{1 - \frac{1}{\alpha}z}{1 - \alpha z} \alpha s_t^* \\ &= \frac{1 - \frac{1}{\alpha}z}{1 - \alpha z} \tilde{s}_t^* \\ &= \tilde{s}_t^* + \frac{1}{\alpha} \tilde{s}_{t-1}^* + \sum_{j=0}^{\infty} \left(\frac{1}{\alpha} \right)^j [\tilde{s}_{t-j}^* - \frac{1}{\alpha} \tilde{s}_{t-j-1}^*], \end{aligned} \quad (13)$$

where $\tilde{s}_t^* = \alpha s_t^*$

Clearly, from just looking at realizations of a time series process one can never tell whether the true shocks are the fundamental ones, recovered from the VAR as in (4), or a particular element of the nonfundamental equivalence class. Since the nonfundamental and the fundamental representations imply the exact same moments, all these representations are equivalent from the perspective of their likelihood. In particular, the innovations to both representations are white noise with the same variance-covariance matrix. This is because the nonfundamental representation written as an VARMA process has roots α and $-\frac{1}{\alpha}$ that cancel out of the spectrum at frequency zero. Note that the issue of non-identification naturally applies not only to VAR models, but to all models that include unobservable explanatory variables, i.e. innovations in the VAR framework as well as factors in dynamic factor models (Stock and Watson, 2001) and dynamic principal component models (Reichlin, 2000).

The existence of this equivalence class of representations hence implies that the relevant task of comparing moments is complicated in the case of conditional moments. Rather than simply comparing the moments of the data to those implicit in calibrated or estimated structural models one has to consider the full class of equivalent representations. From the perspective of the Lucas program this means that one should not reject a particular structural model because it fails to reproduce the conditional moments implicit in a VAR reading of the data, but should do so only if the model's conditional moments violate its analogs in *all* reduced-form readings of the data - fundamental or nonfundamental.¹⁵ Denoting the conditional moment vector of interest by $\varphi(\cdot)$ and the relevant metric to calculate the difference between the theoretical and the data moments

¹⁵At this point one would clearly like to not only consider VARs and VARMA processes that are observationally equivalent to VARs, but also VARMA processes in general. This would be beyond the scope of this paper and is left for future research.

by $H[., .]$, the metric to minimize now becomes $H[\varphi(g(\theta)), \varphi(data, \alpha)]$, where the data moment is denoted by the vector α which identifies a particular element of the equivalence class. This metric then has to be minimized not only with respect to θ but with respect to θ and α jointly.

Let me now turn to the implications of the issue of nonfundamentality for VAR analysis of the effect of monetary shocks on the macroeconomy.

4 The effect of money on output

In a recent survey of the SVAR literature on the effects of monetary shocks Christiano, Eichenbaum and Evans (1999, CEE) document that a large body of work has found a number of fairly robust conditional moments (see CEE for references). While the choice of the monetary aggregate matters, generally the effect of a monetary contraction is a prolonged decline in output, a rise in the interest rate and a (lagged) reduction in the price level. In this section I will focus on a version of the benchmark model of CEE. The vector of variables included in the VAR consists of *industrial production, the CPI, a commodity price index, the federal funds rate, total reserves, nonborrowed reserves* and *M2*.¹⁶ I will make use of the argument expounded in CEE that the exact ordering of the variables does not matter for partial identification. As CEE have shown (see their proposition 4.1), the exact ordering of the variables ordered before and after the monetary policy variable respectively does not matter for the conditional moment statistics of a shock to money. This approach is hence a partial identification approach. CEE measure monetary policy shocks by innovations to the federal funds rate. The argument here is that the federal funds rate, unlike money, is an exogenously controllable process. Hence I would not risk confounding supply and demand shocks. Also, I evade the difficult issue of whether to use M1 or M2. Further, the fed funds rate increases neatly coincide with the narrative Romer and Romer episodes and, finally, using the fed funds rate yields results that are often deemed 'reasonable' - a logic that may be either implicitly Bayesian or just circular (see Uhlig, 2001).

I now check on the range of answers one could get for the question of the effect of money on output. To do so I will explore the bounds of a 'plausible' subset of this equivalence class. By plausible, I mean the following: while, formally, the equivalence class is infinite dimensional, a structural point of view would suggest that the order of the autoregressive dynamics of the true underlying data generating process is probably bounded by some low order. Hence, I will limit my attention to first order Blaschke matrices¹⁷. I also limit the range of α to between 0.8 and 1. Within this class I am minimizing and maximizing the

¹⁶For a detailed description of the data and a discussion of the choice of what variables to include in the VAR see CEE. The data are quarterly observations from 1959.01 to 1995.02 and were kindly provided by Charles Evans.

¹⁷I am denoting the product of J elementary Blaschke matrices an j^{th} -order Blaschke matrix. Thus, a 1st order Blaschke matrix is simply a diagonal matrix with typical element $\frac{\alpha^i - z}{1 - \alpha^i z}$.

variability of output due to unanticipated monetary variability.

$$Var(y_{ij,s}^{\min}) = \min_{\alpha} Var(y_{i,t+s} | s_j^*(\alpha)) \quad (14)$$

and

$$Var(y_{ij,s}^{\max}) = \max_{\alpha} Var(y_{i,t+s} | s_j^*(\alpha)), \quad (15)$$

where α denotes the vector that identifies the nonfundamental representation. The horizon s with respect to which the statistics defined in (14) and (15) are calculated is set equal to 32 periods, or 8 years, but this choice is essentially inconsequential as the results are extremely robust along this dimension. While the restriction to first-order Blaschke matrices is binding in the sense that the solutions optimization problems (14) and (15) could be improved upon by considering higher-order Blaschke-matrices, the qualitative results would not change.¹⁸¹⁹

Before reporting the results of this exercise note that I will report only point estimates. The reason for this is, first, that the optimizations in (14) and (15) focus on point estimates, i.e. the objective is to show how the point estimates of two alternative representations differ, an argument that is valid irrespective of the uncertainty around this estimate for it would hold even in the asymptotic limit. Secondly, and more importantly, the existence of an infinity of nonfundamental representations that imply an important range of alternative impulse response-functions means that we have an element of uncertainty - which representations to choose to calculate the impulse responses - that does not have a known distribution. Actually it does not even have a distribution that could ever be estimated for this is precisely what lack of identification means. As a result the calculation of the variance associated with any given statistic, notably impulse-responses presents theoretical difficulties.

The implied impulse response functions for the output response to a contractionary monetary shock are shown in figure 1. I denote the representation that minimizes the share of money by 'veil' and the one that maximizes the share by 'money matters'. Maybe surprisingly, the latter is essentially the VAR. The 'veil' representation implies a very short-lived and small effect of an unanticipated innovation to the federal funds rate on output. Thus, while the 'veil' response continues to show a hump-shaped response to a shock, the quantitative importance of the shock is much smaller. Arguably, such a response as a

¹⁸The minimal variance bound could be reduced by 45 % if one were to allow for a second order Blaschke matrix and would be reduced by an additional 40 % in the case of a third order Blaschke matrix. Note that these numbers are respectively only inner bounds to the degree that they represent solutions to highly nonlinear optimization routines and as such are likely to be merely local as opposed to global optima. In the case of the upper bound there was no possible improvement.

¹⁹While the VAR literature has found that the qualitative nature of the output response to monetary shocks is very stable across different sample periods, there does seem to be less of a response in more recent data (see Boivin and Giannini, 2001). What this means for the nonfundamental representations studied in this paper is that if I were to use to different data set, for example, limiting myself to post 1982 data, I were to recover a 'veil' representations that would show an even smaller response of output to a monetary shock.

data description would be much easier to reconcile with micro-founded structural models. Given that the puzzling discrepancy between structural DSGE models and the VAR literature is the apparently strong reaction of output to a monetary innovation, I will from now on focus my attention on the VAR reading that minimizes this discrepancy, i.e. the 'veil' representation. Thus, I will investigate to what degree there exist data representations that yield impulse-responses that are in line with the response that standard theoretical models produce.²⁰ It is also interesting to look at the output gap with respect to trend generated by a contractionary policy shock.

The magnitude of the cumulative effect on output of a one percent contractionary shock appears incredible. While the maximum effect in any given quarter is always less than 1 per cent, the cumulative effect after 4 years, for example, is almost 12 per cent for the VAR case. The 'veil' representation on the other hand shows cumulative effects that are quantitatively much closer to those of structural models with little or no exogenous nominal stickiness. Finally, figure 3 plots the point estimates for the cumulative on output after 20 periods for the different representations indexed by α , which indicates the Blaschke matrix that is composed of unity in all diagonal positions with the exception of the equation for the federal funds rate where the diagonal entry is given by $\frac{\alpha - z}{1 - \bar{\alpha}z}$.

Next, figure 4 reports the reaction of the price level to a contractionary response.

While the VAR representation implies a persistent drop in the price level *after* a lag of about 2 years, the 'veil' representation implies that a contractionary monetary shock has *no* effect on the price level at *any* horizon. Would that be reasonable? It might be. Recall that as far as the response of output is concerned the big difference between the VAR and the 'veil' representations is the reaction after about 2-3 years, when in the 'veil' representation output has returned to its trend, whereas in the VAR representation output is still far below its trend. Thus the time horizon over which the two representations' implication for the price level diverge is essentially the same as for output: It is after 2 years that in the veil case the effects of the contractionary monetary shock have vanished, whereas in the VAR representation the effect is still very much present. In order to see to what degree the (non-) reaction of prices in the 'veil' case is reasonable recall that neither representation implies any significant response of the price level over the first 2 years after the shock. Thinking of the VAR as an estimated law of motion of the economy this means that as far as the response to monetary shocks is concerned prices are sticky for two years in general equilibrium. Note that this is not an assumption about price stickiness at a micro-level but an observation given the estimated law of motion of the economy and a shock identification scheme. Thus, conditional on the alternative identification schemes of the VAR and 'veil' representations I

²⁰While this paper does not report a particular benchmark structural model, see the references in footnote 2 for a discussion of the typical responses in models with little or no exogenous price stickiness and either money-in-utility function or cash-in-advance models, e.g. Cooley and Hansen (1998). See also the discussion in Favero (2001).

can regard the price-stickiness after a contractionary shock as a reduced form stylized fact.²¹ But, then why would prices fall after 2 years? Based on microeconomic reasoning they could, if producers are facing a downward-sloping demand-curve. Holding supply constant a reduction in demand would then lead to a drop in prices. This demand-determined reasoning about aggregate output in conjunction with the assumption that prices should react to expected current and future demand then means that prices at a given point in time should react only if current and expected future demand deviates from its trend. But this is the case only in the VAR representation (see figure 1). In the veil representation, output is back to its trend after two years, which means that at that point there is no more incentive for prices to react, hence the 'veil' representation implies that prices do not react at *any* horizon to monetary shocks. As a result the two representations have alternative implications for the reactions of the price level to monetary shocks that are both internally consistent.

One way to look at the differential effect of monetary shocks on output is to look at the implied reduced form policy rules for both the VAR and the 'veil' representation. The reduced form policy rule is given by the appropriate line in (3) or (10), respectively, which yields an equation for the federal funds rate as a function of past macroeconomic variables, current period structural innovations to the variables that appear prior to the policy variable in the assumed block Wold ordering, and, in the case of the 'veil' representation, past monetary innovations.

In the case of the VAR representation I obtain the following policy rule for the federal funds rate, ff_t

$$ff_t = \Phi Y_{t-1} + 0.24s_t^y + 0.05s_t^p + 0.26s_t^{pcom} + 0.83s_t^m \quad (16)$$

where s_t^y , s_t^p , s_t^{pcom} and s_t^m denote the standardized structural innovations to

output, prices, commodity prices and the policy variable and $\Phi * Y_{t-1}$ denotes the projection on past macroeconomic variables. Clearly the reaction in the policy variable is to rise with positive innovations to output, the commodity price level and the price level. Recalling that I am assuming here that the Wold representation of the VAR is correctly specified the policy rule in the 'veil' case differs from (16) merely by the presence of a Blaschke factor associated with the innovation to the policy variable. The resulting policy rule would then be

$$\begin{aligned} (1 - \alpha L)ff_t &= (1 - \alpha L)\Phi Y_{t-1} \\ &\quad + 0.24s_t^y + 0.05s_t^p + 0.26s_t^{pcom} \\ &\quad + 0.83s_t^m(\alpha - L) \end{aligned} \quad (17)$$

The veil representation implies $\alpha = 0.8$. What distinguishes (17) from (16) is

²¹I would like to stress again that this is a reduced form implication that does not reveal anything among its structural causes. It might well be that at the micro-level prices are perfectly flexible since the observation is limited to stickiness at the general-equilibrium aggregate level only.

that today's policy is also a function of last period's policy shock. In other words, the policy maker corrects his observable mistakes. This seems intuitive: If the policy rule describes desired policy, then if policy shocks are observable and where to induce long, hump-shaped responses, why would the policy maker not simply correct them? Given the alternative specifications of the policy rule, figure 5 plots the policy variable's response to an innovation for both representations. Figure 6 shows how in the VAR the policy variable slowly returns to its mean, whereas it overshoots in the 'veil' representation. There, a positive - i.e. contractionary - shock to the federal funds rate is quickly followed by a reduction below its trend level. Intuitively, given the lagged response of the economy, the policy maker can offset the unwanted effects of a previous 'mistake' by channeling the policy variable in the direction that is opposite that of the initial shock.

Under the maintained assumption that only unanticipated policy matters, the average level of the policy maker's target variables on the real side of the economy, in particular output, cannot be affected by the policy maker. The policy maker can, however, affect their variances. Assuming that less variability in output is welfare-improving, as it would be in most standard models with risk-averse consumers, then it would clearly be in the interest of the policy maker to offset policy 'mistakes' by counteracting the contractionary effect of a policy shock by quickly moving the policy variable below trend. Thus, while there is no way to discuss optimal policy in a VAR framework it seems clear that the policy rule implied by the 'veil' representation is superior to the one implicit in the VAR. This fact casts further doubts on the appropriateness of the VAR representation.

Another way to view this issue is to go back to how policy shocks were defined. Policy shocks are unanticipated movements in the policy variable, given a particular filtration. In the VAR case expected policy is set equal to a function of states which are here restricted to past observable macro variables, i.e. $E[ff_t|F_t] = h(F_t)$, where $F_t = \{Y_s, s_{1,t}\}_{s < t}$. and $s_{1,t}$ denotes the structural shocks that occur prior to the policy shock in the Wold ordering. In the 'veil' case, the set of relevant explanatory variables explicitly includes the policy error ('shock'), s_{t-1}^m , committed in the previous period, i.e. $E[ff_t|F_t^*] = h(F_t^*)$, where $F_t^* = \{Y_s, s_{j,t}, s_s^m\}_{s < t}$. Omitting past policy shocks as regressors simply results in omitted variable bias for the implied conditional moments.

The main point to take away from this application is that there exist statistically equivalent representations that yield economically reasonable, yet qualitatively distinct conditional moments of the data. Starting from the observation that there are many stylized facts in the VAR literature that structural models fail to reproduce, this paper has shown that, while VARs are only one way of looking at the data, the theory of Blaschke factors shows how one can find elements of an equivalence class that yield conditional moments that might be much closer to those generated by micro-founded, structural models. More specifically, given that structural models oftentimes fail to produce quantitatively important effects of unanticipated monetary shocks this paper has shown that there is a nonfundamental representation, denoted 'veil', that is econometrically indistin-

guishable from the VAR that shows the effect of monetary shocks on output to be much smaller and much more short-lived than the conventional reading of the VAR would suggest. On the other hand, the hump-shaped response in output is qualitatively robust.

What distinguishes the nonfundamental ‘veil’ representation from a generic VAR is that the innovations recovered from the VAR are not assumed to be linear combinations of current period structural shocks, but of current and past structural shocks. Then under the assumption that the data-generating process is given by the ‘veil’ representation, the econometrician’s information set, which consists of lagged macroeconomic variables, fails to span the relevant information set which defines the true structural innovations. Consequently, the econometrician’s VAR specification suffers from omitted variable bias in the sense that the implied moving-average representation of the VAR fails to be a consistent estimate of the moving-average representation of the data-generating process.

To what degree the nonfundamental reading of the data here presented can be reconciled with structural models is left for future research.²² Also, this paper has simply calculated the bounds of the impulse response function of output to monetary shocks and has shown that the lower bound of this response can narrow the gap between standard structural models and the VAR. This paper has not shown how to identify an element from the equivalence class of fundamental and nonfundamental representations. A companion paper, Klaeffling (2001), proposes an identification criterion in a similar context. That paper looks at the so-called conditional forward excess return, the observation that standard VARs imply that an identified monetary shock generates a statistically significant deviation from uncovered interest rate parity. The identification problem is then solved by choosing the nonfundamental representation that minimizes this conditional forward excess return. Thus, in particular cases there may be theoretical grounds to choose one representation from the equivalence class.

5 Conclusion

DSGE models have been criticized for being unable to reproduce certain stylized facts established in the VAR literature. One of the most prominent examples is the effect of monetary policy shocks on output. This paper has shown that the VAR finding of monetary innovations generating a hump-shaped response

²²While this paper has focused on the case where the Wold ordering implicit in a Choleski identification of the structural shocks is correctly specified, it would also be interesting to extend the model along the lines of Faust (1998) and Faust and Rogers (2000) to explore a more general bounds approach to the effect of money on output. Faust and Rogers define monetary shocks by the signs of their effects on macroeconomic variables at different horizons. They then look at the minimal and maximal effect that monetary shocks could have on output. In their identification scheme they implicitly restrict the impact matrix, denoted B_0 in the text, to be a constant orthonormal matrix, i.e. they assume that one-step ahead forecast errors are linear combinations of the structural shocks. Given the results of this paper, extending their approach to allow one-step ahead errors to be linear combinations of current and past structural innovations might extend the range of their analysis considerably.

of output is qualitatively robust to the extension to nonfundamental representations. Quantitatively, nonfundamental representations can very much reduce the response of output and limit it to the very short horizon, thus moving reduced form data-analysis in the direction of structural theoretical models.

The main point of this paper is that discrepancies between reduced form VAR studies and structural DSGE models should not necessarily lead researchers to abandon the microeconomic rigor of standard structural models by introducing essentially ad hoc elements of nominal stickiness, but rather the natural question to pose should be: Are the standard models wrong or are we misreading the data?

A Invertibility, Fundamentalness and Blaschke matrices

Consider the Wold representation for an n -dimensional time series process, X_t ,

$$X_t = C(L)\epsilon_t \quad (18)$$

where ϵ_t denotes fundamental innovations. They are defined given the econometrician's information set F_t as

$$\epsilon_t = X_t - E[X_t|F_{t-1}], \quad (19)$$

where $F_{t-1} = \{X_s\}_{s < t}$.

VAR modeling is based on inverting (19) and truncating the resulting autoregressive polynomial. Impulse-responses can then be simulated by inverting the estimated VAR for an estimated analog to (18).

Now suppose that the true data generating process was given by

$$X_t = {}^*C(L)\epsilon_t^* \quad (20)$$

If any roots of the polynomial ${}^*C(L)$ are inside the unit circle, then (20) fails to be invertible. The structural shock process, ϵ_t^* , is then called nonfundamental and fails to coincide with ϵ_t .

The possible non-invertibility of the underlying economic structure can be motivated along two different paths. To show this, consider a bivariate economy with an exogenous scalar $x_{1,t}$ and an endogenous scalar $x_{2,t}$.

First, it could of course be that the driving process, $x_{1,t}$, has a noninvertible moving average part. This property would then carry over to the structural moving average representation of the variable of interest. For example, suppose the shock process is given by

$$x_{1,t} = (1 - cL)\epsilon_t, \quad (21)$$

where $|c| > 1$, and model its impact on the variable of interest as

$$x_{2,t} = (1 - dL)x_{1,t} = (1 - dL)(1 - cL)\epsilon_t \quad (22)$$

The structural moving-average representation for $x_{2,t}$, (22), would then trivially be non-invertible as well.

Apart from theoretical time series considerations about the potential non-invertible nature of the moving-average part of the data-generating process, there may be particular reasons to assume that certain underlying economic structures may lead to a noninvertible representations. Examples are given in Hansen and Sargent (1991) and Lippi and Reichlin (1993, 1994). These papers show that noninvertibility is likely to be an issue of particular importance in

forward-looking rational expectations models when the econometrician's information set, F_t is a strict subset of the relevant information set, F_t^* , that defines the structural innovations:

$$\text{span}(F_t) \subset \text{span}(F_t^*) \quad (23)$$

In this case noninvertibility can be an issue even if the data-generating process for the exogenous process, (21), is invertible. To see this suppose that $x_{1,t}$ is observable to the agents of the economy who determine $x_{2,t}$ as a function of their forecasts on $x_{1,t}$, but that $x_{1,t}$ is not observable to the econometrician. For example, suppose that

$$(1 - \alpha_1 L)(1 - \alpha_2 L)x_{2,t} = x_{1,t} + \beta E[x_{1,t+1}] \quad (24)$$

Then, combining (21) and (24) I obtain

$$\begin{aligned} (1 - \alpha_1 L)(1 - \alpha_2 L)x_{2,t} &= x_{1,t+1} + \beta E[x_{1,t+1}] \\ &= (1 - cL - \beta c)\epsilon_t, \end{aligned} \quad (25)$$

which is noninvertible as long as the $|\frac{1-\beta*c}{c}| < 1$, which is the case, for example, for $c = \beta = 0.8$.

To illustrate the difference between the true impulse-response given the data-generating process (25) and a misspecified autoregression approximation take the following example. For the numerical example I assume that $c = \beta = 0.8$; $\alpha_1 = 0.45$ and $\alpha_2 = 0.9$. Obviously, for this parameterization, the process (25) is not invertible. I then simulate this process and subsequently estimate a autoregression on it. Figure 6 plots the impulse response function of for both the structural (nonfundamental) representation and the misspecified autoregression based on the fundamental representation.

What is special about this structure is that the misspecified autoregression is observationally equivalent to the nonfundamental data-generating process. It is not only that there exists a fundamental representation - the Wold representation - that can be inverted and truncated to yield an approximate autoregression. The misspecified autoregression is actually observationally equivalent in that its autocovariance-generating function is exactly that of the nonfundamental data-generating process. The reason for this is that the nonfundamental and the fundamental representation are linked by a Blaschke matrix, i.e. a matrix that reweighs the structural shocks across time to define the fundamental innovation (see section 3). In order to see where the Blaschke matrix enters rewrite (25) as follows

$$(1 - \alpha_1 L)(1 - \alpha_2 L)x_{2,t} = (1 + \theta L)\tilde{\epsilon}_t, \quad (26)$$

where $\theta = -\frac{c}{1-\beta c}\tilde{\epsilon}_t$ and $\sigma_{\tilde{\epsilon}}^2 = (1 - \beta c)^2 \sigma_{\epsilon}^2$. The autocovariance-generating function for x_2 is then

$$g_{x_2}(z) = \frac{(1 + \theta z) * (1 + \theta z^{-1})}{(1 - \alpha_1 z)(1 - \alpha_2 z) * (1 - \alpha_1 z^{-1})(1 - \alpha_2 z^{-1})} * \sigma_{\tilde{\epsilon}}^2. \quad (27)$$

On the other hand the autocovariance-generating function for x_2 given the econometrician's model, a first-order autoregression of $x_{2,t}$, is

$$g_{x_2}^{eco}(z) = \frac{1}{(1 - \alpha_2 z)(1 - \alpha_2 z^{-1})} \sigma_u^2, \quad (28)$$

where $\sigma_u^2 = \theta^2 \sigma_\epsilon^2$. The ratio between these two autocovariance-generating functions is given by $\frac{(\theta+z)(\theta+z^{-1})}{(1-\alpha_1 z)(1-\alpha_1 z^{-1})} * \theta^{-2}$. This ratio describes the transfer function that allows to move from one representation to another. With $\theta = -\frac{1}{\alpha_1}$, as is the case in this numerical example, this transfer function is a Blaschke matrix as discussed in Section 3 of the paper. The reason for the nonidentifiability is thus the fact that the Blaschke matrix is a filter that has a gain at frequency zero of exactly unity.

Finally, note that if x_1 were observable to the econometrician, then combining (21) and (24) I can solve for $x_{2,t}$

$$\begin{aligned} (1 - cL)(1 - \alpha_1 L)(1 - \alpha_2 L)x_{2,t} &= (1 - \beta c - cL)x_{1,t} \\ &= cx_{1,t-1} \\ &\quad + (1 - \beta c)(1 - cL)\epsilon_t, \end{aligned} \quad (29)$$

which is invertible. Thus the question of invertibility in forward-looking rational expectations models is clearly dependent on the econometrician's information set.

B References

References

- [1] Bagliano, C. C. and Favero, C., 1998, *Measuring Monetary Policy with VAR Models: an Evaluation*, IGIER Working Papers, Bocconi University
- [2] Bernanke, B. S. and Mihov, I., 1995, *Measuring Monetary Policy*, NBER Working Paper 5451
- [3] Blanchard, O. J. and Quah, D. T., 1989, *The Dynamic Effects of Aggregate Supply and Demand Disturbances*, American Economic Review
- [4] Boivin, J. and Gianonni, M., 2001, *Has Monetary Policy Become Less Powerful?*, Working Paper, Columbia University
- [5] Calvo, G. A., 1983, *Staggered Prices in a Utility-Maximizing Framework*, Journal of Monetary Economics, 12 (3)
- [6] Canova, F. and Pina, J. P., *Monetary Policy Misspecification in VAR Models*, Working Paper Universitat Pompeu Fabra
- [7] Chari, VV., Kehoe, P. J. and McGrattan, E. R., 1998, *Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?*, NBER Working Paper 5809
- [8] Chari, VV., Kehoe, P. J. and McGrattan, E. R., 2000, *Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?*, NBER Working Paper 7869
- [9] Christiano, L. J., Eichenbaum, M. , and Evans, C. , 1999, *Monetary Policy Shocks: What have we learned and to What End?*, Handbook of Monetary economics, eds., Woodfoord, M., and Taylor, J.
- [10] Christiano, L. J., Eichenbaum, M. , and Evans, C. , 2001, *Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy*, NBER WP #8403
- [11] Cochrane, J., 1994, *Shocks*, Carnegie-Rochester Conference Series on Public Policy.
- [12] Cochrane, J., 1998, *What do the VARs Mean?: Measuring the Output Effects of Monetary Policy*, Journal of Monetary Economics
- [13] Cooley, T. and Hansen, G. D., 1989, *The Inflation Tax in a Real Business Cycle Model*, American Economic Review 79:4
- [14] Dib, A., and Phaneuf, L., 2001, *An Econometric U.S. Business Cycle Model with Nominal and Real Rigidities*, Working Paper, Universite du Quebec a Montreal

- [15] Faust, J. and Leeper, E. M., 1997, *When do Long-Run Identifying Restrictions Give Reliable Results?*, Journal of Business and Economic Statistics
- [16] Faust, J., 1998, *The Robustness of Identified VAR Conclusions about Money*, Carnegie-Rochester Conference Series on Public Policy
- [17] Faust, J., and Rogers, J., 2000, Monetary policy's role in exchange rate behavior , Working Paper, International Finance Division, Federal Reserve Board of Governors
- [18] Favero, C., 2001, *Applied Macroeconometrics*, Oxford University Press
- [19] Goodfried, M., and King, R. , 1997, *The New Neoclassical Synthesis and the Role of Money*, NBER Macroannual
- [20] Hansen, L. P., and Sargent, T. J., 1980, *Formulating and Estimating Linear Rational Expectations Models*, Journal of Economic Dynamics and Control
- [21] Hansen, L. P., and Sargent, T. J., 1981a, *Exact Linear Rational Expectations Models: Specifications and Estimation*, Staff Reprot 71, Federal Reserve Bank of Minnesota
- [22] Hansen, L. P., and Sargent, T. J., 1981b, *Instrumental Variable Procedure for Estimating Linear Rational Expectations Models*, Staff Reprot 70, Federal Reserve Bank of Minnesota
- [23] Hansen, L. P., and Sargent, T. J., 1991, *Two Difficulties in Interpreting Vector Autoregressions*, in Hansen, L. P. and Sargent, T. J. (eds), Rational Expectations Econometrics (Underground Classics in Economics Series), WestView Press
- [24] Ireland, P., 2001, *Sticky-Price Models of the Business-Cycle: Specification and Stability*, Journal of Monetary Economics
- [25] Jeanne, O., 1997, *Generating Real Persistent Effects of Monetary Shocks: How Much Nominal Rigidity Do We Really Need?*, NBER Working Paper 6258
- [26] Kim, J., 1998, *Monetary Policy in a Stochastic Equilibrium Model with Real and Nominal Rigidities*, Working Paper, University of Virginia
- [27] Klaefferling, M. , 2001, *Nonfundamentalness in Macroeconomics: The conditional Excess Return Puzzle Revisited*, Working Paper, Bonn University
- [28] Kollmann, R., 2001, *Macroeconomic Effects of Nominal Exchange Rate Regimes: the Role of Money and Nominal Rigidities*, Working Paper, University of Bonn
- [29] Lippi, M. and L. Reichlin, L., 1993, *The Dynamic Effects of Aggregated Demand and Supply Disturbances: Comment*, American Economic Review

- [30] Lippi, M. and Reichlin, L., 1994, *VAR Analysis, Non-Fundamental Representations, Blaschke matrices*, Journal of Econometrics
- [31] Lucas, R. E. Jr., 1980, *Studies in Business Cycle Theory*, Cambridge: MIT Press
- [32] McGrattan, E. R., Rogerson, R. and Wright, R., 1997, An Equilibrium Model of the Business Cycle with Household Production and Taxation, International Economic Review
- [33] Reichlin, L., 2000, *Extracting Business Cycle Indexes from Large Data Sets: Aggregation, Estimation, Identification*, Manuscript, University Libre de Bruxelles
- [34] Rudebusch, G., 1998, *Do Measures of Monetary Policy in a VAR Make Sense?*, International Economic Review
- [35] Runkle, D., 1987 *Vector Autoregressions and Reality*, Staff Report 107, Federal Reserve Bank of Minneapolis
- [36] Sartre, P.-D. G., 1997, *On the Identification of Structural Vector Autoregressions*, Federal Reserve Bank of Richmond Economic Quarterly
- [37] Sims, C. A., 1980, *Macroeconomics and Reality*, Econometrica
- [38] Sims, C. A., and Zha, T., 1996, *Does Monetary Policy Generate Recessions*, Manuscript, Princeton University
- [39] Stock, J. and Watson, M., 2001, Diffusion Indexes, Manuscript, J. F. Kennedy School of Government
- [40] Townsend, R., 1983, *Forecasting the Forecasting of Others*, Journal of Political Economy
- [41] Uhlig, H., 2001, *What are the effects of monetary policy on output? Results from an Agnostic Identification Procedure*, Working Paper, Humboldt University
- [42] Yun, T., 1999, *Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles*, Journal of Monetary Economics

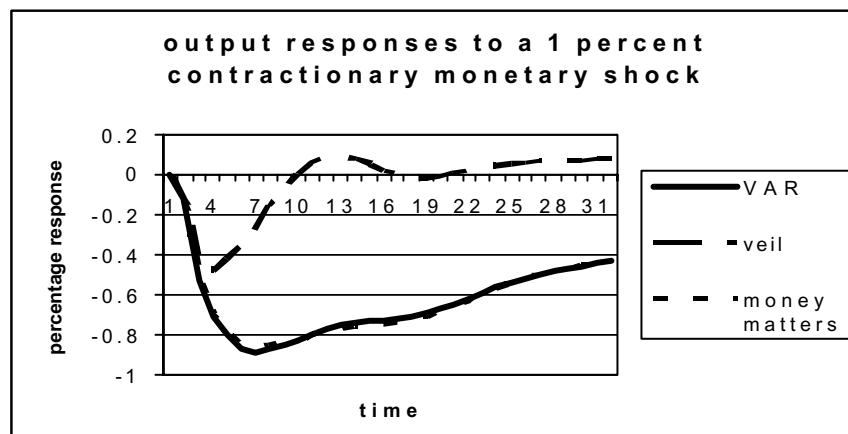


Figure 1:

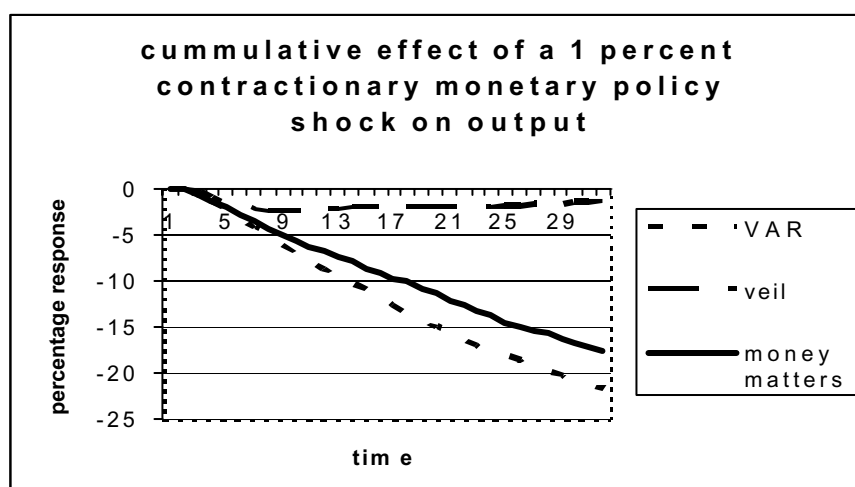


Figure 2:

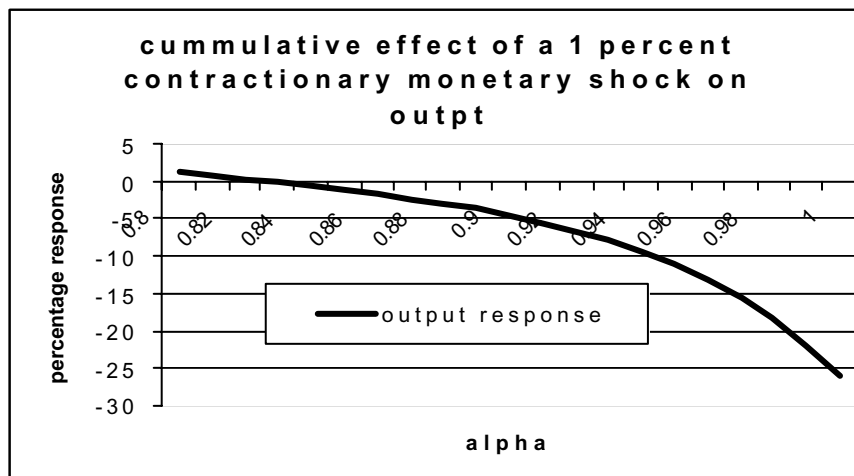


Figure 3:

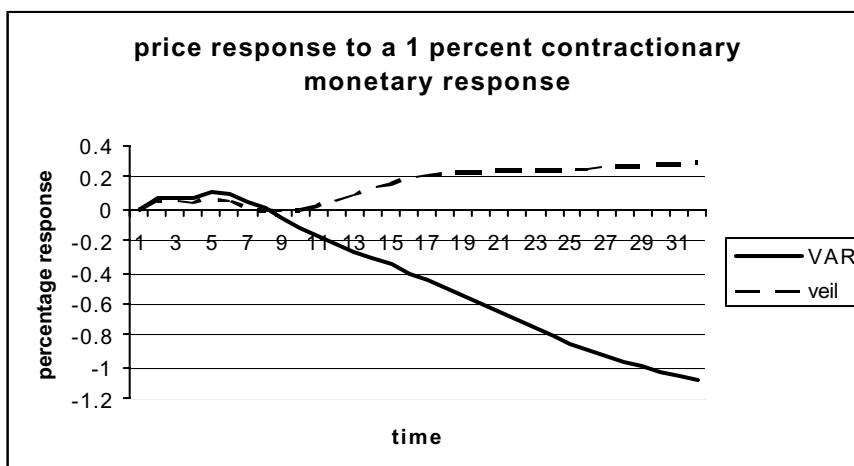


Figure 4:

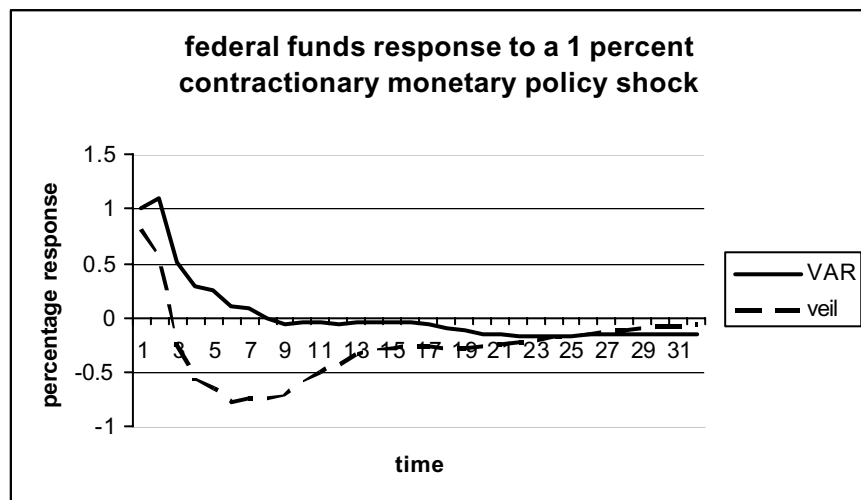


Figure 5:

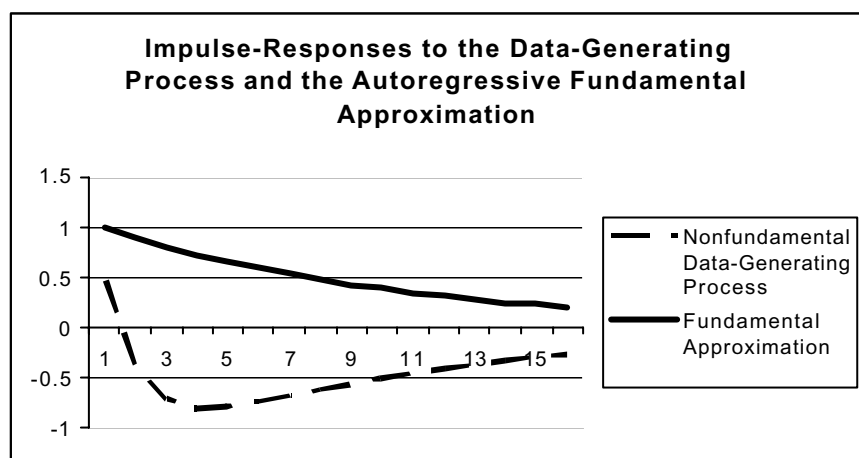


Figure 6:

European Central Bank working paper series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (<http://www.ecb.int>).

- I 13 "Financial frictions and the monetary transmission mechanism: theory, evidence and policy implications" by C. Bean, J. Larsen and K. Nikolov, January 2002.
- I 14 "Monetary transmission in the euro area: where do we stand?" by I. Angeloni, A. Kashyap, B. Mojon, D. Terlizzese, January 2002.
- I 15 "Monetary policy rules, macroeconomic stability and inflation: a view from the trenches" by A. Orphanides, December 2001.
- I 16 "Rent indices for housing in West Germany 1985 to 1998" by J. Hoffmann and C. Kurz., January 2002.
- I 17 "Hedonic house prices without characteristics: the case of new multiunit housing" by O. Bover and P. Velilla, January 2002.
- I 18 "Durable goods, price indexes and quality change: an application to automobile prices in Italy, 1988-98" by G. M. Tomat, January 2002.
- I 19 "Monetary policy and the stock market in the euro area" by N. Cassola and C. Morana, January 2002.
- I 20 "Learning stability in economics with heterogeneous agents" by S. Honkapohja and K. Mitra, January 2002.
- I 21 "Natural rate doubts" by A. Beyer and R. E. A. Farmer, February 2002.
- I 22 "New technologies and productivity growth in the euro area" by F. Visselaar and R. Albers, February 2002.
- I 23 "Analysing and combining multiple credit assessments of financial institutions" by E. Tabakis and A. Vinci, February 2002.
- I 24 "Monetary policy, expectations and commitment" by G. W. Evans and S. Honkapohja, February 2002.
- I 25 "Duration, volume and volatility impact of trades" by S. Manganelli, February 2002.
- I 26 "Optimal contracts in a dynamic costly state verification model" by C. Monnet and E. Quintin, February 2002.
- I 27 "Performance of monetary policy with internal central bank forecasting" by S. Honkapohja and K. Mitra, February 2002.
- I 28 "Openness, imperfect exchange rate pass-through and monetary policy" by F. Smets and R. Wouters, February 2002.

- I 29 “Non-standard central bank loss functions, skewed risks, and certainty equivalence” by A. al-Nowaihi and L. Stracca, March 2002.
- I 30 “Harmonized indexes of consumer prices: their conceptual foundations” by E. Diewert, March 2002.
- I 31 “Measurement bias in the HICP: what do we know, and what do we need to know?” by M. A. Wynne and D. Rodríguez-Palenzuela, March 2002.
- I 32 “Inflation dynamics and dual inflation in accession countries: a “new Keynesian” perspective” by O. Arratibel, D. Rodríguez-Palenzuela and C. Thimann, March 2002.
- I 33 “Can confidence indicators be useful to predict short-term real GDP growth?” by A. Mourougane and M. Roma, March 2002.
- I 34 “The cost of private transportation in the Netherlands, 1992-99” by B. Bode and J. Van Dalen, March 2002.
- I 35 “The optimal mix of taxes on money, consumption and income” by F. De Fiore and P. Teles, April 2002.
- I 36 “Retail bank interest rate pass-through: the new evidence at the euro area level” by G. de Bondt, April 2002.
- I 37 “Equilibrium bidding in the eurosystem’s open market operations” by U. Bindseil, April 2002.
- I 38 “New” views on the optimum currency area theory: what is EMU telling us?” by F. P. Mongelli, April 2002.
- I 39 “On currency crises and contagion” by M. Fratzscher, April 2002.
- I 40 “Price setting and the steady-state effects of inflation” by M. Casares, May 2002.
- I 41 “Asset prices and fiscal balances” by F. Eschenbach and L. Schuknecht, May 2002.
- I 42 “Modelling the daily banknotes in circulation in the context of the liquidity management of the European Central Bank”, by A. Cabrero, G. Camba-Mendez, A. Hirsch and F. Nieto, May 2002.
- I 43 “A non-parametric method for valuing new goods”, by I. Crawford, May 2002.
- I 44 “A failure in the measurement of inflation: results from a hedonic and matched experiment using scanner data”, by M. Silver and S. Heravi, May 2002.
- I 45 “Towards a new early warning system of financial crises”, by M. Fratzscher and M. Bussiere, May 2002.
- I 46 “Competition and stability – what’s special about banking?”, by E. Carletti and P. Hartmann, May 2002.

- I47 “Time-to-build approach in a sticky price, sticky wage optimising monetary model, by M. Casares, May 2002.
- I48 “The functional form of yield curves” by V. Brousseau, May 2002.
- I49 “The Spanish block of the ESCB multi-country model” by A. Estrada and A. Willman, May 2002.
- I50 “Equity and bond market signals as leading indicators of bank fragility” by R. Gropp, J. Vesala and G. Vulpes, June 2002.
- I51 “G7 inflation forecasts” by F. Canova, June 2002.
- I52 “Short-term monitoring of fiscal policy discipline” by G. Camba-Mendez and A. Lamo, June 2002.
- I53 “Euro area production function and potential output: a supply side system approach” by A. Willman, June 2002.
- I54 “The euro bloc, the dollar bloc and the yen bloc: how much monetary policy independence can exchange rate flexibility buy in an interdependent world?” by M. Fratzscher, June 2002.
- I55 “Youth unemployment in the OECD: demographic shifts, labour market institutions, and macroeconomic shocks” by J. F. Jimeno and D. Rodriguez-Palenzuela, June 2002.
- I56 “Identifying endogenous fiscal policy rules for macroeconomic models” by J. J. Perez, and P. Hiebert, July 2002.
- I57 “Bidding and performance in repo auctions: evidence from ECB open market operations” by K. G. Nyborg, U. Bindseil and I. A. Strebulaeu, July 2002.
- I58 “Quantifying Embodied Technological Change” by P. Sakellaris and D. J. Wilson, July 2002.
- I59 “Optimal public money” by C. Monnet, July 2002.
- I60 “Model uncertainty and the equilibrium value of the real effective euro exchange rate” by C. Detken, A. Dieppe, J. Henry, C. Marin and F. Smets, July 2002.
- I61 “The optimal allocation of risks under prospect theory” by L. Stracca, July 2002.
- I62 “Public debt asymmetries: the effect on taxes and spending in the European Union” by S. Krogstrup, August 2002.
- I63 “The rationality of consumers’ inflation expectations: survey-based evidence for the euro area” by M. Forsells and G. Kenny, August 2002.
- I64 “Euro area corporate debt securities market: first empirical evidence” by G. de Bondt, August 2002.

- I 65 “The industry effects of monetary policy in the euro area” by G. Peersman and F. Smets, August 2002.
- I 66 “Monetary and fiscal policy interactions in a micro-founded model of a monetary union” by R. M.W.J. Beetsma and H. Jensen, August 2002.
- I 67 “Identifying the effects of monetary policy shocks on exchange rates using high frequency data” by J. Faust, J.H. Rogers, E. Swanson and J.H. Wright, August 2002.
- I 68 “Estimating the effects of fiscal policy in OECD countries” by R. Perotti, August 2002.
- I 69 “Modelling model uncertainty” by A. Onatski and N. Williams, August 2002.
- I 70 “What measure of inflation should a central bank target?” by G. Mankiw and R. Reis, August 2002.
- I 71 “An estimated stochastic dynamic general equilibrium model of the euro area” by F. Smets and R. Wouters, August 2002.
- I 72 “Constructing quality-adjusted price indices: a comparison of hedonic and discrete choice models” by N. Jonker, September 2002.
- I 73 “Openness and equilibrium determinacy under interest rate rules” by F. de Fiore and Z. Liu, September 2002.
- I 74 “International monetary policy co-ordination and financial market integration” by A. Sutherland, September 2002.
- I 75 “Monetary policy and the financial accelerator in a monetary union” by S. Gilchrist, J.O. Hairault and H. Kempf, September 2002.
- I 76 “Macroeconomics of international price discrimination” by G. Corsetti and L. Dedola, September 2002.
- I 77 “A theory of the currency denomination of international trade” by P. Bacchetta and E. van Wincoop, September 2002.
- I 78 “Inflation persistence and optimal monetary policy in the euro area” by P. Benigno and J.D. López-Salido, September 2002.
- I 79 “Optimal monetary policy with durable and non-durable goods” by C.J. Erceg and A.T. Levin, September 2002.
- I 80 “Regional inflation in a currency union: fiscal policy versus fundamentals” by M. Duarte and A.L. Wolman, September 2002.
- I 81 “Inflation dynamics and international linkages: a model of the United States, the euro area and Japan” by G. Coenen and V. Wieland, September 2002.
- I 82 “The information content of real-time output gap estimates: an application to the euro area” by G. Rünstler, September 2002.

- 183 “Monetary policy in a world with different financial systems” by E. Faia, October 2002.
- 184 “Efficient pricing of large-value interbank payment systems” by C. Holthausen and J.-C. Rochet, October 2002.
- 185 “European integration: what lessons for other regions? The case of Latin America” by E. Dorrucchi, S. Firpo, M. Fratzscher and F. P. Mongelli, October 2002.
- 186 “Using money market rates to assess the alternatives of fixed versus variable rate tenders: the lesson from 1989-98 data for Germany” by M. Manna, October 2002.
- 187 “A fiscal theory of sovereign risk” by M. Uribe, October 2002.
- 188 “Should central banks really be flexible?” by H. P. Grüner, October 2002.
- 189 “Debt reduction and automatic stabilisation” by P. Hiebert, J. J. Pérez and M. Rostagno, October 2002.
- 190 “Monetary policy and the zero bound to interest rates: a review” by T. Yates, October 2002.
- 191 “The fiscal costs of financial instability revisited” by L. Schuknecht and F. Eschenbach, November 2002.
- 192 “Is the European Central Bank (and the United States Federal Reserve) predictable?” by G. Perez-Quiros and J. Sicilia, November 2002.
- 193 “Sustainability of public finances and automatic stabilisation under a rule of budgetary discipline” by J. Marín, November 2002.
- 194 “Sensitivity analysis of volatility: a new tool for risk management” by S. Manganelli, V. Ceci and W. Vecchiato, November 2002.
- 195 “In-sample or out-of-sample tests of predictability: which one should we use?” by A. Inoue and L. Kilian, November 2002.
- 196 “Bootstrapping autoregressions with conditional heteroskedasticity of unknown form” by S. Gonçalves and L. Kilian, November 2002.
- 197 “A model of the Eurosystem’s operational framework for monetary policy implementation” by C. Ewerhart, November 2002.
- 198 “Extracting risk-neutral probability densities by fitting implied volatility smiles: some methodological points and an application to the 3M EURIBOR futures option prices” by A. B. Andersen and T. Wagener, December 2002.
- 199 “Time variation in the tail behaviour of bund futures returns” by T. Werner and C. Upper, December 2002.

- 200 “Interdependence between the euro area and the United States: what role for EMU?” by M. Ehrmann and M. Fratzscher, December 2002.
- 201 “Euro area inflation persistence” by N. Batini, December 2002.
- 202 “Aggregate loans to the euro area private sector” by A. Calza, M. Manrique and J. Sousa, January 2003.
- 203 “Myopic loss aversion, disappointment aversion and the equity premium puzzle” by D. Fielding and L. Stracca, January 2003.
- 204 “Asymmetric dynamics in the correlations of global equity and bond returns” by L. Cappiello, R.F. Engle and K. Sheppard, January 2003.
- 205 “Real exchange rate in an inter-temporal n-country-model with incomplete markets” by B. Mercereau, January 2003.
- 206 “Empirical estimates of reaction functions for the euro area” by D. Gerdesmeier and B. Roffia, January 2003.
- 207 “A comprehensive model on the euro overnight rate” by F. R. Würtz, January 2003.
- 208 “Do demographic changes affect risk premiums? Evidence from international data” by A. Ang and A. Maddaloni, January 2003.
- 209 “A framework for collateral risk control determination” by D. Cossin, Z. Huang, D. Aunon-Nerin and F. González, January 2003.
- 210 “Anticipated Ramsey reforms and the uniform taxation principle: the role of international financial markets” by S. Schmitt-Grohé and M. Uribe, January 2003.
- 211 “Self-control and savings” by P. Michel and J.P. Vidal, January 2003.
- 212 “Modelling the implied probability of stock market movements” by E. Glatzer and M. Scheicher, January 2003.
- 213 “Aggregation and euro area Phillips curves” by S. Fabiani and J. Morgan, February 2003.
- 214 “On the selection of forecasting models” by A. Inoue and L. Kilian, February 2003.
- 215 “Budget institutions and fiscal performance in Central and Eastern European countries” by H. Gleich, February 2003.
- 216 “The admission of accession countries to an enlarged monetary union: a tentative assessment” by M. Ca’Zorzi and R. A. De Santis, February 2003.
- 217 “The role of product market regulations in the process of structural change” by J. Messina, March 2003.
- 218 “The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan” by G. Coenen and V. Wieland, March 2003.

- 219 “Extra-euro area manufacturing import prices and exchange rate pass-through” by B. Anderton, March 2003.
- 220 “The allocation of competencies in an international union: a positive analysis” by M. Ruta, April 2003.
- 221 “Estimating risk premia in money market rates” by A. Durré, S. Evjen and R. Pilegaard, April 2003.
- 222 “Inflation dynamics and subjective expectations in the United States” by K. Adam and M. Padula, April 2003.
- 223 “Optimal monetary policy with imperfect common knowledge” by K. Adam, April 2003.
- 224 “The rise of the yen vis-à-vis the (“synthetic”) euro: is it supported by economic fundamentals?” by C. Osbat, R. Ruffer and B. Schnatz, April 2003.
- 225 “Productivity and the (“synthetic”) euro-dollar exchange rate” by C. Osbat, F. Visselaar and B. Schnatz, April 2003.
- 226 “The central banker as a risk manager: quantifying and forecasting inflation risks” by L. Kilian and S. Manganelli, April 2003.
- 227 “Monetary policy in a low pass-through environment” by T. Monacelli, April 2003.
- 228 “Monetary policy shocks – a nonfundamental look at the data” by M. Klaeffer, May 2003.