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## Uncertain potential output: implications for monetary policy

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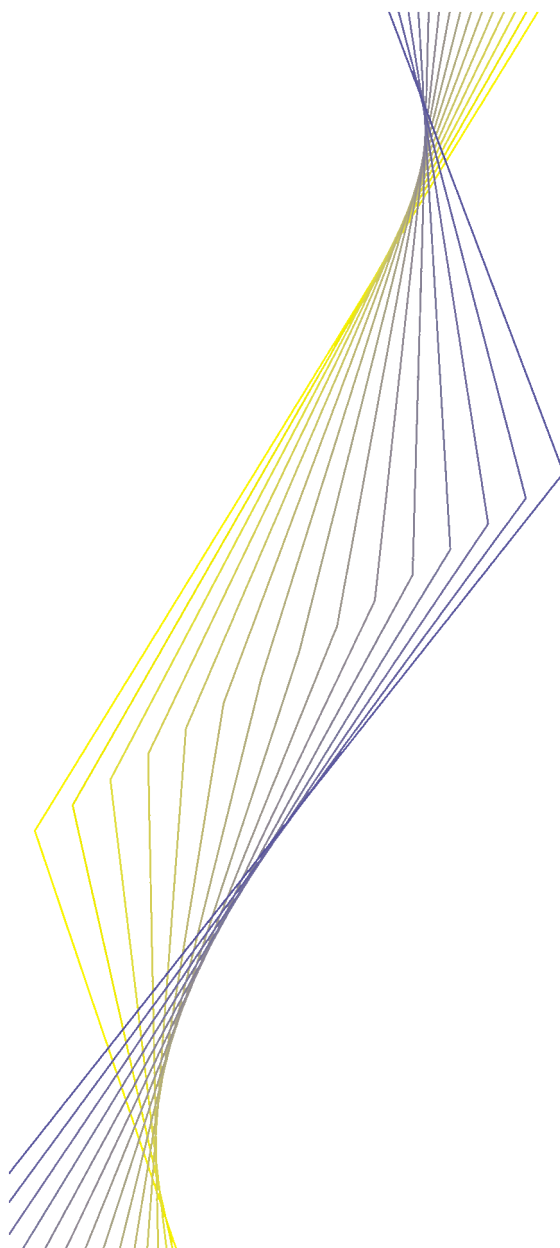
**UNCERTAIN POTENTIAL  
OUTPUT: IMPLICATIONS FOR  
MONETARY POLICY**

**BY MICHAEL EHLMANN  
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**April 2001**



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## Abstract

This paper uses a small, calibrated forward-looking model of the euro-area economy to investigate the implications of incomplete information about potential output for the conduct and the design of monetary policy. Three sets of issues are examined. First, the certainty-equivalent optimal policy under both commitment and discretion is characterised. In both cases, incomplete information about potential output leads to very persistent deviations between the actual and the perceived output gap in response to supply and cost-push shocks. The costs of imperfect information are quite large. Second, the implications for simple policy rules such as a Taylor or inflation-forecast rule are examined. In first-difference form, both rules continue to perform relatively well with imperfect information as long as the output gap and the inflation forecast are optimally estimated. Third, the implications of potential output uncertainty for the optimal delegation to an independent central bank are examined. Incomplete information implies that it is optimal to appoint a more "hawkish" central bank.

Key words: Monetary policy rules; potential output; uncertainty

JEL classifications: E52; E31; E17





# 1 Introduction

In most macro-models that are currently being used for policy analysis the degree of capacity utilisation as, for example, measured by an output gap plays an important role in the determination of inflation. Yet there are no direct measures of the aggregate supply side of the economy or the extent to which the resources in an economy are fully used. A wide variety of both conceptual and empirical methods have been proposed to estimate potential output and make the notion of an output gap operational. As emphasised by McCallum (2000), on the conceptual side, various notions of trend output, potential output, the flexible-price level of output or the output level consistent with stable inflation have all been used. On the empirical side, a wide variety of different methodologies going from simple linear detrending or the more structural production function approach to more sophisticated econometric techniques using multivariate state-space modelling have all been applied.<sup>1</sup>

In this paper we analyse the implications of imperfect information on potential output for the conduct of monetary policy.<sup>2</sup> The discussion above suggests that there are various sources of uncertainty in the measurement of the output gap. Apart from the statistical uncertainty that results from having to estimate a model for potential output, there is uncertainty regarding which model to use. In addition, Orphanides (1998) has highlighted the uncertainty due to the revisions that are made to the real-time estimates of real GDP statistics.

While many researchers have highlighted the complications that uncertain output gaps imply for monetary policy, only recently this issue has been studied more formally in quantitative models of optimal monetary policy. Most of this analysis takes place in a linear-quadratic framework.<sup>3</sup> Estrella and Mishkin (1999) emphasised that uncertainty about the NAIRU would not affect the optimal monetary policy in such a framework because certainty-equivalence holds with respect to shocks that enter the model additively.

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<sup>1</sup>Tetlow (2000) gives a short overview of the history of the measurement of potential output.

<sup>2</sup>Recently, the problems associated with the measurement of potential output have been highlighted in the monetary policy debate in the United States. Uncertainty about the degree of capacity utilisation has been very high as the growth rate of the economy persistently exceeded previous estimates of potential growth and the unemployment rate fell through most previous estimates of the NAIRU without causing a resurgence in inflation. Similar questions also arise in the euro area, where the effects of structural reforms in labour and goods markets on the supply side of the economy are difficult to assess.

<sup>3</sup>One exception is a series of papers by Doug Laxton and co-authors who examine the effects of natural rate uncertainty in a model with a convex Phillips curve. See, for example, Laxton et al (2000).

Smets (1999) integrates the estimation of the output trend in a policy evaluation model, and illustrates the effect of estimation errors on the choice of efficient simple policy rules. He shows that higher uncertainty leads to some policy attenuation in simple Taylor rules. These results are generally confirmed by Tetlow (2000), who extends the analysis to a forward-looking model and also examines inflation-forecast-based rules.

Using estimates of measurement error derived from real-time estimates of the output gap, Orphanides (1998) shows that such errors lead to a significant deterioration of feasible policy outcomes and cause efficient policies to be less activist. Rudebusch (1999) shows that these considerations are essential for reconciling estimated policy reaction functions and optimal policy. However, Svensson and Woodford (2000) and Swanson (2000) argue that these results are due to the fact that the central bank does not use its best estimate of the output gap. While certainty equivalence holds when the optimal policy is expressed in terms of the best estimate of the state variables of the economy, the weights put on various indicators used in deriving such an estimate will depend on how noisy these indicators are.<sup>4</sup>

In this paper, we illustrate and extend these results using a simple backward/forward looking calibrated model of the euro area economy. We systematically compare the implications for the conduct and design of optimal monetary policy of two assumptions regarding the information available to the agents in the economy. Under one assumption the central bank can perfectly derive the nature of the shocks (including those to potential output) that hit the economy. In the other case, the central bank (and the other agents in the economy) do not observe potential output and need to estimate it on the basis of noisy measures of current output and inflation. Following most of the literature we perform the analysis in a linear-quadratic framework where the central bank and the private sector know the structure of the model economy. As a result, certainty equivalence (optimal policies are independent of additive uncertainty) and the separation principle (the estimation problem can be separated from the control problem) hold.<sup>5</sup> As in real policy-making model uncertainty is pervasive, this is an important

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<sup>4</sup>Gaiduch and Hunt (2000) find that there is very little attenuation on the estimated output gap in their simulations. However, in their case the estimation errors are highly serially correlated and procyclical. They do find some attenuation on the coefficient of the inflation forecast. They argue that one reason why attenuation may be found in Smets (1999) and Orphanides et al (2000) may be that those studies put constraints on the interest rate variation. Higher output gap uncertainty increases the variability of the economy and thus may increase interest rate variability. When this is constrained, it may lead to reduced response coefficients on output and inflation.

<sup>5</sup>See Svensson and Woodford (2000).

limitation of the analysis in this paper.

The rest of the paper is organised as follows. In Section 2 we lay out the model and discuss the calibration of the parameters. Careful calibration is necessary because some of the results are likely to be driven by the structure of the economy and in particular the covariance matrix of the shocks and measurement error. In Section 3 we then characterise the certainty-equivalent optimal monetary policy under both commitment and discretion using the results derived by Svensson and Woodford (2000). Section 4 analyses the implications of incomplete information on simple policy rules such as the Taylor rule or inflation-forecast based rules. In Section 5 we examine whether output gap uncertainty affects the optimal delegation to a weight-conservative central bank. Finally, in Section 6 we perform some sensitivity analysis by examining the robustness of the results with respect to changes in the model parameters and the assumption of symmetric information. Section 7 reviews the most important conclusions.

## 2 A simple backward/forward-looking model for policy analysis

In this Section we extend the model estimated in Smets (2000) to include a stochastic process for potential output. Various versions of this model have been used quite extensively to analyse a number of monetary policy issues.<sup>6</sup> We use this model as a laboratory for our analysis of monetary policy rules with incomplete information about potential output.

### 2.1 A backward/forward-looking model

The macro model consists of the following three equations:

$$y_t = \delta y_{t-1} + (1 - \delta) E_t y_{t+1} + \sigma(r_t - E_t \pi_{t+1}) + \varepsilon_t \quad (1)$$

$$\pi_t = \alpha \pi_{t-1} + (1 - \alpha) E_t \pi_{t+1} + \kappa(y_t - \bar{y}_t) + u_t \quad (2)$$

$$\bar{y}_t = \mu + \bar{y}_{t-1} + v_t \quad (3)$$

where  $y_t$  and  $\bar{y}_t$  denote actual and potential output,  $r_t$  the nominal short-term interest rate,  $\pi_t$  the inflation rate and  $E_t$  the expectation operator.

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<sup>6</sup>For a recent application see, for example, McCallum and Nelson (2000).

Equation (1) describes a forward-looking aggregate demand equation. Output depends on its lagged and expected value, the ex-ante real interest rate, and is subject to a demand shock,  $\varepsilon_t$ . The Phillips curve (2) relates inflation to its past and expected values as well as the output gap,  $z_t = y_t - \bar{y}_t$ . The shocks to the inflation equation,  $u_t$ , can be interpreted as cost-push shocks. Finally, potential output is assumed to follow a random walk with drift.<sup>7</sup> We assume that each of the three shocks are independent and serially uncorrelated with variances equal to  $\sigma_\varepsilon^2$ ,  $\sigma_u^2$  and  $\sigma_v^2$  respectively.

For  $\delta$  and  $\alpha$  equal to zero, this model provides the basis of the so-called new neo-classical synthesis, which can be derived from micro foundations (Goodfriend and King, 1997 and Woodford, 1999). In such a model, the forward-looking IS curve can be derived from the consumers' Euler equation, while the inflation equation is consistent with the first-order condition for price setting in a monopolistically competitive goods market with Calvo-style contracts. In such a model, the shocks to potential output can be viewed as shocks to productivity, while demand shocks can arise from shifts in preferences.<sup>8</sup>

The presence of lagged output and inflation in respectively the output and inflation equations is necessary to fit the persistence in the data, as explicitly discussed in Estrella and Fuhrer (1998). In the output equation such persistence can be justified on the basis of a model in which agents' utility functions exhibit habit persistence. Similarly, the presence of lagged inflation in equation (2) can be justified on the basis of a model where agents care about relative wages.

The appendix shows how this model can be written in the following state-space form, where we follow the notation used in Svensson and Woodford (2000) (SW) (See equation (14)).

$$\begin{bmatrix} X_{t+1} \\ x_{t+1/t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t/t} \\ x_{t/t} \end{bmatrix} + Br_t + \theta_t, \quad (4)$$

where  $X_{t+1}$  is the vector of predetermined state variables,  $x_t$  is the vector of forward-looking variables (in our case inflation and output),  $r_t$  is the central bank's policy instrument and  $\theta_t$  the vector of shocks hitting the economy. For any variable  $z_t$ , the index  $\tau/t$  is used to denote the rational expectation of  $z_\tau$ , given the information available at time  $t$ .

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<sup>7</sup>In the simulations, the autoregressive parameter will actually be put equal to a number slightly less than one in order to avoid invertibility problems.

<sup>8</sup>It is less straightforward to derive "micro-foundations" for the cost-push shocks. One possibility is that they arise from credibility problems.

## 2.2 Estimation and calibration

Ideally, we would want to estimate model (4) on annual data for the euro area.<sup>9</sup> One problem with doing so is that not all the state variables are observed. In particular, we do not observe potential output. As shown in the Appendix, the measurement equation corresponding to equation (1) can be written as follows:

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t/t} \\ x_{t/t} \end{bmatrix} + \tau_t \quad (5)$$

where  $Z_t$  is the vector of observable variables consisting of current output and inflation ( $Z_t = [y_t \ \pi_t]'$ ), and  $\tau_t$  are possible measurement errors. A second problem is that the model must be closed with a policy reaction function which determines the evolution of the short-term interest rate. Once such a reaction function is included, estimation can be done by solving the model in terms of the predetermined variables using a standard solution algorithm for linear rational expectations models, applying the Kalman filter to the system of dynamic equations in the predetermined variables and the related measurement equations and estimating the structural parameters by maximising the resulting likelihood.<sup>10</sup>

However, even for this relatively small model, this maximisation algorithm turned out to be rather complicated. The likelihood function is a highly non-linear function of the structural parameters and it turned out that it was difficult to obtain convergence to a reasonable model. Such a model needs to have a negative interest rate effect on output and a positively sloped short-run aggregate supply curve. One reason for these problems may be the limited degrees of freedom. The sample period consists of 24 years of annual euro area data with 7 parameters to be estimated. Another problem with obtaining estimates of this rational expectations model for the euro area is that it requires the estimation of a policy rule. Of course, such a single monetary policy rule was not implemented before 1999. Gerlach and Schnabel (2000) have argued that average monetary policy in the euro area since the 1990s can be described by a simple Taylor rule. However, before this period signs of instability in the average policy reaction function can be detected. It is a well-known short-coming of simultaneous-equations approaches that misspecification in one equation may affect the estimation of the other parameters.

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<sup>9</sup>We prefer annual data in order to keep the dimension of the system relatively small. In order to fit quarterly data a number of additional lags would have to be introduced.

<sup>10</sup>Such a procedure is discussed in Soderlind (1999).

For those reasons, we chose to calibrate the system in three steps. For the parameters in equations (1) to (3), we rely on the estimates provided in Smets (2000), who uses limited-information GMM methods. Using these parameter values, we then estimate the variances of the three structural shocks by running the Kalman filter through model (4) and (5) and maximising the likelihood function. The results (reported in Table 1) suggest that the variance of the shocks to potential output is about  $\sigma_v^2 = 0.13$ , and considerably smaller than the variance of the cost-push shocks ( $\sigma_u^2 = 0.42$ ) and demand shocks ( $\sigma_\varepsilon^2 = 0.63$ ). Finally, we also need to take a stand on the variances of the measurement error in equation (5). For this we rely on recent work by Coenen, Levin and Wieland (2001). They examine the size of revisions in euro area data on output and inflation and find that the typical revision in industrial production and real GDP data is much higher and more persistent than that in price data.<sup>11</sup> This is also consistent with the work on US data revisions by Orphanides (1998) and Rudebusch (1999). As we are dealing with an annual frequency, we assume that the variance of measurement error in current output is about 0.06 ( $\sigma_{\tau,y}^2 = 0.06$ ), while there is no measurement error in current annual inflation. After one year also output is assumed to be perfectly observed.<sup>12</sup>

### 3 Optimal policy under commitment and discretion

In this Section we examine optimal monetary policy under discretion and commitment when the economy can be described as in Section 2. We assume the following loss function

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau [\lambda z_{t+\tau}^2 + \pi_{t+\tau}^2 + \lambda_1 (r_{t+\tau} - r_{t+\tau-1})^2] \quad (6)$$

The central bank sets the short-term interest rate in order to minimise the variability in the output gap, the deviations of inflation from a zero target and the changes in the nominal interest rate. For our benchmark case, we assume equal weights on output gap and inflation stabilisation ( $\lambda = 1$ ) and a small weight on interest rate stabilisation ( $\lambda_1 = 0.1$ ). As in many other papers on optimal monetary policy rules, the interest rate smoothing motive

<sup>11</sup>In fact, there are no revisions in the HICP.

<sup>12</sup>Note that all the variables are expressed in percent. In a quarterly model one would also have to take into account that usually data are released with a considerable lag.

in the loss function is necessary to avoid empirically implausible interest rate variability under the optimal policy.<sup>13</sup>

The analysis of optimal policy in this Section follows SW. We assume that the central bank and the private sector have the same, possibly incomplete, information set.<sup>14</sup> Throughout the paper we compare the outcome in two cases. The first case is one with complete information (*CI*). For this we assume that agents observe current output and inflation, as well as current potential output. This information is sufficient to perfectly derive the three structural shocks to the model economy. The second case is the more realistic one of incomplete information discussed above (*II*). In this case the central bank and the private sector do not observe potential output directly. They do observe current output subject to a measurement error and have perfect observation of inflation. In the Appendix we characterise the dynamics of the economy under the optimal policies and derive an expression for the loss function and variances of the estimated state variables. For the actual derivation of the optimal policies under commitment and discretion we refer to SW.

SW show that in a linear quadratic framework the optimal policies under both discretion and commitment are certainty-equivalent. This extends the well-known certainty-equivalence result to forward-looking models with imperfect information about the state of the economy. More concretely, under both commitment and discretion the optimal policy rule, which relates the policy instrument to the best estimates of the state variables, does not depend on the covariance matrix of the structural shocks or the measurement errors. In addition, SW show that the separation principle continues to hold. This means that the best estimate of the state variables is independent of the policy followed. As a result estimation and control can be separated. In line with this separation result, we first discuss the optimal policy, then look at the information value of the various indicators and finally examine the value of information about potential output in stabilising the economy.

### **3.1 The optimal instrument rule under commitment and discretion**

For the calibration discussed above, the optimal policy under discretion is characterised by the following reaction function:

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<sup>13</sup>See, for example, Rudebusch and Svensson (1999).

<sup>14</sup>In Section 6 we briefly discuss the case of asymmetric information.



$$r_t = F_d X_{t/t} = 1.02y_{t-1} + 0.72\pi_{t-1} - 1.12\bar{y}_{t/t} + 2.32\varepsilon_{t/t} + 1.51u_{t/t} + 0.44r_{t-1} \quad (7)$$

The corresponding policy rule under commitment is described by:

$$r_t = F_c X_{t/t} + \Phi \Xi_{t-1} = 0.77y_{t-1} + 0.72\pi_{t-1} - 0.85\bar{y}_{t/t} + 1.75\varepsilon_{t/t} + 0.67u_{t/t} + 0.55r_{t-1} - 0.57\Xi_{1,t-1} - 0.19\Xi_{2,t-1} \quad (8)$$

where  $\Phi$  is the vector of reaction coefficients to the Lagrange multipliers associated with the forward looking variables ( $\Xi_1$  refers to the Lagrange multiplier on the constraint given by the output equation,  $\Xi_2$  is the multiplier arising from the inflation equation). Under both commitment and discretion nominal interest rates rise in response to positive output and inflation developments and fall in response to a positive shock to potential output. The coefficient on the lagged interest rates is around a half. In addition, as emphasised by Woodford (1999), under commitment interest rates will respond to the whole history of predetermined state variables through its dependence on the Lagrange multipliers.

### 3.2 The Kalman gain matrix and the information value of various indicators

In order to discuss the information value of the various observable variables, it is useful to calculate the Kalman gain matrix ( $K$ ) and the corresponding updating matrix ( $U$ ). The role of the Kalman gain matrix is described in SW. They furthermore develop the derivation of an updating matrix, which shows how new information on the observable variables is used in order to update the estimates of the state variables.<sup>15</sup> The new estimate is then found by  $X_{t/t} = X_{t/t-1} + U(Z_t - Z_{t/t-1})$ . The vector of observable variables is given as  $[y_{t-1} \ \pi_{t-1} \ \bar{y}_t \ y_t \ \pi_t]'$ , and the state variables are  $[y_{t-1} \ \pi_{t-1} \ \bar{y}_t \ \varepsilon_t \ u_t \ r_{t-1}]'$ . An element  $U_{i,j}$  indicates therefore, how information on observable  $j$  is used to update the estimate for state variable  $i$ .

Equations (9) and (10) give the updating matrix under complete and incomplete information. Comparing these equations, it becomes clear that the inference problem is much less trivial for the case of imperfect information. Now, current potential output needs to be estimated from the available

<sup>15</sup>In the notation of SW, this updating matrix is  $K(I + MK)^{-1}$ .

observable variables, which is reflected by the entries in the third row. Additionally, the output measurement is less accurate, which reduces the weight output receives in the estimates (as can be seen by comparing the fourth columns of  $U_{CI}$  and  $U_{II}$ ).

$$U_{CI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -.440 & 0 & 0.491 & 0.959 & 0.202 \\ 0 & -.480 & 0.180 & -.0180 & 0.761 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

$$U_{II} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ .057 & .638 & 0 & .056 & -.261 \\ -.429 & -.296 & 0 & 0.850 & 0.318 \\ .003 & -.368 & 0 & -.154 & .716 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

### 3.3 Economic stabilisation and the value of information

We are now ready to compare the outcome of optimal monetary policy under complete and imperfect information. Table 2 gives the value of the loss function together with the standard deviations of the goal variables in the various cases and for different weights in the loss function. A number of observations can be made. First, it can be seen that both under discretion and commitment, the loss that arises from imperfect information is considerable. The cost of imperfect information is highest when the weight on output gap stabilisation is high. This reflects the fact that it is mostly the standard deviation of the output gap that increases. Somewhat surprisingly, incomplete information about the output gap does not affect the ability of the central bank to stabilise inflation and changes in nominal interest rates very much.

Second, in all cases there is a clear value of commitment. As discussed in Svensson (1997) and Clarida, Gali and Gertler (1999), the value of commitment in this model does not follow from avoiding an average inflation bias, but from eliminating a stabilisation bias. This bias arises from the fact that a central bank would like to promise to keep being tough in the future in response to a current cost push shock in order to stabilise current inflation expectations and thus current inflation. However, this promise is

time-inconsistent. Once inflation expectations adjust and time passes, the central bank has an incentive to renege on its promise in order to avoid the output cost. From Table 2 there are, however, no clear indications that the gains from commitment are relatively larger when there is less complete information. It is interesting to note that the gains in a shift from discretion to commitment are mostly due to a fall in the variability of inflation, while the volatility of the output gap actually increases somewhat. In sum, while the gains from commitment are mostly reflected in an improved stabilisation of inflation, in our framework better information is mostly beneficial in terms of reduced variability in the output gap.

[Insert Table 2: Incomplete information and economic stabilisation]

Figures 1a and b plot the response of the actual and perceived output gap, output, inflation and the nominal interest rate to each of the four shocks under *II*.<sup>16</sup> The most striking feature of these impulse response functions is how persistent the prediction errors in the output gap are in response to a potential output shock or a cost-push shock. When the central bank observes a rise in output and a fall in prices, this could be due to either a positive supply shock or a negative cost push shock. However, the implications for the response of the output gap are opposite. In the case of a negative cost-push shock the output gap is expected to rise, while in the case of a positive shock to potential output the output gap should fall. As a result, the central bank under-predicts the output gap in the case of a cost-push shock, while it over-predicts the output gap in response to a shock to potential output. In the latter case, the central bank persistently perceives a positive output gap, while the actual gap is negative. As a result, even after ten years the prediction error is still more than 10 basis points.<sup>17</sup>

[Insert Figure 1a and b: Impulse responses: actual versus perceived]

In contrast, in response to demand shocks the actual and perceived output gap respond very similarly. Only during the first period, there is a small perception error due to the presence of measurement error in output. This is due to the assumption that these measurement errors only last for one period. As can be seen from the last row of Graph 1a and b, an output measurement error does lead to some instability in the economy as the central bank responds to a perceived positive output gap.

<sup>16</sup>The impulse responses are depicted for positive shocks to potential output, demand and the measurement error, but to a negative cost-push shock.

<sup>17</sup>Of course, if the central bank observed other variables which would be related to potential output, but not the cost-push shock the updating could be much faster.

[Insert Figures 2a and b: Impulse responses: complete versus incomplete information]

Figures 2a and 2b directly compare the impulse responses of the actual output gap, inflation and interest rate under *CI* and *II*. It is clear that *II* unambiguously leads to a magnified response of the output gap to both cost push and supply shocks. The effects on inflation are, however, opposite. The negative response of inflation to a positive supply shock is larger in absolute value under *II* than under *CI*, while it is smaller in response to a negative cost push shock. The intuition is clear: in response to a negative cost-push shock the central bank assigns some probability that this is actually a positive supply shock. As a result, it will lower the real rate by more than it would otherwise have done. This leads to a larger response in the output gap, and a smaller fall in inflation. The opposite occurs in response to a positive supply shock. In this case, the central bank assigns some probability to the fact that this is actually a negative cost push shock. As a result, it will lower real interest rates less than it would have done under *CI*. In turn, this leads to a larger fall in the output gap and a larger fall in inflation. This intuition explains the result, discussed above, that the variability of inflation is more or less unaffected by the large uncertainty regarding the output gap. The reason is that the misperception of the output gap has offsetting effects on inflation depending on whether it is caused by cost push or supply shocks.

Figures 2a and 2b also show the effect of incomplete information on the setting of the nominal interest rate. In response to a supply shock, the slow updating of the central bank's estimate of potential output leads to a much more persistent effect on inflation and thus the nominal interest rate. In contrast, in response to a cost-push shock, not the persistence but the amplitude of the interest rate response is affected.

Finally, Figure 3 compares the commitment and discretionary outcome in response to a supply and cost push shock under both *CI* and *II*. The second column shows that in all cases the lack of a commitment mechanism leads to a stronger response of inflation to shocks. This is a manifestation of the so-called stabilisation bias and explains why the cost of discretion is mostly in terms of higher inflation variability.<sup>18</sup> However, this difference is most striking under incomplete information when a shock to potential output occurs. The combination of slow updating of the new potential output level and policy discretion leads to a more persistent effect on inflation.

[Insert Figure 3: Impulse responses: commitment versus discretion]

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<sup>18</sup>See, for example, Svensson (1999).

Overall, the analysis in this Section suggests that incomplete information about potential output has substantial effects on the stabilisation and the dynamics of the economy. Even if central banks continuously update their estimate of potential output and thus do not make any systematic estimation errors, *ex post* the misperception of the output gap will be substantial and very persistent.<sup>19</sup> This leads to a substantial increase in output gap fluctuations. In particular in response to shocks to potential output, such misperception errors may lead to a very persistent effect on inflation, which will be exacerbated when the central bank optimises under discretion. These results suggest that the findings of Orphanides (2000) that the Federal Reserve Board made very persistent serially correlated estimation errors in the output gap in the 1970s is not necessarily evidence of systematic *ex ante* mistakes. Instead, it could be the outcome of a slow, but rational updating process in the face of an unobservable shift in the potential output growth rate. Nevertheless, we also find that the overall variance of inflation is not very much affected. This is because under incomplete information, the response of inflation to cost-push shocks will be mitigated.

## 4 Potential output uncertainty and simple instrument rules

Following the discussion of optimal monetary policy in the previous Section, we investigate in this Section the effect of incomplete information about potential output on the stabilisation properties of simple instrument rules. As we assume that the central bank can commit to such a simple rule, the stabilisation outcome of these rules should be compared with the outcome when the monetary authorities can commit to a policy that minimises society's loss function. Our benchmark loss function is the same as in Section 3. We think it is useful to analyse simple instrument rules for three reasons. First, there are several papers which have shown that simple rules yield similar macroeconomic stability to optimal rules.<sup>20</sup> Second, simple rules have been shown to be more robust to model uncertainty than complicated optimal rules.<sup>21</sup> Finally, the analysis of Clarida, Gali and Gertler (1997) suggests that simple

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<sup>19</sup>Lansing (2000) argues that some of the low frequency movements in output and inflation following the slow-down in productivity growth in the early 1970s can be due to the gradual learning by the central bank of the shift in potential output growth. However, he does not consider optimal signal extraction.

<sup>20</sup>See, for example, Rudebusch and Svensson (1999) or Williams (2000).

<sup>21</sup>See, for example, Levin, Wieland and Williams (1999).

rules are a reasonable approximation of how policy-makers actually behave.<sup>22</sup> It is therefore of interest to see how the performance of simple rules is affected by incomplete information about potential output.

## 4.1 Taylor rules and nominal income growth rules

Tables 3a and 3b compare the results for efficient Taylor rules with those for efficient inflation and output growth rules under both *CI* and *II*. The first column reports the value of the loss function, while the other columns report the value of the optimal reaction coefficients. The arguments in these rules are inflation and the central bank's best estimates of the output gap or output growth. We report the results of both rules with and without a response to the lagged interest rate. In the case of the inflation and output growth rule, assuming equal reaction coefficients on inflation and real growth would result in a nominal income growth rule as, for example, advocated by McCallum (1997). Here we allow for separate coefficients in order not to bias the results against the nominal income growth rule. A comparison between level and growth rules is of interest because several researchers have argued that in view of the uncertainty about the level of potential output it is advisable to respond to the growth rate of output rather than its level.<sup>23</sup>

[Insert Table 3a and b: Taylor and income growth rules]

Several results deserve to be highlighted. First, it is clear that the rules that include a response to the lagged interest rate do much better than those without such a response. Both under *CI* and *II*, the optimal simple interest rate rule involves a reaction coefficient on the lagged interest rate which is close to one. In other words the optimal rule can be written as a first-difference rule. Under complete information this is consistent with the results reported in Williams (2000) that simple first-difference rules do much better than simple level rules. The reason is that in forward-looking models a commitment to first-difference rules has stabilising effects on current inflation because it promises to increase interest rates in the future as long as the goal variables deviate from their target.<sup>24</sup> What may be more surprising is that this feature of an efficient simple rule is maintained under incomplete information about potential output. Second, even with the considerable degree of uncertainty regarding the output gap, the first-difference Taylor rule

<sup>22</sup>See Gerlach and Schnabel (2000) for an application to the euro area.

<sup>23</sup>See, for example, McCallum (1988, 1997) and Orphanides (2000). For a recent discussion of the pros and cons of nominal income growth rules, see also Rudebusch (2000).

<sup>24</sup>See also the discussion in Clarida et al (1999).

delivers outcomes that are quite close to the optimal policy under commitment. The first-difference growth rule does not perform as well, although incomplete information does improve its relative performance.

Third, incomplete information does lead to some attenuation in the Taylor rule as the reaction coefficients on the output gap falls. This is clearest in the case of the Taylor rules without a lagged interest rate. The reaction coefficient on the output gap falls by about a half from 0.81 to 0.41. It is less clear in the case of the first-difference rules where the weight on the output gap falls only marginally from 1.33 to 1.29. The reaction coefficient on inflation remains largely unaffected. This contrasts somewhat with the results reported by Smets (1999), Tetlow (2000) and Orphanides et al (2000), who find that attenuation also applies to the reaction coefficient on inflation.

SW and Swanson (2000) have highlighted that while certainty equivalence holds with respect to the reaction coefficients in the optimal policy rule written in terms of the best estimates of the state variables, the weight put on noisy indicators in estimating those state variables will fall as the signal-to-noise ratio increases. As emphasised by Swanson (2000) in a simple model, it may also be the case that higher noise in one indicator increases the weight on other indicators with less noise (This is, however, not a general result). The results in this Section show that simple Taylor rules appear quite robust in the face of considerable uncertainty about the output gap as long as the central bank uses its best estimate of the output gap. However, at the same time it is to be expected that this robustness breaks down when the estimated output gap is mis-specified. By way of example, the last two rows of Table 3a report the results when the central bank thinks potential output follows a deterministic path. In that case it will associate its best estimate of the output gap with its best estimate of current output. As one can see, this mis-specification of the output gap leads to an even larger fall in the reaction coefficient on output.

In sum, the analysis in this Section shows that an efficient Taylor rule in first-difference form performs very well even with incomplete information about potential output, as long as the central bank uses its best estimate of the output gap. However, as this estimate is a quite complicated function of past observable variables and, moreover, model dependent, the Taylor rule loses a bit of its attraction as a “simple” policy benchmark. This loss of simplicity could be overcome by using a Taylor rule written only in terms of observable variables that can be easily verified. However, in this case the performance of the Taylor rule in stabilising the economy will deteriorate significantly in the face of high uncertainty about the true output gap.

## 4.2 Simple inflation-forecast-based rules

A second class of simple instrument rules that we want to analyse are so-called inflation-forecast-based (IFB) rules as advocated by Black et al (1997) and Batini and Haldane (1999). In those rules, the policy rate responds to its own lag and an inflation forecast at a particular horizon.

[Insert Table 3c: Simple IFB rules]

Table 3c compares the results of a simple one-year-ahead IFB rule with a rule which feeds back on current inflation. Both an unconditional inflation forecast and a forecast conditional on a constant interest rate are used. Under both *CI* and *II*, the IFB rules dominates the simple current inflation rule, although the difference in loss is smaller under *II*. The IFB rule is, however, dominated by the simple Taylor rule in first-difference form discussed in the previous Section. By and large, the reaction coefficients on the inflation forecast remain unaffected by the output gap uncertainty. In addition, it is worth noting that the conditional IFB rule performs somewhat better than the unconditional IFB rule. This is consistent with the results reported in Rudebusch and Svensson (1999).

## 5 Output gap uncertainty and optimal delegation

### 5.1 The optimal degree of “conservativeness”

Recently, Svensson (1997) and Clarida et al (1999) have revived Rogoff’s (1985) argument that delegating monetary policy to a more weight-conservative central banker may improve on the discretionary equilibrium in a model without an average inflation bias similar to the one used in this paper. In these models, cost-push shocks feed through into future inflation and thereby lead to a deterioration of the future inflation/output variability trade-off. Ex ante the central bank has an incentive to promise to continue being tough in the future in response to such a cost-push shock. However, ex-post a discretionary central banker will re-optimize and respond only gradually faced with this deteriorated trade-off. The private sector will realise this ex-post incentive and accordingly increase its inflation expectations, which in turn leads to higher response of current inflation to such a shock. This will lead to a so-called stabilisation bias, whereby for given shocks to the economy inflation variability is higher in the discretionary versus the commitment equilibrium. As discussed before, the results reported in Table 2 confirm this cost of the



lack of a commitment mechanism. Appointing a more weight-conservative central banker (i.e. a central banker which puts less weight on output gap stabilisation) may alleviate this stabilisation bias, because such a central banker will care less about output gap variability faced with higher inflation in the future. The cost of doing so is that the stabilisation of the real economy in response to cost-push shocks may be sub-optimal. The optimal weight will be achieved by balancing the marginal cost and benefit.

In this Section we examine whether output gap uncertainty provides a rationale for appointing an even more conservative central banker. Such a result would suggest that the value of commitment is even larger under *II* than under *CI*. As discussed in Section 3, under incomplete information the inflation response to a potential output or cost-push shock becomes indeed much more persistent under discretion than under commitment.

[Insert Figure 4: Incomplete information and the optimal weight on output gap stabilisation]

Figure 4 summarises the solution to the optimal delegation problem in our calibrated model. It plots the loss as a function of the relative weight on output gap stabilisation assigned to the central bank. We assume that the central bank's assigned loss function is given by

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[ (1 - \lambda_1) \left[ \lambda z_{t+\tau}^2 + (1 - \lambda) \pi_{t+\tau}^2 \right] + \lambda_1 (r_{t+\tau} - r_{t+\tau-1})^2 \right] \quad (11)$$

When the assigned weight on output gap stabilisation is  $\lambda = 0.5$ , the central bank has the same weights as society and the resulting outcome is the one in a discretionary equilibrium.<sup>25</sup> The loss under discretion and *CI* or *II* is given by the two horizontal lines. Appointing a more conservative central banker (i.e. one with a lower weight on output gap stabilisation) leads to an improvement in the loss. This is an example of the result emphasised by Clarida et al (1999). Moreover, we find that the optimal weight on output gap stabilisation is lower under *II* ( $\lambda = 0.1$ ), and hence the central bank is more conservative, than it is under *CI* ( $\lambda = 0.2$ ). This appears to confirm the intuition that in the absence of a commitment mechanism, incomplete information about potential output does provide a rationale for appointing a central bank that puts less weight on output gap stabilisation.

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<sup>25</sup>The weight on interest rate stabilisation does not change.

## 5.2 The optimal weight on price level stability

A number of authors (e.g. Vestin (2000), Svensson (1999)) have argued that giving a price level stabilisation objective to a discretionary central bank may also alleviate the time-inconsistency problem and lead to a welfare improvement compared to the discretionary equilibrium. Indeed, Gaspar and Smets (2000) show that, in a model with complete information very similar to the one examined in this paper, putting a small weight on price level stabilisation in the central bank's loss function leads to an inward shift of the output/inflation variability efficiency frontier. The loss function in such a case can be described by

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[ (1 - \lambda_1)(1 - \lambda_2) \left[ \lambda z_{t+\tau}^2 + (1 - \lambda) \pi_{t+\tau}^2 \right] + \lambda_1 (r_{t+\tau} - r_{t+\tau-1})^2 + \lambda_2 p_{t+\tau}^2 \right] \quad (12)$$

Figure 5 examines how incomplete information about potential output affects the optimal weight on price level stability. Again we find that output gap uncertainty leads to a more conservative mandate. In our specific model, the weight on price level stability increases from  $\lambda_2 = 0.08$  under *CI* to  $\lambda_2 = 0.09$  under *II*. Comparing Figures 4 and 5, it is worth noting that giving the central bank a price stability objective is more effective than reducing the weight on output gap stabilisation.

[Insert Figure 5: Incomplete information and the optimal weight on price level stability]

## 6 Some sensitivity analysis

One of the striking features of the analysis in this paper is that the misperception of the output gap in response to cost-push or potential output shocks is very persistent. In this Section we analyse the robustness of this result with respect to changes in the parameters of the model and the assumption of symmetric information.

### 6.1 Changing the relative variance of the potential output and cost-push shocks

The results so far have been calculated using a set of calibrated parameter values. Given the uncertainty of such parameter values, this section checks

the results obtained in the preceding sections for their sensitivity with respect to parameter changes. In general, we find that most results carry through qualitatively if the structural parameters are varied. For example, doubling the slope of the Phillips curve and/or the interest rate sensitivity of output leaves the results basically unchanged.

More interesting is whether a change in the relative variances of the shocks affects the results. In the benchmark case, cost-push shocks are much more volatile than shocks to potential output, which is reflected in the central bank's response to shocks: facing uncertainty about the nature of a shock hitting the economy, it will be led to believe for a long time that the shock was indeed a cost-push shock, thus creating the persistent perception errors mentioned in section 3. It is interesting to see whether this persistence will continue to hold with different relative variances of the shocks.

[Insert Figure 6: Perception errors in the output gap and the relative variance of cost-push and potential output shocks]

Figure 6 reports, for the discretionary case, the perception error on the output gap following a shock to potential output and a cost-push shock for various relative variances. The variances have been calculated such that the initial perception error for a shock to potential output is found to be the same. The solid line represents the benchmark case. The lines above result from cases where the relative variances of cost-push shocks are lower than in the benchmark case, the line below shows a case where cost-push shocks are even more volatile than before. It becomes clear that the persistence of perception errors is indeed a feature of the relative variances. The dotted line represents a case where both variances are equal, and the short dashed line a case where the variance of cost-push shocks is half of the one of potential output shocks. It can be seen that the persistence of the perception error continuously decreases, rather than finding a minimum when both variances are equal. On the other hand, the errors remain fairly persistent for all cases; several years are needed to reduce the perception bias regardless of the relative variances.

## 6.2 The case of asymmetric information

Following SW, we have up to now analysed the case of symmetric information. In this Section we briefly discuss the case when the private sector does observe potential output and thus can distinguish between the various shocks. We continue to assume that the central bank only observes output and inflation. However, the central bank does know that the private sector has knowledge

about the underlying shocks.<sup>26</sup> Dotsey and Hornstein (2000) have shown that in this case certainty equivalence continues to hold when the central bank optimises under discretion. In other words, the optimal discretionary policy is independent of the estimation errors and the estimation problem more generally. However, the estimation problem and more specifically the Kalman gain matrix will depend on the optimal policy. In other words, the separation principle does not hold. In order to solve the estimation problem we follow Dotsey and Hornstein (2000).<sup>27</sup>

[Insert Figure 7: Symmetric versus asymmetric incomplete information (under discretion)]

Figure 7 compares the impulse response functions under optimal discretionary policy in the three cases: complete information (CI), symmetric incomplete information (SCI) and asymmetric incomplete information (ACI). First, it is clear from Figure 7 and a comparison of the loss function and the unconditional variances that the central bank can use the private sector's knowledge to improve on the stabilisation of the output gap and the economy more generally. The loss function lies in between that of CI and SCI. This is mostly due to a fall in the variance of the output gap.<sup>28</sup> The first column of Figure 7 clearly shows that the effect of the potential output and cost-push shock on the output gap is smaller in size and much less persistent.

[Insert Figure 8: Perception errors in the output gap and asymmetric information]

Figure 8 shows that also the perception error is much less persistent when the private sector observes potential output. The half-life of the perception error falls from about 8 years to 4 years. Interestingly, this ranking of the three cases does not hold for the initial response of inflation. When the private sector knows the underlying shock, inflation initially responds more

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<sup>26</sup>The assumption that the private sector has complete information, while the central bank has not is of course extreme, but is, as such, useful as a benchmark. The assumption that the private sector has more information than the central bank may not be so unrealistic, once one observes that the private sector is atomistic. Each individual consumer and/or firm can observe which shock is relevant and this information is aggregated in economy wide output and inflation. However, it is difficult for the central bank to survey each of these private agents to find out which shocks have hit each of them.

<sup>27</sup>See Appendix A3 of Dotsey and Hornstein (2000).

<sup>28</sup>The loss under AII is 80.11 which compares with 71.04 under CI and 90.45 under SII. This difference is mostly due to the different stabilisation of the output gap. The standard deviation of the output gap under AII is 1.16 compared to 1.00 under CI and 1.31 under SII.

strongly to a positive supply shock. The intuition is the following. When the private sector knows a positive potential output shock has hit the economy, it also knows that the central bank will underestimate this shock and, as a result, will reduce its policy rate less than it would have done under complete information. As a result, the private sector will expect a greater disinflation than when it did not know which shock had hit the economy. This will feed through in a larger fall of current inflation. As the central bank observes this larger than expected disinflation, the central bank will upgrade its probability that a potential output shock has hit the economy and pursue a more aggressive monetary policy. This will be reflected in a smaller and less persistent response of the output gap. The opposite occurs when a negative cost push shock affects the economy.

## 7 Conclusions

In this paper we have investigated the implications of incomplete information about potential output for the conduct and design of monetary policy using a small backward/forward looking model calibrated to fit annual euro area data. The model features three shocks: a shock to potential output, a cost-push shock and a demand shock. Using a standard quadratic loss function in inflation and output gap stabilisation, we systematically compare optimal monetary policy under two assumptions regarding the information available to the central bank and the private sector. In the first case the information set contains both current and potential output and inflation. As a result, the central bank can perfectly deduce which of the three shocks is hitting the economy and adjust its policy rate accordingly. In the second case, information is incomplete. The central bank does observe current inflation, but does not observe potential output and has only a noisy measure of current output. The measurement error is calibrated to fit revisions in euro area output data. As a result, the central bank faces a signal extraction problem when setting the optimal interest rate.

Our main results can be summarised under three headings. First, we characterise and compare the optimal monetary policy under both commitment and discretion in the two informational cases using recent results of Svensson and Woodford (2000). We find that the loss due to incomplete information is substantial and mainly results from a significant increase in the variability of the output gap. In our benchmark case, the standard deviation of the output is about one third higher under incomplete information. Even if a central bank continuously updates its estimate of potential output and thus *ex ante* does not make any systematic estimation errors, *ex post* the

misperception of the output gap is substantial and very persistent in particular in response to a shock to potential output. This result would suggest that the empirical findings of Orphanides (2000) that the Federal Reserve Board made serially correlated estimation errors in the output gap in the 1970s may not be evidence of systematic *ex ante* mistakes. We also find that in response to a shock to potential output, such misperception errors lead to a very persistent effect on inflation, which is exacerbated when the central bank optimises under discretion. Nevertheless, the overall variance of inflation is not very much affected, because under incomplete information, the response of inflation to cost-push shocks is mitigated.

Second, we investigate the implications of incomplete information for simple policy rules such as a Taylor rule or an inflation-forecast based rule. Imperfect information about potential output leads to some limited attenuation in the Taylor rule, but does not affect the inflation-forecast based rule. Both rules (and in particular the Taylor rule) continue to perform relatively well compared with the commitment equilibrium as long as the output gap and the inflation forecast are optimally estimated. However, when the central bank misspecifies the output gap (for example, because it does not use the correct model for potential output) the performance of the Taylor rule deteriorates considerably.

Finally, we examine the implications of the unobservability of potential output for the optimal delegation of monetary policy to a discretionary central bank. Using two examples, we find that incomplete information about the output gap implies that it is optimal to appoint a relatively more conservative central bank than in the complete information case. In the first example following Rogoff (1985) and Clarida, Gali and Gertler (1999), this is reflected in the fact that society would like to appoint a more weight-conservative central banker (i.e. a central banker with a smaller weight on output gap stabilisation in his/her loss function). In the second example following Vestin (2000), this is reflected in society's desire to put a larger weight on a price stability objective under incomplete information.

Overall, our results suggest that incomplete information about potential output does not significantly overturn results regarding optimal monetary policy that are derived in models with perfect information, as long as the central bank uses its best estimates of the state of the economy and has the true model of the economy at its disposal. Of course, these results very much depend on the linear-quadratic framework used in this paper and are likely to break down when model uncertainty and/or non-linearities are considered. The results do strongly indicate that there are potentially large gains to be made from trying to identify the particular shocks that hit the economy at any point in time. An early identification of the nature of the shocks

that affect the economy allows for a more appropriate policy response and improves the stabilisation of the economy significantly.

## A Appendix

Much of the analysis in this paper is an application of the work by Svensson and Woodford (2000) (SW). In a general linear-quadratic set-up, SW discuss the optimal policy and the associated signal extraction problem when a central bank optimises under discretion (i.e. it reoptimises every period) and under commitment. Here we also consider efficient simple policy rules such as an optimised Taylor rule or a nominal income growth rule. The first Section of this Appendix shows how the model can be written in state space form. Section A.2 describes the optimal policy for commitment and discretion. Section A.3. shows how to calculate the loss function for a given policy. In what follows, we try to use the same notation as in SW. We also refer to SW for the derivation of the optimal policy under commitment and discretion.<sup>29</sup>

### A.1 The model in state space form

Defining  $\gamma = (1 - \delta)(1 - \alpha)$ , the model described in equations (1) to (3) can be written in state space form as follows:

$$\begin{bmatrix} X_{t+1} \\ x_{t+1/t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t/t} \\ x_{t/t} \end{bmatrix} + Br_t + \theta_{t+1}, \quad (13)$$

where

$$X_{t+1} = \begin{bmatrix} y_t & \pi_t & \bar{y}_{t+1} & \varepsilon_{t+1} & u_{t+1} & r_t \end{bmatrix}, x_{t+1/t} = \begin{bmatrix} E_t y_{t+1} & E_t \pi_{t+1} \end{bmatrix},$$

$$\theta_{t+1} = \begin{bmatrix} v_{t+1} & \varepsilon_{t+1} & u_{t+1} \end{bmatrix}, A_2 = 0,$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\delta}{1-\delta} & -\frac{\alpha\sigma}{\gamma} & \frac{\sigma\kappa}{\gamma} & -\frac{1}{1-\delta} & -\frac{\sigma}{\gamma} & 0 & \frac{1-\alpha-\sigma\kappa}{\gamma} & \frac{\sigma}{\gamma} \\ 0 & -\frac{\alpha}{1-\alpha} & \frac{\kappa}{1-\alpha} & 0 & -\frac{1}{1-\alpha} & 0 & -\frac{\kappa}{1-\alpha} & \frac{1}{1-\alpha} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{\sigma}{1-\delta} \\ 0 \end{bmatrix}$$

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<sup>29</sup>In order to calculate the optimal policies, we modified the Gauss-routines of Soederlind (1999) to take into account the imperfect information case discussed in SW(1999).



The observation equation (corresponding to equation (5) in the text) for this model is given by:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \bar{y}_t \\ \varepsilon_t \\ u_t \\ r_{t-1} \\ y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \tau_t^y \\ 0 \end{bmatrix} \quad (14)$$

Hence the matrix  $D^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ , and  $D^2 = 0$ . The goal variables are given by:

$$\begin{bmatrix} z_t \\ \pi_t \\ \Delta r_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \bar{y}_t \\ \varepsilon_t \\ u_t \\ r_{t-1} \\ y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r_t \quad (15)$$

## A.2 Optimal monetary policy

### A.2.1 Discretionary case

When the central bank optimises under discretion, the optimal policy results in a set of dynamic equations for the predetermined variables:

$$X_{t+1} = HX_t + JX_{t/t} + u_{t+1} \quad (16)$$

$$X_{t/t} = X_{t/t-1} + K [L(X_t - X_{t/t-1}) + v_t] \quad (17)$$

$$X_{t+1/t} = (H + J)X_{t/t} \quad (18)$$

In addition, the law of motion of the actual and perceived forward-looking variables and the policy instrument are given by:

$$x_t = G^1 X_t + G^2 X_{t/t} \quad (19)$$

$$x_{t/t} = G X_{t/t} \quad (20)$$

$$r_t = F X_{t/t} \quad (21)$$

where the matrices  $H$ ,  $J$ ,  $K$ ,  $L$ ,  $G^1$ ,  $G^2$ ,  $G$  and  $F$  are defined in SW (1999).

### A.2.2 Simple rules

A similar set of dynamic equations can be derived when the central bank commits to a simple instrument rule of the following form,

$$r_t = F^1 X_{t/t} + F^2 x_{t/t} \quad (22)$$

Using (A.14) and () the dynamics of the perceived state variables can be described by:

$$\begin{bmatrix} X_{t+1/t} \\ x_{t+1/t} \end{bmatrix} = (A^1 + A^2 + B[F^1 + F^2]) \begin{bmatrix} X_{t/t} \\ x_{t/t} \end{bmatrix} \quad (23)$$

Using, for example, Klein's algorithm for solving linear rational expectations models, this yields the following expression for the forward-looking variables:

$$x_{t/t} = G X_{t/t} \quad (24)$$

$$r_t = (F^1 + F^2 G) X_{t/t} = F X_{t/t} \quad (25)$$

Taking into account the new expression for  $G$  and  $F$ , equations identical to those above can be derived.

### A.2.3 Commitment

Finally, when the central bank optimises under commitment, the corresponding set of dynamic equations can be derived:

$$X_{t+1} = H X_t + J X_{t/t} + \Psi \Xi_{t-1} + u_{t+1} \quad (26)$$

$$X_{t/t} = X_{t/t-1} + K [L(X_t - X_{t/t-1}) + v_t] \quad (27)$$

$$X_{t+1/t} = (H + J)X_{t/t} + \Psi\Xi_{t-1} \quad (28)$$

$$\Xi_t = SX_{t/t} + \Sigma\Xi_{t-1} \quad (29)$$

where  $\Xi_t$  is the vector of Lagrange multipliers associated with the equations for the forward-looking variables. The forward-looking variables and the policy instrument are defined by:

$$x_t = G^1 X_t + G^2 X_{t/t} + \Gamma\Xi_{t-1} \quad (30)$$

$$x_{t/t} = GX_{t/t} + \Gamma\Xi_{t-1} \quad (31)$$

$$r_t = FX_{t/t} + \Phi\Xi_{t-1} \quad (32)$$

Again the matrices are defined in SW (1999). One of the main differences between the outcome under discretion versus the outcome under commitment is that all the variables now also depend on the Lagrange multipliers associated with the equations for the forward-looking variables. As discussed in SW(1999) this leads to history-dependence as the Lagrange multipliers are themselves functions of past predetermined variables.

### A.3 The value of the loss function:

In this Section we derive an expression for the value of the loss function in the case of a discretionary central bank or with simple rules. A similar expression can be derived for the case of commitment.

The value of the loss function is given by:

$$J_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} \quad (33)$$

This can be rewritten as:

$$J_t = L_{t/t} + \beta J_{t+1/t} \quad (34)$$

Because the problem is linear-quadratic, the loss-function can be written as a quadratic form in the perceived predetermined state variables:

$$J_t = X_{t/t}^T V X_{t/t} + w \quad (35)$$

Substituting (A.13) into (A.14) yields:

$$X_{t/t}^T V X_{t/t} + w = L_{t/t} + \beta E_t \{ X_{t+1/t+1}^T V X_{t+1/t+1} + w \} \quad (36)$$

Using equations (A.4-6) in equation ( ) yields:

$$Y_t = Ch^1 X_{t/t} + Ch^2 (X_t - X_{t/t}) \quad (37)$$

where  $Ch^1 = C_1 + C_2 G + C_i F$  and  $Ch^2 = C_1^1 + C_2^1 G^1$ .

Therefore, the first term on the right-hand-side of equation (A.14) can be written as:

$$L_{t/t} = X_{t/t}^T Ch^{1T} W Ch^1 X_{t/t} + E_t \{ (X_t - X_{t/t})^T Ch^{2T} W Ch^2 (X_t - X_{t/t}) \} \quad (38)$$

Using equations (A.1-3) yields the following expression:

$$X_{t+1/t+1} = (H + J)X_{t/t} + KLH(X_t - X_{t/t}) + KLu_{t+1} + Kv_{t+1} \quad (39)$$

Because each of the terms in (A.17) are independent of each other, the second term in equation (A.14) can be written as:

$$\begin{aligned} X_{t/t}^T (H + J)^T V (H + J) X_{t/t} + E_t \{ (X_t - X_{t/t})^T (KLH)^T V (KLH) (X_t - X_{t/t}) \} \\ + E_t \{ u_{t+1}^T (LK)^T V KLu_{t+1} \} + E_t \{ v_{t+1}^T K^T V Kv_{t+1} \} \end{aligned} \quad (40)$$

Substituting equations (A.16) and (A.18) in (A.14) and collecting terms, the following expressions for  $V$  and  $w$  can be derived:

$$V = Ch^{1T} W Ch^1 + \beta (H + J)^T V (H + J) \quad (41)$$

$$w = \frac{1}{1 - \beta} \{ \text{trace}(w^1 P_{t/t}) + \beta \text{trace}(w^2 P_{t/t}) + \beta \text{trace}(w^3 \Sigma_{uu}) + \beta \text{trace}(w^4 \Sigma_{vv}) \} \quad (42)$$

where  $w^1 = Ch^{2T} W Ch^2$ ,  $w^2 = (KLH)^T V (KLH)$ ,  $w^3 = K^T L^T V K L$  and  $w^4 = K^T V K$  and  $P_{t/t}$  is the covariance matrix of the within-period prediction errors.

**Table I**

Parameter values used in the calibration

Parameter values used in the calibration								
$\delta$	$\kappa$	$\alpha$	$\sigma$	$\sigma_\varepsilon^2$	$\sigma_u^2$	$\sigma_v^2$	$\sigma_{\tau,y}^2$	$\beta$
0.44	0.18	0.48	-0.06	0.63	0.42	0.13	0.06	0.96

**Table 2****Incomplete information and economic stabilisation for different loss functions**

Commitment case		Loss	Standard deviation of output gap	Standard deviation of inflation	Standard deviation of interest rate changes
$\lambda = 1.0, \lambda_1 = 0.0$	<i>CI</i>	27.04	0.65	0.86	19.27
	<i>II</i>	46.41	1.11	0.83	18.70
$\lambda = 1.0, \lambda_1 = 0.1$	<i>CI</i>	57.55	1.07	0.97	2.03
	<i>II</i>	76.25	1.37	0.97	2.03
$\lambda = 1.0, \lambda_1 = 0.25$	<i>CI</i>	67.00	1.16	1.03	1.43
	<i>II</i>	85.74	1.43	1.04	1.43
$\lambda = 0.0, \lambda_1 = 0.1$	<i>CI</i>	24.31	1.40	0.90	1.63
	<i>II</i>	25.33	1.61	0.92	1.67
$\lambda = 0.5, \lambda_1 = 0.1$	<i>CI</i>	28.61	1.34	0.90	1.63
	<i>II</i>	52.64	1.45	0.93	1.81
$\lambda = 1.0, \lambda_1 = 0.1$	<i>CI</i>	57.55	1.07	0.97	2.03
	<i>II</i>	76.25	1.37	0.97	2.03
$\lambda = 2.0, \lambda_1 = 0.1$	<i>CI</i>	80.32	0.93	1.05	2.50
	<i>II</i>	118.32	1.28	1.04	2.46

Discretionary case		Loss	Standard deviation of output gap	Standard deviation of inflation	Standard deviation of interest rate changes
$\lambda = 1.0, \lambda_1 = 0.0$	<i>CI</i>	35.39	0.53	1.11	20.33
	<i>II</i>	54.21	1.05	1.07	20.38
$\lambda = 1.0, \lambda_1 = 0.1$	<i>CI</i>	71.04	1.01	1.19	2.41
	<i>II</i>	90.41	1.31	1.21	2.45
$\lambda = 1.0, \lambda_1 = 0.25$	<i>CI</i>	87.37	1.11	1.31	1.77
	<i>II</i>	107.27	1.39	1.33	1.80
$\lambda = 0.0, \lambda_1 = 0.1$	<i>CI</i>	35.36	1.21	1.07	1.89
	<i>II</i>	37.06	1.45	1.10	1.94
$\lambda = 0.5, \lambda_1 = 0.1$	<i>CI</i>	54.97	1.09	1.13	2.15
	<i>II</i>	65.27	1.37	1.15	2.20
$\lambda = 1.0, \lambda_1 = 0.1$	<i>CI</i>	71.04	1.01	1.19	2.41
	<i>II</i>	90.41	1.31	1.21	2.45
$\lambda = 2.0, \lambda_1 = 0.1$	<i>CI</i>	97.37	0.90	1.30	2.89
	<i>II</i>	135.54	1.25	1.31	2.90

**Table 3a**

Taylor rule, with varying information

Type of rule	Loss	$r_{t-1}$	$\pi_t$	$gap_{t/t}$
Case 1: <i>CI</i>				
Without smoothing	94.94	-	3.00	0.81
With smoothing	60.39	0.98	1.48	1.33
Case 2: <i>II</i>				
Without smoothing	113.2	-	3.05	0.41
With smoothing	78.67	0.99	1.46	1.29
Misspecified rule without smoothing	113.6	-	3.13	0.18
Misspecified rule with smoothing	81.57	1.03	1.85	0.88

**Table 3b**

Income growth rule, with varying information

Type of rule	Loss	$r_{t-1}$	$\pi_t$	$\Delta y_{t/t}$
Case 1: <i>CI</i>				
Without smoothing	97.58	-	3.10	-0.03
With smoothing	71.97	0.95	1.97	1.07
Case 2: <i>II</i>				
Without smoothing	113.8	-	3.13	0.04
With smoothing	86.04	0.98	2.03	1.21

**Table 3c**

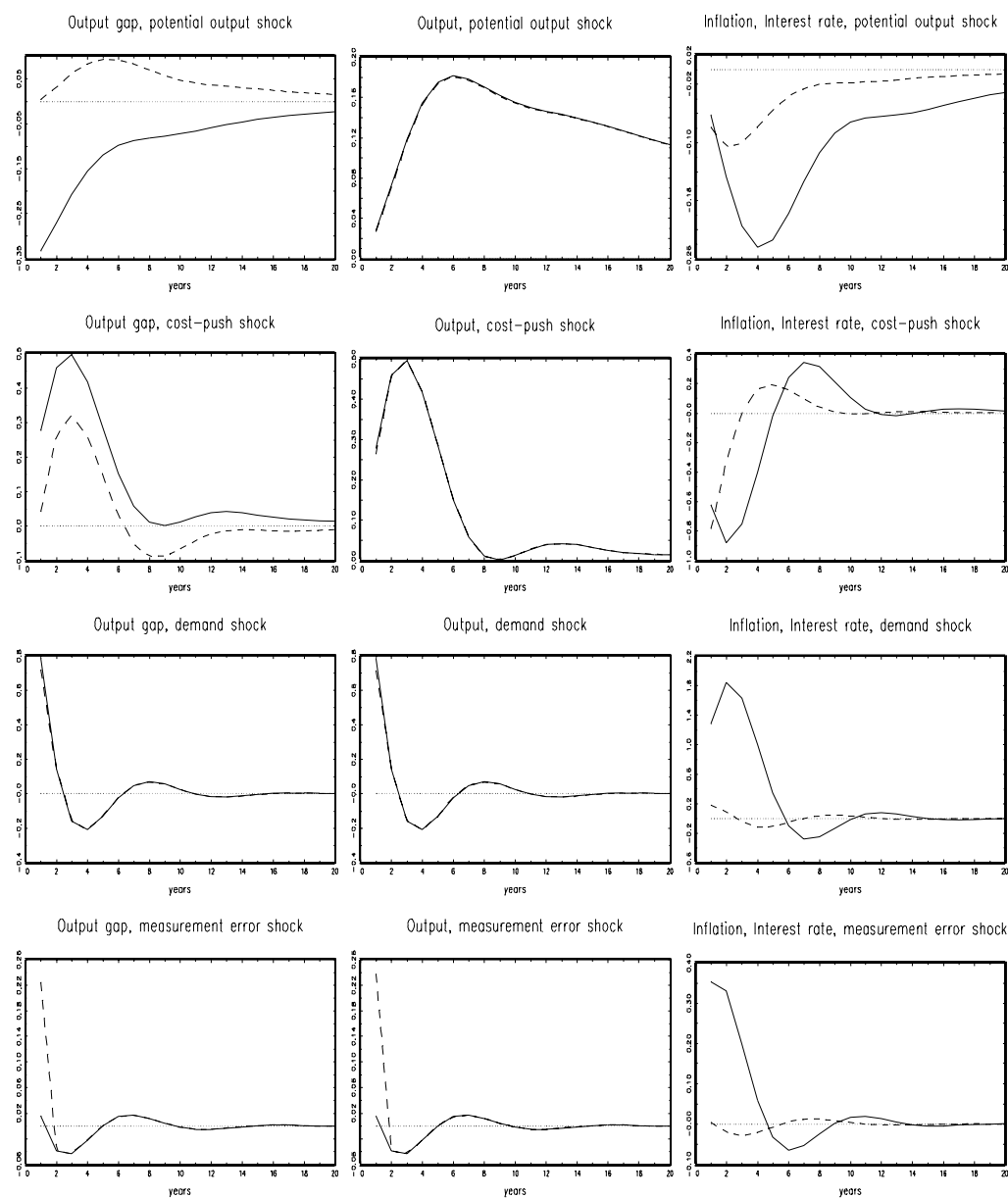
Simple inflation-forecast based rules, with varying information

Type of rule	Loss	$r_{t-1}$	$\pi_t$ or $\pi_{t+1/t}$
Case 1: <i>CI</i>			
$\pi_t$	75.15	0.76	1.99
$\pi_{t+1/t}$	65.68	1.14	4.45
$\pi_{t+1/t}^c$	63.26	1.31	4.25
Case 2: <i>II</i>			
$\pi_t$	89.91	0.77	1.98
$\pi_{t+1/t}$	82.69	1.16	4.45
$\pi_{t+1/t}^c$	80.89	1.32	4.17

**Reminder:** loss under commitment, *CI*: 57.55, *II*: 76.25; loss under discretion, *CI*: 71.04, *II*: 90.41

**Figure 1a**

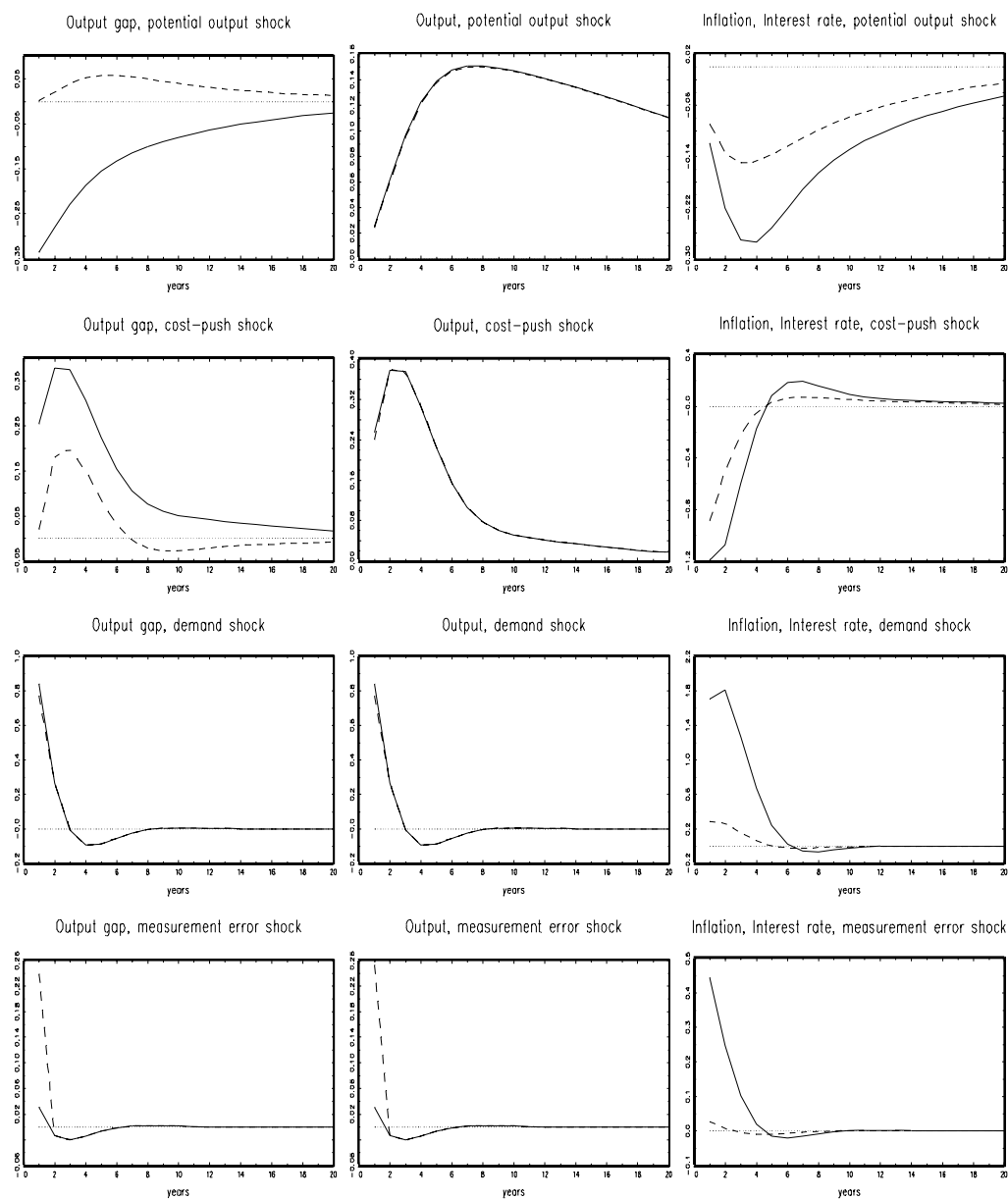
Actual versus perceived under commitment



solid lines: actual; dashed lines: perceived; last column: solid lines: interest rate;  
dashed: actual inflation

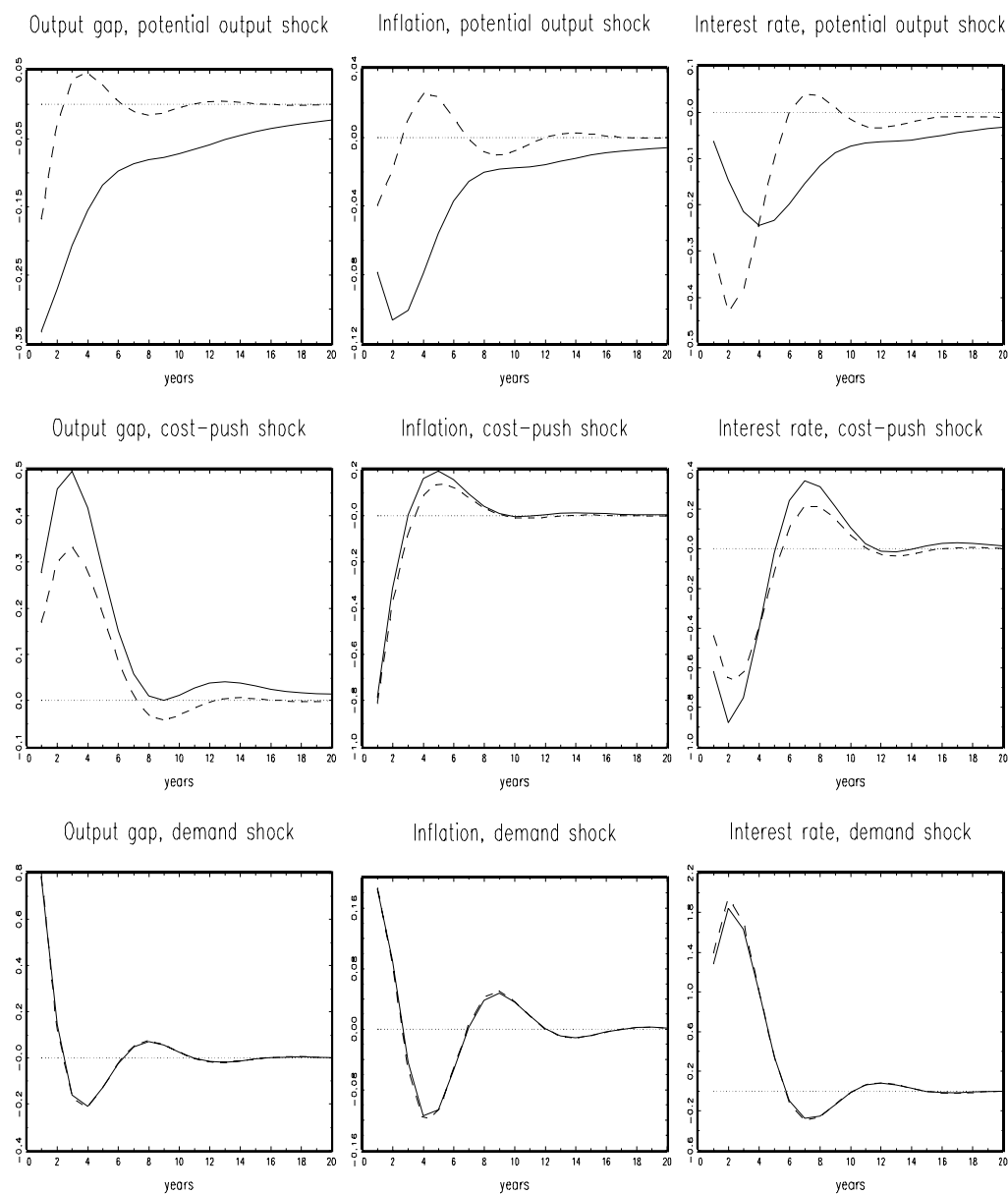


**Figure 1b**  
Actual versus perceived under discretion



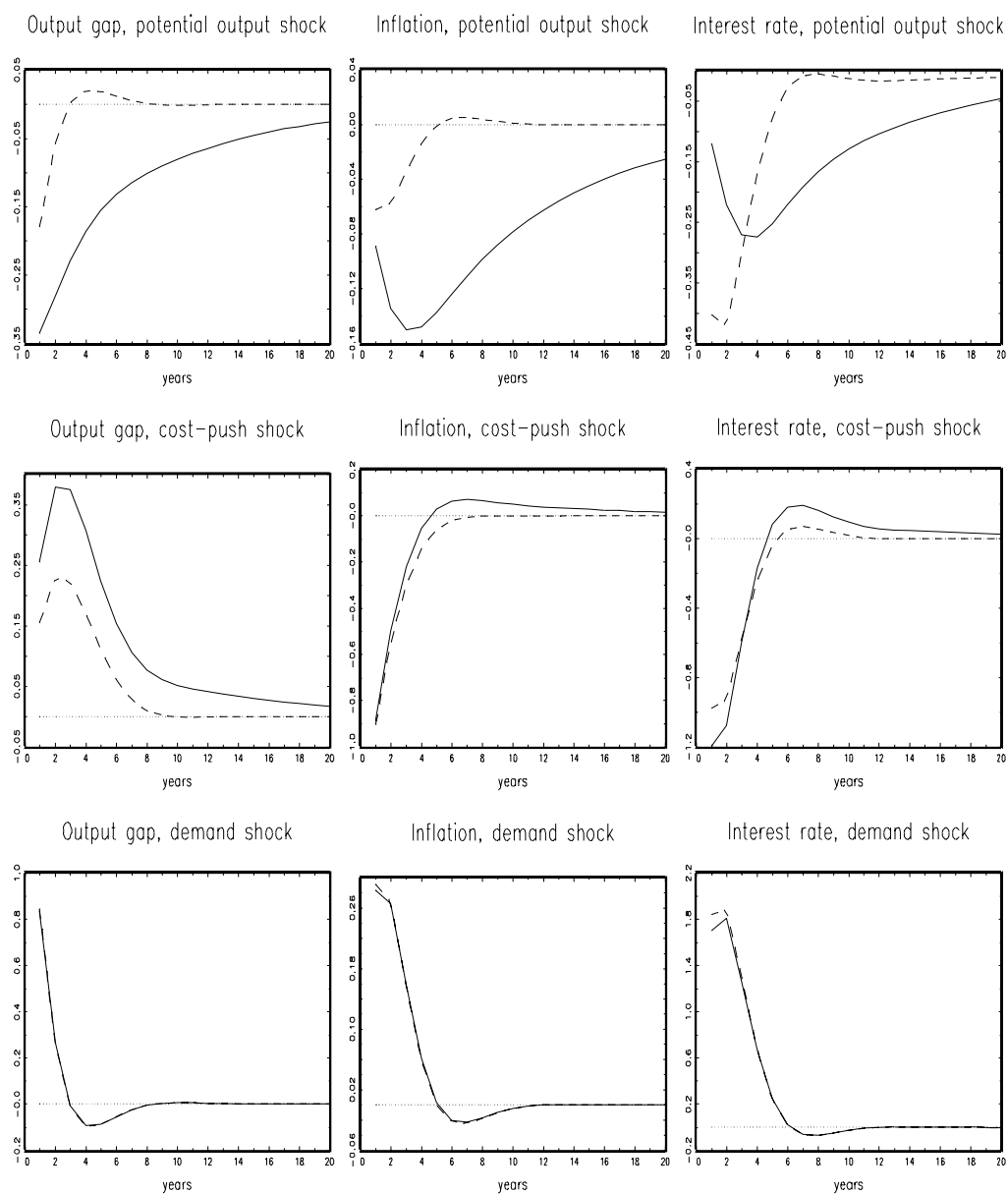
solid lines: actual; dashed lines: perceived; last column: solid lines: interest rate;  
dashed: actual inflation

**Figure 2a**  
Complete versus incomplete information under commitment



solid lines: incomplete information; dashed: complete information

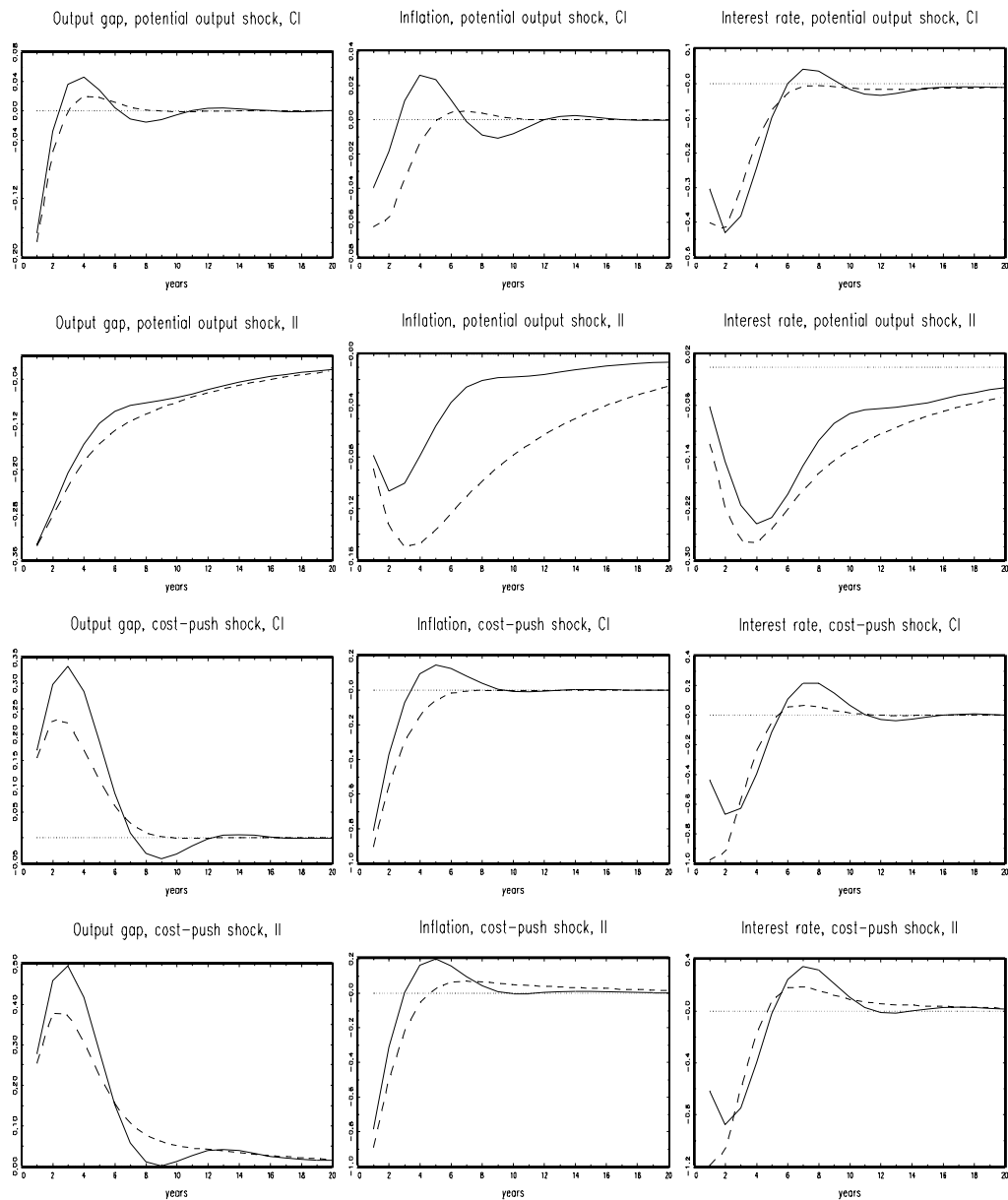
**Figure 2b**  
Complete versus incomplete information under discretion



solid lines: incomplete information; dashed: complete information

**Figure 3**

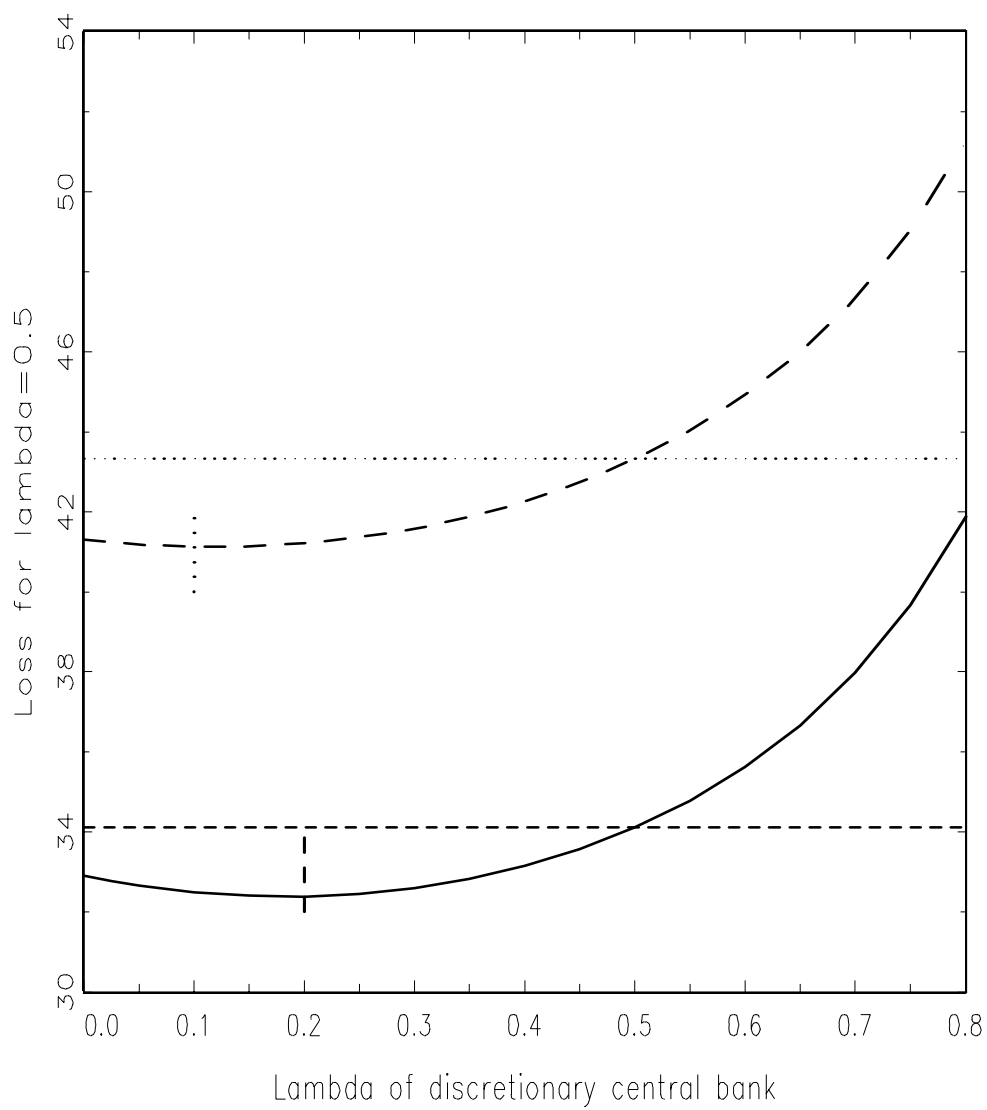
**Commitment versus discretion**



solid lines: commitment; dashed: discretion

**Figure 4**

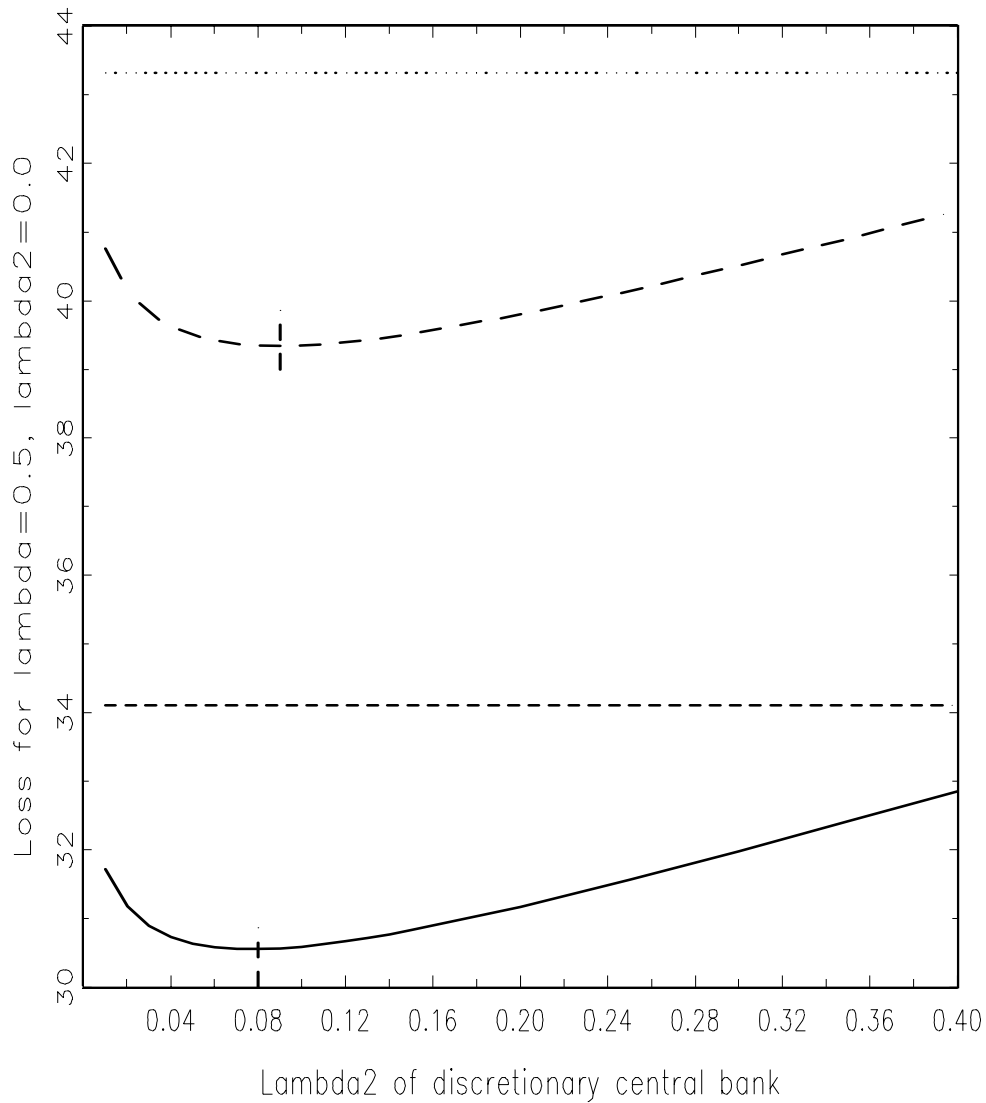
Incomplete information and the optimal weight on output gap stabilisation



dashed (dotted) horizontal line: discretionary equilibrium with  $\lambda = 0.5$ ,  
complete (incomplete) information; solid (dashed) line: loss for varying  $\lambda$ ,  
complete (incomplete) information

**Figure 5**

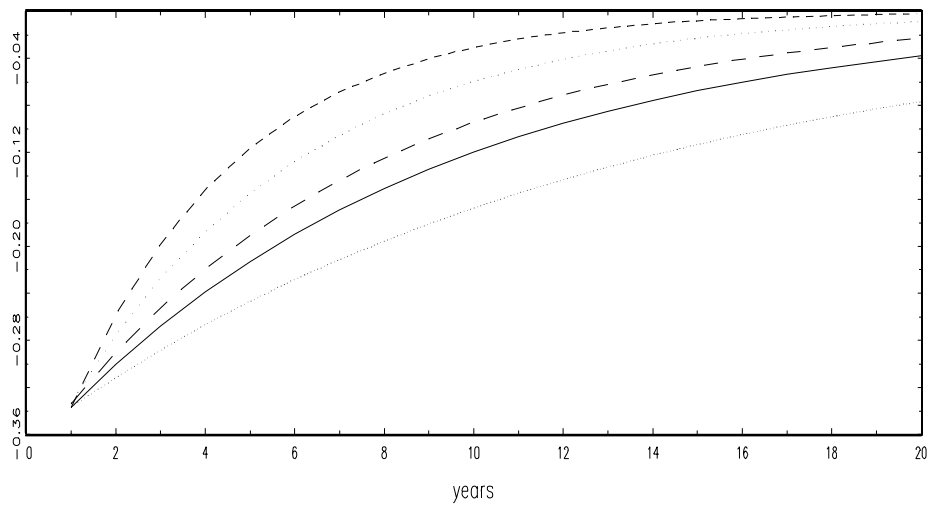
Incomplete information and the optimal weight on price level objectives



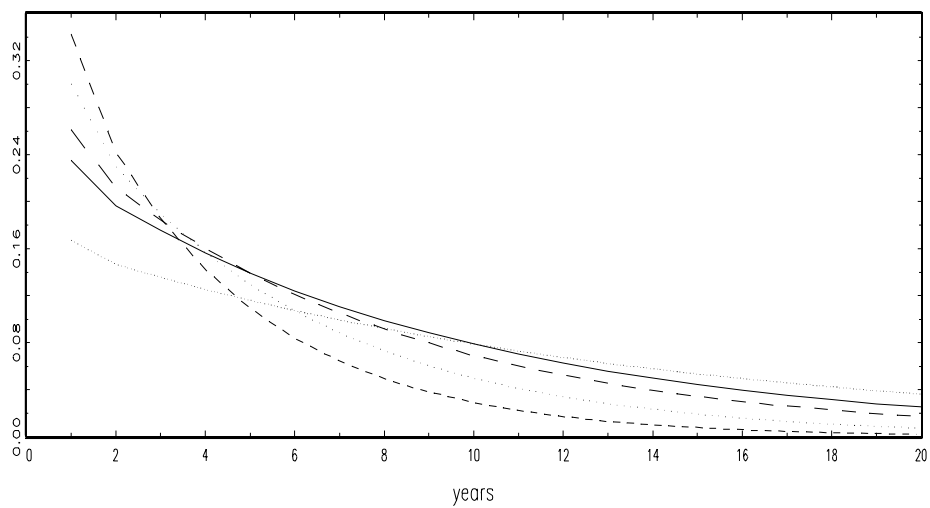
dashed (dotted) horizontal line: discretionary equilibrium with  $\lambda = 0.5$ ,  $\lambda_2 = 0.0$ , complete (incomplete) information; solid (dashed) line: loss for varying  $\lambda_2$ , complete (incomplete) information

**Figure 6**

Perception errors in the output gap and the relative variance of cost-push and potential output shocks

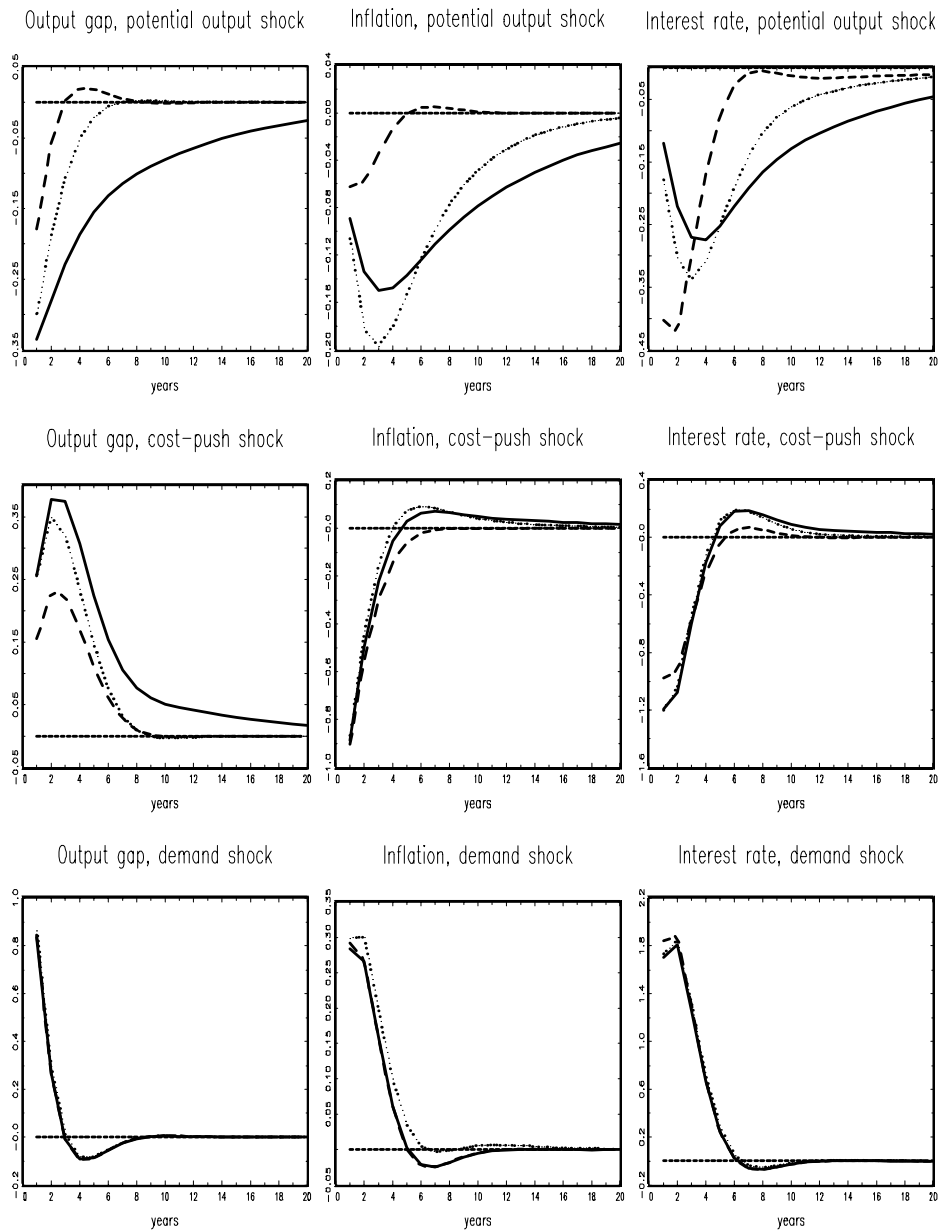


output gap perception error, cost-push shock



**Figure 7**

Symmetric versus asymmetric incomplete information (under discretion)

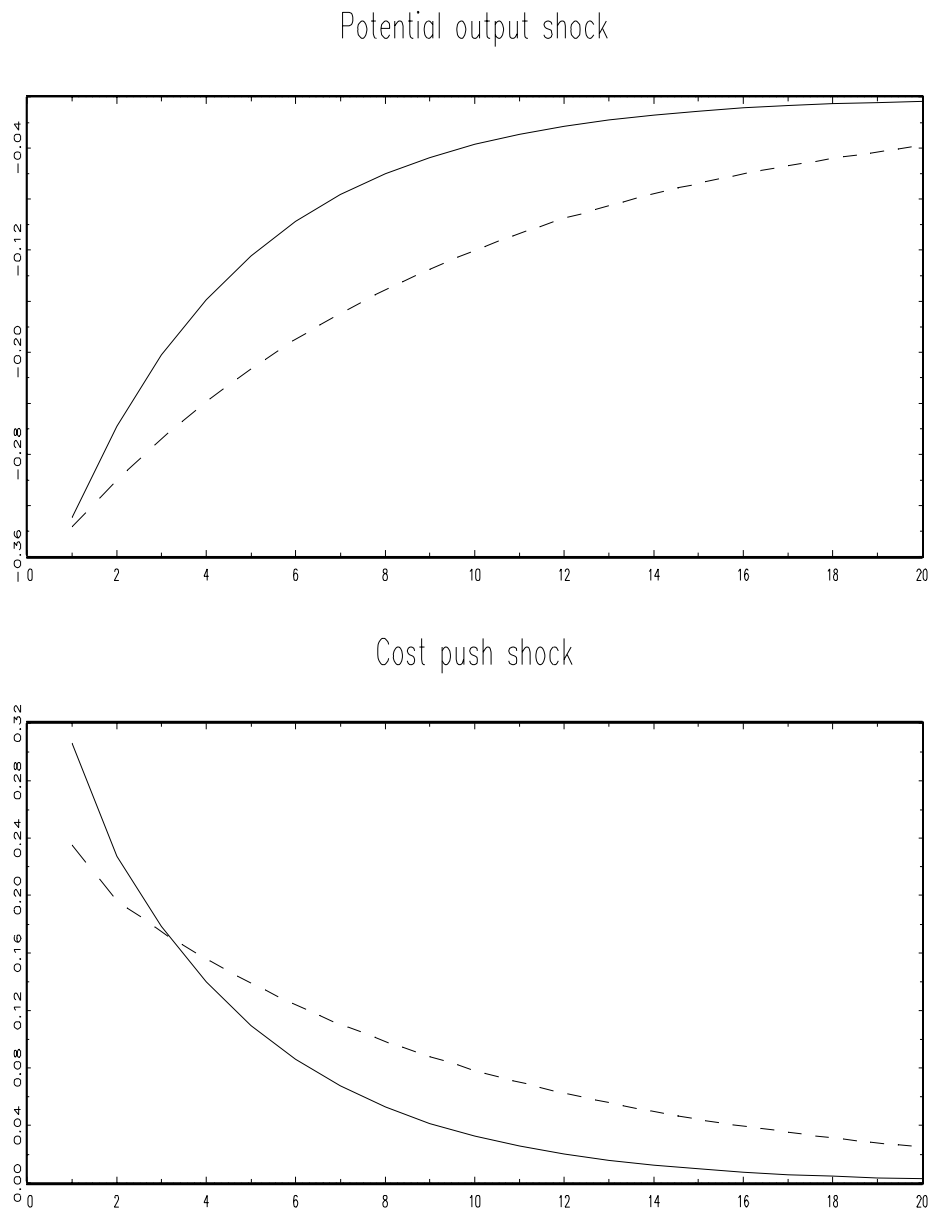


Solid lines: SII; dashed lines: CI; dotted lines: AII



**Figure 8**

Perception errors in the output gap and asymmetric information



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## Non-technical Summary

In most macro-models that are currently being used for policy analysis the degree of capacity utilisation as, for example, measured by an output gap plays an important role in the determination of inflation. Yet there are no direct measures of the aggregate supply side of the economy or the extent to which the resources in an economy are fully used. A wide variety of both conceptual and empirical methods have been proposed to estimate potential output and make the notion of an output gap operational. On the conceptual side, various notions of trend output, potential output, the flexible-price level of output or the output level consistent with stable inflation have all been used. On the empirical side, a wide variety of different methodologies going from simple linear de-trending or the more structural production function approach to more sophisticated econometric techniques using multivariate state-space modelling have all been applied.

In the second half of the 1990s, the problems associated with the measurement of potential output have been highlighted in the monetary policy debate in the United States. Uncertainty about the degree of capacity utilisation has been very high as the growth rate of the economy persistently exceeded previous estimates of potential growth and the unemployment rate fell through most previous estimates of the NAIRU without causing a resurgence in inflation. Similar questions also arise in the euro area, where the effects of structural reforms in labour and goods markets on the supply side of the economy are difficult to assess.

In this paper we investigate the implications of incomplete information about potential output for the conduct and design of monetary policy using a small backward/forward looking model calibrated to fit annual euro area data. The model features three shocks: a shock to potential output, a cost-push shock and a demand shock. Using a standard quadratic loss function in inflation and output gap stabilisation, we systematically compare optimal monetary policy under two assumptions regarding the information available to the central bank and the private sector. In the first case the information set contains both current and potential output and inflation. As a result, the central bank can perfectly deduce which of the three shocks is hitting the economy and adjust its policy rate accordingly. In the second case, information is incomplete. The central bank does observe current inflation, but does not observe potential output and has only a noisy measure of current output. The measurement error is calibrated to fit revisions in euro area output data. As a result, the central bank faces a signal extraction problem when setting the optimal interest rate.

Our main results can be summarised under three headings. First, we characterise and compare the optimal monetary policy under both commitment and discretion in the two informational cases using recent results of Svensson and Woodford (2000). We find that the loss due to incomplete information is substantial and mainly results from a significant increase in the variability of the output gap. In our benchmark case, the standard deviation of the output is about one third higher under incomplete information. Even if a central bank continuously updates its estimate of potential output and thus *ex ante* does not make any systematic estimation errors, *ex post* the misperception of the output gap is substantial and very persistent in particular in response to a shock to potential output. This result could suggest that the empirical findings of Orphanides (2000) that the Federal Reserve Board made serially correlated estimation errors in the output gap in the 1970s may not be evidence of systematic *ex ante* mistakes. We also find that in response to a shock to potential output, such misperception errors lead to a very persistent effect on inflation, which is exacerbated when the central bank optimises under discretion. Nevertheless, the overall variance of inflation is not very much affected, because under incomplete information, the response of inflation to cost-push shocks is mitigated.

Second, we investigate the implications of incomplete information for simple policy rules such as a Taylor rule or an inflation-forecast based rule. We find that, in first-difference form, these rules (and in particular the Taylor rule) perform relatively well compared to the commitment equilibrium. Imperfect information about potential output leads to some limited attenuation in the Taylor rule, but does not affect the inflation-forecast based rule. Both rules continue to perform relatively well compared as long as the output gap and the inflation forecast are optimally estimated. However, when the central bank misspecifies the output gap (for example, because it does not use the correct model for potential output) the performance of the rules deteriorates considerably.

Finally, we examine the implications of the unobservability of potential output for the optimal delegation of monetary policy to a discretionary central bank. Using two examples, we find that incomplete information about the output gap implies that it is optimal to appoint a relatively more “hawkish” central bank than in the complete information case. In the first example following Rogoff (1985) and Clarida, Gali and Gertler (1999), this is reflected in the fact that society would like to appoint a more weight-conservative central banker (i.e. a central banker with a smaller weight on output gap stabilisation in his/her loss function). In the second example following Vestin (1999), this is reflected in society’s desire to put a larger weight on a price stability objective under incomplete information.

Overall, our results suggest that incomplete information about potential output does not significantly overturn results regarding optimal monetary policy that are derived in models with perfect information, as long as the central bank uses its best estimates of the state of the economy and has the true model of the economy at its disposal. Of course, these results very much depend on the linear-quadratic framework used in this paper and are likely to break down when model uncertainty and/or non-linearities are considered. The results do strongly indicate that there are potentially large gains to be made from trying to identify the particular shocks that hit the economy at any point in time. An early identification of the nature of the shocks that affect the economy allows for a more appropriate policy response and improves the stabilisation of the economy significantly.

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