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## Duration, volume and volatility impact of trades

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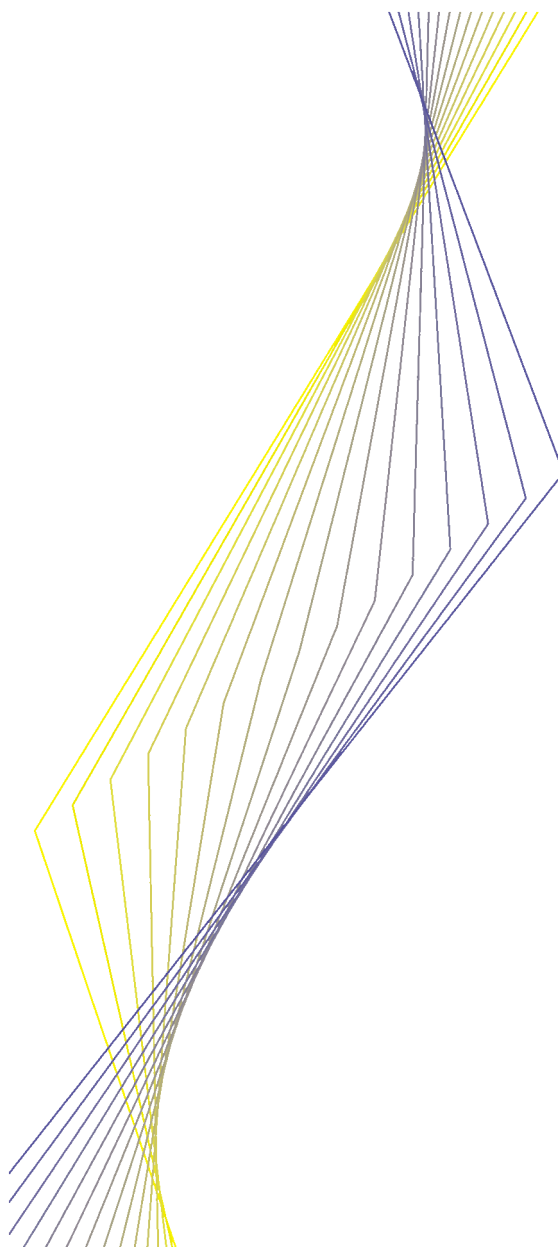
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## **Abstract**

This paper develops a new econometric framework to model duration, volume and volatility simultaneously. We obtain an econometric reduced form that incorporates causal and feedback effects among these variables. We construct impulse-response functions that show how the system reacts to a perturbation of its long-run equilibrium. The methodology is applied to two groups of stocks from NYSE, classified according to their trade intensity. We document how the two groups of stocks are characterised by different dynamics: 1) volume is more persistent for frequently traded stocks than for the infrequently traded ones; 2) the well-known positive relationship between volume and price variability holds only for the frequently traded stocks at the ultra high frequency level; 3) the trade arrival process can be considered exogenous only for the not frequently traded stocks; 4) the more frequently traded the stock, the faster the market returns to its full information equilibrium after a perturbation.

**JEL Classification Codes:** C32, G14

**Key words:** Autoregressive Conditional Duration, GARCH, Ultra High Frequency Data, Empirical Market Microstructure

## **Non-technical summary**

How does the presence of insider traders affect the functioning of financial markets? How many trades and how many hours does it take for private information to be incorporated into prices? In which markets information is revealed faster? This paper presents an econometric framework that can be used to address questions of this kind. Each financial transaction can be viewed as the realisation of a stochastic process, whose variables of interest are the timing between trades (duration), prices and volumes. We introduce the VAR (Vector AutoRegression) modelling idea into a volatility framework, so that it becomes possible to elaborate a complete system, where returns and volatilities directly interact with duration and volume. We allow expected duration, expected volume and variance of returns to depend on current and lagged values of the variables under study. In this way, we can build a system that incorporates causal and feedback effects among these variables. We also construct impulse-response functions that show how the system reacts to a perturbation of its long-run equilibrium.

The econometric methodology proposed in this paper can be used to study the characteristics and long-run equilibrium properties of different markets. The model is applied to a sample of ten stocks from New York Stock Exchange, which covers the period from January 1998 to June 1999. The stocks are divided into two groups according to the intensity with which they are traded. In the empirical analysis, we find strong evidence that the dynamics of frequently traded stocks differ significantly from those of the infrequently traded ones. We find that volume is a high persistent process. This confirms the common intuition that volume (as well as duration and volatility) might be driven by an unobserved factor such as “private information intensity”. Our results show also that the well-known positive relationship between volume and volatility holds only for the frequently traded stocks. Finally, the analysis of the system with the impulse-response functions indicates that the more traded the stock the faster the market returns to its full information equilibrium after an initial perturbation. This suggests that not frequently traded stocks might be characterised by a different mechanism of information transmission with respect to the more frequently traded stocks. In particular, this is consistent with the (plausible) assumption that the more frequently traded the stock the higher the number of insider traders.

## 1. Introduction

The main goal of both theoretical and empirical market microstructure literatures is to explain how prices are formed in the economy. A critical factor to accomplish this task is to understand how traders behave, how information is disclosed and how that affects volume and volatility. Many researchers have developed theoretical models that try to explain how the arrival of news and the interactions among informed and uninformed traders can affect the prices of the stocks, the spreads posted by the market makers, the transaction rate and the volumes exchanged in each trade.

Each transaction can be viewed as the realisation of a stochastic process, whose variables of interest are, say, the timing between trades (duration), prices and volumes. Hasbrouck (1991) and Dufour and Engle (2000) use a vector autoregressive system to model the interactions among the variables of interest. Engle and Russell (1998) introduce the Autoregressive Conditional Duration model to study directly the duration between trades. Engle (2000) incorporates this approach into a volatility framework. Zhang, Russell and Tsay (2001) improve upon the original Engle and Russell's model by allowing the expected duration to depend nonlinearly on past information variables. None of these approaches, however, models the interactions among trade intensity, volume and price variability. More recently, Grammig and Wellner (2002) suggest a model for the interdependence of infra-day volatility and trade duration processes, whose unknown parameters are estimated via GMM. A drawback of this approach is that it assumes that returns follow a weak GARCH model (see Drost and Nijman (1993)). Since in a weak GARCH setting only linear projections (and not conditional expectations) of the squared innovation process are considered, the results cannot be strictly interpreted as estimates of conditional variances.

This paper introduces the VAR idea of Hasbrouck (1991) into a volatility framework, so that it becomes possible to elaborate a complete system, where returns and volatilities directly interact with duration and volume.<sup>1</sup> This is accomplished in two steps. First, the modelling idea behind the Autoregressive Conditional Duration by Engle and Russell (1998) is extended to model volumes. We treat volumes as a stochastic process and model it using an autoregressive specification that multiplies an i.i.d.

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<sup>1</sup> Since we are dealing with ultra high frequency data, throughout the paper we use the term volume as a synonym of trade order size.



error term. Since the volume can take only positive values, the support of the distribution of the error terms is the positive real line.

Next, duration, volume and returns are modelled simultaneously, using a special type of vector autoregression. We allow expected duration, expected volume and variance of returns to depend on current and lagged values of the variables under study. In this way, we can build a system that incorporates causal and feedback effects among these variables. We also construct impulse-response functions that show how the system reacts to a perturbation of its long-run equilibrium. The impulse-response function analysis can be helpful to understand market “resiliency”, i.e. the speed with which prices tend to converge to the underlying liquidation value of the asset.

The econometric methodology proposed in this paper can be applied to study the characteristics and long-run equilibrium properties of different markets. The model is applied to a sample of ten stocks from NYSE, which covers the period from January 1998 to June 1999. The stocks are divided into two groups according to the intensity with which they are traded. In the empirical analysis, we find strong evidence that the dynamics of frequently traded stocks differ significantly from those of the infrequently traded ones.

There are four findings in particular. First, we find that volume is a high persistent process. That is, the empirical regularities found for duration and volatility models are confirmed also for high frequency volumes. The strong persistence of these three variables is consistent with the common intuition that all three variables might be driven by an unobserved factor such as “information intensity” or “private information intensity” (Clark (1973), Easley and O’Hara (1992)). The persistence of the three variables is substantially weaker for the not frequently traded stocks. The results are particularly striking for volumes.

Second, our results show that the well-known positive relationship between volume and volatility (Karpoff (1987)) holds only for the frequently traded stocks. In particular, we find that this positive relationship holds also at the ultra high frequency level, whereas the previous findings were limited to daily averages. In the case of the not frequently traded stocks, instead, no strong relationship is found between volume and variance.

A third interesting finding is that the exogeneity of the trade arrival process imposed by some theoretical models (Glosten and Milgrom (1985), Easley and O’Hara (1987)) may be a reasonable assumption only when dealing with not frequently traded stocks. Indeed, we find that for the frequently

traded stocks both lagged volumes and squared returns affect the expected duration of the next trade, with a very significant negative coefficient.

Finally, the analysis of the system with the impulse-response functions indicates that the more traded the stock the faster the market returns to its full information equilibrium after an initial perturbation. This suggests that not frequently traded stocks might be characterised by a different mechanism of information transmission with respect to the more frequently traded stocks. In particular, this is consistent with the (plausible) assumption that the more frequently traded the stock the higher the number of informed traders. For example, Holden and Subrahmanyam (1992) show that with multiple informed traders there will be more aggressive trading in the early periods, causing more information to be revealed earlier. Our results are also consistent with those provided by Zhang, Russell and Tsay (2001), who find strong evidence that fast and slow transacting periods have quite different dynamics, although their empirical analysis is limited to the IBM stock.

The paper is structured as follows. In the next section we describe the theoretical models of market microstructure relevant for the analysis. In section 3 we introduce our econometric model. Section 4 presents the empirical application. Section 5 concludes and suggests new directions for future research.

## 2. Economic Theories

Trading in financial markets occurs either for information or for liquidity reasons. Informed agents are motivated by relevant private information about the real value of the asset not known to others, while the needs of liquidity traders are generally modelled as exogenous. Consequently, most theoretical research that explores the relation among trades, volume and stock price dynamics has concentrated on asymmetric information models. The basic framework was set up by Bagehot (1971) and assumes that in the market there exist traders with superior information who try to transform their informational advantage into net profits. A common feature of this type of models is that new information is incorporated into prices through the interaction of informed and uninformed traders. The underlying assumption is that prices are only semi-strong form efficient, that is they reflect all public information, but not the private one. If this private information is valuable, then informed traders can make positive profits. Many possible extensions of this basic idea have been considered. See, for example, Copeland and Galai (1983), Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987 and 1992), Diamond and Verrecchia (1987), Admati and Pfleiderer (1988). See O'Hara (1995) for a comprehensive survey of the argument.

There are at least three possible ways through which market makers and uninformed traders can learn from observing market information: from prices, volumes and times between trades. Market microstructure researchers have devoted considerable efforts in explaining why and how these variables can convey information.

The early microstructure models (Copeland and Galai (1983) and Glosten and Milgrom (1985)) do not recognise any explicit role for volume, as all trades involve only one unit of the asset. Nevertheless, empirical research on the price-volume relation has shown that there is a positive correlation between the absolute values of daily price changes and daily volumes of stocks (see Karpoff (1987)). The explanations of this phenomenon revolve around the idea of the random arrival of new pieces of information, as in the mixture of distribution hypothesis (MDH) developed by Clark (1973)<sup>2</sup>. MDH assumes that the same underlying latent event (the information arrival) drives both the returns and volume processes. The intuition here is that volatilities are changing over time because information is available to traders at a varying rate. On days when not much information is available, trading is slow and there are only few price

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<sup>2</sup> See also Epps and Epps (1976), Tauchen and Pitts (1983), Andersen (1996) and Liesenfeld (2001).

changes. On the contrary, when new unexpected information hits the markets, trades become more frequent and prices move much faster.

On a more theoretical side, Easley and O'Hara (1987) were the first to acknowledge the role of trade size, by introducing two key differences with respect to previous models: 1) traders are allowed to transact different trade sizes and 2) the existence of new information is not assumed. Their basic intuition is that volume influences price because it is correlated with the private information about the stock's true value. Since informed traders wish to trade larger quantities when they have valuable information, a rational market maker will interpret large orders as evidence of trading by informed agents and will adjust beliefs and prices accordingly. Blume, Easley and O'Hara (1994) consider a model in which two groups of traders receive informative signal of different quality in each period. It is the fact that both the level and the quality of these signals are unknown that makes volume informative. The authors show that, given the price, traders can use volume to make inferences about the quality of the signal and hence the real value of the asset. This suggests that the role of volume in the price adjustment process is to give information about the underlying uncertainty and hence that volume is correlated to the quality and quantity of traders' information. Admati and Pfleiderer (1988) offer an alternative explanation for the volume-price variability relation. They suggest that frequent trading signals that the market is very liquid. Indeed, the most active liquidity traders are likely to be large financial institutions that trade for liquidity needs of their clients or to rebalance their portfolios. The idea of Admati and Pfleiderer (1988) is that liquidity traders can choose when to trade and that they will prefer to trade when the market is very liquid, in order to minimise transaction costs. For the same reasons, informed traders also want to trade when the market is thick. The main result of the paper is that if information acquisition is endogenous, then in equilibrium more people become privately informed in periods when liquidity trading is high, and as a consequence prices become more informative in these periods.

The third factor that can convey information about the true asset's value is the time between trades. The motivation of this line of research is that if time can be correlated with any factor that affects asset prices (such as the arrival of new information), then the timing among trades may be informative to market participants. Indeed, if we think of the series of transaction prices as being produced by a sampling of the underlying true stochastic price process, then it might be reasonable to assume that more samples are observed when there is new information, since informed traders are more likely to trade than uninformed

ones. On the other hand, another plausible explanation could be that a higher rate of trade simply coincides with a concentration of liquidity traders and might not be associated to any news arrival.

Diamond and Verrecchia (1987) use a rational expectation model to study the effects of constraints on short sales on the distribution and speed of adjustment to private information of stock prices. The empirically testable conclusion of the model is that periods of absence of trade need to be interpreted as the symptom that bad news have arrived, since they indicate an increased chance of informed traders with bad news who are constrained from selling short. Hence, absence of trade should be positively correlated with price volatility.

Easley and O'Hara (1992) develop a framework in which time by itself can affect prices, in the sense that both periods of trading and non-trading can cause price changes. The intuition is that the lack of trade can be associated to the event that no new information exists and that prices efficiently incorporate all the available information. More precisely, after a period in which no trade occurred, the market maker updates her beliefs, raising the probability she attaches to no information event and to the prior expected value of the asset. Hence, periods of low variance tend to occur in periods where there is little trade.

We now turn our attention to the econometric models that might be used to evaluate these predictions.

### **3. Econometric Models**

When studying market microstructure, a major problem faced by the econometrician is that transaction data arrive at irregular times. The practice was to ignore the time occurring between two trades and analyse the data with fixed time interval econometric techniques, in many cases by taking averages of the values of interest over a given, arbitrary interval.<sup>3</sup> However, as many theoretical works have shown, there might be a significant loss of information by adopting this modelling strategy, since the time elapsing between two trades may be informative about the behaviour of the traders.

The Autoregressive Conditional Duration (ACD) model by Engle and Russell (1998) provides a possible solution to this problem. Engle and Russell model the arrival times of trades as random variables

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<sup>3</sup> See, for example, Hartmann (1999), Fleming and Remolona (1999), Ederington and Lee (1993), Jones, Kaul and Lipson (1994).

that follow a point process. Associated with each arrival time there are random variables called marks, such as volume, spread and price. It is often the case that the variables of interest are those in the marks, for example the price and the volatility of the asset under study. If this is the case, then the attention and the efforts of the researcher will be concentrated, for example, on some sort of volatility model. ACD models will be necessary to formulate the model correctly, perhaps to construct explanatory variables that might enter the specification of the variance process and to compute impulse-response functions in calendar time (as in Dufour and Engle (2000)). In particular, this modelling strategy provides a convenient way to test some of the hypothesis from the market microstructure theories (for example, if shorter durations are due to the arrival of unexpected news and increase the volatility of the asset).

The objective of this paper is to push further the modelling idea behind the ACD, extending it to model components of the mark other than prices and to construct a general framework to study the interaction among them.

Let  $d_t = z_t - z_{t-1}$  be the duration between two consecutive trades. At the  $t^{\text{th}}$  transaction, the  $t^{\text{th}}$  mark  $x_t$  is realised. Typically,  $x_t$  will be a vector containing the price and the volume of the transaction, plus the bid ask spread posted by the market maker. These data can be viewed as the sample realisation from a hypothetical stochastic process. We assume that the true DGP generating each pair  $(d_t, x_t)$  can be written as follows:

$$(1) \quad (d_t, x_t) \sim f(d_t, x_t \mid \Omega_t; \theta)$$

where  $\Omega_t$  denotes the information available at time  $t$  and  $\theta$  is a (finite) vector of parameters of interest.

Engle (2000) points out that this joint density can be rewritten as the product of the marginal density of  $d$  times the conditional density of  $x$  given  $d$ . Assuming, throughout the paper, that the marks of interest are just returns ( $y$ ) and volume ( $v$ ), we can write the conditional density of  $x$  given  $d$  as the marginal conditional density of  $v$  given  $d$  times the density of  $y$  conditional on  $d$  and  $v$ :

$$(2) \quad \begin{aligned} (d_t, v_t, y_t) &\sim f(d_t, v_t, y_t \mid \Omega_t; \theta) \\ &= g(d_t \mid \Omega_t; \theta_1) h(v_t \mid d_t, \Omega_t; \theta_2) k(y_t \mid d_t, v_t, \Omega_t; \theta_3) \end{aligned}$$

Although other parameterisations are possible, the one proposed here seems natural, given the widespread use of strategic models in the market microstructure literature. In Kyle's (1985) model, for example, insider traders act strategically, i.e. they take into account the effect their orders have on price, by conditioning on the behaviour of both the market maker and uninformed traders. At the moment in which new private information becomes available, informed traders face a given supply/demand schedule from the market maker. They will thus choose the amount of shares to trade in order to maximise their informational advantage, discounting the market maker reaction to their trade. Since the timing of the trade as well can convey information to the market maker (see Easley and O'Hara (1992)), informed traders will take also this aspect into account, when determining the timing of their trades. This implies that both duration and volume will determine the price at time  $t$ . A similar argument can be used regarding the relationship between duration and volume. When new private information arrives, the informed trader would like to exploit it by trading a larger amount of shares. However, by following this strategy she would immediately reveal her type of informed trader to the market maker, who would then adjust prices accordingly. In order to hide his type, the informed trader will split her buys and sells orders in many trades. It is then this trading intensity that determines the amount of shares to be sold or bought at each trade. These arguments support the parameterisation used in (2), suggesting a plausible causality relation running from duration to volume and from duration and volume to prices.

By modelling each marginal and conditional distribution in (2), it is possible to develop a complete framework for the triple  $(d_t, v_t, y_t)$ . Economic theory (Clark (1973), Easley and O'Hara (1987 and 1992), for example) suggests that clusters of activity should be observed in the market every time some unexpected piece of information arrives, or simply when the market is very liquid. In terms of duration, this translates in periods in which we observe very frequent transactions, with very short durations. The ACD models pick up this aspect.

The typical ACD model can be stated as follows:

$$(3) \quad \begin{aligned} d_t &= \psi_t \varepsilon_t & \varepsilon_t &\sim i.i.d.(1, \sigma_\varepsilon^2) \\ \psi_t &\equiv E(d_t \mid \Omega_t; \theta_d) \end{aligned}$$

where  $\Omega_t$  is, as usual, the information available at time  $t$ , and  $\theta_d$  is a vector of parameters that need to be estimated.

Since  $\psi_t$  is the expectation of  $d_t$ ,  $\varepsilon_t$  must be a random variable with mean 1. Moreover, both  $\psi_t$  and the support of the distribution of  $\varepsilon_t$  must be positive, as duration cannot be negative. Engle and Russell (1998) suggest to model  $\psi_t$  as an autoregressive process. This choice is reasonable, as the autoregression can be interpreted as the statistical way to model the clustering of activity discussed above. Moreover, the long memory characteristic of autoregressive models is consistent with the fact that the whole sequence of past trades might become informative. Pushing along these lines, it seems plausible to model in a similar fashion the volume. A possible model for volume could be the following:<sup>4</sup>

$$(4) \quad \begin{aligned} v_t &= \phi_t \eta_t & \eta_t &\sim i.i.d.(1, \sigma_\eta^2) \\ \phi_t &\equiv E(v_t | \Omega_t; \theta_v) \end{aligned}$$

As in the ACD model,  $\eta_t$  must be an i.i.d. error term with mean 1. Since volume is a variable that assumes only positive values, the distribution of the error term will be defined only for positive values. By imposing different distributional assumptions on this error term and specifying the process followed by the expected volume  $\phi_t$ , we can get a variety of models. Moreover, the quasi-maximum likelihood results of GARCH and ACD models can be directly extended to this model: imposing an exponential distribution for  $\eta_t$ , it is possible to obtain consistent estimates of the parameters  $\theta_v$ , provided that the expected volume  $\phi_t$  is correctly specified.<sup>5</sup>

Given the predictions of economic theories and the known empirical regularities that characterise durations and volatilities, we model expected volume as an autoregressive process. To stress the analogy with duration and volatility models, we call these models Autoregressive Conditional Volume (ACV). The simplest possible specification is an ACV(1,1):

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<sup>4</sup> Here we assume that volume is a stationary process. In practice, however, volume might have a tendency to increase over time. For example, while the average daily trading volume in the broad-based S&P 500 index was around 190 million shares back in 1993, it has since increased steadily to around 1.5 billion shares at present. Given the relatively short sample period used in our empirical analysis, this is unlikely to create any problem. Nevertheless, when longer time series are studied, one might want to take this aspect into consideration when data are cleaned for the analysis.

<sup>5</sup> See Bollerslev and Wooldridge (1992) and Engle and Russell (1998) for details.



$$(5) \quad \phi_t = \omega + \alpha v_{t-1} + \beta \phi_{t-1}$$

Other more general ACV specifications can be introduced. However, instead of pursuing this strategy, we propose a more general approach. Modelling the returns with a GARCH process whose variance is denoted by  $\sigma_t^2$  and specifying a model for each of the marginal and conditional density functions in (2), we can get a complete and general framework for the random variables of interest. More precisely, we assume the following<sup>6</sup>:

$$(6) \quad \begin{aligned} d_t &= \psi_t(\theta_d; \Omega_t) \varepsilon_t & \varepsilon_t &\sim i.i.d. (1, \sigma_\varepsilon^2) \\ v_t &= \phi_t(\theta_v; d_t, \Omega_t) \eta_t & \eta_t &\sim i.i.d. (1, \sigma_\eta^2) \\ y_t &= \sigma_t(\theta_y; d_t, v_t, \Omega_t) \zeta_t & \zeta_t &\sim i.i.d. (0,1) \end{aligned}$$

Note that in this framework the error terms are uncorrelated with each other.

Taken separately, these are, respectively, the ACD, ACV and GARCH models. However, under this framework it is possible to allow interactions among the three variables under study. According to how we specify the functional forms for expected duration, expected volume and variance, we can get many different models and construct an econometric reduced form to evaluate the theoretical predictions from market microstructure. Moreover, the old issue of the relationship between volume and volatility can be addressed directly under this framework.<sup>7</sup>

One possible specification is the following:

$$(7) \quad \begin{aligned} \psi_t &= a_0 + \sum_{i=1}^q (a_{1i} \psi_{t-i} + a_{2i} \phi_{t-i} + a_{3i} \sigma_{t-i}^2) & + \sum_{j=1}^p (a_{4j} d_{t-j} + a_{5j} v_{t-j} + a_{6j} y_{t-j}^2) \\ \phi_t &= b_0 + \sum_{i=1}^q (b_{1i} \psi_{t-i} + b_{2i} \phi_{t-i} + b_{3i} \sigma_{t-i}^2) + b_4 d_t & + \sum_{j=1}^p (b_{5j} d_{t-j} + b_{6j} v_{t-j} + b_{7j} y_{t-j}^2) \\ \sigma_t^2 &= c_0 + \sum_{i=1}^q (c_{1i} \psi_{t-i} + c_{2i} \phi_{t-i} + c_{3i} \sigma_{t-i}^2) + c_4 d_t + c_5 v_t & + \sum_{j=1}^p (c_{6j} d_{t-j} + c_{7j} v_{t-j} + c_{8j} y_{t-j}^2) \end{aligned}$$

<sup>6</sup> Here we also implicitly assume, without loss of generality, that the conditional mean of the returns has been modelled appropriately.

<sup>7</sup> This provides a strong motivation to model volume as in (4). Modelling volume using logs (as done for example by Hartmann (1999)) makes it more difficult to study directly the interaction between expected volume and volatility.

The nice feature of this specification is that it has a vector autoregressive moving average representation. Indeed, rewriting (7) in matrix form, we get, in more compact notation:

$$(8) \quad \mu_t = \gamma + A_1\mu_{t-1} + \dots + A_q\mu_t + B_0\tau_t + B_1\tau_{t-1} + \dots + B_p\tau_{t-p}$$

where  $\mu_t' = (\psi_t, \phi_t, \sigma_t^2)$ ,  $\tau_t' = (d_t, v_t, y_t^2)$ ,  $\gamma$  is a (3,1) vector of coefficients and  $A_1, \dots, A_q, B_0, B_1, \dots, B_p$  are (3,3) matrices of coefficients.

To keep things simple, in this paper we concentrate our attention on a first order model of (8):

$$(9) \quad \mu_t = \gamma + A\mu_{t-1} + B\tau_t + C\tau_{t-1}$$

The parametrisation in (2) imposes that  $B$  is lower diagonal with all the elements on the diagonal equal to zero. Further restrictions can be imposed. For example, by imposing that  $A$  and  $C$  are diagonal and  $B$  is equal to zero, we get back the three independent models ACD, ACV and GARCH. Note that assuming that  $A$  is diagonal is equivalent to assuming that the parameters of this system are variation free as in Engle, Hendry and Richard (1983). Under this assumption, the likelihood in (2) can be rewritten as:

$$(10) \quad f(d_t, v_t, y_t | \Omega_t; \theta) = g(d_t | \Omega_t; \theta_d) h(v_t | d_t, \Omega_t; \theta_v) k(y_t | d_t, v_t, \Omega_t; \theta_y)$$

where  $\theta' = (\theta_d', \theta_v', \theta_y')$ . This makes it possible to estimate the three models separately.

Proposition 1 gives conditions under which the VAR in (8) is covariance stationary. All the proofs are in Appendix A.

**Proposition 1** - *The process in (8) is covariance stationary if and only if  $|\lambda| < 1$  for all values of  $\lambda$  satisfying  $|I_n\lambda^p - H_1\lambda^{p-1} - H_2\lambda^{p-2} \dots - H_p| = 0$ , where  $H_i = (B_0 - I)^{-1}(B_i + A_i)$ ,  $i=1, \dots, p$ .*

The model described by (6) and (9) allows one to compute the effect that an unexpected trade today has on future expected duration, volume and volatility. The following proposition shows how to compute the impulse response function of the system.

**Proposition 2** - *The impulse-response function of the model (6) and (9) for  $t > 0$  is:*

$$\frac{\partial E(\mu_t | \Omega_t)}{\partial \tau_0} \equiv \Phi_t = D^{t-1} (I - B)^{-1} (AB + C)$$

where  $D = (I - B)^{-1} (A + C)$ .

The standard errors for the impulse-response are given by Proposition 3.

**Proposition 3** - *Consider the model formed by (6) and (9). Let  $\theta = [\theta_d', \theta_v', \theta_y']'$  and  $\theta$  is  $(p \times 1)$*

$\kappa_t \equiv \text{vec}(\Phi_t(\theta))$ . If  $\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{a} N(0, Q)$ , then  $\sqrt{T}(\hat{\kappa}_t - \kappa_t) \xrightarrow{a} N(0, G_t Q G_t')$ , where  $G_t = \frac{\partial \kappa_t}{\partial \theta'}$  is  $(9 \times p)$ .

The  $i^{\text{th}}$  column of the matrix of derivatives  $G_t$  can be easily estimated numerically as follows:

$$(11) \quad G_t^i = \frac{\kappa_t(\hat{\theta} + e_i \Delta) - \kappa_t(\hat{\theta})}{\Delta}$$

where  $e_i$  denotes the  $i^{\text{th}}$  column of the  $(p \times p)$  identity matrix and  $\Delta$  is a small number.

#### 4. Empirical Analysis

The econometric models discussed in the previous section were tested on a sample of ten stocks. All stocks are quoted in NYSE and cover the period from January 1, 1998 to June 30, 1999. They were randomly chosen using the following procedure. We constructed ten deciles on the basis of the 1997 total number of trades of all the stocks quoted on NYSE. Then we randomly picked five stocks from the eighth decile and five from the second decile. All the stocks that didn't survive the whole period of study were

discarded and substituted by other randomly chosen stocks from the same decile. The tickers and names of the ten stocks are reported in Appendix B.

To prepare the data for the analysis, first we dropped any transaction that occurred before the opening time, 9:30 am, and after the closing time, 16:00. Then, we computed the durations between trades treating the overnight period as if it didn't exist, so that, for example, the time elapsing between 15:59:30 and 9:30:10 of the following day is only 40 seconds. We adopted this strategy, because in our sample we have stocks that are very rarely traded. Eliminating the durations for the overnight period would have caused the loss of important data for these stocks. Third we computed the price of each transaction as the average of the bid and ask quotes that appear at least five seconds before the transaction itself. This procedure is standard in market microstructure studies. Taking the average of the bid and ask quotes limits the bid-ask bounce problems, while considering only 5 seconds old quotes was originally proposed by Lee and Ready (1991) and its purpose is to correct reporting errors in the sequence of trades and quotes.<sup>8</sup> Fourth, we eliminated all the transaction data with zero duration. We treated these transactions as one single transaction, summing up all the volumes. Fifth, we adjusted the data for stock splits, by multiplying prices and volumes by the stock split ratio. Sixth, the returns were computed as the difference of the log of the prices. Seventh, we adjusted the data for dividend payments and trading halts, by simply deleting the first observation whose return incorporated the dividend payment or the trading halt.

It is a well-known fact that both durations and volatilities exhibit a typical daily pattern over the course of the trading day, with very high trading activity at the beginning and at the end of the day. In order to remove this feature from the data, the time series of durations, prices and volumes were diurnally adjusted as in Engle (2000). We regressed durations, volumes and absolute values of returns on a piecewise linear spline with knots at 9:30, 10:00, 11:00, 12:00, 13:00, 14:00, 15:00, 15:30 and 16:00 and constructed the adjusted series by dividing each original series by the spline forecast. The two extra knots in the first and last half-hour reflect the typical different trading intensity during the day.

Throughout the empirical section, we concentrate our attention on volume and variance per transaction. Even if volatility per unit of time is the natural and commonly used measure of volatility, in

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<sup>8</sup> In the following empirical analysis, all the variables of interest are measured whenever a transaction occurs. However, in the real world, there is a difference between a transaction and a print. Consider, for example, a 1000 share market order for which the specialist takes 600 shares and the rest is passed onto the limit order book. This will show up in the TAQ data set as two prints, but in fact it was just one transaction with three counterparties. This is a limitation of the data set that is hard to overcome. The only way to get around this problem is use order data, which are rarely available.

market microstructure we observe only transaction data. Transforming these data into per unit data would bias the analysis. Indeed, some of the theories predict that the absence of trade must be interpreted as if bad news has arrived, and hence trades with longer durations should have a higher price impact, measured in transaction (not calendar) time. In this case, if we divide returns by the square root of the durations (in order to get volatility per second), the price impact of the transaction will be considerably reduced, since long durations will almost always be associated with low returns per second.

In table 1 we report some summary statistics for the ten stocks used in the analysis. For each stock we report the number of observations, and average and median of duration and volume. The sharp difference between the number of observations for the two groups of stocks reflects the criterion used to select them. It is interesting to note that this difference is reflected in the average and median duration of the trades, but not in the volumes, whose average magnitude seems comparable across the two groups of stocks.

First we estimated the three independent ACD, ACV and GARCH models. In terms of the general model described in the previous section, we set the matrix  $B$  equal to zero and impose the matrices  $A$  and  $C$  to be diagonal. In table 2 we report the estimated autoregressive coefficients (the diagonal elements of  $A$ ) of the three models. All these coefficients are highly significant. We omit to report the t-statistics in order to present a cleaner picture of the results.

This exercise is novel in two aspects. First the ACV model is estimated and second these models are estimated with two groups of stocks, frequently and not frequently traded stocks. Volume appears to be a very persistent process for the frequently traded stocks, as indicated by the autoregressive coefficients, which is always above 0.9, and above 0.95 for four out of five stocks. This finding confirms that the empirical regularities found for duration and volatility models hold for volume as well. It is also consistent with the predictions of market microstructure theories, according to which markets should be very active every time an unexpected piece of information arrives or when there is a clustering of liquidity traders.<sup>9</sup> The persistence, however, is significantly lower for the five less frequently traded stocks. Here the autoregressive coefficient drops to 0.7-0.8, and down to 0.3 in one case. This is a first piece of evidence that the dynamics of frequently traded stocks differ significantly from those of the infrequently traded

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<sup>9</sup> Although the persistence of volume at the ultra high frequency level is a new finding in the empirical literature of market microstructure, similar evidence was found by Hartmann (1999) who shows how the log of daily foreign exchange market trading volumes displays conditional heteroscedasticity.

stocks. Looking also at the ACD and GARCH models, we notice an analogous drop in the level of the autoregressive coefficients for the not frequently traded stocks, but much less dramatic.

With only this framework at hand, it is not possible to distinguish among different market microstructure theories. For example, it is not possible to determine whether an increase in market activity increases the trade variance of the stock (and hence is consistent with the arrival of new information story) or simply to liquidity reasons (with no effect on variance per trade). However, by estimating models that allow interactions among the three variables under study, we can address this issue. In the estimation, we imposed the assumption of weak exogeneity (that is we assume that the matrix  $A$  in (9) is diagonal), so that the optimisation could be done separately for each variable, and we estimated the following model:

$$\begin{aligned}
 \psi_t &= a_0 + a_1\psi_{t-1} + a_2d_{t-1} + a_3v_{t-1} + a_4\mu_{t-1}^2 \\
 (12) \quad \phi_t &= b_0 + b_1\phi_{t-1} + b_2v_{t-1} + b_3d_t + b_4d_{t-1} + b_5\mu_{t-1}^2 \\
 \left| \begin{array}{l} y_t = X_t' \beta + u_t \quad u_t = \rho u_{t-1} + \mu_t + \theta \mu_{t-1} \\ \sigma_t^2 = c_0 + c_1\sigma_{t-1}^2 + c_2\mu_{t-1}^2 + c_3d_t + c_4d_{t-1} + c_5v_t + c_6v_{t-1} \end{array} \right.
 \end{aligned}$$

In (12) we model the mean of  $y_t$  as an ARMA(1,1). The vector  $X_t$  contains current and lagged values of  $d_t$  and  $v_t$ . We don't report the results for the mean because our interest lies principally in the second moment dynamics.

In table 3 we report the Ljung-Box statistics with 15 lags for raw and fitted data for duration, volume and returns. The statistics for the raw data are overwhelmingly significant, indicating the existence of strong autocorrelation in the variables. The analysis of the LB statistics for the residuals of the fitted models reveals that these models succeed in capturing most of the autocorrelation, as indicated by the sharp drop in value of the statistic. In some cases, the Ljung-Box statistic is still significant, but this is common with such large time series (see, for example, Engle (2000)).

To evaluate the effect that duration and volume have on prices, we need to look at their lagged values, because we are interested in the reaction of the market maker to the last trade. For this purpose, we need to take into account the effect that lagged duration and volume have on lagged expected volume and variance per trade. Rewriting (12) in matrix form as in (9), we get:

$$\begin{aligned}
(13) \quad \mu_t &= \gamma + A\mu_{t-1} + B\tau_t + C\tau_{t-1} \\
&= \sum_{k=0}^{t-1} A^k \gamma + B\tau_t + \sum_{k=1}^{t-1} A^{k-1} (AB + C)\tau_{t-k} + A^{t-1} C\tau_0 + A^t \mu_0
\end{aligned}$$

The matrix of coefficients of  $\tau_{t-1}$  is  $(AB+C)$  and the null hypothesis that these coefficients are equal to 0 can be easily tested as shown in Proposition 3. The results are reported in table 4.

Let's start by looking at the variance equations. The coefficient of lagged duration in the variance equation is negative for 8 out of 10 stocks, but significant only in three cases. The sign of the duration coefficient is consistent with the results typically found in the empirical microstructure literature (see, for example, Engle (2000) and Dufour and Engle (2000)) and suggests that times of greater activity coincide with a higher fraction of informed traders present in the market. The existing results, however, are limited to very liquid blue chips stocks. Our findings provide evidence against the robustness of such results when the analysis is extended to less frequently traded stocks.

The volume coefficient is always positive, but strongly significant only for the frequently traded stocks. This supports some of the predictions of Easley and O'Hara (1987, 1992), for example suggesting that trades with larger sizes are more likely to be executed by informed traders and thus have a greater impact on the price of the stock.

A very different picture emerges from the results for the not frequently traded stocks. In this case neither duration nor volume affect the variance. Thus the relationships among trading intensity, trade sizes and volatilities predicted by many market microstructure theories do not find confirmation for the less frequently traded stocks.

A second striking difference between the two groups of stocks is found in the duration equation. For the frequently traded stocks, both lagged volumes and squared returns affect the expected duration of the next trade. The signs of the two variables are always negative, implying that more frequent trading follows big price movements and high volumes. These results confirm those of Dufour and Engle (2000), who found that "short durations and thus high trading follow large returns and large trades". However, the coefficients for the not frequently traded stocks are almost never significant. This suggests that the exogeneity of the trade arrival process imposed by Dufour and Engle (2000) and by some theoretical models may be a reasonable assumption only when dealing with not frequently traded stocks.

The weak exogeneity assumption imposed in (12) could be restrictive, although this strategy was adopted in a similar context by most empirical microstructure papers. In table 5, we test this restriction as suggested by Dufour and Engle (2000). We regress the standardised residuals of the three models against lagged expected variables. More precisely, we run the following three regressions:

$$\begin{aligned}\hat{\varepsilon}_t &= \alpha_0 + \alpha_1 \hat{\phi}_{t-1} + \alpha_2 \hat{\sigma}_{t-1}^2 \\ \hat{\eta}_t &= \beta_0 + \beta_1 \hat{\psi}_{t-1} + \beta_2 \hat{\sigma}_{t-1}^2 \\ \frac{\hat{\mu}_t}{\hat{\sigma}_t^2} &= \gamma_0 + \gamma_1 \hat{\psi}_{t-1} + \gamma_2 \hat{\phi}_{t-1}\end{aligned}$$

If the model was correctly specified, then the standardised residuals should be i.i.d, and the estimated coefficients of the above regressions should be not significantly different from zero. On the other hand, if the expected variables belong to the model, than we should find significant correlation between the estimated residuals and the omitted variables. White (1980) type standard errors are reported in parenthesis. The results show that the coefficients of the lagged expected variables are almost always not significantly different from zero, especially for the not frequently traded stocks. Only the test for the variance specification shows evidence of misspecification for the frequently traded stocks, suggesting that lagged expected volume should also be taken into consideration.

We can use the model developed in this paper also to evaluate the effect that trades have on the forecasts of expected duration, expected volume and variance per trade. To forecast variance and expected volume of the next trade, we need to substitute in the system the variables realised at time  $t$  with their expectation. Taking the conditional expectation in (13) and solving for the conditional expectation, we get:

$$(14) \quad \begin{aligned}E(\mu_t | \Omega_t) &= (I - B)^{-1} (I - A)^{-1} \gamma + (I - B)^{-1} \sum_{k=1}^{t-1} A^{k-1} (C + AB) \tau_{t-k} \\ &\quad + (I - B)^{-1} A^{t-1} C \tau_0 + (I - B)^{-1} A^t \mu_0\end{aligned}$$

The object of interest is the matrix of coefficients that multiplies  $\tau_{t-1}$ ,  $(I - B)^{-1} (C + AB)$ . In table 6 we report these coefficients together with their standard errors. Figures 1 and 2 report also the impulse-response functions derived in Proposition 2, for the two representative stocks, COX and JAX. Note that the



coefficients of the variables that enter the expected duration ( $\psi_i$ ) are the same as those in tables 4, by construction.

In the variance equation, volume has, as expected, positive sign and also duration usually enters with a positive coefficient. Although the sign on the duration seems to contradict the result of table 4, the explanation is that a high lagged duration increases the expected duration of the next trade. In a geometric Brownian motion world, for example, the variance per trade would be proportional to the time elapsed since the last trade.

The impulse-response functions in figures 1 and 2 give a graphical representation of the results of table 6. Since the impulse-response functions are plotted in transaction time, they are not directly comparable among different stocks. However, we can approximate the time the system takes to return to its long-run equilibrium, by multiplying the number of transactions by their average duration. The average duration per trade of the two representative stocks is 98 seconds for COX and 3,165 seconds for JAX. This implies, for example, that a shock to the duration of COX is absorbed by the expected duration after about 2,000 trades, or, on average, after 55 hours. In the case of JAX, instead, the same shock is absorbed after 500 transaction, which correspond, on average, to a period of 440 hours. Similar results hold for the other impulse-responses, indicating that the more traded the stock, the faster the market returns to its full information equilibrium after an initial perturbation. In particular, this is consistent with the (plausible) assumption that the more frequently traded the stock the higher the number of informed traders. For example, Holden and Subrahmanyam (1992) show that with multiple informed traders there will be more aggressive trading in the early periods, causing more information to be revealed earlier.

The ACD model can be used to compute directly the impulse-responses in calendar time, as suggested by Dufour and Engle (2000). However, in this case the duration is not exogenous to the volume and return processes and the impulse-responses can be computed only through stochastic simulation. To do this, we start the system in steady state and simulate the model described by (6) and (9) for 10 steps ahead, using the following algorithm:

1. Generate a  $(3 \times 10)$  matrix of random error terms, using an exponential distribution with mean 1 for  $\varepsilon_t$  and  $\eta_t$ , and a standard normal for  $\zeta_t$ .

2. Compute  $\mu_t$  and  $\tau_t$  for  $t = 1, \dots, 10$  using the error terms generated in point (1) and put back the deterministic component.<sup>10</sup> At the same time compute the volatility per second as  $\sigma_t^2/d_t$  and tabulate  $\mu_t$  and the volatility per second as a function of calendar time.<sup>11</sup>

3. Shock one element of  $\tau_0$  and repeat the procedure of point (2) using the same matrix of error terms generated in point (1).

4. Repeat steps (1) to (3) 60,000 times. Sample  $\mu_t$ ,  $\sigma_t^2/d_t$  and the corresponding shocked values every second and take the averages.

The impulse-response function is given by the difference, at each point in time, between the average of the shocked values and the average of the original values.

In figure 3 and 4, we report the impulse-responses in calendar time of COX and JAX during the first minute after the shock. Note that the impulse-response becomes noisier as time elapses. This is due to a curse of dimensionality problem. Since there are three variables to simulate, the greater the number of steps taken into consideration (that is the higher the number of transactions occurred) the less accurate the simulation will be. Nevertheless, the response over the first few seconds should be accurate enough. The noise of the impulse-response for JAX is lower because this stock has a much higher average duration. This implies a lower expected number of trades over the first few seconds and thus a much more accurate stochastic approximation.

As expected, the top 6 graphs in figures 3 and 4 are perfectly consistent with those of figures 1 and 2 respectively. The bottom three graphs, instead, show the impulse-response of the expected volatility per second. From these plots we can see that a shock to the volatility of COX is absorbed much faster than a shock to the volatility of JAX, confirming the finding that not frequently traded stocks are characterised by a different mechanism of information transmission.

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<sup>10</sup> Remember that we diurnally adjusted the series before estimating the models. The only part of the deterministic component that we put back was the average of each series.

<sup>11</sup> Time was computed as the cumulative sum of the preceding durations, multiplied by the average trade duration of the stock and rounded to the next nearest integer.

## 5. Conclusion

We presented a new econometric framework to analyse ultra high frequency data, that allows expected duration, expected volume and variance of returns to depend on current and lagged values of the variables under study. We arrived to an econometric reduced form that incorporates causal and feedback effects among these variables. We constructed impulse-response functions that showed how quickly the system returns to its long-run equilibrium after a shock.

The methodology was applied to two groups of stocks, differing from each other according to their trade intensity. We found significant evidence that the frequency with which the stock is traded is important in determining its dynamic behaviour. Shocks are absorbed more quickly for frequently traded stocks than for infrequently traded ones. This is consistent with many insights from market microstructure theories, according to which, for example, the resiliency of prices is determined by the number of insider traders active in the market.

An interesting extension of the econometric framework constructed in this paper is to incorporate depth and spread, by modelling them in a similar way to volume and duration. By adding these two explanatory variables to the model, one could get a clearer picture about the trading environment of a specific market.

A second interesting possibility is a systematic study that addresses the issue of the relationship between the characteristics of different markets and their dynamic properties. Many market microstructure models show that prices eventually converge to full information value, but provide very little insight into how long the adjustment process might take. We believe that the impulse-response framework suggested in this paper might prove a very valuable tool for this kind of analysis. For example, one could compare the estimates of the model across market structures (electronic versus specialist markets) or across different periods (crashes versus calm markets).

## Appendix A: Proofs

### Proof of Proposition 1

In general, a  $(n,1)$ -vector process  $x_t$  is said to be covariance-stationary if its first and second moments  $E(x_t)$  and  $E(x_t, x_{t-j}')$ ,  $j=1,2,\dots,n$ , are independent of the date  $t$ . To see under what conditions process (8) is covariance stationary, assume without loss of generality that  $p>q$  and rewrite (8) as follows:

$$(13) \quad \mu_t - \tau_t = \gamma + A_1(\mu_{t-1} - \tau_{t-1}) + \dots + A_q(\mu_t - \tau_{t-q}) + (B_0 - I)\tau_t + (A_1 + B_1)\tau_{t-1} + \dots + (A_s + B_p)\tau_{t-p}$$

where  $A_s=0$  for  $s=q+1, q+2, \dots, p$ . Since  $\mu_t = E(\tau_t)$ , the random variables  $\mu_t - \tau_t$  will form a white noise process. Rewrite (A1) as

$$(B_0 - I)\tau_t = -\gamma + (A_1 + B_1)\tau_{t-1} + \dots + (A_s + B_p)\tau_{t-p} + (\mu_t - \tau_t) - A_1(\mu_{t-1} - \tau_{t-1}) - \dots - A_q(\mu_t - \tau_{t-q})$$

or 
$$\tau_t = -(B_0 - I)^{-1}\gamma + H_1\tau_{t-1} + \dots + H_p\tau_{t-p} + v_t + G_1v_{t-1} + \dots + G_qv_{t-q}$$

where  $v_t = \mu_t - \tau_t$ ,  $H_i = (B_0 - I)^{-1}(A_i + B_i)$ ,  $i=1,\dots,p$ ,  $G_j = -(B_0 - I)^{-1}A_j$ ,  $j=1,\dots,q$ .

This is a VARMA(p,q) process that can be rewritten as a VARMA(1,q) (see Hamilton (1994), page 259). Define

$$\xi_t = \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \vdots \\ \mu_{t-p} \end{bmatrix}, \quad \kappa = \begin{bmatrix} -(B_0 - I)^{-1}\gamma \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} H_1 & H_2 & \dots & H_{p-1} & H_p \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_n & 0 \end{bmatrix}, \quad v_{t-j} = \begin{bmatrix} G_j v_{t-j} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad j=0,1,\dots,q, \quad \text{with } G_0=I_n.$$

Hence, the VARMA(p,q) in (A1) can be rewritten as

$$\xi_t = \kappa + F\xi_{t-1} + v_t + \dots + v_{t-q}$$

To have stationarity the absolute value of the eigenvalues of F must be less than 1. Proposition 10.1 of Hamilton (1994) assures that the eigenvalues of F satisfy

$$\left| I_n \lambda^p - H_1 \lambda^{p-1} - H_2 \lambda^{p-2} - \dots - H_p \right| = 0.$$

Q.E.D.

### Proof of Proposition 2

Suppose that the system is in steady state up to time  $t=0$ . That is, all the innovations before  $t=0$  are equal to 1, so that

$$\mu_t = \bar{\mu} = (I - A - B - C)^{-1}\gamma \quad \text{for } t < 0$$

$$\tau_t = \mu_t$$

Suppose now that at time  $t=0$  a shock occurs to  $\tau_0$ . This has an immediate effect on  $\mu_0$  and a lagged effect on  $\mu_t$ ,  $t>0$ . The effect on  $\mu_0$  is simply:

$$\mu_0 = \gamma + B\tau_0 + (A + C)\bar{\mu}$$

To evaluate the effect of  $\tau_0$  on future expected  $\mu_t$ , note that (9) implies the following:

$$E(\mu_t | \Omega_0) = \gamma + AE(\mu_{t-1} | \Omega_0) + BE(\mu_t | \Omega_0) + CE(\mu_{t-1} | \Omega_0)$$

because  $E(\tau_t | \Omega_0) = E(\mu_t | \Omega_0)$  by the law of iterated expectations. Noting further that

$$E(\mu_1 | \Omega_0) = (I - B)^{-1}(\gamma + A\mu_0 + C\tau_0)$$

by recursive substitution, we get the result.

Q.E.D.

### Proof of Proposition 3

Since  $\kappa_j$  is continuous in  $\theta$ , the result is a straightforward consequence of the properties of sequences of random vectors.

## References

- Andersen, T.G. (1996), Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility, *Journal of Finance* **51**, 169-204.
- Admati, A. and P. Pfleiderer (1988), A Theory of Intraday Patterns: Volume and Price Variability, *Review of Financial Studies* **1**, Spring, 3-40.
- Bagehot, W. (1971), The Only Game in Town, *Financial Analysts Journal* **27**, 12-14, 22.
- Blume, L.E., D. Easley and M. O'Hara (1994), Market Statistics and Technical Analysis: The Role of Volume, *Journal of Finance* **49**, 153-182.
- Bollerslev, T. and J.M. Wooldridge (1992), Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances, *Econometric Reviews* **11(2)**, 143-172.
- Brown, D. P. and R.H. Jennings (1989), On Technical Analysis, *Review of Financial Studies* **2**, 527-552.
- Clark, P. (1973), A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices, *Econometrica* **41**, 135-155.
- Copeland, T.E. (1976), A Model of Asset Trading Under the Assumption of Sequential Information Arrival, *Journal of Finance* **31**, 4: 1149-1168.
- Copeland, T.E. and D. Galai (1983), Information Effects and the Bid-Ask Spread, *Journal of Finance* **38**, 1457-1469.
- Diamond, D.W. and R. E. Verrecchia (1987), Constraints on Short-selling and Asset Price Adjustments to Private Information, *Journal of Financial Economics* **82**, 33-53.
- Drost, F.C. and T.E. Nijman (1993), Temporal Aggregation of GARCH processes, *Econometrica* **61**: 907-927.
- Dufour, A. and R. Engle (2000), Time and Price Impact of a Trade, *Journal of Finance* **55**, 2467-98.
- Easley, D and M. O'Hara (1987), Price, Trade Size, and Information in Securities Markets, *Journal of Financial Economics* **19**, 69-90.
- Easley, D. and M. O'Hara (1992), Time and the Process of Security Price Adjustment, *Journal of Finance* **47**, 577-606.
- Ederington, L.H. and J.H. Lee (1993), How Markets Process Information: News Releases and Volatility, *Journal of Finance* **48**, 1161-91.
- Engle, R. (2000), The Econometrics of Ultra High Frequency Data, *Econometrica* **68**, 1-22.
- Engle R., D.F. Hendry and J.-F. Richard (1983), Exogeneity, *Econometrica* **51**, 277-304.
- Engle, R. and J. Russell (1998), Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data, *Econometrica* **66**, 1127-1162.
- Epps and Epps (1976), The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distributions Hypothesis, *Econometrica* **44**, 305-321.
- Fleming, M.J. and E.M. Remolona (1999), Price Formation and Liquidity in the U.S. Treasury Market: The Response to Public Information, *Journal of Finance* **54**, 1901-1915.

- Glosten, L. and P. Milgrom (1985), Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics* **13**, 71-100.
- Grammig, J. and M. Wellner (2002), Modeling the Interdependence of Volatility and Inter-transaction Duration Processes, *Journal of Econometrics* **106**: 369-400.
- Grossman, S.J. and J.E. Stiglitz (1980), On the Impossibility of Informationally Efficient Markets, *American Economic Review* **70**, 393-408.
- Grundy, B.D. and M. McNichols (1989), Trade and Revelation of Information Through Prices and Direct Disclosure, *Review of Financial Studies* **2**, 495-526.
- Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press.
- Hartmann, P. (1999), Trading Volumes and Transaction Costs in the Foreign Exchange Market. Evidence from Daily Dollar-Yen Spot Data, *Journal of Banking and Finance* **23**, 801-824.
- Hasbrouck, J. (1991), Measuring the Information Content of Stock Trades, *Journal of Finance* **46**, 1: 179-207.
- Holden, C.W. and A. Subrahmanyam (1992), Long-Lived Private Information and Imperfect Competition, *Journal of Finance* **47**, 247-270.
- Jones, C.M., G. Kaul and M.L. Lipson (1994), Transactions, Volume and Volatility, *The Review of Financial Studies* **7**, 631-651.
- Karpoff, J. (1987), The Relation Between Price Change and Trading Volume: A Survey, *Journal of Financial and Quantitative Analysis* **22**, March, 109-126.
- Kyle, A.S. (1985), Continuous Auctions and Insider Trading, *Econometrica* **53**, 1315-1336.
- Lee, C.M. and M.J. Ready (1991), Inferring Trade Direction from Intraday Data, *Journal of Finance* **46**, 733-747.
- Liesenfeld, R. (2001), A Generalized Bivariate Mixture Model for Stock Price Volatility and Trading Volume, *Journal of Econometrics* **104**: 141-178.
- O'Hara, M. (1995), *Market Microstructure Theories*, Blackwell Publisher Inc, Massachusetts.
- Tauchen, G.E. and M. Pitts (1983), The Price Variability-Volume Relationship on Speculative Markets, *Econometrica* **51**, 485-505.
- White, H. (1980), A Heteroscedasticity-consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity, *Econometrica*, **48**, 351-372.
- Zhang, M. Y., J. Russell and R. S. Tsay (2001), A Nonlinear Autoregressive Conditional Duration Model with Applications to Financial Transaction Data, *Journal of Econometrics* **104**: 179-207.

**Appendix B: Stocks Used in the Analysis****A. Frequently traded**

CP	CDN PACIFIC
GAP	GREAT A & P
COX	COX COMMUNICATION
DLP	DELTA & PINELAND
AVT	AVNET INC

**B. Not frequently traded**

DTC	DOMTAR INC
FTD	FORT DEARBN INCM
GBX	GREENBRIER CO
GSE	GUNDLE/SLT ENV
JAX	J ALEXANDERS

**Table 1** – Summary statistics for the ten stocks used in the analysis. The sharp difference between the number of trades for the two groups of stocks reflects the selection criterion used. Notice that this difference is reflected in the average and median duration, but not in volume.

Ticker	# Obs	Duration		Volume	
		<i>Average</i>	<i>Median</i>	<i>Average</i>	<i>Median</i>
<b>DLP</b>	65304	134.15	45	1483.84	500
<b>GAP</b>	46826	187.3	80	827.03	300
<b>CP</b>	71672	122.43	65	2978.55	500
<b>COX</b>	88917	98.6	35	2692.57	800
<b>AVT</b>	58389	150.02	64	1075.55	400
<b>JAX</b>	2761	3165.7	1032	1002.32	500
<b>GSE</b>	1968	4438.42	1523	1523.53	600
<b>GBX</b>	5154	1695.49	620	1483.95	500
<b>FTD</b>	3264	2416.44	1390	736.95	500
<b>DTC</b>	4161	2094.22	526	2143.91	800

**Table 2** – Autoregressive coefficients ( $\beta$ ) of the three models for duration, volume and trade variance.

- $d_t = \psi_t \varepsilon_t \quad \varepsilon_t \sim i.i.d.(1, \sigma_\varepsilon^2)$   
 $\psi_t = \omega + \alpha d_{t-1} + \beta \psi_{t-1}$
- $v_t = \phi_t \eta_t \quad \eta_t \sim i.i.d.(1, \sigma_\eta^2)$   
 $\phi_t = \omega + \alpha v_{t-1} + \beta \phi_{t-1}$
- $y_t = \sigma_t \zeta_t \quad \zeta_t \sim i.i.d.(0,1)$   
 $\sigma_t^2 = \bar{\omega} + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$

These models were estimated imposing the strong exogeneity assumption. In terms of the notation developed in the paper, we set the matrix  $B$  equal to zero and imposed the matrices  $A$  and  $C$  to be diagonal. Standard errors are omitted, because all the coefficients resulted overwhelmingly significant.

Ticker	ACD	ACV	GARCH
<b>DLP</b>	0.9324	0.9525	0.9618
<b>GAP</b>	0.9521	0.9726	0.9658
<b>CP</b>	0.9651	0.9707	0.9583
<b>COX</b>	0.9373	0.9718	0.9505
<b>AVT</b>	0.8950	0.9154	0.9508
<b>JAX</b>	0.9533	0.7163	0.9072
<b>GSE</b>	0.8303	0.8496	0.9493
<b>GBX</b>	0.9295	0.7154	0.9149
<b>FTD</b>	0.8764	0.2925	0.9487
<b>DTC</b>	0.8857	0.8004	0.9519



**Table 3** – Ljung-Box (LB) statistics for raw data and fitted residuals. The fitted residuals are obtained from estimation of the full model of equation (12):

- $\psi_t = a_0 + a_1\psi_{t-1} + a_2d_{t-1} + a_3v_{t-1} + a_4\mu_{t-1}^2$
- $\phi_t = b_0 + b_1\phi_{t-1} + b_2v_{t-1} + b_3d_t + b_4d_{t-1} + b_5\mu_{t-1}^2$
- $$\begin{cases} y_t = X_t' \beta + u_t & u_t = \rho u_{t-1} + \mu_t + \theta \mu_{t-1} \\ \sigma_t^2 = c_0 + c_1\sigma_{t-1}^2 + c_2\mu_{t-1}^2 + c_3d_t + c_4d_{t-1} + c_5v_t + c_6v_{t-1} \end{cases}$$

The sharp drop in value of the LB statistics reveals that this model successfully captures most of the autocorrelation present in the raw data.

	Duration		Volume		Return (mean)		Return (variance)	
	LB Raw	LB Fitted	LB Raw	LB Fitted	LB Raw	LB Fitted	LB Raw	LB Fitted
<b>CP</b>	3704.96	118.07	2513.63	31.94	376.17	12.18	3610.92	15.09
<b>GAP</b>	5052.96	56.16	357.02	58.16	45.78	19.62	3539.06	42.14
<b>COX</b>	30163.28	87.65	361.96	25.19	864.21	15.10	694.16	48.29
<b>DLP</b>	20027.87	114.03	2010.17	84.81	604.70	16.16	7922.59	65.76
<b>AVT</b>	6781.7526	80.1084	484.3870	36.1466	484.0792	12.6921	659.2740	39.5419

	Duration		Volume		Return (mean)		Return (variance)	
	LB Raw	LB Fitted	LB Raw	LB Fitted	LB Raw	LB Fitted	LB Raw	LB Fitted
<b>DTC</b>	161.73	10.46	401.08	10.80	78.68	15.66	182.09	11.38
<b>FTD</b>	17.68	10.63	82.88	22.63	46.89	33.67	326.16	9.82
<b>GBX</b>	1023.15	15.05	389.99	12.25	31.09	13.84	77.60	11.79
<b>GSE</b>	85.56	6.81	82.69	25.89	37.77	18.77	71.15	15.57
<b>JAX</b>	104.33	7.50	95.01	14.84	31.66	17.46	330.35	11.84

**Table 4** - Coefficients of the matrix (AB +C). These coefficients indicate the effect that lagged values of duration, volume and squared trade returns have on expected variables. Bollerslev-Wooldridge robust t-statistics in parenthesis. Variables significant at the 1% confidence level formatted in bold. The asterisc denotes significance at 5%.

	CP			GAP			COX			DLP			AVT		
	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$
$\psi_t$	<b>0.0274</b> (23.90)	<b>-0.0004</b> (-2.79)	<b>-0.0001</b> (-3.19)	<b>0.0450</b> (27.50)	<b>-0.0013</b> (-3.37)	-0.0001 (-1.12)	<b>0.0608</b> (44.70)	<b>-0.0006</b> (-6.74)	<b>-0.0002</b> (-3.48)	<b>0.0679</b> (39.30)	<b>-0.0026</b> (-5.36)	<b>-0.0006</b> (-2.90)	<b>0.0450</b> (27.50)	<b>-0.0013</b> (-3.37)	-0.0001 (-1.12)
$\phi_t$	-0.0011 (-1.55)	<b>0.0227</b> (12.00)	0.0001 (0.36)	<b>-0.0037</b> (-4.10)	<b>0.0190</b> (10.40)	<b>-0.0001</b> (-3.14)	0.0009 (1.34)	<b>0.0241</b> (13.60)	0.0000 (-0.17)	0.0002 (0.33)	<b>0.0408</b> (17.30)	0.0001 (1.60)	<b>-0.0037</b> (-4.10)	<b>0.0190</b> (10.40)	<b>-0.0001</b> (-3.14)
$\sigma_t^2$	<b>-0.0432</b> (-15.1)	<b>0.0195</b> (3.68)	<b>0.0421</b> (17.30)	-0.0043 (-0.88)	<b>0.0198</b> (4.27)	<b>0.0390</b> (14.30)	<b>-0.0161</b> (-5.54)	<b>0.0114</b> (4.91)	<b>0.0407</b> (21.40)	-0.0033 (-1.18)	0.0013 (0.44)	<b>0.0399</b> (9.31)	-0.0043 (-0.88)	<b>0.0198</b> (4.27)	<b>0.0390</b> (14.30)

	DTC			FTD			GBX			GSE			JAX		
	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$
$\psi_t$	<b>0.0850</b> (8.05)	0.0001 (0.03)	-0.0009 (-1.26)	0.032 * (2.29)	<b>-0.0430</b> (-46.9)	-0.0020 (-1.51)	<b>0.0679</b> (10.03)	0.0000 (0.03)	0.0003 (0.80)	<b>0.0964</b> (4.71)	-0.0089 (-1.59)	<b>-0.0021</b> (-8.30)	<b>0.0388</b> (5.92)	-0.0012 (-0.62)	-0.0002 (-0.67)
$\phi_t$	-0.0110 (-1.58)	<b>0.1210</b> (2.33)	0.0003 (0.19)	-0.0203 (-1.65)	<b>0.1190</b> (5.02)	-0.0018 (-0.98)	-0.0087 (-1.33)	<b>0.1429</b> (5.74)	0.0004 (0.21)	0.0012 (0.18)	<b>0.0988</b> (4.90)	0.0016 (0.99)	-0.0078 (-0.91)	<b>0.1530</b> (5.06)	0.0064 (1.09)
$\sigma_t^2$	0.045 * (2.30)	0.0381 (1.90)	<b>0.0650</b> (4.24)	-0.04 * (-1.96)	0.068 * (2.25)	<b>0.0424</b> (5.49)	-0.0010 (-0.13)	0.0004 (0.11)	<b>0.0588</b> (5.20)	0.0038 (0.17)	0.0282 (1.53)	<b>0.0486</b> (3.94)	<b>-0.0521</b> (-7.61)	0.029 * (2.06)	<b>0.0632</b> (7.25)

**Table 5** – Tests for omitted variables. We regressed the standardised residuals from model (12) on lagged expected variables:

$$\hat{\varepsilon}_t = \alpha_0 + \alpha_1 \hat{\phi}_{t-1} + \alpha_2 \hat{\sigma}_{t-1}^2$$

$$\hat{\eta}_t = \beta_0 + \beta_1 \hat{\psi}_{t-1} + \beta_2 \hat{\sigma}_{t-1}^2$$

$$\hat{\mu}_t / \hat{\sigma}_t^2 = \gamma_0 + \gamma_1 \hat{\psi}_{t-1} + \gamma_2 \hat{\phi}_{t-1}$$

White robust t-statistics in parenthesis. Variables significant at the 1% confidence level are formatted in bold. The asterisc denotes significance at 5%.

	Exp Dur			Exp Vol			Volatility		
	c	Exp Vol	Volatility	c	Exp Dur	Volatility	c	Exp Dur	Exp Vol
<b>CP</b>	<b>1.0068</b> (104.49)	0.0153 * (2.13)	<b>-0.0050</b> (-3.33)	<b>0.9473</b> (17.70)	0.0228 (0.49)	0.0069 (1.70)	<b>1.2054</b> (19.99)	-0.0846 (-1.46)	<b>-0.1136</b> (-6.23)
<b>GAP</b>	<b>0.9737</b> (44.58)	0.0004 (0.16)	0.0253 (1.23)	<b>1.0011</b> (29.53)	0.0028 (0.10)	-0.0010 (-0.42)	<b>1.3936</b> (15.18)	<b>-0.1096</b> (-2.40)	<b>-0.2840</b> (-5.88)
<b>COX</b>	<b>0.9985</b> (120.59)	0.0123 * (2.05)	-0.0027 * (-2.16)	<b>0.9676</b> (29.13)	0.0077 (0.30)	0.0059 (1.49)	<b>1.0783</b> (45.57)	0.0082 (0.48)	<b>-0.0800</b> (-6.46)
<b>DLP</b>	<b>1.0079</b> (81.09)	-0.0019 * (-2.31)	-0.0005 (-0.04)	<b>1.0165</b> (67.34)	-0.0140 (-1.03)	-0.0005 (-0.52)	<b>1.0682</b> (12.27)	-0.0264 (-1.33)	-0.0284 (-0.55)
<b>AVT</b>	<b>0.9944</b> (67.13)	0.0054 (0.53)	0.0000 (0.01)	<b>1.0259</b> (28.64)	-0.0075 (-0.29)	-0.0051 (-0.91)	<b>1.0851</b> (24.91)	-0.0212 (-0.70)	-0.0546 * (-2.01)

	Exp Dur			Exp Vol			Volatility		
	c	Exp Vol	Volatility	c	Exp Dur	Volatility	c	Exp Dur	Exp Vol
<b>DTC</b>	<b>1.0129</b> (21.68)	0.0034 (0.10)	-0.0052 (-0.46)	<b>0.9948</b> (9.72)	-0.0018 (-0.02)	0.0023 (0.12)	<b>1.0260</b> (9.36)	0.0166 (0.17)	<b>-0.0426</b> (-2.96)
<b>FTD</b>	<b>0.9831</b> (8.73)	0.0427 (0.39)	-0.0054 (-0.61)	<b>0.5641</b> (2.52)	0.3732 (1.76)	0.0181 * (2.21)	-0.5320 (-0.47)	0.4833 (0.65)	<b>1.0702</b> (2.39)
<b>GBX</b>	<b>1.0209</b> (27.26)	-0.0246 (-1.15)	0.0010 (0.15)	<b>1.0076</b> (15.20)	-0.0291 (-0.46)	0.0061 (0.60)	<b>1.0299</b> (11.80)	0.0329 (0.50)	-0.0636 * (-2.14)
<b>GSE</b>	<b>0.9708</b> (13.77)	0.0185 (0.26)	0.0023 (0.16)	<b>1.2034</b> (7.29)	-0.1797 (-1.37)	-0.0059 (-0.37)	<b>1.1413</b> (4.32)	-0.0570 (-0.28)	-0.0783 (-0.77)
<b>JAX</b>	<b>1.0095</b> (17.29)	0.0298 (0.66)	-0.0129 (-1.08)	<b>1.0518</b> (8.70)	-0.0622 (-0.68)	0.0037 (0.16)	<b>0.9273</b> (6.27)	0.1236 (0.97)	-0.0496 (-1.15)

**Table 6** - Coefficients of the matrix  $(I-B)^{-1}(C+AB)$ . These coefficients indicate the effect that lagged duration, volume and squared trade returns have on the forecasts of expected variables. Bollerslev-Wooldridge robust t-statistics in parenthesis. Variables significant at the 5% confidence level formatted in bold.

	CP			GAP			COX			DLP			AVT		
	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$
$\psi_t$	<b>0.0274</b> (23.90)	<b>-0.0004</b> (-2.79)	<b>-0.0001</b> (-3.19)	<b>0.0450</b> (27.50)	<b>-0.0013</b> (-3.37)	-0.0001 (-1.12)	<b>0.0608</b> (44.70)	<b>-0.0006</b> (-6.74)	<b>-0.0002</b> (-3.48)	<b>0.0679</b> (39.30)	<b>-0.0026</b> (-5.36)	<b>-0.0006</b> (-2.90)	<b>0.0450</b> (27.50)	<b>-0.0013</b> (-3.37)	-0.0001 (-1.12)
$\phi_t$	0.0000 (-0.05)	<b>0.0226</b> (12.00)	0.0001 (0.33)	<b>-0.0023</b> (-2.56)	<b>0.0190</b> (10.40)	<b>-0.0001</b> (-3.19)	<b>0.0063</b> (7.20)	<b>0.0240</b> (13.50)	0.0000 (-0.31)	<b>0.0031</b> (4.27)	<b>0.0407</b> (17.30)	0.0001 (1.14)	<b>-0.0023</b> (-2.56)	<b>0.0190</b> (10.40)	<b>-0.0001</b> (-3.19)
$\sigma_t^2$	<b>-0.0258</b> (-5.21)	<b>0.0277</b> (5.40)	<b>0.0420</b> (17.30)	<b>0.0313</b> (6.39)	<b>0.0221</b> (4.93)	<b>0.0389</b> (14.20)	<b>0.0375</b> (11.40)	<b>0.0177</b> (7.58)	<b>0.0406</b> (21.30)	<b>0.0156</b> (2.90)	0.0001 (0.04)	<b>0.0398</b> (9.23)	<b>0.0313</b> (6.39)	<b>0.0221</b> (4.93)	<b>0.0389</b> (14.20)

	DTC			FTD			GBX			GSE			JAX		
	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$	$d_{t-1}$	$v_{t-1}$	$\mu_{t-1}^2$
$\psi_t$	<b>0.0850</b> (8.05)	0.0001 (0.03)	-0.0009 (-1.26)	0.032 * (2.29)	<b>-0.0430</b> (-46.9)	-0.0020 (-1.51)	<b>0.0679</b> (10.03)	0.0000 (0.03)	0.0003 (0.79)	<b>0.0964</b> (4.71)	-0.0089 (-1.59)	<b>-0.0021</b> (-8.30)	<b>0.0388</b> (5.92)	-0.0012 (-0.62)	-0.0002 (-0.67)
$\phi_t$	-0.0099 (-1.40)	<b>0.1210</b> (2.33)	0.0003 (0.18)	-0.0213 (-1.73)	<b>0.1200</b> (5.06)	-0.0017 (-0.94)	-0.0056 (-0.89)	<b>0.1429</b> (5.74)	0.0004 (0.21)	0.0042 (0.57)	<b>0.0985</b> (4.89)	0.0015 (0.94)	-0.0079 (-0.93)	<b>0.1530</b> (5.06)	0.0064 (1.09)
$\sigma_t^2$	<b>0.0840</b> (3.86)	<b>0.0644</b> (2.78)	<b>0.0647</b> (4.22)	-0.0411 (-1.84)	<b>0.0919</b> (2.88)	<b>0.0414</b> (5.32)	<b>0.0342</b> (3.42)	0.068 * (2.29)	<b>0.0591</b> (5.21)	0.0089 (0.37)	0.0330 (1.74)	<b>0.0486</b> (3.93)	<b>-0.0374</b> (-3.97)	<b>0.0692</b> (2.79)	<b>0.0648</b> (7.27)

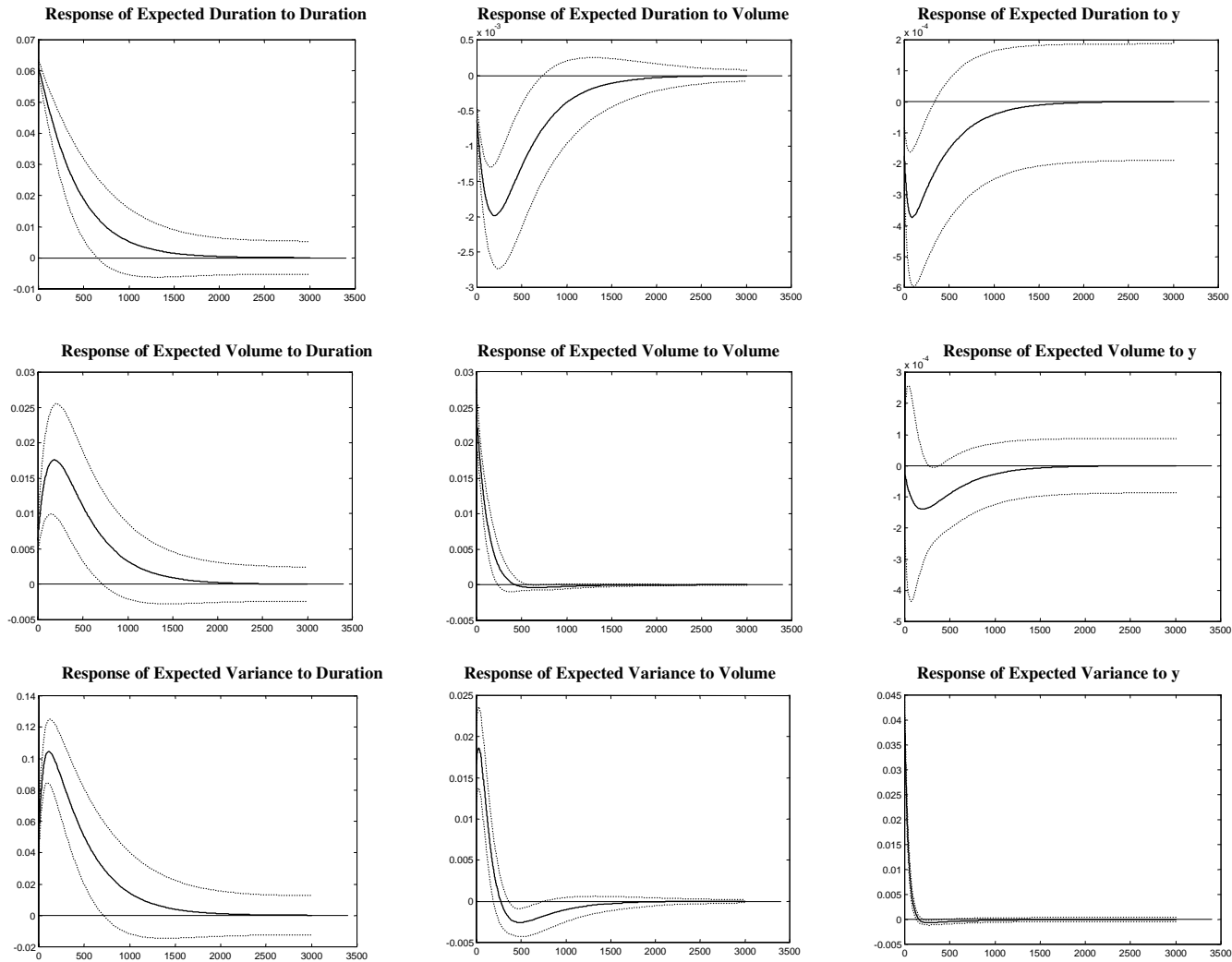


Figure 1 - Impulse-response function for COX (frequently traded). Dotted lines are 95% confidence intervals.

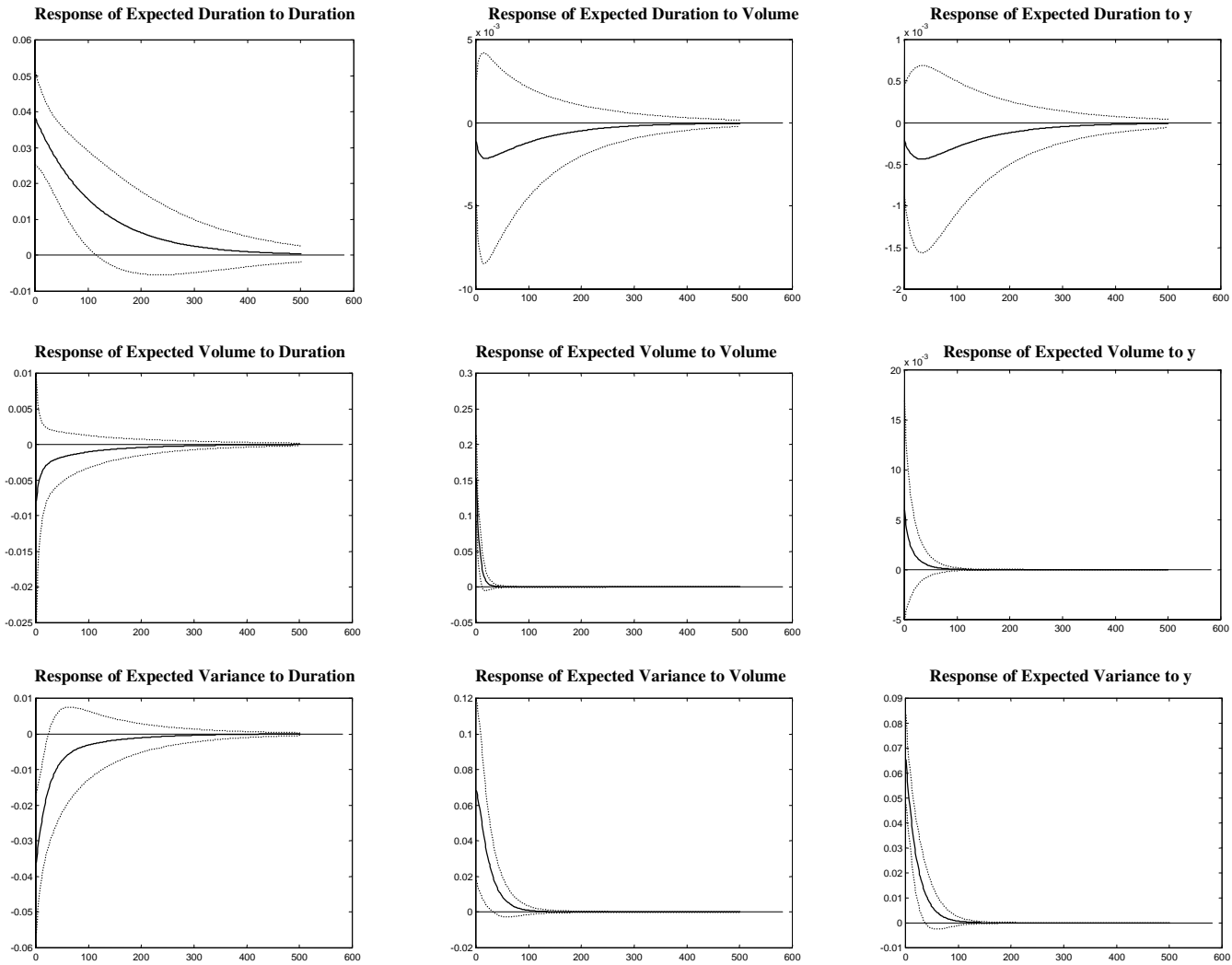


Figure 2 - Impulse-response function for JAX (not frequently traded). Dotted lines are 95% confidence intervals.

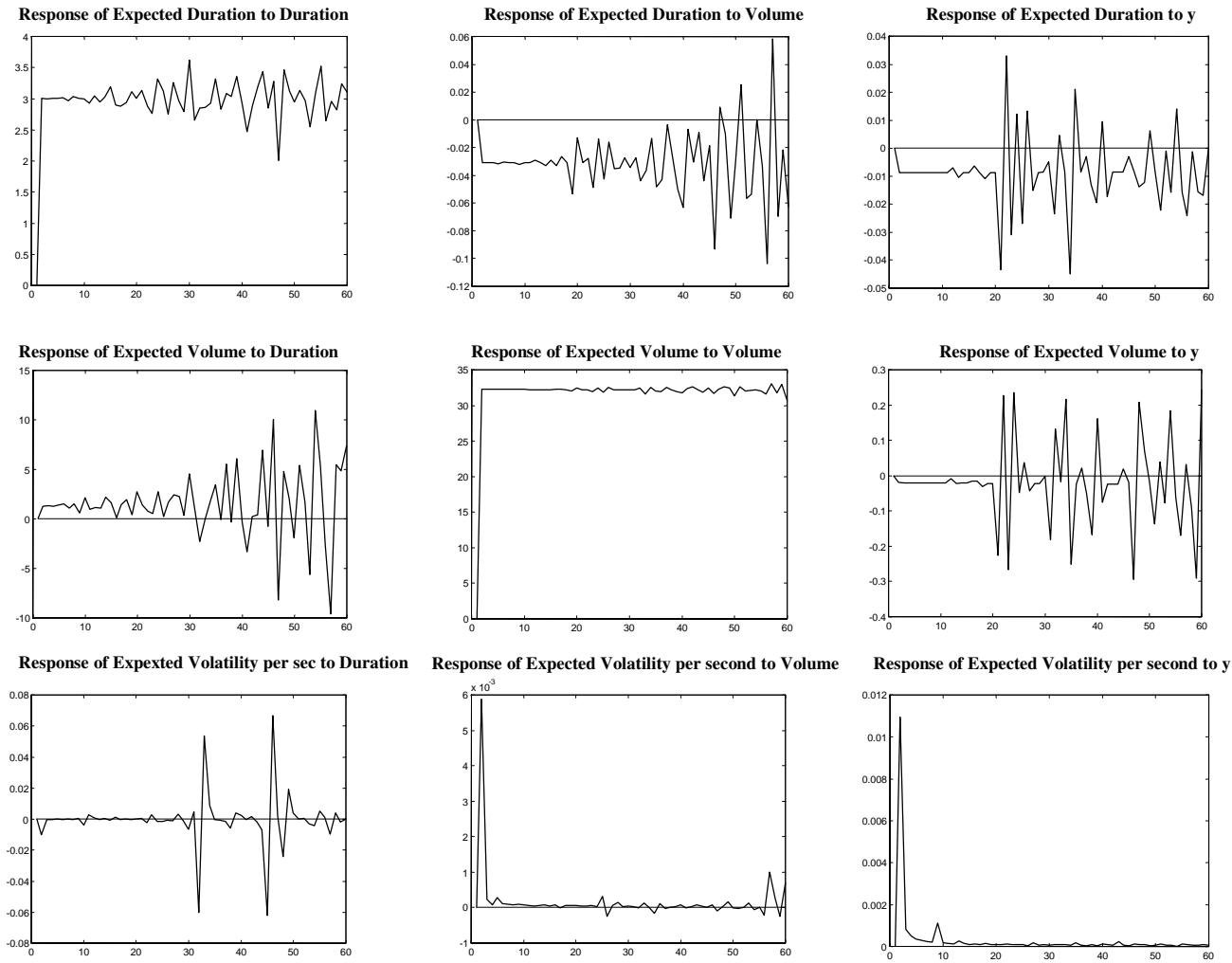
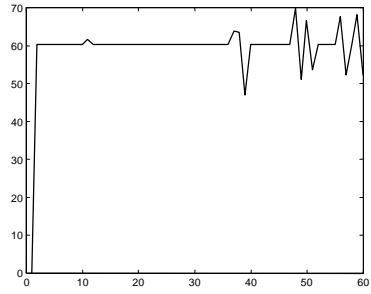
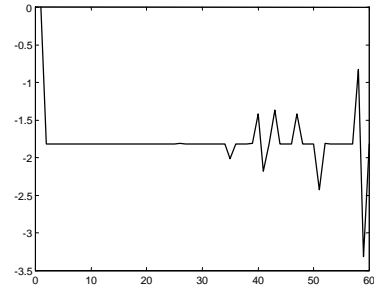


Figure 3 - Impulse-response function for COX in calendar time (seconds).

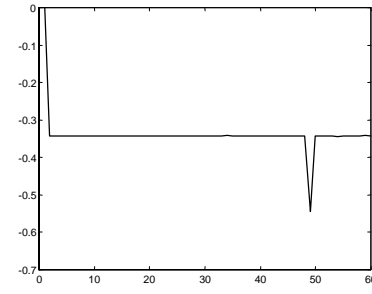
Response of Expected Duration to Duration



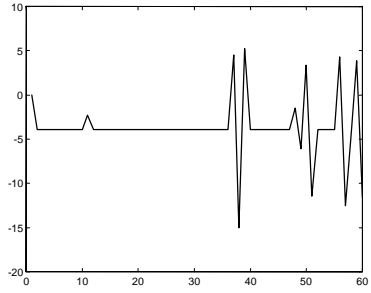
Response of Expected Duration to Volume



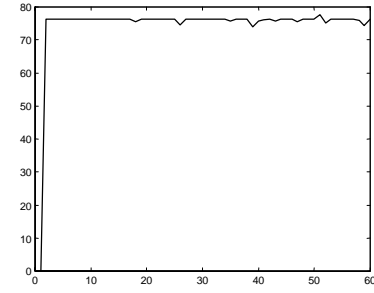
Response of Expected Duration to y



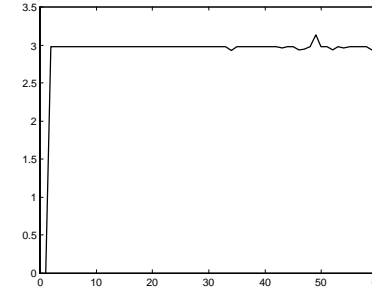
Response of Expected Volume to Duration



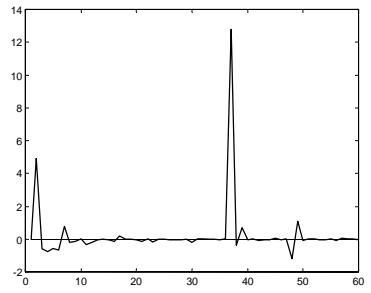
Response of Expected Volume to Volume



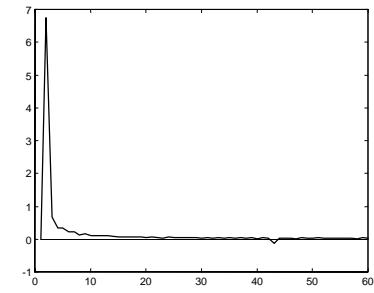
Response of Expected Volume to y



Response of Expected Volatility per sec to Duration



Response of Expected Volatility per second to Volume



Response of Expected Volatility per second to y

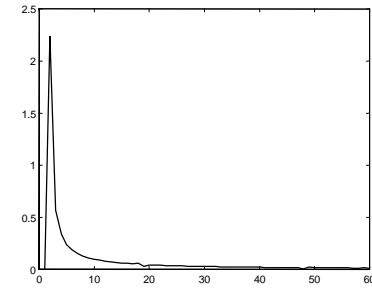


Figure 4 - Impulse-response function for JAX in calendar time (seconds).



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