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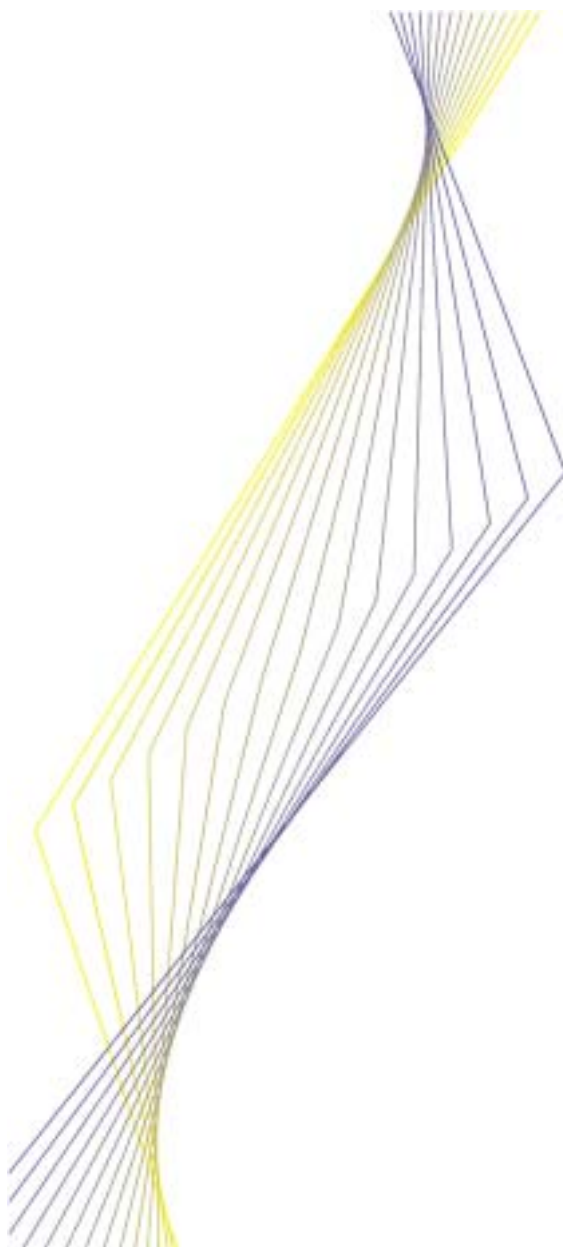
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**WORKING PAPER NO. 161**  
**THE OPTIMAL ALLOCATION  
OF RISKS UNDER  
PROSPECT THEORY**  
**BY LIVIO STRACCA**

**July 2002**

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**THE OPTIMAL ALLOCATION  
OF RISKS UNDER  
PROSPECT THEORY**

**BY LIVIO STRACCA\***

**July 2002**

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## Abstract

This paper deals with the optimal allocation of risks for an agent whose preferences may be represented with prospect theory (Tversky and Kahneman, 1992). A simple setting is considered with  $n$  identically distributed and symmetric sources of risk. Under expected utility, equal diversification of risks is optimal in this setting ("do not put your eggs in the same basket"). Conversely, under prospect theory, provided that the subjective probability of obtaining a *perfect* hedge is negligible, risk concentration is optimal ("do put your eggs in the same basket"). The intuitive reason behind this result is that a prospect theory agent is risk-seeking over losses, with the consequence that the property of diversification of *averaging* downside risks is welfare-reducing rather than welfare-improving.

**Keywords:** Cumulative prospect theory, loss aversion, diminishing sensitivity, diversification, home bias puzzle.

**JEL codes:** D81

## Executive summary

Prospect theory (first introduced by Kahneman and Tversky in 1979) is a well established descriptive theory of human behaviour under risk. The theory postulates that agents form their decisions in two steps. First, a certain decision problem is *framed* ("editing" phase), i.e. considered as a self-contained decision problem, often in a very narrow setting. Subsequently, in a second step the decision is taken by maximizing the *prospective value function* defined for the problem.

Four key features of the prospective value function distinguish it from expected utility:

- First, *changes* in wealth (or other economic variables), rather than levels, matter most for a prospect theory agent (levels have a second-order importance). In turn, changes are defined in terms of a *reference point* which is determined in the editing phase, and broadly reflects the (uniquely given) agent's expectations and norms.
- Second, changes are evaluated as gains and losses compared with the reference point, with losses looming larger than gains (*loss aversion*).
- Third, deviations from the reference point are evaluated with *diminishing sensitivity*, i.e. a marginal deviation from the reference point is more important close to the reference point than far away from it. Diminishing sensitivity reflects a general principle of human perception (Kahneman and Tversky, 2000).
- Finally, probabilities of events are weighted *non-linearly*, generally with large probabilities being undervalued and small probabilities being overvalued compared with the standard linear case. Therefore, agents evaluate risky prospects according to their *subjective* expected value, i.e. the expected value computed using subjectively weighted probabilities.

This paper builds on this strand of literature to apply prospect theory to a classical problem of the finance literature, namely the optimal allocation of the agent's stakes (e.g., wealth) among  $n > 1$  identically distributed and symmetric sources of risk.

The most striking result of this simple analysis is that, under the fairly general assumption that the subjective probability of obtaining a *perfect hedge* is negligible, the optimal allocation of risks by a prospect theory agent requires risk *concentration*, rather than *diversification* as held in mainstream finance and economics. Thus, the agent, instead of "not putting the eggs in the same basket" as is taught in economics textbooks, will optimally "put the eggs in the same basket".

The features of prospect theory crucially driving this result are loss aversion and in particular diminishing sensitivity to gains and losses, which makes the representative agent risk-seeking over losses. Under the assumption of diminishing sensitivity, agents are supposed to care

more -- compared, for instance, with a mean-variance specification of preferences -- about small shocks with high probability than about large shocks with low probability. While risk diversification reduces the likelihood of events very distant from the reference point, it may actually *increase* the noise in its neighborhood, which has an important negative bearing on agents' welfare under prospect theory. As a consequence, the property of diversification of averaging existing downside risks is welfare-reducing for a prospect theory agent, whilst it is welfare-improving for an agent with a standard concave utility function. This explains why risk concentration, as opposed to risk diversification popular in the finance literature, turns out to be optimal in the risk allocation problem, provided that the subjective probability of obtaining a perfect hedge is negligible.



# 1 Introduction

Prospect theory, first introduced by Kahneman and Tversky (1979), stands out prominently in applied and theoretical research. It is fair to say that it ranks second only to expected utility as a positive theory of human attitudes towards risk. Derived initially from theoretical reasoning, it has found important confirmation in experimental studies. The advanced, rank-dependent version of the theory (Tversky and Kahneman, 1992) has been used by Benartzi and Thaler (1995), Barberis, Huang and Santos (2000) and Barberis and Huang (2001) to provide intriguing explanations to some of the most enduring puzzles in finance, such as the equity premium puzzle of Mehra and Prescott (1985) and the predictability of equity returns at low frequency. These papers pave the way to a promising research agenda in financial economics.<sup>1</sup>

This paper builds on this strand of literature to apply cumulative prospect theory (hereafter for brevity CPT) to a classical problem of the finance literature, namely the optimal allocation of the agent's stakes (e.g., wealth) among  $n > 1$  identically distributed and symmetric sources of risk.<sup>2</sup> According to CPT, losses matter more than gains in the agent's value function (loss aversion); the value function is mildly concave for gains and mildly convex for losses (i.e., there is diminishing sensitivity to gains and losses); and the agent weighs non-linearly probabilities of events according to their ranking, thus departing from expected utility theory. In this paper, it is assumed that the optimal selection of allocation weights among  $n$  risky payoff functions represents a "framed prospect" in the sense of Tversky and Kahneman (1986), namely a self-contained decision problem which is analyzed independently by our representative agent. Hence, the agent assigns the allocation weights so as to maximize the prospective value function defined for the problem.

The most striking result of this simple analysis is that, under the fairly general assumption that the subjective probability of obtaining a *perfect hedge* is negligible, the optimal allocation of risks by our CPT agent requires risk *concentration*, rather than *diversification* as held in mainstream finance and economics (Samuelson, 1967). Thus, the CPT agent, instead of "not putting the eggs in the same basket" as is taught in

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<sup>1</sup>On the potential of prospect theory in explaining existing puzzles in economics and finance, see Rabin (1998) and Thaler (2000).

<sup>2</sup>The optimal allocation between a safe and a risky asset for an investor whose preferences may be modeled with prospect theory has been already investigated quite extensively in the literature (see for example Barberis, Huang and Santos, 2000, Berkelaar and Kouwenberg, 2000a and 2000b, and Hwang and Satchell, 2001). The focus of the present paper is, instead, the optimal allocation of stakes among  $n > 1$  risky payoff functions.

economics textbooks, will optimally "put the eggs in the same basket". The features of CPT crucially driving this result are loss aversion and in particular diminishing sensitivity to gains and losses, which makes the representative agent risk-seeking over losses. Under the assumption of diminishing sensitivity, agents are supposed to care more – compared, for instance, with a mean-variance specification of preferences – about small shocks with high probability than about large shocks with low probability. While risk diversification reduces the likelihood of events very distant from the reference point, it may actually *increase* the noise in its neighborhood, which has an important negative bearing on agents' welfare under CPT. As a consequence, the property of diversification of averaging existing downside risks is welfare-reducing for a CPT agent, whilst it is welfare-improving for an agent with a standard concave utility function. This explains why risk concentration, as opposed to risk diversification popular in the finance literature, turns out to be optimal in the risk allocation problem, provided that the subjective probability of obtaining a perfect hedge is negligible.

To grasp the intuition behind this result, consider this simple example. Assume that you have two credit cards and two wallets. You might decide to spread the two credit cards in the two wallets, or to keep both in one wallet. The probability of losing each of the two wallets is  $\frac{1}{4}$ , and the event of losing one wallet is independent of the event of losing the other one (thus, the probability that both wallets go lost is  $\frac{1}{16}$ ). If you diversify your risks and you keep the two credit cards separate, you will have a probability of  $\frac{1}{16}$  to lose *both* credit cards, and a probability of  $\frac{6}{16}$  to lose *one* credit card (therefore, a total probability of  $\frac{7}{16}$  of losing *at least one* credit card). If you, instead, put your eggs in the same basket (i.e., the two credit cards in the same wallet), you have a probability of  $\frac{1}{4}$  of losing both credit cards, and zero probability of losing only one. Under expected utility theory, the event of losing two credit cards carries proportionally more dis-utility than the event of losing only one credit card; therefore, you will be better off by diversifying your risks, i.e. by putting your credit cards in separate wallets. By contrast, with diminishing sensitivity to losses as postulated in CPT, the dis-utility associated with the loss of both credit cards is *proportionally* smaller than that associated with the loss of one credit card (in other words, the marginal dis-utility of losing one credit card is smaller if you have already lost one credit card, for instance because you have to call your bank anyway). Thus, if you behave like a CPT agent, you will be better off by concentrating your risks (in one wallet), reflecting your risk-seeking attitude for losses. After all, you have a  $\frac{3}{4}$  probability of getting off scot-free if you concentrate risks, compared with only  $\frac{9}{16}$  if you diversify risks. Overall,

keeping the two credit cards in the same wallet does not seem to be a counter-intuitive behaviour, although it is certainly in contrast with economics textbooks and mean-variance optimization.

Is it a plausible idea that *rational* risk (loss) averse agents may want to concentrate, rather than diversify, risks, and is it relevant? The answer seems to be positive to both questions. First, there is indeed a large body of evidence showing that agents tend to refrain from diversification in a variety of contexts important for economics (French and Poterba, 1991; Shiller, 1998), and that, when they diversify, they do so in a naive manner (Benartzi, 2001). Thus, the idea that risk concentration might be optimal, at least in some contexts, for rational and risk averse agents, although it is inconsistent with mainstream finance and economics, is not necessarily in contrast with the available evidence. Second, the finding that refraining from diversification might reflect an essential feature of human preferences under risk might in turn help to *explain* why agents (as observed) do not seem to have, in a variety of situations, the strong incentives to diversify risks that economics textbooks attribute them, a tendency which has so far found no convincing and sufficiently general explanation in the literature. Thus, in this respect the results of this paper appear to be of relevance. Probably the most famous example of lack of risk diversification is the observed "home bias" in financial investment, which has been widely documented, and yet found no convincing explanation, in the literature (Lewis, 1999). Against this background, it can be argued that prospect theory might contribute, together with other factors, to solve the home bias puzzle, which is one of the most enduring in the international finance literature.

The paper is structured as follows. The optimal allocation of risks for a representative agent whose preferences may be modelled with prospect theory is laid down in Section 2. Section 3 generalizes the results of the analysis to the situation where the sources of risk are not identically and symmetrically distributed. The relevance of the results of this section is then discussed in Section 4. Section 5 concludes.

## **2 The optimal allocation of risks under cumulative prospect theory**

### **2.1 The environment**

In the ensuing analysis we consider a simple environment in which our agent has to allocate his stakes (e.g., wealth) among  $n > 1$  separate and identical sources of risk, each of them having zero expected return compared with a reference point. While it would be straightforward to present a more realistic setting where the carriers of risk were also

carriers of a non-zero expected return and the sources of risk were not identical, this would make the problem less easily tractable, without implying a real loss of generality. Moreover, a symmetric probability distribution around a reference point is arguably the most intuitive notion of "risk" that agents have in mind, and it is a situation traditionally prominent in finance. Nonetheless, in Section 3 we consider the more general case of not identically and symmetrically distributed sources of risk, which is however slightly more cumbersome and less intuitive from an analytical perspective.

From now on, the  $n$  sources of risk have stochastic payoffs  $x_i^\theta$ ,  $i = 1, \dots, n$ , vis-a-vis a reference point for the agent  $\theta$  (the role of which will be discussed below), with  $E(x_i^\theta) = 0$ ,  $x_i^\theta$  identically distributed and the joint probability density  $\Pr(x_1^\theta, \dots, x_n^\theta)$  symmetric in its arguments, with finite variances and covariances.<sup>3</sup>

Our representative agent cares about the following variable  $y_\theta$  :

$$y_\theta = \sum \alpha_i x_i^\theta, \quad (1)$$

where  $\alpha_i$  are allocation weights set by the agent,  $0 \leq \alpha_i \leq 1$  and  $\sum \alpha_i = 1$ . In vector notation, let  $x' = [x_1^\theta, \dots, x_n^\theta]$  and  $\alpha' = [\alpha_1, \dots, \alpha_n]$ , whereby  $y_\theta = \alpha'x$ . Due to the symmetry of the probability distribution of  $x$ ,  $y_\theta$  is symmetrically distributed around zero (this assumption will be relaxed in Section 3).

This is a situation well known in the finance literature at least since the work of Samuelson (1967), where it is optimal for an agent with expected utility defined over  $y_\theta$  to spread his stakes evenly over the  $n$  sources of risk (equal diversification, i.e.  $\alpha_i = \frac{1}{n}$ ). Hence, the agent "should not put the eggs in the same basket". However, while this conclusion is mandatory for an agent maximizing expected utility, it is not always warranted for a prospect theory agent. Indeed, as it is shown later, under very general conditions a prospect theory agent with loss aversion will find it optimal to concentrate, rather than diversify risks.

## 2.2 The prospective value function for the problem

Prospect theory is a well established descriptive theory of human behaviour under risk. Its success in explaining phenomena which are puzzling for the mainstream approach based on expected utility maximization is already impressive (see, e.g., Benartzi and Thaler, 1995).

Prospect theory postulates that agents form their decisions in two steps. First, a certain decision problem is "framed" (editing phase),

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<sup>3</sup>In the remainder of this paper, payoffs in excess of the reference point  $\theta$  will generally have the superscript  $\theta$ , to emphasize that they are defined in terms of that particular reference point and not in abstract terms.

i.e. considered as a self-contained decision problem, often in a very narrow setting. Subsequently, in a second step the decision is taken by maximizing the prospective value function defined for the problem (Kahneman and Tversky, 1979).

Four key features of the prospective value function distinguish it from expected utility. First, *changes* in wealth (or other economic variables), rather than levels, matter most for a prospect theory agent (levels have a second-order importance). In turn, changes are defined in terms of a *reference point* which is determined in the editing phase, and broadly reflects the (uniquely given) agent's expectations and norms. Second, changes are evaluated as gains and losses compared with the reference point, with losses looming larger than gains (*loss aversion*). Third, deviations from the reference point are evaluated with *diminishing sensitivity*, i.e. a marginal deviation from the reference point is more important close to the reference point than far away from it. Diminishing sensitivity reflects a general principle of human perception (Kahneman and Tversky, 2000) and it is the key property of prospect theory driving the results in this paper. Finally, probabilities of events are weighted non-linearly, generally with large probabilities being undervalued and small probabilities being overvalued compared with the standard linear case.

In this analysis we assume that the allocation of the agent's stakes over the  $n$  sources of risk is a *framed prospect* in the sense of Tversky and Kahneman (1986), namely a self-contained decision problem for the agent. As in Kahneman and Tversky (1979), we assume the following *value function*:

$$V(y_\theta) = \begin{cases} y_\theta^b, & \text{if } y_\theta \geq 0 \\ -a(-y_\theta)^b, & \text{if } y_\theta < 0 \end{cases}, \quad (2)$$

where  $0 < b < 1$  (diminishing sensitivity),  $a > 1$  (loss aversion).<sup>4</sup> The *prospective value function* is the *subjective* expected value of the value function, where the word "subjective" signals that it is computed by using subjectively weighed probabilities, rather than the original probabilities. In line with the literature, we assume that the probability weighing function,  $\pi(\cdot)$ , is a function of the probability density of  $y_\theta$  defined from  $[0, 1]$  into  $[0, 1]$ , such that  $\pi(\Pr(y_\theta)) \geq 0$ ,  $\pi(0) = 0$ , and  $\pi(\Pr(y_\theta)) = \pi(\Pr(-y_\theta))$  if  $\Pr(y_\theta) = \Pr(-y_\theta)$ , due to the reflection property (Tversky and Kahneman, 1992).<sup>5</sup> In the original version of prospect

<sup>4</sup>This value function has a "kink" at the reference point and it is concave over gains and convex over losses. In general, in this paper it is assumed  $b < 1$  (Tversky and Kahneman, 1992), but the case  $b = 1$  is also touched upon later on.

<sup>5</sup>The property of *reflection* may be defined as preferences between *negative* prospects (i.e., prospects involving only losses) being the "mirror image" of preferences between *positive* prospects (i.e., prospects involving only gains). Besides

theory (Kahneman and Tversky, 1979),  $\pi(\cdot)$  is a function of the probability density, while in the more advanced, cumulative version of the theory (Tversky and Kahneman, 1992)  $\pi(\cdot)$  is a function of the cumulative probability distribution. For simplicity of notation, we will refer to the original version of the theory where  $\pi(\cdot)$  is defined over the probability density, which in our setting implies no loss of generality, as it will be shown later.

Against this background, the prospective value function for our problem (henceforth PVF) may be written as follows:

$$PVF = \int_{-\infty}^{\infty} V(y_{\theta})\pi(y_{\theta})dy_{\theta}, \quad (3)$$

where  $V(y_{\theta})$  is defined as in (2). It is immediate to show that, owing to the symmetric probability distribution of  $y_{\theta}$  and to the property of reflection of  $\pi(\cdot)$ , the PVF can be written as a function of losses only. In fact, developing the PVF we obtain:

$$PVF = -a \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta} + \int_0^{\infty} y_{\theta}^b \pi(y_{\theta}) dy_{\theta}, \quad (4)$$

whereby:

$$PVF = (1-a) \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta} - \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta} + \int_0^{\infty} y_{\theta}^b \pi(y_{\theta}) dy_{\theta} \quad (5)$$

Due to the property of reflection,  $\pi(y_{\theta}) = \pi(-y_{\theta})$ , and  $y_{\theta}^b$  for  $y_{\theta} \geq 0$  is equal to  $(-y_{\theta})^b$  for  $y_{\theta} < 0$ . Thus,  $[- \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta} + \int_0^{\infty} y_{\theta}^b \pi(y_{\theta}) dy_{\theta}] = 0$ . Therefore:

$$PVF = (1-a) \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta} \quad (6)$$

Without loss of generality and in order to simplify slightly the notation, we assume  $a = 2$  (a value broadly in line with much experimental evidence; Kahneman and Tversky, 2000), so  $PVF = - \int_{-\infty}^0 (-y_{\theta})^b \pi(y_{\theta}) dy_{\theta}$ .

The specification of the PVF in (6) makes it clear why considering the original version of prospect theory does not imply any loss of generality. For our CPT agent and given our assumptions about  $\Pr(x)$ , the framed prospect  $\{y_{\theta}, \Pr(y_{\theta})\}$  can be mapped into a one-sided prospect  $\{y_{\theta}, \Pr(y_{\theta})\}_{y_{\theta} < 0}$ . For one-sided prospects (i.e., prospects which involve only losses or only gains), it does not matter if one considers the original or the cumulative version of the theory (Tversky and Kahneman, 1992).<sup>6</sup>

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simplifying the notation considerably, the property of reflection has also found broad confirmation in experimental studies (Kahneman and Tversky, 2000).

<sup>6</sup>One might argue that the agent should wish to *withdraw* from a lottery involving

To sum up, our agent has to select the weights  $\alpha_i$  so as to maximize:

$$PVF = - \int_{\sum \alpha_i x_i < 0} (-\sum \alpha_i x_i)^b \pi(\sum \alpha_i x_i) d\sum \alpha_i x_i \quad (7)$$

It is useful for the ensuing analysis to write down the PVF in (7) in vector notation and as a  $n$ -dimensional multiple integral defined over  $x$ :

$$PVF = - \int_{\alpha'x < 0} (-\alpha'x)^b \pi(x) dx \quad (8)$$

After having laid down the environment and the assumptions underlying our analysis, the next section studies the CPT agent's attitude towards risk concentration or diversification, which is the key objective of this paper.

### 2.3 Conditions for optimal risk concentration or diversification

In our simple setting with symmetric and identically distributed risks, only two allocations make sense, namely equal diversification ( $\alpha_i = \frac{1}{n}$ ) and full concentration ( $\alpha_j = 1$  for one  $j$ ,  $\alpha_i = 0 \forall i \neq j$ ). To assess the relative desirability of diversification and concentration, let us consider the PVF in the two cases:

$$PVF_{div} = - \int_{\bar{x} < 0} (-\bar{x})^b \pi(x) dx, \quad (9)$$

$$PVF_{con} = - \int_{x_j < 0} (-x_j)^b \pi(x) dx, \quad (10)$$

where  $PVF_{div}$  is the PVF for diversification,  $PVF_{con}$  is the PVF for concentration, and  $\bar{x} = \sum \frac{x_i}{n}$ . Hence, diversification is optimal if  $PVF_{div} > PVF_{con}$ , while the opposite holds true if  $PVF_{div} < PVF_{con}$ .

$PVF_{div}$  and  $PVF_{con}$  are statistics derived from the joint probability distribution of  $x$ . Thus, whether diversification or concentration is optimal depends on this probability distribution,  $\Pr(x)$ . As it is shown in two examples below, diversification and concentration can both be optimal, depending on  $\Pr(x)$ .

Before moving closer to the examples, however, it may be useful to explain the intuition behind the idea that diminishing sensitivity is

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only losses and no gains. However, this possibility is ruled out by the assumption that the allocation of risks is a framed prospect for the agent. In other words, for our agent the allocation of risks is a self-contained decision problem, which *has to be dealt with* (otherwise, it would not have survived the editing phase). It is implicitly assumed that other benefits (for example, a positive expected return on the lottery) may overcome the perceived losses associated to the allocation of risks, and that the agent analyzes these benefits separately from the allocation of risks.



likely to shift the balance in favor of risk concentration, rather than diversification. Essentially, risk diversification can have two consequences, namely *removing* (perfect hedging) or *averaging* existing risks (in particular, for any given probability density of  $x$  the averaging mechanism reduces the probability of "large" risks and increases the probability of "small" risks). In our context, only downside risks matter (as it is shown in equation (8), the PVF may be transformed into a function of losses only). Clearly, if diversification *removes* downside risks, our agent will be better off than he would be under risk concentration. In fact, a loss averse agent is also risk averse, i.e. he would prefer not to play a lottery with zero expected value. However, if diversification simply *averages* existing *downside* risks without eliminating them, it will reduce, rather than improve the agent's welfare. In fact, a distinct feature of prospect theory is risk-seeking behaviour over losses, which is a consequence of the property of diminishing sensitivity.<sup>7</sup> Conversely, under expected utility the agent is risk averse everywhere and diversification is always welfare-improving, even in the absence of perfect hedging.

To pin down the intuition behind these considerations, let us consider two very simple examples. The first example is of optimal risk diversification. Suppose that our agent has to invest his financial wealth and can select two risky assets with payoffs  $x_1^\theta$  and  $x_2^\theta$  compared with the reference point (which is, say, the current level of wealth). The joint probability distribution is the following:

	$x_2^\theta = -10\%$	$x_2^\theta = 0\%$	$x_2^\theta = 10\%$
$x_1^\theta = -10\%$	1/9	1/9	1/9
$x_1^\theta = 0\%$	1/9	1/9	1/9
$x_1^\theta = 10\%$	1/9	1/9	1/9

Our agent has to select the decision weight  $\alpha_1$  in order to maximize the PVF defined over  $y_\theta = \alpha_1 x_1^\theta + (1 - \alpha_1) x_2^\theta$ . As in Tversky and Kahneman (1992), we posit  $a = 2.25$  and  $b = 0.88$ . For the probability weighing function, we assume for simplicity a linear weighing,  $\pi(p) = p$ . It is immediate to find that the diversified portfolio strictly dominates the concentrated portfolio ( $PVF_{div} = -2.19 > PVF_{con} = -3.16$ ). Thus, risk diversification is optimal.

In a second example, let us assume the same setting, but with the following probability distribution for  $x = [x_1^\theta, x_2^\theta]$ :

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<sup>7</sup>The non-linear weighing of probabilities postulated in CPT may also determine a risk-seeking behavior over small-probability losses (see, for example, Prelec, 1998).



	$x_2^\theta = -10\%$	$x_2^\theta = 0\%$	$x_2^\theta = 10\%$
$x_1^\theta = -10\%$	1/9	1/9	0
$x_1^\theta = 0\%$	1/9	1/3	1/9
$x_1^\theta = 10\%$	0	1/9	1/9

The chart below reports the probability distribution of the payoff on the diversified portfolio ( $y_\theta^{div} = \frac{x_1^\theta + x_2^\theta}{2}$ ) and of the concentrated portfolio ( $y_\theta^{con} = x_1^\theta$ ). In this example the property of averaging risks of diversification comes out clearly. The concentrated portfolio has a downward risk  $\{-10\%, 0.2\}$ ; if one multiplies the size of the risk ( $-10\%$ ) by its probability (0.2), the total expected downside risk is equal to 2%. The diversified portfolio "averages" this total downside risk of 2% as follows:  $\{-10\%, 0.1; -5\%, 0.2\}$ . Owing to the property of risk-seeking over losses, the CPT agent does not like such averaging of the total downside risk. In fact, under the same assumptions for the parameters of the PVF of the above example, it is found that risk concentration is optimal ( $PVF_{con} = -2.11 > PVF_{div} = -2.20$ ).

[insert chart here]

This latter example is an illustration that risk concentration may be optimal under CPT, in contrast with the standard approach based on expected utility maximization. A general sufficient condition for optimal risk concentration is laid down in the following Proposition:

**Proposition 1** *If:*

$$\int_{All\ x} \pi(\alpha x = 0) dx = 0 \quad \forall \alpha, \quad (11)$$

*then risk concentration ( $\alpha_j = 1$  for one  $j$ , and  $\alpha_i = 0$  for  $i \neq j$ ) is always optimal for a cumulative prospect theory agent with PVF defined as in (8).*

**Proof.** Let  $D_\alpha^+$ ,  $D_\alpha^0$  and  $D_\alpha^-$  be subsets of the domain  $D$  of  $x$  defined as follows:

$$D_\alpha^+ : \{x/\alpha'x > 0\} \quad (12)$$

$$D_\alpha^0 : \{x/\alpha'x = 0\} \quad (13)$$

$$D_\alpha^- : \{x/\alpha'x < 0\} \quad (14)$$

The PVF in (8) for a generic value of  $\alpha$  can be thus written down as:

$$PVF(\alpha) = - \int_{x \in D_\alpha^-} (-\alpha'x)^b \pi(x) dx \quad (15)$$

Due to the symmetry of the probability distribution of  $y_\theta = \alpha'x$ ,  $\int_{x \in D_\alpha^-} (-\alpha'x)^b \pi(x) dx = \int_{x \in D_\alpha^+} (\alpha'x)^b \pi(x) dx$ . Thus, it is possible to write the PVF in (15) as follows:

$$PVF = -\frac{1}{2} \left[ \int_{x \in D_\alpha^-} (-\alpha'x)^b \pi(x) dx + \int_{x \in D_\alpha^+} (\alpha'x)^b \pi(x) dx \right] \quad (16)$$

The function  $(-\alpha'x)^b$  for  $x \in D_\alpha^-$  is *concave* in  $\alpha$ , and the same holds true for  $(\alpha'x)^b$  for  $x \in D_\alpha^+$ , given that  $b < 1$ . A property of any concave function  $f$  is that  $f(\sum \alpha_i x_i) \geq \sum \alpha_i f(x_i)$ . Thus:

$$\int_{x \in D_\alpha^-} (-\alpha'x)^b \pi(x) dx \geq \int_{x \in D_\alpha^-} \alpha'(-x)^b \pi(x) dx, \quad (17)$$

where  $(-x)^b = [(-x_1^\theta)^b, \dots, (-x_n^\theta)^b]$ . Equally:

$$\int_{x \in D_\alpha^+} (\alpha'x)^b \pi(x) dx \geq \int_{x \in D_\alpha^+} \alpha'(x)^b \pi(x) dx, \quad (18)$$

with  $(x)^b = [(x_1^\theta)^b, \dots, (x_n^\theta)^b]$ .

Consequently:

$$PVF \leq -\frac{1}{2} \left[ \int_{x \in D_\alpha^-} \alpha'(-x)^b \pi(x) dx + \int_{x \in D_\alpha^+} \alpha'(x)^b \pi(x) dx \right] \quad (19)$$

This expression may be rewritten as an integral over the whole domain  $D$  of  $x$  as follows:

$$PVF \leq -\frac{1}{2} \alpha' \int_{x \in D} |x|^b \pi(x) dx, \quad (20)$$

owing to the assumption that  $\int_{x \in D_\alpha^0} \pi(x) dx = 0$ . Writing the right hand side term of expression (20) differently:

$$-\frac{1}{2} \alpha' \int_{x \in D} |x|^b \pi(x) dx = -\frac{1}{2} \sum \alpha_i \int_{x \in D} |x_i^\theta|^b \pi(x) dx \quad (21)$$

Due to the assumption that the  $x_i^\theta$  are identically distributed and noting that  $\int_{x \in D} |x_j^\theta|^b \pi(x) dx$  is a moment of the probability distribution of a generic  $x_j^\theta$ , we can define:

$$x_b^* = \int_{All \ x} |x_j^\theta|^b \pi(x) dx, \quad (22)$$

which takes the same value for every  $j$ . Thus, the following holds:

$$PVF \leq -\frac{1}{2} \sum \alpha_i \int_{x \in D} |x_i^\theta|^b \pi(x) dx = -\frac{1}{2} \sum \alpha_i x_b^* = -\frac{1}{2} x_b^* \quad (23)$$

It is immediate to show that the right hand side term is the PVF of the concentrated solution, as:

$$-\frac{1}{2}x_b^* = -\frac{1}{2} \int_{x \in D} |x_j^\theta|^b \pi(x) dx = -\int_{-\infty}^0 |x_j^\theta|^b \pi(x_j^\theta) dx_j^\theta = PVF_{con} \quad (24)$$

Thus, taking into account (19), it has been shown that:

$$PVF(\alpha) \leq PVF_{con} \quad \forall \alpha, \quad (25)$$

which leads necessarily to the conclusion that risk concentration is optimal, as it maximizes the PVF of the agent for  $\alpha$ . ■

A special case which is worth investigating is *linear* sensitivity to gains and losses, i.e.  $b = 1$ . Although prospect theory models normally postulate  $b < 1$ , in empirical studies  $b$  is found to be close to one (Tversky and Kahneman, 1992), so studying the case  $b = 1$  makes sense. As the following Proposition shows (and as it is quite intuitive in this setting), if our agent has a linear sensitivity to gains and losses, it will be indifferent between concentrating and diversifying risks:

**Proposition 2** *If:*

$$\int_{All \ x} \pi(\alpha x = 0) dx = 0 \quad \forall \alpha, \quad (26)$$

*then, if  $b = 1$ , risk concentration ( $\alpha_j = 1$  for one  $j$ , and  $\alpha_i = 0$  for  $i \neq j$ ) and risk diversification ( $\alpha_i = \frac{1}{n}$ ,  $\forall i$ ) provide the same utility to the CPT agent.*

**Proof.** Let us write the PVF as in (15), with  $b = 1$ :

$$PVF = \int_{\alpha' x < 0} (\alpha' x) \pi(x) dx = \alpha' \int_{\alpha' x < 0} x \pi(x) dx = \sum \alpha_i \int_{\alpha' x < 0} x_i^\theta \pi(x) dx \quad (27)$$

Due to the symmetry of the  $x_i^\theta$  and the assumption that  $\int_{All \ x} \pi(\alpha x = 0) dx = 0 \quad \forall \alpha$ , the following condition holds:

$$\int_{\alpha' x < 0} x_j^\theta \pi(x) dx = -\frac{1}{2} \int_{All \ x} |x_j^\theta| \pi(x) dx, \quad \forall j, \quad (28)$$

and the PVF is equal to:

$$PVF = -\frac{1}{2} \sum \alpha_i \int_{All \ x} |x_i^\theta| \pi(x) dx \quad (29)$$

As noted above, due to the assumption that the  $x_i^\theta$  are identically distributed and noting that  $\int_{All \ x} |x_j^\theta| \pi(x) dx$  is a moment of the probability distribution of the generic  $x_j^\theta$ , we can define:

$$x^* = -\int_{All \ x} |x_j^\theta| \pi(x) dx, \quad (30)$$

which is independent of  $j$ . Thus:

$$PVF = \frac{1}{2}x^* \sum \alpha_i = \frac{1}{2}x^*, \forall \alpha_i \quad (31)$$

The conclusion is that the  $PVF$  does not depend on the values chosen for  $\alpha_i$ , i.e. the agent is indifferent between risk concentration and diversification. ■

In intuitive terms, Proposition 1 states that if perfect hedging is not possible with strictly positive weighted probability (assumption in (11)), it is always optimal for a CPT agent to concentrate risks. Only if  $b = 1$  (linear sensitivity, i.e. neither decreasing nor increasing) is the agent indifferent between risk concentration and diversification (Proposition 2). This seems to be a rather striking result, as (at least some) risk diversification is instead *always* optimal in the standard setting, i.e. under expected utility theory (Samuelson, 1967). Moreover, the assumption in (11) is far from being a mere theoretical *curiosum*. There are indeed many examples in real life where the weighted probability of obtaining a *perfect* hedge is negligible. Notably, the probability of having a perfect hedge is always zero with continuous probability distributions. Assume, for instance, that a CPT investor has to allocate his wealth among  $n$  identical risky assets, the payoff on each of them Normally distributed with zero mean. This is a standard finance textbook problem, which has found solution in mean-variance diversification since the work of Markowitz (1952). Under prospect theory, irrespective of the correlation matrix among the  $n$  returns<sup>8</sup>, the agent will be better off by concentrating risk on only one asset. Another example is, of course, the situation facing you with your two credit cards mentioned in the Introduction of this paper.

It has to be emphasized that (11) is a *sufficient*, but not a *necessary* condition for optimal risk concentration. There might be cases in which perfect hedging has a strictly positive weighted probability and yet risk concentration remains optimal. To show this, assume that (11) does not hold and, therefore, that:

$$\int_{-\infty}^{\infty} \pi(\bar{x} = 0) dx > 0, \quad (32)$$

which shifts the balance in favor of risk diversification (now there is a strictly positive probability that diversification eliminates downside risks, which is clearly beneficial for our CPT agent exactly as for an agent

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<sup>8</sup>Unless, of course, two returns are perfectly negatively correlated – a situation very uncommon in practice.

maximizing expected utility). The difference between the concentrated and the diversified PVF may be derived straightforwardly as follows:

$$PVF_{con} - PVF_{div} = PVF_{AVE} - PVF_{HED}, \quad (33)$$

where:

$$PVF_{AVE} = \int_{x/\bar{x} < 0} (-\bar{x})^b \pi(x) dx - \int_{x/x_j < 0} (-x_j)^b \pi(x) dx, \quad (34)$$

$$PVF_{HED} = \int_{x/\bar{x}=0, x_j < 0} (-x_j)^b \pi(x) dx \quad (35)$$

From Proposition 1, we know that  $PVF_{AVE} \geq 0$  (i.e., the property of diversification of averaging downside risks decreases the agent's utility), while  $PVF_{HED} \leq 0$  (i.e., the perfect hedging property of diversification increases the agent's welfare). The relative desirability of risk concentration or diversification depends on whether the downside risks averaging effect ( $PVF_{AVE}$ ) prevails over the perfect hedging effect ( $PVF_{HED}$ ). If  $|PVF_{AVE}| > |PVF_{HED}|$ , risk concentration remains optimal even if perfect hedging has a strictly positive weighted probability. However, given that  $b$  is normally found to be close to (albeit slightly smaller than) one (Kahneman and Tversky, 2000), the term  $PVF_{AVE}$  is unlikely to be large (because the value function is only mildly convex over losses, i.e. it is very close to a piecewise linear function). Thus, even a small probability of having a perfect hedge is likely to put the odds in favor of risk diversification.

### 3 The more general case

In this section the assumptions regarding the distribution of  $x$  previously imposed are relaxed, so as to consider a more general setting than in the previous section. From now on, the  $x_i^\theta$  can have whatever probability distribution, i.e. also non-symmetric and non-identically distributed. The purpose of this section is to identify the conditions under which risk concentration or diversification is optimal for our CPT agent in this general case. As this setting is slightly more complicated to deal with from an analytical perspective, the results will be less neat and general than in the previous section. Nonetheless, the basic intuition behind the results remains the same.

Using the same notation as in the proof of Proposition 1, we can write the  $PVF$  as follows:

$$PVF(\alpha) = PVF_+ - \alpha PVF_-, \quad (36)$$

where  $PVF_+ = \int_{x \in D_\alpha^+} (\alpha'x)^b \pi(x) dx$ ,  $PVF_- = \int_{x \in D_\alpha^-} (-\alpha'x)^b \pi(x) dx$ .  
Let us consider:

$$PVF^* = \sup_{\alpha} PVF, \quad (37)$$

which is the maximum value for  $\alpha$  of the PVF. In general, the optimal  $\alpha$  cannot be derived analytically as the first and second order conditions cannot be solved in closed form, but it is easy to compute it using numerical methods. At the optimal value of  $\alpha$ , say  $\alpha^*$ , one obtains:

$$PVF^* = PVF_+^* - aPVF_-^* \quad (38)$$

There follows:

**Proposition 3** *If:*

$$\int_{All\ x} \pi(\alpha x = 0) dx = 0 \quad \forall \alpha, \quad (39)$$

*then risk concentration ( $\alpha_j = 1$  for one  $j$ , and  $\alpha_i = 0$  for  $i \neq j$ ) is optimal for a cumulative prospect theory agent with PVF defined as in (8) if:*

$$PVF_+^* - aPVF_-^* < 0, \quad (40)$$

*while at least some risk diversification (there are at least two  $i$  and  $j$  for which  $\alpha_i > 0$  and  $\alpha_j > 0$ ) is optimal if:*

$$PVF_+^* - aPVF_-^* > 0 \quad (41)$$

**Proof.** If (40) holds, then:

$$PVF^* = -a'PVF_-^*, \quad (42)$$

with  $a' > 0$ . Hence, the  $PVF^*$  may be written as a function of losses only as in (8) and it is therefore a convex function in  $\alpha$ , at least in the neighbourhood of  $\alpha = \alpha^*$ . Thus, the maximum value for the  $PVF$  must be necessarily found at the boundary of the parameter space of  $\alpha$ , implying that risks concentration has to prevail.

By contrast, if (41) holds, then:

$$PVF^* = a'PVF_+^*, \quad (43)$$

with  $a' > 0$ . In this case, the  $PVF^*$  may be written as a function of gains only and it is therefore a concave function in  $\alpha$ , at least in the neighbourhood of  $\alpha = \alpha^*$ . Under these conditions, Theorem II of Samuelson (1967) applies and at least some diversification is mandatory.

■

It should be emphasised that Proposition 3 is a generalisation of Proposition 1, where the assumption of identically distributed and symmetric  $x_i^\theta$  ensures that the condition (40) holds, leading to the result that risk concentration is optimal for the CPT agent.

In intuitive terms, whether risk concentration or diversification prevails depends on the position of the reference point relative to the probability distribution of the  $n$  sources of risk. If the reference point is "high" and losses tend to prevail, agents are willing to be risk-seeking and therefore prefer risk concentration. For example, in the case of the credit cards mentioned in the Introduction the (implicitly assumed) reference point (no credit card lost) is an upper bound for the possible states of the world, and any event different from it would be perceived as a loss, prompting risk-seeking behaviour by our CPT agent. Conversely, if the reference point is "low" and most states of the world are perceived as a gain, risk-averse behaviour would prevail and risk diversification with it. Suppose, for instance, that in the same example of the credit cards our agent chose the worst possible outcome (both credit cards lost) as the reference point. In such case, all events different from the reference point would be perceived as a gain, for which a CPT agent is risk averse. Hence, it would be optimal for this agent not to keep the eggs in the same basket, namely to spread the credit cards between the two wallets.<sup>9</sup>

It might be argued that, in most circumstances, the condition in (40) is more likely to hold than that in (41), because losses are more important than gains to a CPT agent ( $a > 1$ ). This is the reason why, for instance, the condition (40) prevails if the probability distributions of the  $x_i^\theta$  are symmetric around zero. Ultimately, the likelihood that condition (40) and thereby risk concentration hold hinges on whether the assumption that the allocation of risks is treated as an independent decision problem (framed prospect) is justified or not. Were this assumption not justified, it would not be possible to postulate that the agent willingly enters in a lottery involving mainly losses.<sup>10</sup> Nevertheless, while the psychological process leading to any decision depends uniquely on

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<sup>9</sup>As the value function is only mildly concave for gains, the incentives for risk diversification would in most cases be small, e.g. much smaller than with a mean-variance specification of preferences. Of course, also the incentives towards concentration (when condition (40) holds) will be rather small. With the value function close to a piecewise linear function, the issue of risk concentration or diversification loses much of its importance, as Proposition 2 suggests, unless diversification makes perfect hedging possible.

<sup>10</sup>It is interesting to note that risk concentration can never be preferable to not entering the lottery at all, i.e. no gain and no loss (and  $PVF = 0$ ), as expressions (40) and (41) in Proposition 3 show.

the nature of the problem and no generalization can be made, it is fair to say that the assumption that the allocation of risks can be treated as a framed prospect makes sense in many circumstances (for instance, where the agent cannot avoid to take risks, a situation very common in life). Overall, the conclusion seems warranted that risk concentration is likely to be optimal at least in some circumstances under CPT, and it therefore deserves serious consideration as far as its consequences for economic behaviour are concerned.

#### 4 Prospect theory: a solution to the "home bias" puzzle?

The key result of the previous sections is that risk concentration may be optimal for a *rational* loss averse agent displaying risk-seeking behaviour on losses, provided that the probability of obtaining a perfect hedge is negligible and the agent sees the allocation of risks as a self-contained decision problem. In other words, risk diversification does not lead to risk (loss) minimization for our CPT agent, in contrast with a standard expected utility agent. The two next interesting questions are, first, whether this result is plausible, and, second, whether it is relevant.

As to the plausibility of the result that rational loss averse agents may prefer risk concentration to risk diversification, it should be noted that lack of risk diversification is a tendency which is often found among economic agents and has been widely documented in the literature. For example, French and Poterba (1991) and Shiller (1998) reported that agents seem to have no appetite for diversification, be it in financial investment or in real estate acquisition. Rode (2000) reported survey data indicating that diversification is normally seen by agents as being a different thing from risk minimization. Indeed, while agents in some occasions appear to have a vague feeling that diversification is beneficial (for instance in financial investment), they fail to diversify appropriately (Goetzmann and Kumar, 2001). Moreover, when they actually diversify risks, they tend to do so in a very naive manner, for instance by following the  $1/n$  heuristic (Benartzi, 2001). Overall, it appears that the idea that agents in a variety of contexts do not regard risk diversification as contributing to risk minimization, in contrast with the normative implications of expected utility theory, is broadly in keeping with the available empirical evidence. It is therefore an interesting and plausible result that prospect theory (namely, a theory which is based on strong psychological foundations and which has received a substantial amount of empirical support) implies that risk diversification is not mandatory, but rather predicts that its opposite, risk concentration, is likely to emerge in a variety of situations. All in all, the analysis in this paper seems to have



identified a key difference between the implications of prospect theory and those of standard expected utility theory.

As to the relevance of the results in this study to explain real world phenomena, perhaps the most famous example of a lack of diversification is the "home bias" in international financial investment (French and Poterba, 1991). As noted by Lewis (1999), the so-called "home bias puzzle" is still far from having received a satisfactory explanation despite repeated efforts in the literature. It has to be noted that one of the key alleged benefits of international diversification is the minimization of risk for a given expected return, which emerges naturally in a mean-variance context (Markowitz, 1952; Grubel, 1968). In other words, international diversification should be no more than an application of the general law "do not put your eggs in the same basket" (in this case, country). Noting that in financial investment the probability of obtaining a perfect hedge is probably nil and that the expected return on the portfolio may be considered as an appropriate reference point for the normal investor, loss aversion with diminishing sensitivity suggests that concentrating risks on a *single* asset might be optimal from the point of view of risk (loss) minimization, as noted in the previous sections. Hence, the investor should "put all his eggs in one basket", i.e. country. Thus, if prospect theory is an accurate description of human attitudes towards risk, the benefits of international diversification would be reduced to a significant extent. In other words, holding internationally diversified portfolios might not be as "desirable" for economic agents as it is commonly regarded.

Yet, while prospect theory could explain the tendency to concentrate risks on a single asset rather than to hold a well diversified portfolio, it cannot explain why the single asset the investor chooses is a *domestic* one. Clearly, the home bias in financial investment is probably a complex and multi-faceted phenomenon, which reflects the influence of various factors. For instance, transaction costs (Tesar and Werner, 1992), a greater familiarity with domestic assets (Gehrig, 1993), and the fact that holding a portfolio concentrated on domestic assets was mandatory in the past due to restrictions to international capital flows, are all likely to play an important role. However, it is important to stress that, as shown in this paper, risk diversification aimed at risk minimization (for a given level of expected return) could be far from being the powerful force to remove these obstacles to international diversification that has been hypothesized in the past (Grubel, 1968), if investors indeed behave like prospect theory agents. Thus, even if prospect theory *per se* cannot account for the home bias, it might be a key element in its overall explanation.

## 5 Conclusions

Cumulative prospect theory posits that agents care about losses against a reference point comparatively more than about gains of equal size (loss aversion), and that the importance of marginal gains and losses is higher close to the reference point than away from it.

This paper has studied the optimal allocation of a representative CPT agent's stakes among  $n$  identical sources of risk (which it is assumed to be a "framed prospect" for the agent in the definition of Tversky and Kahneman, 1986). This is a classical problem of the finance literature since at least Markowitz (1952). The key result of this analysis is that, due to the prevalence of losses in the agent's value function and to risk-seeking behaviour for losses, the property of diversification of averaging downside risks is welfare-reducing rather than welfare-improving. Therefore, provided that the subjective probability of obtaining a perfect hedge is negligible, our CPT agent is better off by *concentrating* rather than by *diversifying* risks, which is in contrast with the normative prediction of standard expected utility models normally found in economics and finance textbooks. Noting that there is ample evidence that agents refrain from diversifying risks in a variety of contexts (French and Poterba, 1991; Shiller, 1998), the overall conclusion of the paper is that optimal risk concentration is an interesting and rather realistic feature of prospect theory, and an important point of departure of this theory from expected utility theory. The paper has also argued that the optimality of risk concentration, at least in some circumstances, might be one of the factors (albeit certainly not the only one) which explain the widely observed "home bias" in international financial investment.

The analysis in this paper might be expanded in several directions, and two of them might be mentioned here. First, it might be useful to link prospect theory with other observed behavioral biases to assess the overall importance of psychological factors for the incentives for (or against) risk diversification. For example, the "event-splitting" heuristic described in Starmer and Sugden (1993) – namely, the tendency for risks to appear bigger under a disaggregated description – is likely to further contribute to shift the balance against risk diversification. Conversely, the naive diversification strategies described in Benartzi (2001) are expression of an equally naive agents' preference for diversification as an "end in itself". Second, it would be interesting to study models in which the reference point evolves endogenously depending on the allocation of risks. In this respect, disappointment aversion introduced by Gul (1991), with its focus on the endogenous formation of reference points, seems to be a good place to begin.

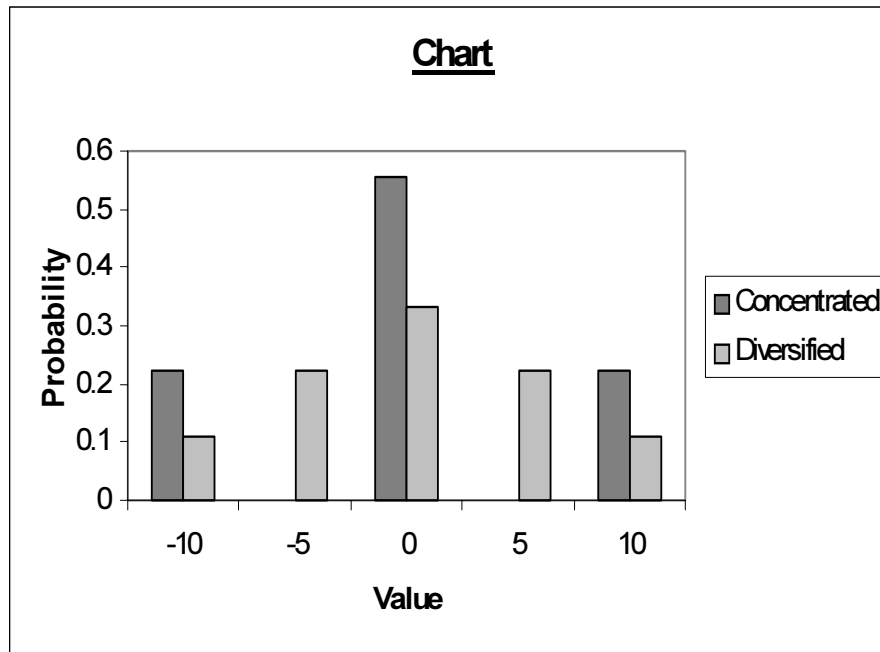


Figure 1:

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