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Abstract
In this paper we study risk-neutral densities (RNDs) for the German stock market. The use of option prices allows us to quantify the risk-neutral probabilities of various levels of the DAX index. For the period from December 1995 to November 2001, we implement the mixture of log-normals model and a volatility-smoothing method. We discuss the time series behaviour of the implied PDFs and we examine the relations between the moments and observable factors such as macroeconomic variables, the US stock markets and credit risk. We find that the risk-neutral densities exhibit pronounced negative skewness. Our second main observation is a significant spillover of volatility, as the implied volatility and kurtosis of the DAX RND are mostly driven by the volatility of US stock prices.

Key words: Option prices, risk-neutral density, volatility, spillover;

JEL Classification: C22, C51, G13, G15;
Non-technical summary

In Financial Economics, many researchers have studied option prices, because these derivatives contain unique information that is not available from the prices of other financial instruments. A call option gives the buyer the right to purchase in the future a certain asset at a price fixed today. The value of such an option is determined by the distance between the current stock price and the exercise price. When market participants price option contracts in the course of trading, they use forecasts of the probability of different asset prices for the period until the derivative expires. The perception of market participants about the movement of the asset price, in particular the probability density until expiry, is thus incorporated into the market price of the puts and calls through transactions made on the derivatives exchange. Therefore, the observed market prices of the options convey information about the market operators’ assessment of the price process of the underlying instrument.

By means of econometric methods, the information contained in options prices can be extracted. In the literature, two methods are most frequently chosen, namely the implied volatility and the risk-neutral density. The latter approach extends the frequently used concept of the volatility implicit in option prices to modelling the probabilities that market participants assign to all possible price levels of the underlying instrument. The entire RND offers a wider information set as it includes the third (skewness) and the fourth (kurtosis) moment of a distribution. The implied skewness measures the asymmetry in the expectations of the operators in the option markets around the mean. The kurtosis computed from the RND indicates how frequently market participants expect extreme price changes of either sign to occur. Overall, the information conveyed by option prices is more comprehensive than that contained in a time series of stock returns.

The purpose of this paper is to analyse the risk-neutral density derived from prices of DAX options. We first estimate two specifications of the RND. Then, we focus on observable factors that may drive changes in the moments of the RND. For this purpose, we investigate the impact of various macroeconomic and financial variables on risk-neutral densities of stock market movements. In this way, we attempt to uncover relationships between the implied volatility, skewness and kurtosis computed from the RND and the underlying fundamentals of the stock market.

Our paper offers two contributions to the literature: First, we investigate RNDs for the German stock market, which is the largest stock market in the euro area. Second, we evaluate whether a comprehensive set of factors can explain the changes in the uncertainty about future equity prices. Hence, we analyse which types of information affect the perceptions about future stock market movements as contained in DAX option prices. So far, the literature has concentrated on issues of estimations, but there has been no attempt to analyse the potential linkages of RNDs to fundamental factors by means of an econometric analysis.

Regarding these two issues, we obtain the following results. First, we report strong negative skewness in the risk-neutral density, which indicates that the probability of a large decrease in stock prices exceeds the probability of a large increase. In the literature on US equity derivatives, this finding has been termed “crashophobia”. Our second result is that the implied volatility of the US stock market has the strongest effect on changes in the DAX RNDs. Therefore the expectation about future stock market movements is less influenced by economic activity in Germany, but more by perceptions about the variability of US stock prices. We also document that the explanatory value for the third and fourth moments falls relative to the second moment. This finding indicates the existence of an unobserved
component in the determinants for skewness and kurtosis. Overall, our observations indicate that the German stock market is influenced to a considerable extent by US information. So far, work on the integration or segmentation of continental European stock markets has been confined to methodologies based on returns. Therefore, our derivatives-based approach broadens the perspective because it uses the forward-looking nature of option prices instead of the backward-looking characteristic of time series models.
Introduction

The recent fall in stock markets after the end of the period of “irrational exuberance” has again demonstrated that large price changes are not rare events in the asset markets of developed economies. For a central bank, there are several motivations for studying measures of the movement of stock markets. In the context of monetary stability, the information that can be extracted from asset prices has been gaining in importance. One reason is that households have been investing more heavily in the stock market. Therefore, negative wealth effects caused by stock price declines may worsen the economic climate. In addition, the fluctuations on the stock market also affect the financing conditions of firms. For preserving financial stability, it is also important to analyse the downside risk of the portfolios of institutional investors. After all, sudden price changes can cause large losses in the trading books of financial institutions, thus possibly endangering the stability of the financial system.

The econometric analysis of the movements of stock prices is commonly based on the probability density function (PDF), because this function shows estimates for the probability of particular levels of the asset price. In the literature, two methodologies can be discerned. First, the actual (or statistical) density function is estimated from time series of historical returns by means of a parametric model, such as Student’s t density. Second, the risk-neutral density (RND) is estimated from daily cross-sections of option prices. This approach extends the frequently used concept of the volatility implicit in option prices to modelling the probabilities that market participants assign to all possible price levels of the underlying instrument. Volatility is a narrow measure of uncertainty because it describes only the width or dispersion of the implied PDF. In contrast, the entire RND offers a wider information set as it includes the third (skewness) and the fourth (kurtosis) moment of a distribution. The implied skewness measures the asymmetry in the expectations of the operators in the option markets around the mean. The kurtosis computed from the RND indicates how frequently market participants expect extreme price changes of either sign to occur. Overall, the information conveyed by option prices is more comprehensive than that contained in a time series of returns and therefore allows unique insights into the perceptions of option traders.

The purpose of this paper is to analyse the risk-neutral density derived from prices of DAX options. We focus on observable factors that may influence changes in the moments of the RND. For this purpose, we investigate the impact of various macroeconomic and financial variables on risk-neutral densities of stock market movements. In this way, we attempt to uncover relationships between the implied volatility, skewness and kurtosis computed from the RND and the underlying fundamentals of the stock market. Our sample runs from December 1995 to November 2001. The period under review includes both the strong rise and the subsequent fall of the German stock market.

To extract the market beliefs on the future movement of the DAX, we use the mixture of log-normals model and a volatility smoothing method. These two specifications can reproduce different levels of skewness and kurtosis and therefore capture a variety of shapes of the density. We construct the RNDs for a constant horizon of 45 days. In the second step, we compute from our implied PDFs the volatility, skewness and kurtosis and subsequently examine how economic and financial variables affect these moments. In a regression

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3 For a recent example, see Peiro (1999).
5 In our context, the term “uncertainty” represents volatility, skewness and kurtosis, but captures also risk premia and the uncertainty about these moments, e.g. the possibility that volatility is stochastic.
framework, we evaluate to what extent interest rates, inflation, measures of economic activity and the US stock markets determine the changes in the market perception about future stock price dynamics. Our separate analysis of the second, third and fourth moments allows us to evaluate which factors influence the dispersion, asymmetry and tail mass of the density. Hence, our analysis can show whether, for instance, a fall in German economic activity only affects the implied volatility or the higher moments as well.

Our paper offers two contributions to the literature: First, we investigate RNDs for the German stock market, which is the largest stock market in the euro area. Second, we evaluate whether a comprehensive set of factors can explain the changes in the uncertainty about future equity prices. Hence, we analyse which types of information affect the perceptions about future stock market movements as contained in DAX option prices. So far, the literature has concentrated on issues of estimations, but there has been no attempt to analyse the potential linkages of RNDs to fundamental factors by means of an econometric analysis. Our study is closely related to Mixon (2002). This author also studies determinants of the information extracted from options prices. But in contrast to our focus on RNDs, Mixon (2002) investigates variables influencing the implied volatility surface.

Regarding these two issues, we obtain the following results. First, we report strong negative skewness in the risk-neutral density, which indicates that the probability of a large decrease in stock prices exceeds the probability of a large increase. In the literature on US equity derivatives, this finding has been termed “crashophobia”. Our second result is that the implied volatility of the US stock market has the strongest effect on changes in the DAX RNDs. Therefore the expectation about future stock market movements is less influenced by economic activity in Germany, but more by perceptions about the variability of US stock prices. We also document that the explanatory value for the third and fourth moments falls relative to the second moment. This finding indicates the existence of an unobserved component in the determinants for skewness and kurtosis. Overall, our observations indicate that the German stock market is influenced to a considerable extent by US information. So far, work on the integration or segmentation of continental European stock markets has been confined to methodologies based on returns. Therefore, our derivatives-based approach broadens the view because it uses the forward-looking nature of option prices instead of the backward-looking characteristic of time series models.

The rest of this paper is organised as follows: The second section describes the methods to estimate the implied densities and outlines our sample. In section three, we report the estimation results from the two specifications and then we analyse the determinants of the moments. Section four summarises our main results and concludes.

2. Methodology

2.1 Models

Options are derivative instruments, which give the owner of a call option (put option) the right to buy (sell) a certain asset at a fixed exercise price over a fixed time period (American option) or at a fixed date (European option). The primary characteristic of option contracts is their moneyness, which is defined as the exercise price (= strike price) divided by the current market price of the underlying instrument, e.g. the value of the stock index. Three categories

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7 The RND is a more elaborate estimation procedure than the volatility surface and hence our paper provides a methodological extension to Mixon (2002).
of moneyness are distinguished: Out of the money (OTM) if the strike exceeds the current price, at the money (ATM) if equal and in the money (ITM) if the strike is lower than the current price. A central result is that the theoretical price for a European call option equals:

\[ c(S, X, T) = e^{-rT} \int_{X}^{\infty} (S - X) f(S) dS \]

with
\[ c(.) \text{ price for a European call option} \]
\[ S \text{ price of the underlying asset} \]
\[ X \text{ exercise price} \]
\[ T \text{ time to maturity of the option} \]
\[ f(S) \text{ risk-neutral density of the price of the underlying asset} \]
\[ r \text{ risk-free interest rate} \]

Hence, the value of a European call option is determined by the difference between the current price of the underlying asset and the strike price. If this distance is positive, i.e. if the option is in the money, then the current payoff of the position is positive. In case the strike price is higher than the current stock price, i.e. if the contract is OTM, the value of the call option is still larger than zero, because until maturity, the price difference can become positive. When market participants value option contracts, they use forecasts of the probability of different asset prices for the period until the derivative expires. The perception of market participants about the movement of the asset price, in particular the probability density until expiry, is thus incorporated into the market price of the puts and calls in the process of trading. Therefore, the observed prices of the options convey information about the market operators' assessment of the price process of the underlying instrument, in our case the DAX index.

Among practitioners, the seminal model of Black and Scholes (1973) is commonly used. It assumes that the dynamics of the asset price follow a geometric Brownian motion (GBM). In this case, returns follow a normal (Gaussian) density, and the theoretical price of a call option is

\[ c(S, X, T) = SN(d_1) - Xe^{-rT} N(d_2) \]

with
\[ d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]
\[ N(.) \text{ cumulative Gaussian density} \]
\[ \sigma \text{ volatility of stock returns} \]

Given that all other parameters are known, the Black-Scholes model makes it possible to estimate the volatility implicit in put and call prices by means of a numerical iteration method.

To reduce the impact of misspecification problems, we employ two alternative estimation procedures. As Bliss and Panigirtzoglou (2002) discuss, the RND can be obtained by means of five methods: Specification of the stochastic process, implied trees, finite differences,

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\[ \text{See Hull (2000).} \]
approximation methods and smoothing of volatility. Among these alternatives, we choose the last two models. Hence, we use an extension of the Black-Scholes model and a semiparametric method, based on implied volatility. The following points support our choice of methods. First, the specification of the stochastic process is complex due to the variety of possible parameterisations and requires a considerable number of parameters, which increases the potential impact of estimation errors. Second, the estimation by means of finite differences requires equally spaced strike prices. For our sample, this is a considerable difficulty in the implementation, as section 2.2 demonstrates. Third, regarding the performance of the tree approach, there is little evidence of superior results. Finally, the two approaches that we use are frequently applied in the literature. Despite the different modelling strategies, both approaches are flexible enough to generate a variety of shapes that deviate from the normal (Gaussian) model. This property is required because the empirically observed densities of returns contrast with the Gaussian model [see Pagan 1996]. This rejection results from two stylised facts. First, large price changes appear more frequently than the normal density would lead to expect. Second, there are indications of significant asymmetry in stock returns. In other words, negative and positive price changes do not have the same probability. These two stylised facts are also apparent in implied volatilities. The plot of the volatilities and their corresponding strike prices shows a U-shaped or inverted J-shaped relation. In the literature, this empirical observation has been termed the smile or smirk effect. It reflects the fact that in contrast to the Black-Scholes assumptions, options with a distant strike price are given higher values in trading because the probability of a large price change is higher than in the Gaussian model.

In the first approach, the risk neutral densities extracted from observed market prices based on a theoretical pricing model. As a specification, we use the mixture of log-normal densities, which has proved to be flexible and computationally stable and the parameters of which offer straightforward interpretations. The mixture of two log-normal distributions is defined as

\[
f(S) = \theta \log N(a_1, b_1, S) + (1-\theta) \log N(a_2, b_2, S)
\]

(3) \[a_i = \ln S + (\mu_i - 0.5\sigma_i^2)T\]
\[b_i = \sigma_i \sqrt{T}\]

with

- \(S\) current price of the underlying asset
- \(i\) index of state (\(i = 1, 2\))
- \(\theta\) weight on the log-normal distributions (\(0 \leq \theta \leq 1\))
- \(\mu_i, \sigma_i\) mean and variance of the normal distributions
- \(a_i, b_i\) location and dispersion parameters of the log-normal distributions

The above specification of the stochastic process for the price of the underlying instrument is based on two states with different moments, governed by the weights \(\theta\) and 1-\(\theta\). In each state, the stock price is log-normally distributed. The estimation relies on nonlinear least squares (NLS): We minimise the squared distance between the observed market price and the theoretical price based on the mixture model. This estimation procedure is commonly used for

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9 One possible approach is the modelling of the conditional density by means of GARCH, cf. Lehar, Scheicher and Schittenkopf (2002).
10 In his survey, Jackwerth (1999), p.79 “concludes: “In empirical tests, implied trees perform as well (or as poorly) as parametric models and naive trader rules.”
11 The model has been introduced by Melick and Thomas (1997).
extracting parameters from option prices [cf. Engle and Mustafa (1992)]. In case the weight takes the boundary values of 0 or 1, the mixture model collapses to the Black-Scholes model. The second approach to obtain a RND does not rely on distributional assumptions. Based on the result derived by Breeden and Litzenberger (1978), the RND is obtained from the second derivative of the price of a call option with respect to the corresponding strike price:

\[
(4) \quad f(S) = e^{rT} \frac{\partial^2 C(S, T, X)}{\partial X^2}
\]

To implement this method a problem is that the observed option prices do not provide a continuous range, so that the resulting RND is not a well-behaved function. We overcome this problem by using the smoothed volatility smile. From the observed option prices, the implied volatilities are extracted by means of the Black-Scholes pricing function. To obtain a smoothed volatility smile we then transform our data set of implied volatilities from the volatility/strike space to the volatility/delta space. In the delta space, more weight is allocated to the at-the-money options, which are more actively traded; thus the more liquid prices have more prominence in determining the shape of the smile curve. After this change of dimension we fit a piecewise cubic spline to the data points. The smoothing spline is estimated by minimising the following function:

\[
(5) \quad L(\theta) = \sum_{i=1}^{N} w_i (\sigma_i - \sigma(\Delta_i, \theta))^2 + \lambda \int_{-\infty}^{\infty} g''(x; \theta)^2 \, dx
\]

with

- \( \theta \) parameters of the smoothing spline
- \( w_i \) weight of the error of the ith implied volatility
- \( \sigma_i \) implied volatility from observed option price “i”
- \( \Delta_i \) delta from observed option price “i”
- \( \lambda \) smoothing parameter
- \( g(\cdot) \) cubic spline function

The loss function has two components: The first part serves as the estimation function for the parameters of the spline function. The second component is a roughness measure to control the smoothness of the fitted spline whose knot points are situated at the observed deltas. The penalty is given by the parameter \( \lambda \) and our smoothness criterion is the integrated second derivative of the spline. We take the second derivative because this function determines that the shape of the RND is well behaved. As \( \lambda \), we choose a value of 6 after having tried out a number of alternative values. The experiment showed that this value gave a smooth RND for all months in our sample. In the loss function, the volatility errors are weighted according to the Vega coefficient of the corresponding option prices, that is their sensitivity to volatility. This choice can be motivated by the fact that for a call option, Vega declines with the distance of the strike price to the current price of the underlying asset. Having fitted the spline, we convert 5000 data points from the volatility/delta space to the price/strike space through application of the Black-Scholes formula. The resulting option prices are twice differentiated numerically. After controlling for the discount factor, the RND is obtained. In this context, it is important to keep in mind that the semiparametric approach does not rely on the validity of the Black-Scholes model. Instead, the theoretical formula only serves as a function to back out the volatility from the observed market prices. The sample for the spline procedure only

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12 See appendix A.2 in Bliss and Panigirtzoglou (2002) for more details.
includes call prices. Given that the put/call parity holds quite well, the information content of put prices can be neglected.

2.2 Sample

Our sample consists of the daily trade statistics of the options and futures on the DAX index. On EUREX, DAX options are traded European style; hence there is no need to account for the impact of early exercise. Every day, maturities of up to two years with at least five strike prices are traded. For contracts with a maturity of up to six months, there are at least nine strikes. The contract value is EUR 5 per index point. The minimum price movement is EUR 0.5. The payment of the option premium is due on the first trading day after the transaction has taken place. Options expire every third Friday in each month.

In order to eliminate time-to-maturity effects from our estimates, an RND with constant maturity is required. If this effect is neglected, the problem is that parameters change due to approaching of the expiry date; the volatility decreases with each time increment as the uncertainty about the asset price on the day of the maturity is reduced. We construct RNDs that are free of these erroneous effects by using monthly option prices with a maturity of 45 days. There are two reasons for choosing a monthly sample. First, this horizon provides a constant maturity implied PDF without interpolation. Therefore, the impact of estimation errors on the RNDs is kept as small as possible. Second, to analyse which factors determine the changes in the risk neutral moments, a monthly frequency allows us to use as many explanatory variables as possible. In particular, we can include macroeconomic variables that are only available as monthly data. Our estimations are based on put and call prices for all strike prices sampled on the trading day 45 days before expiry. As options expire each month, the period between December 1995 and November 2001 results in 72 data sets. Every month, we collect an array of put and call prices with equal maturity but different strikes as the basis for computing the risk-neutral densities. In our sample, the median number of options is 69, with a minimum of 30 options in January 1996 and a maximum of 121 options in November 2001 (see table 1). Since October 1996, the number of available option prices has always exceeded 40. Therefore, the number of data points from which we estimate the RNDs is quite high, compared to studies that analyse FX options. The size of our monthly sample is important for representing the behaviour of market participants with enough precision.

As a proxy for the risk-free rate we use interbank interest rates, specifically FIBOR rates before the introduction of the euro and EURIBOR rates thereafter. To match the 45-day horizon of the options with the maturity of the sampled interest rates, a linear interpolation is applied. The choice of interbank interest rates as input into the valuation of options rests on the liquidity and widespread use of this instrument by banks acting as option traders. For the estimation of the implied PDF, data quality is a key concern. Our search for errors in the database proceeded in three steps. As a first step, we discarded all options with a price below EUR 0.5 and all those where the numerical routine failed to generate an implied volatility. Then, we undertook a visual inspection of the put and call smirks for each month. The result of this procedure was that the put/call parity holds for almost all prices. Only a minor fraction had differing volatilities for the same moneyness. All of these were at the margins of the moneyness range – for calls, deep ITM; for puts, deep OTM. We therefore also discarded all options with moneyness below 0.75. As a test, we finally computed from our data set the VDAX index, which is published by EUREX and distributed in Datastream. Comparing the two series, we found an almost perfect match\textsuperscript{13}. To provide an overview of our data base,

\textsuperscript{13} The correlation between the Datastream VDAX and our estimated series is 0.9970 in levels and 0.9924 in changes.
table 1 shows the 0, 25, 50, 75 and 100% quantiles of the DAX index as well as strike prices, option prices, Black-Scholes implied volatilities and risk-free rates. The strikes vary between 2000 and 10200 index points, implied volatility ranges from 7.9% to 66.4%, and interest rates show comparatively little variation.

The behaviour of implied volatilities computed from our filtered sample is quite stable. As a representative example, graph 1 shows the pattern for our last date, 6 November 2001, with expiry on 21 December 2001. The relation between volatility and moneyness differs from the familiar smile effect, because volatility decreases as the strike price increases. As the estimations results will show, the RND corresponding to this volatility pattern has a thinner right tail and a fatter left tail. Jackwerth and Rubinstein (1996) document that the inverted J shape is also apparent in the US stock market after the crash of October 1987. We can observe that put and call volatilities coincide for the majority of strike prices. The small deviations to the right of the curve have no effect on the spline method as it relies exclusively on call prices. For the mixture models, the deviations have a small impact because of the construction of the loss function in the nonlinear least squares method.

Based on the 72 monthly data sets, we perform monthly estimates of the RNDs in a rolling window technique. As the value of the underlying instrument for each month, we use an estimate that is based on the DAX future. Given the quarterly expiry cycle of the DAX future, we interpolate in the other months. Depending on the state of the maturity cycle, there are three cases: a single future, DAX index and future, or two futures. These values enter as a starting point in the procedure to compute the ATM point. This point is obtained as the average of calls and puts for 2 strikes above and below the current value using the put/call parity. Having estimated the two RNDs, we compute the associated risk-neutral moments.

### 3. Empirical results

In this section, we start by discussing the estimated RNDs. Then we analyse the higher moments. Finally, we study the determinants of the changes in the moments of the risk neutral densities.

#### 3.1 Estimation results

We first analyse how the RNDs evolve over time. Graph 2 summarises the information from the monthly RND estimates by plotting the mean together with the 5% and 95% estimated percentiles from both methods. During our sample period, the mean of the implied PDF moved between 2000 and 8000 points, and the allocation of the probability mass between the centre and the tails changed. In 1996, the distance between the 5% and 95% percentiles was comparatively small so that the RNDs had more probability mass around the mean. From December 1997 onwards, the width of the interval increased, corresponding to rising uncertainty about the behaviour of the German stock market. In the first half of 2000, the bands widened as market operators were expecting a larger range of possible values for the DAX index. Concerning the differences between the two methods, the graph shows that the estimates from the mixture model and from the semiparametric method are quite close. So the two methods generate only small differences in the allocation of the 90% probability mass. An indication for the negligible difference is given by the relative percentage deviation between the upper and lower percentiles. Here the median values are 0.4% for the lower and 0.1% for the upper quantile. Hence, the description of the market assessment about the development of the DAX index does not strongly depend on which extraction method is used.
Our sample contains two tumultuous periods, namely autumn 1998 and autumn 2001. During both episodes, a pronounced widening of the distance between the two percentiles took place. Another observation from graph 2 concerns the symmetry of the 90% confidence interval. The distance between the mean and the upper and lower percentiles is varying over time. In some periods, the lower percentile is also more distant from the mean. This asymmetry reflects the fact that the left and the right tails of the RNDs do not contain the same amount of probability mass, i.e. the risk-neutral perception about directional moves differs according to the sign.

Graph 3 depicts the estimated RND’s in yearly intervals from January 1996 until the last estimate, for the expiry date in December 2001. Because the results of the two models are similar, we only show the mixture results. As mentioned earlier, the shift in the allocation of the relative probability mass manifests itself quite clearly in the estimated RNDs. Due to the fact that probability has moved from the centre towards the tails, the densities have become flatter. This shift means that a wider range of index values is now expected. Furthermore, the upward and subsequent downward movement in the probability of given index values is visible. The primary cause for this movement is the general fall in the value of stocks contained in the DAX index since spring 2000. At that time, the right-hand tail was above 8000 points and the probability of an index at 4000 points was close to zero. In November 2001, the right tail was situated at 6500 and the left at 3500. Thus, the density estimated for December 1998 and the estimate for November 2001 coincide quite closely, as in the three intervening years, the DAX had risen and subsequently lost all its earlier gains.

3.2 The higher moments of the RNDs

Of particular interest is the information contained in the higher moments of the RNDs. In graphs 4, 5 and 6 we plot the time series of volatility, skewness and kurtosis from the mixture of log-normals and the smoothed volatility smile methods, with table 2 showing some descriptive statistics.

As a measure of skewness, we use the Pearson statistic, which is defined as (mean–mode) / standard deviation. The commonly used skewness coefficient, namely the central third moment divided by the standard deviation, is very sensitive to the mass in the tails, and as the tails are not fully represented in the volatility smoothing method, some probability mass is omitted. In contrast, the Pearson measure\(^{14}\) is less sensitive to the tails and hence produces a more robust picture of the changes in the asymmetry.

In graph 4, we observe peaks of the implied volatility in 1998 and in 2001. The highest level is recorded for October 1998, when volatility rose to 54 %. In the aftermath of the events on September 11, volatility rose from 25 % to 37 %. Graph 5 indicates that the estimates for the skewness do not differ considerably between the mixture and the smoothed volatility, as both models produce a negative asymmetry. This relatively larger size of the left tail is in accordance with the downward shaped pattern in the relation between volatility and moneyness shown in graph 1. This pattern in implied volatilities leads to a negative skewness of the implied density because option contracts where the exercise price is below the current index level have higher volatilities and hence assign more probability mass in the left tail.\(^{15}\) The following descriptive analysis of the higher moments of the RNDs is based on the mixture model. However, results for the spline approach were quite similar.

\(^{14}\) Similar results are obtained for the Pearson II statistic, which uses the median instead of the mode.

\(^{15}\) See e.g. chapter 14 in Hull (2000).
The risk-neutral density from the mixture model deviates from the hypothesis of Gaussian price changes, because the skewness is persistently negative and there is excess kurtosis. The median of the asymmetry measure is –0.17, the minimum –0.38 and the maximum –0.03. The excess kurtosis, which reproduces the relative size of probability mass in the tails, lies between –0.08 and 2.60, with the median situated at 0.79. The single negative excess kurtosis arises from a particularly large distance between the means of the two regimes. The resulting RND has a hump that resembles a bimodal shape. This phenomenon may have arisen due some disagreement among operators in the DAX options market on the outlook of the index. Besides this episode, it can be concluded that there was relative unanimity among market participants for the outlook of the German stock market, because in all other cases, no bimodal RND appeared.

The finding of persistent negative skewness gives an insight into the assessments of market operators. DAX options traders expect that large downward jumps in the value of the German stock market appear more often than large increases. This result from the RND is the mirror image of the smirk in the implied volatility, which we discussed in section two. For the US stock market this observation has been documented by Jackwerth and Rubinstein (1996) and termed crashophobia. The economic rationale for this term is that put options are used as hedging instruments to protect against large downward movements in stock prices. This demand by investors due to portfolio insurance strategies has increased the price of protection and therefore the left tail of the RND receives more weight.16

An important caveat in this analysis is that the option-implied PDFs are derived under the assumption of risk-neutrality. Therefore, there may be differences between the options-based estimates and the actual (statistical) density of returns. As we aim at analysing the changes in the perceptions of market participants, these potential differences have no direct impact on our methodology.

3.3 Analysing the determinants of changes in the RND moments

Having discussed the time series behaviour of the moments, we now study which economic variables influence the changes in volatility, skewness and kurtosis. We focus on these three moments because they illustrate the market perception of the uncertainty about future DAX movements. Our separate analysis of the second, third and fourth moments enables us to evaluate which factors affect the width, asymmetry and tail mass of the implied PDF. Hence we can distinguish whether, for instance, a rise in German interest rates only affects the variance or also the skewness of the DAX RND. The theory that we outline in the following paragraphs deals with determinants of the second moments, as there is no theoretical framework for the determinants of the third and fourth moments.17 Hence, our analysis of the factors influencing measures of the asymmetry and tail mass of the implied PDF has a tentative, rather provisional character.

A starting point for choosing factors, which might explain changes in the RND moments, is the comprehensive work on linkages between stock markets and economic fundamentals in general. The seminal paper on this topic is Chen, Roll and Ross (1986).18 These authors use

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16 Chen, Hong and Stein (2000) give an alternative interpretation of the negative skewness. They argue that differences of opinions among investors and the possible existence of stock market bubbles may explain the negative asymmetry in US stock returns.

17 To quantify the effects of FX market interventions, regressions using RND moments have also been used by Galati and Melick (2002).

18 For a recent study on factor models see Flannery and Protopapadakis (2002).
the framework of arbitrage pricing theory (APT) to evaluate which macroeconomic risks are priced in the US stock market. The starting point for the construction of the factor set is the hypothesis that the current stock price equals the discounted present value of expected future dividends:

\[ E_{t-1}P_t = E_{t-1}\sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+r_{t+i})} \]

with
- \( P \) stock price
- \( D \) dividends and capital gains
- \( r \) discount rate

From equation (6) we can observe that factors, which influence the discount rate and the future cash flows should ultimately have an impact on stock prices. The above relation leads to a factor model where stock returns are determined by a set of \( k \) factors, summarised by the following equation:

\[ R_t = a + \sum_{i=1}^{k} \beta_i f_{it} + e_t \]

with
- \( R \) stock return
- \( \beta \) factor loading
- \( f \) factor
- \( e \) idiosyncratic error term

To estimate equation (7), the sample of Chen et al. (1986) consists of industrial production, the default and term spreads, consumption and the oil price. Chen et al. find a strong impact on US stock portfolios from industrial production and interest rates.

Our analysis is similar to the study by Schwert (1989), because it investigates the determinants of the time variation in the volatility of US stock returns. The paper by Schwert focuses not on the determinants of returns as in (7), but on factors affecting a measure of the uncertainty of returns. Starting from the relations given in equation (6), it follows that, when a given factor influences the returns, its volatility should affect the volatility of stock returns:19

\[ \sigma_{st} = a + \sum_{i=1}^{k} \gamma_i \sigma_{it} + e_t \]

with
- \( \sigma_{st} \) volatility of stock returns
- \( \sigma_{it} \) volatility of factor \( i \)
- \( e \) idiosyncratic error term

By means of the specification given in (8), Schwert (1989) investigates the impact of a comprehensive set of real and nominal macroeconomic variables on a time series measure of

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19 Another approach to derive this relation is the Factor ARCH model, cf. Ng et al. (1992).
the volatility of returns from 1859 to 1987. He concludes that the variability of stock prices is related to the general situation of the economy, financial leverage and trading activity.

An alternative approach to investigate determinants of stock market uncertainty has been taken by Gemmill and Kamyiama (2000). These authors focus on the transmission of international information in equity derivatives markets. Similar to our approach, Gemmill and Kamyiama (2000) use index options, but their risk neutral measures are not based on the implied PDF and the information set is smaller. Their main finding is that significant spillovers exist among US, UK and Japanese volatilities, but not among the measures of skewness. Hence, they conclude that local factors drive the changes in the third moments.

From this brief survey, we can conclude that a comprehensive factor set is needed, which contains both macroeconomic and international information. Therefore, our set of explanatory variables consists of the following seven categories: German stock market, German economic activity, monetary stability, interest rates, the exchange rate, the US stock market and credit spreads. The factors are outlined in detail in the following.

1. **German stock market**: The principal influence on the changes of the moments is the movement of the DAX index itself. Depending on the situation, a pronounced decline may lead market participants to expect that further falls are likely. This reaction could be based on the assumption that the stock market is overvalued. Therefore, besides the index we also include an indicator of the valuation of the stock market. Here we choose the price/earnings ratio, which represents, albeit in a simple manner, the relation between the profitability of a firm and the current market price of equity.

2. **German economic activity**: Due to the business cycle, the cash flows of companies are influenced by the macroeconomic climate. A downturn may reduce aggregate demand and hence, reduce the profits of firms in the near future. Thus, a deteriorating economic outlook should lead to higher uncertainty in financial markets. We include three different measures of the state of the economy: German industrial production, the IFO overall business climate index and the unemployment rate. The change in industrial production is a measure of the development of output, which frequently leads the GDP growth cycle. Hence, a decline in industrial production is expected to increase the uncertainty about company profits and therefore also about the development of stock prices. The IFO index is a frequently observed climate indicator, and it is attributed leading indicator characteristics. In addition to output and consumer confidence, we include unemployment as an alternative measure of the economic climate, which may also affect market expectations. The inclusion of three diverse variables in the regression should indicate links between the real economy and the perceptions of market participants. Due to the release calendar, two lags are specified for each factor.

3. **Changes in monetary stability**: The potential impact of inflation arises from the fact that a worsening of monetary stability leads to higher nominal interest rates. From equation (6), we see that this rise lowers the discounted expected future dividends and hence may lead to a reaction of stock prices. Because of its significant information content for inflation prospects,20 we use the growth in the key monetary aggregate, the German contribution to M 3, as an indicator. Due to the release calendar, two lags are specified.

4. **Changes in interest rates**: In equation (6), interest rates represent the discount factor, but they are also important due to the close linkages between equities and fixed income. Evidence for this linkage is the “flight to quality” effect, where a falling stock market leads investors to

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20 See e.g. Nicoletti Altimari (2001).
choose the lower risk and higher liquidity of government bonds. To cover the entire term structure, we use the three-month interbank interest rate (FIBOR and EURIBOR) and the ten-year benchmark Bund yield.

5. Changes in the exchange rate: The exchange rate has two potential effects, namely on the asset allocation of international investors and on the profits of companies trading internationally. Many companies in the DAX index generate substantial parts of their cash flows abroad, which is why the movement of exchange rates also influences their earnings. A sharp drop in the exchange rate may affect the beliefs of traders in the option market by increasing the uncertainty about future profits. For the German economy, the key exchange rate is the US dollar, which we include relative to Deutsche mark and, since January 1999, to the euro.

6. US stock market: The integration of national financial markets has led to a higher correlation among stock markets. This process has strengthened the transmission of foreign shocks into the German market. The US stock market has the largest market capitalisation; hence, its movements have a strong influence on many other capital markets. To measure the movements in the US stock market, we choose the broad S&P 500 index.

7. Changes in credit spreads: The literature on credit risk shows the existence of a strong interaction between stock and credit markets. In particular, the valuation of default-risky debt depends on the movements of the stock price of the respective firm. More generally, due to the interdependence of market risk and default risk, a rise in the default risk premium may affect the assessment of participants in the DAX option market. As a proxy for credit risk, we choose the swap spread, i.e. the yield differential between a ten-year interest rate swap and a German government bond with the same maturity. A more refined measure of credit risk would be the yield spread of corporate bonds, but this measure is not available because the markets for euro-denominated corporate bonds have only been active for the second half of our sample. In addition to a premium for default risk, the swap spread also accounts for liquidity risk, as it is affected by the funding operations of banks in the interbank market.

In testing for the impact of explanatory factors on the moments, four methodological issues emerge. The first issue is whether the regressions ought to be estimated on the levels of the variables, as given in equation (8) or on the first differences. In order to find the appropriate specification we have tested for the evidence of unit roots. The results from the augmented Dickey-Fuller tests for the hypothesis of I(1) in the first column of table 5 show strong evidence of nonstationarity, as at the 5% level, most series contain unit roots. Thus, all further analysis is based on first differences, interpretable as growth rates or returns, respectively. Therefore we evaluate how a change in a certain factor affects e.g. the change in the skewness. The second issue is which factors show heteroscedasticity. This distinction arises again from equation (8), where the return volatilities are driven by the factor variances. To investigate which factors have a time-varying volatility, we computed the ARCH(1) test, which regresses the squared change in the factor on its lag. The results from this test, given in the last column of table 3, indicate heteroscedasticity for the stock market variables DAX, DAX P/E and S&P. The rejection of time-varying second moments for the macroeconomic variables probably arises from the small sample, namely 71 observations. Therefore, we restricted our volatility measures to financial variables and omitted macroeconomic variables. To include a large variety of potential factors, we estimated GARCH models for the daily returns of two clearly heteroscedastic series, namely the long yield and the DEM/USD

21 See e.g. Longin and Solnik (1995).
22 See e.g. Collin-Dufresne et al. (2001).
exchange rate. These daily series were then aggregated to a monthly frequency. To represent the variability of the US stock market, we do not use a GARCH estimate, but instead we include the VIX implied volatility calculated by CBOE. This option-based measure of stock market volatility ensures homogeneity with our risk-neutral density. Third, as it is quite restrictive to assume that there are only simultaneous effects, we also evaluate the existence of lead-lag relations. Accordingly, we apply Granger causality tests to uncover possible dynamic relations. Finally, pronounced interdependence in the set of regressors complicates the separation of the effects of the individual factors. For example, the highest correlation with a value of -0.7 is observed between US stock returns and their volatility. The significant correlation structure among the set of factors complicates the selection of an optimal specification because it leads to multicollinearity among the explanatory variables. To investigate the robustness of our findings, we therefore apply a principal components analysis to construct a set of orthogonal factors.

The following table summarises our set of 14 explanatory variables and descriptive statistics of the set of factors are given in table 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>Level of DAX index</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>Price/earnings ratio of DAX index</td>
</tr>
<tr>
<td>Ind.prod.</td>
<td>Index of industrial production</td>
</tr>
<tr>
<td>IFO</td>
<td>IFO overall business climate index</td>
</tr>
<tr>
<td>Unemployment</td>
<td>Unemployment rate in %</td>
</tr>
<tr>
<td>M3</td>
<td>German contribution to M3</td>
</tr>
<tr>
<td>Fibor</td>
<td>Short rate on German money market</td>
</tr>
<tr>
<td>Yield</td>
<td>Yield on 10 year bond of German government</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>GARCH estimate for volatility of yield</td>
</tr>
<tr>
<td>USDM</td>
<td>Exchange rate of US$ to DM and €</td>
</tr>
<tr>
<td>Vola (USDM)</td>
<td>GARCH estimate for volatility of US$</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>Level of S&amp;P 500 index</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td>VIX implied volatility of S&amp;P</td>
</tr>
<tr>
<td>Swap</td>
<td>Spread of 10 year interest rate swap to yield</td>
</tr>
</tbody>
</table>

### 3.4 Results for factor regressions

Table 4 shows the regressions for the first differences of the three moments, each obtained from the mixture and the smoothed volatility, together with their adjusted $R^2$ and Durbin-Watson tests. All regressions are estimated with Newey-West standard errors (lag truncation equals 3). Overall, two results emerge. First, US stock market factors have a strong impact on the changes in all three moments. The change in the US implied volatility has a significant effect on all three moments estimated from the mixture model and on the spline volatility and spline kurtosis. For the spline skewness, the US stock market is represented by S&P returns. Second, the regression estimates show some impact from interest rates, namely the short-term rate and the volatility of Bund yields affect the implied third and fourth moments. We can hence conclude that the hypothesis of Schwert (1989) is supported: The volatility of a variable in our factor set, namely the Bund yield, indeed influences the uncertainty about the future development of the DAX index. However, this relation is between yield volatility and higher moments, as there is no significant impact on the second moments. The coefficients on the changes in yield volatility are positive; thus, a rise in uncertainty on the fixed income market increases the third and fourth moments of the implied PDF.
When we analyse the differences among the three moments, we observe that volatility, skewness and kurtosis are all influenced by the same factor, namely the US stock market. In addition, the short rate and the variance of the German long interest rates affect the third and fourth moments. This finding is notable, as it shows that the asymmetry of the RND is also partly influenced by a key German variable. The comparison of the estimates from the mixture and the spline method shows limited differences, as the skewness of the spline is not affected by the VIX, but rather by the S&P returns. The Durbin-Watson tests indicate small residual autocorrelation. The R² measures range from 5% for the spline kurtosis to 64%, respectively 65%, for the two volatility series and indicate that our OLS procedure achieves a satisfying level of explanatory value for the changes in second moments. The explanatory value of the regressions for the changes in the third and fourth moments is therefore considerably smaller than for the second moments.

Another perspective on the determinants of the RND moments is obtained when we search for the single input factor with the largest explanatory value. The fourteen bivariate regressions of all factors on the first differences of the moments are presented in table 5. For reasons of space we show the results only for the mixture model. The resulting R²s are plotted in graph 7. The single largest R² is recorded for the implied volatility of the US stock market. Among the entire set of variables, we observe that the volatility of the US stock market accounts for the highest explanatory value. For this series, the measure of determination with a value of 62% for the second moment is marginally below the adjusted R² of the regression with the entire set of factors (64%). The variable with the second highest R² is the return on the S&P 500, accounting for around 44%. The third highest values are recorded for the DAX variables with R²s of 24 and 40%. The individual impact of interest rate variables is quite small, as the R² is below 5% 23. For the third and fourth moments, graph 7 shows that the highest R² measures are between 20 and 30%, hence about half of those for the second moments. The fact that up to 75% of the variance is not explained in our regressions shows that the determinants of skewness and kurtosis are not well covered by our regressors. In this context, the results of Gemmill and Kamyama (2000) are of considerable interest. This study finds that the skewness contained in British, Japanese and US equity options is not affected by international spillovers, but rather by local factors. Given that our approach also included local factors, our results indicate that the skewness and kurtosis of the DAX RNDs are also affected by an unobserved component, which cannot be linked to German macroeconomic or financial variables.

The positive sign of the coefficient on VIX indicates that rising volatility in the US stock market is directly imported into the volatility of the DAX index. The regression coefficient shows that 85% of an increase in the S&P variance is transmitted into the DAX variance. What is also notable is that both skewness and kurtosis have a negative relation to VIX. Therefore, falling US volatility raises the – negative – skewness, which means that the RND becomes more symmetric. Additionally, falling US volatility raises the kurtosis. This observation seems counterintuitive, but it can be explained in a statistical framework: A rise in the US variance makes the implied PDF flatter and more asymmetric. The change in skewness therefore seems to lead to a fall in kurtosis, as probability mass is shifted away from the right tail of the RND towards the centre and toward the left tail. Furthermore, the table indicates that, due to the negative sign of the S&P return, a fall in the US stock market also raises the volatility in the German stock market. Regarding the other relations, we do not find clear results. Some divergence among the variables is noteworthy; for instance, the proxies for economic activity do not all have a positive effect on the uncertainty in the stock market.

23 In this context, we have also estimated the impact of the volatility of the swap spread. We found no significant effects (e.g. the P value for the regression on the change in the volatility of the mixture model is 0.48).
However, given that their significance is quite low in comparison to VIX, the contradictory results might be statistical artefacts.

The fact that the two US variables have the highest $R^2$ documents the extent of integration among the two major stock markets in the USA and the Euro area and gives further support to the interdependence observed e.g. by Longin and Solnik (1995), DeSantis and Gerard (1997), or Peiro et al. (1998). In this context, the issue of nonsynchronous data arises: The US market closes after the German market and therefore there is a time gap between the recording of the two variables. In order to account for this issue, the equation from table 5 was estimated with the previous day’s US volatility. The results were unchanged, indicating that the time gap has in fact no impact. Another test for the robustness of results is the estimation of the regressions with principal components, i.e. orthogonalised factors. This method shows whether our results are afflicted by multicollinearity. The results given in the appendix support the finding that the US volatility is the key determinant.

After the OLS analysis we also applied Granger causality tests to uncover possible lead/lag effects. The test results in table 6, given for the mixture model and lag 1, show that the interest rate variables significantly influence skewness and the DAX variables appear in the tests on the second and fourth moments. Hence, we find no clear causal relationship between volatility and factors other than the DAX itself. Regarding the interpretation of the differences between Granger tests and OLS, we can therefore conclude that the existing interaction between German and US second moments is limited to a simultaneous type. The finding of causal relationships between the third moment and interest rates is difficult to interpret in the theoretical framework that we outlined above and hence forms a topic for future research.

4. Conclusion

This paper has analysed two methods for measuring the market perception of the uncertainty about the future dynamics in the German stock market. Our sample extended from December 1995 to November 2001. We evaluated two alternative approaches to obtain a risk-neutral density, namely the mixture of log-normals and a smoothing spline based on implied volatilities. After the discussion of the estimation results, we focused on the higher moments of the DAX RND. We analysed the time series behaviour of volatility, skewness and kurtosis and we evaluated linkages to macroeconomic variables, the US stock markets and credit risk. Our two principal results are as follows. First, the prices on DAX options imply a pronounced negative skewness of the RND. Second, the higher moments of the RND estimated for the German stock market are strongly affected by volatility in US stock market, with the largest impact observed for DAX volatility. This linkage is of a simultaneous nature as there is no evidence for causality of the volatilities. Our second observation therefore underlines the extent of integration between the two major stock markets in the USA and the Euro area. Thus, the determinants of the DAX moments indicate that the US and German stock market are showing a pronounced interdependence. Hence, our paper supports the hypothesis of integrated stock markets from an option-based perspective. A practical implication for investors is that the same risk factors have a simultaneous impact in both countries, which complicates the diversification of portfolios.

For future research, three directions seem promising. First, the robustness of our results with respect to the RND specifications could be examined. Alternative approaches are the Hermite model used by Coutant et al. (2001) or the implied tree method discussed by Jackwerth (1999). These methods could complement our semiparametric and mixture of log-normal
models. Second, as the dynamic relationship between the factors and the third and fourth moments is difficult to interpret in the currently available theoretical framework, extensions of the theoretical models towards higher moments would be appropriate. Third, the linkages between the RND and risk appetite indices might be a promising extension. In this context, the indices published by some investment banks might be a potential measure for investor behaviour towards risk.

References


Table 1: Quartiles of sample variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Quantile 25</th>
<th>Median</th>
<th>Quantile 75</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strike</td>
<td>2000</td>
<td>3900</td>
<td>5050</td>
<td>6200</td>
<td>10200</td>
</tr>
<tr>
<td>Option price</td>
<td>0.500</td>
<td>38.200</td>
<td>162.000</td>
<td>454.400</td>
<td>3058.600</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.079</td>
<td>0.198</td>
<td>0.247</td>
<td>0.312</td>
<td>0.664</td>
</tr>
<tr>
<td>Moneyness</td>
<td>0.751</td>
<td>0.904</td>
<td>0.982</td>
<td>1.059</td>
<td>1.455</td>
</tr>
<tr>
<td>Number of options</td>
<td>30</td>
<td>54</td>
<td>69</td>
<td>86</td>
<td>121</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.025</td>
<td>0.032</td>
<td>0.034</td>
<td>0.038</td>
<td>0.049</td>
</tr>
</tbody>
</table>

This table summarises the descriptive statistics for our sample.

Table 2: Descriptive statistics for volatility, skewness and kurtosis

<table>
<thead>
<tr>
<th></th>
<th>Mixture of log-normals</th>
<th>Volatility smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volatility</td>
<td>Skewness</td>
</tr>
<tr>
<td>Mean</td>
<td>28.38</td>
<td>-0.1773</td>
</tr>
<tr>
<td>Median</td>
<td>27.15</td>
<td>-0.1755</td>
</tr>
<tr>
<td>Maximum</td>
<td>64.00</td>
<td>-0.0345</td>
</tr>
<tr>
<td>Minimum</td>
<td>12.01</td>
<td>-0.3813</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>10.17</td>
<td>0.0635</td>
</tr>
</tbody>
</table>

This table shows the descriptive statistics for the volatility (annual percentage points), Pearson 1 skewness and excess kurtosis (i.e. minus 3);

Table 3: Descriptive statistics of factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>ADF</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
<th>ARCH test</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>-1.68</td>
<td>0.010</td>
<td>0.018</td>
<td>0.124</td>
<td>-0.191</td>
<td>0.072</td>
<td>5.944</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>-2.08</td>
<td>0.037</td>
<td>0.300</td>
<td>3.400</td>
<td>-5.400</td>
<td>1.693</td>
<td>8.526</td>
</tr>
<tr>
<td>Ind.prod</td>
<td>-1.06</td>
<td>0.192</td>
<td>0.300</td>
<td>3.500</td>
<td>-3.300</td>
<td>1.462</td>
<td>0.986</td>
</tr>
<tr>
<td>IFO</td>
<td>-1.13</td>
<td>-0.185</td>
<td>0.000</td>
<td>7.000</td>
<td>-11.600</td>
<td>3.348</td>
<td>0.312</td>
</tr>
<tr>
<td>Unemployt</td>
<td>-1.01</td>
<td>-0.006</td>
<td>0.000</td>
<td>0.200</td>
<td>-0.200</td>
<td>0.089</td>
<td>1.317</td>
</tr>
<tr>
<td>M3</td>
<td>0.35</td>
<td>6.241</td>
<td>5.400</td>
<td>24.600</td>
<td>-7.500</td>
<td>6.150</td>
<td>0.337</td>
</tr>
<tr>
<td>Fibor</td>
<td>-1.59</td>
<td>-0.008</td>
<td>0.004</td>
<td>0.555</td>
<td>-0.607</td>
<td>0.181</td>
<td>0.243</td>
</tr>
<tr>
<td>Yield</td>
<td>-1.18</td>
<td>-0.025</td>
<td>-0.045</td>
<td>0.507</td>
<td>-0.452</td>
<td>0.206</td>
<td>0.131</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>-2.96</td>
<td>0.007</td>
<td>-0.040</td>
<td>2.074</td>
<td>-1.817</td>
<td>0.719</td>
<td>NA</td>
</tr>
<tr>
<td>USDM</td>
<td>-1.02</td>
<td>0.006</td>
<td>0.009</td>
<td>0.059</td>
<td>-0.077</td>
<td>0.029</td>
<td>0.514</td>
</tr>
<tr>
<td>Vola (USDM)</td>
<td>-1.80</td>
<td>0.001</td>
<td>-0.009</td>
<td>0.127</td>
<td>-0.097</td>
<td>0.049</td>
<td>NA</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>-1.76</td>
<td>0.008</td>
<td>0.008</td>
<td>0.122</td>
<td>-0.125</td>
<td>0.051</td>
<td>7.400</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td>-3.80</td>
<td>0.255</td>
<td>0.230</td>
<td>17.520</td>
<td>-17.540</td>
<td>6.041</td>
<td>NA</td>
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<tr>
<td>SWAP</td>
<td>-1.87</td>
<td>0.000</td>
<td>0.008</td>
<td>0.155</td>
<td>-0.241</td>
<td>0.081</td>
<td>0.349</td>
</tr>
</tbody>
</table>

In the first column this table shows the ADF test for unit roots in the levels (5% Critical Value=2.9035). The remaining columns report descriptive statistics for the first differences of the explanatory factors and the ARCH(1) test for heteroscedasticity again in first differences, where NA denotes not applicable.
### Table 4: OLS regressions of changes in moments on all factors

<table>
<thead>
<tr>
<th>Dep.Variable</th>
<th>Vola MofL</th>
<th>Skewness MofL</th>
<th>Kurtosis MofL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff</td>
<td>t-Stat</td>
<td>Coeff</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.086</td>
<td>-0.19</td>
<td>-0.004</td>
</tr>
<tr>
<td>Fibor</td>
<td>-1.223</td>
<td>-0.40</td>
<td>0.075</td>
</tr>
<tr>
<td>Yield</td>
<td>-1.402</td>
<td>-0.40</td>
<td>-0.022</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>-0.438</td>
<td>-0.54</td>
<td>0.027</td>
</tr>
<tr>
<td>Swap</td>
<td>4.952</td>
<td>0.89</td>
<td>-0.021</td>
</tr>
<tr>
<td>DAX</td>
<td>-23.481</td>
<td>-1.37</td>
<td>0.083</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>-0.321</td>
<td>-0.79</td>
<td>0.001</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td><strong>0.601</strong></td>
<td><strong>3.47</strong></td>
<td><strong>-0.006</strong></td>
</tr>
<tr>
<td>S&amp;P</td>
<td>-3.602</td>
<td>-0.18</td>
<td>0.141</td>
</tr>
<tr>
<td>M3</td>
<td>0.109</td>
<td>1.32</td>
<td>0.001</td>
</tr>
<tr>
<td>Ind.prod.</td>
<td>-0.046</td>
<td>-0.14</td>
<td>0.002</td>
</tr>
<tr>
<td>IFO</td>
<td>0.058</td>
<td>0.34</td>
<td>-0.001</td>
</tr>
<tr>
<td>Un employt</td>
<td>8.716</td>
<td>1.56</td>
<td>0.069</td>
</tr>
<tr>
<td>USD M</td>
<td>-6.403</td>
<td>-0.30</td>
<td>-0.147</td>
</tr>
<tr>
<td>Vola (USD M)</td>
<td>10.288</td>
<td>0.81</td>
<td>0.092</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.64</td>
<td></td>
<td>0.312</td>
</tr>
<tr>
<td>D-W test</td>
<td>2.02</td>
<td></td>
<td>2.705</td>
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<table>
<thead>
<tr>
<th>Dep.variable</th>
<th>Vola VS</th>
<th>Skewness VS</th>
<th>Kurtosis VS</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Coeff</td>
<td>t-Stat</td>
<td>Coeff</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.079</td>
<td>-0.17</td>
<td>-0.004</td>
</tr>
<tr>
<td>Fibor</td>
<td>-0.995</td>
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<td>0.089</td>
</tr>
<tr>
<td>Yield</td>
<td>-1.261</td>
<td>-0.36</td>
<td>0.059</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>-0.409</td>
<td>-0.51</td>
<td><strong>0.055</strong></td>
</tr>
<tr>
<td>Swap</td>
<td>4.734</td>
<td>0.86</td>
<td>-0.109</td>
</tr>
<tr>
<td>DAX</td>
<td>-23.200</td>
<td>-1.37</td>
<td>-0.007</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>-0.317</td>
<td>-0.79</td>
<td>-0.004</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td><strong>0.607</strong></td>
<td><strong>3.57</strong></td>
<td>0.002</td>
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<tr>
<td>S&amp;P</td>
<td>-1.810</td>
<td>-0.09</td>
<td><strong>1.117</strong></td>
</tr>
<tr>
<td>M3</td>
<td>0.105</td>
<td>1.267</td>
<td>0.000</td>
</tr>
<tr>
<td>Ind.prod.</td>
<td>-0.026</td>
<td>-0.08</td>
<td>0.010</td>
</tr>
<tr>
<td>IFO</td>
<td>0.049</td>
<td>0.29</td>
<td>-0.004</td>
</tr>
<tr>
<td>Un employt</td>
<td>8.800</td>
<td>1.60</td>
<td>0.088</td>
</tr>
<tr>
<td>USD M</td>
<td>-8.47</td>
<td>-0.40</td>
<td>-0.307</td>
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<tr>
<td>Vola (USD M)</td>
<td>10.487</td>
<td>0.84</td>
<td>0.259</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.65</td>
<td></td>
<td>0.148</td>
</tr>
<tr>
<td>D-W test</td>
<td>2.011</td>
<td></td>
<td>2.593</td>
</tr>
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</table>

This table shows the regressions of the changes in moments on all 14 factors with Newey-West standard errors; MofL represents mixture of log-normals, VS volatility smoothing. The coefficients significant at the 5% level are marked in bold.
Table 5: OLS regressions of changes in moments on single factors

<table>
<thead>
<tr>
<th>Dep. variable</th>
<th>Volatility</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Coeff</td>
<td>t-Stat</td>
<td>R²</td>
</tr>
<tr>
<td>Fibor</td>
<td>-5.80</td>
<td>-1.36</td>
<td>0.03</td>
</tr>
<tr>
<td>Yield</td>
<td>-4.34</td>
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<tr>
<td>Vola (Yield)</td>
<td>2.07</td>
<td>1.58</td>
<td>0.05</td>
</tr>
<tr>
<td>Swap</td>
<td>15.89</td>
<td>1.00</td>
<td>0.04</td>
</tr>
<tr>
<td>DAX</td>
<td>-57.42</td>
<td>-4.60</td>
<td>0.40</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>-1.88</td>
<td>-2.53</td>
<td>0.24</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td>0.85</td>
<td>9.80</td>
<td>0.62</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>-84.64</td>
<td>-6.22</td>
<td>0.44</td>
</tr>
<tr>
<td>M3</td>
<td>0.11</td>
<td>1.21</td>
<td>0.01</td>
</tr>
<tr>
<td>Ind.prod.</td>
<td>0.90</td>
<td>1.76</td>
<td>0.04</td>
</tr>
<tr>
<td>IFO</td>
<td>0.30</td>
<td>1.68</td>
<td>0.02</td>
</tr>
<tr>
<td>Unemployt</td>
<td>6.93</td>
<td>0.87</td>
<td>0.01</td>
</tr>
<tr>
<td>USDm</td>
<td>-71.82</td>
<td>-2.22</td>
<td>0.10</td>
</tr>
<tr>
<td>Vola (USDM)</td>
<td>24.70</td>
<td>1.45</td>
<td>0.03</td>
</tr>
</tbody>
</table>

This table shows the regressions of the changes in moments from the mixture model on the individual factors. The coefficients significant at the 5% level are marked in bold.

Table 6: Granger causality tests of changes in moments on all factors

<table>
<thead>
<tr>
<th>Dep. variable</th>
<th>Vola MofL</th>
<th>Skewness MofL</th>
<th>Kurtosis MofL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>F-test</td>
<td>Prob</td>
<td>F-test</td>
</tr>
<tr>
<td>Fibor</td>
<td>0.15</td>
<td>0.70</td>
<td>5.87</td>
</tr>
<tr>
<td>Yield</td>
<td>0.90</td>
<td>0.35</td>
<td>0.82</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>0.31</td>
<td>0.58</td>
<td>4.97</td>
</tr>
<tr>
<td>Swap</td>
<td>0.08</td>
<td>0.78</td>
<td>1.54</td>
</tr>
<tr>
<td>DAX</td>
<td>6.06</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>3.60</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td>1.37</td>
<td>0.25</td>
<td>0.47</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>1.90</td>
<td>0.17</td>
<td>0.68</td>
</tr>
<tr>
<td>M3</td>
<td>0.00</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>Ind.prod.</td>
<td>1.10</td>
<td>0.30</td>
<td>0.86</td>
</tr>
<tr>
<td>IFO</td>
<td>0.00</td>
<td>0.95</td>
<td>3.91</td>
</tr>
<tr>
<td>Unemployt</td>
<td>0.18</td>
<td>0.67</td>
<td>0.07</td>
</tr>
<tr>
<td>USDm</td>
<td>0.30</td>
<td>0.59</td>
<td>1.58</td>
</tr>
<tr>
<td>Vola (USDM)</td>
<td>0.10</td>
<td>0.75</td>
<td>3.37</td>
</tr>
</tbody>
</table>

This table shows the Granger causality tests from the mixture on the individual factors. The coefficients significant at the 5% level are marked in bold.
Graph 1: Implied volatilities on 6 November 2001

Graph 2: Index and 5%, 95% RND confidence bands for both models
Graph 3: Yearly implied PDFs for mixture model

Graph 4: RND estimate for volatility and DAX index
Graph 7: $R^2$ of bivariate regressions on changes in MofL moments
Appendix: Principal Components Analysis

In this appendix, we demonstrate the robustness of our results with respect to correlation among the set of regressors. In order to deal with potential multicollinearity, we have transformed the set of fourteen regressors into fourteen orthogonal variables by means of principal components analysis. The PCA summarises the interdependence among the variables in a simplified form. The principal components are calculated by orthogonalising the 14 original, standardised series. The series are grouped into pair wise uncorrelated linear combinations that depend on their weights in the initial data. The resulting factors have the same variability as the initial data and are arranged in a way that the first series explains the highest share (in percent) in the variability of the original time series; the second series explains the second-highest share, etc.

To interpret the factors, table a reports for all 14 principal components the eigenvalues, the variance proportion explained by each component and the weight allocated to the 14 input factors. These have been standardised to allow for direct comparisons.

Table a: Principal Components Analysis

<table>
<thead>
<tr>
<th></th>
<th>F 1</th>
<th>F 2</th>
<th>F 3</th>
<th>F 4</th>
<th>F 5</th>
<th>F 6</th>
<th>F 7</th>
<th>F 8</th>
<th>F 9</th>
<th>F 10</th>
<th>F 11</th>
<th>F 12</th>
<th>F 13</th>
<th>F 14</th>
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</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
<td>3.280</td>
<td>1.856</td>
<td>1.650</td>
<td>1.418</td>
<td>1.155</td>
<td>0.953</td>
<td>0.815</td>
<td>0.718</td>
<td>0.597</td>
<td>0.498</td>
<td>0.377</td>
<td>0.331</td>
<td>0.184</td>
<td>0.166</td>
</tr>
<tr>
<td>Variance Prop.%</td>
<td>23.4%</td>
<td>13.3%</td>
<td>11.8%</td>
<td>10.1%</td>
<td>8.3%</td>
<td>6.8%</td>
<td>5.8%</td>
<td>5.1%</td>
<td>4.3%</td>
<td>3.6%</td>
<td>2.7%</td>
<td>2.4%</td>
<td>1.3%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Fibor</td>
<td>-0.059</td>
<td>0.302</td>
<td>0.263</td>
<td>0.238</td>
<td>0.586</td>
<td>0.078</td>
<td>0.082</td>
<td>0.345</td>
<td>-0.162</td>
<td>-0.353</td>
<td>0.273</td>
<td>0.275</td>
<td>-0.052</td>
<td>0.006</td>
</tr>
<tr>
<td>Yield</td>
<td>-0.075</td>
<td>0.404</td>
<td>-0.029</td>
<td>-0.448</td>
<td>0.204</td>
<td>0.424</td>
<td>-0.195</td>
<td>-0.065</td>
<td>-0.250</td>
<td>0.029</td>
<td>-0.490</td>
<td>-0.040</td>
<td>0.226</td>
<td>0.100</td>
</tr>
<tr>
<td>Vola (Yield)</td>
<td>0.153</td>
<td>-0.483</td>
<td>0.268</td>
<td>-0.283</td>
<td>0.101</td>
<td>0.164</td>
<td>-0.157</td>
<td>0.123</td>
<td>0.027</td>
<td>0.351</td>
<td>0.039</td>
<td>0.620</td>
<td>0.068</td>
<td>-0.010</td>
</tr>
<tr>
<td>Swap</td>
<td>0.149</td>
<td>-0.272</td>
<td>0.420</td>
<td>0.264</td>
<td>0.313</td>
<td>-0.088</td>
<td>0.399</td>
<td>-0.093</td>
<td>-0.065</td>
<td>0.154</td>
<td>-0.526</td>
<td>-0.266</td>
<td>-0.010</td>
<td>0.096</td>
</tr>
<tr>
<td>DAX</td>
<td>-0.463</td>
<td>-0.056</td>
<td>0.297</td>
<td>-0.113</td>
<td>-0.013</td>
<td>-0.013</td>
<td>0.051</td>
<td>-0.397</td>
<td>-0.151</td>
<td>0.184</td>
<td>-0.062</td>
<td>-0.177</td>
<td>-0.096</td>
<td>0.009</td>
</tr>
<tr>
<td>DAX P/E</td>
<td>-0.397</td>
<td>-0.056</td>
<td>0.297</td>
<td>-0.113</td>
<td>-0.013</td>
<td>-0.013</td>
<td>0.051</td>
<td>-0.397</td>
<td>-0.151</td>
<td>0.184</td>
<td>-0.062</td>
<td>-0.177</td>
<td>-0.096</td>
<td>0.009</td>
</tr>
<tr>
<td>Vola (S&amp;P)</td>
<td>0.438</td>
<td>-0.094</td>
<td>0.075</td>
<td>-0.221</td>
<td>-0.032</td>
<td>0.037</td>
<td>0.245</td>
<td>-0.473</td>
<td>-0.047</td>
<td>0.092</td>
<td>0.137</td>
<td>0.050</td>
<td>-0.115</td>
<td>-0.046</td>
</tr>
<tr>
<td>S&amp;P</td>
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<td>0.404</td>
<td>-0.029</td>
<td>-0.448</td>
<td>0.204</td>
<td>0.424</td>
<td>-0.195</td>
<td>-0.065</td>
<td>-0.250</td>
<td>0.029</td>
<td>-0.490</td>
<td>-0.040</td>
<td>0.226</td>
<td>0.100</td>
</tr>
<tr>
<td>M3</td>
<td>0.192</td>
<td>0.374</td>
<td>0.150</td>
<td>0.102</td>
<td>-0.363</td>
<td>0.064</td>
<td>0.423</td>
<td>0.205</td>
<td>0.374</td>
<td>0.150</td>
<td>0.102</td>
<td>-0.363</td>
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<td>Ind.prod.</td>
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<td>-0.239</td>
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<td>-0.064</td>
<td>-0.400</td>
<td>-0.315</td>
<td>-0.281</td>
<td>0.290</td>
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<td>0.263</td>
</tr>
<tr>
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<td>-0.162</td>
<td>-0.016</td>
<td>-0.064</td>
<td>-0.784</td>
<td>0.150</td>
<td>-0.163</td>
<td>-0.448</td>
<td>0.039</td>
<td>-0.705</td>
<td>0.264</td>
<td>-0.010</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.345</td>
<td>-0.162</td>
<td>-0.353</td>
<td>0.273</td>
<td>0.275</td>
<td>-0.052</td>
<td>0.006</td>
<td>0.345</td>
<td>-0.162</td>
<td>-0.353</td>
<td>0.273</td>
<td>0.275</td>
<td>-0.052</td>
<td>0.006</td>
</tr>
<tr>
<td>USDM</td>
<td>-0.273</td>
<td>0.068</td>
<td>0.155</td>
<td>-0.237</td>
<td>-0.252</td>
<td>0.371</td>
<td>0.517</td>
<td>-0.273</td>
<td>0.068</td>
<td>0.155</td>
<td>-0.237</td>
<td>-0.252</td>
<td>0.371</td>
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<tr>
<td>Vola (USDM)</td>
<td>0.147</td>
<td>-0.116</td>
<td>0.488</td>
<td>-0.125</td>
<td>-0.357</td>
<td>-0.006</td>
<td>-0.333</td>
<td>0.147</td>
<td>-0.116</td>
<td>0.488</td>
<td>-0.125</td>
<td>-0.357</td>
<td>-0.006</td>
<td>-0.333</td>
</tr>
</tbody>
</table>
In table a, we observe that factor 1 has a variance proportion of 23% and it includes all variables related to German and US stock markets. Due to the strong correlation, it jointly represents the returns on the DAX and the S&P index, the VIX and the price/earnings ratio.

Table b shows the regressions for the higher moments on the set of principal components. We use the results from the mixture RND. Across all measures a clear result emerges. Unanimously, the principal component 1 is significant at a level of 5%. We see that the \( R^2 \)s for the variance regression is below 0.7, so around 65% of the variability in the changes of the moments is explained by the set of factors. As the principal components 4, 7 and 10 have variance proportions of at most 10%, the moments are most strongly related to factor 1, which contains stock prices and US stock market volatility. Factors 7 and 10 are significant only for volatilities. Third moments are affected also by other variables, without a clear trend in results. For instance, factor 10, which contains the short rate, the yield volatility and again some stock market variables, indirectly affects second and third moments. This observation is in accordance with our regressions on the original factor set and shows that multicollinearity has only small effects on our main findings.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff</th>
<th>t-Stat</th>
<th>Coeff</th>
<th>t-Stat</th>
<th>Coeff</th>
<th>t-Stat</th>
</tr>
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<tbody>
<tr>
<td>C</td>
<td>0.409</td>
<td>0.87</td>
<td>0.000</td>
<td>0.11</td>
<td>-0.005</td>
<td>-0.12</td>
</tr>
<tr>
<td>F1</td>
<td>2.750</td>
<td>10.59</td>
<td>-0.018</td>
<td>4.38</td>
<td>-0.112</td>
<td>3.55</td>
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<tr>
<td>F2</td>
<td>-0.556</td>
<td>-1.65</td>
<td>-0.006</td>
<td>1.16</td>
<td>-0.032</td>
<td>0.84</td>
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<tr>
<td>F3</td>
<td>-0.372</td>
<td>-0.95</td>
<td>0.008</td>
<td>1.58</td>
<td>-0.036</td>
<td>0.98</td>
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<tr>
<td>F4</td>
<td>-0.961</td>
<td>-1.82</td>
<td>0.004</td>
<td>0.86</td>
<td>0.017</td>
<td>0.37</td>
</tr>
<tr>
<td>F5</td>
<td>0.063</td>
<td>0.12</td>
<td>0.009</td>
<td>1.57</td>
<td>0.053</td>
<td>1.07</td>
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<tr>
<td>F6</td>
<td>-0.500</td>
<td>-1.08</td>
<td>-0.004</td>
<td>1.04</td>
<td>0.033</td>
<td>0.65</td>
</tr>
<tr>
<td>F7</td>
<td>1.119</td>
<td>2.40</td>
<td>-0.009</td>
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<td>0.45</td>
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<tr>
<td>F8</td>
<td>-0.095</td>
<td>-0.15</td>
<td>0.012</td>
<td>2.46</td>
<td>0.094</td>
<td>1.05</td>
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<tr>
<td>F9</td>
<td>0.028</td>
<td>0.05</td>
<td>-0.016</td>
<td>2.14</td>
<td>0.044</td>
<td>0.63</td>
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<tr>
<td>F10</td>
<td>-1.410</td>
<td>-2.62</td>
<td>0.017</td>
<td>2.44</td>
<td>0.116</td>
<td>1.51</td>
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<tr>
<td>F11</td>
<td>0.741</td>
<td>1.25</td>
<td>0.004</td>
<td>0.37</td>
<td>0.018</td>
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<tr>
<td>F12</td>
<td>-0.873</td>
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<td>F13</td>
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<tr>
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<td>-0.005</td>
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<td>-0.108</td>
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<td>Adj. R²</td>
<td>0.647</td>
<td>0.312</td>
<td>0.141</td>
<td>0.141</td>
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<td>D-W stat</td>
<td>2.026</td>
<td>2.705</td>
<td>2.814</td>
<td>2.814</td>
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</table>
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