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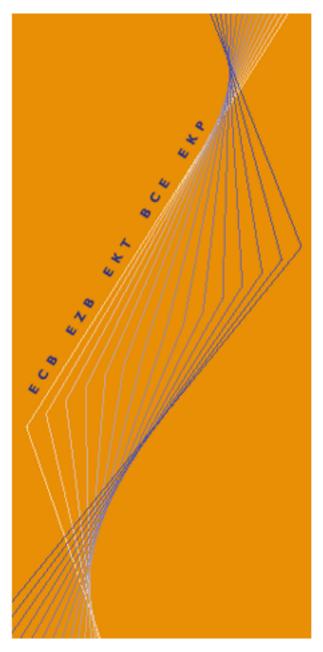
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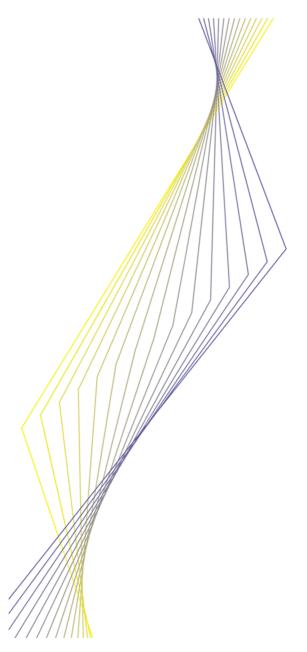


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A SMALL ESTIMATED EURO AREA MODEL WITH RATIONAL EXPECTATIONS AND NOMINAL RIGIDITIES

BY GÜNTER COENEN AND VOLKER WIELAND

September 2000



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## BY GÜNTER COENEN AND VOLKER WIELAND

## September 2000

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### Abstract

The objective of this paper is to estimate a small model of the euro area to be used as a laboratory for evaluating the performance of alternative monetary policy strategies. We start with the relationship between output and inflation and investigate the fit of the nominal wage contracting model due to Taylor (1980) and three different versions of the relative real wage contracting model proposed by Buiter and Jewitt (1981) and estimated by Fuhrer and Moore (1995a) for the United States. While Fuhrer and Moore reject the nominal contracting model and find strong evidence in favor of the relative contracting model which induces a higher degree of inflation persistence, we find that both types of contracting models fit euro area data reasonably well. The best fitting specification is a version of the relative contracting model, but one that is theoretically more plausible than the one preferred by Fuhrer and Moore for U.S. data.

A drawback of the euro area estimation is that the data are averaged over the member economies, which experienced different monetary policy regimes prior to the formation of EMU. Whereas Germany enjoyed stable inflation with fairly predictable monetary policy, countries such as Italy and France experienced a long-drawn out and probably imperfectly anticipated disinflation. To investigate the validity of our results, we also obtain estimates for France, Germany and Italy separately. We find that the relative contracting model does quite well in countries which transitioned out of a high inflation regime such as France and Italy, while the nominal contracting model fits German data better. Thus, an optimist may conclude that the independent European Central Bank will face a similar environment in the future as the Bundesbank did in Germany and pick the nominal contracting specification, while a pessimist, who suspects that stabilizing euro area inflation will require higher output losses, may want to pick the relative contracting specification. We close the model by estimating an aggregate demand relationship and investigate the consequences of the different wage contracting specifications for the output costs associated with stabilizing inflation, when interest rates are set according to Taylor's rule.

#### JEL classification system: E31, E52, E58, E61

Keywords: European Monetary Union, euro area, macroeconomic modelling, rational expectations, nominal rigidities, overlapping wage contracts, inflation persistence, monetary policy rules

## 1 Introduction

With the formation of European Monetary Union (EMU) in 1999, the eleven countries that adopted the euro began to conduct a single monetary policy oriented towards unionwide objectives.<sup>1</sup> As prescribed by the Maastricht Treaty the primary goal of this policy is to maintain price stability within the euro area. The operational definition of this goal announced by the European Central Bank (ECB) is to aim for year-on-year increases in the euro area inflation rate below 2 percent.<sup>2</sup> To evaluate alternative strategies for achieving such a euro-area-wide objective, it is essential to build empirical models that can be used to assess the area-wide impact of policy on key macroeconomic variables such as output and inflation. Thus, the objective of this paper is to construct a small model of the euro area, which may serve as a laboratory for evaluating the performance of alternative monetary policy strategies in the vein of recent studies for the United States.<sup>3</sup>

One possible approach to building a model of the euro area is to start by constructing separate models of the individual member economies and then link them together in a multicountry model. The main alternative is to first aggregate the relevant macroeconomic time series across member economies, and then estimate a model for the euro area as a whole. In this paper, we pursue the latter approach, the reason being that the objectives as well as the instruments of Eurosystem monetary policy are defined on the euro area level. Of course, a problem with this approach is that the data used in aggregation stem from periods

<sup>&</sup>lt;sup>1</sup>Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. Denmark, Sweden, Greece and the U.K. have not adopted the euro. Their central banks are also members of the European System of Central Banks, but not of the Europystem.

<sup>&</sup>lt;sup>2</sup>As measured by the Harmonized Index of Consumer Prices (HICP). It was further clarified that this definition excludes decreases and thus deflation. A detailed discussion of these and other issues regarding the ECB's strategy can be found in Angeloni, Gaspar and Tristani (1999).

<sup>&</sup>lt;sup>3</sup>The recent literature on policy rules for the U.S. economy has used a variety of models: (i) small-scale backward-looking models such as Rudebusch and Svensson (1999); (ii) large-scale backward-looking models such as Fair and Howrey (1996); (iii) small-scale models with rational expectations and nominal rigidities (cf. Fuhrer and Moore (1995a), (1995b), Fuhrer (1997), Orphanides, Small, Wieland and Wilcox (1997), and Orphanides and Wieland (1998)); (iv) large-scale models of this type such as Taylor (1993a) and the Federal Reserve Board's FRB/US model (cf. Brayton et al. (1997), Reifschneider et al. 1999); and (v) small models with optimizing agents such as Rotemberg and Woodford (1997, 1999), Clarida, Galí and Gertler (1999, 2000) and McCallum and Nelson (1999). Recent comparative studies of interest include Bryant, Hooper and Mann (1993) and Taylor (1999a).

prior to EMU, when the different member economies experienced different monetary regimes and policies. Therefore, we also estimate every model specification separately for the three largest member economies, France, Germany and Italy, which together comprise over 70% of economic activity in the euro area. By comparing the estimates obtained with French, German and Italian data to the euro area estimates, we can assess to what extent the choice of model specification for the euro area is influenced by the aggregation itself. Furthermore, by comparing France and Italy, which experienced a convergence process prior to EMU, with Germany, which enjoyed stable inflation and interest rates, we can see whether the choice of specification is influenced by differences in the monetary regime prior to EMU.

In building our small-scale euro area model we start with the relationship between inflation and output. In this respect we make two modelling assumptions, which are central to the key policy tradeoff between inflation and output variability. We assume that market participants form expectations in a forward-looking, rational manner and that monetary policy has short-run real effects due to the existence of overlapping wage contracts. In the long run, however, monetary policy is neutral. The assumption of rational expectations constitutes a useful benchmark for policy evaluation, because the alternative assumption of backward-looking expectations would imply that the central bank can exploit systematic expectational errors by market participants.<sup>4</sup>

As to overlapping wage contracts, we explore the empirical fit of the nominal wage contracting model due to Taylor (1980) as well as three different versions of the relative real wage contracting model first proposed by Buiter and Jewitt (1981) and investigated empirically by Fuhrer and Moore (1995a) for the United States. The nominal contracting model belongs to the class of New-Keynesian sticky-price models which are consistent with intertemporal optimization by imperfectly competitive firms.<sup>5</sup> However, Fuhrer and Moore (1995a) have argued that the nominal contracting model cannot explain the degree of in-

<sup>&</sup>lt;sup>4</sup>Thus, our analysis accounts for the Lucas critique in the narrow sense that market expectations take into account the decision rule of the policymaker. However, it violates the Lucas critique in the wider sense, because it does not explicitly take into account the optimizing behavior of individual and possibly heterogeneous market participants.

<sup>&</sup>lt;sup>5</sup>See Goodfriend and King (1997) for a comprehensive survey.

flation persistence observed in U.S. data, while the relative real wage contracting model instead induces sufficient inflation stickiness. This difference has important policy implications, in particular regarding the costs of stabilizing inflation in terms of increased output variability.

Comparing these alternative specifications, we find that the relative real wage contracting model fits euro area inflation dynamics better than the nominal contracting model. However, we also note that the nominal contracting model is not rejected by the data. Among the three different versions of the relative real wage contracting model, it is not the simplified specification preferred by Fuhrer and Moore, but a theoretically more plausible specification, which obtains the best fit. Comparing the estimates for the individual countries, we find that the same relative wage contracting model fits Italian and French data quite well, but not the German data, which exhibits a substantially lower degree of inflation persistence. Only the nominal contracting model seems to have a shot at explaining inflation dynamics in Germany.

Fuhrer and Moore's empirical findings with U.S. data have generated a continuing debate on the sources of inflation stickiness. For example, Roberts (1997) showed that a stickyinflation model with rational expectations is observationally equivalent to a sticky-price model with expectations that are imperfectly rational. Using data on survey expectations in the U.S., Roberts found evidence of backward-looking behavior. More recently, Sbordone (1998) and Galí and Gertler (1999) have argued that the New-Keynesian sticky-price model is capable of explaining U.S. inflation dynamics, if one uses a measure of marginal costs rather than the output gap as the determinant of inflation.<sup>6</sup> Finally, Taylor (1999b) has pointed out that the level of inflation influences the pricing power of firms, and argued that inflation is more persistent in a high inflation regime than in a low inflation regime with credible monetary policy.

Our comparative analysis with European data contributes some new results to this de-

<sup>&</sup>lt;sup>6</sup>As the authors show, a model with price-stickiness is sufficient in this case, because marginal costs themselves exhibit persistence. An open question, which needs to be settled in order to construct a complete macro model, concerns the source of the observed persistence in marginal costs.

bate. By assuming rational expecations, our estimation approach attributes the large degree of inflation persistence in France and Italy and the euro area as a whole to structural nominal rigidities. An alternative interpretation of this finding is to consider it evidence of adaptive expectations as suggested by Roberts (1997) in the context of the U.S. inflation process. This interpretation is also a plausible explanation of the observed degree of persistence, because the convergence process experienced by those countries may at best have been imperfectly anticipated by market participants. Thus, as far as the future of the EMU is concerned, the estimation based on historical euro area data might overstate the case for the relative real wage contracting model. Further support for this interpretation of our results is provided by the better fit of the nominal contracting model in Germany, where inflation was rather stable and monetary policy fairly predictable. The estimation results with German data also provide indirect empirical support for the thesis that the degree of inflation persistence is lower in a stable monetary policy regime with low average inflation, because of the change in the pricing power of firms as suggested by Taylor (1999b).

In estimating euro area, French and Italian inflation dynamics with pre-EMU data, we use the deviations of inflation from the downward trend rather than the inflation rate itself. This downward trend was generated by the convergence of inflation in Italy, France, Spain and other EMU member countries to German levels from the late 1970s until the late 1990s. Our estimates for Germany, however, are based on actual inflation rates since Germany experienced rather stable inflation. The downward trend is a unique feature of historical euro area data and should not be expected to persist nor to be reversed in the future, if the ECB achieves its policy objective. The short-run variations around this trend, however, to the extent that they were due to structural rigidities, may still help predicting the inflation dynamics after the formation of EMU. We discuss the use of inflation deviations from trend in more detail when we describe the data and investigate its implications for the estimation results later on.

In terms of evaluating alternative monetary policy strategies for the euro area, an analyst who is pessimistic about the output losses associated with stabilizing inflation might prefer to use the relative contracting model, while an optimist might prefer the nominal contracting model. A robust monetary policy strategy, however, should perform reasonably well under both specifications. We provide an illustrative example for the case of Taylor's rule.

The remainder of the paper proceeds as follows. Section 2 reviews the overlapping contracts specifications. The data is discussed in section 3. Section 4 summarizes inflation and output dynamics in form of unconstrained VAR models, while the structural estimates obtained by means of simulation-based indirect inference methods are reported in section 5. In section 6 we close the model with an aggregate demand equation, a term structure relationship and a policy rule. Impulse responses and disinflation scenarios under alternative specifications are compared in section 7. Section 8 concludes and the appendix provides the details of the indirect estimation methodology.

## 2 Modelling inflation dynamics with overlapping contracts

We estimate four different specifications of overlapping wage contracts, the nominal wage contracting model of Taylor (1980) and three variants of the relative real wage contracting model estimated by Fuhrer and Moore (1995a) for the United States. The essential persistence-inducing mechanism in these models is that workers, which are negotiating long-term contracts, compare the contract wage to contracts that have been negotiated previously and are still in effect and to future contracts that will be negotiated over the life of their contracts. The existence of longer-term wage contracts in European countries bolsters the case for applying this framework to euro area data. One could argue though that European labor markets were geographically fragmented, so that comparisons were more likely made with contract wages previously negotiated in the same country rather than in the euro area as a whole.<sup>7</sup> We take this as a further reason to investigate the robustness of our modelling approach by estimating the contracting specifications also separately for the

<sup>&</sup>lt;sup>7</sup>Nevertheless, worker migrations to European high-wage countries may have exerted some influence on wage setting across countries. Furthermore, just as empirical investigations for the U.S. average over wage settlements in different sectors of the economy, our investigation of the euro area averages over wage settlements in different member economies.

three largest euro area economies.

While the models we consider emphasize wage contracting, the implications for price dynamics are essentially the same as for wage dynamics if prices are related to wages by a fixed markup. Thus, we follow Fuhrer and Moore in using price instead of wage data in estimation and from here on use the terms "contract price" and "contract wage" interchangeably.<sup>8</sup>

A common feature of the four specifications is that the log aggregate price index in the current quarter,  $p_t$ , is a weighted average of the log contract wages,  $x_{t-i}$  (i = 0, 1, ...), which were negotiated in the current and the preceding quarters and are still in effect. The sticky price index can be observed directly, while the flexible contract wage is an unobserved variable. As a benchmark we consider the case of a one-year weighted average:

$$p_t = f_0 x_t + f_1 x_{t-1} + f_2 x_{t-2} + f_3 x_{t-3}.$$
 (1)

The weights  $f_i$  (i = 0, 1, 2, 3) on contract wages from different periods are assumed to be time-invariant and subject to  $f_i \ge 0$  and  $\sum_i f_i = 1$ . As shown in Taylor (1980), these weights would be equal to .25, if 25 percent of all workers sign contracts each quarter and if each contract lasts one year. Taylor (1993a) provides an interpretation for the more general case with unequal weights in terms of the distribution of workers by lengths of contracts. He shows that the weights  $f_i$  are time-invariant, if the distribution of workers by contract length is time-invariant and if the variation of average contract wages over contracts of different length is negligible.<sup>9</sup> We restrict the number of lags in (1) to three, which is consistent with a maximum contract length of four quarters.<sup>10</sup> Rather than estimating each of the weights

<sup>&</sup>lt;sup>8</sup>For recent studies considering wage and price stickiness separately, see Taylor (1993a), Erceg, Henderson and Levin (1999) and Amato and Laubach (1999).

<sup>&</sup>lt;sup>9</sup>For the derivation see Taylor (1993a), pp. 35-38. Further restrictions could be imposed on the weights  $f_i$ , if collective long-term wage settlements are synchronized, i.e. concentrated in a single quarter of the year, as may be the case in some European economies. Given sufficient information on the percentage of synchronized contracts, these restrictions could be implemented in estimation as shown by Taylor (1993a) for the case of Japan. In the present context, however, we will leave the question regarding the relative importance of synchronized wage setting for future research.

<sup>&</sup>lt;sup>10</sup>Fuhrer and Moore (1995a) found this lag length sufficient to explain the degree of persistence in U.S. inflation data. Similarly, Taylor (1993a) estimated the nominal contracting model for all G-7 countries with such a lag length.

 $f_i$  separately, we follow Fuhrer and Moore and assume that the weights are a downwardsloping linear function of contract length, given by  $f_i = .25 + (1.5 - i) s$  with  $s \in (0, 1/6]$ . This distribution depends on a single parameter, the slope s.

The determination of the nominal contract wage  $x_t$  for the different specifications is best explained starting with Taylor's nominal wage contracting model (the NW model in the following). In this case,  $x_t$  is negotiated with reference to the price level that is expected to prevail over the life of the contract, as well as the expected degree of excess demand over the life of the contract, which is measured in terms of the deviations of output from its potential,  $q_t$ :

$$x_t = \mathcal{E}_t \left[ \sum_{i=0}^3 f_i \, p_{t+i} + \gamma \sum_{i=0}^3 f_i \, q_{t+i} \right] + \sigma_{\epsilon_x} \, \epsilon_{x,t}. \tag{2}$$

The structural shock term,  $\epsilon_{x,t}$ , is scaled by the parameter  $\sigma_{\epsilon_x}$ , which denotes its standard deviation. Since the price indices  $p_{t+i}$  are functions of contemporaneous and preceding contract wages, equation (2) implies that in negotiating the current contract wage, agents look at an average of the nominal contract wages that were negotiated in the recent past as well as those that are expected to be negotiated in the near future. In other words, they take into account nominal wages that apply to overlapping contracts. In addition, wage setters take into account expected demand conditions. For example, when they expect demand to exceed potential,  $q_{t+i} > 0$ , the current contract wage is adjusted upwards relative to contracts negotiated recently or expected to be negotiated in the near future. The parameter  $\gamma$  measures the sensitivity of contract wages to the future excess demand term.

Next, we turn to the relative real wage contracting specification (the RW specification in the following). In this case, wage setters compare the real wage over the life of their contract with the real wages negotiated on overlapping contracts in the recent past and near future.<sup>11</sup> While this comparison is carried out in real terms, it is still the nominal wage that is negotiated. It remains to define the elements of this comparison. The average real contract wage is defined using the weighted average of current and future price indices

<sup>&</sup>lt;sup>11</sup>See Ascari and Garcia (1999) for an analysis of relative wage concerns on the part of representative households in a dynamic general equilibrium setting with staggered wages and their implications for the propagation of monetary policy shocks.

prevailing over the life of the contracts, denoted by  $\bar{p}_t = \sum_{i=0}^3 f_i p_{t+i}$ . To summarize real wages on nearby contracts it is helpful to define an index of real contract wages negotiated on the contracts that are currently in effect:

$$v_t = \sum_{i=0}^{3} f_i (x_{t-i} - \mathcal{E}_{t-j}[\bar{p}_{t-i}]).$$
(3)

The current nominal contract wage under the RW specification is then determined by:

$$x_t - \mathcal{E}_t\left[\bar{p}_t\right] = \mathcal{E}_t\left[\sum_{i=0}^3 f_i \, v_{t+i} + \gamma \sum_{i=0}^3 f_i \, q_{t+i}\right] + \sigma_{\epsilon_x} \, \epsilon_{x,t}.$$
(4)

In this case, agents negotiate the real wage under contracts signed in the current period with reference to the average real contract wage index expected to prevail over the current and the next three quarters. Thus, in negotiating current contracts agents compare the current real contract wage to an average of the real contract wages that were negotiated in the recent past and those expected to be negotiated in the near future. Again, agents also adjust for expected demand conditions and push for a higher real contract wage when they expect output above potential.

For the RW specification a subtle but important question arises with respect to the timing of the price expectations  $E_{t-j}[\bar{p}_{t-i}]$  in the real contract wage indices  $v_t$ . For example, the current nominal contract wage  $x_t$  depends on the index of real contract wages currently in effect,  $v_t$ , which in turn is a function of the real contract wages from periods t-1, t-2 and t-3. One possibility is that the relevant reference points for the determination of the current contract wage are the *ex-post realized real contract wages* from these periods , which are now known to wage setters, and therefore j = 0 in (3). The other possibility is that current wage negotiations refer to the *ex-ante expected real contract wages*, which formed the basis of the negotiations in earlier periods and therefore j = i in (3). To give an example, the average real contract wage from period t-1, which enters the index  $v_t$  in (3) conditional on period t information, would then be defined as  $x_{t-1} - (f_0 p_{t-1} + f_1 p_t + f_2 E_t[p_{t+1}] + f_3 E_t[p_{t+2}])$ . In period t-1 information,  $x_{t-1} - (f_0 p_{t-1} + f_1 E_{t-1}[p_t] + f_2 E_{t-1}[p_{t+1}] + f_3 E_{t-1}[p_{t+2}])$ .

Since both definitions seem plausible, we will consider both in estimation. We refer to the relative contracting specification with price expectations conditioned on historically available information as the RW-C specification.

Fuhrer and Moore (1995a) discussed the RW and RW-C specification in the appendix of their paper. Their preferred specification for U.S. data, which is the main focus of their paper, was instead a simplified version of the RW model, which they chose based on a specification test. The simplification concerns the definition of the real contract wage. Instead of using the average price level expected to prevail over the life of the contracts,  $E_t[\bar{p}_t] = E_t[\sum_{i=0}^3 f_i p_{t+i}]$ , they simply use the current price level,  $p_t$ . Thus, the current real wage simplifies to  $x_t - p_t$  and the index of real contract wages that are in effect,  $v_t$ , simplifies to  $\sum_{i=0}^3 f_i(x_{t-i} - p_{t-i})$ . We refer to this case as the RW-S specification. In this case the index  $v_t$  no longer uses price expectations. Consequently, the point regarding the timing of these expectations discussed above is mute.

To the extent that the three alternative relative real wage contracting specifications entail different degrees of forward-looking behavior when forming price expectations, they have different implications for the persistence of the inflation process. Since the RW-S specification does not take into account forward-looking price expectations it will induce a higher degree of inflation persistence than the RW and RW-C specifications. By conditioning price expectations on historically available information, the RW-C specification should in turn imply somewhat higher persistence for inflation than the RW specification.

Before turning to the data used in estimation, we note that although the above specifications are written in terms of the price level and the nominal contract wage, they can be rewritten in terms of the quarterly inflation rate and the real contract wage. Thus, either price levels or inflation rates can be used in estimation while nominal as well as real contract wages are unobservable. Furthermore, we note that the contracting specifications only pin down the steady-state real contract wage, but not the steady-state inflation rate. Steady-state inflation will eventually be determined by the central bank's inflation target, once we close the model in section 6.

## 3 The data

The data we use are quarterly series of inflation, output and the short-term nominal interest rate. As noted previously, using price instead of wage data in estimating staggered contracting specifications may be motivated by linking prices to wages with a fixed markup. The measures we use for output and prices are real GDP and the GDP deflator. The interest rate is the three-month money market rate. To obtain measures for the euro area we aggregate over the data for the euro area member countries using fixed GDP weights at PPP rates.<sup>12</sup>

The historical path of these euro area aggregates between 1974:Q1 and 1998:Q4 is shown in **Figure 1**. As displayed in the top left-hand panel average inflation in the euro area steadily declined over the last 25 years. The panel also shows estimates of the underlying trend in inflation which is specified alternatively as a linear and an exponential function of time. Similarly, the average short-term nominal interest rate depicted in the top right-hand panel tended to decline from the mid 1980s onwards except for the crises of the European Monetary System (EMS) in the early 1990s. This downward trend is a special feature of euro area data, which complicates the empirical investigation of European inflation dynamics relative to similar analyses for the United States. We will return to this issue below.

The contracting models in section 2 relate the short-run dynamics of inflation to the output gap. While a measure for actual real GDP in the euro area is available and shown in the bottom left-hand panel of **Figure 1** (solid line), we need to estimate potential output. Constructing a structural estimate of potential output for the euro area prior to EMU goes beyond the objective of this paper. Even for the individual member countries this would be rather difficult. A common alternative estimate used in the macroeconomic modelling literature is the log-linear trend (see for example Fuhrer and Moore (1995a) and Taylor (1993a) among many others), which is shown as the dashed line in the bottom left panel. The bottom right-hand panel compares the output gap implied by the log-linear trend to the

 $<sup>^{12}</sup>$ These data are drawn from the ECB area-wide model database (see Fagan et al. (1999)).

OECD's (1999) estimate of the euro area output gap (dotted line). Since these estimates are surprisingly similar, except for a small difference in the 1990s, we will follow Taylor and Fuhrer and Moore in using output gaps based on a log-linear trend for estimating the overlapping contracts model.<sup>13</sup>

The source of the downward trend in euro area inflation noted previously is directly apparent from **Figure 2**. As shown in the top left-hand panel inflation rates in the early 1970s were much higher in countries such as France and Italy than in Germany due to oil price shocks and accommodative monetary policy. It took 10 to 15 years, respectively, for French and Italian inflation rates to decline to German levels. Convergence in inflation rates was accompanied by convergence in nominal interest rates in the late 1990s as can be seen from the upper right-hand panel. Over time, as economic convergence and the future formation of a monetary union became more widely expected the inflation premium incorporated in Italian and French short-term nominal interest rates relative to German rates eventually disappeared.

This convergence process and the role of the EMS in its context have been widely debated and analyzed in the academic and policy literature of the last decade.<sup>14</sup> There is little doubt that the decline of inflation has largely been due to the growing commitment on the part of monetary policy makers in the euro area to achieve and maintain low inflation. The credibility of this commitment, however, likely varied over time, probably being rather low in the early stages of the EMS in the early and mid 1980s and higher during the "hard" EMS period in the late 1980s up to the EMS crises in 1992 and 1993. Following these crises credibility regarding the central banks' commitment to achieve low inflation likely increased again with the progress of preparations for EMU. To the extent that disinflation during these periods was credible and expected by wage and price setters, the associated

<sup>&</sup>lt;sup>13</sup>Other alternatives include estimates based on the HP filter or unobserved components methods. We have conducted some sensitivity studies in this respect. We stick with the linear trend as our benchmark for comparability with the results obtained by Fuhrer and Moore and Taylor for the U.S. and because of the similarity to the OECD estimate of the euro area output gap.

<sup>&</sup>lt;sup>14</sup>For more detailed discussions of this convergence process see for example Gros and Thygesen (1992), Giavazzi and Pagano (1994), De Grauwe (1996, 1997), Favero et al. (1997), Angeloni and Dedola (1999).

output losses would have been rather low. In fact, a casual comparison of the extent of disinflation in Italy and France relative to Germany and the output gap estimates for these three economies based on the log-linear trends that are shown in the bottom right panel of **Figure 2**, suggests that the disinflations in Italy and France did not require large and protracted recessions and thus may have been partly anticipated.

In principle, a complete model of the European inflation process prior to EMU would need to account for both, the long-run convergence process as well as the short-run variations around this downward trend. However, modelling the convergence process would require taking into account the varying degree of credibility of exchange rate pegs, the possibility of crises and realignments and learning by market participants about the long-run inflation objectives of European policymakers. Such an analysis would be beyond the objective of this paper.

Instead we take a shortcut by approximating the implicit time-varying inflation objective with a linear trend, and then estimate the overlapping contracts models using inflation deviations from this trend. We detrend the average inflation rate for the euro area as well as the French and Italian inflation rates in this manner. Similar approaches have been used by other researchers with regard to European inflation data, notably Gerlach and Svensson (1999) and Cecchetti, McConnell and Quiros (2000).<sup>15</sup> As an additional robustness check regarding our euro area estimates, we also consider the exponential trend preferred by Gerlach and Svensson. Estimates of both the linear and the exponential trend are depicted in the top left-hand panel of **Figure 1**. An advantage of the exponential trend estimate is that it flattens out in the late 1990s and is consistent with stable average inflation from then on. A drawback is the unreasonably high trend level of inflation at the beginning of the sample.

Focusing on inflation deviations from trend would be appropriate, if the source of disin-

<sup>&</sup>lt;sup>15</sup>Gerlach and Svensson use an exponential trend for the euro area inflation rate in estimating a P-star model of inflation dynamics à la Hallman, Porter and Small (1991) for the euro area. Cecchetti et al. construct inflation and output deviations from a 12-month moving average of actual values and estimate inflation-output tradeoffs based on this data for a number of euro area economies.

flation had been a credible, fully anticipated, gradually phased-in reduction in the policymakers' inflation target. However, given this was not the case, this approach introduces an error that may bias our estimation results. In particular, it could influence the estimated degree of inflation persistence and the implied case for the relative real wage specification. Again, the estimation with German data will provide a useful benchmark for comparison, because it is the only case for which the inflation series exhibits no strong trend. In addition, we will conduct a sensitivity study to assess how our euro area estimates would change if market participants had been consistently surprised by the downward trend.

## 4 Empirical inflation and output dynamics

Our empirical analysis proceeds in two stages. In the first stage, we estimate an unconstrained bivariate VAR model of output and inflation. In the second stage, we use this unconstrained VAR as an auxiliary model in estimating the structural overlapping wage contracting specifications by simulation-based indirect inference methods. These are methods for calibrating the parameters of the structural model by matching its reduced form, which constitutes a constrained VAR, as closely as possible with the estimated unconstrained VAR model.

The unconstrained VAR provides an empirical summary description of euro area inflation and output dynamics.<sup>16</sup> We estimate the short-run dynamics jointly with a deterministic linear trend for inflation and the logarithm of output over the sample period. Following Fuhrer and Moore (1995a) we then compute the autocorrelation functions implied by the VAR including the associated asymptotic confidence bands.<sup>17</sup> These autocorrelation functions serve as an indication whether the lead-lag relationship between inflation and output is consistent with a short-run tradeoff, that is, with a short-run Phillips curve. Furthermore,

<sup>&</sup>lt;sup>16</sup>Although interest rates are important determinants of output and inflation, we restrict attention to bivariate VARs without including an interest rate, primarily because it is unclear what would be an appropriate interest rate for the euro area. We return to this problem later on in section 6 when estimating an aggregate demand equation that closes the small macroeconomic model.

<sup>&</sup>lt;sup>17</sup>For a detailed discussion of the methodology and the derivation of the asymptotic confidence bands for the estimated autocorrelation functions the reader is referred to Coenen (2000).

they form a benchmark against which we can evaluate the ability of the alternative overlapping contracts specifications to explain the dynamics of inflation in euro area data. Such an approach has also been recommended by McCallum (1999), who argued that autocovariance and autocorrelation functions are a more appropriate device for confronting macroeconomic models with the data than impulse response functions because of their purely descriptive nature.

The empirical model for output and inflation, written in terms of the level of inflation,  $\Pi_t$ , and the logarithm of output,  $Q_t$ , corresponds to

$$\begin{bmatrix} \Pi_t \\ Q_t \end{bmatrix} = \begin{bmatrix} a_{0,\Pi} \\ a_{0,Q} \end{bmatrix} + \begin{bmatrix} a_{1,\Pi} \\ a_{1,Q} \end{bmatrix} t + \begin{bmatrix} \pi_t \\ q_t \end{bmatrix},$$
(5)

where  $\pi_t$  and  $q_t$  refer to the inflation and the output gap respectively, which are determined by an unconstrained VAR of lag order 3:

$$\begin{bmatrix} \pi_t \\ q_t \end{bmatrix} = A_1 \begin{bmatrix} \pi_{t-1} \\ q_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} \pi_{t-2} \\ q_{t-2} \end{bmatrix} + A_3 \begin{bmatrix} \pi_{t-3} \\ q_{t-3} \end{bmatrix} + \begin{bmatrix} u_{\pi,t} \\ u_{q,t} \end{bmatrix}.$$
 (6)

The  $A_i$  matrices (i = 1, 2, 3) contain the coefficients on the first three lags of the inflation and the output gap.<sup>18</sup> The error terms  $u_{\pi,t}$  and  $u_{q,t}$  are assumed to be serially uncorrelated with mean zero and positive definite covariance matrix  $\Sigma_u$ .

We fit this model to the aggregated output and inflation data for the euro area as a whole for the period from 1974:Q1 to 1998:Q4. First, we detrend the data by a simple projection technique and then we estimate the parameters of the VAR model, that is the coefficient matrices  $A_i$  and the covariance matrix  $\Sigma_u$  by Quasi-Maximum-Likelihood (QML) methods.<sup>19</sup> The estimates of the unconstrained VAR model are shown in **Table 1**. Standard lag selection procedures based on the HQ and SC criteria suggest that a lag order of 2 would be sufficient to capture the empirical inflation and output dynamics. The Ljung-Box Q(12) statistic indicates serially uncorrelated residuals with a marginal probability value of 42.8%. The estimates of the parameters of the VAR(2) model are shown in panel A. Our point

<sup>&</sup>lt;sup>18</sup>Here, we use a maximum lag order of 3, simply because this corresponds to the reduced-form VAR representation of the overlapping contract models of section 2 with a contract length of 4 quarters.

<sup>&</sup>lt;sup>19</sup>For some more detail the reader is referred to the appendix, section A.2.

estimates imply that the smallest root of the characteristic equation  $\det(I_2 - A_1 z - A_2 z^2) = 0$  is 1.2835, thereby suggesting that the inflation and output gaps are stationary. This conclusion is roughly supported by the results of standard univariate Dickey-Fuller tests for the presence of unit roots.

We also estimate an unconstrained VAR(3) model. This is of interest, because all contracting specifications discussed in section 2 have a reduced form which is a constrained VAR of order 3 if the maximum contract length is one year. To assess the sensitivity of our results to the lag length, we will use the VAR(2) and VAR(3) models in parallel in the estimation of the contracting specifications in the following section. On a statistical basis, the third lag would not be absolutely necessary, as can be seen from panel B, which shows that the  $A_3$  coefficients are insignificant. As a further robustness check we also estimate a VAR(3) model based on inflation deviations from the exponential trend. As can be seen by comparing the estimates in panel B with those in panel C the VAR(3) coefficients differ little between the linear trend and exponential trend specification, with the differences well below one standard deviation.

The autocorrelation functions associated with the unconstrained VAR(3) model of the euro area based on the linear trend are depicted in **Figure 3**. The diagonal elements show the autocorrelations of the detrended inflation rate and the output gap, the off-diagonal elements the lagged cross correlations. The solid lines represent the point estimates, while the dotted lines indicate 95% confidence bands. Both inflation and output are quite persistent with positive autocorrelations out to lags of about 5 and 8 quarters which are highly significant. The cross correlations in the off-diagonal panels confirm much of conventional wisdom about inflation and output dynamics. For example, in the second panel of the top row, a high level of output is followed by a high level of inflation a year later and again these cross correlations are statistically significant. In the first panel of the bottom row a high level of inflation is followed by a low level of output a year later. These lead-lag interactions are highly indicative of the existence of a conventional short-run tradeoff between output and inflation. All in all these correlations are stylized facts which any structural model of

output and inflation dynamics ought to be able to explain. The autocorrelation functions of the VAR(3) model based on inflation deviations from the exponential trend are very similar and thus not shown in the figure.

The results for the bivariate VARs of order 3 for France, Germany and Italy are summarized in **Table 2**. Note that for Germany the estimates are obtained without a trend in inflation  $(a_{1,\Pi} = 0)$ . The estimated autocorrelation functions for output and inflation in France and Italy display qualitatively similar characteristics as for the euro area as a whole, in particular regarding the persistence in inflation and output variations. The cross correlations, however, are somewhat smaller.<sup>20</sup> As to Germany, the degree of persistence in inflation is substantially lower and the correlations between current output and lagged inflation have the opposite sign, albeit statistically insignificant. We return to these issues in the next section, when we use the empirical autocorrelation functions as a benchmark to evaluate the fit of alternative structural overlapping contracts specifications.

## 5 Estimating the overlapping contracts specifications

In the following we use the unconstrained VAR models as approximating probability models in estimating the coefficients of the different overlapping contracts specifications discussed in section 2. As can be seen from equations (1) through (4) there are only three structural parameters to estimate for each specification: (i) the slope of the contracting distribution s that determines the series of contract weights  $f_i$ ; (ii) the sensitivity of the contract wage to expected future aggegrate demand over the life of the contract  $\gamma$ ; and (iii) the standard deviation of the contract wage shock  $\sigma_{\epsilon_x}$ .

#### 5.1 The reduced-form representation

Of course, the overlapping contracts specifications discussed in section 2 do not represent a complete model of inflation determination. Since the contract wage equations (2) and (4)

 $<sup>^{20}</sup>$ To save space, we do not report separate figures for the estimated autocorrelation functions and their associated asymptotic confidence bands. They are plotted jointly with the autocorrelation functions of the constrained VARs obtained under alternative overlapping contracting specifications in **Figures 5**, **6** and **7**.

contain expected future output gaps, we need to specify how the output gap is determined in order to solve for the reduced-form representation of inflation and output dynamics under each of the contracting specifications. A full-information estimation approach would require a complete macroeconomic model and estimate all the model's structural parameters jointly. A version of such a model in the spirit of Fuhrer and Moore (1995b) would include an aggregate demand equation, which relates output gaps to ex-ante long-term real rates, as well as a Fisher equation, a term structure relationship and a monetary policy rule.

While our ultimate objective is to build precisely such a model, we take a less ambitious approach in estimating the contracting parameters. We simply use the output gap equation from the unconstrained VAR models, which corresponds to the second row in (6), as an auxiliary equation for output determination. This limited-information approach is close to the estimation approaches used by Taylor (1993a) and Fuhrer and Moore (1995a) and is likely to be more robust to specification errors in the part of the model describing output determination than a full-information approach. We estimate the aggregate demand equation later on by single-equation methods and discuss those results in the next section.

Using the output equation from the unconstrained VAR together with the wage-price block from section 2, we can solve for the reduced-form inflation and output dynamics under each of the four different contracting specifications (RW, RW-C, RW-S and NW).<sup>21</sup> For this purpose it is convenient to rewrite the wage-price block, which was originally defined in levels of nominal contract wages and prices, in terms of the real contract wage  $(x-p)_t$ and the annualized quarterly inflation rate  $\pi_t$ . The reduced-form solution of this rational expectations model is a trivariate *constrained* VAR. While the quarterly inflation rate  $\pi_t$  and the output gap  $q_t$  are observable variables, the real contract wage  $(x-p)_t$  is unobservable.<sup>22</sup> For a contracting specification with a one-year maximum contract length this constrained

 $<sup>^{21}</sup>$ We assume that the expectations in the contract wage equations are formed rationally, and use the AIM algorithm of Anderson and Moore (1985) which solves for the reduced-form dynamics of linear rational expectations models by employing the Blanchard and Kahn (1980) method.

<sup>&</sup>lt;sup>22</sup>Note that for some models such as the RW-C specification it is helpful to define further auxiliary state variables that are unobservable. A more detailed discussion is provided in the appendix.

VAR can be written as

$$\begin{bmatrix} (x-p)_t \\ \pi_t \\ q_t \end{bmatrix} = B_1 \begin{bmatrix} (x-p)_{t-1} \\ \pi_{t-1} \\ q_{t-1} \end{bmatrix} + B_2 \begin{bmatrix} (x-p)_{t-2} \\ \pi_{t-2} \\ q_{t-2} \end{bmatrix} + B_3 \begin{bmatrix} (x-p)_{t-3} \\ \pi_{t-3} \\ q_{t-3} \end{bmatrix} + B_0 \epsilon_t, \quad (7)$$

where the  $B_i$  matrices (i = 0, 1, 2, 3) contain the coefficients of the constrained VAR and  $\epsilon_t$ is a vector of serially uncorrelated error terms with mean zero and positive (semi-) definite covariance matrix which is assumed to be diagonal with its non-zero elements normalized to unity.

The coefficients in the bottom row of the  $B_i$  matrices coincide exactly with the coefficients of the output equation of the unconstrained VAR, with the  $B_0$  cofficients obtained by means of a Choleski decomposition of the covariance matrix  $\Sigma_u$ . The reduced-form coefficients in the upper two rows of the  $B_i$  matrices, which are associated with the determination of the real contract wage and inflation, are functions of the structural parameters  $(s, \gamma, \sigma_{\epsilon_x})$ as well as the coefficients of the output equation of the unconstrained VAR.

#### 5.2 Estimates of the structural parameters

We estimate the structural parameters of the overlapping contracts specifications s,  $\gamma$  and  $\sigma_{\epsilon_x}$  using the indirect inference methods proposed by Smith (1993) and Gouriéroux, Monfort and Renault (1993) and developed further in Gouriéroux and Monfort (1995, 1996). The estimation procedure, including its asymptotic properties, is discussed in detail in the appendix of this paper. In the appendix we also compare this procedure to the Maximum-Likelihood methods used by Taylor (1993a) and Fuhrer and Moore (1995a).

Indirect inference is a simulation-based procedure for calibrating a structural model with the objective of finding parameter values such that its dynamic characteristics match the dynamic properties of the observed data as summarised by an approximating probability model. The latter should fit the empirical dynamics reasonably well, but need not necessarily nest the structural model. In the case at hand, the unconstrained VAR model discussed in section 4 is a natural candidate for such an approximating probability model. By serving as the approximating probability model for the estimation of the structural parameters by indirect inference and by providing a summary description of the inflation and output gap dynamics which the structural model should be able to explain the unconstrained VAR model provides a single, consistent benchmark for the empirical analysis carried out in this paper.

Of course, one cannot directly match the parameters of the constrained VAR model (7) with the parameters of the unconstrained VAR model (6) because the constrained VAR model also includes the unobservable real contract wage variable. Instead, for given values of the structural parameters  $(s, \gamma, \sigma_{\epsilon_x})$  and the parameters of the output gap equation from the unconstrained VAR model, we simulate the reduced form of the structural model, that is the constrained VAR model, to generate "artificial" series for the real contract wage, the inflation rate and the output gap. All that is needed for simulation are three initial values for each of these variables and a sequence of random shocks.<sup>23</sup> Subsequently we fit the unconstrained VAR model to the artificial series of inflation and the output gap and match the simulation-based estimates of the inflation equation as closely as possible with the empirical estimates by searching over the feasible space of the structural parameters.<sup>24</sup>

Euro area estimation results for the baseline version of the relative real wage contracting model (RW), the version with price expectations conditioned on historically available information (RW-C), the simplified version preferred by Fuhrer and Moore (RW-S) and the nominal wage contracting model (NW) are reported in **Table 3**. As a sensitivity check we consider both the VAR(2) and VAR(3) models estimated in section 4 as approximating probability models.<sup>25</sup> Furthermore, we investigate whether our findings based on the VAR(3) model are robust to the use of inflation deviations from the exponential instead of

 $<sup>^{23}</sup>$ In estimation we use steady-state values as initial conditions and are careful to only use simulation data for later periods that are essentially unaffected by this choice of initial conditions. This issue is discussed in more detail in the appendix, section A.3.

<sup>&</sup>lt;sup>24</sup>We do not need data for the unobservable real contract wage since the unconstrained VAR is only fitted to the observable data for inflation and the output gap.

<sup>&</sup>lt;sup>25</sup>In the case of the VAR(2) model, we restrict the maximum contract length in the structural contracting specification to three quarters instead of one year, such that its lag order corresponds to that of the structural model's reduced-form solution. In this case, the slope parameter s is restricted to lie in the interval (0, 1/3]. Note, because of the difference in its domain the magnitude of the slope parameter will not be directly comparable across the specifications with three-quarter and one-year maximum contract length, respectively.

the linear trend. The estimation results indicate that all four contracting models fit the euro area inflation dynamics reasonably well, in particular when we allow for a maximum contract length of one year and thus three lags in the VAR. As can be seen from the standard errors given in parentheses, the estimates of the structural parameters are in almost all cases statistically significant, with the appropriate sign and economically significant magnitude.

We also compute the probability (P-) values of the test for the over-identifying restrictions, which were imposed when estimating the structural parameters. According to this test, none of the four contracting specifications is rejected by the data, when we use the VAR(3) as approximating probability model and allow for a one-year maximum contract length. The coefficient estimates we obtain when using inflation deviations from the exponential trend are with one exception within one standard deviation of the estimates based on the linear trend in inflation. When we use the VAR(2) model and constrain the maximum contract length to three quarters, both, the RW-C and the RW-S specification can be rejected at convenient confidence levels, but not the RW or the NW specifications. Though the estimates of the real wage contracting specifications are not directly comparable, since the latter imply structures with different degrees of forward-lookingness, it is worthwhile to note that the RW-S specification implies stronger rigidities than the RW and the RW-C specifications as measured by the smaller estimates of the slope parameter *s* of the contracting distributions.

While neither the RW nor the NW specification can be rejected, we can use the associated P-values of the test of overidentifying restrictions to discriminate between these two specifications. In the case of our preferred setup with one-year maximum contract length and the VAR(3) as approximating probability model, the RW specification implies a higher P-value than the NW specification, whether we use inflation deviations from a linear or an exponential trend. For the estimation based on the VAR(2) model, however, the NW specification entails a higher P-value. Overall, the RW specification with one-year maximum contract length performs best. Thus, our findings for the euro area differ quite a bit from the results in Fuhrer and Moore (1995a), who reject the nominal wage contracting model for U.S. data and find that the RW-S specification fits U.S. inflation dynamics better than the theoretically more plausible RW specification.

To provide further insight regarding these estimation results, we compare the autocorrelation functions implied by the constrained VAR(3) representation of each of the four contracting models with the autocorrelation functions from the unconstrained VAR. As shown in **Figure 4**, the autocorrelation functions for all four models tend to remain inside the 95% confidence bands (dotted lines) associated with the autocorrelation functions of the unconstrained VAR. The three relative real wage contracting specifications (RW: solid line with bold dots, RW-C: dash-dotted line, RW-S: solid line) are rather similar. They exhibit substantial inflation persistence and quite pronounced cross correlations that are indicative of a short-run Phillips curve tradeoff. The upper right-hand panel indicates that high levels of output are followed by high inflation, while the lower left-hand panel shows that high levels of inflation are followed by low levels of output. The only noticeable difference from the unconstrained VAR is that the latter set of cross correlations for the nominal contracting model (NW: dashed line) indicate a lower degree of inflation persistence and less pronounced cross correlations than for the different relative real wage contracting models.

As noted in the introduction to this paper, the estimation results with euro area data may be questioned for a number of reasons. First of all, the data are artificial in the sense that they are only averages of the data from the member economies prior to the formation of EMU. Furthermore, the member economies experienced different monetary policy regimes. While Germany enjoyed stable inflation with fairly predictable monetary policy, other countries such as France and Italy experienced a long-drawn out convergence process, which was probably not fully anticipated by market participants. As a result, euro area inflation data exhibits a longlasting decline which we removed from the data by subtracting a linear trend.

To investigate the validity of our results with respect to aggregation, we also estimate the different contracting models for France, Germany and Italy separately. The results are summarized in **Table 4**. Here we only focus on the case of the VAR(3) model. For France we reject the RW-C and the RW-S specifications, but not the RW and the NW specification. The NW model exhibits the highest *P*-value. However, in this case the parameter measuring the sensitivity to aggregate demand,  $\gamma$ , is statistically insignificant. The parameter estimates for the RW specification are significant and relatively close to the values obtained for the euro area. For Italy, which experienced the most dramatic transition process, the estimation of the NW model did not converge. Instead, the RW and the RW-C model seem to fit Italian inflation data reasonably well and imply statistically significant parameter estimates. For Germany, where inflation exhibited no long-run trend, we find that all three relative real wage contracting models are strongly rejected by the data. While the nominal contracting model is also rejected, it does fit better in the sense of implying a higher *P*-value. The parameter estimates for the NW model with German data are surprisingly close to the NW estimates obtained with euro area data.

Again, a promising alternative approach for evaluating the fit of the RW and NW specifications is to compare the autocorrelation functions of the constrained and unconstrained VAR models. As shown in **Figure 5** for France, the NW specification does better than the RW specification in terms of fitting the empirical autocorrelations consistent with the result of the test of overidentifying restrictions presented in **Table 4**. In the case of Italy the RW model comes close to matching the empirical autocorrelations of inflation in the top lefthand panel of **Figure 7**, and also does reasonably well with regard to the cross correlations. The results for Germany in **Figure 6** are, as expected, quite different. The autocorrelation functions of the unconstrained VAR (solid line with dotted confidence bands) indicated a much smaller degree of inflation persistence and a counterintuitive, albeit statistically insignificant, positive correlation between output and lagged inflation. Consequently, the nominal contracting model has a better chance at fitting German inflation dynamics than the relative contracting models.

We conclude from these results that, both the RW and the NW specifications are plausible alternatives for the euro area. On the one hand, the estimation with aggregated euro area data indicates a slight preference for the relative wage contracting model. On the other hand, the comparison between France, Germany and Italy suggests that this preference may partly be due to the high-inflation regime in countries such as Italy and France and the fact that the subsequent long-run decline in inflation was not fully anticipated. Thus, an optimist would conclude that the independent European Central Bank will likely face a similar environment in the future as the Bundesbank did in Germany or possibly the French central bank in the latter part of the EMS, that is in the "Franc fort" period. In this case, the inflation-output relationship for the euro area would be best characterized by the nominal contracting specification. A skeptic, who suspects that stabilizing euro area inflation will require higher output losses, would instead prefer to use the RW specification for the euro area. A robust monetary policy strategy, however, should perform reasonably well under both specifications.

#### 5.3 Further sensitivity analysis

As discussed in sections 3 and 4, we used inflation deviations from trend rather than the inflation rate itself in estimating the alternative wage contracting specifications with French, Italian and euro area data. In setting up the empirical VAR model in equations (5) and (6) in section 4, we defined the inflation gap  $\pi_t$  as the difference between the level of inflation  $\Pi_t$  and trend inflation. Throughout our analysis we used a linear trend but for the euro area we considered in addition an exponential trend as a robustness check. The reason for detrending inflation was to separate the short-run dynamics in inflation that are due to nominal rigidities from the convergence process that was driven by changes in long-run inflation objectives in countries such as France and Italy. The question remains how our estimation results are affected by detrending the inflation series. Our approach would have been a fully anticipated and credible, gradually phased-in reduction in the policymakers' inflation target. However, given this was not the case, this approach introduces an error that may bias our estimation results.

In order to assess whether the detrending procedure may have introduced a significant error in our estimation, we have conducted a small Monte Carlo Study. In this study we compare the small-sample properties of the indirect inference estimation procedure under two alternative scenarios. The first scenario assumes that the downward linear trend in inflation was perfectly anticipated by wage and price setters. This scenario forms a benchmark for comparison that is needed to determine the small sample properties of the estimation procedure, when the model is correctly specified, before investigating the effect of model mis-specification on those estimates. In the second scenario, we consider the more realistic alternative that wage and price setters were consistently surprised by the downward trend in inflation. Technically, this means we assume that wage and price setters expect trend inflation to follow a random walk. Thus, denoting trend inflation by  $\pi^*$ , market participants expect trend inflation to be determined according to  $\pi_t^* = \pi_{t-1}^* + \epsilon_{\pi^*,t}$ , where  $\epsilon_{\pi^*}$ is a random shock. The motivation for this scenario is the following. Although market participants were not able to perfectly predict the downward trend in inflation, they were certainly able to take into account the possibility of unpredictable changes in the underlying long-run inflation target of monetary policymakers.

First, we generate a large number of data samples under each of these two scenarios. Then, we apply the detrending and estimation procedure to each simulated history. While the estimation procedure is correct in the first scenario, it suffers from mis-specification in the case of the random-walk scenario. We then assess to what extent the estimates obtained under either of the two scenarios differ from the true parameter values. Of course, the random-walk expectations scenario is only one of many possible scenarios, but it should provide some indication whether detrending the inflation series induces a danger of significantly under- or overstating the degree of inflation persistence due to structural nominal rigidities.

We decided to use the RW-S contracting specification for this Monte Carlo study, because it induces the highest degree of inflation persistence. Specifically we use the parameter estimates obtained with euro area data for the specification with a maximum contract length of four quarters, i.e. s = 0.0742,  $\gamma = 0.0212$  and  $\sigma_{\epsilon_x} = 0.0024$ . The sample sizes we consider are  $T \in \{100, 500\}$ . For each sample size 100 replications are carried out and for each simulated sample 100 initial observations are discarded to minimize the effect of the initial values that are set to the model's steady-state values. For the simulations with random walk expectations we draw the shocks  $\epsilon_{\pi^*}$  from a normal distribution with mean -0.001 and a standard deviation of the same absolute magnitude. As a result the simulated histories exhibit a negative drift, which on average is equal to the historical downward trend in euro area inflation.

The small-sample performance of the simulation-based indirect estimator is evaluated by means of some simple statistics: the average deviation of the estimates from the structural parameters  $(s, \gamma, \sigma_{\epsilon_x})$  used for generating the data (BIAS), their standard deviation (STD) and their root mean-squared error (RMSE). The statistics are expressed as fractions of the structural parameters (reported in percentage points). The results of the Monte Carlo experiment are summarized in **Table 5**.

Panel A refers to the baseline scenario where the downward trend in inflation is fully anticipated by wage and price setters, that is, the case where our approach of detrending the inflation series is correct. In this case, we find that with a sample size of T = 100 the estimates of the slope parameter of the contracting distribution s are biased upwards by 13.3% on average. This means, the indirect estimation method somewhat under-estimates the persistence of the inflation process in the RW-S model. The estimates of the sensitivity of the contract wage to expected future aggregate demand  $\gamma$  are biased upwards by 42.0% and the estimates of the standard deviation of the contract wage shock  $\sigma_{\epsilon_x}$  are biased downwards by -13.7%. Once we increase the sample size to T = 500 the biases of the estimates and similarly their standard deviation and their root mean-squared error turn out to be much smaller. We conclude from this exercise that our estimation procedure has reasonably good small-sample properties at least when the underlying model is correctly specified.

The outcome of the Monte Carlo study under the second scenario, where wage and price

setters' expectations incorporate the possibility of unpredictable changes in trend inflation (ultimately the policymaker's long-run inflation target), is reported in panel B. Although, the estimation procedure is still based on detrended inflation series and consequently not fully appropriate, the biases, standard deviations and root mean squared errors of the resulting parameter estimates are generally of the same magnitude as in panel A. In other words, the error introduced by detrending the inflation series under this scenario is rather small. In some cases the error even turns out to offset the small-sample bias observed in panel A to some extent. For example, the biases regarding the estimates of s and  $\sigma_{\epsilon_x}$  are a bit smaller than in panel A. Thus, the assumption implicit in our estimation procedure, namely that the downward-trend in inflation was fully anticipated by price setters, does not seem to introduce significant distortions in our estimation. While this is only a limited sensitivity study, we find the outcome rather encouraging.

Given the estimates of euro area inflation dynamics, as reported in **Table 3**, we now proceed to close the model by estimating an aggregate demand equation and imposing a term structure relationship and an interest rate rule.

### 6 Closing the model: Output gaps and interest rates

We model aggregate demand with a simple reduced-form IS equation, which relates the current output gap,  $q_t$ , to two lags of itself and to the lagged long-term ex-ante real interest rate,  $r_{t-1}^l$ :

$$q_t = \delta_0 + \delta_1 q_{t-1} + \delta_2 q_{t-2} + \delta_3 r_{t-1}^l + \sigma_{\epsilon_d} \epsilon_{d,t}.$$
(8)

 $\epsilon_{d,t}$  denotes an unexpected demand shock scaled with the parameter  $\sigma_{\epsilon_d}$ , which measures the standard deviation of the demand shock. The rationale for including lags of output is to account for habit persistence in consumption as well as adjustment costs and accelerator effects in investment. We use the lagged instead of the contemporaneous value of the real interest rate to allow for a transmission lag of monetary policy. For now we neglect the possibility of effects of the real exchange rate and expected future income on aggregate demand. Fuhrer and Moore (1995b) found that a similar aggregate demand specification fits U.S. output dynamics quite well. Since the euro area is a large, relatively closed economy just like the United States, the exchange rate is likely to play a less important role than it did in the individual member economies prior to EMU.

Next we turn to the financial sector and relate the long-term ex-ante real interest rate, which affects aggregate demand, to the short-term nominal interest rate, which is the principal instrument of monetary policy. Three equations determine the various interest rates in the model. The short-term nominal interest rate,  $i_t^s$ , is set according to a Taylor-type interest rate rule (see Taylor (1993b)). This rule incorporates policy responses to inflation deviations from target and output deviations from potential output, and allows for some degree of partial adjustment:

$$i_t^s = \alpha_r \, i_{t-1}^s + (1 - \alpha_r)(r^* + \pi_t^{(4)}) + \alpha_\pi(\pi_t^{(4)} - \pi^*) + \alpha_q \, q_t. \tag{9}$$

 $r^*$  denotes the long-run equilibrium real rate, while  $\pi^*$  refers to the policymaker's target for inflation. The inflation measure is the four-quarter moving average of the annualized quarterly inflation rate, that is  $\pi_t^{(4)} = \frac{1}{4} \sum_{j=0}^3 \pi_{t-j} = p_t - p_{t-4}$ , and the interest rate is annualized.

As to the term structure, we rely on the accumulated forecasts of the short rate over two years which, under the expectations hypothesis, will coincide with the long rate forecast for this horizon:

$$i_t^l = \mathcal{E}_t \left[ \frac{1}{8} \sum_{j=0}^7 i_{t+j}^s \right],$$
 (10)

where the term premium is assumed to be constant and equal to zero. By subtracting inflation expectations over the following 8 quarters, we then obtain the long-term ex-ante real interest rate:

$$r_t^l = i_t^l - \mathcal{E}_t \left[ \frac{1}{2} (p_{t+8} - p_t) \right].$$
 (11)

To estimate the parameters of the aggregate demand equation (8) we first construct the ex-post real long-term rate as defined by equations (10) and (11) but replacing expected

future with realized values. We then proceed to estimate the parameters by means of Generalized Method of Moments (GMM) using lagged values of output, inflation and interest rates as instruments. The estimation results are reported in **Table 6**. The sample period for this estimation is 1974:Q4 to 1997:Q1.

Panel A refers to the estimates for the euro area. In the first row, the output gap and interest rate data are area-wide GDP-PPP-weighted averages. The coefficients on the two lags of the output gap are significant and exhibit an accelerator pattern. The interest rate sensitivity of aggregate demand has the expected negative sign, however the parameter estimate is only borderline significant and rather small. It is not clear however, what is the appropriate real interest rate measure for the euro area. For example, instead of GDP weights it may be a more appropriate to use the relative weights in debt financing for aggregating national nominal interest rates. Or, one could make the argument that the relevant real rate for the euro area is the German one. After all, movements in German interest rates presumably had to be mirrored eventually by the other countries to the extent that they intended to maintain exchange rate parities within the European Monetary System. For this reason we also use the German real interest rate to estimate the interest rate sensitivity of euro area aggregate demand. In this case, as shown in the second row, we find similar coefficients on the lags of the output gap, but the estimate of the interest rate sensitivity is highly significant and three times as large.

We have subjected this specification of aggregate demand to a battery of sensitivity tests. For example, we have investigated alternative specifications of potential output, alternative horizons on the term structure equation, including the use of average long-term rates instead of a term structure based on short-term rates and we have varied the length of the sample period. At least qualitatively the estimation results remain the same.

For comparison, we have also estimated the same specification for France, Germany and Italy. In each case we use the domestic real interest rate. For France and Italy we obtain qualitatively similar estimates as for the euro area. For Germany however, the estimate of the interest rate sensitivity is not significant and the lags of output do not exhibit an accelerator-type pattern.

It remains to discuss the deterministic steady state of this model. In steady state, the output gap is zero and the long-term real rate is equal to the equilibrium real rate,  $r^*$ . This equilibrium rate is determined as a function of the parameters of the aggregate demand curve (8), given by  $r^* = -\delta_0/\delta_3$ . The steady-state value of inflation is determined exclusively by monetary policy. Since the overlapping contracts specifications of the wageprice block do not impose any restriction on the steady-state inflation rate, steady-state inflation will be equal to the inflation target,  $\pi^*$ , in the policy rule.

# 7 Evaluating monetary policy rules: An example

In the last few years it has become quite common to represent alternative monetary policy strategies in terms of rules for setting the short-term nominal interest rate. As far as European countries are concerned, empirical work by Clarida and Gertler (1997) and Clarida, Galí and Gertler (1998) suggests that German interest rate policy since 1979 is summarized quite well by an interest rate rule that responds to the forecast of inflation and the current output gap and exhibits some degree of partial adjustment. Clarida et al. (1998) also argue that German monetary policy had a strong influence on interest rate policy in the U.K., France and Italy throughout this period and may have led to higher interest rates in those countries than warranted by domestic conditions at the time of the EMS collapse.<sup>26</sup> More recently, Gerlach and Schnabel (2000) have suggested that average interest rates in the EMU countries in 1990-98, with the exception of the period of exchange market turmoil in 1992-93, moved very closely with average output gaps and inflation as implied by Taylor's rule.

In the remainder of this section we explore the inflation and output dynamics that would arise in our model of the euro area when the coefficients in the interest rate rule (9) are set equal to the values proposed in Taylor (1993b), that is  $\alpha_r = 0$ ,  $\alpha_{\pi} = 0.5$  and  $\alpha_q = 0.5$ . To this end, we simulate the impulse responses of inflation and output to unexpected demand

<sup>&</sup>lt;sup>26</sup>For an analysis of these tensions based on a multi-country model see Wieland (1996).

and supply shocks as well as a disinflation. We compare the simulation outcomes with the euro area estimates of the RW, RW-S, RW-C and NW specifications as well as the German estimate of the NW specification. For euro area aggregate demand we use the estimates obtained with the German real interest rate as reported in the second row of **Table 6**.

We start with an unexpected temporary supply shock, which in our framework is a shock to the contract wage equation. The standard deviation of this type of shock,  $\sigma_{\epsilon_x}$ , was estimated as part of the structural estimation of each of the contracting specifications in section 5. The effect of a standard deviation contract wage shock when policy follows Taylor's rule is depicted in **Figure 8.** The top and bottom left-hand panels show the response of inflation and the output gap given euro area estimates of the three different relative real wage contracting specifications. The solid line with bold dots refers to the RW specification, which we found to fit best with euro area data. The RW-S and RW-C specifications correspond to the solid and dashed-dotted lines respectively.

The shock occurs in the first quarter of the second year (period 5). As a result of this shock inflation increases over the next four quarters by almost a full percentage point. Monetary policy, which follows Taylor's rule, responds to this increase in inflation by raising short-term nominal interest rates sufficiently so as to increase the long-term real interest rate. This policy tightening induces a slowdown in aggregate demand which lasts for about four years. Since future aggregate demand affects contract wage setting and through this channel the inflation rate, inflation returns back to target after little more than two years and even undershoots for a few periods thereafter.

The panels on the right-hand side of **Figure 8** report the inflation and output responses for the nominal wage contracting specifications for Germany (dashed line) and the euro area (dotted line). Qualitatively, the impact on output and inflation is the same as for the relative wage contracting specifications. For the German estimates the impact on inflation is of similar magnitude, while it is somewhat smaller with the euro area estimates. Of course, the differences between the latter two simulations may partly be due to differences in the parameters of aggregate demand. The important result is that the magnitude of the shortfall of aggregate demand is much smaller under the nominal wage specifications than under the relative real wage specifications. Thus, the cost of stabilizing inflation in terms of reduced output is much larger under the relative real wage specifications.

**Figure 9** reports output and inflation responses in the case of an unexpected temporary demand shock of 0.5 percentage points of potential output. Again the outcomes under the relative real wage contracting specifications are shown in the panels on the left and under nominal wage contracting on the right-hand side. As demand increases above potential, inflation also rises above target up to 1 percentage point under the RW and RW-C specificiations. The impact on inflation is somewhat smoother but more drawn out for the RW-S specification. Monetary policy responds to this increase in inflation and output by raising nominal interest rates sufficiently to induce an increase in the long-term real interest rate which reduces aggregate demand and ultimately returns both inflation and output to steady state. Comparing the relative real wage contracting specifications with the nominal wage contracting specification, it is directly apparent that the inflationary impact of the demand shock is much smaller under nominal contracting.

Finally, **Figure 10** reports the effect of a disinflation by 2 percentage points under the alternative contracting specifications. This disinflation is simulated as an unanticipated, perfectly credible change in the policymaker's inflation target,  $\pi^*$  in the policy rule (9), from 2 percent to 0 percent. Initial conditions are consistent with a steady-state inflation rate of 2 percent. As shown in the two right-hand side panels, inflation declines rather quickly, within a year, to the new target value when contract wages are set according to Taylor's nominal contracting model. This decline is only accompanied by very small output losses. The solid lines in the left-hand side plane indicate that the output losses necessary to achieve this disinflation is much higher when contract wages are set according to the RW-S specification preferred by Fuhrer and Moore (1995a). A more surprising finding, however, is that the cost of disinflation under the RW and the RW-C specifications (solid line with bold dots and dashed-dotted line) is much lower than under the RW-S specification. In fact it is not too different from the disinflation in the NW model. The reason for the lower

output cost of the disinflation is that wage setters in the RW and RW-C setup set contract wages with respect to the average price level expected to prevail over the life of the contract. In the RW-S model contract wages are instead set with respect to the current price level. Thus, wage setters in the RW and RW-C models more quickly incorporate the disinflation in their decisions.

As far as future European monetary policy is concerned, however, the disinflation scenario shown in **Figure 10** may not be as relevant, simply because euro area inflation was already below the ECB's upper threshold of 2 percent at the outset of monetary union. Instead, the more relevant question may be how to keep inflation low in the event of unforeseen shocks such as the temporary demand and supply shocks depicted in **Figures 8** and **9** respectively.

# 8 Summary and concluding remarks

We have estimated a small-scale empirical model of the euro area where short-run real effects of monetary policy arise due to overlapping wage contracts. In particular, we focused on the nominal wage contracting model due to Taylor (1980) and three different versions of the relative real wage contracting model first proposed by Buiter and Jewitt (1981) and investigated empirically by Fuhrer and Moore (1995a) for the United States. Contrary to Fuhrer and Moore, who reject the nominal contracting model and find strong evidence in favor of the relative contracting model which induces a higher degree of inflation persistence, we find that both types of contracting models fit euro area data reasonably well. The best fitting specification is a version of the relative contracting model, which is theoretically more plausible than the simplified version preferred by Fuhrer and Moore.

To investigate the validity of our results, we also estimated the contracting models for France, Germany and Italy separately. Our findings show that the relative contracting model does quite well in countries which transitioned out of a high inflation regime such as France and Italy, while the nominal contracting model fits German data better.

One interpretation of these findings is that nominal rigidities in wage and price setting

are more pronounced in countries such as France and Italy than in Germany. If so, one might expect that the euro area as a whole will also be characterized by a higher degree of nominal rigidity than in Germany as suggested by our estimates based on euro area averages. Another interpretation would attribute the higher degree of inflation persistence in euro area, French and Italian data to expectations about imperfectly credible monetary policy and less than fully anticipated disinflation. In this case, the better fit of the relative wage contracting specification would be considered misleading. And consequently, the nominal wage contracting model may be viewed as a more accurate measure of nominal rigidities in the euro area in the future.

We close the model by estimating aggregate demand equations and positing a term structure relationship as well as a policy rule. Using the response coefficients recommended by Taylor (1993b) in this policy rule, we investigate the impact of unexpected demand and price shocks on output and inflation. As expected, we find that stabilizing inflation in the event of adverse price shocks is much more costly under the relative real wage contracting specifications than the nominal contracting specification. Furthermore, unanticipated demand shocks have a much stronger inflationary impact in the relative wage contracting models. The findings for unanticipated, credible disinflations are more surprising, since only one of the relative real wage specifications requires a substantial output loss to achieve such a disinflation.

For future research on evaluating alternative monetary policy strategies for the euro area with this model we plan to use both, the best fitting relative real wage and nominal wage contracting specifications. Given the high degree of uncertainty about the determination of euro area inflation in the future, it seems particularly important to identify robust monetary policy strategies that perform reasonably well under different types of nominal rigidities.

# References

Amato, J. and T. Laubach, 1999, Monetary policy in an estimated optimization-based model with sticky prices and wages, Manuscript.

Anderson, G. S. and G. R. Moore, 1985, A linear algebraic procedure for solving linear perfect foresight models, Economics Letters, 17, 247-252.

Angeloni, I. and L. Dedola, 1999, From the ERM to the euro: New evidence on economic and policy convergence among EU countries, European Central Bank, Working Paper No. 4, May.

Angeloni, I., V. Gaspar and O. Tristani, 1999, The monetary policy strategy of the ECB, in D. Cobham and G. Zis (eds.), From EMS to EMU, London: Macmillan.

Ascari, G. and J. A. Garcia, 1999, Relative wage concern and the Keynesian contract multiplier, EUI Working Paper ECO No. 99/5.

Blanchard, O. J. and C. Kahn, 1980, The solution of linear difference models under rational expectations, Econometrica, 48, 1305-1311.

Branch, M. A. and A. Grace, 1996, Optimization toolbox: For use with MATLAB, Natick: The Math Works.

Brayton, F., A. Levin, R. W. Tryon and J. C. Williams, 1997, The evolution of macro models at the Federal Reserve Board, Carnegie-Rochester Conference Series on Public Policy, 42, 115-167.

Bryant, R. C., P. Hooper and C. Mann (eds.), 1993, Evaluating policy regimes: New research in empirical macroeconomics, Washington DC: Brookings Institution.

Buiter, W. H. and I. Jewitt, 1981, Staggered wage setting with real wage relativities: Variations on a theme of Taylor, The Manchester School, 49, 211-228, reprinted in W. H. Buiter (ed.), 1989, Macroeconomic theory and stabilization policy, Manchester: Manchester University Press.

Cecchetti, S., M. McConnell and G. Perez Quiros, 2000, Policymakers' revealed preferences and the output-inflation variability tradeoff: Implications for the European System of Central Banks, forthcoming, European Economic Review.

Clarida, R. and M. Gertler, 1997, How the Bundesbank conducts monetary policy, in C. Romer and D. Romer (eds.), Reducing inflation, Chicago: NBER and University of Chicago Press.

Clarida, R., J. Galí and M. Gertler, 1998, Monetary policy rules in practice: Some international evidence, European Economic Review, 42, 1033-1067. Clarida, R., J. Galí and M. Gertler, 1999, The science of monetary policy: A new Keynesian perspective, Journal of Economic Literature, 37, 1661-1707.

Clarida, R., J. Galí and M. Gertler, 2000, Monetary policy rules and macroeconomic stability: Evidence and some theory, Quarterly Journal of Economics, 115, 147-180.

Coenen, G., 2000, Asymptotic confidence bands for the estimated autocovariance and autocovariance functions of vector autoregressive models, European Central Bank, Working Paper No. 9, January.

De Grauwe, P., 1996, Inflation convergence during the transition to EMU, in P. De Grauwe, S. Micossi and G. Tullio (eds.), Inflation and wage behaviour in Europe, Oxford: Oxford University Press.

De Grauwe, P., 1997, The economics of monetary integration, 3rd edition, Oxford: Oxford University Press.

Duffie, D. and K. J. Singleton 1993, Simulated moments estimation of Markov models of asset prices, Econometrica, 61, 929-952.

Erceg, C., D. Henderson, and A. Levin, 1999, Optimal monetary policy with staggered wage and price contracts, forthcoming in Journal of Monetary Economics.

Fagan, G., J. Henry and R. Mestre, 1999, An area-wide model (AWM) for the EU11, European Central Bank, Manuscript.

Fair, R. C. and E. P. Howrey, 1996, Evaluating alternative monetary policy rules, Journal of Monetary Economics, 38, 173-193.

Favero, C. A., F. Giavazzi and L. Spaventa, 1997, High yields: the spread on German interest rates, Economic Journal, 107, 956-985.

Fuhrer, J. C., 1997, Inflation/output variance trade-offs and optimal monetary policy, Journal of Money, Credit, and Banking, 29, 214-234.

Fuhrer, J. C. and G. R. Moore, 1995a, Inflation persistence, Quarterly Journal of Economics, 110, 127-159.

Fuhrer, J. C. and G. R. Moore, 1995b, Monetary policy trade-offs and the correlation between nominal interest rates and real output, American Economic Review, 85, 219-239.

Galí, J. and M. Gertler, 1999, Inflation dynamics: A structural econometric analysis, Journal of Monetary Economics, 44, 195-222.

Gallant, R. A. and G. Tauchen, 1996, Which moments to match?, Econometric Theory, 12, 657-681.

Giavazzi, F. and M. Pagano, 1994, The advantage of tying one's hands: EMS discipline and

central bank credibility, in T. Persson and G. Tabellini (eds.), Monetary and fiscal policy, Volume 1, Credibility, Cambridge: MIT Press (previously published in 1988, European Economic Review, 32, 1055-1075).

Gerlach, S. and L. E. O. Svensson, 1999, Money and inflation in the euro area: A case for monetary indicators?, Manuscript, July.

Gerlach, S. and G. Schnabel, 2000, The Taylor rule and interest rates in the EMU area, Economics Letters, 67, 165-171.

Goodfriend, M. and R. King, 1997, The new neoclassical synthesis and the role of monetary policy, NBER Macroeconomics Annual, Cambridge: MIT Press.

Gouriéroux, C. and A. Monfort, 1995, Testing, encompassing and simulating dynamic econometric models, Econometric Theory, 11, 195-228.

Gouriéroux, C. and A. Monfort, 1996, Simulation-based inference methods, Oxford: Oxford University Press.

Gouriéroux, C., A. Monfort and E. Renault, 1993, Indirect inference, Journal of Applied Econometrics, 8, Special issue: Econometric inference using simulation techniques, S85-S118.

Gros, D. and N. Thygesen, 1992, European monetary integration, London: Longman.

Hallman, J. J., R. D. Porter and D. H. Small, 1991, Is the price level tied to the M2 aggregate in the long run?, American Economic Review, 81, 841-858.

McCallum, B. T., 1999, Analysis of the monetary transmission mechanism: methodological issues, Manuscript, June.

McCallum, B. T. and E. Nelson, 1999, Performance of operational policy rules in an estimated semiclassical structural model, in Taylor, 1999a.

Newey, W. K. and K. D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, Econometrica, 55, 703-708.

OECD, 1999, OECD economic outlook No. 65, Organisation for Economic Co-Operation and Development, June.

Orphanides, A, D. Small, V. Wieland and D. Wilcox, 1997, A quantitative exploration of the opportunistic approach to disinflation, Finance and Economics Discussion Series, 97-36, Board of Governors of the Federal Reserve System, June.

Orphanides, A. and V. Wieland, 1998, Price stability and monetary policy effectiveness when nominal interest rates are bounded at zero, Finance and Economics Discussion Series, 98-35, Board of Governors of the Federal Reserve System, June. Reifschneider, D., R. Tetlow and J. C. Williams, 1999, Aggregate disturbances, Monetary policy, and the macroeconomy: The FRB/US perspective, Federal Reserve Bulletin, 85, 1-19.

Roberts, J., 1997, Is inflation sticky?, Journal of Monetary Economics, 39, 173-196.

Rotemberg, J. and M. Woodford, 1997, An optimization-based econometric framework for the evaluation of monetary policy, NBER Macroeconomics Annual, Cambridge: MIT Press.

Rotemberg, J. and M Woodford, 1999, Interest-rate rules in an estimated sticky price model, in Taylor, 1999a.

Rudebusch, G. and L. E. O. Svensson, 1999, Policy rules for inflation targeting, in Taylor, 1999a.

Sbordone, A., 1998, Price and unit labor costs: A new test of price stickiness, Manuscript, August.

Smith, A. A., 1993, Estimating nonlinear time-series models using simulated vector autoregressions, Journal of Applied Econometrics, 8, Special issue: Econometric inference using simulation techniques, S63-S84.

Toda, H. Y. and T. Yamamoto, 1995, Statistical inference in vector autoregressions with possibly integrated processes, Journal of Econometrics, 66, 225-250.

Taylor, J. B., 1980, Aggregate dynamics and staggered contracts, Journal of Political Economy, 88, 1-24.

Taylor, J. B., 1993a, Macroeconomic policy in the world economy: From econometric design to practical operation, New York: W.W. Norton.

Taylor, J. B., 1993b, Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

Taylor, J. B. (ed.), 1999a, Monetary policy rules, Chicago: NBER and University of Chicago Press.

Taylor, J. B., 1999b, Low inflation and the pricing power of firms, Manuscript, October.

White, H., 1994, Estimation, inference and specification analysis, Cambridge: Cambridge University Press.

Wieland, V., 1996, Monetary policy targets and the stabilization objective: A source of tension in the EMS, Journal of International Money and Finance, 15, 95-116.

# A Estimation by indirect inference

To estimate the structural parameters of the overlapping contracts specifications we apply the indirect inference methods proposed by Smith (1993), Gouriéroux, Monfort and Renault (1993) and developed further in Gouriéroux and Monfort (1995, 1996). Indirect inference is a simulation-based procedure for calibrating a structural model with the objective of finding values of its parameters such that its dynamic characteristics match the dynamic properties of the observed data as summarized by the estimated parameters of an approximating probability model.<sup>27</sup> The latter should fit the empirical dynamics reasonably well, but need not necessarily nest the structural model. The unconstrained VAR model, which we used as a descriptive device to explore the dynamic interaction of the inflation and output data in section 4 is a natural candidate for such an approximating probability model.

We start with a brief review of standard Maximum-Likelihood (ML) methods used by Taylor (1993a) and Fuhrer and Moore (1995a) for estimating the structural models at hand in subsection A.1, and then motivate and describe the indirect estimation method in subsection A.2. We also discuss the asymptotic properties of the indirect estimator as well as a global specification test which may be used to assess whether the structural model is consistent with the data. Finally, subsection A.3 provides more detail on the implementation of the indirect estimation method.

#### A.1 The ML estimator

Both, the Taylor and Fuhrer-Moore models have a stationary reduced-form vector autoregressive representation

$$z_t = B_1 z_{t-1} + \dots + B_p z_{t-p} + B_0 \epsilon_t, \tag{A.1}$$

where the K-dimensional vector of endogenous variables  $z_t = [x'_t, y'_t]'$  comprises a kdimensional vector of observable variables  $y_t$  and a (K - k)-dimensional vector of nonobservable variables  $x_t$  such as the contract wage. The K-dimensional vector  $\epsilon_t$  is serially uncorrelated with mean zero and positive semi-definite covariance matrix. The covariance matrix is assumed to be diagonal with its non-zero elements normalized to unity. The  $(K \times K)$ -dimensional coefficient matrices  $B_i = B_i(\theta)$   $(i = 0, 1, \ldots, p)$  are non-linear functions of an *m*-dimensional vector of structural parameters  $\theta \in \Theta$ , where  $\Theta \subset \mathbf{R}^m$  denotes the feasible parameter space. The maximum lag length p of the endogenous variables  $z_t$  is equal to the maximum length of the wage contracts minus one. We refer to (A.1) as our constrained VAR(p) model, but also note that it contains unobservable variables such as the contract wage  $x_t$  in addition to output and inflation which are the only variables in the empirical VAR.

<sup>&</sup>lt;sup>27</sup>See Duffie and Singleton (1993) and Gallant and Tauchen (1996) for related approaches relying either on selected sample moments or on the scores of an approximating probability model.

The reduced-form representation of the structural model can also be characterized by a family of parametric conditional density functions

$$\mathbf{F}(\theta) = \left\{ f(z_{-p+1}, \dots, z_0; \theta), \left\{ f(y_t | z_{t-p}, \dots, z_{t-1}; \theta) \right\}_{t=1}^{\infty} : \theta \in \Theta \right\}$$

where  $\mathbf{f}(\theta)$  denotes an element of  $\mathbf{F}(\theta)$ . The observed data  $\{y_t\}_{t=-p+1}^T$  is then presumed to be a sample from  $f(z_{-p+1}, \ldots, z_0; \theta) \prod_{t=1}^T f(y_t | z_{t-p}, \ldots, z_{t-1}; \theta)$  for some  $\theta \in \Theta$ . Usually, the structural parameter vector  $\theta$  may be estimated by Maximum-Likelihood (ML) methods relying on Kalman filtering techniques (see Fuhrer and Moore (1995a, 1995b)). In particular, if we condition on fixed pre-sample values  $z_{-p+1}, \ldots, z_0$ , the conditional ML estimator  $\hat{\theta}_T$ for  $\theta$  is

$$\hat{\theta}_T = \arg \max_{\theta \in \Theta} \sum_{t=1}^T \ln f(y_t | z_{t-p}, \dots, z_{t-1}; \theta).$$

Under appropriate regularity conditions the ML estimator  $\hat{\theta}_T$  is consistent for the "true" structural parameter vector  $\theta_0$ ,

$$\lim_{T \to \infty} \hat{\theta}_T = \theta_0,$$

and asymptotically normal,

$$\sqrt{T} \left( \hat{\theta}_T - \theta_0 \right) \stackrel{d}{\longrightarrow} \mathrm{N}[0, \Sigma_{\hat{\theta}}(\theta_0)],$$

where  $\Sigma_{\hat{\theta}}(\theta_0)$  is the asymptotic covariance matrix of  $\sqrt{T} (\hat{\theta}_T - \theta_0)$ .

In practice, however, the ML estimator may be sensitive with respect to the choice of the partially unobserved initial conditions  $z_{-p+1}, \ldots, z_0$  and may lack robustness if the presumed data-generating process  $\mathbf{f}(\theta)$  is mis-specified.

#### A.2 The Indirect Estimator

Instead of estimating the parameter vector  $\theta$  of the structural model directly, indirect estimation starts from an approximating probability model – henceforth auxiliary model – which is capable of summarizing the dynamic characteristics of the sequence of observed data  $\{y_t\}_{t=-p+1}^T$ .

In the case at hand, we use the unconstrained k-dimensional VAR(p) model

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \tag{A.2}$$

with  $u_t$  being serially uncorrelated with mean zero and positive-definite covariance matrix  $E[u_t u'_t] = \Sigma_u$  for approximating the structural model, because the latter has a constrained VAR(p) representation as given by (A.1).

Throughout, we assume that the unconstrained VAR(p) model (A.2) is stable, i.e.

$$\det(I_k - A_1 z - \dots - A_p z^p) = 0 \quad \Rightarrow \quad |z| > 1,$$

where  $|\cdot|$  denotes the absolute value operator.

The unconstrained VAR(p) model, in turn, can be characterized by a parametric family of conditional densities

$$\mathbf{F}(\beta) = \left\{ \tilde{f}(y_{1-p}, \dots, y_0; \beta), \{ \tilde{f}(y_t | y_{t-p}, \dots, y_{t-1}; \beta) \}_{t=1}^{\infty} : \beta \in B \right\}$$

with an element of  $\mathbf{F}(\beta)$  being denoted by  $\mathbf{f}(\beta)$  and where

$$\beta = \left[ \operatorname{vec}(A_1, \dots, A_p)', \operatorname{vech}(\Sigma_u)' \right]',$$

is the *n*-dimensional parameter vector of the auxiliary VAR(*p*) model with  $n = pk^2 + k(k + 1)/2$ ).  $B \subset \mathbf{R}^n$  denotes the feasible parameter space. The vec(·)-operator stacks the columns of a matrix in a column vector and the vech(·)-operator stacks the elements on and below the principal diagonal of a square matrix.

Given a sample  $\{y_t\}_{t=1}^T$  with fixed pre-sample values  $y_{-p+1}, \ldots, y_0$ , its dynamic characteristics can be summarized by computing the conditional Quasi-Maximum-Likelihood (QML) estimator  $\hat{\beta}_T$  for  $\beta$ ,<sup>28</sup>

$$\hat{\beta}_T = \arg \max_{\beta \in B} \sum_{t=1}^T \ln \tilde{f}(y_t | y_{t-p}, \dots, y_{t-1}; \beta).$$

Under general regularity conditions the QML estimator  $\hat{\beta}_T$  is consistent for the "true" reduced-form parameter vector  $\beta_0$ ,

$$\lim_{T \to \infty} \hat{\beta}_T = \beta_0,$$

and asymptotically normal,

$$\sqrt{T} \left( \hat{\beta}_T - \beta_0 \right) \stackrel{d}{\longrightarrow} \mathrm{N}[0, \Sigma_{\hat{\beta}}(\beta_0)],$$

$$Y_t = \alpha_0 + \alpha_1 t + y_t$$

Substituting (A.2), it becomes obvious that  $\{Y_t\}$  is assumed to follow a VAR(p) process around a deterministic linear trend,

$$Y_t - \alpha_0 - \alpha_1 t = A_1 (Y_{t-1} - \alpha_0 - \alpha_1 (t-1)) + \dots + A_p (Y_{t-p} - \alpha_0 - \alpha_1 (t-p)) + u_t.$$

Since we are merely interested in the parameters of the VAR(p) model, i.e.  $A_1, \ldots, A_p$  and  $\Sigma_u$ , we proceed in two steps. First, we detrend the data by a simple projection technique to obtain the sample  $\{y_t\}$ . Second, using this sample, we compute the QML estimates of the parameters of interest.

<sup>&</sup>lt;sup>28</sup>As layed out in Section 4, we assume that the available data  $\{Y_t\}_{t=-p+1}^T$  are a sample of the 2dimensional vector of variables  $Y = [\Pi, Q]'$  being generated by the linear model

with  $\{y_t\}$  following a stable VAR(p) process as represented by (A.2) above. This type of model has been advocated by Toda and Yamamoto (1995) for conducting statistical inference in vector autoregressions with possibly integrated variables.

where  $\Sigma_{\hat{\beta}}(\beta_0) = \mathcal{H}(\beta_0)^{-1} \mathcal{I}(\beta_0) \mathcal{H}(\beta_0)^{-1}$  is the asymptotic covariance matrix of  $\sqrt{T} (\hat{\beta}_T - \beta_0)$ .  $\mathcal{I}(\beta_0)$  denotes the asymptotic information matrix and  $\mathcal{H}(\beta_0)$  is the asymptotic expected Hessian of the appropriately normalized quasi-log-likelihood function evaluated at  $\beta_0$ .<sup>29</sup>

If the auxiliary model  $\mathbf{F}(\beta)$  approximates the structural model  $\mathbf{F}(\theta)$  sufficiently well, it makes sense to estimate the vector of structural parameters  $\theta$  indirectly by minimizing the "distance" between the structural model  $\mathbf{F}(\theta)$  and the auxiliary model fitted to the data, i.e.  $\mathbf{f}(\hat{\beta}_T) \in \mathbf{F}(\beta)$ . Thus, the indirect estimator brings the information in  $\hat{\beta}_T$  to bear on the task of estimating the structural parameter vector  $\theta$ .

To make this approach operational Gouriéroux, Monfort and Renault (1993) and Gouriéroux and Monfort (1995, 1996) start from the well-known Kullback-Leibler information criterion (KLIC),

$$\mathrm{KLIC}(\beta,\theta) = \mathrm{E}_{\theta} \left[ \ln \left( \frac{f(y_t | z_{t-p}, \dots, z_{t-1}; \theta)}{\tilde{f}(y_t | y_{t-p}, \dots, y_{t-1}; \beta)} \right) \right],$$

and introduce the so-called binding function  $b: \Theta \to B$ ,

$$b(\theta) = \arg \min_{\beta \in B} \text{KLIC}(\beta, \theta)$$
  
=  $\arg \max_{\beta \in B} \text{E}_{\theta} \Big[ \ln \tilde{f}(y_t | y_{t-p}, \dots, y_{t-1}; \beta) \Big],$ 

which, for a given  $\theta \in \Theta$ , identifies the density  $\mathbf{f}(\beta) \in \mathbf{F}(\beta)$  minimizing the distance between the particular density  $\mathbf{f}(\theta) \in \mathbf{F}(\theta)$  and the elements of the family of densities  $\mathbf{F}(\beta)$ .

For a given  $\beta \in B$ , the expectation of the logarithm of the conditional density  $\ln \tilde{f}(y_t|y_{t-p},\ldots,y_{t-1};\beta)$  has to be determined with respect to the conditional density  $f(y_t|z_{t-p},\ldots,z_{t-1};\theta)$  from  $\mathbf{f}(\theta) \in \mathbf{F}(\theta)$ . While this expectation cannot be obtained analytically in general, it may easily be approximated for given parameters  $\theta$  and  $\beta$  by the simulated sample moment

$$\frac{1}{S}\sum_{s=1}^{S}\ln \tilde{f}(y_s(\theta)|y_{s-p}(\theta),\ldots,y_{s-1}(\theta);\beta),$$

where the simulated sample  $\{y_s(\theta)\}_{s=-p+1}^S$  is generated by the structural model characterized by  $\mathbf{f}(\theta) \in \mathbf{F}(\theta)$ .

Obviously, the approximation of the binding function  $b(\theta)$  coincides with the QML estimator  $\hat{\beta}_{S}(\theta)$  of the auxiliary model using the simulated sample,

$$\hat{\beta}_{S}(\theta) = \arg \max_{\beta \in B} \frac{1}{S} \sum_{s=1}^{S} \ln \tilde{f}(y_{s}(\theta) | y_{s-p}(\theta), \dots, y_{s-1}(\theta); \beta),$$

where, without loss of generality, the simulated conditional quasi-log-likelihood function has been normalized by 1/S.

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 $<sup>^{29}</sup>$ See White (1994) for a thorough treatment of QML theory and covariance estimation.

Subsequently, a simulation-based indirect estimator  $\hat{\theta}_{S,T}$  for  $\theta$  is obtained by minimizing a criterion function  $Q_{S,T}: B \times \Theta \to \mathbf{R}_+$  which is defined as a quadratic form measuring the distance between the empirical QML estimator  $\hat{\beta}_T$  and the simulated QML estimator  $\hat{\beta}_S(\theta)$ :

$$\hat{\theta}_{S,T} = \arg\min_{\theta \in \Theta} Q_{S,T}(\hat{\beta}_T, \theta)$$

with

$$Q_{S,T}(\hat{\beta}_T,\theta) = \left(\hat{\beta}_T - \hat{\beta}_S(\theta)\right)' W_T(\hat{\beta}_T) \left(\hat{\beta}_T - \hat{\beta}_S(\theta)\right),$$

where  $W_T(\hat{\beta}_T)$  is a positive definite  $(n \times n)$ -dimensional weighting matrix which possibly depends on the QML estimate  $\hat{\beta}_T$ . The two subscripts S and T indicate that the particular object depends on both the empirical sample  $\{y_t\}_{t=-p+1}^T$  and the simulated sample  $\{y_s(\theta)\}_{s=-p+1}^S$ .

Under appropriate assumptions, the following results can be established.

#### **Proposition:** (Asymptotic properties of the indirect estimator)

*i.* The indirect estimator  $\hat{\theta}_{S,T}$  is consistent for  $\theta_0$ ,

$$\lim_{T \to \infty} \hat{\theta}_{S,T} = \theta_0,$$

and asymptotically normal,

$$\sqrt{T} \left( \hat{\theta}_{S,T} - \theta_0 \right) \stackrel{d}{\longrightarrow} \mathrm{N}[0, (1 + c^{-1}) (\mathcal{B}' W \mathcal{B})^{-1} \mathcal{B}' W \Sigma_{\hat{\beta}} W \mathcal{B} (\mathcal{B}' W \mathcal{B})^{-1}],$$

where S = cT with  $c \in \mathbf{N}$ ,  $\mathcal{B} = \mathcal{B}(\theta_0) = (\partial / \partial \theta') \mathbf{b}(\theta_0)$ ,  $\Sigma_{\hat{\beta}} = \Sigma_{\hat{\beta}}(\beta_0) = \lim_{T \to \infty} \operatorname{Var}[\sqrt{T}(\hat{\beta}_T - \beta_0)]$  and  $W = W(\beta_0) = \operatorname{plim}_{T \to \infty} W_T(\hat{\beta}_T)$  with  $\beta_0 = \operatorname{plim}_{T \to \infty} \hat{\beta}_T$ .

ii. The asymptotically efficient indirect estimator  $\hat{\theta}_{S,T}^*$  is obtained by using a consistent estimate  $\left(\hat{\Sigma}_{\hat{\beta},T}\right)^{-1}$  of the optimal asymptotic weighting matrix  $W^* = \Sigma_{\hat{\beta}}^{-1}$  and is asymptotically normal,

$$\sqrt{T} \left( \hat{\theta}_{S,T}^* - \theta_0 \right) \stackrel{d}{\longrightarrow} \mathrm{N}[0, (1 + c^{-1}) (\mathcal{B}' \Sigma_{\hat{\beta}}^{-1} \mathcal{B})^{-1}].$$

- iii. If the auxiliary model  $\mathbf{F}(\beta)$  nests the structural model  $\mathbf{f}(\theta_0) \in \mathbf{F}(\theta)$ , then the asymptotically efficient indirect estimator  $\hat{\theta}_{ST}^*$  is as efficient as the ML estimator  $\hat{\theta}_T$ .
- iv. Under the null hypothesis that the structural model  $\mathbf{f}(\theta_0) \in \mathbf{F}(\theta)$  is correctly specified, the statistic

$$\mathcal{Z}_{S,T}(\hat{\beta}_T, \hat{\theta}_{S,T}^*) = \frac{ST}{S+T} Q_{S,T}^*(\hat{\beta}_T, \hat{\theta}_{S,T}^*)$$

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with

$$Q_{S,T}^*(\hat{\beta}_T, \hat{\theta}_{S,T}^*) = \min_{\theta \in \Theta} \left( \hat{\beta}_T - \hat{\beta}_S(\theta) \right)' \left( \hat{\Sigma}_{\hat{\beta},T} \right)^{-1} \left( \hat{\beta}_T - \hat{\beta}_S(\theta) \right),$$

is asymptotically  $\chi^2$ -distributed with n-m degrees of freedom.

*Proof:* Parts *i.*, *ii.* and *iv.* of the proposition are alternatively established in Smith (1993), Gouriéroux, Monfort and Renault (1993) and Gouriéroux and Monfort (1996), chapter 4.5. Part *iii.* follows from Gallant and Tauchen (1996), pp. 665-666.

Among the assumptions underlying this proposition is the identification condition that the equation  $b(\theta) = \beta_0$  has a unique root at  $\theta_0 \in \Theta$ . This condition resembles the standard identification condition in a non-linear equation system. The necessary (order and rank) condition for identification is  $n \ge m$ . If n > m holds, n - m overidentifying restrictions are imposed when estimating  $\theta$ . For the asymptotically efficient indirect estimator  $\hat{\theta}_{S,T}^*$ these overidentifying restrictions can be tested by means of the statistic  $\mathcal{Z}_{S,T}(\hat{\beta}_T, \hat{\theta}_{S,T}^*)$ . Its probability value  $P(\mathcal{Z} > z)$  gives the probability that the re-scaled minimized criterion function takes a value larger than that obtained in the estimation exercise. Hereby, a probabilistic assessment of the consistency of the structural model and the data is obtained.

#### A.3 Implementation

To estimate the parameters of the different wage contracting specifications, it is necessary to close the model in a way that makes it possible to solve for expected future output gaps. Rather than specifying a complete macroeconomic model for forecasting future output gaps that enter the contract wage equations, we instead use the reduced-form output gap equation from the unconstrained VAR(p) model fitted to the data. Thus, we confine ourselves to the parameters of the inflation equation in conducting indirect inference. This approach is very much in the spirit of the limited-information ML procedure applied by Fuhrer and Moore (1995a). They use the output gap and interest rate equations from a three-dimensional VAR model to generate the output gap forecasts in the relative real wage contracting equation. Similarly, Taylor (1993a) used such a reduced-form output gap equation in applying limitedinformation ML methods for estimating the structural parameters of the nominal wage contracting equation.

Let S denote the  $((pk+1) \times (pk^2 + k(k+1)/2))$ -dimensional (0, 1) selection matrix which picks the elements of  $\beta \in B$  corresponding to the inflation equation, then our asymptotically efficient indirect estimator  $\hat{\theta}_{S,T}^*$  for  $\theta \in \Theta$  is obtained by minimizing the criterion function

$$Q_{S,T}^*(\hat{\beta}_T,\theta) = \left(\hat{\beta}_T - \hat{\beta}_S(\theta)\right)' \mathcal{S}' \left(\mathcal{S}\hat{\Sigma}_{\hat{\beta},T} \mathcal{S}'\right)^{-1} \mathcal{S} \left(\hat{\beta}_T - \hat{\beta}_S(\theta)\right),$$

where  $\hat{\beta}_T$  and  $\hat{\beta}_S(\theta)$  are the empirical and the simulated QML estimates of the parameters

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of the unconstrained VAR(p) model and  $(\hat{\Sigma}_{\hat{\beta},T})^{-1}$  is a consistent estimate of the optimal asymptotic weighting matrix  $W^* = (\Sigma_{\hat{\beta}})^{-1}$ .

In order to minimize the criterion function  $Q_{S,T}^*(\hat{\beta}_T, \theta)$  with respect to  $\theta \in \Theta$ , we rely on the sequential dynamic programming algorithm provided by the MATLAB Optimization Toolboox.<sup>30</sup> This algorithm allows to take account of the constraints to be imposed on the parameter space  $\Theta$ , thereby guaranteeing the existence of a unique rational expectations solution. When minimizing  $Q_{ST}^*(\hat{\beta}_T, \theta)$ , we repeatedly have to simulate samples of the observable variables  $\{y_s(\theta)\}_{s=-p+1}^S$  from our structural model in order to compute the simulated QML estimate  $\hat{\beta}_{S}(\theta)$ . These samples are generated by drawing a normally distributed random sequence  $\{\epsilon_s\}_{s=-(\tilde{p}+p)+1}^S$  and recursively computing the accompanying sequences of endogenous variables  $\{z_s(\theta)\}_{s=-(\tilde{p}+p)+1}^S$  by using the structural model's reduced-form representation (A.1) for varying parameter vectors  $\theta \in \Theta$ . The recursions may start from arbitrary initial values  $\bar{z}$ , but a sufficiently large number of simulated values, say  $\tilde{p} = 100 - p$ , should be discarded in order to guarantee that the effect of the initial values die out. The sequences of the observable variables  $\{y_s(\theta)\}_{s=-p+1}^S$  are subsequently retained from the sequences  $\{z_s(\theta)\}_{s=-p+1}^S$ . Note that in repeated simulations the employed random number generator must always start from the same random seed. Similarly, always the same initial values  $\bar{z}$  must be chosen.

Reporting standard errors for the indirect estimate  $\hat{\theta}_{S,T}^*$  requires estimation of the asymptotic covariance matrix  $\Sigma_{\hat{\theta}^*} = (1 + c^{-1})(\mathcal{B}' \mathcal{S}' (\mathcal{S} \Sigma_{\hat{\beta}} \mathcal{S}')^{-1} \mathcal{S} \mathcal{B})^{-1}$ . This matrix may be consistently estimated by replacing the unknown quantities by consistent estimates,

$$\hat{\Sigma}_{\hat{\theta}^*,S,T} = (1+c^{-1}) \left( B_S(\hat{\theta}^*_{S,T})' \mathcal{S}' \left( \mathcal{S} \hat{\Sigma}_{\hat{\beta},T} \mathcal{S}' \right)^{-1} \mathcal{S} B_S(\hat{\theta}^*_{S,T}) \right)^{-1}$$

with  $B_S(\theta) = (\partial/\partial \theta') \hat{\beta}_S(\theta)$  being computed by finite difference methods.

Our experience has been, however, that it is more convenient to estimate the asymptotic covariance matrix by the inverse of the appropriately normalized Hessian of the criterion function that is returned by the numerical algorithm,

$$\hat{\Sigma}_{\hat{\theta}^*,S,T} = (1+c^{-1}) \left( \frac{1}{2} \frac{\partial Q_{S,T}^*(\hat{\beta}_T, \hat{\theta}_{S,T}^*)}{\partial \theta \, \partial \theta'} \right)^{-1}.$$

 $<sup>^{30}\</sup>mathrm{See}$  Branch and Grace (1996) for technical details.

# Table I

#### Estimates of the Unconstrained VAR Model for the Euro Area

$A_1$		$A_2$		$A_3$		$\Sigma_u   imes  10^4$	
A. VAR(2	:):						
0.4879 (0.0963)	$0.3890 \\ (0.1709)$	$0.0989 \\ (0.0899)$	-0.2190 (0.1688)			0.9871 (0.1518)	
$\begin{array}{c} 0.0481 \\ (0.0571) \end{array}$	$1.1236 \\ (0.0928)$	-0.2159 (0.0366)	-0.1605 (0.1094)			-0.0686 (0.0528)	$\begin{array}{c} 0.2736 \\ (0.0584) \end{array}$
B. VAR(3	):						
$0.4763 \\ (0.0968)$	$0.4038 \\ (0.1661)$	$0.0995 \\ (0.1035)$	-0.2272 (0.2644)	$0.0181 \\ (0.1043)$	-0.0105 (0.1774)	$0.9826 \\ (0.1556)$	
$\begin{array}{c} 0.0430 \\ (0.0561) \end{array}$	$1.0758 \\ (0.1041)$	-0.1804 (0.0502)	-0.0450 (0.1345)	-0.0426 (0.0533)	-0.0740 (0.0767)	-0.0701 (0.0527)	$0.2728 \\ (0.0619)$
C. VAR(3	) [exponentia	al trend in in	flation]:				
$0.5086 \\ (0.0897)$	0.4916 (0.2006)	0.1171 (0.1127)	-0.2751 (0.2909)	0.0758 (0.1042)	-0.0646 (0.1834)	1.0139 (0.1687)	
0.0708 (0.0438)	1.1373 (0.1383)	-0.1643 (0.0469)	-0.0843 (0.1581)	0.0015 (0.0756)	-0.1134 (0.0624)	-0.0456 (0.0568)	$0.2889 \\ (0.0728)$

Note: Estimates of the asymptotic standard errors in parentheses with the asymptotic information matrix being estimated by the Newey-West (1987) estimator with the lag truncation parameter set equal to 3.

Estimates of the Unconstrained VAR(3) Model for France, Germany and Italy

$A_1$		$A_2$		$A_3$		$\Sigma_u \times 10^4$	
A. France	<u>:</u>						
$0.4898 \\ (0.1201)$	$\begin{array}{c} 0.4551 \\ (0.3697) \end{array}$	-0.0645 (0.1465)	-0.4035 (0.5265)	$0.1262 \\ (0.0990)$	$\begin{array}{c} 0.0506 \ (0.3305) \end{array}$	$3.1883 \\ (0.6399)$	
$\begin{array}{c} 0.0069\\ (0.0274) \end{array}$	$\begin{array}{c} 1.1412 \\ (0.1135) \end{array}$	-0.0804 (0.0384)	-0.0661 (0.1496)	$\begin{array}{c} 0.0291 \\ (0.0415) \end{array}$	-0.1499 (0.0889)	-0.2390 (0.1020)	$\begin{array}{c} 0.3151 \ (0.0524) \end{array}$
B. Germa	ny:						
$0.0334 \\ (0.0873)$	$0.4474 \\ (0.1867)$	$\begin{array}{c} 0.2035 \\ (0.0990) \end{array}$	-0.0508 (0.2240)	$\begin{array}{c} 0.1402 \\ (0.0899) \end{array}$	-0.1270 (0.2125)	$3.5367 \\ (0.4390)$	
-0.0190 (0.0658)	0.7480 (0.0812)	-0.1061 (0.0561)	$\begin{array}{c} 0.1380 \\ (0.0891) \end{array}$	-0.0197 (0.0622)	$0.0692 \\ (0.0901)$	-0.2614 (0.2357)	$1.1826 \\ (0.1623)$
C. Italy:							
0.7137 (0.1186)	$0.5620 \\ (0.3391)$	-0.0715 (0.2425)	-0.5580 (0.7027)	0.0074 (0.0986)	$0.0502 \\ (0.4881)$	4.4426 (1.1728)	
0.0005 (0.0313)	$1.3220 \\ (0.1312)$	-0.0215 (0.0367)	-0.3212 (0.1855)	-0.0711 (0.0273)	-0.0362 (0.0877)	$\begin{array}{c} 0.2931 \\ (0.1389) \end{array}$	$0.4077 \\ (0.0848)$

Note: Estimates of the asymptotic standard errors in parentheses with the asymptotic information matrix being estimated by the Newey-West (1987) estimator with the lag truncation parameter set equal to 3.

	Rel	Nominal Wage		
	RW	RW-C	RW-S	Contracts (NW)
A. VAR(2): $a$				
8	.0658 $(.0283)$	.1344 $(.0330)$	$( - )^{b}$	0( — )
$\gamma$	.0016 $(.0000)$	.0026 $(.0006)$	.0126 $(.0033)$	.0070 (.0025)
$\sigma_{\epsilon_x}$	.0002 $(.0000)$	.0009 $(.0002)$	.0018 $(.0001)$	.0027 $(.0001)$
$P(\mathcal{Z} > z)^{c}$	.3265 [2]	.0510 [2]	.0197 [2]	.5743[2]
B. VAR(3):				
s	.1276 (.0401)	.1372 (.0129)	$.0742 \\ (.0245)$	$.0456 \\ (.0465)$
$\gamma$	.0022 (.0011)	.0046 $(.0008)$	.0212 (.0048)	.0115 $(.0053)$
$\sigma_{\epsilon_x}$	.0003 $(.0001)$	.0012 (.0002)	.0024 $(.0003)$	.0038 $(.0005)$
$P(\mathcal{Z} > z)$	.7993 [4]	.3326 [4]	.2602 [4]	.3186[4]
C. VAR(3) [expo	onential trend in inf	lation]:		
S	.1441 (.0315)	.0928 $(.0211)$	.0718 $(.0228)$	.0113 $(.0283)$
$\gamma$	.0018 $(.0007)$	.0049 $(.0009)$	$.0209 \\ (.0048)$	.0160 (.0056)
$\sigma_{\epsilon_x}$	.0001 $(.0001)$	.0016 $(.0003)$	.0024 $(.0003)$	.0039 $(.0003)$
$P(\mathcal{Z} > z)$	.8316 [4]	.5194 [4]	.1028 [4]	.1429[4]

## Estimates of the Staggered Contracts Models for the Euro Area

Notes: <sup>a</sup> Estimated standard errors in parantheses. <sup>b</sup> Estimate at the boundary of the parameter space. <sup>c</sup> Probability value of the test of overidentifying restrictions. Number of overidentifying restrictions in brackets.

	Rela	Nominal Wage			
VAR(3)	RW	RW-C	RW-S	Contracts (NW)	
<u>A. France:</u> $^{a}$					
s	.1085 $(.0500)$	$( - )^{b}$	.0564 $(.0230)$	.1189 (.0370)	
$\gamma$	.0036 $(.0020)$	.0108 (.0000)	$.0296 \\ (.0066)$	.0041 $(.0041)$	
$\sigma_{\epsilon_x}$	.0004 $(.0001)$	.0052 (.0000)	.0046 (.0005)	.0048 (.0010)	
$P(\mathcal{Z} > z)^{c}$	.1156[4]	.0073 [4]	.0002 [4]	.5435[4]	
B. Germany:					
S	.0487 $(.0209)$	.0376 (.0195)	0()	.0501 (.0296)	
$\gamma$	.0061 $(.0017)$	.0084 $(.0013)$	.0273 $(.0064)$	$.0195 \\ (.0057)$	
$\sigma_{\epsilon_x}$	.0008 $(.0001)$	.0054 $(.0007)$	.0063 $(.0003)$	.0074 $(.0007)$	
$P(\mathcal{Z} > z)$	$< 10^{-5} [4]$	.0001 [4]	$< 10^{-7} [4]$	.0026 [4]	
C. Italy:					
8	$( \begin{array}{c} 1/6 \\ ( \begin{array}{c} - \end{array} )$	.1244 $(.0111)$	.0970 $(.0162)$	n.c. $^d$	
$\gamma$	.0006 $(.0003)$	.0046 (.0010)	.0141 $(.0043)$	n.c.	
$\sigma_{\epsilon_x}$	.0002 $(.0000)$	.0023 (.0003)	.0038 $(.0005)$	n.c.	
$P(\mathcal{Z} > z)$	.1575[4]	.1574[4]	.0709[4]		

## Estimates of the Staggered Contracts Models for France, Germany and Italy

Notes: <sup>*a*</sup> Estimated standard errors in parantheses. <sup>*b*</sup> Estimate at the boundary of the parameter space. <sup>*c*</sup> Probability value of the test of overidentifying restrictions. Number of overidentifying restrictions in brackets. <sup>*d*</sup> No convergence.

		T = 100			T = 500			
	$BIAS^{a}$	STD	RMSE	BIAS	STD	RMSE		
A. Fully anticipated trend decline in inflation: $^{b}$								
s	13.3	53.2	54.8	13.2	24.9	28.2		
$\gamma$	42.0	72.2	83.5	-0.2	20.4	20.4		
$\sigma_{\epsilon_x}$	-13.7	22.7	26.5	-10.4	12.0	15.9		
B. Unanticipated trend decline in inflation: $c$								
s	12.1	52.4	53.8	1.5	27.3	27.4		
$\gamma$	41.6	66.3	78.2	1.1	20.9	20.9		
$\sigma_{\epsilon_x}$	-12.4	23.0	26.1	-5.4	12.4	13.6		

#### **Small-Sample Properties of the Indirect Estimation Procedure**

Note: <sup>a</sup> All statistics are reported as fractions of the structural parameters (in percentage points). <sup>b</sup> The structural parameters used for generating the samples are s = 0.0742,  $\gamma = 0.0212$  and  $\sigma_{\epsilon_x} = 0.0024$ . <sup>c</sup> In this scenario price and wage setters expect trend inflation, which is ultimately determined by the policymakers' long-run inflation target, to follow a random walk. The innovations to trend inflation  $\pi^*$  are drawn from a normal distribution with mean -0.001 and a standard deviation of the same absolute magnitude. As a result annualized inflation, on average, exhibits a downward drift of 10% over the course of 25 years as in the historical euro area sample.

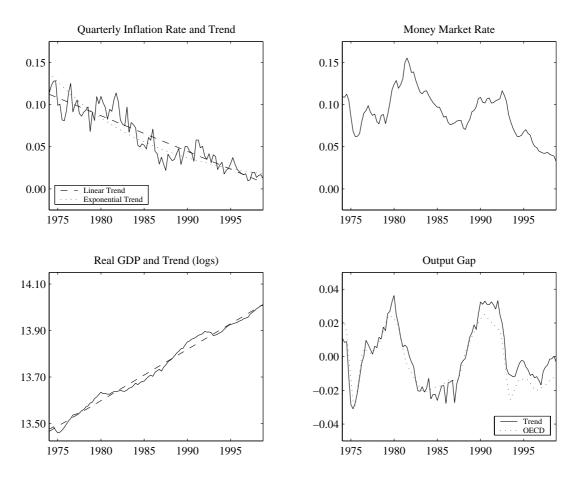
## Estimates of the IS Curve for the Euro Area, France, Germany and Italy

	$\delta_0$	$\delta_1$	$\delta_2$	$\delta_3$	$\sigma_{\epsilon_d} \times 10^4$	$P(\mathcal{J} > j)^{b}$
<u>A. Euro Area:</u> $^{a}$						
A.1 area-wide rate:	0.0012 (0.0007)	1.2347 (0.0916)	-0.2737 (0.1004)	-0.0364 (0.0224)	0.3185	0.1209[5]
A.2 German rate:	0.0027 (0.0012)	1.1807 (0.1006)	-0.2045 (0.1065)	-0.0947 (0.0333)	0.3176	0.2307[5]
B. France:	0.0024 (0.0008)	1.2247 (0.1275)	-0.2708 (0.1284)	-0.0638 (0.0234)	0.3460	0.1977[5]
C. Germany:	0.0012 (0.0027)	$0.7865 \\ (0.0686)$	$0.1395 \\ (0.0825)$	-0.0365 (0.0874)	1.2289	0.2518[5]
D. Italy:	0.0023 (0.0009)	$1.3524 \\ (0.0845)$	-0.3852 (0.0804)	-0.0544 (0.0236)	0.3913	0.4210 [5]

Notes: <sup>a</sup> GMM estimates using a vector of ones and lagged values of the output gap  $(q_{t-1}, q_{t-2})$ , the quarterly inflation rate  $(\pi_{t-1}, \pi_{t-2}, \pi_{t-3})$ , and the short-term nominal interest rate  $(i_{t-1}^s, i_{t-2}^s, i_{t-3}^s)$  as instruments. The weighting matrix is estimated by means of the Newey-West (1987) estimator with the lag truncation parameter set equal to 7. Estimated standard errors in parametheses. <sup>b</sup> Probability value of the  $\mathcal{J}$ -test of overidentifying restrictions. Number of overidentifying restrictions in brackets.

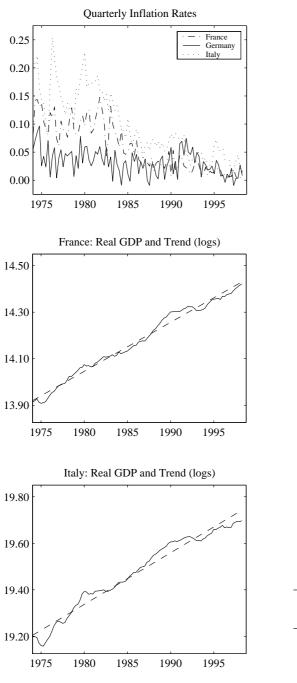
# Figure I

#### The Euro Area Data

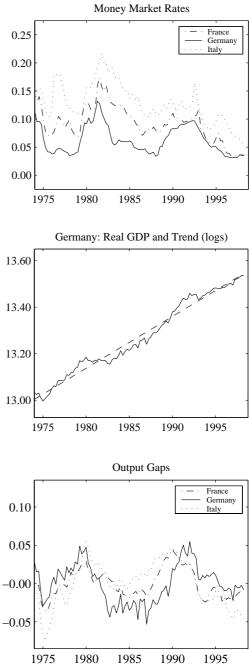


Source: ECB area-wide model database (see Fagan et al. (1999)). Aggregation over data for the member countries of the euro area using fixed GDP weights at PPP rates. The OECD output gap is obtained by interpolating the annual figures reported in OECD (1999).

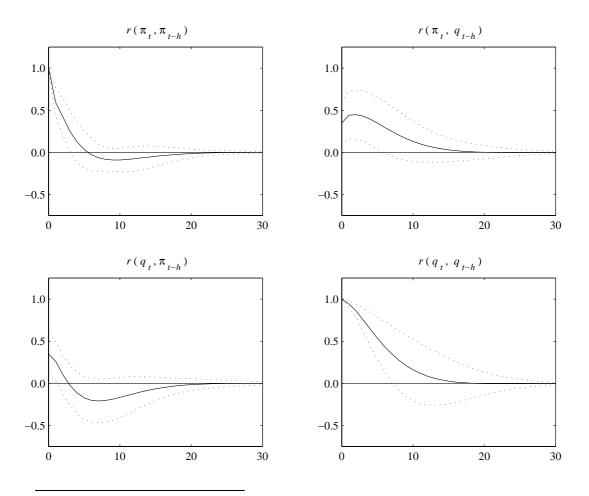
## The Data for France, Germany and Italy



Source: ECB multi-country model database.



#### Estimated Autocorrelations of the Unconstrained VAR(3) Model for the Euro Area

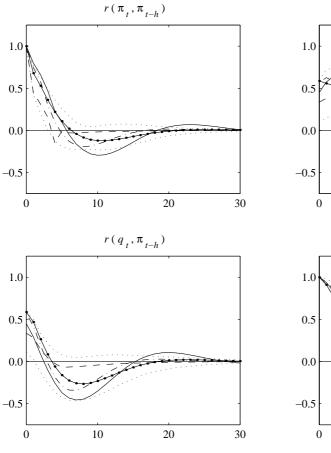


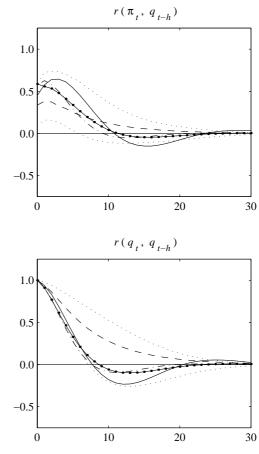
Notes:

Solid line: Estimated autocorrelations.

Dotted lines: Estimated autocorrelations plus/minus twice their estimated asymptotic standard errors.

## Estimated Autocorrelations of the Constrained VAR(3) Models for the Euro Area





Notes:

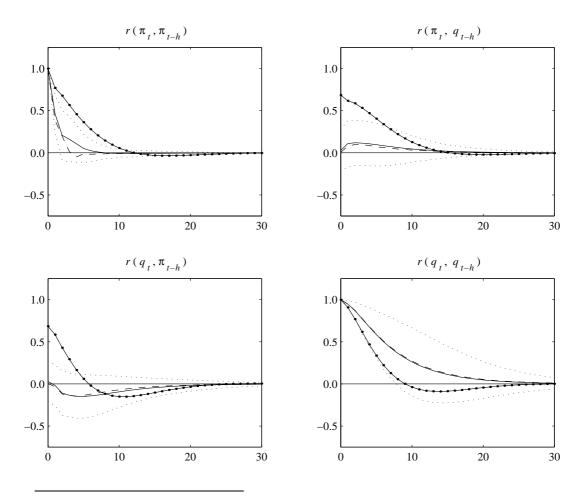
Solid line with bold dots: RW model.

Dash-dotted line: RW-C model.

Solid line: RW-S model.

Dashed line: NW model.

## Estimated Autocorrelations of the VAR(3) Models for France



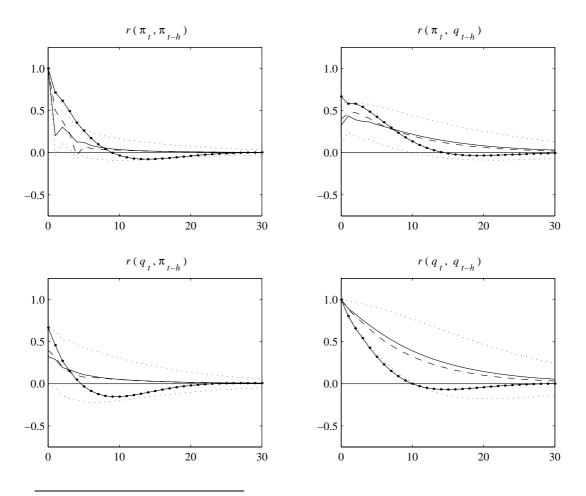
Notes:

Solid line with bold dots: RW model.

Dashed line: NW model.

Solid line: Estimated autocorrelations of the unconstrained VAR(3) model.

#### Estimated Autocorrelations of the VAR(3) Models for Germany



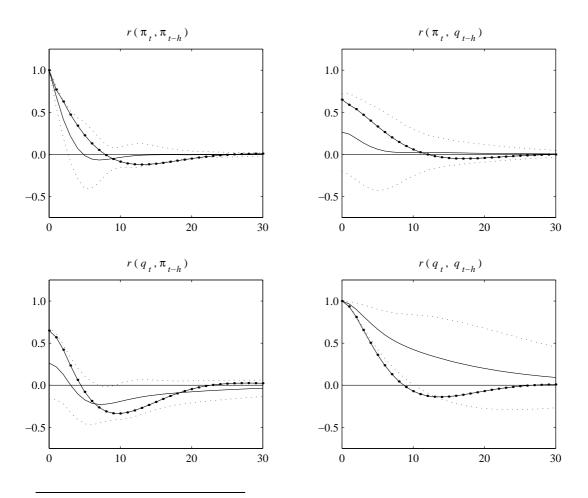
Notes:

Solid line with bold dots: RW model.

Dashed line: NW model.

Solid line: Estimated autocorrelations of the unconstrained VAR(3) model.

## Estimated Autocorrelations of the VAR(3) Models for Italy

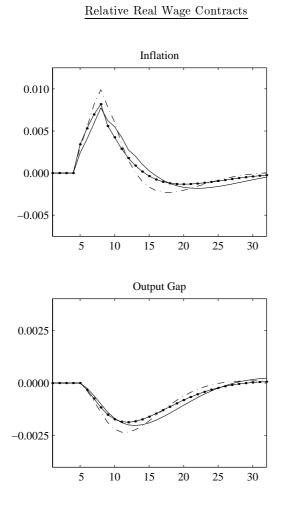


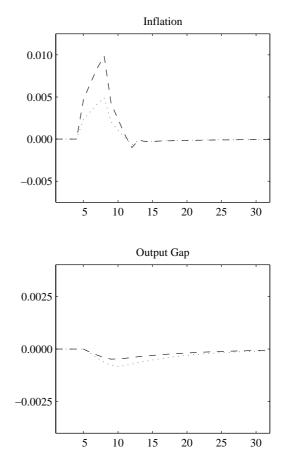
#### Notes:

Solid line with bold dots: RW model.

Solid line: Estimated autocorrelations of the unconstrained VAR(3) model.

## Contract Wage Shock [+1 Standard Deviation]





#### Notes:

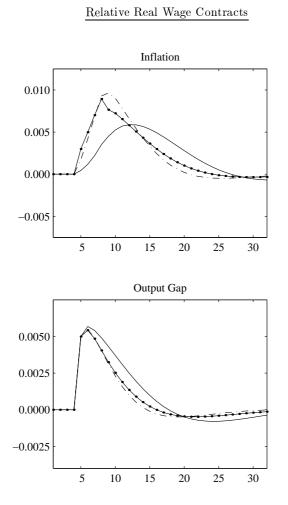
Solid line with bold dots: Euro area RW model. Solid line: Euro area RW-S model. Dashed-dotted line: Euro area RW-C model.

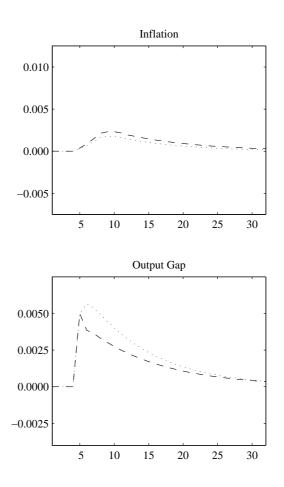
Dashed line: German NW model.

Dotted line: Euro area NW model.

Nominal Wage Contracts

## Demand Shock [+0.5 Percentage Points]





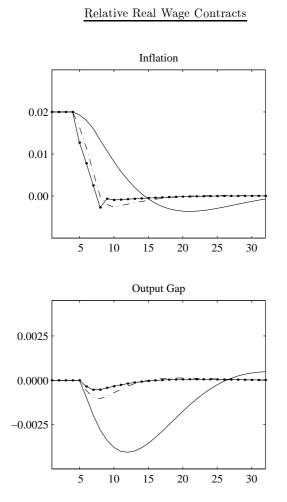
#### Notes:

Solid line with bold dots: Euro area RW model. Solid line: Euro area RW-S model. Dashed-dotted line: Euro area RW-C model. Dashed line: German NW model.

Dotted line: Euro area NW model.

## Nominal Wage Contracts

## **Disinflation** [-2 Percentage Points]



# Inflation 0.02 0.01 0.00 5 10 15 20 25 30 Output Gap 0.0025 0.0000 -0.0025

5

10

15

20

25

30

#### Notes:

- Solid line with bold dots: Euro area RW model. Solid line: Euro area RW-S model. Dashed-dotted line: Euro area RW-C model.
- Dashed line: German NW model.

Nominal Wage Contracts

Dotted line: Euro area NW model.

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