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## Non-standard central bank loss functions, skewed risks, and certainty equivalence

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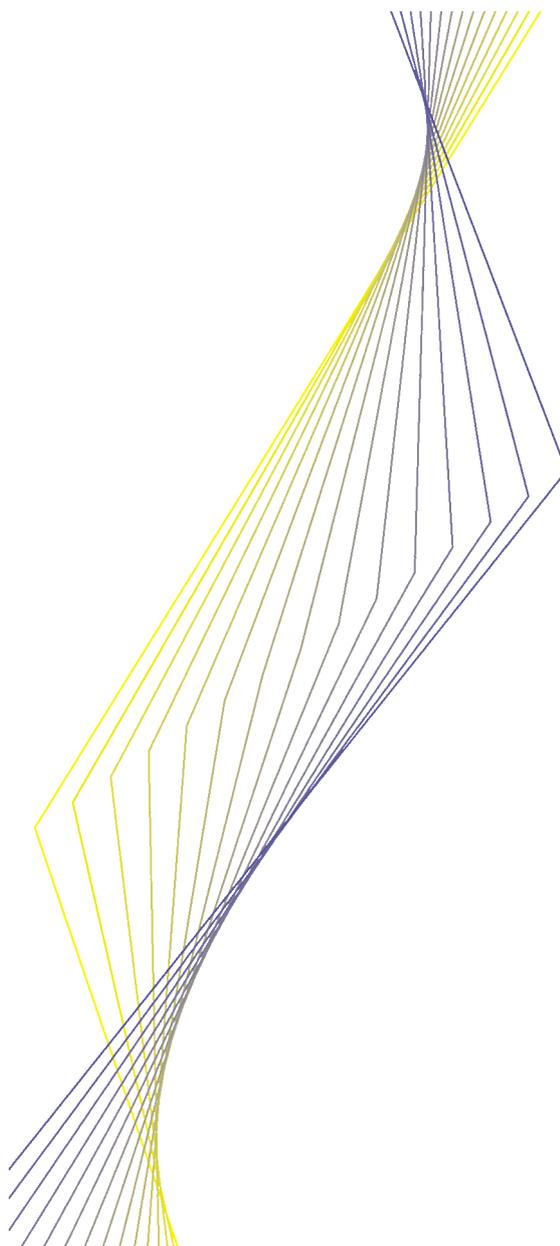
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BANK LOSS FUNCTIONS,  
SKEWED RISKS, AND  
CERTAINTY EQUIVALENCE**

**BY ALI AL-NOWAIHI  
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**March 2002**

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**BY ALI AL-NOWAIHI\*  
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**March 2002**

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## Abstract

This paper sets out to investigate the role of additive uncertainty under plausible *non-standard* central bank loss functions over future inflation. Building on a substantial body of evidence in the economic psychology literature, this paper postulates (i) period-by-period loss functions that are non-convex, i.e. displaying diminishing or non-increasing sensitivity to losses, and (ii) non-linear weighing of probabilities, hence departing from the expected utility paradigm. The main conclusion of the study is that if the additive uncertainty is caused by a non-Normal distributed additive shock, for instance if the probability distribution of the shock is skewed, then with these departures from the quadratic function *the principle of certainty equivalence does not hold anymore*. Thus, it appears that with additive uncertainty of the non-Normal type the assumption of a quadratic loss function for the central banker may not be as innocuous as it is commonly regarded.

**Keywords:** Monetary policy, non-quadratic loss functions, economic psychology, certainty equivalence

**JEL codes:** E52, E58

## Non-technical summary

The role of uncertainty in monetary policy-making has attracted considerable interest in the literature in recent years. It has long been known that uncertainty (namely, over the true "value" of the interest rate elasticity of output and inflation) is not neutral for the policy-maker and generally leads to caution and policy gradualism (Brainard, 1967). There is, however, much less consensus on the role of *additive* uncertainty, which denotes the uncertainty over the true "state of the economy", despite the obvious importance of this matter for policy-makers, who are confronted with this type of uncertainty practically every day.

On the one hand, the view is prevailing in the academia that additive uncertainty should not matter, a principle known as "certainty equivalence" (Theil, 1958). Recently, Svensson and Woodford (2000) have provided a general proof that with a quadratic objective function for the central banker, the optimal policy is unaffected by uncertainty about the state of the economy (Svensson and Woodford characterise this situation as the orthogonality of estimation and policy). Moreover, the principle of certainty equivalence does not seem to be a mere artefact of the use of a quadratic loss function. Chadha and Schellekens (1999) have shown that the certainty equivalence principle holds for a general class of convex loss functions, and the coefficients of the optimal policy rule are not affected by additive uncertainty even if the preferences of the central banker are asymmetric.

On the other hand, policy-makers generally do not seem to think that additive uncertainty is irrelevant (see for example Blinder, 1998). A casual look at central banks' external communication tends to lend support to this assessment. Thus, there seems to be an important discrepancy of views between policy-makers and the academia over this key aspect of monetary policy-making.

Against this background, this paper sets out to analyze the interaction, if any, between non-standard and yet analytically tractable and behaviorally plausible central bank loss functions and uncertainty modelled as a non-Normal distributed additive shock to the inflation process. This paper, in particular, relaxes two commonly maintained assumptions on the central bank loss function:

- (i) A curvature of the loss function different from, and more general than, the quadratic is considered.
- (ii) The effect of a non-linear weighing of probabilities by the central bank is analyzed.

In devising plausible and tractable non-quadratic central bank loss functions, this paper builds on the substantial body of evidence made available by the literature on economic psychology under uncertainty, especially by the strand linked to the names of Daniel Kahneman and

Amos Tversky (see Kahneman and Tversky, 2000, for a review and assessment of this literature).

Ultimately, the objective of the analysis is to establish whether the principle of certainty equivalence carries through to the non-standard loss functions examined here, and hence whether the assumption of a quadratic loss function evaluated according to the expected utility criterion is indeed innocuous or not.

The main result of the paper is that the interaction of a non-Normal additive disturbance to inflation and the set of non-quadratic preferences postulated in the economic psychology literature leads to situations where the principle of certainty equivalence does not hold. Thus, additive uncertainty seems to matter. Only if the disturbance is Normal distributed is the usual result of certainty equivalence still valid. If one observes that non-Normal distributed shocks are an essential element which monetary policy-makers have to deal with, the overall policy message of this paper is that additive uncertainty matters, and that the assumption of a quadratic loss function may not be as innocuous as it is often regarded.

## 1 Introduction

The role of uncertainty in monetary policy-making has attracted considerable interest in the literature in recent years (see, e.g., the review by Batini, Martin and Salmon, 1999). It has long been known that *multiplicative* uncertainty (namely, over the true "value" of the interest rate elasticity of output and inflation) is not neutral for the policy-maker and generally leads to caution and policy gradualism (Brainard, 1967).

There is, however, much less consensus on the role of *additive* uncertainty, which denotes the uncertainty over the true "state of the economy", despite the obvious importance of this matter for policy-makers, who are confronted with this type of uncertainty practically every day.<sup>1</sup> On the one hand, the view is prevailing in the academia that additive uncertainty should not matter, a principle known as "certainty equivalence" (Theil, 1958).<sup>2</sup> Recently, Svensson and Woodford (2000) have provided a general proof that with a quadratic objective function for the central banker, the optimal policy is unaffected by uncertainty about the state of the economy (Svensson and Woodford characterise this situation as the orthogonality of estimation and policy). Moreover, the principle of certainty equivalence does not seem to be a mere artefact of the use of a quadratic loss function. Chadha and Schellekens (1999) – henceforth CS – have shown that the certainty equivalence principle holds for a general class of convex loss functions, and the coefficients of the optimal policy rule are not affected by additive uncertainty even if the preferences of the central banker are asymmetric.<sup>3</sup> On the other hand, policy-makers generally do not seem to think that additive uncertainty is irrelevant (see for example Blinder, 1998). A casual look at central banks' external communication tends to lend support to this assessment. For instance, in the Sveriges Riiksbank's Inflation Report (quoted in Blix and Sellin, 2000), it was reported that:

"The element of *uncertainty in the inflation assessment* can accordingly influence monetary policy's construction. A high degree of

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<sup>1</sup>Data measurement problems, uncertainty over the economy's natural rate of employment (or the natural rate of interest) at any point in time, shocks to the inflation rate and/or to the output gap which occur *after* a certain monetary policy decision but *before* its impulse has fully worked through the economy, are all prominent examples of additive uncertainty.

<sup>2</sup>Formally, let  $x$  be a control variable and  $y$  a state variable, with  $y = f(x) + e$ ,  $e$  being a zero mean additive disturbance and  $f$  a deterministic function. If certainty equivalence holds, the optimal value of  $x$  (for instance, the value which minimizes  $y$ ) is independent of any moment of the probability distribution of  $e$ .

<sup>3</sup>Orphanides and Wieland (2000) also analysed the properties of inflation zone targeting using a nonlinear central bank loss function. Orphanides and Wieland, however, did not deal with the issue of additive uncertainty, at least not directly.

uncertainty can be a reason for giving policy a more cautious turn”  
[emphasis ours],

and in the Bank of England’s Inflation Report (again quoted from Blix and Sellin):

”in the light of the central projection and *the risks surrounding it*, the Bank continues to see the need for a moderate tightening of policy”.  
[emphasis ours]

Finally, this statement can be retrieved from the European Central Bank’s website:

The European Central Bank (ECB) confirms its position of ‘wait and see’ with regard to its monetary policy stance. In an environment of *increased uncertainty over the global economy and its impact on the euro area*, the Governing Council is carefully assessing whether and to what extent upward risks to price stability will continue to decline.”  
[emphasis ours]

In all cases, central banks seem to refer to additive uncertainty (i.e., uncertainty over the state of the economy, not over the effect of the monetary policy levers) as an important element in the determination of policy. Thus, there seems to be an important discrepancy of views between policy-makers and the academia over this key aspect of monetary policy-making. This divergence, in turn, should lead one to wonder whether simple and plausible monetary policy models alternative to those traditionally used in the academic literature on optimal monetary policy can be worked out to give account of the seemingly important role of additive uncertainty in actual policy-making.

Against this background, this paper sets out to analyze the interaction, if any, between non-standard and yet analytically tractable and behaviorally plausible central bank loss functions and *additive* uncertainty modelled as a non-Normal distributed additive shock to the inflation process. Throughout the paper, the assumption will be maintained that the policy-maker has no multiplicative uncertainty, namely he has a perfect knowledge of the effect of the monetary policy instrument on the target variable(s). This paper relaxes two commonly maintained assumptions on the central bank loss function. First, a curvature different from, and more general than, the quadratic is considered. Second, the effect of a non-linear weighing of probabilities by the central bank is analyzed. Ultimately, the objective of the analysis is to establish whether the principle of certainty equivalence carries through to

the non-standard loss functions examined here, and hence whether the assumption of a quadratic loss function evaluated according to the expected utility criterion – which permeates the bulk of the literature on optimal monetary policy – is indeed innocuous or not. That no assumption on the probability distribution of the additive shock is maintained is worth stressing, as it distinguishes the analysis in this paper from that in CS.<sup>4</sup> Indeed, the type of uncertainty central bankers are routinely confronted with is certainly not always Normal distributed. Goodhart (2001), for instance, stressed that skewed risks represent a challenge for monetary policy-makers and that the profession should come up with an analytical framework to study how such skewed risks may be meaningfully incorporated in the policy assessment. Overall, this paper seems to be a first step in that direction.

In devising plausible and tractable non-quadratic central bank loss functions, this paper builds on the substantial body of evidence made available by the literature on economic psychology under uncertainty, especially by the strand linked to the names of Daniel Kahneman and Amos Tversky (see Kahneman and Tversky, 2000, for a review and assessment of this literature). With no evidence available thus far on the "typical central banker"'s psychology, the working assumption of this paper is that the patterns and tendencies that the Kahneman-Tversky literature has identified in a number of experimental studies are also valid for those agents in charge of monetary policy.<sup>5</sup> Among the key elements identified in this literature, diminishing sensitivity to losses (i.e., non-convex loss functions) and non-linear weighing of probabilities (i.e., departures from the expected utility paradigm) seem to be plausible and interesting also as a characterization of central bank preferences.

In sum, the paper finds that the interaction of a non-Normal additive disturbance to inflation and the set of non-quadratic preferences postulated here leads to situations where the principle of certainty equivalence does *not* hold. Thus, additive uncertainty seems to matter. Instead, if the disturbance is Normal distributed (as assumed in CS), the usual re-

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<sup>4</sup>CS maintain the assumption of a Normal distributed additive shock to inflation throughout their paper.

<sup>5</sup>It should be stressed that throughout the paper the emphasis will always be on the central bank's *positive* (i.e., descriptive) preferences. The analysis abstains from the determination of the *normative* preferences, for example those that would maximise society's welfare (see, e.g., Svensson, 2001). Of course, if it were possible for society to write down explicitly the central banker's loss function, this would lead us a long way towards the identification of the "true" central bank preferences. However, even in this (rather unrealistic) case the "ex post" preferences will not correspond to the "ex ante" preferences (those which matter for monetary policy-making), for the latter also involve the central banker's attitude towards risk.

sult of certainty equivalence continues to be valid. If one observes that non-Normal distributed shocks are an essential element which monetary policy-makers have to deal with, the overall policy message of this paper is that additive uncertainty matters, and that the assumption of a quadratic loss function may not be as innocuous as it is often regarded.

The paper is organized as follows. In Section 2 the usual setting is outlined of a monetary policy-maker aiming at minimizing a loss function defined in terms of the inflation rate, with the structure of the economy acting as a constraint on behaviour. The effect of considering non-quadratic central bank loss functions on the role of additive uncertainty is analyzed in Section 3. Finally, Section 4 concludes.

## 2 A simple optimal control model for discretionary monetary policy

This section lays down a standard optimal control problem for discretionary monetary policy.<sup>6</sup> The structure of the economy includes an IS curve, whereby the monetary authority can influence the output gap by steering the nominal interest rate, and a backward-looking Phillips curve, linking current inflation to past inflation and to past output gap (see, e.g., Clarida, Gali and Gertler, 1999, and Mankiw, 2001a).<sup>7</sup>

The IS curve is specified as follows:

$$x_t = a_1 x_{t-1} - a_2 (i_t - \pi_t) + u_t, \quad (1)$$

where the output gap  $x$  is affected by the real interest rate  $(i - \pi)$ , and  $u$  is a disturbance term, unknown to the central bank at time  $t$ , with  $E_t u_t = 0$ . Parameters in this equation are  $0 < a_1 < 1$  and  $a_2 > 0$ , known to the central bank.

The Phillips curve is:

$$\pi_{t+1} = b_1 \pi_t + b_2 x_t + v_{t+1}, \quad (2)$$

where  $v$  is an additive disturbance, for instance capturing a cost-push shock unknown to the policy-maker at time  $t$ , with  $E_t v_{t+1} = 0$ , and  $0 < b_1 < 1$  and  $b_2 > 0$  are parameters, of which the central bank

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<sup>6</sup>Throughout the paper, the assumption is always maintained that the central bank cannot credibly commit to follow a policy rule; monetary policy is thus carried out in a discretionary manner.

<sup>7</sup>The choice of a backward-looking specification of the Phillips curve is motivated by the fact that it squares better with the available empirical evidence (see in particular Mankiw, 2001a, and Rudebusch, 2001). The line of argumentation in the paper, however, would not be substantially changed with a forward-looking Phillips curve, as long as inflationary expectations at time  $t$  are exogenous for the central bank (to avoid the simultaneity problems discussed by Svensson and Woodford, 2000).

has again full knowledge.<sup>8,9</sup> No further assumption on the probability distribution of  $v$  is added at this stage. For simplicity, a zero drift is assumed; this, together with the assumption that  $b_1 < 1$ , implies that the steady state level of inflation is zero.

For notational simplicity, it is convenient to consolidate the IS and the Phillips curves to obtain a reduced form for inflation as follows:

$$\pi_{t+1} = c_1\pi_t + c_2x_{t-1} - c_3i_t + \varepsilon_{t+1}, \quad (3)$$

where  $c_1 = b_1 + b_2a_2$ ,  $c_2 = b_2a_1$ ,  $c_3 = b_2a_2$ , and  $\varepsilon_{t+1} = b_2u_t + v_{t+1}$  is a zero mean (but possibly non-Normal and non-symmetrically distributed) random disturbance (comprising output gap and cost-push shocks).

Simplifying it further:

$$\pi_{t+1} = z_t - c_3i_t + \varepsilon_{t+1}, \quad (4)$$

with  $z_t = c_1\pi_t - c_2x_{t-1}$ , known to the central bank at time  $t$  and exogenous to  $i_t$ , representing the "state of the economy" or equivalently the "inflationary pressures" at time  $t$ . In general, policy rules are specified as feedback rules  $i_t = f(z_t)$ , whereby the central bank sets its monetary policy instrument in reaction to changes in the state of the economy.

For simplicity of notation, let us consider the variable  $\tilde{i}_t$  defined as follows:

$$\tilde{i}_t = c_3i_t - z_t \quad (5)$$

If  $\tilde{i}_t = 0$ ,  $i_t = \frac{z_t}{c_3}$  and  $E_t\pi_{t+1} = 0$ . Therefore,  $\tilde{i}_t$  is the deviation of the monetary policy instrument from the value which offsets at time  $t + 1$  the expected impact of the inflationary pressures observed at time  $t$  (i.e.,  $z_t$ ). There follows that the inflation process may also be expressed as:

$$\pi_{t+1} = \varepsilon_{t+1} - \tilde{i}_t, \quad (6)$$

with  $\varepsilon_{t+1}$  independent of  $\tilde{i}_t$ . Intuitively, the inflation process is equal to the difference between the realization of the additive shock at time  $t + 1$  and the deviation of the monetary policy instrument from the value for which  $E_t\pi_{t+1} = 0$ . Thus, the value  $-\tilde{i}_t$  may also be interpreted as the inflation target of the central bank at time  $t$ . In the continuation of this paper, we will often refer to  $\tilde{i}_t$  when speaking about monetary policy, without any loss of generality as the monetary policy instrument may be derived straightforwardly as  $i_t = \frac{\tilde{i}_t + z_t}{c_3}$ .

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<sup>8</sup>That the monetary policy instrument  $i$  affects inflation with a longer lag than it affects output is consistent with the bulk of the available empirical evidence.

<sup>9</sup>More complex Phillips curve equations may be conceived (see, e.g, the non-linear specification in Clark et al, 2001), but such complications should *prima facie* be immaterial for the purpose of the present analysis.

The task of monetary policy is to select a value of  $\tilde{i}_t$  which minimizes an intertemporal loss function defined in terms of inflation levels (assuming for simplicity – and without loss of generality – that the inflation objective of the central bank is zero).<sup>10</sup> The principle of certainty equivalence – the main focus of the present analysis – stipulates that the probability distribution of  $\varepsilon$  should *not* matter in the determination of  $\tilde{i}_t$  (or,  $i_t$ ).

The standard approach to deal with monetary policy problems is to specify the objective function of the central banker, with the structure of the economy that acts as a constraint on behaviour. Given a period-by-period loss function  $L(\pi)$ , the central bank is normally assumed to minimize a time separable intertemporal loss function expressed as the expected value of a discounted sum  $L$  (where  $0 < \gamma < 1$  is the discount factor):

$$E_t L = E_t \sum_{j=1}^{\infty} \gamma^{j-1} L(\pi_{t+j}), \quad (7)$$

under the constraint  $\pi_{t+1} = \varepsilon_{t+1} - \tilde{i}_t$ . It is clear from the way the problem is formulated that the central bank's task may be reduced straightforwardly to the minimization of  $E_t L(\pi_{t+1})$ . In fact, the problem is specified recursively and the monetary policy instrument at time  $t$  only affects inflation one period ahead.

Under quadratic preferences, we have:

$$E_t L(\pi_{t+1}) = E_t \pi_{t+1}^2 = E_t (\varepsilon_{t+1} - \tilde{i}_t)^2, \quad (8)$$

and thus, after solving the first order condition:

$$\tilde{i}_t = 0, \quad (9)$$

given that  $E_t \varepsilon_{t+1} = 0$ . Of course, this result may be derived also by force of the statistical law that the arithmetic mean (zero, in this case) is the measure of central tendency minimizing the average of the squared deviations from it. Hence,  $i_t = \frac{z_t}{c_3} = \frac{c_1 \pi_t - c_2 x_{t-1}}{c_3}$ , *irrespective* of the probability distribution of  $\varepsilon$ ; this is the standard result of certainty equivalence. In general, a value of  $\tilde{i}_t$  different from zero depending on (some moments

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<sup>10</sup>As in CS, we do not consider the possibility that the monetary authority may also care about the output gap. While this is certainly a restrictive and rather unrealistic assumption, it greatly simplifies the algebra. Moreover, the inclusion of the output gap may under certain conditions determine a *departure* from certainty equivalence (see in particular Smets, 1998). Thus, given that the focus of the present analysis is to ascertain whether additive uncertainty *on inflation* matters *per se* under non-quadratic central bank loss functions, we leave this complication aside in order to balance the odds in favour of certainty equivalence as much as possible.

of) the probability distribution of  $\varepsilon_{t+1}$  would signal a departure from certainty equivalence.

Having laid down the framework and the notation for the central bank's optimal control problem, in the ensuing section we move to analyze the effect of imposing non-quadratic loss functions onto the optimal choice of the monetary policy instrument  $\tilde{i}_t$ .

### 3 Non-standard central bank loss functions

In this section two assumptions underlying the standard central bank intertemporal loss function normally considered in the literature are relaxed with a view to studying the effect of these departures on the role of additive uncertainty. First, we consider the possibility that the loss function may have a non-quadratic functional form, possibly being non-convex over its dominion. Second, we analyze the effect of a non-linear weighing of probabilities by the central banker, thus departing from the expected utility paradigm. For each departure from the standard setting, some behavioral justification will be provided.

In general, in order to devise *reasonable* alternatives to the quadratic as a central bank loss function, it should be first pondered about the behavioral assumptions underlying the quadratic function. A key element of the quadratic function is that "large" shocks are penalized proportionally more heavily than "small" shocks.<sup>11</sup> However, it appears *prima facie* useful to analyze functional forms that do not penalize large shocks as much as the quadratic function does (or, equivalently, do not attribute so little weight to small shocks). For example, Goodhart (2001) criticizes the assumption that large shocks should be penalized proportionally more than small shocks and reports that "I could never see why a 2% deviation from desired outcome was 4x as bad as a 1% deviation, rather than just twice as bad". Moreover, Rabin (2000) and Rabin and Thaler (2001) have shown (albeit in a context unrelated to monetary policy) that with a convex (e.g., quadratic) loss function and even a small aversion to small shocks, the aversion to moderate or large shocks may easily reach astronomical (behaviourally absurd) levels. Indeed, an important strand of the economic psychology literature, mainly popularized by Daniel Kahneman and Amos Tversky, building on basic psychological intuition and on a substantial body of experimental evidence, points to the fact that agents generally show a tendency to a mildly *decreasing* (instead of *increasing* as implied by the quadratic function)

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<sup>11</sup>A standard argument in favour of the quadratic loss function is that for "small" shocks it approximates relatively well any differentiable loss function as a second order Taylor expansion. However, in our analysis we assume that "large" shocks do exist, albeit maybe with low probability.

sensitivity to shocks (computed against a "reference point") as the size of the shock rises (see Kahneman and Tversky, 2000, Rabin, 1998, and Thaler, 2000, for a review of this literature). If the results of this literature are to be taken seriously, this would suggest loss functions with a *curvature* different from the quadratic, and in particular functions that are *non-convex*.<sup>12</sup>

Another key element of this literature is the observed widespread tendency for agents to weigh probabilities in a non-linear manner, hence departing from the expected utility paradigm (see, e.g., Starmer, 2000). A non-linear weighing of probabilities by central banks is very realistic. Cecchetti (2000), for instance, reports that "[...] we would expect policy-makers to take action when the mean and variance of forecast distributions are likely to stay the same, while the probability of some extreme bad event increases. [...] even if the variance is unchanged, an increase in the possibility of a severe economic downturn is likely to prompt action." While this is somewhat at odds with Goodhart (2001), who claims that "[...] main characteristic of risks which policy should *not* try to pre-empt is that they are low-probability events with a high pay-off", a non-linear weighing of probabilities is suggested in both cases. Non-linear weighing of probabilities – such as a disproportionate weight attached to changes in probabilities around zero, a phenomenon known as the "certainty effect" – is sometimes associated with anticipatory feelings and anxiety (see, for instance, Caplin and Leahy, 2001). While central bankers may be cooler than normal human beings, it is a fair assumption that at times they can certainly become anxious and mis-calculate (or willingly distort) probabilities according to their emotional state.

In sum, central bankers are, above all, humans; the possibility should be at least considered that they display the same attitudes towards risk that have been documented for other types of agents in the economic psychology literature.<sup>13</sup> Hence the main focus of this section of the paper is evaluating the effect of incorporating these attitudes (decreasing sensitivity and non-linear weighing of probabilities) in the central bank's loss function on the optimal setting of the monetary policy instrument and, in particular, on whether certainty equivalence continues to hold.

Turning first to the issue of the curvature of the period-by-period loss function, a relatively general and simple functional form to be considered in this analysis can be the following (see Kahneman and Tversky,

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<sup>12</sup>The Kahneman-Tversky literature favours loss functions that have a "kink" on the reference value, and a different curvature above and below this value.

<sup>13</sup>Another key trait identified by the Kahneman-Tversky literature is the asymmetric treatment of gains and losses. This feature, however, should not matter for our central banker, who can only lose (and not gain) from a shock to inflation.

1992):<sup>14</sup>

$$L(\pi_{t+1}) = |\pi_{t+1}|^\beta \quad (10)$$

If  $\beta = 2$ , this is the standard quadratic loss function. In the Kahneman-Tversky literature (see in particular Tversky and Kahneman, 1992)  $0 < \beta < 1$  (although  $\beta$  is normally estimated to be very close to one<sup>15</sup>); thus, the loss function is not convex and has a "kink" at zero (clearly the "reference value" in our setting). A loss function specified as in (10) is able to encompass a large number of functional forms and behavioral assumptions. The parameter  $\beta$ , in particular, drives the curvature of the loss function. Values  $\beta < 1$  identify non-convex loss functions, namely functions where the value of the loss grows *less than proportionally* with the size of the shock (as in the function postulated in Tversky and Kahneman, 1992).

According to Kahneman and Tversky, diminishing sensitivity is a general trait and a basic element of human perception. In the case of monetary policy-making, some special circumstances have to be taken into consideration. If our central banker behaves in a Kahneman-Tversky manner (i.e.,  $\beta < 1$ ), he will be more upset by a 1% *marginal* change in inflation from 2% to 3% than by a change from, say, 15% to 16% (a quadratic central banker will be more upset by the latter). However, the presence of thresholds and "target zones" for inflation (see Orphanides and Wieland, 2000), as it is common in some way or another in many countries, is likely to impart severe non-linearities to the central bank loss function, because overcoming the threshold(s) may be particularly costly. Thus, a central banker may be *more* concerned by a marginal move of inflation from 2% to 3% than from 1% to 2% if the target zone has an upper bound at, say, 2.5%. Nevertheless, *within* and *outside* the target zone the general principle of diminishing sensitivity should prevail.

These considerations lead us to think that a realistic central bank loss function, at least in those countries where the inflation target is specified in terms of a "target zone" of symmetric width  $\bar{\pi}$ , is the following:

$$L(\pi_{t+1}) = a(\pi_{t+1}) + |\pi_{t+1}|^\beta, \quad (11)$$

where  $a(\pi_{t+1}) = a > 0$  if  $|\pi_{t+1}| \geq \bar{\pi}$ , and zero otherwise. Thus, there is a fixed cost  $a$  to be incurred if inflation goes outside the target zone. Chart 1 depicts this loss function for three key values of  $\beta$ , namely  $\beta = 2$  (quadratic),  $0 < \beta < 1$  (diminishing sensitivity) and  $\beta = 0$

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<sup>14</sup>The same loss function is considered in Vickers (1998).

<sup>15</sup>For example, Tversky and Kahneman (1992) provided the estimate  $\beta = 0.88$ .

(perfectionism). The case  $\beta = 2$  is studied in Orphanides and Porter (2000). This loss function implies that larger deviations of inflation from the target have an increasingly greater weight, especially when outside the target zone. For the considerations outlined above, we do not regard this loss function as very plausible. Instead, we argue that psychological insight and evidence, together with the standard institutional setting of monetary policy, determines quite naturally a loss function as in (11) with  $0 \leq \beta < 1$  (diminishing sensitivity), with  $\beta$  is plausibly somewhat greater than, but close to, zero.

In any case, it is important to stress that the analysis in this paper is not limited to the case  $0 < \beta < 1$ , even if there are quite strong behavioural grounds to support it, and a full range of possibilities is considered. At one extreme,  $\beta = 0$  indicates that the central bank cares about *any* shock, independent of the size of the shock.<sup>16</sup> Alternatively,  $\beta > 1$  indicates that losses rise *more than proportionally* with the size of the shock, i.e. the loss function is convex (and it is also differentiable in the whole domain, having no "kink" at zero). As  $\beta \rightarrow \infty$ , the central bank becomes much more concerned with large shocks; in the limit, it only cares about the largest possible shock. This type of central banker can be labelled as "minimax".

In this setting, abstaining from considerations related to the target zone for simplicity and analytical tractability, the central bank's problem may be expressed as the minimization of:

$$E_t L(\pi_{t+1}) = E_t |\varepsilon_{t+1} - \tilde{i}_t|^\beta \quad (12)$$

It is immediate to realize that, in general, the measure of central tendency  $\tilde{i}_t$  that minimizes expression (12) will *differ* from the expected value of  $\varepsilon_{t+1}$  (i.e., zero), as it is the case under quadratic preferences (for it is well known in probability theory that  $\tilde{i}_t = E_t \varepsilon_{t+1} = 0$  minimizes (12) only if  $\beta = 2$ ). In general,  $E_t L(\pi_{t+1})$  will be minimised by the measure of statistical central tendency of order  $\beta$ .

Let us consider, for instance, the case  $\beta = 1$ .<sup>17</sup> The central bank's loss function collapses to the absolute value of the deviations of inflation from its target (zero):

$$E_t L(\pi_{t+1}) = E_t |\varepsilon_{t+1} - \tilde{i}_t| \quad (13)$$

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<sup>16</sup>This is the "perfectionist" central banker referred to above and in Vickers (1998).

<sup>17</sup>Goodhart (2001) describing his own experience at the Bank of England's MPC reports that "I believe that I could, more or less, interpret my loss function when I was at the MPC (*symmetrically linear* in the deviation from target at the six to eight quarter horizon)" [emphasis ours]. This would imply that for Goodhart  $\beta = 1$ .

Under this specification, the optimal value for  $\tilde{i}_t$  is given by  $\tilde{i}_t = M_t \varepsilon_{t+1}$ , where  $M$  represents the *median* value of the probability distribution of  $\varepsilon$ . Hence:

$$i_t = \frac{z_t}{c_3} + \frac{1}{c_3} M_t \varepsilon_{t+1} \quad (14)$$

In general,  $M_t \varepsilon_{t+1}$  will be different from zero, and the optimal policy will deviate from that identified under the quadratic loss function ( $\tilde{i}_t = \frac{z_t}{c_3}$ , or  $\tilde{i}_t = 0$ ). In particular, if the probability distribution of  $\varepsilon_{t+1}$  is positively skewed, the median will be negative (i.e., smaller than the mean) and the optimal policy will imply a *smaller* value of  $i_t$  than it would have been the case under the quadratic loss function (the opposite, i.e. a *higher* value of  $i_t$ , will hold true if the probability distribution of  $\varepsilon_{t+1}$  is negatively skewed). Only if the probability distribution of  $\varepsilon_{t+1}$  is symmetric is the standard result recovered ( $i_t = \frac{z_t}{c_3}$ ). More generally, the value  $\tilde{i}_t$  – interpreted as a measure of central tendency minimizing the loss function in (12) for an arbitrary value of  $\beta$  – will certainly depend on the probability distribution of  $\varepsilon_{t+1}$ , i.e. *the principle of certainty equivalence will not hold (unless  $\beta = 2$ )*. In particular, if  $\beta = 0$ , the interest rate is set to be equal to the *mode* of  $\varepsilon_{t+1}$ .<sup>18</sup> The opposite case is  $\beta \rightarrow \infty$  (i.e., only the largest shock matters), which leads to  $\tilde{i}_t = \varepsilon_{t+1}^*$ , where  $\varepsilon_{t+1}^*$  is the value of  $\varepsilon$  for which  $|\varepsilon_{t+1}|$  is largest. Clearly, this corresponds to a "robust control" or "minimax" solution to optimal monetary policy.

If, however,  $\varepsilon_{t+1}$  is Normally distributed, then all measures of central tendency collapse to the mean (at least for any finite  $\beta$ ), thus to zero. In this case, the principle of certainty equivalence holds and nothing is lost by using the quadratic function.<sup>19</sup>

A second extension which appears to be both interesting and plausible (as discussed above) is a non-linear weighing of probabilities. Now, instead of minimizing  $E_t L(\pi_{t+1})$ , the central bank will aim at minimizing  $E_t^\delta L(\pi_{t+1})$ , with  $E_t^\delta$  defined as follows:

$$E_t^\delta L(\pi_{t+1}) = \int L(\pi_{t+1}) \delta(P(\pi_{t+1})) d\pi_{t+1}, \quad (15)$$

where  $P$  is either the cumulative probability distribution of  $\pi_{t+1}$  or the probability density, and  $\delta$  is a weighing function which satisfies  $0 \leq \delta(P(\pi_{t+1})) \leq 1$ . It should be noted that  $\delta(P)$  is a function of

<sup>18</sup>The emphasis of many central banks on the mode of the probability distribution of future inflation (see, for instance, the fan chart in the Bank of England quarterly Inflation Report) might suggest that a low (near-zero)  $\beta$  lies behind. However, the emphasis on the mode might also simply reflect a presentational advantage (see Wallis, 1999).

<sup>19</sup>This appears to reconcile our results with those of CS, who assumed a Normal distributed additive shock to inflation at time  $t + 1$ .

$P$  and not of  $\pi_{t+1}$ , with the consequence that if  $P(\pi_{t+1}) = P(-\pi_{t+1})$ , then  $\delta(P(\pi_{t+1})) = \delta(P(-\pi_{t+1}))$ . In plain words, the  $\delta$  transformation preserves symmetry (reflection property).

If the  $\delta$  function is the identity function and  $P$  is the probability density, the standard expected utility formulation is recovered. Otherwise, the  $\delta$  function may be any non-linear transformation of either the cumulative probability distribution or of the probability density. The expression in (15) can be thought of as the mathematical expectation of  $L(\pi_{t+1})$  computed according to the *transformed* "probability law"  $\delta(P)$ .<sup>20</sup> In the Kahneman-Tversky literature,  $P$  is normally the cumulative probability distribution (this property is often referred to as "rank dependence") and the function  $\delta(\cdot)$  is normally found to give more weight to "small" probabilities and less weight to "large" probabilities compared with the linear case (see Kahneman and Tversky, 2000). For instance, in Kahneman and Tversky (1992) the following function is postulated:

$$\delta(P) = \frac{P^\omega}{[P^\omega + (1 - P)^\omega]^{\frac{1}{\omega}}}, \quad (16)$$

$\omega > 0$ , which encompasses the linear weighing of expected utility models as a special case when  $\omega = 1$ .<sup>21</sup> If  $0 < \omega < 1$ , this weighing function is first concave and then convex, crossing the linear weighing in a point which is also determined by the value of  $\omega$ . This function is only an example, as many other weighing functions have been proposed in the literature.<sup>22</sup>

In order to deal with one complication at a time and to isolate the effect of a non-linear weighing of probabilities *in itself*, we assume that the period loss function is quadratic, i.e.  $L(\pi_{t+1}) = \pi_{t+1}^2$ . Our central banker is thus called to minimize:

$$E_t^\delta \pi_{t+1}^2 = E_t^\delta (\varepsilon_{t+1} - \tilde{i}_t)^2, \quad (17)$$

which developing the quadratic term in parentheses leads to:

$$E_t^\delta \pi_{t+1}^2 = E_t^\delta (\varepsilon_{t+1}^2 - 2\tilde{i}_t \varepsilon_{t+1} + i_t^2) \quad (18)$$

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<sup>20</sup>The caveat has to be borne in mind that the weighted probabilities do not necessarily add up to one over the domain of the variable. In particular, the weighted probabilities are normally sub-additive (i.e.,  $\int \delta(P(\pi_{t+1})) d\pi_{t+1} < 1$ ). Thus, the weighted probabilities cannot be interpreted *strictu sensu* as a probability distribution, although we adopt this simplification in loose terms for illustrative purposes.

<sup>21</sup>In Kahneman and Tversky (1992) the parameter  $\omega$  is estimated to be close to .6 for gains and to .7 for losses.

<sup>22</sup>Another popular weighing function is the one proposed by Prelec (1998),  $\delta(P) = \exp[-(-\ln P)^\alpha]$ ,  $0 < \alpha < 1$ .

The first order condition is thus:

$$\frac{\partial E_t^\delta \pi_{t+1}^2}{\partial \tilde{i}_t} = 2\tilde{i}_t - 2E_t^\delta \varepsilon_{t+1} = 0, \quad (19)$$

whereby:

$$\tilde{i}_t = E_t^\delta \varepsilon_{t+1} \quad (20)$$

Recalling that  $i_t = \frac{\tilde{i}_t + z_t}{c_3}$ , it follows:

$$i_t = \frac{z_t}{c_3} + \frac{1}{c_3} E_t^\delta \varepsilon_{t+1}, \quad (21)$$

which corresponds to the canonical solution  $i_t = \frac{z_t}{c_3}$  only if  $E_t^\delta \varepsilon_{t+1} = 0$ . If the probability distribution of  $\varepsilon_{t+1}$  is *symmetric*, then  $E_t^\delta \varepsilon_{t+1} = E_t \varepsilon_{t+1} = 0$ , due to the property of symmetry preservation of the  $\delta(P)$  function, and the usual solution  $i_t = \frac{z_t}{c_3}$  (thereby the irrelevance of additive uncertainty) is recovered.<sup>23</sup> In general, however, the non-linear weighing will not be neutral, i.e.  $E_t^\delta \varepsilon_{t+1} \neq E_t \varepsilon_{t+1} = 0$ , hence the probability distribution of  $\varepsilon_{t+1}$  will not be irrelevant, in other words, the principle of certainty equivalence will *not* hold. In particular, the interplay between a non-linear weighing of probabilities and a skewed probability distribution of  $\varepsilon_{t+1}$  determines a departure from the certainty equivalence principle.<sup>24</sup>

As a simple numerical example to illustrate the kind of situation that the analysis in this section refers to, consider a central banker who observes  $z_t = 2$ ,  $c_3 = \frac{1}{2}$  (expressed in percentage points). Assume further that the probability distribution of  $\varepsilon_{t+1}$  is the following:

$\varepsilon_{t+1}$	Prob
-2	.4
-1	.3
1	.1
5	.2

Of course, the assumption is satisfied that  $E_t \varepsilon_{t+1} = 0$ . Whilst this probability distribution is purely hypothetical, it is nevertheless representative at least of the *type* of uncertainty that central banks often

<sup>23</sup>Of course, a Normal distribution is symmetric and certainty equivalence, as in CS, is also recovered.

<sup>24</sup>A very good example of this is in Goodhart (2001): "You should not run a systematically mildly inflationary policy because there is a non-zero risk of a 1929 (or a Japanese) collapse in asset prices". This is clearly a recommendation for non-certainty equivalence behaviour based on a non-linear weighing of probabilities (the small probability of a 1929 collapse in asset prices is neglected).

have to deal with in practice. This probability distribution features high-probability and small-size downside risks (-2 and -1, respectively with probabilities .4 and .3), a low-probability, small-size upside risk (+1, with probability .1) and a low-probability, large-size upside risk (+5, with probability .2). For instance, the event "+5" might be associated with "extreme" circumstances such as, say, the collapse of an exchange rate peg. If our central bank has a quadratic loss function and weighs probabilities linearly, it will pick  $\tilde{i}_t = E_t \varepsilon_{t+1} = 0\%$  (and  $i_t = \frac{z_t + c_3 \tilde{i}_t}{c_3} = 4\%$ ). However, central bankers with a different  $\beta$  will pick other values of  $i_t$ . For instance, Goodhart (2001) – for whom  $\beta = 1$  – will select  $i_t = \frac{z_t}{c_3} + \frac{1}{c_3} M_t \varepsilon_{t+1} = 4\% - 2\% = 2\%$  (the median value of  $\varepsilon_{t+1}$  is, in fact, -1). A "minimax" type of central banker ( $\beta \rightarrow \infty$ ) will select  $i_t = \frac{z_t}{c_3} + \frac{1}{c_3} \varepsilon_{t+1}^*$  (where  $\varepsilon_{t+1}^*$  is the value of  $\varepsilon$  for which  $|\varepsilon_{t+1}|$  is largest); hence  $i_t = 4\% + 10\% = 14\%$ . Finally, a central banker for who  $\beta = 0$  will pick  $i_t = \frac{z_t}{c_3} + \frac{1}{c_3} \text{mod}(\varepsilon_{t+1})$  – where  $\text{mod}(\varepsilon_{t+1})$  is the mode of  $\varepsilon_{t+1}$ , in this case -2 – and then  $i_t = 4\% - 4\% = 0\%$ . To sum up:

Central banker type	Optimal policy
Quadratic ( $\beta = 2$ )	$i_t = 4\%$
Linear ( $\beta = 1$ )	$i_t = 2\%$
Minimax ( $\beta \rightarrow \infty$ )	$i_t = 14\%$
$\beta \rightarrow 0$	$i_t = 0\%$

A similar reasoning (and a similar discrepancy of policy outcomes) would follow by imposing non-linear methods of weighing probabilities. Assume, for instance, that our central banker weighs probabilities following the weighing function in Tversky and Kahnman (1992), as recalled above in (16), with  $\omega = \frac{1}{2}$ . The modified "probability distribution" (again bearing in mind the caveat that it does not necessarily add up to one) is the following:

$\varepsilon_{t+1}$	Prob
-2	.32
-1	.29
1	.20
5	.25

There follows that  $E_t^\delta \varepsilon_{t+1} = 0.52 \neq 0$ . Hence, even under the quadratic loss function  $\tilde{i}_t = 1.04\%$ , and  $i_t = 5.04\% \neq 4\%$ . Another possibility is that the high-payoff, low probability event  $\{5, .2\}$  is completely neglected by the central banker (Goodhart, 2001, mentions this explicitly, as already noted), while all other probabilities are weighted linearly. In that case, it is immediate to find that  $E_t^\delta \varepsilon_{t+1} = -1$ , and thus  $i_t = -2\%$ ,

$i_t = 2\% \neq 4\%$ . Of course, many other realistic nonlinear weighing systems might be conceived.

Clearly, the large variation in the policy outcomes is the result of the particular set up of the example, but it is certainly not far-fetched – as already argued above – to think of a real-world probability distribution of future inflation resembling, at least *qualitatively*, to our hypothetical distribution. When the probability distribution of future inflation is very concentrated around the target and/or approximately Normal, however, the importance of the specification of the central bank loss function for actual policy outcomes becomes quite limited. One real world example might be drawn using the probability distribution for future inflation underlying the Bank of England’s fan chart. The Bank of England, in particular, releases some moments of the distribution of the inflation forecast two years ahead, which is supposed to drive the MPC’s policy decisions. For instance, in the November 2001 Inflation Report it is indicated that the mean of the probability distribution of RPIX inflation (based on the assumption of unchanged short-term interest rates) is 2.52, the median 2.56, and the mode 2.36. Thus, a policy based on a loss function with  $\beta = 0$  would significantly depart from certainty equivalence (as the mode significantly deviates from the mean), while  $\beta = 1$  (i.e., targeting the median) would essentially replicate the certainty equivalence outcome. It is also interesting to note that the probability of inflation being outside the “loose target zone” of 1% above or below the inflation target of 2.5%<sup>25</sup> is estimated to be asymmetric (the probability of being above 3.5% is 0.13, while the probability of being below 1.5% is 0.09). One might surmise that the high-payoff upward risk is related to a possible sharp depreciation of the sterling, a classic high payoff / low probability event which is likely to be weighted non-linearly by the central banker, as we have argued above (and as it is claimed in Goodhart, 2001).

More in general, the probability distributions underlying the Bank of England’s fan chart are an interesting tool from the point of view of the analysis of this paper. With the complete probability distribution of future inflation available, it would certainly be possible to minimize even a non-differentiable loss function as the one proposed in (11) using numerical techniques. At the same time, the probability distribution underlying the fan chart is generally quite concentrated around 2.5% and approximately Normal, which makes the selection and analysis of the central bank loss function not very relevant from a practical perspective. However, it should be also noted that, first, the probability distribution

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<sup>25</sup>See Vickers (1998) on the role of the 1% deviation from 2.5% in the Bank of England’s mandate.

of future inflation may not be so well-behaved in all countries, especially in those where inflation is on average still high, and, second, there may be "spurious" technical reasons which explain why the probability distribution is approximately Normal (for instance, because it is in part derived from econometric methods based on the assumption of Normality), while the MPC members' "true" subjective probability distribution of future inflation may be non-Normal. It is worth stressing here that, if certainty equivalence does not hold, estimation and policy cannot be separated, and the econometric technique used for forecasting should be optimally tailored to the user's loss function (see Granger, 1999). It is only in the particular case that certainty equivalence holds and the loss function is quadratic that forecasting methods based on standard inference are optimal. Thus, if the central banker has a non-quadratic loss function, he should invest in a forecasting technique tailored to his loss function, rather than relying on a (for his preferences) sub-optimal forecasting model based on standard inference.

In synthesis, this paper has found that (i) a non-quadratic curvature of the loss function can determine a departure from the principle of certainty equivalence unless the probability distribution of  $\varepsilon_{t+1}$  is *Normal*, and (ii) a non-linear weighing of probabilities (alone) will bring about the same result if the probability distribution of  $\varepsilon_{t+1}$  is not *symmetric*. Therefore, additive uncertainty matters. It should be added that none of these departures from the principle of certainty equivalence suggests the optimality of policy gradualism: the optimal response to  $z_t$  (the "state of the economy" at time  $t$ ) is independent of the probability distribution of  $\varepsilon_{t+1}$ , for  $\frac{\partial i_t}{\partial z_t} = \frac{1}{c_3}$  in all the considered cases.

## 4 Conclusions

This paper has investigated the robustness of the principle of certainty equivalence, i.e. the irrelevance of the uncertainty over additive shocks, to simple and behaviorally plausible departures of the central bank's loss function from the standard time separable expected quadratic loss. The analysis essentially follows up a paper by Chadha and Schellekens (1999). Compared to that study, this paper relaxes a key assumption, namely that the additive shock to inflation is Normal distributed. This seems to be quite an important innovation because non-Normal probability distributions of future inflation (e.g., skewed risks as stressed by Goodhart, 2001) are arguably the "daily bread" of central bankers.

The analysis in this paper has considered a main departure from the expected time separable quadratic loss, which is a main analytical tool of the literature on optimal monetary policy. Building on a substantial body of literature on economic psychology mainly linked to the names

of Daniel Kahneman and Amos Tversky, this paper has considered the possibility that the central bank loss function displays diminishing sensitivity to losses (namely, be non-convex over its argument) and that the central bank weighs probabilities in a non-linear manner (thus departing from the expected utility paradigm). This analysis appears to be interesting and to make sense, because these tendencies are firmly grounded in the economic psychology literature and have been tested and confirmed in a large number of experimental studies (Kahneman and Tversky, 2000). Therefore, it appears *prima facie* reasonable to assume that central bankers – who are humans after all – may also display such attitudes. From this analysis it is found that this form of non-quadratic loss functions generally brings about a departure from the principle of certainty equivalence, unless the probability distribution of the additive shocks to inflation is *Normal* (as far as the property of diminishing sensitivity is concerned, and in general if the curvature of the function is not of the quadratic type) or *symmetric* (as far as a non-linear weighing of probabilities is concerned). Overall, thus, the assumption of a quadratic loss function does not seem to be innocuous when considering the effect of non-Normal distributed additive shocks to inflation onto the determination of the optimal policy.

In sum, this paper has shown that with additive uncertainty of a non-Normal type, the assumption of quadratic loss functions may not be completely innocuous. Thus, this paper tends to limit the generality of the conclusions of Chadha and Schellekens, who favored the idea that the assumption of a quadratic loss function is not so restrictive as far as the role of additive uncertainty is concerned.<sup>26</sup> This result should caution against an a-critical reliance on simple policy rules, such as the Taylor rule, in which certainty equivalence holds and the uncertainty on the state of the economy does not play any role in the optimal setting of monetary policy instruments.

In addition, this paper provides an analytical framework to study the role of the higher moments of (non-Normal) probability distribution of the additive shock to inflation, for example its skewness, for optimal policy-making. This seems to be a very interesting line of research because policy-makers are continuously confronted with this type of uncertainty.<sup>27</sup> At the same time, this paper does not shed any light

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<sup>26</sup>The difference in results between this paper and CS is entirely explained by the fact that here the assumption of Normality of the probability distribution of the additive shock to inflation is relaxed.

<sup>27</sup>Goodhart (2000) again reports that "unlike uncertainty and variance, skew and risk mapped directly into the interest rate decision". We could not agree more with this statement.

on explaining the alleged positive correlation between policy gradualism and uncertainty over the state of the economy in actual policy-making. It seems that imposing some form of multiplicative uncertainty is necessary to explain central banks' tendency to act gradually and to react cautiously to new information.

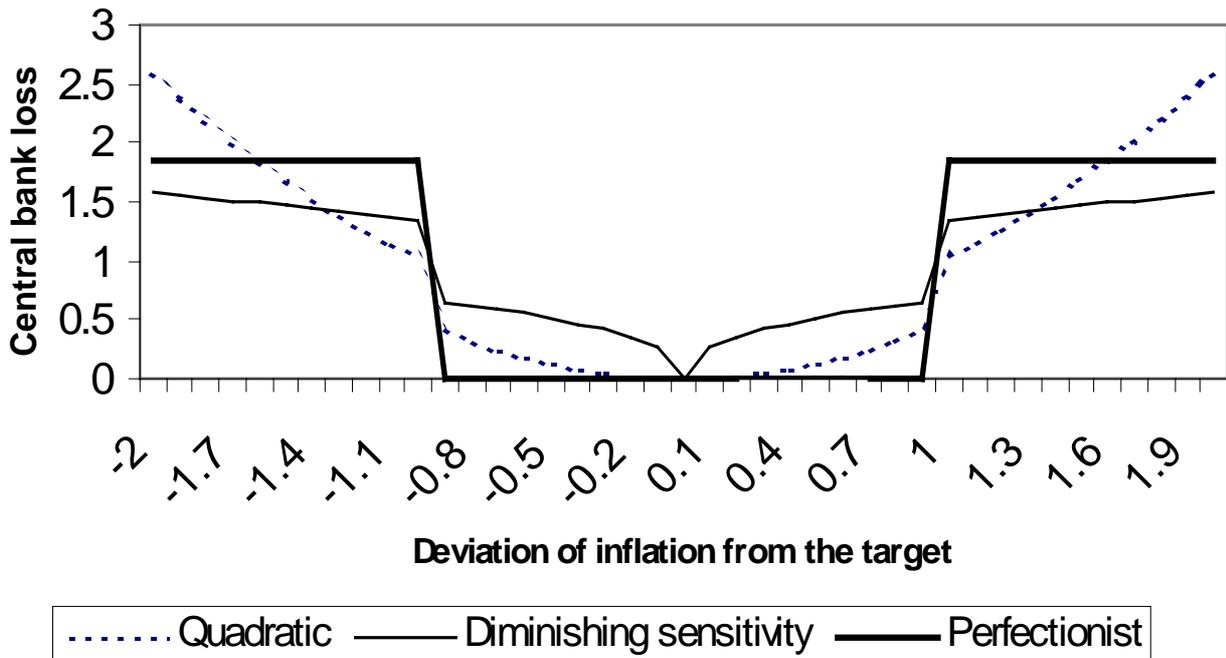
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**Chart 1: Central bank loss functions with a 1% target zone around the central target**



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