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**Working Paper**

## Fiscal policy effectiveness and neutrality results in a non-Ricardian world

ECB Working Paper, No. 3

**Provided in Cooperation with:**

European Central Bank (ECB)

*Suggested Citation:* Detken, Carsten (1999) : Fiscal policy effectiveness and neutrality results in a non-Ricardian world, ECB Working Paper, No. 3, European Central Bank (ECB), Frankfurt a. M.

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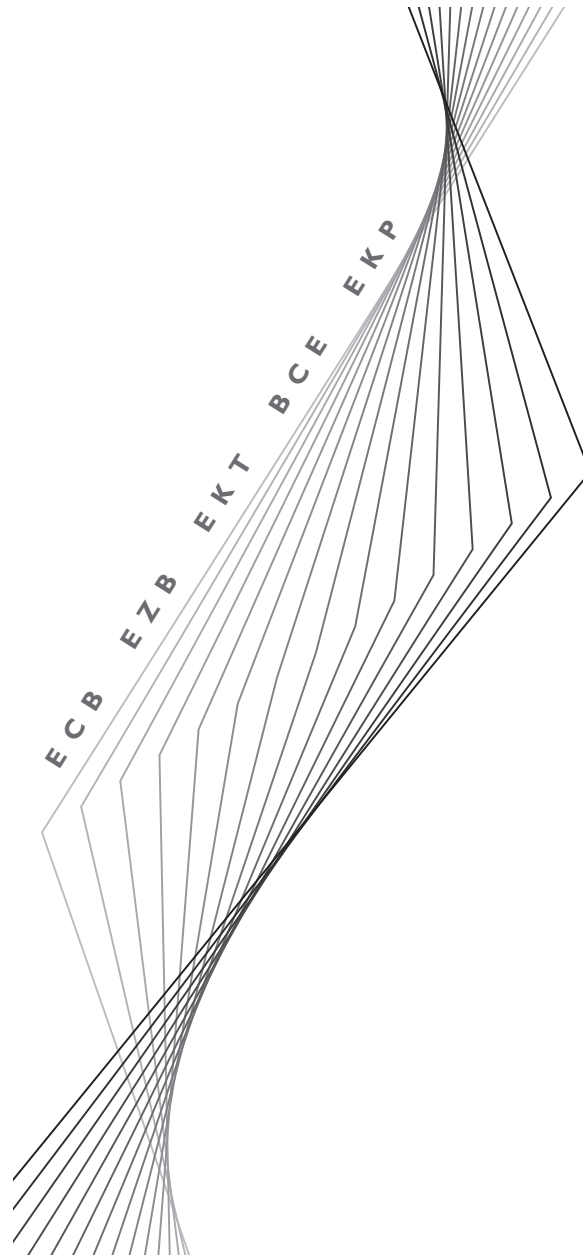


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**FISCAL POLICY EFFECTIVENESS  
AND  
NEUTRALITY RESULTS IN A NON-  
RICARDIAN WORLD  
BY  
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**May 1999**

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\* I would like to thank M. Canzoneri, M. Taylor, J. van Hagen and participants of the 1998 ZEI Summer School for comments on a first draft. I benefited from very useful suggestions from I. Angeloni, O. De Bandt, F. Browne, V. Gaspar, P. Hartmann, J. Marin, F. Mongelli and K. Tuori. I am also grateful for comments received by R. Marimon and other participants of the CÉPR/CREi/EUI conference on the Political Economy of Fiscal and Monetary Stability in EMU, December 1998, as well as an anonymous referee of the Working Paper Series while retaining responsibility for all remaining errors. Views expressed represent exclusively the opinion of the author and do not necessarily correspond to those of the European Central Bank.

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ISSN 1561-0810

## **Abstract**

The paper introduces monetary and fiscal regimes into a Blanchard-Weil overlapping generations model. Contrary to intuition, it is shown that fiscal policy becomes more effective, the less the central bank monetises government debt. Furthermore, there is a degree of debt monetisation at which Ricardian equivalence seems to hold in this "non-Ricardian" model, as fiscal policy is neutral with respect to agent's net wealth. At the origin of these results are the opposite intergenerational wealth effects of money and debt financing.

Since, on average, central bank independence increased through EMU, the analysis suggests that fiscal policy might have become a more powerful instrument for euro-area countries. It is further argued that given the Stability and Growth Pact, governments will find it wise to run budget positions 'close to balance or in surplus' in order to maintain the increased fiscal policy effectiveness.

JEL classification: E63

Key words: monetary and fiscal policy regimes, fiscal policy effectiveness, Ricardian equivalence, central bank independence, Stability and Growth Pact.

## I Introduction

The fact that countries joining the European Monetary Union loose monetary policy as a discretionary policy instrument, is a standard argument to advocate a larger stabilisation role for fiscal policy. The Stability and Growth Pact (SGP), enforcing the 3% deficit ratio ceilings agreed upon in the Maastricht Treaty, constrains national fiscal policies. This is the reason why the SGP has recently been called a nuisance at best and a threat to European growth at worst (Eichengreen and Wyplosz, 1998)<sup>1</sup>. The heart of this is that the SGP is claimed to severely curtail the automatic stabilising function of the government budget deficit and the economy might therefore be trapped in a low-level-equilibrium<sup>2</sup>. The theoretical results presented below have no implications for the automatic stabilising function but for one thus far neglected aspect of fiscal policy in EMU, which is the possibility of an increasing short-run effectiveness of deficit financing, which would nevertheless dampen the above cited criticism of the SGP. The importance of the interaction of monetary and fiscal policy regimes for the effectiveness of fiscal policy<sup>3</sup> has received little theoretical attention so far (an exception is Liviatan, 1982). This is even more surprising as, in the light of the ongoing major monetary and fiscal regime changes, the question of changes in the degree to which the economy deviates from Ricardian equivalence has major policy implications. If, for example, Ricardian equivalence holds, the SGP will clearly be superfluous in any possible sense.

We combine an overlapping generations model à la Blanchard-Weil with money in the utility function with fiscal and monetary regimes defined by Liviatan (1988) and used by him in a representative agent framework. The contribution of this paper is to show how the relationship of fiscal and monetary policy regimes interact with the effectiveness of fiscal policy. The results shed some light on the debate on Ricardian equivalence in a monetary economy. It is shown that there exists a degree of monetisation where the non-Ricardian overlapping generations world behaves Ricardian. Ricardian behaviour is defined when for given government expenditures, an initial tax cut has no effects on total wealth and thus aggregate demand. This definition encompasses the possibility of monetary financing. Reducing the degree of monetisation from this neutral level, leads to increasing deviations from the Ricardian result. This is counterintuitive compared to the IS/LM model, where fiscal policy is more effective the more monetary policy accommodates it. The result is also in contrast to opinions found in the literature that either money does not matter at all for Ricardian equivalence (Barro 1989) or that the higher the share of public debt monetised by the central bank, the stronger the breakdown of the Ricardian equivalence theorem (Liviatan 1982; Leiderman and Blejer 1988; Sargent 1982). Also, Woodford (1995)<sup>4</sup> promoting the new fiscal theory of price determination, contrasts with the result of this paper as he considers a non-Ricardian regime, in which prices and not the primary deficit adjust to satisfy the intertemporal budget constraint<sup>5</sup>. The results are then applied to the ongoing debate on the Stability and Growth Pact.

Section 2 presents the overlapping generations model of the Blanchard-Weil mould, section 3 analyses fiscal policy effectiveness. Section 4 gives the intuition for the results while section 5

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<sup>1</sup> See also Buiter et al. (1993) for early scepticism towards the SGP and Buiter and Kletzer (1991) for a discussion of possible externalities of fiscal policy.

<sup>2</sup> On the other hand, Buti et al. (1997a) and Thygesen (1998) are more optimistic about the automatic stabilising functions within the SGP as long as governments manage to run a balanced or surplus budget in the medium term.

<sup>3</sup> This is in contrast with the huge amount of studies analysing the effects of fiscal and monetary regimes on inflation. See section 5 for references.

<sup>4</sup> See also Canzoneri et al. (1997).

<sup>5</sup> In the new fiscal theory of price determination, prices adjust without being monetised in a traditional sense. But the point is that in the model presented below, a regime where the primary deficit is constant and which has been labelled non-Ricardian by Woodford, is compatible with full Ricardian effects.

focuses on the debate on Ricardian equivalence. Section 6 dwells on implications for European Monetary Union before section 7 concludes.

## 2 The model

### 2.1 Households

In the following the Blanchard (1985) model is used with money in the utility function like in Weil (1987). As in Weil (1989), agents live infinitely to highlight the crucial aspect of the Blanchard overlapping generations model, i.e. the disconnectedness of new generations.

Agents face the following optimisation problem:

$$(1) \quad \max_{(c_s[z], m_s[z])} \left\{ \int_{s=t}^{\infty} \ln(c_s[z]^\gamma m_s[z]^{1-\gamma}) e^{-\theta(s-t)} ds \right\}$$

$$\text{s.t.} \quad \dot{a}_t[z] = r_t a_t[z] - i_t m_t[z] + y_t - \tau_t - c_t[z]$$

The index indicates time and the square brackets give the birth date of an individual to identify the generation. Real consumption is denoted  $c$ ,  $m$  are real cash balances,  $b$  inflation indexed government bonds whereas  $a$  is real total financial wealth, i.e.  $m + b$ .  $\gamma$  is the weight of consumption in the utility function and  $\theta$  the rate of time preference.  $r$  stands for the real and  $i$  for the nominal interest rate. Real non-interest income is denoted  $y$  and real lump sum taxes by  $\tau$ . The log-utility function has an intertemporal elasticity of substitution of 1. This will not significantly restrict the generality of the results as the focus will be on the steady state and not on the transition path (Fischer 1979, Cohen 1985). New generations are born with zero non-human wealth, which is reflected by the fact that financial wealth is accumulating at rate  $r$  and not  $r-n$  in the differential equation for the state variable in equation (1). Individuals optimise intertemporally with  $c$  and  $m$  as control and  $a$  as state variables. The transversality condition, stating that the value of any individual's financial wealth in the infinite future discounted at the real interest rate has to converge to zero is given by:

$$(2) \quad \lim_{s \rightarrow \infty} (a_s[z] e^{-\int_{u=t}^s r_u du}) \rightarrow 0$$

From the other three first order conditions the consumption function can be derived. Agents consume with propensity  $\gamma \theta$  out of their total wealth, consisting of financial and human wealth,  $h$ . Human wealth is the present value of all future labour income and is the same for all generations, independent from their time of birth.

$$(3) \quad c_t[z] = \gamma\theta[a_t[z] + h_t] \quad \text{with} \quad h_t \equiv \int_{s=t}^{\infty} \left( y_s e^{-\int_{u=t}^s r_u du} \right) ds$$

Equation (4) shows the procedure to derive aggregate per capita values. This procedure is necessary because different generations will have accumulated different amounts of financial wealth and thus will have different levels of consumption. Population is growing at rate  $n$ . The total population size in period  $t$  is  $e^{nt}$  assuming that the population size at the beginning of time was 1. The size of a generation born in period  $t$  is  $n e^{nt}$ . Each variable is first summed up over all generations as  $z$  is running from minus infinity until today and the sum is then divided by the current population size. Capital letters denote these transformed variables, i.e. real aggregate per capita variables.

$$(4) \quad X_t = \frac{\int_{z=-\infty}^t x_t[z] n e^{nz} dz}{e^{nt}}$$

Equation (5) describes the dynamics of aggregate per capita consumption on the optimal path. Note that the second term on the RHS of equation (5) makes up the difference to the dynamics of individual consumption.

$$(5) \quad \dot{C}_t = (r_t - \theta) C_t - \gamma\theta n A_t$$

Remember that  $\gamma\theta$  is the individual propensity to consume out of total wealth, and  $n$  is the growth rate of disconnected agents. The additional term in equation (5), which reduces the growth of aggregate per capita consumption is thus the additional consumption the new born individuals would have chosen, if they had owned the average per capita level of financial wealth at birth. At birth the new generation can only consume out of their human wealth. Thus, aggregate per capita consumption grows slower than individual consumption of those agents alive, because the former is depressed by the lower consumption of the newcomers. If there is no population growth, so that  $n$  equals zero, the Blanchard-Weil model is reduced to the Sidrauski model without population growth.

Equation (6) is the steady state version of (5) and defines the real interest rate, which will always exceed the rate of time preference.

$$(6) \quad r_t = \theta + \frac{\gamma\theta n A_t}{C_t}$$

## 2.2 Monetary and Fiscal Policy

Equation (6) and the following four equations describe the current set up:

$$(7) \quad \dot{B} = (r - n) B + G - T - \mu M$$



$$(8) \quad \dot{M} = (r - n + \mu) M - \frac{1 - \gamma}{\gamma} C$$

$$(9) \quad \bar{D} \equiv G - T + \delta rB \quad 0 \leq \delta \leq 1$$

$$(10) \quad \alpha \equiv \frac{\dot{M} + M(n + \pi)}{\dot{B} + nB}$$

Equation (7) is the government budget identity, revealing the dynamics of government bonds.  $m$  stands for the nominal money growth rate and thus  $mM$  signifies real, aggregate per capita seigniorage revenues of the government. The debt ratio increases with the real interest payments on the debt,  $rB$ , and government expenditures,  $G$ , and decreases with the new debt allowed for by population growth,  $nB$ , lump sum tax revenues,  $T$ , and seigniorage revenues,  $mM$ . Equation (8) determines the development of real cash balances and is obtained directly from the first order conditions. Note that the term in brackets on the right hand side is the nominal interest rate. It can thus be interpreted as the aggregate dynamic money demand function. Equations (9) and (10) depict the Liviatan (1988) fiscal and monetary regime specifications, respectively, formerly used in a representative agent framework. In equation (9),  $\bar{D}$  is the constant deficit. Which deficit is held constant by the government, the primary or the overall deficit, is determined by the value of  $d$ . If  $d$  equals 1, the government's target will be the overall deficit, for  $d$  equal to zero it will be the primary deficit. Equation (9) can also be seen as the tax reaction function where  $d$  varying between 0 and 1 describes the degree to which the real interest payments of the government influence the general tax level. Equation (10) depicts the degree of monetary financing according to changes in the government debt level. The nominator and denominator show the real value of the nominal change in cash balances and the real value of nominal changes in government bonds, respectively. Other regime specifications are certainly possible. There are two reasons why equations (9) and (10) were chosen. First of all, the fiscal regime has been used by Liviatan (1982) and both regimes by Liviatan (1988). The former is the only study focusing directly on the subject of Ricardian equivalence. Weil (1987) used equations (6), (7) and (8) but with a constant primary deficit. Thus with the regimes given above, comparability to earlier results is immediately possible. Liviatan used representative agent models so that by replicating his regimes, the difference in the results has to come from the use of the Blanchard-Weil OLG framework. Second, the parameters in the regimes specified here are easily related to changes occurring with EMU. The decrease in the degree of monetary financing towards practically zero as well as the definition of the 3% deficit ceiling for the overall deficit, allows for a discussion of EMU related aspects in terms of  $a$ ,  $d$  and  $\bar{D}$ .

It remains to be mentioned that the system of five equations (6)-(10) has five endogenous variables,  $r$ ,  $M$ ,  $B$ ,  $T$  and  $p$ , where  $p$  is the inflation rate<sup>6</sup>.

## 2.3 Equilibrium

We deal with a closed economy where demand equals supply permanently.

$$(11) \quad Y = G + C$$

Thus, with constant real per capita non-interest income,  $Y$ , and constant real per capita government expenditures,  $G$ , aggregate real per capita consumption has to be constant as well. As consumption increases continuously for each individual during their lifetime, it is the appearance of new agents with initially low consumption levels, which allows aggregate consumption to remain constant in the steady state. Given the rate of time preference, the real interest rate determines the size of the individual increase in consumption so that (11) holds.

## 2.4 Solution

We will derive the results without the closed form solution, and construct a three differential equation system in three variables  $r$ ,  $M$  and  $B$ , analogously to Weil (1987).

The differential equation for  $r$ , equation (12), is constructed by differentiating equation (6) and inserting the sum of equations (7) and (8) for the differential of the term  $(M+B)$ . Then, equation (6) is substituted for  $(M+B)$ . The differential equation for  $B$  equation (13) is obtained by substituting equations (9) and (10) in (7). By using equations (10), (9) and the differential equation for  $B$  equation (13), one gets the differential equation for  $M$ , equation (14)<sup>7</sup>.

$$(12) \quad \dot{r} = r^2 - (n + \theta + \frac{\gamma\theta n\delta}{Y-G}B)r + \theta n + \frac{\gamma\theta n}{Y-G}(\bar{D} - \frac{1-\gamma}{\gamma}(Y-G))$$

$$(13) \quad \dot{B} = \left[ \frac{1-\delta}{1+\alpha}r - n \right] B + \frac{\bar{D}}{1+\alpha}$$

$$(14) \quad \dot{M} = \left[ r \left( 1 - \frac{(1-\delta)\alpha}{1+\alpha} \right) - n \right] M + \frac{(1-\delta)\alpha(Y-G)}{(1+\alpha)\gamma\theta n} r (r - \theta) + \frac{\alpha}{1+\alpha} \bar{D} - \frac{1-\gamma}{\gamma} (Y-G)$$

The three steady state equations follow from the system (12)-(14).

$$(15) \quad r = \theta + \frac{\gamma\theta n}{Y-G} (M+B)$$

$$(16) \quad M = \frac{\frac{1-\gamma}{\gamma}(Y-G) - \alpha n B}{r - n}$$

$$(17) \quad B = \frac{\bar{D}}{(1+\alpha)n - (1-\delta)r}$$

By solving for  $r$ , a third order polynomial is derived.

<sup>6</sup> Note that throughout the presentation we will use  $i = r + \mathbf{p}$  and  $\mathbf{p} = \mathbf{m} \cdot n$ .

<sup>7</sup> Note that  $\mathbf{p}$  vanishes from the equations using  $\mathbf{p} = \mathbf{m} \cdot n$  and the monetary regime equation (10).

(18)

$$(1 - \delta) r^3 - [(1 - \delta)(n + \theta) + (1 + \alpha)n] r^2 + \left[ (1 + \alpha)n(n + \theta) + (1 - \delta)\theta n + \frac{\gamma\theta n}{Y - G} \left( \bar{D} - (1 - \delta) \frac{1 - \gamma}{\gamma} (Y - G) \right) \right] r - (1 + \alpha)n \left[ \theta n + \frac{\gamma\theta n}{Y - G} \left( \bar{D} - \frac{1 - \gamma}{\gamma} (Y - G) \right) \right] = 0$$

We thus have to deal with three equilibria. Demanding that  $r$ ,  $M$ , and  $B$  are all positive will lead to a restriction on the size of the constant deficit  $\bar{D}$ , which, given regime specifications, cannot be too large, compared to the size of the economy<sup>8</sup>. Assuming a further restriction on  $\bar{D}$  ensuring that we necessarily get a dynamic efficient outcome, i.e. a real interest rate exceeding the population growth rate, it is only the second of the three equilibria, which exhibits a saddle path property. In this case, the first equilibrium does not exist for positive  $r$ ,  $M$  and  $B$  and the third is an unstable node. Appendix 1 shows these results with the help of separate phase diagrams and the eigenvalues derived from the linearized system. Thus, the following, will focus on the second equilibrium:

### 3 The effectiveness of fiscal policy – a Ricardian experiment

The way to investigate fiscal policy effectiveness in the Blanchard-Weil cum regimes world is to derive the real interest rate effects of a change in the constant deficit  $\bar{D}$  for given government expenditures. To see this, one could think of the fiscal regime equation (9) as being a tax reaction function.

$$(19) \quad T = G - \bar{D} + \delta r B$$

An increase in the constant deficit  $\bar{D}$  will initially signify a tax reduction. Then taxes, the government debt and seignorage will adjust endogenously. With full Ricardian equivalence, this policy experiment would not have any effect on the real interest rate. In this model without capital, an increase in  $\bar{D}$  which augments the real interest rate, should be considered expansionary as the real interest rate rises to counterbalance positive demand effects due to an increase in net financial wealth<sup>9</sup> (see equation (15)). Thus, we are interested in the steady state partial derivative of  $\frac{\partial r}{\partial \bar{D}}$ , for given  $G$ , where a positive sign reveals a positive wealth effect.

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<sup>8</sup> See appendix 1.

<sup>9</sup> In a model with capital, the increase in the steady state interest rate would eventually lead to a lower real aggregate per capita capital stock and decrease real aggregate per capita consumption. But the reason for this is an initially positive wealth effect, which, in the model with capital, would increase consumption immediately after the positive shock in  $\bar{D}$ . The exclusion of capital thus does not change the results in the short run. But it should be kept in mind that when capital is introduced, an increase in the deficit when  $\frac{\partial r}{\partial \bar{D}} > 0$  depresses steady state consumption.

*Proposition 1*

$$\frac{\partial r}{\partial \bar{D}} > 0 \text{ for } \mathbf{a} < \mathbf{a}^*, \quad \frac{\partial r}{\partial \bar{D}} = 0 \text{ for } \mathbf{a} = \mathbf{a}^*, \quad \frac{\partial r}{\partial \bar{D}} < 0 \text{ for } \mathbf{a} > \mathbf{a}^*,$$

with  $\mathbf{a}^* = r/n - 1$

Proposition 1 states that there exists a degree of monetary financing,  $\mathbf{a}^*$ , for which a change in the constant deficit would not have any wealth effects. Were the degree of monetary financing smaller than  $\mathbf{a}^*$ , an increase in  $\bar{D}$  would increase aggregate demand if the rise in the real rate did not compensate this effect. Were the degree of monetary financing larger than  $\mathbf{a}^*$ , an increase in  $\bar{D}$  would lower aggregate demand.

*Proof of proposition 1*

Applying the implicit function theorem to the polynomial for  $r$ , equation (18) gives

$$(20) \quad \frac{\partial r}{\partial \bar{D}} = \frac{\gamma \theta n [(1 + \alpha)n - r]}{(Y - G) \zeta} \quad \text{with}$$

$$\zeta =$$

$$3(1 - \delta)r^2 - 2[(1 + \alpha)n + (1 - \delta)(n + \theta)]r + (1 + \alpha)n(n + \theta) + \gamma \theta n \left( \frac{\bar{D}}{Y - G} + 1 - \delta \right)$$

Appendix 2 shows that  $\mathbf{z}$ , evaluated at the second equilibrium, is negative. Thus, the sign of the above derivative depends inversely on the sign of  $(1 + \alpha)n - r$  and is zero when the latter term is zero. Thus,  $\mathbf{a}^* = r/n - 1$ . As  $\frac{\partial r}{\partial \bar{D}} < 0$ , see appendix 2, the sign of the partial derivative (20) can only be as stated in the proposition. With an increase in  $\mathbf{a}$ ,  $r$  will be smaller and thus the nominator is positive when  $\mathbf{a} > \mathbf{a}^*$  and negative when  $\mathbf{a} < \mathbf{a}^*$ .

*Proposition 2*

$$\frac{\partial \left( \frac{\partial r}{\partial \bar{D}} \right)}{\partial \alpha} < 0 \text{ for } \alpha \neq \alpha^*$$

Proposition 2 states that the deviations from the neutral or Ricardian outcome are larger the smaller the degree of monetary financing when  $\mathbf{a} < \mathbf{a}^*$  and larger, the higher the degree of monetary financing when  $\mathbf{a} > \mathbf{a}^*$ .

*Proof of proposition 2*

Applying Weil's (1991) definition of net wealth of individuals alive today,  $\mathbf{F}$ , to our economy with bonds and money would give the following definition<sup>10</sup>:

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<sup>10</sup> Net wealth of individuals currently alive is net of the present value of non-interest income, which is uninteresting in the present setting as  $Y$  is constant. The wealth defined as above is proportional to consumption.

$$(21) \quad \Phi = M + B - \frac{T}{\text{discount factor}} - \frac{\text{opportunity costs of cash balances}}{\text{discount factor}}$$

After substituting the fiscal and monetary regime equations (9) and (10), as well as the steady state values for M equation (16) and B equation (17) and the polynomial for r equation (18) and deriving  $iM/q$  as the present value of opportunity costs of cash holdings for individuals currently alive one gets<sup>11</sup>:

$$(22) \quad \Phi = \frac{n[(1-\gamma)(Y-G)[(1+\alpha)n - (1-\delta)r] + \gamma\bar{D}[r - (1+\alpha)n]]}{r(r-n)[(1+\alpha)n - (1-\delta)r]}$$

Taking the partial derivative with respect to  $\bar{D}$  with  $r$  assumed to be constant leads to:

$$(23) \quad \frac{\partial \Phi}{\partial \bar{D}} = \frac{\gamma n [r - (1+\alpha)n]}{r(r-n)[(1+\alpha)n - (1-\delta)r]}$$

Again leaving  $r$  constant, the partial derivative of (23) with respect to  $\alpha$  is

$$(24) \quad \frac{\partial^2 \Phi}{\partial \bar{D} \partial \alpha} = - \frac{\delta \gamma n^2}{(r-n)[(1+\alpha)n - (1-\delta)r]^2} < 0$$

Since we know that  $r$  always exceeds  $n$  for the second equilibrium, the partial derivative, equation (24), is necessarily negative. To derive from this result the reaction of the real interest rate it is enough to show that  $\frac{\partial r}{\partial \bar{D}}$  has the same sign as  $\frac{\partial \Phi}{\partial \bar{D}}$ . To see this one can transform the consumption function (6) together with  $iM = (1-g)C/g$  stemming from the first order conditions into

$$(25) \quad C = \theta \left[ \Phi + \frac{Y}{r} \right]$$

It is thus evident that any change in  $\Phi$  will have to be neutralized by a parallel movement in the real interest rate because real aggregate per capita consumption is constant. Thus, knowing the sign of the partial derivatives for  $\Phi$  is equivalent to knowing the sign for  $r$ .

With the above results and the additional knowledge of the sign of the following partial derivative<sup>12</sup>

$$(26) \quad \frac{\partial \left( \frac{\partial r}{\partial \bar{D}} \right)}{\partial \delta} < 0 \quad \text{for } \mathbf{a} < \mathbf{a}^*$$

<sup>11</sup> See appendix 3.

<sup>12</sup> See appendix 3. Equation (26) states the intuitive result that deviations from Ricardian equivalence are smaller, the larger  $\mathbf{d}$ . Thus when taxes react strongly to increases in debt service, a bond financed tax cut will trigger weaker expansionary wealth effects if we are in a non-Ricardian world.

the figure below can be drawn for  $\mathbf{a} < \mathbf{a}^*$ , showing Iso-Ric's, which are loci of similar deviation from the Ricardian, i.e. neutrality result.

Figure 1

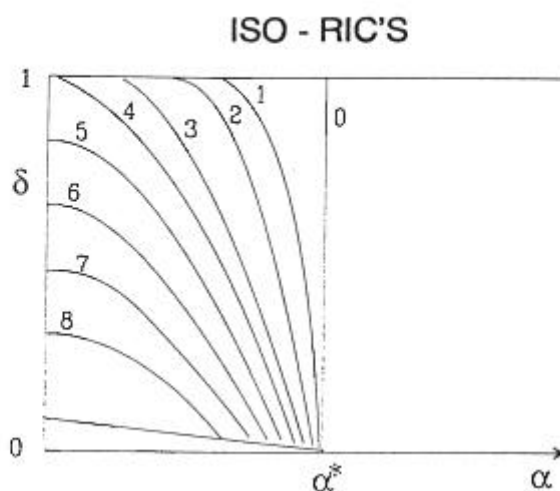


Figure 1 shows lines of equal deviation from the Ricardian result. The higher the number on the Iso-Ric, the stronger the deviation from the Ricardian result. In other words, the lower the tax responsiveness  $\mathbf{d}$  to interest payments on the debt, and the lower the degree of monetary financing  $\mathbf{a}$ , the more significant is the deviation from the Ricardian outcome, i.e. the stronger is the demand effect of fiscal policy as described above.

#### 4 Intuition

At the heart of the results are two different kinds of wealth effects. It is by now common knowledge that in the non-Ricardian world of the Blanchard-Weil model, government indebtedness redistributes wealth from future to current generations. The reason that bonds are net wealth to individuals living today is that new generations are born to share the burden of future taxation. Current generations profit from the tax cut, as future generations will contribute to shoulder the burden of future tax increases. In contrast and less obvious, money creation has just the opposite effect. Inflation redistributes wealth from current to future generations<sup>13</sup>. In the model presented here, the inflation related to the sudden increase in the constant deficit lowers the real value of current generations' non-indexed financial wealth. The real loss only happens to generations currently alive, as generations born in the new steady state will only profit from lower tax payments without having to bear the loss on any asset.<sup>14</sup>

The lower the degree of monetisation, the stronger the "traditional", non-Ricardian, overlapping generations model effect. A fiscal deficit increases aggregate demand, as the compensating negative inflation effect is less pronounced. At  $\mathbf{a}^*$  those two wealth effects cancel completely, so that the non-Ricardian world in which bonds and money constitute net wealth performs as if Ricardian equivalence would hold. This might be another explanation why empirical tests of Ricardian equivalence usually

<sup>13</sup> See Weil (1991).

<sup>14</sup> In this setting it is only the value of money holdings which is reduced as bonds are indexed.

have problems to reject equivalence even if the real world seems quite remote from the assumptions necessary to derive the neutrality result in a strict sense, which are, for example, non-distortionary taxes, the bequest motive, non-credit constraint individuals, see, for example, Bernheim (1987) and Evans (1991).

A closer look at  $\mathbf{a}^*$  reveals that in this monetary regime the central bank monetises all the “net” interest obligation on the debt, where “net” means net of the additional indebtedness allowed for by population growth. To see this we write the steady state equation for  $B$ , equation (17) in the following way:

$$(27) \quad \bar{D} = \alpha nB + nB - rB + \delta rB$$

The first term on the RHS equals monetisation  $mM$  and the first three terms add up to zero at  $\mathbf{a}^*$ , as  $r = (1 + \mathbf{a}^*)n$ , thus the central bank monetises  $(r - n)B$ . For EU countries since the 1980's this would be a very high degree of monetisation, which is why the focus has been laid on the case where  $\mathbf{a} < \mathbf{a}^*$ .

## 5 The debate on Ricardian equivalence

Though the literature on fiscal and monetary regimes is quite extensive and triggered by Sargent's and Wallace's 1981 unpleasant monetarist arithmetics article<sup>15</sup>, the focus has mainly been on the inflationary impact of monetary policy with different fiscal constraints. The only theoretical paper, according to the author's knowledge, which explicitly tries to investigate Ricardian equivalence in the context of a monetary regime is Liviatan (1982). Liviatan uses the Sidrauski model, i.e. a representative agent with money in the utility function, and employs the same monetary regime equation as given in equation (11). He then argues that any policy measure which leads to a change in the stock of government bonds will tend to be non-neutral as a change in  $B$  will lead to a change in the money growth rate and thus inflation. Inflation is an argument of the utility function and, therefore, he argues, non-neutrality results as the utility of agent's changes in the new steady state. The more the inflation tax is used, the more severe is the breakdown of Ricardian equivalence. Whether one should consider this particular change in utility as a deviation from Ricardian equivalence is not obvious, and at first sight looks like a question of definition<sup>16</sup>. In the Sidrauski model a change in the money growth rate is considered to be superneutral as no real variables change. The change in inflation and thus agent's utility is not deemed a violation of neutrality. Superneutrality and Ricardian equivalence have a twin relationship as pointed out by Weil (1991), for example, they usually fail due to the same intergenerational channels. It would be peculiar in the context of the same model to define a monetary policy experiment to be superneutral and a fiscal policy experiment to be non-neutral when both policy experiments give exactly the same result. Still, discussing whether Liviatan's results really represent deviations from the Ricardian outcome, more than semantics is involved. At the heart of the debate on Ricardian equivalence is the question whether bonds should be considered as net wealth or not<sup>17</sup>. Cohen (1985) shows that neither bonds nor real cash balances would be considered net wealth by a representative agent in the Sidrauski model. This can easily be

<sup>15</sup> Follow-ups have been, for example, McCallum (1984), Drazen (1984) and Liviatan (1984, 1988) in a representative agent setting and Darby (1984), Miller and Sargent (1984) and Weil (1987) in overlapping generations models.

<sup>16</sup> Admittedly, already the definition of Ricardian equivalence in a monetary economy is not obvious. Most definitions of Ricardian equivalence do not involve money and simply require that given government expenditures, a bond financed tax cut does not alter private budget constraints because the present value of future higher tax payments equals exactly the current tax cut. But there is no economic rationale why the definition of taxes should not include the inflation tax. Nevertheless, when speaking about Ricardian equivalence in a monetary economy, the possibly reserved term “equivalence” would best be avoided in favour of “effects”, or “outcome” or “as if” sentences.

<sup>17</sup> Thus, on this subject, Barro's (1974) title cannot be improved upon.

seen using the net wealth definition of equation (23) for  $n$  equal to zero, as then  $F$  will be zero. In this case there are no disconnected generations born and the Blanchard-Weil model collapses into the representative agent Sidrauski model without population growth. Thus, in the representative agent economy, no policy or regime which leads to exchanges of bonds against money or vice versa would have any real effect on consumption investment or the real interest rate - except the utility change due to the inclusion of money in the utility function. Thus, one can argue, in disagreement with Liviatan, that in the representative agent model money does not change a thing concerning Ricardian equivalence. Buiter's (1990, p 219-220) statement fits particularly well at this point: "The analysis of the consequences of 'deficit financing'...requires the abandonment of the representative consumer model if it is not to beg all the important questions....In order to be policy-relevant, the profession will have to invest in macroeconomics without a representative agent." With the heterogeneous agents of the Blanchard-Weil model, Ricardian effects result as shown in section 3. When the government monetises the net interest obligations on the debt, the economy behaves as if Ricardian equivalence would hold. Decreasing the degree of tax responsibility to interest payments and/or lowering the degree of monetisation by the central bank diminishes the "Ricardianess" of the economy. This result does not only contradict Leviatan (1982) but is also in opposition to a simple closed economy IS/LM model as well as opinions on the subject found in the literature and cited in the introduction.

## 6 Implications for EMU

If one judges the described wealth effects as being important in reality, there will be implications for European Monetary Union. First one should note that the average central bank independence in the euro-area has certainly increased. Therefore, less responsiveness to national debt developments, if any at all, would be expected from the European Central Bank. Or put differently,  $\alpha$  decreases for the euro-area and would probably be close to zero<sup>18</sup>. Thus, ceteris paribus, fiscal policy becomes more effective. Given the SGP, the model would create an incentive for governments to run a budget position well below the 3% deficit ceiling. The reason is that a budget deficit close to the 3% ceiling would immediately call for  $d$  to be increased to 1. This is so because as the SGP definition of the deficit is the overall deficit, the government would have to adjust taxes to keep  $\overline{D}$  constant at 3% whatever the interest payments on the debt. Thus the economy would turn more Ricardian as at the limit people could reasonably expect any deficit financed tax cut to be reversed as soon as the debt ratio starts to rise. This is shown in the decreasing deviation from the Ricardian outcome when  $d$  increases in figure 1. A government which prefers to maintain the increased effectiveness of fiscal policy for demand management, should care for a low  $d$  which is only possible when the budget balance is sufficiently remote from the 3% deficit ceiling. Buti et al. (1997b) make a related point in a very different framework. They argue that governments should run a budget position close to balance to let the automatic stabilizers play in recessions while still avoiding excessive deficits. Given the cyclical sensitivity of budgets in EU member states to the outputgap, mostly lying in the range of 0.5-0.6 and given the unweighted average of the lowest negative outputgaps of EU countries over the period 1961-1997 being just above 4% of trend GDP, a deficit buffer zone for normal times of approximately 2%-2.5% of GDP is derived. Recall that this argument refers to the automatic stabilizing function only. Retrospectively applying the SGP over the last 36 years, the same authors reinforce their point as they find that given past behavior a cyclically balanced starting position would have avoided an excessive deficit in 19 out of 24 cases of recessions entailing a fall in real GDP by at least 0.75%<sup>19</sup>. This number is reduced to 8 when the starting position would have been a cyclically

<sup>18</sup> Note that according to the Maastricht Treaty, the ECB's primary objective is to maintain price stability and it is not allowed to conduct open market operations for other purposes than monetary policy. It is thus not allowed to monetise government debt.

<sup>19</sup> Those cases which led to an excessive deficit had prolonged recessions where real GDP fell for at least two consecutive years.



adjusted deficit of 2% of GDP. So while the Buti et al. results would lead to the same policy recommendation, the derivation is very different. Whereas they rely on the information content of past behavior, the above model hinges on theoretical effects resulting from changing monetary and fiscal regimes<sup>20</sup>.

Trying to rationalize the Stability and Growth Pact many authors have interpreted the Pact as a device to safeguard the ECB's credibility to pursue the primary goal of maintaining price stability. Without fiscal constraints pressure might arise to monetize national debt one way or the other<sup>21</sup>. A second line of reasoning could welcome the SGP as a device to contain the "deficit bias of democracies" (see Roubini and Sachs, 1989). Beetsma and Uhlig (1997), for example, show how a common currency can exacerbate the "deficit bias" without the SGP, because individual governments would have an only muted incentive to restrain debt for the benefit of lower future inflation. The logic of the Blanchard-Weil OLG model with regimes, introduces the possibility to discuss the SGP under the heading of the "political economy of intergenerational transfers". One could see the increased central bank independence or rather the fact of foregoing the possibility of monetising public debt as a benefit accruing mainly to current generations, because of the positive wealth effect going along with a decrease in  $a$ . The SGP would then re-establish intergenerational fairness by redistributing wealth back to future generations. This happens as the SGP could be characterized by increasing  $d$  for a given  $\bar{D}$  or when the incentive for governments described above works, by lowering  $\bar{D}$ <sup>22</sup>. Thus in both cases the SGP effectively protects the interests of future taxpayers.

## 7 Conclusions

The contribution of the paper is to combine a Blanchard-Weil overlapping generations model with a fiscal and monetary regime as in Liviatan (1988). The results shed new light on the interaction between monetary and fiscal policies. It can be shown that, first of all, there exists one degree of monetary financing, i.e., when the central bank monetises the net interest obligations on the government debt, where the non-Ricardian world of the overlapping generations model exhibits Ricardian results. This means a tax cut has no wealth effects at all. Second, one can derive the result that the smaller the degree of monetary financing or the more independent the central bank, the stronger the deviations from Ricardian equivalence. This result contrasts with opinions about Ricardian equivalence and monetary financing found in the literature so far. These different opinions stem, first of all, from the intuition given by the closed economy IS/LM model, where expansionary fiscal policy is more effective when accommodated by expansionary monetary policy. Second, seemingly there is some notion that the more seigniorage is used to service the debt instead of lump sum taxes, the world would be less Ricardian (Liviatan, 1982 and Sargent, 1982) or that the means of financing is not important at all (Barro, 1989).

The explanation for the opposite results in this paper rely on the combination of two different kinds of wealth effects. Debt finance redistributes wealth from future to current generations (Blanchard, 1984). Money finance does the opposite and redistributes wealth from current to

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<sup>20</sup> Buti et al. (1997a) mention themselves in their concluding remarks the possibility of a change in the effectiveness of fiscal policy. Concerning the latter, they refer to a Mundell-Fleming framework where two opposing effects, higher substitutability of assets and higher import ratios, are responsible for the ambiguity of the sign of a change in fiscal policy effectiveness in EMU. Thygesen (1998), arguing also with the Mundell-Fleming model, believes as well that fiscal policy will become more effective, due to weaker crowding-out effects in EMU. He does not give any weight to the possibility of increased positive spillovers due to higher import ratios.

<sup>21</sup> See Buti et al. (1997b).

<sup>22</sup> Note that  $\frac{\partial \bar{D}}{\partial d} > 0$  and  $\frac{\partial \bar{D}}{\partial a} < 0$  for  $a < a^*$ .

future generations as only agents currently alive will suffer a loss in the real value of their assets whereas future generations would also benefit from lower future tax payments (Weil, 1991). The less the central bank monetizes government debt, the stronger is the former "traditional" effect, according to which debt financing imposes a burden on future generations.

Implications for EMU are that according to these wealth channels and given the Stability and Growth Pact, fiscal policy will be more effective as long as the government keeps a budget position 'close to balance or in surplus'. Only if the deficit ratio is small enough not to trigger expectations about immediate reversal of debt financed tax cuts due to the proximity of the 3% ceiling, can deficit financing remain effective. This theoretical argument reinforces the recommendation by Buti et al. (1997a), which had been derived from empirical observation of past budgetary behaviour. In the spirit of the analysis of this paper, the Stability and Growth Pact itself could be interpreted as an intergenerational contract safeguarding the interests of future generations.

## Appendix

### A1 Equilibria

The following two possibilities for the separate phase diagrams will be motivated below:

Figure (A1) Case I

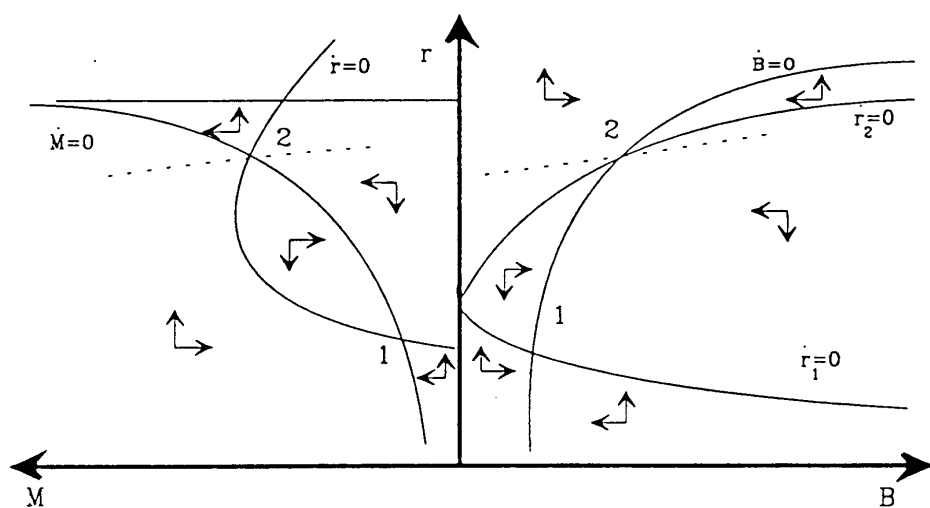
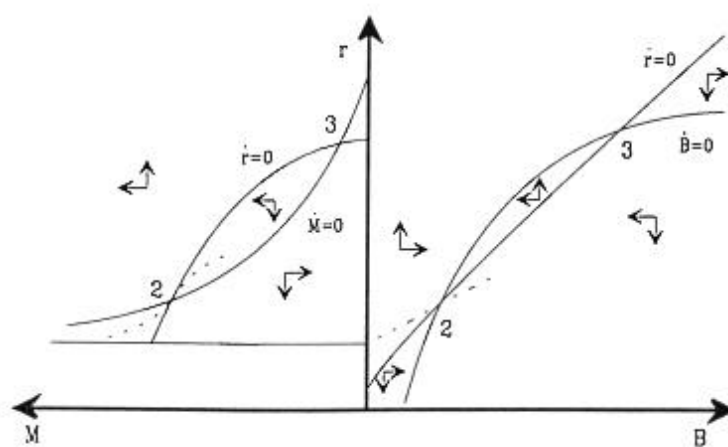


Figure (A2) Case II



The separate phase diagrams have the following demarcation lines in  $[r; B]$  space:

$$(A1) \quad \begin{aligned} \dot{B} = 0: \quad r &= \frac{1+\alpha}{1-\delta}n - \frac{\bar{D}}{1-\delta}B^{-1} \\ \dot{r} = 0: \quad r_{1,2} &= \frac{n+\theta + \frac{\delta\gamma\theta n}{Y-G}B}{2} \pm \sqrt{\frac{(n+\theta + \frac{\delta\gamma\theta n}{Y-G}B)^2}{4} - \gamma\theta n\left(\frac{\bar{D}}{Y-G} + 1\right)} \end{aligned}$$

The  $B$  demarcation line has the following  $B$  value at  $r = 0, Z_r$ :

$$(A2) \quad Z_r: B = \frac{\bar{D}}{(1+\alpha)n}$$

The  $B$  demarcation line exhibits

$$(A3) \quad \lim_{B \rightarrow \infty} r = \frac{(1+\alpha)n}{(1-\delta)}$$

The  $r$  demarcation line has the following two  $r$  values at  $B = 0$ :

$$(A4) \quad Z_B^{1,2}: r_{1,2} = \frac{n+\theta}{2} \pm \sqrt{\frac{(n+\theta)^2}{4} - \gamma\theta n\left(\frac{\bar{D}}{Y-G} + 1\right)}$$

which means

$$(A5) \quad 0 < Z_B^1 < \frac{n+\theta}{2} < Z_B^2 < n+\theta$$

furthermore, assuming throughout the paper  $\bar{D} > 0$  the following slope characteristics of the  $r$  demarcation line can be derived:

$$(A6) \quad \frac{\partial(\dot{r}_1 = 0)}{\partial B} < 0 \quad \wedge \quad \frac{\partial^2(\dot{r}_1 = 0)}{\partial^2 B} > 0$$

$$(A7) \quad \frac{\partial(\dot{r}_2 = 0)}{\partial B} > 0 \quad \wedge \quad \frac{\partial^2(\dot{r}_2 = 0)}{\partial^2 B} < 0$$

The demarcation lines in  $[M; r]$  space are:

$$(A8) \quad \begin{aligned} \dot{M} = 0 : \quad M &= \frac{\frac{1-\gamma}{\gamma}(Y-G) - \frac{\alpha}{1+\alpha}\bar{D} - \frac{\alpha(1-\delta)(Y-G)}{(1+\alpha)\gamma\theta n} r(r-\theta)}{r\left(\frac{1+\delta\alpha}{1+\alpha}\right)^{-n}} \\ \dot{r} = 0 : \quad M &= \frac{(Y-G)}{\delta\gamma\theta n} \left[ (1+\delta)\theta + n - (1-\delta)r - \frac{\gamma\theta n\left(\frac{\bar{D}}{Y-G} + 1\right)}{r} \right] \end{aligned}$$

The  $r$  demarcation line was obtained from the differential equation (12) after  $M$  has been substituted for  $B$  using equation (6).

The  $r$  demarcation line has the following  $r$  values at  $M=0$ :

(A9)

$$Z_M^{1,2} : r_{1,2} = \frac{\theta + \frac{n}{1-\delta}}{2} \pm \sqrt{\frac{\left(\theta + \frac{n}{1-\delta}\right)^2}{4} - \frac{\gamma\theta n\left(\frac{\bar{D}}{Y-G} + 1\right)}{1-\delta}}$$

it follows:

(A10)

$$0 < Z_M^1 < \frac{\theta + \frac{n}{1-\delta}}{2} < Z_M^2 < \theta + \frac{n}{1-\delta}$$

Those zero- $M$ -values should exist and be real numbers. Otherwise  $M$  will never be positive. Therefore the condition that the term under the square root is positive is one of the fundamental conditions for equilibria to exhibit positive values for all endogenous variables, see below. The maximal  $M$  value will be attained at

(A11)

$$r = \sqrt{\gamma\theta n\left(\frac{\bar{D}}{Y-G} + 1\right)}$$

The  $M$  demarcation line will have the following  $r$  values at  $M=0$ :

(A12)

$$Z_M^{1,2}: r_{1,2} = \frac{\theta}{2} - + \sqrt{\frac{\theta^2}{4} + \frac{\theta n}{1-\delta} \left[ \frac{(1-\gamma)(1+\alpha)}{\alpha} - \gamma \frac{\bar{D}}{Y-G} \right]}$$

The  $M$  demarcation line will have the following  $M$  value at  $r=0$ :

(A13)

$$Z_r: M = \frac{\frac{\alpha}{1+\alpha} \bar{D} - \frac{1-\gamma}{\gamma} (Y-G)}{n}$$

so that

(A14)

$$Z_M^1 < 0 < \theta < Z_M^2 \wedge Z_r < 0 \text{ for } \bar{D} < \frac{(1-\gamma)(1+\alpha)}{\gamma\alpha} (Y-G)$$

$$0 < Z_M^1 < \frac{\theta}{2} < Z_M^2 < \theta \wedge Z_r > 0 \text{ for } \bar{D} > \frac{(1-\gamma)(1+\alpha)}{\gamma\alpha} (Y-G)$$

The first fundamental restriction on  $D$  comes from the fact that in order for equilibria to exist with positive values for  $M$ ,  $B$  and  $r$ , it is necessary that the terms under the square root in (A4) and A(9) are positive:

$$(A15) \quad \bar{D} < (Y-G) \times \min \left\{ \frac{(1-\delta)(\theta + \frac{n}{1-\delta})^2}{4\gamma\theta n} - 1; \frac{(n+\theta)^2}{4\gamma\theta n} - 1 \right\}$$

Furthermore, we need a case distinction concerning the size of  $\bar{D}$  relative to some other parameters of the economy.

Case I

$$\bar{D} > \frac{(1+\alpha)(1-\gamma)}{\alpha\gamma} (Y-G)$$

which is derived from (A14).

Case II

$$\bar{D} < \frac{1+\alpha}{\gamma}(Y-G) \times \min \left\{ \frac{1-\gamma}{\alpha} - \frac{1-\delta}{(1+\delta\alpha)\theta} \left( \frac{(1+\alpha)n}{1+\delta\alpha} - \theta \right); \frac{1-\gamma}{\alpha} \right\}$$

The restriction for  $\bar{D}$  defining case II is obtained by substituting for  $r$  the value which lets the denominator of the  $M$  demarcation line in (A8) be zero. Then the nominator of the same equation is constrained to be positive. As the denominator decreases with an increasing  $r$ , the condition for case two is a conservative restriction ensuring that  $M$  is positive when  $r$  exceeds its value at the asymptote. To obtain positive  $M$  values in case two it is necessary that the denominator of the  $M$  demarcation line in (A8) is positive which is only the case when  $r$  exceeds  $(1+a)n/(1+da)$ . As  $d$  is less than 1 it follows that  $r$  necessarily exceeds  $n$ , and all equilibria in case II are necessarily dynamic efficient.

For values of  $D$  in the interval  
(A16)

$$\frac{1+\alpha}{\gamma}(Y-G) \left[ \frac{1-\gamma}{\alpha} - \frac{1-\delta}{(1+\delta\alpha)\theta} \left( \frac{(1+\alpha)n}{1+\delta\alpha} - \theta \right) \right] < \bar{D} < \frac{(1-\gamma)(1+\alpha)}{\gamma\alpha}(Y-G)$$

cases I and II are possible. Only if  $D$  is smaller than the left boundary of this interval are we clearly in case II.

The second term on the RHS of the case II condition assures that  $D$  really is smaller than  $(1-g)(1+a)(Y-G)/ga$  which would not necessarily be the case when  $q$  is very large. Otherwise  $Z_M^2$  would be smaller than  $q$  and the equilibrium value for  $r$ , which must exceed  $q$ , cannot coexist with positive cash holdings.

Trajectories:

From equation (12) it follows:  
(A17)

$$\frac{\partial \dot{r}}{\partial B} = - \frac{\delta\gamma\theta n}{Y-G} < 0$$

and from equation (13):  
(A18)

$$\frac{\partial \dot{B}}{\partial r} = \frac{1-\delta}{1+\alpha} B > 0$$

Moreover by equation (14):  
(A19)

$$\left. \begin{aligned} \frac{\partial \dot{M}}{\partial M} = \frac{1+\delta\alpha}{1+\alpha} r - n \end{aligned} \right\} \begin{aligned} &> 0 \text{ for } r > \frac{(1+\alpha)n}{1+\delta\alpha} \\ &< 0 \text{ for } r < \frac{(1+\alpha)n}{1+\delta\alpha} \end{aligned}$$

using:

(A20)

$$\dot{r} = (1 - \delta) r^2 - \left[ (1 - \delta)\theta + n - \frac{\delta\gamma\theta n}{Y - G} M \right] r + \gamma\theta n \left( \frac{\bar{D}}{Y - G} + 1 \right)$$

one gets:

(A21)

$$\frac{\partial \dot{r}}{\partial M} = \frac{\delta\gamma\theta n}{Y - G} r > 0$$

In the following we check the local equilibrium properties of the three equilibria by using the linearized system. The linearization was derived from a Taylor expansion around the equilibrium values of the system.

The linearized system is:

$$(A22) \begin{bmatrix} \dot{r} \\ \dot{M} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} 2r_i - (n + \theta + \frac{\delta\gamma\theta n}{Y - G} B_i) & 0 & -\frac{\delta\gamma\theta n}{Y - G} r_i \\ \frac{1 + \delta\alpha}{1 + \alpha} M_i + \frac{\alpha(1 - \delta)(Y - G)}{\gamma\theta n(1 + \alpha)} (2r_i - \theta) & \frac{1 + \delta\alpha}{1 + \alpha} r_i - n & 0 \\ \frac{1 - \delta}{1 + \alpha} B_i & 0 & \frac{1 - \delta}{1 + \alpha} r_i - n \end{bmatrix} \begin{bmatrix} r - r_i \\ M - M_i \\ B - B_i \end{bmatrix}$$

We will prove the following signs of the three Eigenvalues,  $E_1$ ,  $E_2$  and  $E_3$  of the linearized system for equilibria  $i=1, 2, 3$ :

(A23)

$$i=1: E_1 < 0 \quad E_2 < 0 \quad E_3 < 0$$

$$i=2: E_1 \begin{matrix} > \\ < \end{matrix} 0 \quad E_2 < 0 \quad E_3 > 0$$

$$i=3: E_1 > 0 \quad E_2 > 0 \quad E_3 > 0$$

By using the fact that the product of the Eigenvalues equals the determinant of the underlying matrix and that their sum equals the trace of the matrix, yields the three Eigenvalues of the system:



$$E_{1,i} = \frac{1+\delta\alpha}{1+\alpha} r_i - n$$

$$(A24) \quad E_{2,i} = \frac{2r_i - (n+\theta + \frac{\delta\gamma\theta n}{Y-G} B_i) + \frac{1-\delta}{1+\alpha} r_i - n}{2} - \sqrt{\frac{(2r_i - (n+\theta + \frac{\delta\gamma\theta n}{Y-G} B_i))^2}{4} - \frac{(1-\delta)\delta\gamma\theta n}{(1+\alpha)(Y-G)} r_i B_i}$$

$$E_{3,i} = \frac{2r_i - (n+\theta + \frac{\delta\gamma\theta n}{Y-G} B_i) + \frac{1-\delta}{1+\alpha} r_i - n}{2} + \sqrt{\frac{(2r_i - (n+\theta + \frac{\delta\gamma\theta n}{Y-G} B_i))^2}{4} - \frac{(1-\delta)\delta\gamma\theta n}{(1+\alpha)(Y-G)} r_i B_i}$$

The first Eigenvalue is clearly negative for equilibrium 1 as the latter only exists in case I where  $r$  is by definition smaller than  $(1+a)n/(1+da)$  according to figure A1 and the underlying demarcation line for  $M$ . Equivalently, the first Eigenvalue is definitely positive for equilibria 2 and 3 as can be seen from figure A2 and the formula for the first Eigenvalue above. The following equations result from eliminating  $E_{1,i}$  and will be helpful in determining the sign of the second and third Eigenvalue:

(A25)

$$E_{2,i} + E_{3,i} = 2r_i - \left( n + \theta + \frac{\delta\gamma\theta n}{Y-G} B_i \right) + \frac{1-\delta}{1+\alpha} r_i - n$$

(A25) follows from the fact that the trace equals the sum of the Eigenvalues of a matrix. The product of the Eigenvalues equals the determinant of a matrix. Applying the latter rule and again eliminating  $E_{1,i}$  yields:

(A26)

$$E_{2,i} E_{3,i} = \left[ 2r_i - \left( n + \theta + \frac{\delta\gamma\theta n}{Y-G} B_i \right) \right] \left[ \frac{1-\delta}{1+\alpha} r_i - n \right] + \frac{(1-\delta)\delta\gamma\theta n}{(1+\alpha)(Y-G)} r_i B_i$$

For the first equilibrium,  $i=1$ , the sum of the first two terms on the RHS of equation (A25) is negative which can be seen from figure A1 and with the  $r$  demarcation line in (A1). The first equilibrium lies on the downward sloping branch of the  $r$  demarcation line. The third term on the RHS is negative for all equilibria which follows from equation (17) and the condition that the steady state value of  $B$  has to be positive. Thus for  $i=1$ , the sum of the second and third Eigenvalue is negative. In equation (A26) both terms in square brackets on the RHS are negative for the reasons just mentioned. The product will thus be positive as with the remaining term on the RHS. The product of the second and third Eigenvalue is positive. A positive product and a negative sum of two terms are only possible if both are negative.

For equilibria 2 and 3 the proof is slightly more costly. Let us define the second and third Eigenvalue to have the following structure:

$$E_{2,3,i} = F_i - J_i$$

where  $J$  stands for the square root term of the Eigenvalues. The following equation delivers the condition that the value of  $F$  is equal to the value of  $J^{23}$  using the equations defining  $E_2$  and  $E_3$ :

$$(A27) \quad |F| = |J| \quad \text{iff} \quad \frac{1-\delta}{1+\alpha} r_i - n + \frac{(1-\delta)\delta\gamma\theta n}{2(1+\alpha)(Y-G)\sqrt{\Omega_i}} r_i B_i = 0$$

$$\text{with} \quad \Omega_i = \frac{(n+\theta + \frac{\delta\gamma\theta n}{(Y-G)} B_i)^2}{4} - \gamma\theta n \left( \frac{\bar{D}}{Y-G} + 1 \right)$$

Using the slopes of the demarcation lines in the  $[r;B]$  space one can show that if the first term in equation (A27) is greater than the second one, the slope of the  $B$  demarcation line exceeds the  $r$  demarcation line which defines equilibrium 3. Thus, for equilibrium 3  $|F| > |J|$  and for equilibrium 2  $|F| < |J|$  hold, respectively. From this it follows immediately for the second equilibrium that  $E_2$  is negative and  $E_3$  positive. For the third equilibrium there are two possibilities. If  $F$  is negative  $E_2$  and  $E_3$  will both be negative, which can be excluded from the phase diagrams. If  $F$  is positive then  $E_2$  and  $E_3$  are positive. For equilibrium 1 the three Eigenvalues are negative. Equilibrium 1 will thus be locally, and due to the separate phase diagram also globally, a stable node. Equilibrium 2 has either the saddle path property or is a stable-2 manifold, depending on whether case II or I applies. This is established locally because the second Eigenvalue is negative and the third positive. The first Eigenvalue is negative in case I and positive in case II. Thus for instance in case II, one finds two positive and one negative Eigenvalue opposed to one predetermined variable,  $B$ , which yields a saddle path equilibrium. Equilibrium 3 is locally, and due to the phase diagram analysis, also globally unstable. All three Eigenvalues are positive.

Fortunately, the problem of having to deal with multiple equilibria can be resolved in a straightforward manner. It will be shown later that the central bank monetises all the interest payments on the government debt when the real interest rate equals  $(1+a)n$ . As shown in the text this is a very high degree of monetisation. Thus, as  $r$  depends negatively on  $a^{24}$ , the real interest rate will exceed  $(1+a)n$ . The latter term is greater than  $(1+a)n/(1+da)$ , i.e., the  $r$  value for which the  $M$ -demarcation line has an asymptote. Therefore, we necessarily are in case II. Since equilibrium 3 is unstable and equilibrium 2 is a saddle path, the following analysis will only consider the latter equilibrium. Let us assume for the moment, the degree of monetary financing would nevertheless exceed the interest payments on government debt. Case I would then be possible, but one would still choose equilibrium 2 instead of equilibrium 1. The reason being that the first equilibrium is a stable node and the second a stable-2 manifold. In a perfect foresight framework, agents will have more co-ordination problems in equilibrium 1. After a shock they would have to agree on the impact effect in  $M$  and  $r$ . These variables could jump basically to any values from which the equilibrium would be reached in finite time. In the second equilibrium, the rational agents only have to agree on either  $M$  or  $r$  as the other one would then be given by the stable manifold. Also, the latter would involve a co-ordination problem, but this problem would only be one-dimensional. A last reason why equilibrium 2 would be the one to investigate, is the fact that for some values of  $\bar{D}$  relative to the given parameters of the model, the

<sup>23</sup> The derivation implied taking both sides to the second power which explains why the condition is stated in values and not in absolute numbers.

<sup>24</sup> This is shown below.

second equilibrium would be the only one possible with positive steady state values for all the endogenous variables.

*A2 Proof of Proposition 1*

We will show that  $z$  evaluated at the second equilibrium is always negative. To see this we investigate the effect of a change in  $a$  on the stock of outstanding government bonds. We equate the two demarcation lines in  $[r;B]$  space and apply the implicit function theorem:

$$(A28) \quad \frac{\partial B}{\partial \alpha} = \frac{\frac{n}{1-\delta}}{\frac{\delta\gamma\theta n}{2(Y-G)} \left[ 1 + \frac{n + \theta + \frac{\delta\gamma\theta n}{Y-G} B}{2 \sqrt{\frac{(n + \theta + \frac{\delta\gamma\theta n}{Y-G} B)^2}{4} - \gamma\theta n \left( \frac{\bar{D}}{Y-G} + 1 \right)}} \right] - \frac{\bar{D}}{(1-\delta)B^2}} < 0$$

The numerator is clearly positive. The first and the second term in the denominator are the slopes of the  $r$  and  $B$  demarcation lines in  $[r;B]$  space, respectively. Equilibrium 2 is characterized by a  $B$  demarcation line steeper than the  $r$  demarcation line. It follows that the partial derivative is negative. An increase in the degree of monetary financing,  $a$ , lowers the government debt stock. As the  $r$  demarcation line in the  $[r;B]$  space has a positive slope and is unaffected by a change in  $a$ , a reduction of  $B$  also signifies a reduction in the real interest rate. The partial derivative would be:

$$(A29) \quad \frac{\partial r}{\partial \alpha} = \frac{n \left[ r^2 - (n + \theta)r + \gamma\theta n \left( \frac{\bar{D}}{Y-G} + 1 \right) \right]}{3(1-\delta)r^2 - 2[(1+\alpha)n + (1-\delta)(n + \theta)]r + (1+\alpha)n(n + \theta) + \gamma\theta n \left( \frac{\bar{D}}{Y-G} + 1 - \delta \right)} < 0$$

The denominator is nothing other than  $z$ . The nominator is positive for any values of  $B$  and  $d$  and only be zero when  $d$  or  $B$  are equal to zero as follows from the discussion of the  $r$  demarcation line in  $[r;B]$  space. As the sign of  $\partial r / \partial a$  is known from the analysis of (A29) it is obvious that  $z$  is always negative if evaluated at the second equilibrium.

*A3 Proof of proposition 2*

It still remains to be shown that the present value of opportunity costs of cash balances of individuals currently alive equals  $iM/q$ . From individual optimization and omitting the time index we get:

$$(A30) \quad \dot{m} = \left( r - \theta - \frac{i}{q} \right) m$$

$i$  is constant in the steady state. We also know that the real interest rate will always exceed the rate of time preference so that individual real cash balances will grow at the rate  $(r-q)$ . Accordingly, the term  $M_t e^{(r-q)t}$  represents the aggregate per capita real cash balances from generations present in

period  $t$ , accumulated till period  $t+v$ <sup>25</sup>. Equation A31) then represents the present value of the aggregate per capita opportunity costs of money for individuals alive at period  $t$ :

(A31)

$$\int_{v=0}^{\infty} i_{t+v} (M_t e^{(r-\theta)v}) e^{-rv} dv = \frac{iM}{\theta}$$

We still need to prove the sign of the partial derivative of equation (26) in the text. The same logic applied to prove the sign of

$$\frac{\partial \left( \frac{\partial r}{\partial \bar{D}} \right)}{\partial \alpha} < 0 \text{ we get}$$

$$(A32) \quad \frac{\partial^2 \Phi}{\partial \bar{D} \partial \delta} = - \frac{\gamma n [r - (1 + \alpha)n]}{(r - n) [(1 + \alpha)n - (1 - \delta)r]^2} \begin{matrix} > \\ < \end{matrix} 0 \text{ for } \alpha \begin{matrix} > \\ < \end{matrix} \alpha^*$$

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<sup>25</sup>  $M_t e^{(r-\theta)v} > M_{t+v}$  because the aggregate real per capita money stock in period  $t+v$  includes the lower cash balances of the generations born between  $t$  and  $t+v$ .

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