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Article

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Quantitative Economics

Provided in Cooperation with:

The Econometric Society

Suggested Citation: Ejrnæs, Mette; Browning, Martin (2014) : The persistent–transitory representation for earnings processes, Quantitative Economics, ISSN 1759-7331, The Econometric Society, New Haven, CT, Vol. 5, Iss. 3, pp. 555-581, <https://doi.org/10.3982/QE239>

This Version is available at:

<https://hdl.handle.net/10419/150374>

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The persistent–transitory representation for earnings processes

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We consider the decomposition of shocks to a dynamic process into a persistent and a transitory component. Without additional assumptions (such as zero correlation) the decomposition of shocks into a persistent and transitory component is indeterminate. The assumption that is conventional in the earnings literature is that there is no correlation. The Beveridge–Nelson decomposition that is widely used in time series analysis assumes a perfect correlation. Without restrictions on the correlation, the persistent–transitory decomposition is only set-identified. For reasonable autoregressive moving average (ARMA) parameters the bounds for widely used objects of interest are very wide. We illustrate that these disquieting findings are of considerable practical importance, using a sample of male workers drawn from the Panel Study of Income Dynamics (PSID).

KEYWORDS. Earnings process, persistent–transitory shocks, ARMA model, permanent–transitory shocks.

JEL CLASSIFICATION. C23, D31, J31.

In the literature on income processes, we often seek to decompose shocks into a “persistent” component, which evolves slowly over time, and a “transitory” component, which dies away quickly. This scheme is originally due to Friedman and Kuznets (1954) and has been widely used ever since; recent examples include Blundell, Pistaferri, and Preston (2008), Guvenen (2007), Jappelli and Pistaferri (2010), Meghir and Pistaferri (2004), and Moffitt and Gottschalk (2002). In the panel data literature on dynamic processes, the usual approach is to estimate an ARMA model. The permanent–transitory model has a number of attractive features compared to the general ARMA model. First, it provides a straightforward interpretation of the shocks. Second, it is possible to assess how the variances of the persistent or transitory variance evolve over time (see, e.g., Moffitt and Gottschalk (2002)). Third, the permanent–transitory model make clear how income shock should affect consumption (see Blundell, Pistaferri, and Preston (2008)). In the earnings literature, it is almost universally assumed that the persistent and the

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We thank an editor and two referees for helpful comments. We also thank the Danish Council for independent research in Social Science (FSE) for financial support.

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DOI: 10.3982/QE239

transitory shocks are uncorrelated. This is a very strong assumption. Consider, for example, the loss of a job. This is likely to induce both a persistent effect, if the wage in a new job is lower than in the old job, and a transitory effect, arising from unemployment and a temporary loss of earnings. This in turn gives a positive correlation between the two shocks. This paper lays out the relationship between a generalized version of the permanent–transitory representation and the ARMA approach, taking into account the possibility that the persistent and transitory shocks may be correlated.

From the time series literature, it is known how the Beveridge–Nelson persistent–transitory (PT) decomposition relates to a model similar to the standard PT model (see Morley, Nelson, and Zivot (2003), Oh, Zivot, and Creal (2008), and Proietti (2006)). In the time series literature, the permanent–transitory model is known as the *unobserved component decomposition*, in which the permanent part is the *trend* and the transitory component is named the *cyclical innovation*.

From the time series literature, it is known that for the permanent–transitory (unit root) model, the following statements hold.

- Every permanent–transitory representation has an ARMA representation.
- Every ARMA representation has a permanent–transitory representation with correlation of either -1 or $+1$ between the permanent and transitory shocks.
- The Beveridge–Nelson decomposition also achieves point identification of permanent and transitory shocks by assuming a perfect correlation between the persistent and transitory shocks.

Since evidence is accumulating that no one has an earnings process with a unit root (see, for example, Baker (1997), Guvenen (2009), Browning, Ejrnæs, and Alvarez (2010), and Gustavsson and Osterholm (2010)), we consider a generalization of the usual permanent–transitory model to a persistent–transitory (PT) model. In this model, there are two kinds of shocks: a *persistent shock*, which has an effect that persists forever (albeit with some possible decay if there is no unit root), and a transitory shock, which has only a short run impact (typically one or two periods). For the PT representation, we establish the following results.

- The PT representation with uncorrelated shocks implies restrictions on the parameters from an ARMA model.
- If these restrictions are not rejected, and it is assumed that the persistent and transitory shocks are uncorrelated, then the parameters of the PT representation are point-identified.
- Without an assumption on the correlation, the parameters of the PT representation are only set-identified. The identified set is usually quite wide and, for the leading case, admits *any* correlation between the shocks.
- The ratio of the variances of the persistent and transitory shocks is not point-identified if the shocks are correlated, even if we have a unit root.

- Extra information is required to point-identify persistent and transitory components. However, even if we can observe the reaction of household consumption to the income shock, it requires strong assumptions to actually point-identify the parameters.

These results relate to the set identification for which we characterize the set and derive the bounds on important statistics. Also, we discuss alternative strategies to point-identify the parameters in a PT model.

In this paper, we focus on a ARMA(1, 1), which is equivalent to assuming that the transitory shocks are independent and identically distributed (i.i.d.). For this model, we can derive analytical results. However, a number of the results can be generalized to models that allow for transitory shocks that are (moving average) MA(1).¹ In the [Appendix](#), we show the results for the ARMA(1, 2) model. The paper is organized as follows. Section 2 outlines the PT representation, and Section 3 discusses the identification of shocks for the uncorrelated PT representation and the Beveridge–Nelson decomposition. In Section 3, we present the general case of a PT model with correlated shocks and derive the identified sets for the correlation and for the ratio of variances. Section 4 discusses how additional information may help to point-identify the parameters of the model. Section 5 contains an assessment of the empirical importance of the issues we raise. For a standard data set drawn from the PSID, we show that the bounds on the parameters of interest are very wide. In Section 6, we present our principal conclusion that the PT representation has to be used with extreme caution. Additional data are available in a supplementary file on the journal website, http://qeconomics.org/supp/239/code_and_data.zip.

1. THE PERSISTENT–TRANSITORY REPRESENTATION

Denote log net household income for a given household in period t by y_t . The generalized model² is given by the *persistent–transitory* (PT) representation

$$\begin{aligned} y_t &= \mu + p_t + \tau_t, \\ p_t &= \rho p_{t-1} + \eta_t, \end{aligned} \tag{1}$$

where p_t is the persistent element, η_t is an i.i.d., zero mean shock to the persistent element, and τ_t is an i.i.d., zero mean transitory shock.³ We shall assume that the joint distribution of the shocks is characterized by the mean and covariance matrix, and we denote the (time invariant) variances by σ_η^2 and σ_τ^2 , respectively. The covariance between the shocks is denoted $\sigma_{\eta\tau}$. If $\sigma_{\eta\tau} = 0$, we refer to the model as the *uncorrelated PT* representation.⁴ The parameter ρ governs the persistence of the persistent shock; to avoid excessive special cases, we shall assume $\rho \in (0, 1]$. That is, we assume that there

¹We acknowledge that the literature also contains more advanced models for the transitory shock, for example, an ARMA(1, 1).

²In almost any model of earnings we would want to allow for nonlinear trends. We do not take account of them here to simplify the exposition.

³This transitory shock could also include transitory measurement error.

⁴[Güvenen \(2009\)](#) had a similar formulation of the uncorrelated PT model.

is some positive persistence. The upper bound, $\rho = 1$, gives the widely used *permanent-transitory* representation

$$\begin{aligned} y_t &= \mu + p_t + \tau_t, \\ p_t &= p_{t-1} + \eta_t, \end{aligned} \tag{2}$$

where the parameter μ can, without loss of generality, be suppressed.

2. THE IDENTIFICATION OF PERSISTENT AND TRANSITORY SHOCKS

2.1 The uncorrelated PT representation

To derive the relation between the ARMA model and the persistent-transitory model, we consider an ARMA(1, 1) model

$$y_t = \mu(1 - \rho) + \rho y_{t-1} + \xi_t + \theta \xi_{t-1}, \tag{3}$$

where ξ_t is a zero mean shock with variance ν^2 . If $\rho = 1$, this reduces to a unit root with a MA(1) stochastic component

$$\Delta y_t = \xi_t + \theta \xi_{t-1}.$$

The PT model (1) always has an ARMA representation, which is given by

$$\begin{aligned} y_t &= \mu + (\rho p_{t-1} + \eta_t) + \tau_t \\ &= \mu + \rho(y_{t-1} - \mu - \tau_{t-1}) + \eta_t + \tau_t \\ &= \mu(1 - \rho) + \rho y_{t-1} + \eta_t + \tau_t - \rho \tau_{t-1}. \end{aligned} \tag{4}$$

Comparing (3) and (4), we see that they have the same deterministic components and only differ in the residual term. We can always point-identify ρ and will in the following discussion treat ρ as known.⁵ If (3) and (4) are to be equal, then

$$\xi_t + \theta \xi_{t-1} = \tau_t - \rho \tau_{t-1} + \eta_t. \tag{5}$$

Most researchers who are using the PT representation assume that the shocks are uncorrelated ($\sigma_{\eta\tau} = 0$); we make that assumption in this section. Taking the variance and the first autocovariance gives two equations for the mapping between the two sets of parameters:⁶

$$(1 + \theta^2)\nu^2 = (1 + \rho^2)\sigma_\tau^2 + \sigma_\eta^2, \tag{6}$$

$$\theta\nu^2 = -\rho\sigma_\tau^2. \tag{7}$$

Suppose now that we have estimates of the ARMA parameters (ρ, θ, ν^2) (with $0 < \rho \leq 1$). The following proposition shows the restrictions on these estimates that allow a PT representation.

⁵The parameter ρ is identified from the moment restriction: $E((\Delta y_t - \rho \Delta y_{t-1})(\Delta y_{t-3} - \rho \Delta y_{t-4})) = 0$.

⁶Meghir and Pistaferri (2004) used moments based on a linear combination of variances and first order covariances: $E((\Delta y_{t+1} + \Delta y_t + \Delta y_{t-1})\Delta y_t)$.

PROPOSITION 1. *The ARMA estimates admit an uncorrelated PT representation if and only if*

$$-\rho \leq \theta \leq 0. \tag{8}$$

PROOF. Since $\rho \neq 0$, equations (6) and (7) can be solved to give

$$\sigma_\eta^2 = (\theta + \rho) \left(\theta + \frac{1}{\rho} \right) v^2, \tag{9}$$

$$\sigma_\tau^2 = - \left(\frac{\theta}{\rho} \right) v^2. \tag{10}$$

To ensure $\sigma_\tau^2 \geq 0$, we require $\theta \leq 0$. For $\sigma_\eta^2 \geq 0$, we have $\theta \geq \max(-\rho, -1/\rho) = -\rho$. \square

COROLLARY 2. *If the parameter restriction is satisfied, then the PT parameters are given by (9) and (10).*

Given the parameter restrictions, we have three important special cases:

CASE 1. The permanent–transitory model: The ARMA model with $\rho = 1$, in which case we require $-1 \leq \theta \leq 0$.

CASE 2. There are no persistent shocks if $\theta = -\rho$, in which case the ARMA(1, 1) model is given by

$$\begin{aligned} y_t &= (1 - \rho)\mu + \rho y_{t-1} + \xi_t - \rho \xi_{t-1} \\ \implies y_t &= \mu + \rho^t (y_0 - \mu - \xi_0) + \xi_t. \end{aligned}$$

CASE 3. There are no transitory shocks if $\theta = 0$, in which case the ARMA(1, 1) reduces to an autoregressive (AR(1)) model.

Many users of the PT model assume that the transitory component is itself a MA(1) process for which the corresponding ARMA model is an ARMA(1, 2) process. The restrictions on the three ARMA parameters that allow us to infer a corresponding PT representation are given in Appendix A.

2.2 The Beveridge–Nelson decomposition

In time series analysis, a Beveridge–Nelson (BN) decomposition is used to decompose a nonstationary process ξ_t into a stationary part and a random walk. In this paper, we use the spirit of the BN decomposition to decompose the shock into a transitory and a persistent component; see, for example, Hamilton (1994, Section 17.5). For an ARMA(1, 1), the BN decomposition is

$$\begin{aligned} \tau_t &= -\frac{\theta}{\rho} \xi_t, \\ \eta_t &= \left(1 + \frac{\theta}{\rho} \right) \xi_t \end{aligned} \tag{11}$$

(see Appendix B for the derivation for the more general ARMA(1, 2) case). This decomposition has two notable features. First, it does not impose any restrictions on the ARMA parameters since the implied variances will always be nonnegative:

$$\begin{aligned}\sigma_{\tau}^2 &= \left(\frac{\theta}{\rho}\right)^2 \nu^2, \\ \sigma_{\eta}^2 &= \left(1 + \frac{\theta}{\rho}\right)^2 \nu^2.\end{aligned}\tag{12}$$

The other notable feature of this decomposition is that it does *not* impose a zero covariance between the shocks. Instead, we have

$$\sigma_{\eta\tau} = -\frac{\theta}{\rho} \left(1 + \frac{\theta}{\rho}\right) \nu^2,\tag{13}$$

which implies a correlation coefficient of +1 if $\theta \in (-\rho, 0)$ and -1 if $\theta > 0$ or $\theta < -\rho$.⁷ Thus, the identification of the four BN parameters $\{\rho, \sigma_{\tau}^2, \sigma_{\eta}^2, \sigma_{\eta\tau}\}$ from the three ARMA parameters $\{\rho, \theta, \nu^2\}$ is achieved by implicitly assuming a perfect correlation between τ_t and η_t (as long as $\theta \neq 0$ or $\theta \neq -\rho$).

Given that the two procedures for identifying the permanent and transitory shocks from a ARMA process differ in their treatment of the covariance between the shocks, we turn now to the general case.

3. THE PT REPRESENTATION WITH CORRELATED SHOCKS

For earnings processes, the conventional assumption that the transitory and persistent shocks are uncorrelated is very restrictive. Often, unemployment is considered to be an important shock to the income process. The change in hours of work is often assumed to be a transitory shock, while the changes in hourly wage could be seen as the persistent part of the shock; this clearly induces a positive correlation between the two shocks. Conversely, the correlation would be negative if, for example, a layoff yields a severance pay that is recorded as a temporary increase in earnings. As another example, Hryshko (2013) suggested a negative correlation if being promoted entails losing a bonus. This would be important if the income process is to be used in a consumption simulation model with liquidity constraints.

We now illustrate how the restrictions on the parameters are affected if we allow for correlated shocks. From equation (5), we have two equations that generalize (6) and (7):

$$(1 + \theta^2)\nu^2 = (1 + \rho^2)\sigma_{\tau}^2 + \sigma_{\eta}^2 + 2\sigma_{\eta\tau},\tag{14}$$

$$\theta\nu^2 = -\rho(\sigma_{\tau}^2 + \sigma_{\eta\tau}).\tag{15}$$

⁷This is immediate from (11), which gives a functional linear relationship between the shocks:

$$\tau_t = \frac{-\theta}{\theta + \rho} \eta_t.$$

Allowing for correlated shocks removes the restrictions in Proposition 1, but this comes at the cost of losing point identification. The following proposition gives the solutions for the correlated PT model.

PROPOSITION 3. *The solutions for the correlated PT model are given by*

$$\sigma_\tau^2 \in \left[\left(\frac{\theta}{\rho} \right)^2 \nu^2, \left(\frac{1}{\rho} \right)^2 \nu^2 \right], \tag{16}$$

$$\sigma_\eta^2 = (1 + \theta^2) \nu^2 + \left(\frac{2\theta}{\rho} \right) \nu^2 + (1 - \rho^2) \sigma_\tau^2, \tag{17}$$

$$\sigma_{\eta\tau} = \left(-\frac{\theta}{\rho} \right) \nu^2 - \sigma_\tau^2. \tag{18}$$

PROOF. The restrictions on σ_τ^2 are derived from the fact that the variance of σ_η^2 has to be positive, and the correlation between η and τ has to lie between -1 and $+1$. \square

Morley, Nelson, and Zivot (2003) and Oh, Zivot, and Creal (2008) also showed the lack of (point) identification by showing how the unobserved components decomposition is related to the BN decomposition. The bounds on the variance of the persistent shock and the covariance are given by

$$\sigma_\eta^2 \in \left[\left(\frac{1}{\rho} + \theta \right)^2 \nu^2, \left(1 + \frac{\theta}{\rho} \right)^2 \nu^2 \right], \tag{19}$$

$$\sigma_{\eta\tau} \in \left[-\frac{1}{\rho} \left(\frac{1}{\rho} + \theta \right) \nu^2, -\frac{\theta}{\rho} \left(1 + \frac{\theta}{\rho} \right) \nu^2 \right]. \tag{20}$$

COROLLARY 4. *We can always find a PT representation.*

This corollary follows since the intervals in (16), (19), and (20) are always nonempty if $\theta \in [-1, 1]$ and $\rho \in (0, 1]$. If $\theta \in (-1, 1)$, then point identification fails. For the end points, we do have point identification.

COROLLARY 5. *The PT parameters are all point-identified if and only if $|\theta| = 1$.*

The corollary follows since the interval in (16) is a point if and only if $\theta^2 = 1$. The final corollary is that the ratio of variances is generally not point-identified.

COROLLARY 6. *If $\theta \neq 0$, the bounds on the ratio of persistent variance to the transitory variance are given by*

$$\frac{\sigma_\eta^2}{\sigma_\tau^2} \in \left[(1 + \theta\rho)^2, \left(1 + \frac{\rho}{\theta} \right)^2 \right]. \tag{21}$$

This follows since the ratio of the variances is given by

$$\frac{\sigma_\eta^2}{\sigma_\tau^2} = \frac{(1 + \theta^2)\nu^2}{\sigma_\tau^2} + \frac{2\theta\nu^2}{\rho\sigma_\tau^2} + (1 - \rho^2). \tag{22}$$

The ratio is a decreasing function of σ_τ^2 and the bounds follow from the bounds on σ_τ^2 .

One implication of (21) is that the ratio if the correlation is set to zero (given by the ratio of the variances in (9) and (10)) is the harmonic mean of the end points of the identified set given in (22).

The fact that the ratio between the persistent variance to the transitory variance is not point-identified has also been discussed intensively in the literature on the decomposition of gross domestic product (see Morley, Nelson, and Zivot (2003)). Here they found that the BN decomposition and an uncorrelated PT model give very different results in terms of characterizing the cyclical innovations and the trends.

For any solution, the correlation between the shocks is given by

$$\chi_{\eta\tau} = \frac{\sigma_{\eta\tau}}{\sqrt{\sigma_\eta^2}\sqrt{\sigma_\tau^2}}.$$

This is not point-identified; the following proposition gives the bounds on the identified set.

PROPOSITION 7. *If $-\rho < \theta < 0$, then we can find a solution to (14) and (15) with $\chi_{\eta\tau} \in [-1, 1]$.*

If $\theta > 0$ or $\theta < -\rho$, then we have $\chi_{\eta\tau} \in [-1, \lambda]$, where

$$\lambda = -2 \frac{\sqrt{(\theta/\rho)(\theta + \rho)(\theta + 1/\rho)}}{|1 + \theta^2 + (2\theta)/\rho|}. \tag{23}$$

PROOF. To show this, we assume, without loss of generality, that $\nu^2 = 1$. Suppose $-\rho < \theta < 0$. Taking the lower bound in (16), we have

$$\begin{aligned} \sigma_\tau^2 &= \left(\frac{\theta}{\rho}\right)^2, \\ \sigma_\eta^2 &= (1 + \theta^2) + \left(\frac{2\theta}{\rho}\right) + (1 - \rho^2)\left(\frac{\theta}{\rho}\right)^2 = \left(1 + \frac{\theta}{\rho}\right)^2, \\ \sigma_{\eta\tau} &= -\frac{\theta}{\rho} - \left(\frac{\theta}{\rho}\right)^2 = -\frac{\theta}{\rho}\left(1 + \frac{\theta}{\rho}\right), \\ \chi_{\eta\tau} &= \frac{-\frac{\theta}{\rho}\left(1 + \frac{\theta}{\rho}\right)}{\left(1 + \frac{\theta}{\rho}\right)\left(-\frac{\theta}{\rho}\right)} = 1. \end{aligned}$$

Note that we take $\sqrt{\sigma_\tau^2} = -\theta/\rho$ to ensure that the standard deviation is nonnegative. If we take the upper bound in (16), we can show that $\chi_{\eta\tau} = -1$. The correlation coefficient

is a continuous function of σ_τ^2 between these bounds and, hence, any value of σ_τ^2 corresponding to an arbitrary $\chi_{\eta\tau} \in [-1, 1]$ is a solution. To prove the second statement, both the lower and the upper bound for $\sigma_{\eta\tau}$ in (20) always give a correlation of -1 . We can find the upper bound for the correlation in the case where $\theta > 0$ or $\theta < -\rho$. The correlation is given by

$$\chi_{\eta\tau} = \frac{-(\theta/\rho) - \sigma_\tau^2}{\sqrt{(1 + \theta^2) + (2\theta)/\rho + (1 - \rho^2)\sigma_\tau^2}\sqrt{\sigma_\tau^2}}.$$

We can find the maximum correlation by solving the first order condition. The solution to the first order condition is

$$\sigma_\tau^2 = \frac{\theta(1 + \theta^2 + (2\theta)/\rho)}{\rho(1 + \theta^2 + 2\theta\rho)}.$$

The maximum correlation is

$$\lambda = \chi_{\eta\tau} = -2\frac{\sqrt{(\theta/\rho)(\theta + \rho)(\theta + 1/\rho)}}{|1 + \theta^2 + (2\theta)/\rho|}. \quad \square$$

These are striking results. The first statement implies that if $\theta \in [-\rho, 0]$, there are no bounds on the correlation between shocks.⁸ The case with no correlation corresponds to the usual PT identifying assumption, whereas the case with a correlation of $+1$ corresponds to the BN decomposition. The various cases are illustrated in the top panel of Figure 1. The second statement also has significant implications: if θ is outside the bounds given for the uncorrelated PT representation, then the covariance between the two shocks is always negative, with perfect negative correlation (the BN case) always being a solution. However, having a negative correlation is exactly the case that we would often wish to exclude. The bounds get tighter as θ approaches -1 or $+1$.⁹ In Figure 2, we show the upper bound for four different values of ρ .

Although not all of the parameters of the PT representation are point-identified for the unit root model ($\rho = 1$), the variance of the permanent shock is point-identified with $\sigma_\eta^2 = (1 + \theta)^2\nu^2$; see equation (17). However, the two remaining parameters σ_τ^2 and $\sigma_{\eta\tau}$ are only set identified. In the bottom panel of Figure 1, we show the set identification for a unit root model ($\rho = 1$) with different values of θ and $\nu^2 = 1$. In many applications of the unit root permanent–transitory model, the ratio of the variance of the transitory shocks to the variance of the permanent shocks is of importance. These calculations show that this ratio is not identified and can vary substantially; for example, if $\theta = -0.5$, the ratio can be between unity and 4.¹⁰

To illustrate the importance of the nonidentification, we have constructed two series of transitory and permanent shocks ($\rho = 1$), which generate exactly the same y process.

⁸The coincidence of this with the parameter values that yield the uncorrelated PT representation (Proposition 1) is particular to the ARMA(1, 1) case. For example, it does *not* hold for the ARMA(1, 2) case.

⁹This also follows from Corollary 5, which states that the parameters are point-identified if $|\theta| = 1$.

¹⁰This follows from Corollary 6.

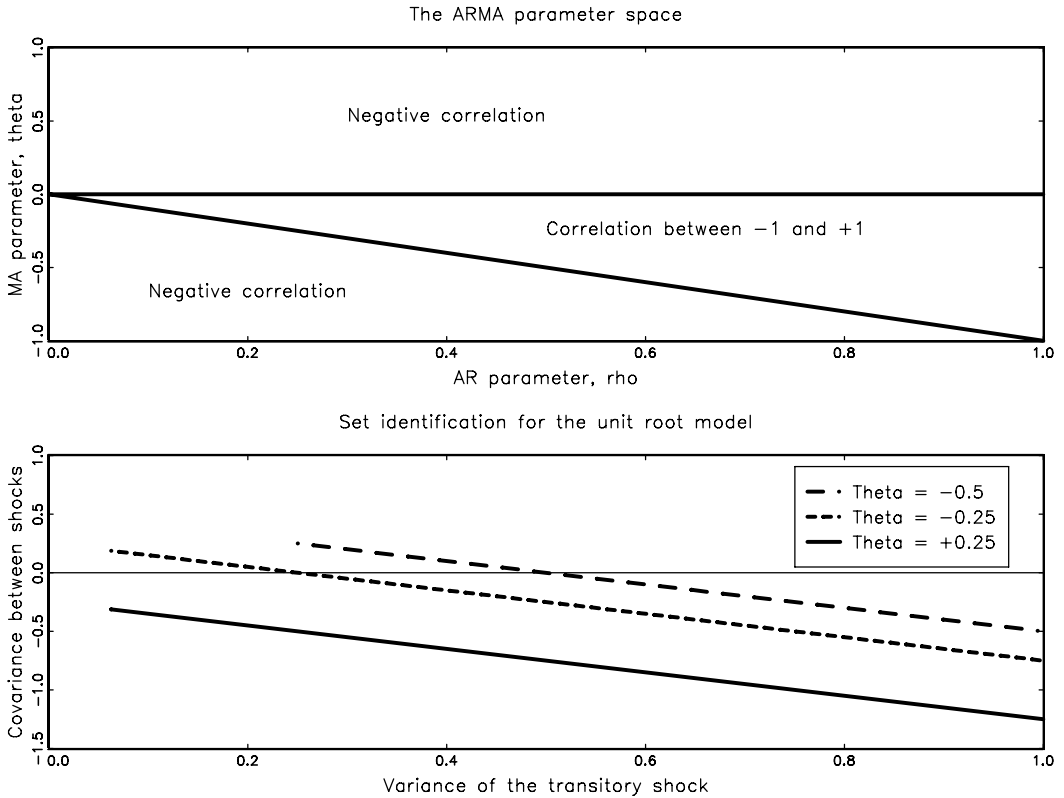


FIGURE 1. Set identification for the ARMA model.

Both sets of transitory and permanent shocks satisfy the conditions stated in Section 1.¹¹ The first set is with uncorrelated transitory and permanent shocks; this is shown in the upper panel of Figure 3. The lower panel of Figure 3 corresponds to transitory and permanent shocks that are perfectly positively correlated (the BN decomposition). The figure shows that two very different set of shocks can generate with exactly the same time series. Note that the variance of the transitory shocks is much lower when the shocks are correlated. This also shows that the nonidentification cannot be resolved by including additional moments of the y -process. If we want to obtain point identification, we need additional assumptions on the process and in some cases also additional moments.

4. ALTERNATIVE IDENTIFICATION STRATEGIES

In the previous section, we showed that in the general PT model with correlated shocks, the variances are only set-identified. Point identification requires either stronger assumptions or additional information or data. In this section, we show how either additional structure imposed by the theory, consumption data, or applying higher order moments can help in point-identifying the parameters. However, all the alternative strategies require strong additional assumptions.

¹¹A description of how the figures are generated can be found in Appendix C.

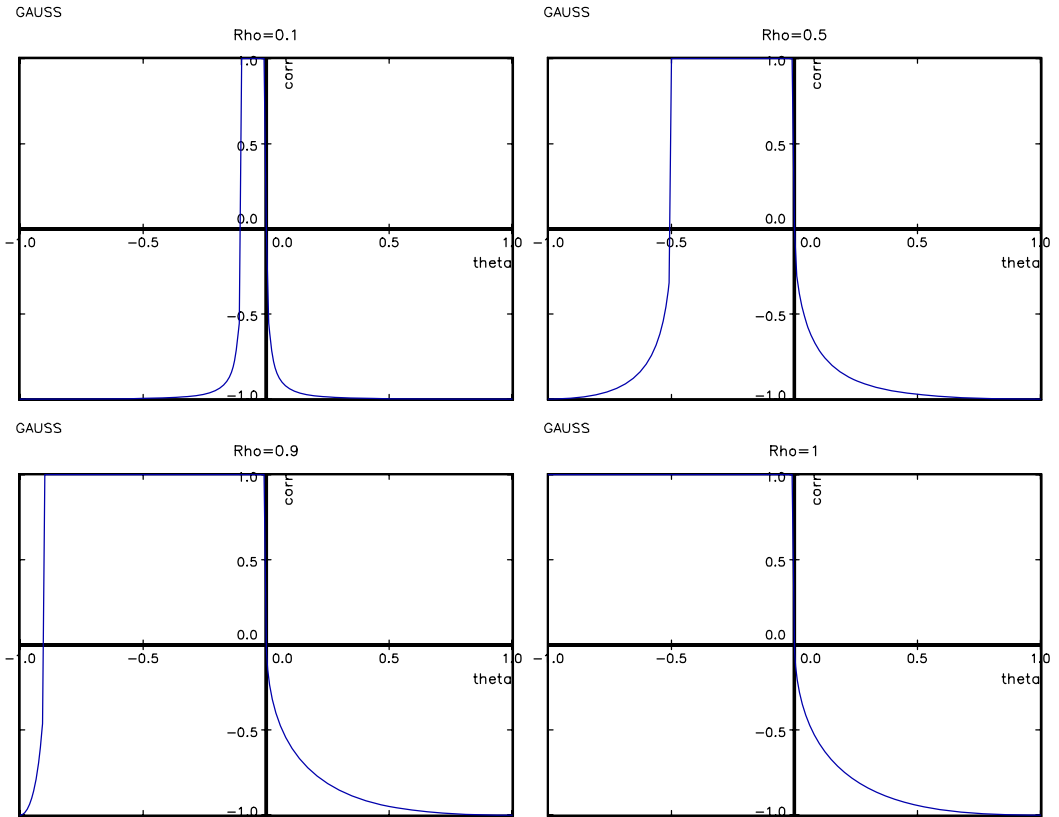


FIGURE 2. Bounds on the correlation for different values of ρ .

4.1 Identification using structural wage and labor supply models

One alternative way to identify the variances is to impose more structure on the earnings process by using theory on labor supply, wages, and earnings. By using a (semi-) structural model on labor supply, wages, and earnings, one can decompose the shocks into transitory and persistent shocks. [Altonji, Smith, and Vidangos \(2013\)](#) explicitly modelled both labor supply and the accumulation of tenure together with wages and earnings. In such a model, the event of losing a job will generate both a transitory and a persistent effect on earnings. The transitory effect arises because of the reduction in labor supply, while the loss of tenure generates a persistent effect on the wage and thereby affects earnings. By using the structure imposed by the wage and labor supply model, one may be able to identify an observable event such as displacement that can be seen as a transitory and/or persistent shock, and to pin down the correlation between the two shocks. Other potentially observable events include promotion, a temporary layoff, a health shock, occupational mobility, and geographical mobility.

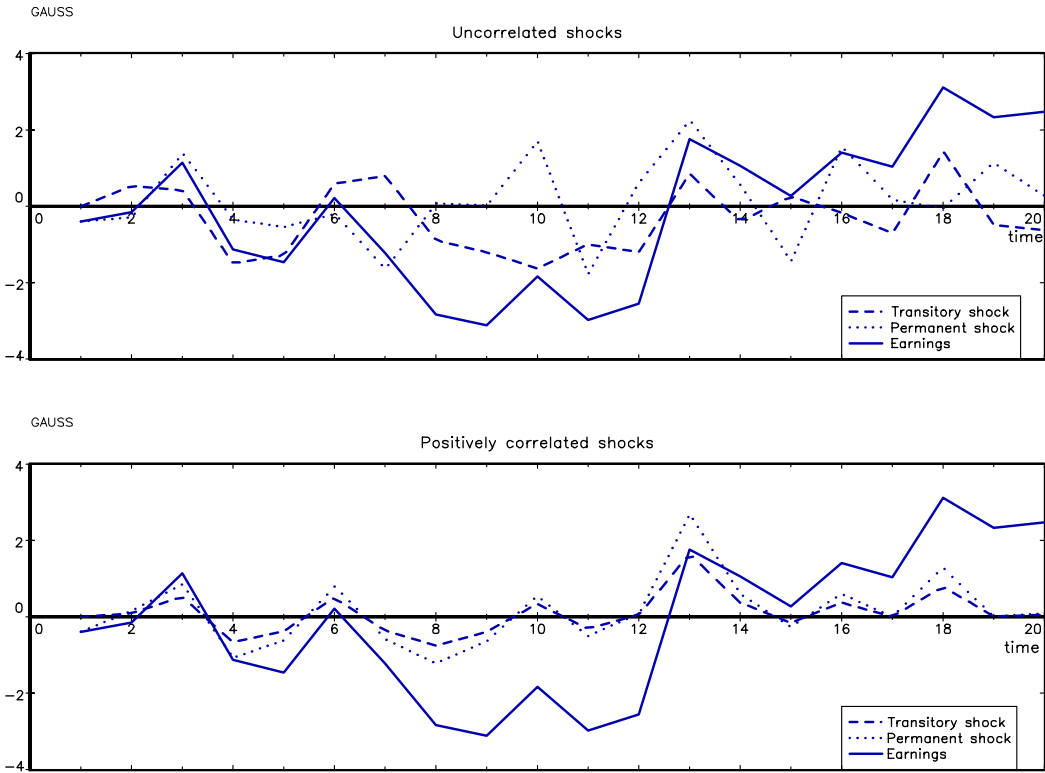


FIGURE 3. PT model with uncorrelated shock and positively correlated shock.

4.2 Information on expectations

An alternative route to point identification was given by Pistaferri (2001), who suggested using additional information in a panel of income and wealth (the Survey of Italian Households’ Income and Wealth (SHIW)). The extra information is a direct question, asked in period t , about expected income growth between t and $t + 1$. This is interpreted as giving a direct measure of $E_t(\Delta y_{i,t+1})$, the conditional mean of income growth conditional on information at time t . Applying this to the permanent–transitory model (2),¹² we have

$$\begin{aligned}
 E_t(\Delta y_{i,t+1}) &= E_t(p_{t+1} + \tau_{t+1} - p_t - \tau_t) \\
 &= E_t(\eta_{t+1} + \tau_{t+1} - \tau_t) = -\tau_t.
 \end{aligned}
 \tag{24}$$

Thus, the conditional mean of income growth yields an estimate of the (negative) transitory shock itself. Given this, the permanent shock can be recovered from

$$\begin{aligned}
 \Delta y_{it} + E_t(\Delta y_{i,t+1}) - E_{t-1}(\Delta y_{it}) &= (\eta_t + \tau_t - \tau_{t-1}) - \tau_t + \tau_{t-1} \\
 &= \eta_t.
 \end{aligned}
 \tag{25}$$

¹²Similar manipulations can be carried out for the persistent–transitory model, but for clarity we here take the model with $\rho = 1$.

Two points arise. First, the informational requirement here is much higher than in most income panels. Even in the SHIW, complications arise since the survey is run every second year, but the expectations question only asks about the next year. Having only $(y_{i,t+2} - y_{it})$ requires supplemental assumptions to identify τ_t directly. More importantly, the method requires that the expectations measure be free of measurement error. It can be shown that measurement error in the expectations response will lead to a downward bias in the estimate of the correlation between the two shocks.

Pistaferri (2001) used these data in a model of consumption and income but did not present an estimate of the correlation between the shocks. Hryshko (2013) adopted the Pistaferri method and used the Italian SHIW to calculate the correlation between the persistent shock and the permanent shock. He found a strong negative correlation.

4.3 Identification using consumption data

If consumption data are available, they can be used to point-identify the parameters. We assume a setup as in Blundell, Pistaferri, and Preston (2008), in which consumption changes depend on the transitory and persistent shock. To simplify the exposition, we assume $\rho = 1$ with a model for income and consumption growth given by

$$\begin{aligned}\Delta y_t &= \eta_t + \tau_t - \tau_{t-1}, \\ \Delta c_t &= \phi \eta_t + \psi \tau_t + v_t.\end{aligned}\tag{26}$$

Using the moment conditions given in the Appendix of Blundell, Pistaferri, and Preston (2008), we have

$$\begin{aligned}E((\Delta y_{it+1} + \Delta y_{it} + \Delta y_{it-1})\Delta y_{it}) &= \sigma_\eta^2, \\ E(\Delta y_{it}\Delta y_{it-1}) &= -\sigma_\tau^2 - \sigma_{\eta\tau}, \\ E((\Delta y_{it+1} + \Delta y_{it} + \Delta y_{it-1})\Delta c_{it}) &= \phi \sigma_\eta^2 + \psi \sigma_{\tau\eta}, \\ E(\Delta c_{it}\Delta y_{it+1}) &= -\psi \sigma_\tau^2 - \phi \sigma_{\tau\eta}, \\ V(\Delta c_{it}) &= \phi^2 \sigma_\eta^2 + \psi^2 \sigma_\tau^2 + 2\phi\psi \sigma_{\tau\eta} + V(v_t).\end{aligned}\tag{27}$$

The equations show that the variance of the persistent shock, σ_η^2 , is point-identified (as it is for any unit root model). Using second through fourth equation in (27), we cannot point-identify $(\sigma_\tau^2, \sigma_{\eta\tau}, \phi, \psi)$.¹³ To point-identify the remaining parameters, we need at least one of the following conditions to be imposed:

$$\begin{aligned}\sigma_{\tau\eta} &= 0, \\ \psi &= 0, \\ \phi &= 0.\end{aligned}$$

¹³Where the consumption information is directly useful for estimating income processes is in identifying measurement error in income. In a model with only income information, classical measurement error is indistinguishable from the transitory shock.

The first restriction is uncorrelated shocks (as assumed in [Blundell, Pistaferri, and Preston \(2008\)](#)). The second restriction states that there is no impact of transitory shocks on consumption; this is the most natural assumption. The third restriction is the counter-intuitive assumption that there is no impact of permanent shocks on consumption.

[Hryshko \(2013\)](#) showed what happens in a consumption model such as (26) if we mistakenly assume zero correlation (as in most studies of income and consumption). In the case where there is a negative correlation ($\sigma_{\tau\eta} < 0$), this will lead to a downward bias in ϕ and ψ . In this case, one will find “excess smoothness of consumption.” On the other hand, if the correlation is positive, the estimates of ϕ and ψ are upward biased. Furthermore, [Hryshko \(2013\)](#) argued that a consumption–income model that allows for negatively correlated permanent and transitory shocks in the income process better explains the “excess smoothness” that is often found in consumption data. [Hryshko \(2013\)](#) used a structural model of income and consumption (with assumption imposed by the structural relation) and was thereby able to point-identify the parameters. He estimated the correlation between the transitory and the permanent income shock to be -0.6 .

4.4 Identification from higher order moments

Finally, we show how assumptions on higher order moments can help to pin down the correlation between the transitory and the persistent shock. Assume that we have a correlated PT model. We can write the persistent and the transitory shocks as

$$\begin{aligned}\eta_t &= z_t + v_t, \\ \tau_t &= az_t + u_t,\end{aligned}$$

where z_t is the common part of the shock. The parameters of the shocks are given by a , σ_z^2 , σ_u^2 , and σ_v^2 , where a identifies the covariance. For known ρ , we have

$$\begin{aligned}y_t - \rho y_{t-1} &= \eta_t + \tau_t - \rho\tau_{t-1} \\ &= (1+a)z_t + v_t + u_t - \rho az_{t-1} - \rho u_{t-1}.\end{aligned}$$

If z_t is drawn from an asymmetric distribution but u and v are symmetric, we can identify the a (and thereby the covariance). Assuming

$$\begin{aligned}E(z_t^3) &\neq 0, \\ E(u_t^3) &= E(v_t^3) = 0,\end{aligned}$$

we have

$$E((y_t - \rho y_{t-1})^3) = ((1+a)^3 - \rho^3 a^3)E(z_t^3),$$

and we can then identify a if $E(z^3)$ is a known or a function of first and second order moments. Similarly we can identify the covariance if either $E(u_t^3)$ or $E(v_t^3)$ was different from zero. Higher order moments such as fourth order moments can also be used, but again it requires that $E(z^4)$ is known or at least a known function of first and second order moments.

5. QUANTITATIVE IMPLICATIONS

5.1 Empirical specification

The analysis above shows that for given values of the ARMA parameters, the bounds on the estimates of the ratio of the persistent and the transitory variances can be quite wide. In this section, we consider whether the possibility of wide bounds is actually realized for a given sample of workers. To quantify the implications, we follow closely [Browning, Ejrnæs, and Alvarez \(2010\)](#) (BEA); readers are referred to that paper for a detailed rationale of the empirical approach we use here. BEA estimated an ARMA(1, 2) model with a quadratic trend and an allowance that the reversion to the trend (the autoregressive parameter) is time dependent. To make our point cleanly, we here take a simplified version of this model with an ARMA(1, 1) process with a linear trend and a time independent AR parameter. For household i ($= 1, \dots, H$) in time t ($= 2, \dots, T$), log earnings, y_{it} , are given by

$$y_{it} = [\mu_i(1 - \rho_i) + \rho_i\alpha_i] + \rho_i y_{i,t-1} + [\alpha_i(1 - \rho_i)](t - 1) + \nu_i \xi_{it} + \theta_i \nu_i \xi_{i,t-1}, \quad (28)$$

where the ξ_{it} 's are independent standard normals. If $\rho_i = 1$, this reduces to a unit root model with a drift equal to α_i . The important element of this specification is that we allow that there is pervasive heterogeneity; that is, all of the parameters ($\mu, \alpha, \rho, \theta, \nu$) are allowed to vary across workers. Apart from the gain in the fit to the data, this is particularly useful in the current context since it allows us to examine the cross-section distribution of the identified sets we are interested in.

To start the process, we model the initial condition by

$$y_{i1} = a_0 + a_1 d_i + c_0 \xi_{i1} + c_1 \xi_{i0}, \quad (29)$$

where d_i is the year of birth of worker i to allow for age/cohort effects in the starting value; again, see BEA.

To model the heterogeneity, we adopt a two factor structure for the parameters in (28).¹⁴ Letting η_{ki} be independent standard normals for $k = 1, 2$ and $i = 1, \dots, H$, we take

$$\begin{aligned} \mu_i &= \phi_1 + \exp(\psi_{11})\eta_{1i}, \\ \nu_i &= \exp(\phi_2 + \psi_{21}\eta_{1i} + \exp(\psi_{22})\eta_{2i}), \\ \alpha_i &= \phi_3 + \psi_{31}\eta_{1i} + \psi_{32}\eta_{2i}, \\ \rho_i &= \ell(\phi_4 + \psi_{41}\eta_{1i} + \psi_{42}\eta_{2i}), \\ \theta_i &= 2\ell(\phi_5 + \psi_{51}\eta_{1i} + \psi_{52}\eta_{2i}) - 1, \end{aligned} \quad (30)$$

¹⁴This specification is the end result of a specification search that began with a five factor model. The $\chi^2(6)$ test statistic for dropping three factors was 3.6.

where $\ell(x) = \exp(x)/(1 + \exp(x))$ so that $\rho \in (0, 1)$ and $\theta \in (-1, 1)$.¹⁵ This structure allows for a good deal of heterogeneity with dependence across parameters. In all, we have 18 parameters to estimate: (a_0, a_1, c_0, c_1) from (29) and $(\phi_1, \dots, \phi_5, \psi_{11}, \dots, \psi_{52})$.

5.2 Estimation method

We use indirect inference to estimate the parameters that govern the distribution of parameters. Gouriéroux, Phillips, and Yu (2010) provided a strong defense for using indirect inference to estimate the parameters of a parametric dynamic panel model. The main motivation is that this method provides a bias reduction estimation method to allow for the well known bias in dynamic panel data estimation.

Indirect inference requires us to specify auxiliary parameters (a.p.'s) that can be calculated on the data at hand and on simulated data that purport to model the empirical generating process. Indirect inference chooses parameter estimates to minimize the distance between the two sets of auxiliary parameters. In our estimation procedure, we rely heavily on auxiliary parameters that are based on regressions for each worker; this follows the literature on testing for unit roots in panel data (see Levin, Lin, and Chu (2002) or Im, Pesaran, and Shin (2003)). Of course, the estimates from individual regressions based on short time series do not give unbiased estimates of anything of interest. However, the use of the same auxiliary process for the data sample and the simulated sample introduces similar biases in both; it is in this sense that Gouriéroux, Phillips, and Yu (2010) saw indirect inference as a bias reduction method. As well as auxiliary parameters based on individual regressions, we also use moments that have been widely used in the earnings process literature. Details of the construction of the auxiliary parameters are given in Appendix D. These statistics provide a rich description of the time series and cross-section features of the original data. In all, we have 42 auxiliary parameters for the 24 distribution parameters to be estimated.

So as to estimate, we have to simulate from the model in (28) and (29). To reduce the impact of the misspecification in the initial value, we start the process at $t = -3$ (using (29)) and then recursively generate $y_{i,t-2}, \dots, y_{iT}$. We then discard the first three observations for each worker. Additionally, we have to allow that the panel we use is unbalanced, with some workers not in the first observation period and some dropping out before the final observation period. To take account of this, we replicate the actual data with the values not observed for each replicated household masked out as in the data. For example, if household i is only observed for periods 4–16, then for each replicated household for that particular household, we simulate from 1 to T and set the periods 1–3 and 17– T as missing. Thus the simulated data have the same imbalance as the original data.

5.3 Sample

We estimate using a subsample of the sample drawn from the PSID as used in Meghir and Pistaferri (2004) (MP). This is an unbalanced sample of male workers followed from

¹⁵The restriction on ρ explicitly rules out that anyone has a unit root. Guvenen (2009) and Browning, Ejrnæs, and Alvarez (2010) provided evidence that this is not rejected if we allow for heterogeneous deterministic trends (the α_i 's).

(survey years) 1968 to 1993. We select on being aged between 25 and 57, and being in the sample for at least 9 years. The original MP sample consists of 2069 individuals, with 31,631 observations. The earnings variable includes all income from labor, deflated to the year 1992. For individuals in this sample, the variables we use to control for observable heterogeneity are education, race, age, and birth cohort. We deal with some of the observable heterogeneity by stratifying on education and working by selecting whites who have a high school education. This gives a sample size of 749, with workers being observed between 9 and 26 years, which gives in total 11,503 observations. Following MP, we run a first round regression of log earnings on year dummies and age dummies, and treat the residuals from this regression as earnings in (28).

5.4 Empirical estimates

The $\chi^2(24)$ test statistic for the overidentifying restrictions has a value of 38.3 with an associated p -value of 2.9%. Although marginal, the fit is quite good and we deem it unlikely that a marginal improvement in the fit from generalizing the model would change the qualitative implications below. The $\chi^2(4)$ test statistic for shutting down the heterogeneity in (ρ, θ) (that is, imposing $\psi_{41} = \psi_{42} = \psi_{51} = \psi_{52}$) is 24.0, which indicates that there is significant heterogeneity in the ARMA parameters. Table 1 presents the marginal distributions of the heterogeneous model parameters.¹⁶ As can be seen, all of the parameters are quite dispersed. Since these distributions are similar to those in BEA, we only discuss the ARMA parameters that are the focus of this study. The AR parameter is quite dispersed, with no significant bunching near unity (see BEA for details of how this relates to a test for any one having a unit root). The MA parameters are mostly negative, but a considerable proportion have a positive value.

Table 2 presents the correlations of the heterogeneous model parameters. This shows that there is considerable dependence between the model parameters; this is to be expected in a low dimensional factor model. Of particular interest for us, the ARMA parameters ρ and θ are strongly negatively correlated; this is also clear from the scatter plot of the estimates in Figure 4. Most of the estimates have $-\rho < \theta < 0$, so that for a majority of the population, any correlation between -1 and $+1$ is admissible. However, for a significant proportion (mostly with $\theta > 0$), the PT representation only allows a negative correlation or no representation if we impose zero correlation.

TABLE 1. Marginal distributions of model parameters.

Parameter	10%	25%	50%	75%	90%
μ	-0.31	-0.20	-0.09	0.02	0.13
α	-0.10	-0.07	-0.03	0.00	0.04
ρ	0.48	0.66	0.82	0.91	0.96
θ	-0.48	-0.37	-0.22	-0.07	0.07
ν	0.08	0.11	0.16	0.22	0.31

¹⁶Estimates of the parameters in (29) and (30) are given in Appendix D.2.

TABLE 2. Correlations of model parameters.

	μ	α	ρ	θ	ν
μ	1	–	–	–	–
α	–0.95	1	–	–	–
ρ	–0.6	0.39	1	–	–
θ	0.26	–0.15	–0.57	1	–
ν	0.93	–0.88	–0.58	0.24	1

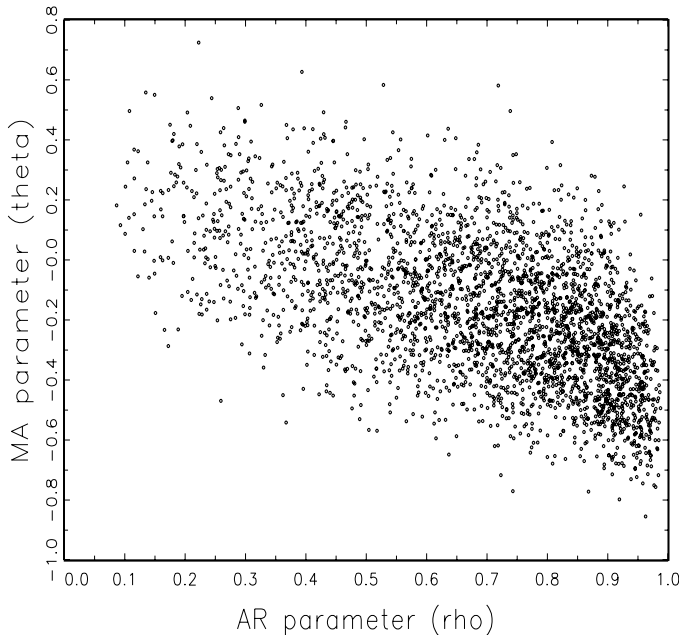


FIGURE 4. The joint distribution of the ARMA parameters.

5.5 Implications

Finally we can turn to the implications of these estimates for the identification issues we are primarily concerned with. We first present the implications for the mean of the ARMA parameters, which are $\rho = 0.69$ and $\theta = -0.17$. These estimates are unremarkable for (stationary) models that assume no heterogeneity in the ARMA coefficients. The restrictions from Proposition 1 require $-\rho \leq \theta \leq 0$ so that at the mean, the ARMA model admits a PT representation with zero correlation between the shocks. It thus follows that the identified set for the correlation between the persistent shock and the transitory shock is $[-1, 1]$. With these mean values, the identified set for the ratio of the variance of the persistent shock to the variance of the transitory shock is $[0.70, 6.96]$ so that the highest possible ratio is 10 times the lowest possible. If we set the correlation to zero, we have a point estimate of 2.21 (the harmonic mean of the end points of the identified set). Comparing to the literature, which all assume zero correlation, [Gottschalk and Moffitt \(1994\)](#) and [Guvenen \(2009\)](#) also found the ratio of the variances close to 2, while [Meghir](#)

TABLE 3. The distribution of identified sets.

	10%	25%	50%	75%	90%
$\sigma_{\eta}^2/\sigma_{\tau}^2$	1.2	3.1	8.5	38.0	219.6
Upper bound for correlation	-0.91	-0.82	-0.65	-0.48	-0.34

and Pistaferri (2004), Blundell, Pistaferri, and Preston (2008), and Hryshko (2012) found the ratio in the range 0.5–1.¹⁷

Having allowed for heterogeneous ARMA parameters, we can also display the distributions of the identified sets. For all values of (ρ, θ) , we can construct the ratio of the variance of the persistent shock to the variance of the transitory shock from (21). The first row of Table 3 shows the distribution of the ratio of the upper bound to the lower bound for the ratio of variances. The median value is 8.5, which is slightly lower than the ratio given above at the mean of the estimates, but still very high. Since the upper bound is unbounded above ($\sigma_{\eta}^2/\sigma_{\tau}^2 \rightarrow \infty$ as $\theta \rightarrow 0$), we see very high values for the upper tail. The second row of Table 3 shows the upper bound on the correlation for the 17.7% of the sample who do not satisfy the condition that $-\rho \leq \theta \leq 0$ (as shown above, the lower bound is always -1). Even in the tail of this distribution, the identified set is quite wide.

6. CONCLUSION

This paper has derived the relationship between ARMA estimates of a dynamic model and the representation that allows us to identify a transitory and a persistent component for shocks. The main result is that the decomposition is critically dependent on the assumed correlation between the two shocks. In the absence of such an assumption, the parameters of the PT representation are only set-identified. Moreover, there are no bounds on the correlation between the shocks if we have a moderate negative MA parameter. A quantitative assessment of the seriousness of this lack of point identification suggests that it is very serious with very wide bounds for both the correlation and the ratio of the variances of the shocks.

The nonidentification has important implications for analyses made on earnings dynamics. For example, analyses that examine how the variance of transitory and persistent shocks have changed over time are only valid under the assumption that the correlation remains unchanged. Thus, a finding on a time varying ratio of variances (see Moffitt and Gottschalk (2012)) could be generated by constant variances and time varying covariances. An apparent increase in the transitory variance in an uncorrelated PT model could instead have been induced by a decrease in the correlation. Also analyses that examine the ability of households to smooth consumption are heavily influenced by the assumption on the correlation with wildly varying values for the variance of the persistent shock relative to the transitory shock. Given this, it is advisable for future re-

¹⁷The ratio when we assume zero correlation is very sensitive to the value taken for ρ .

searchers to present identified sets for outcomes of interest that explicitly take into account the nonidentification of the correlation between the shocks.

APPENDIX A: THE ARMA(1, 2) CASE WITH ZERO COVARIANCE

In this appendix, we consider the ARMA(1, 2) model. For simplicity, we assume a process with mean zero and no trend, and again we assume that $0 < \rho \leq 1$. The ARMA(1, 2) is given by

$$y_t = \rho y_{t-1} + \xi_t + \theta_1 \xi_{t-1} + \theta_2 \xi_{t-2}. \tag{31}$$

This model can, under certain parameter restrictions, be written as a PT model with uncorrelated shocks, where the transitory shock τ_t is an MA(1) process with parameter λ :

$$\begin{aligned} \xi_t + \theta_1 \xi_{t-1} + \theta_2 \xi_{t-2} &= \varepsilon_t + \lambda \varepsilon_{t-1} - \rho(\varepsilon_{t-1} + \lambda \varepsilon_{t-2}) + \eta_t \\ &= \varepsilon_t + (\lambda - \rho)\varepsilon_{t-1} - \rho\lambda \varepsilon_{t-2} + \eta_t. \end{aligned}$$

Taking covariances gives three equations that give the mapping between the two sets of parameters:

$$\begin{aligned} (1 + \theta_1^2 + \theta_2^2)\nu^2 &= (1 + (\lambda - \rho)^2 + (\rho\lambda)^2)\sigma_\varepsilon^2 + \sigma_\eta^2, \\ \theta_1(1 + \theta_2)\nu^2 &= (\lambda - \rho)(1 - \lambda\rho)\sigma_\varepsilon^2, \\ \theta_2\nu^2 &= -\lambda\rho\sigma_\varepsilon^2. \end{aligned}$$

We consider the unit root case, $\rho = 1$, and we restrict θ_1 , θ_2 , and λ to lie within -1 and 1 . The equations are given by

$$\begin{aligned} (1 + \theta_1^2 + \theta_2^2)\nu^2 &= 2(1 + \lambda^2 - \lambda)\sigma_\varepsilon^2 + \sigma_\eta^2, \\ \theta_1(1 + \theta_2)\nu^2 &= -(\lambda - 1)^2\sigma_\varepsilon^2, \\ \theta_2\nu^2 &= -\lambda\sigma_\varepsilon^2. \end{aligned}$$

From the middle equation, we can immediately see that θ_1 has to be negative; θ_2 can be positive, zero, and negative depending on the sign of λ . We now consider three cases:

	$\lambda < 0$	$\lambda = 0$	$\lambda > 0$
θ_1	< 0	< 0	< 0
θ_2	> 0	$= 0$	< 0

PROPOSITION 8. *The ARMA(1, 2) model with $\rho = 1$ has an uncorrelated PT representation if the parameters θ_1 and θ_2 satisfy the restrictions*

$$\begin{aligned} -1 \leq \theta_1 \leq \frac{-4\theta_2}{(1 + \theta_2)}, \quad \theta_2 > 0, \\ \theta_1 \leq 0, \quad \theta_2 \leq 0. \end{aligned}$$

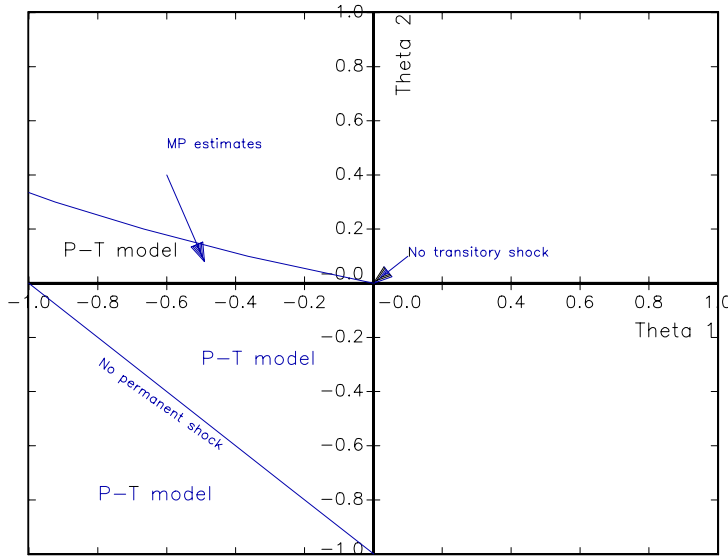


FIGURE 5. The restrictions of the ARMA(1, 2) parameters when $\rho = 1$.

The PT parameters are given, if $\theta_2 \neq 0$, by

$$\sigma_\eta^2 = (1 + \theta_1 + \theta_2)^2 v^2,$$

$$\lambda = \frac{1}{2\theta_2} (\theta_1 + 2\theta_2 + \theta_1\theta_2 \pm \sqrt{\theta_1(\theta_2 + 1)(\theta_1 + 4\theta_2 + \theta_1\theta_2)}),$$

$$\sigma_\varepsilon^2 = \frac{\theta_2 v^2}{-\lambda} = -\frac{2\theta_2^2 v^2}{(\theta_1 + 2\theta_2 + \theta_1\theta_2 \pm \sqrt{\theta_1(\theta_2 + 1)(\theta_1 + 4\theta_2 + \theta_1\theta_2)})}.$$

If $\theta_2 = 0$, then $\lambda = 0$ and $\sigma_\varepsilon^2 = -\theta_1 v^2$.

In Figure 5, we show the parameter space of ARMA(1, 2) with $\rho = 1$ that is consistent with a PT representation. On the figure, we also show the ARMA(1, 2) parameter implied by the Meghir and Pistaferri (2004) estimation.

COROLLARY 9. *In the unit root model $\rho = 1$, there are no transitory shocks if $\theta_1 = \theta_2 = 0$. If $\theta_1 + \theta_2 = -1$, there are no permanent shocks.*

Moving on to the more general case with $\rho \in (0, 1)$, and θ_1, θ_2 , and $\lambda \in (-1, 1)$, we now consider the different cases.

From Table 4, one can conclude that if both θ_1 and θ_2 are positive, then the earnings process cannot be represented with a standard (uncorrelated) PT model.

TABLE 4. The parameter space of the ARMA model.

	$\lambda < \rho$	$\lambda = \rho$	$\lambda > \rho$
$\lambda < 0$	$\theta_1 < 0$ $\theta_2 > 0$		
$\lambda = 0$	$\theta_1 < 0$ $\theta_2 = 0$		
$\lambda > 0$	$\theta_1 < 0$ $\theta_2 < 0$	$\theta_1 = 0$ $\theta_2 < 0$	$\theta_1 > 0$ $\theta_2 < 0$

APPENDIX B: THE BEVERIDGE–NELSON APPROACH

We also use the Beveridge–Nelson approach to decompose the ARMA(1, 2) into a persistent shock and a transitory shock.¹⁸ However, it turns out that this decomposition is different than the PT model with uncorrelated shock. For simplicity, we assume a process with mean zero and no trend, and again we assume that $0 < \rho \leq 1$. The ARMA(1, 2) is given by

$$y_t = \rho y_{t-1} + \xi_t + \theta_1 \xi_{t-1} + \theta_2 \xi_{t-2}. \tag{32}$$

By using the time series representation, we can write the ARMA model as

$$A(L)y_t = \theta(L)\xi_t,$$

where

$$A(L) = 1 - \rho L \quad \text{and} \quad \theta(L) = 1 + \theta_1 L + \theta_2 L^2.$$

We use the BN decomposition of a general ARMA model into a persistent and a transitory component. Under the condition that the $A(L)$ and $\theta(L)$ have no common roots, we can write $y_t = (A(L))^{-1} \theta(L) \xi_t$. Since the root in the $A(L)$ is $r = 1/\rho$, the condition of no common roots implies that $1/\rho$ cannot be a root in $\theta(L)$:

$$\theta(1/\rho) = 1 + \theta_1/\rho + \theta_2/\rho^2 \neq 0.$$

We then make the decomposition as

$$y_t = (A(L))^{-1} \theta(L) \xi_t = \sigma (A(L))^{-1} \xi_t + (A(L))^{-1} (\theta(L) - \sigma) \xi_t,$$

where σ is a scalar that indicates how much of the shock can be attributed to a persistent shock. The first term $\sigma (A(L))^{-1} \xi_t$ is the persistent part of the process:

$$\sigma (A(L))^{-1} \xi_t = \sigma \sum_{i=0}^{\infty} \rho^i \xi_{t-i}.$$

¹⁸The decomposition is strictly speaking not a BN decomposition, but does use the same ideas.

So as to make the second term a transitory shock, we require that this process is an MA(q) process. To be this, we need that $A(L)$ and $(\theta(L) - \sigma)$ have common roots; that is, that $1/\rho$ should be a root in $(\theta(L) - \sigma)$:

$$\begin{aligned} 1 - \sigma + \theta_1/\rho + \theta_2/\rho^2 &= 0, \\ 1 + \theta_1/\rho + \theta_2/\rho^2 &= \sigma. \end{aligned}$$

Using the factorization of $(\theta(L) - \sigma)$, we get

$$(\theta(L) - \sigma) = (L - 1/\rho)(\theta_1 + \theta_2(L + 1/\rho)).$$

The transitory shock is given by

$$\begin{aligned} (A(L))^{-1}(\theta(L) - \sigma)\xi_t &= (1 - \rho L)^{-1}(L - 1/\rho)(\theta_1 + \theta_2(L + 1/\rho))\xi_t \\ &= -1/\rho(\theta_1 + \theta_2(L + 1/\rho))\xi_t \\ &= -1/\rho * (\theta_1 + \theta_2/\rho)\xi_t - 1/\rho * \theta_2\xi_{t-1}. \end{aligned}$$

The persistent shock is given by

$$\eta_t = (1 + \theta_1/\rho + \theta_2/\rho^2)\xi_t.$$

The decomposition is given by

$$y_t = (1 + \theta_1/\rho + \theta_2/\rho^2) \sum_{i=0}^{\infty} \rho^i \xi_{t-i} - 1/\rho * (\theta_1 + \theta_2/\rho)\xi_t - 1/\rho * \theta_2\xi_{t-1}.$$

If we restrict the scalar to $0 \leq \sigma \leq 1$, which implies that σ can be interpreted as the part of the shock that is the persistent part, we can derive the restrictions on the ARMA parameters:

$$\begin{aligned} 0 < 1 + \theta_1/\rho + \theta_2/\rho^2 < 1, \\ -1 < \theta_1/\rho + \theta_2/\rho^2 < 0. \end{aligned} \tag{33}$$

If $1 + \theta_1/\rho + \theta_2/\rho^2 = 0$, we have no persistent shock, and if $1 + \theta_1/\rho + \theta_2/\rho^2 = 1$, we have no transitory shocks.

The main difference between the time series approach and the PT model is that the time series approach does not require that the persistent and the transitory shocks are uncorrelated. In the time series approach, the covariance of the shocks is given by

$$\begin{aligned} \text{Cov}(\eta_t, 1/\rho * (\theta_1 + \theta_2/\rho)\xi_t - 1/\rho * \theta_2\xi_{t-1}) \\ = -1/\rho * (\theta_1 + \theta_2/\rho) * (1 + \theta_1/\rho + \theta_2/\rho^2) * \nu^2. \end{aligned}$$

Given the restrictions (33), the covariance will always be positive, but otherwise the covariance will be negative.

APPENDIX C: DECOMPOSITION OF SHOCKS

We start by simulating a PT model with uncorrelated shocks. The parameters of this model is given by $(\rho, \sigma_\eta^2, \sigma_\tau^2)$. We simulate a sequence of independent transitory and persistent shocks, $(\tau_1^0, \dots, \tau_T^0)$ and $(\eta_1^0, \dots, \eta_T^0)$, and the process is given by

$$\begin{aligned} y_t &= p_t + \tau_t^0, \\ p_t &= \rho p_{t-1} + \eta_t^0, \quad t > 1, \\ p_1 &= \eta_1^0, \quad \tau_1^0 = 0. \end{aligned}$$

We know that we can write this process as

$$\begin{aligned} y_t - \rho y_{t-1} &= \xi_t + \theta \xi_{t-1}, \quad t > 1, \\ y_1 &= \xi_1, \end{aligned}$$

where the parameters in the model are ρ , θ , and ν^2 can be found as

$$\begin{aligned} \theta &= \frac{1}{2\rho\sigma_\tau^2} \left[-(1 + \rho^2)\sigma_\tau^2 - \sigma_\eta^2 + \sqrt{(1 - \rho^2)^2\sigma_\tau^4 + \sigma_\eta^4 + 2(1 + \rho^2)\sigma_\tau^2\sigma_\eta^2} \right], \\ \nu^2 &= (-\rho/\theta) * \sigma_\tau^2. \end{aligned}$$

We can then recursively determine the shocks in the ARMA(1, 1) model:

$$\begin{aligned} \xi_1 &= y_1, \\ \xi_t &= y_t - \rho y_{t-1} - \theta \xi_{t-1}, \quad t > 1. \end{aligned}$$

We can then define the transitory and the persistent shock by using the BN decomposition:

$$\begin{aligned} \eta_1^P &= y_1, \quad \tau_1^P = 0, \\ \tau_t^P &= \frac{-\theta}{\rho} \xi_t, \quad t > 1, \\ \eta_t^P &= \left(1 + \frac{\theta}{\rho} \right) \xi_t, \quad t > 1. \end{aligned}$$

Figure 3 is generated for $\rho = 1$, $\sigma_\tau^2 = \sigma_\eta^2 = 1$.

APPENDIX D: EMPIRICAL DETAILS

D.1 *The auxiliary parameters*

We have five heterogeneous model parameters $(\mu, \alpha, \rho, \theta, \nu)$. We wish to define Individual Regression Based (IRB) a.p.'s that are "bound" to these.

The original data are denoted y_{it} for $i = 1, \dots, H$ and $t = t_{1i}, \dots, t_{Ti}$, where the latter values are the first and last periods for i .

STEP 1. Take deviations about the mean: $x_{it} = y_{it} - \bar{y}_i$.

STEP 2. Detrend by regressing the deviations about the mean on a trend,

$$x_{it} = b_{1i} * t + u_{1it}, \quad (34)$$

and record the estimated residuals $\hat{u}_{1it} = x_{it} - \hat{b}_{1i}t$ for $t = t_{1i}, \dots, t_{Ti}$.

STEP 3. Regress these residuals on their lagged values,

$$\hat{u}_{1it} = b_{2i}\hat{u}_{1i,t-1} + u_{2it}, \quad (35)$$

and record the estimated residuals $\hat{u}_{2it} = \hat{u}_{1it} - \hat{b}_{2i}\hat{u}_{1i,t-1}$ for $t = t_{1i} + 1, \dots, t_{Ti}$. The estimated b_{2i} is bound to the autoregressive parameter.

STEP 4. Perform the regression

$$x_{it} - b_{2i}x_{it-1} = b_{3i} * t + u_{3it}. \quad (36)$$

The estimated \hat{b}_{3i} 's are bound to the trend parameter.

STEP 5. Calculate

$$\hat{b}_{4i} = (1 - \hat{b}_{2i})\bar{y}_i - \hat{b}_{3i} * t. \quad (37)$$

The parameters are bound to the intercept of the process.

STEP 6. Calculate residuals for $t = t = t_{1i} + 1, \dots, t_{Ti}$,

$$\hat{u}_{3it} = y_{it} - \hat{b}_{4i} - b_{2i} * y_{i,t-1} - \hat{b}_{3i} * t, \quad (38)$$

and record the standard deviation and autocorrelation (denoted \hat{b}_{5i} and \hat{b}_{6i} , respectively). These are for the variance and the MA parameters.

The a.p.'s for the five model parameters $(\mu, \alpha, \rho, \theta, \nu)$ are, respectively, $(\hat{b}_{4i}, \hat{b}_{3i}, \hat{b}_{2i}, \hat{b}_{6i}, \hat{b}_{5i})$. We also record the initial value y_{i1} , and take means, standard deviations, and correlations of the six values for each worker.

D.2 Parameter estimates

TABLE 5. Parameter estimates.

	Estimate	Std. Error	<i>t</i> -Value
ϕ_1	-0.089	0.314	0.283
ϕ_2	-1.844	0.033	55.668
ϕ_3	-0.033	0.063	0.517
ϕ_4	1.518	0.402	3.772
ϕ_5	-0.448	0.110	4.088
c_0	-3.581	0.997	3.590
c_1	0.010	6.372	0.002
ψ_{11}	-1.776	1.084	1.639
ψ_{21}	0.488	0.105	4.649
ψ_{22}	-1.620	1.217	1.331
ψ_{31}	-0.051	0.029	1.771
ψ_{32}	-0.017	0.032	0.523
ψ_{41}	-1.096	0.504	2.176
ψ_{42}	-0.610	0.710	0.860
ψ_{51}	0.367	0.179	2.050
ψ_{52}	-0.303	0.212	1.428
a_0	-0.026	0.442	0.059
a_1	0.313	0.850	0.369

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