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The Sequential Trading Approach to Disequilibrium Dynamics

Ungleichgewichtsmodelle mit sequentieller Transaktionsstruktur

By Thomas Lux*), Würzburg

This paper is concerned with a certain branch of disequilibrium models characterized by sequential trading, rather than the standard Barro/Grossman-Malinaud disequilibrium framework where there is instantaneous interaction between agents in all markets. A special outcome of the first class of models is the so-called non-Walrasian equilibria. That is, states of the economy where both the labour and goods markets clear but the usual marginal productivity conditions for Walrasian equilibria are not fulfilled. The sequential trading approach developed mainly by Varian (1977), Eckalbar (1980, 1981), and Svensson/Weibull (1984) has the advantage unlike the standard models that no violation of the agents' budget constraints can occur¹⁾. The sequential trading approach, as is pointed out by Lorenz (1982), also overcomes the unrealistic assumption of a quantity tâtonnement equilibrating effective market functions and the perceived constraints in any temporary equilibrium. The aim of the present paper is twofold: First, we present a generalized version of Varian's original contribution. As our set-up of the model is based on the differentiation of regimes, it is more easily comparable to the standard models. Since we are using only standard mathematical tools in our stability analysis, our representation of the sequential trading framework will also be more easily accessible than the original contributions without loss of any economically meaningful information. Second, we review some extensions of the model as presented in the literature and show how sensitively it reacts to variations of the underlying structure. This paper may also contribute to a recent discussion taking place between Heubes (1982), Englmann (1983, 1984), and Homburg (1984, 1985) on the correct notion of effective functions and the validity of

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¹⁾ The possibility of a violation of the budget constraints in the simultaneous trading models is due to the fact that, following Benassy's concept of market functions (*Benassy, 1975*), we have to consider n (number of markets) separate maximization programmes in every period, and these are not guaranteed to be consistent. It was also mentioned by *Martin/Portes* in their recent survey on effective market functions in empirical work that sequential trading overcomes this infeasibility problem (*Martin/Portes, 1990*).

Walras' law in disequilibrium. The main difference in this controversy seems to be the implicit assumption of a sequential (Englmann) or simultaneous (Heubes, Homburg) trading model. We show that these different set-ups also generate very different long-run behaviour of the economy.

1. The model

First, the framework of the model is presented – there are two decision making units. The representative household is characterized by a strictly quasi-concave utility function

$$U = U(C, H - L, \tilde{m})$$

+ + +

$$\text{subject to: } m + wL = C + \tilde{m} \quad (1)$$

where $w = W/p$ is the real wage, $L(C)$ is the quantity of labour (goods) transacted, $H - L$ is leisure time, and $m = M/p$ ($\tilde{m} = \tilde{M}/p$) are the real money balances at the beginning and at the end of the period. The result of this maximization program is, in the first stage, notional supply of labour $L^s = L^s(w)$. When transactions on the labour market have been carried out another maximization with respect to consumption demand takes place, which gives: $C^d = C^d(wL + m)$. This is a notional (effective) function, if $L = L^s$ ($L < L^s$). The representative firm is described by a neoclassical production function $Y = F(L)$; profit maximization gives the notional demand for labour $\bar{L}^d(w)$, and the notional supply of output $\bar{Y}^s = F(\bar{L}^d(w))$. As there is trading first in the labour market, the firm must form sales expectations e . If $\bar{Y}^s > e$, then there is a spill-over effect on the labour market, so that expected demand of labour is $L^d(w, e) = \min\{\bar{L}^d(w), F^{-1}(e)\}$.

Expressed supply on the goods market is what has been produced, i.e. $Y^s(w, e) = \min\{\bar{Y}^s(w), F(L^s(w)), e\}$.

This is a notional function, if $\min\{ \} = \bar{Y}^s$, and an effective function, if $\min\{ \} = F(L^s)$ or $\min\{ \} = e$. In the latter expression the spill-over effect on the labour market caused by expected demand failures is transmitted to the goods market.

The following transactions result after incorporating supply and demand into the labour market:

$$L = \min\{F^{-1}(e), \bar{L}^d(w), L^s(w)\} \quad (2)$$

Depending on which restriction is binding, three *labour market regimes* can be differentiated: a Keynesian (K): $L = F^{-1}(e)$, where production is constrained by the expectation of deficient demand, a classical (Cl): $L = \bar{L}^d(w)$, and an inflationary (I): $L = L^s(w)$, where production is constrained either by labour demand or supply. The three regimes are separated by the thick lines in Fig. 1. The slopes of the frontiers are easily obtained by setting equal each pair of restrictions and differentiating with respect to w and e :

$$dw/de|_{K \cap Cl} = (F' \delta \bar{L}^d / \delta w)^{-1} < 0; \quad dw/de|_{K \cap I} = (F' \delta L^s / \delta w)^{-1} > 0;$$

$$dw/de|_{Cl \cap I} = 0 \quad (3)$$

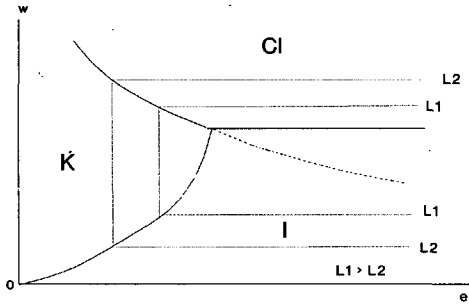


Figure 1

Employment depends only on sales expectations in (K) or on the real wage in (Cl) and (I). Therefore the iso-employment loci in regions (I) and (Cl) are parallels to the abscissa, whereas in (K) they are parallels to the ordinata. Note that this would be the only possible regime classification in a simultaneous trading model (cf. Benassy 1986, chap. 3) or in a model with “rational” or correct expectations (in both cases $e = C^d(wL + m)$), for a formal proof of their equivalence cf. Michel, 1982).

But, here there exists a sequential trading world with given expectations which may turn out to be incorrect. We therefore also have to distinguish between different situations on the goods market. Transactions on the goods market (C) are:

$$C = \min\{F(L), C^d(wL + m)\} = \min\{e, F(\bar{L}^d(w)), F(L^s), C^d(wL + m)\} \quad (4)$$

For every labour market regime we now have to differentiate two subregimes according to the situation in the goods market. For the Keynesian regime there are the following possibilities:

$$(K1) \quad C = e < C^d(wF^{-1}(e) + m) \quad (5)$$

$$(K2) \quad C = C^d(wF^{-1}(e) + m) < e \quad (6)$$

These are separated by a line $C^d(.) = e$ in the interior of labour market regime (K), which has a positive slope because higher w means higher labour income (directly and indirectly via higher labour demand, when consumption rises), so that sales expectations must also rise to be correct.

The classical regime contains the following subregimes:

$$(Cl1) \quad C = F(\bar{L}^d(w)) < e, C^d(.) \quad (7)$$

$$(Cl2) \quad C = C^d(w\bar{L}^d(w) + m) < F(\bar{L}^d(w)) < e \quad (8)$$

The two subregimes are separated by a horizontal line $F(\bar{L}^d(w)) = C^d(.)$.

The inflationary regime has the following subregimes:

$$(I1) \quad C = F(L^s(w)) < e, \quad C^d(.) \quad (9)$$

$$(I2) \quad C = C^d(wL^s(w) + m) < F(L^s(w)) < e \quad (10)$$

which are also separated by a horizontal line: $F(L^s) = C^d(.)$. The division of the parameter space with respect to the goods market situation will become clearer in the following dynamical analysis.

2. Dynamics of the model

Like Varian (1977), Eckalbar (1980), and Svensson/Weibull (1984) we now take into account that nominal wages, prices and sales expectations all change in disequilibrium – assuming that prices and wages adjust according to the “law of supply and demand” (using effective functions) and assuming that expectations adjust adaptively. We therefore have to consider the following dynamic system in R^2 :

$$\begin{aligned} \dot{e} &= C^d(wL + m) - e \\ \dot{w}/w &= \dot{W}/W - \dot{p}/p = k_w \{ \min(\bar{L}^d(w), F^{-1}(e)) - L^s(w) \} - k_p \{ C^d(.) - \\ &\quad \min(e, F(\bar{L}^d), F(L^s)) \} \end{aligned} \quad (11)$$

whereby k_w , k_p are adjustment coefficients. We assume that the firms' interest in correcting their expectations concerning goods market demand does not depend on the prevailing regime (for another specification see Eckalbar (1980); the results presented here do not depend on the hypothesis used for the correction of sales expectations).

A stationary state is a solution to the system $\dot{e} = \dot{w}/w = 0$. Therefore, there exists in equilibrium: $e = C^d$, which is the case along the frontier between (K1) and (K2) and at certain combinations (e, w) in the interior of subregimes (CI1) and (I1). On the other hand, $\dot{w}/w = 0$ is possible only in (K2), (CI2), and (I1), where excess demand in the labour and the goods market have the same sign. Stationary states of system (11) may therefore lie in the interior of (I1) with positive excess demand in both markets (existence of such equilibria depends on the slopes of $\dot{e} = 0$ and $\dot{w}/w = 0$) or in one or more point(s) on the frontier between (K) and (I), where $e = C^d(.) = F(L^s)$. This could be identically described by: $\dot{e} = \dot{W}/W = \dot{p}/p = 0$. Points on the curve $K \cap I$ are non-Walrasian equilibria as described above. Equilibria in (I1) are so-called quasi-equilibria, because there the nominal variables (W, p) change at the same proportionate rate²). For further investigation of the number and stability of steady states we must examine the functions $\dot{e} = 0$ and $\dot{w}/w = 0$ applying the relevant min-conditions. For the adjustment of expectations we have the following piecewise defined function:

²) All the above mentioned contributions seem to overlook the possibility of quasi-equilibria.

$$\dot{e} = \begin{cases} C^d(wF^{-1}(e) + m) - e & \text{in K} \\ C^d(w\bar{L}^d(w) + m) - e & \text{in Cl} \\ C^d(wL^s(w) + m) - e & \text{in I} \end{cases} \quad (12)$$

For the slopes of $\dot{e} = 0$, we obtain:

$$\begin{aligned} dw/de|_K &= (1 - C^{d'}w/F')/C^{d'}F^{-1}(e) > 0 \\ dw/de|_{Cl} &= \{C^{d'}(\bar{L}^d + w\delta\bar{L}^d/\delta w)\}^{-1} > (<) 0? \\ dw/de|_I &= \{C^{d'}(L^s + w\delta L^s/\delta w)\}^{-1} > 0 \end{aligned} \quad (13)$$

As a result one can state that $\dot{e} = 0$ has positive slope in regimes (K) and (I), whereas in (Cl) the slope will be positive in the neighbourhood of full employment, but will become negative at low levels of employment. It is important to note that the $\dot{e} = 0$ curve must intersect the abscissa, because with $w = 0$ there exists an e so that: $e = C^d(0 + m) \Leftrightarrow \dot{e} = 0$. Therefore, the $\dot{e} = 0$ curve is located in the inflationary region for low values of the real wage. We now have to differentiate the following possibilities:

(1) $\dot{e} = 0$ does not intersect the $K \cap I$ -curve. This will be the case for high values of the real money balances. Then there exist no non-Walrasian equilibria.

(2) There exists a point of intersection. Because of the assumptions concerning the underlying utility and production functions we know that $K \cap I$ (that is $e = F(L^s(w))$) is concave and $\dot{e} = 0$ (which is identical with the notional demand function in I) is convex. If both functions were defined for the whole parameter space, one would obtain either zero or two intersections (we omit the borderline case of a tangency point). But here the second intersection point may or may not exist. If it does not exist, then there must be instead an intersection between $\dot{e} = 0$ and $K \cap Cl$. Varian himself examined another borderline case where the second intersection is identical with the Walrasian equilibrium E_w .

We can now also identify the goods market subregimes in w - e -space. In labour market regime (K), $\dot{e} = 0$ is identical with the existence of a goods market equilibrium. To the left (right) of the curve we have situations of deficient supply (demand). In the inflationary region $\dot{e} = 0$ is only possible for subregime (I1). In the intersection points between $\dot{e} = 0$ and $K \cap I$ we have: $e = C^d(\cdot) = F(L^s(w))$ and, consequently, they give us the values of the real wage which guarantee equilibrium in the goods market. As a consequence, the (I1) region may consist of two separate parts. If there is an intersection between $\dot{e} = 0$ and $K \cap Cl$ this gives us, analogously, the value of w for which we have goods market equilibrium in the classical region. Higher (lower) values of the real wage indicate deficient supply (demand)³. Summarizing the preceding considerations, we can state that demand failures will take place in the area in Fig. 2 to the right of the $\dot{e} = 0$ segment in (K), whereas, in the rest of the parameter space, goods market demand exceeds supply.

³ If we have no intersection on $K \cap Cl$ (two intersections on $K \cap I$), then all of the classical labour market regime is of the Cl1 kind. This is a consequence of the fact that $w = F'$ in the classical regime. It follows that $F'\delta\bar{L}^d/\delta w < C^{d'}(\bar{L}^d + w\delta\bar{L}^d/\delta w)$, i.e. with rising real wage supply of goods decreases more rapidly than demand (which may even increase).

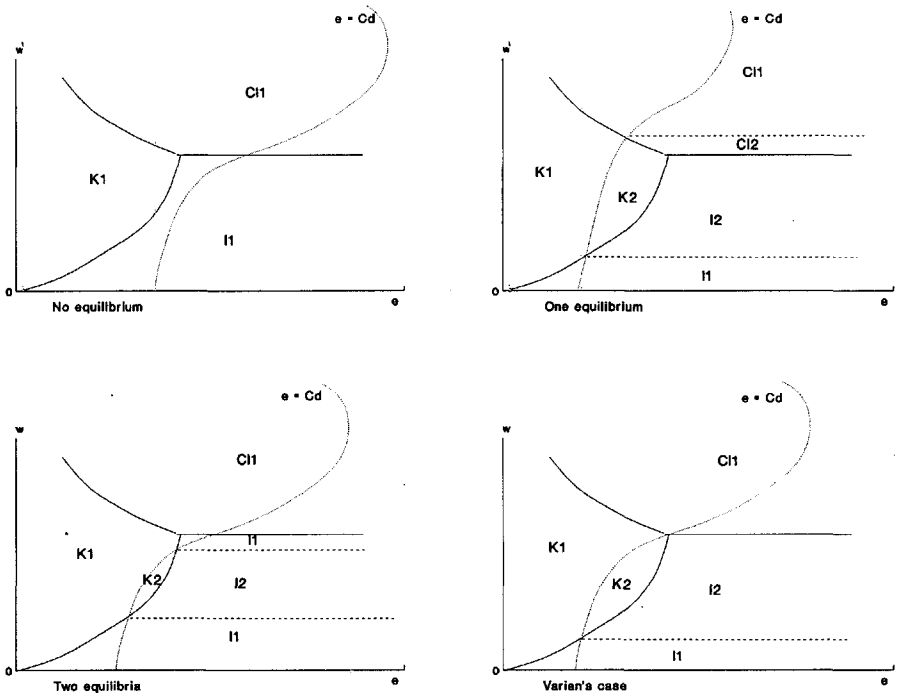


Figure 2

For the change in the real wage we have the following concrete forms:

$$\dot{w} = w \begin{cases} k_w \{F^{-1}(e) - L^s(w)\} - k_p \{C^d(wF^{-1}(e) + m) - e\} & \text{in K} \\ k_w \{\bar{L}^d(w) - L^s(w)\} - k_p \{C^d(w\bar{L}^d(w) + m) - F(\bar{L}^d(w))\} & \text{in Cl} \\ k_w \{\min[F^{-1}(e), \bar{L}^d(w)] - L^s(w)\} - k_p \{C^d(wL^s(w) + m) - F(L^s(w))\} & \text{in I} \end{cases} \quad (14)$$

The slopes of the $\dot{w} = 0$ curve are:

$$dw/de|_K = \frac{k_p(1 - C^{d'}w/F') + k_w/F'}{k_p C^{d'}F^{-1}(e) + k_w \delta L^s/\delta w} > 0$$

$$dw/de|_{Cl} = 0 \quad (15)$$

$$dw/de|_I = \begin{cases} \frac{k_w/F'}{k_p C^{d'}L^s(w) + \delta L^s/\delta w [k_w + k_p(C^{d'}w - F')]} & \text{if } F^{-1}(e) < \bar{L}^d \\ 0 & \text{if } F^{-1}(e) > \bar{L}^d \end{cases}$$

Note that labour demand in (I) can in general be either $F^{-1}(e)$ or $\bar{L}^d(w)$ depending on whether we are under or above the broken line in Fig. 1, which is a continuation of the frontier $K \cap CI^4$). We, therefore had to pay attention to another min-condition in the inflationary region.

The signs of the slope are unambiguous except for the $\dot{w} = 0$ -segment in (I) under the broken line. Here it depends crucially on the speed of adjustment of prices and nominal wages. If the labour market reacts much more rapidly, the slope will be positive and $\dot{w} = 0$ lies near the frontier $K \cap I$. On the other hand, if the goods market adjusts much more rapidly, we have negative (positive) slope in the (I1) part characterized by low (high) employment respectively⁵). The real wage will be unambiguously declining in the CI1 and K1 regime because of the simultaneous decline of the nominal wage and rise of the price level whereas the opposite holds in I2. In the other subregimes we have conflicting effects from the labour and goods markets, so that the change in the real wage depends on the relative strength of disequilibria in both markets. In general, the real wage will decline to the left and rise to the right of the curve $\dot{w} = 0$.

In Fig. 3 we have restricted our attention to the case of two intersection points between $\dot{e} = 0$ and $K \cap I$ because this situation reveals all interesting features of the w -dynamics.

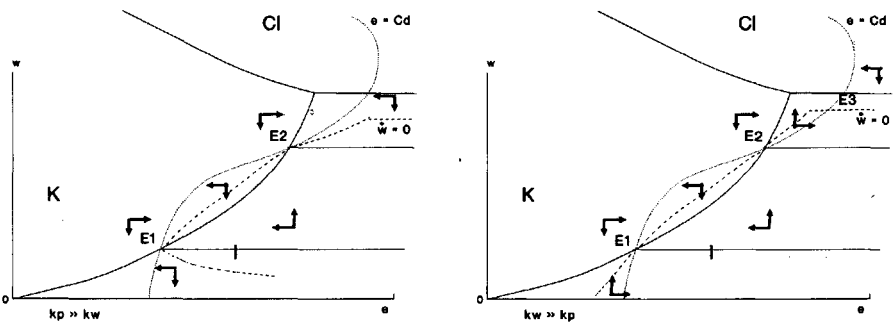


Figure 3

As is indicated by the arrows, the low employment non-Walrasian equilibrium (E1) will be stable with respect to the K -dynamics, whereas the high employment one (E2 which might be the Walrasian equilibrium in the borderline case) is unstable. The same holds for the I -dynamics provided $k_w \gg k_p$. In this case there may also exist a (stable) inflationary quasi-equilibrium (E3). If the reverse is true, the stability properties concerning the I -regime are also reversed⁶).

⁴) The frontier $K \cap CI$ and its continuation separates the “Keynesian” and “Walrasian” regimes in the papers by Varian and Svensson/Weibull. It is clear from the preceding analysis that such a classification does not correspond to the usual labelling of rationing regimes.

⁵) Assuming k_w to be small, the sign of the slope depends on $C^d(L^s + w\delta L^s/\delta w) - F'\delta L^s/\delta w$ which gives the change of the excess demand on the goods market. Approaching the lower $I1 \cap I2$ frontier, the term will be negative, as one approaches equilibrium, departing from the second frontier (from equilibrium to excess demand) it will be positive.

⁶) For a formal treatment see appendix 1.

Therefore local asymptotic stability applies, in the case of k_w great compared to k_p , only for the low employment non-Walrasian equilibrium E1⁷). In this case equilibria of the E2-type are saddle points with respect to both the K- and I-dynamics. In the more realistic case of a low adjustment speed in the labour market both equilibria are stable with respect to one adjoint regime and unstable with respect to the other. Although they are not overall stable, they may be both attractors for trajectories starting in a certain (possibly large) subset of the parameter space. Concluding we can say that the stability result for quasi-equilibria obtained by Varian is not confirmed in the general setting presented here. The difference, however, is mainly due to the fact that Varian examines only the dynamics in K for E1 equilibria.

3. Some extensions

In the existing literature, Eckalbar (1980) and Svensson/Weibull (1984) are very close to the original contribution by Varian. The first author modifies the model by assuming that goods market demand is always equal to production, i.e. individuals spend all their income. As a consequence $\dot{e} = 0$ and $\dot{w} = 0$ is the case in all points on the frontier $K \cap I$. One, therefore, obtains a continuum of (stable) steady states which are all non-Walrasian equilibria. A generalization concerning the n commodity case is presented by Raymon (1982). Interestingly enough, the underlying structure of his model is a mixture of sequential and simultaneous trading: there is first trading on the labour market, then transactions are carried out on the n goods markets.

The modification in the Svensson/Weibull-paper seems to be of minor importance: unlike Varian they assume that the representative firm supplies in the inflationary labour market regime an amount of goods equal to expected demand e , instead of $F(L^s(w))^8$. But this modification is responsible for a qualitative change in the stability results: Independent of the relative size of k_p and k_w , low employment non-Walrasian equilibria are now stable on both sides, whereas the high employment equilibria are unstable.

We shortly present another example of the sensitivity in the stability results for this kind of model. Assume that labour supply is inelastic: $L^s = \bar{L}^s$. As the first consequence the boundary $K \cap I$ will become a vertical line $e = F(\bar{L}^s)$, cf. fig. 4. The locus of $e = C^d(\cdot)$ is still convex in K and I, so that we now have, at most, one intersection point (i.e. non-Walrasian equilibrium). Setting $\delta L^s / \delta w = 0$ in the Jacobians (see appendix 2), it is straightforward to show that the stability behaviour of this unique equilibrium is the E2 type analysed above.

Beside these modifications, there are at least three possible extensions which are worth mentioning:

⁷) In general, stability of each subsystem in a disequilibrium model is neither necessary nor sufficient for overall stability. However, Ito (1980, appendix 3) and Honkapohja/Ito (1983, p. 38ff.) have shown that for R^2 in the case of a steady state on the boundary between two piecewise defined systems of differential equations stability with respect to both systems is a sufficient condition for overall stability. The only additional condition is that at the boundary the solutions of both systems must point into the same regime.

⁸) Note that this assumption is in contrast to the spirit of the sequential trading approach: firms supply more than they are able to produce.

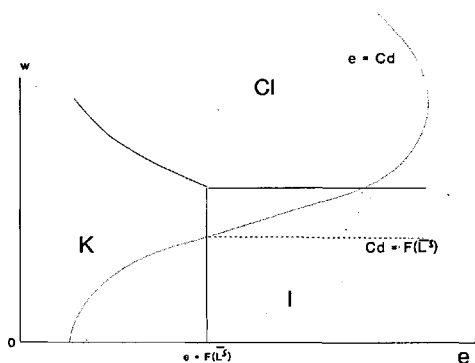


Figure 4

(1) Investigation of the intrinsic dynamics of the real money balances, first investigated in the simultaneous transaction approach by Böhm (1978) and Honkapohja (1979). The change in the household's monetary resources caused by the transactions in temporary equilibrium are here:

$$\begin{aligned} \dot{m} = wL - C - m \dot{p}/p &= w \min\{F^{-1}(e), \bar{L}^d, L^s\} - \\ &- \min\{e, F(\bar{L}^d), F(L^s), C^d\} - m \dot{p}/p \end{aligned} \tag{16}$$

In the steady state of the three-dimensional w - e - m -dynamic we still have $\dot{e} = \dot{w} = 0$, and consequently this condition is fulfilled by non-Walrasian equilibria. As one has to take into account the transacted quantity on the goods market (C) one obtains different dynamical systems, not only for the three labour market regimes but also for their goods market subregimes. Since there are no easily applicable theorems in the literature that allow inference from regime (in-)stability to overall (in-)stability in R^3 , the formal analysis does not seem very promising, and indeed it reveals no clear tendencies⁹⁾.

(2) The analysis of the standard sequential trading model reveals one striking asymmetry: while the firm forms expectations concerning the goods market situation, the household behaves as though its demand will always be satisfied. As this is, in general, not the case in a disequilibrium model, it would be more appropriate to also take into consideration the household's expectations. The anticipation of rationing in the goods market (at an expected level e_H) would necessarily imply that one has to take into account an effective labour supply function $L^s = L^s(w, e_H)$. For a symmetric treatment one now has to add an adjustment function for the household's expectations:

$$\dot{e}_H = C - e_H = \min\{e, F(L^s), F(\bar{L}^d), C^d\} - e_H \tag{17}$$

⁹⁾ There seems to be a clear result in Yannelis (1986) who claims that the incorporation of the real balance dynamics destabilize non-Walrasian equilibria. However, his instability result is due to the assumption of inelastic labour supply which can be easily shown by eliminating the m -dynamics from his system. Another shortcoming of his paper lies in the fact that he examines only the dynamics in the two Keynesian subsystems, so that his analysis is incomplete (this omission may be due to the above mentioned confusion concerning the regime descriptions).

Here exactly the same considerations apply as in the case of the real balance dynamics¹⁰).

(3) A very natural extension in dynamic disequilibrium models is to allow for firms to hold buffer stocks. This seems to be especially interesting under the assumption of sequential trading where production and trade take place at different points in time. Here we have the contributions by Simonovits (1982), elaborated further by Hommes/Nusse (1989), Hommes (1989), and Eckalbar (1985), who examine the dynamics of inventories and sales expectations¹¹). Their works add important insights into the traditional inventory models, showing that in a disequilibrium framework instable Metzlerian equilibria may be stabilized to limit cycle oscillations between the Keynesian and the inflationary region. Unfortunately, these studies do not take into consideration the dynamics of prices and wages, which may alter significantly the results.

For summary, one can state the following: By introducing into dynamic disequilibrium models the very realistic feature that production deserves time (this is in fact the only difference between the model presented here and the one in e.g. Honkapohja (1979)), one gets a totally new kind of steady states for an economy where prices and wages are flexible, but their adjustment speed is finite. Moreover, the existence of non-Walrasian equilibria *depends* crucially on the assumption of a *competitive* adjustment of prices and nominal wages. They are not present in models with intertemporal rigid prices and wages (the buffer stock models, Yannelis (1983), Eckalbar (1981)) nor in the simulation experiments by Lorenz (1982), who assumes monopolistic price-setting by firms. The stability results, on the other hand, depend on the exact specification of the model in question.

As consumption demand in this non-Walrasian equilibria is lower than profit maximizing production, they seem to be very near to traditional Keynesian thoughts. The structure of the dynamic sequential trading model can explain why price and wage flexibility may not be able to preserve the economy from suffering such a (suboptimal) situation. As the underlying structure also avoids the main criticism in the disequilibrium macroeconomics, namely the violation of the budget constraints, it is astonishing to note that there are few contributions in the literature. This fact may be due to the exponentially rising difficulties one faces if one tries to incorporate additional features of reality (e.g. in the cases (1) and (2) described above one has to take into consideration eight (!) dynamical subsystems because we have six subregimes and one additional discontinuity in the inflationary region).

Appendix

1. Following Ito (1980, p. 401ff.) and Honkapohja/Ito (1983, p. 38ff.) in the case of piecewise continuous differential equations with two subsystems separated by a smooth boundary in R^2 one can infer overall stability from stability of both subsystems provided the solutions of both systems point into the same regime at the boundary. The validity of this additional condition is shown at the end of appendix 1.

¹⁰) Yannelis (1983, chap. IV) investigates a two-dimensional system of adaptive expectations correction, whereby prices and wages are assumed to be constant.

¹¹) Both models seem to be totally identical, except for the fact, that Simonovits uses difference and Eckalbar differential equations.

For the examination of local (asymptotic) stability of steady states, we therefore have to investigate the Jacobians of system (11). As the non-Walrasian equilibria are located on the frontier $K \cap I$, we have to consider the dynamics of both regimes. For the K-dynamics the Jacobian is:

$$J_K = \begin{vmatrix} C^d{}_w/F' - 1[-] & C^d{}_F^{-1}(e)[+] \\ w\{k_w/F' - k_p(C^d{}_w/F' - 1)\}[+] & w\{-k_w\delta L^s/\delta w - k_p C^d{}_F^{-1}(e)\}[-] \end{vmatrix}$$

and therefore:

$$\text{tr } J_K < 0$$

The sign of the determinant is in general ambiguous, but as $\dot{w} = 0$ lies in the interior of (K2) it is clear from the slopes of $\dot{w} = 0$ and $\dot{e} = 0$ that it is positive for E1 and negative for E2.

For the I-dynamics we have:

$$J_I = \begin{vmatrix} -1 & C^d{}_F(L^s(w) + w\delta L^s/\delta w) \\ w k_w/F' & w\{-k_w\delta L^s/\delta w - k_p(C^d{}_F(L^s(w) + w\delta L^s/\delta w) - F'\delta L^s/\delta w)\} \end{vmatrix}$$

$$\text{tr } J_I = -1 - w k_w\delta L^s/\delta w - w k_p(C^d{}_F(L^s(w) + w\delta L^s/\delta w) - F'\delta L^s/\delta w)$$

$$\det J_I = w(k_p - k_w/F')(C^d{}_F(L^s(w) + w\delta L^s/\delta w) - F'\delta L^s/\delta w)$$

Taking account of footnote 5 the conclusions in the main text are easily verified:

a) $k_p \gg k_w \Rightarrow \text{tr } J_I \leq 0$ (?) for E1, < 0 for E2,

$$\det J_I > (<) 0 \text{ for E2 (E1)}$$

b) $k_p \ll k_w \Rightarrow \text{tr } J_I < 0$, $\det J_I > (<) 0$ for E1 (E2).

It remains to be shown that the "crossing-over" condition mentioned above holds. It reads:

$$\left[(\mu_e, \mu_w) J_I \begin{pmatrix} \mu_w \\ -\mu_e \end{pmatrix} \right] \left[(\mu_e, \mu_w) J_K \begin{pmatrix} \mu_w \\ -\mu_e \end{pmatrix} \right] > 0$$

where μ_e, μ_w are the partial derivatives of the boundary $\mu = F^{-1}(e) - L^s(w) = 0$ between the Keynesian and inflationary regimes evaluated at the equilibrium:

$$\mu_e = F^{-1}'(e); \quad \mu_w = \delta L^s/\delta w$$

Solving both brackets we derive:

$$[a_{11}\mu_e\mu_w + a_{21}\mu_w^2 - a_{12}\mu_e^2 - a_{22}\mu_e\mu_w] \cdot$$

$$[b_{11}\mu_e\mu_w + b_{21}\mu_w^2 - b_{12}\mu_e^2 - b_{22}\mu_e\mu_w]$$

where a_{ij} and b_{ij} are the elements of J_I and J_K , respectively. After some manipulations we obtain:

$$[(F^{-1} + w k_p\delta L^s/\delta w)(\delta L^s/\delta w - C^d{}_F^{-1}(L^s(w) + w\delta L^s/\delta w))] \cdot$$

$$[(F^{-1} + w k_p\delta L^s/\delta w)(\delta L^s/\delta w - C^d{}_F^{-1}(F^{-1}(e) + w\delta L^s/\delta w))] > 0$$

Taking account of the fact that $L^s(w) = F^{-1}(e)$ at the boundary both vectors are identical and positivity holds.

2. Inelastic labour supply:

For the K-dynamics the Jacobian is now:

$$J_K = \begin{vmatrix} C^{d'}w/F' - 1[-] & C^{d'}F^{-1}(e)[+] \\ w\{k_w/F' - k_p(C^{d'}w/F' - 1)\}[+] & -wk_pC^{d'}F^{-1}(e)[-] \end{vmatrix}$$

and therefore:

$$\text{tr } J_K < 0$$

$$\det J_K = -wk_pC^{d'}F^{-1}(e)(C^{d'}w/F' - 1) -$$

$$-C^{d'}F^{-1}(e)w\{k_w/F' - k_p(C^{d'}w/F' - 1)\} = -wC^{d'}F^{-1}(e)k_w/F' < 0$$

For the I-dynamics we have:

$$J_I = \begin{vmatrix} -1 & C^{d'}L^s[+] \\ wk_w/F'[+] & -wk_pC^{d'}L^s[-] \end{vmatrix}$$

$$\text{tr } J_I < 0$$

$$\det J_I = w(k_p - k_w/F')C^{d'}L^s \begin{cases} < 0, \text{ if } k_p \ll k_w \\ > 0, \text{ if } k_p \gg k_w \end{cases}$$

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Summary

This paper presents a generalization of Varian's original contribution on disequilibrium models characterized by sequential trading. By building up the model on the differentiation of regimes we make the subject more easily comparable to standard disequilibrium models. Compared to the latter (the simultaneous trading models) one obtains a totally new kind of steady states where both the labour and goods market clear but the usual marginal productivity conditions for Walrasian equilibria are not fulfilled. It is shown that the stability of these "non-Walrasian equilibria" depends on the adjustment speeds of prices and wages whereas their existence depends on the assumption of a competitive adjustment on both markets.

Zusammenfassung

Der Aufsatz behandelt eine Verallgemeinerung von H. Varians Originalbeitrag zu Ungleichgewichtsmodellen mit sequentieller Transaktionsstruktur. Da die Präsentation des Modells auf der Unterscheidung der Rationierungsregimes aufbaut, sind die Ergebnisse hier leichter mit denen der häufiger verwendeten Modelle mit simultaner Transaktionsstruktur zu vergleichen. Im Gegensatz zu letzteren erhält man eine völlig neue Art von Steady states, die gekennzeichnet sind durch das Vorliegen eines Gleichgewichts auf Güter- und Arbeitsmarkt, ohne daß die üblichen Marginalbedingungen für Walrasianische Gleichgewichte erfüllt sind. Die Stabilität solcher „Nicht-Walrasianischen Gleichgewichte“ hängt wesentlich von der Reaktionsgeschwindigkeit der Preise und Löhne ab, ihre Existenz beruht auf der Annahme eines kompetitiven Anpassungsprozesses auf beiden Märkten.

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