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Working Paper

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CHOPE Working Paper, No. 2015-16

Provided in Cooperation with:

Center for the History of Political Economy at Duke University

Suggested Citation: Plassard, Romain (2015) : The origins, development, and fate of Clower's 'stock-flow' general-equilibrium program, CHOPE Working Paper, No. 2015-16, Duke University, Center for the History of Political Economy (CHOPE), Durham, NC

This Version is available at:

<https://hdl.handle.net/10419/149743>

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THE ORIGINS, DEVELOPMENT, AND FATE OF CLOWER'S
'STOCK-FLOW' GENERAL-EQUILIBRIUM PROGRAM

BY

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CHOPE WORKING PAPER No. 2015-16

DECEMBER 2015



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The origins, development, and fate of Clower's 'stock-flow' general-equilibrium program¹

Abstract:

Before becoming the hallmark of macroeconomics *à la* Wynne Godley, the 'stock-flow' analysis was already developed in microeconomics and general equilibrium theory. Basically, the goal was to study the formation of economic plans and the determination of market prices when individuals were supposed to consume, produce, and hold commodities. It is acknowledged that Robert W. Clower was a central figure in this theoretical context. Yet, for both his contemporaries and for historians, his contributions remained essentially technical. No attention was paid to the theoretical project underlying the statics and dynamics analyses of his 'stock-flow' price theory. My paper aims to fill this gap. In light of his doctoral dissertation, I show that the elaboration of 'stock-flow' market models was part of a project aiming at offering sound microfoundations to a Keynesian business cycle model. I analyze the origins of this microfoundation program, trace its development, and discuss its fate.

JEL codes: B2, E12, E32, D4

Keywords: microfoundations of macroeconomics, trade cycle, stock-flow analysis, Bushaw and Clower.

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Introduction:

Before becoming the hallmark of macroeconomics *à la* Wynne Godley, the ‘stock-flow’ analysis was already developed in microeconomics and general equilibrium theory.² Basically, the goal was to study the formation of economic plans and the determination of market prices when individuals were supposed to consume, produce, and hold commodities for future disposal. It is acknowledged that Robert W. Clower was a central figure in this context because of his pioneering and extensive works on ‘stock-flow’ market models (Glenn W. Harrison, 2008). Yet, for both his contemporaries and for historians, his contributions remained essentially technical. No attention was paid to the theoretical project underlying the statics and dynamics analyses of his ‘stock-flow’ price theory. My paper aims to fill this gap. To do so, I make an extensive use of the archival material found at Duke University. Particular attention will be given to the doctoral dissertation that Clower prepared under John R. Hicks’s supervision at Oxford, from 1949 to 1952. This unpublished manuscript is the central piece needed to solve the theoretical puzzle. The elaboration of ‘stock-flow’ market models was part of a project aiming at offering sound microfoundations to a business cycle theory inspired by John M. Keynes (1936) and supposed to include the models of Roy F. Harrod (1939) and Hicks (1950) as special cases. Since the ‘stock-flow’ structure was used to support this unification, the incorporation of the relations of stock and flows in standard microeconomics and in the general equilibrium theory became the cornerstone of the project. My paper offers a detailed presentation of the resulting ‘stock-flow’ general equilibrium program of microfoundations.³ I analyze its origins, trace its development and discuss its fate.

² Godley is often considered as one of the founding fathers of the “stock-flow consistent models” (Claudio H. Dos Santos and Gennaro Zezza, 2004).

³ The expression “general equilibrium program” was coined by Kevin D. Hoover (2012) to characterize the way Hicks and his followers addressed the microfoundations of macroeconomics.

The ‘stock-flow’ market models were developed in partial and general equilibrium frameworks in a series of papers published in the early 1950s, and in a book titled *Introduction to Mathematical Economics* (1957). In 1953, Clower set the basic structure of the ‘stock-flow’ price theory. His point was that when commodities were consumed, produced and held by individuals (e.g., capital goods stored by entrepreneurs), the determination of equilibrium prices required taking into account current activities and the resulting effects on the stocks held by individuals. This paper paved the way for formal investigations on the statics and dynamics properties of ‘stock-flow’ market models. They were exposed in three articles: “An investigation into the Dynamic of Investment” (1954), “Productivity, Thrift and the Rate of Interest” (1954a) and “Price Determination in a Stock-Flow Economy” (1954b). The last two papers were written with a mathematician specialized in dynamics, Donald W. Bushaw. This marked the beginning of a collaboration which culminated with the writing of *Introduction to Mathematical Economics*, a book almost entirely devoted to ‘stock-flow’ market analyses.

This market theory was developed with no clear reference to a search for microfoundations of macroeconomics. Macroeconomic issues were always put in the background, and in *Introduction to Mathematical Economics*, Bushaw and Clower devoted only a short appendix to the derivation of a Keynesian macromodel from the ‘stock-flow’ price theory. In view of this, it is not surprising that neither Clower’s contemporaries nor the historians perceived the theoretical project underlying the analysis of this market structure. Reviewers of *Introduction to Mathematical Economics* (Diran Bodenhorn, 1958; John A. Nordin, 1958; Victor E. Smith, 1958; Allen Spivey, 1958; and William J. Baumol, 1959) essentially praised the clarity and rigor of the mathematical treatment of price determination processes. Those who used the ‘stock-flow’ price theory considered that Bushaw and Clower provided only a general analysis that needed to be oriented, either to ground Keynes’ theory

(Cliff L. Lloyd, 1960; Baumol, 1962) or to enhance the understanding of microeconomic behaviors of entrepreneurs (Vernon L. Smith, 1961; Sam Jr. Chase, 1963; Richard S. Higgins, 1972) and of consumers (Josef Hadar, 1965). With regard to historians, emphasis was given to the role played by Bushaw and Clower in dynamics since they pioneered the Lyapunov technique to study the stability of competitive equilibrium (E. Roy Weintraub, 1991; Giancarlo Gondolfo, 2010; Roger E. Backhouse and Mauro Boianovsky, 2013). Therefore, until now, a technical rather than theoretical interpretation has prevailed among reviewers, theorists, and historians.

Yet, Clower's 'stock-flow' market models were an outgrowth of a theoretical project outlined in his doctoral dissertation. The project can be summarized as an attempt of synthesis between Hicks's *Value and Capital* (1939) and Hicks's *Contribution to the Theory of the Trade Cycle* (1950). At the macroeconomic level, Clower also drew inspiration from Keynes's *General Theory* to build his own theory of capital accumulation. The resulting macromodel was structured around the articulation of stocks and flows, a feature considered as the essence of capital accumulation processes. This supported Clower's claims to have found a "general theory of capital accumulation" that could include the models of Harrod (1939) and Hicks (1950) as special cases. At the microeconomic level, Clower set about revising the framework found in *Value and Capital* to ground the relations of stocks and flows. But he did not manage to formulate a general theory of markets. Accordingly, Clower could neither clarify the logical properties of his business cycle model nor demonstrate the consistency between his theory of choices and aggregates. The program of microfoundation was incomplete. This was one of the reasons why Clower failed to obtain his Oxford D. Phil. Thereafter, Clower developed the 'stock-flow' market models in view of carrying out his project to provide microfoundations to Keynesian business cycle models.

1. Clower's PhD dissertation as the origins of the 'stock-flow' general-equilibrium program

Clower's doctoral dissertation sketched a program to microfound Keynesian macroeconomics. In the introduction of his dissertation, Clower indicated his intention to follow in Hicks's (1939) footsteps to "lay the foundations of a general theory of capital accumulation" (1952: p. 11). This theory, inspired by Keynes (1936), would include the kind of business cycle models developed by Harrod (1939) and Hicks (1950) as special cases. The reason was that its structure, the articulation of stocks and flows, was considered as the essence of capital accumulation processes. Since the relation of stocks and flows was not taken into account in standard microeconomics, Clower set about revising the theory of choices. He formulated the "producer-consumer" theory of the firm and used it to justify some properties of Keynes's and Keynesian business cycle models. But the resulting connections with macroeconomics remained limited because a general theory of markets was lacking.

1.1 Microfoundations of Keynesian macroeconomics

Clower's doctoral dissertation, "Theories of capital accumulation with special reference to their ability to explain the experience of the U.S since 1870" (1952), was separated into three sections: macroeconomics, empirical testing and microeconomics. After the introduction (chapter 1), Clower expounded a model of aggregate supply (chapter 2). Then, he presented the Keynesian theories developed by Harrod (1939) and Hicks (1950) (chapter 3) and his reformulation of Keynes' theory of the trade cycle (chapters 4-5). The last two chapters were devoted to a statistical study aiming at testing the empirical content of his macromodel. After the conclusion (chapter 8), in appendices, Clower dealt with microeconomic issues: the introduction of assets in the standard theory of the firm (appendix I); the validity of the "traditional" law of demand when individuals were supposed to

consume and hold commodities (appendix II); and the introduction of interdependent preferences in the standard theory of the consumer (appendix III).

In the introduction, Clower claimed that the dissertation was written in reverse order. This indicated his intention to provide sound microfoundations to Keynes's and Keynesian theories of capital accumulation:

The writer began by examining the general pure theory of economic behavior (as expressed e.g., in *Value and Capital*) in an attempt to discover whether that theory was in any way inadequate as a foundation for capital accumulation theory. After making appropriate alterations to the general theory, the writer tried to fit various recent theories of capital accumulation [Reference to Keynes (1936), Harrod (1939) and Hicks (1950)] into it as special cases (1952: p. 8).

Clower believed that the understanding of capital accumulation processes required starting from individual behaviors (1952: p. 12). But since practical results could hardly be obtained at this level of analysis, he also expressed the need to formalize aggregative models similar to those developed by Harrod and Hicks (1952: p.12). Of course, Clower raised the issue of the compatibility between these two levels of analysis. In the introduction of his dissertation, he questioned in particular the compatibility between Keynes's *General Theory* and standard microeconomics, as expressed by Hicks in *Value and Capital*. According to him, Walrasian and Keynesian theories were fundamentally compatible. But this compatibility was conditional on modifications of Walrasian microeconomics:

From a formal point of view, is the *General Theory* a special case of established general equilibrium theory? Once again, there are essential differences between the two levels of analysis, differences which may not be reconcilable until the foundations of general equilibrium theory are broadened (1952: p. 5).

In brief, Clower intended to follow in Hicks's (1939) footsteps.⁴ Influenced by the “pure logical analysis of capitalism” ([1939] 1946: p. 4) expounded in *Value and Capital*, he considered i) that macroeconomics had to be deduced from sound and reliable microeconomics; and ii) that the formulation of a general equilibrium model was necessary to prove the compatibility between these two levels of analysis. Viewed from this microfoundational angle, Clower's originality lay in his intention to modify Walrasian microeconomics in order to offer “the foundations of a general theory of capital accumulation” (1952: p. 11).

1.2 The “general theory of capital accumulation” and the articulation of stocks and flows

To understand the general nature of Clower's theory of capital accumulation, it is necessary to go beyond the explanations of trade cycles to focus on the basic structure of the model. Clower started from Keynes' liquidity preference theory and deduced a macromodel with an explicit ‘stock-flow’ architecture. He wanted to show that the cyclical dynamic was ultimately related to the variations of the stock of capital assets, variations due to the difference between capital inflows and outflows. According to him, the same was true in the models developed by Harrod and Hicks. The cyclical dynamic was mainly explained by the accelerator, a relation linking the flow of output and the stock of capital assets. Because of that, the relation of stocks and flows was perceived as the dynamic essence of trade cycles. Since this relation was literally at the heart of his macromodel, Clower claimed to have found a “general theory of capital accumulation.”

1.2.1 From Keynes (1936) to the articulation of stocks and flows

In contrast with Keynes, Clower did not focus on the marginal efficiency of the capital but on the liquidity preference to build his business cycle model. In chapter 22 of the *General*

⁴ For an exhaustive presentation of Hicks' method, see Weintraub (1979). For a short presentation, see Hoover (2012).

Theory, Keynes (1936) considered that the marginal efficiency of the capital was the only component of the effective demand that fluctuated repeatedly and cyclically. But according to Clower, he neither gave decisive arguments to discard other components (the propensity to consume and the liquidity preference) nor did he succeed in explaining why the marginal efficiency of the capital fluctuated cyclically (1952: p. 80-83).⁵ All of this justified a reorientation. The starting point was an extension of the liquidity preference theory. Clower tried to show that Keynes' monetary theory could be applied to physical assets. Transactions, precautionary, and speculative motives remained relevant to analyze investment decisions. The transaction motive was equivalent to a production motive. Firms needed to hold physical assets to produce (1952: p. 69). The precautionary motive was at work when entrepreneurs decided to use only part of their production capacity, what Clower called "desired excess-capacity" (1952: p. 71). Finally, entrepreneurs held capital assets for the sake of their expected yield, which characterized the speculative motive (1952: p. 70). Keynes stressed the role of speculative behaviors in the determination of the rate of interest and therefore in the determination of income. Clower added that speculative behaviors could also have an effect on capital accumulation processes and therefore on fluctuations:

The theory of liquidity preference was linked by Keynes to the theory of output in such a way that economic activity in the real sphere could be shown to be "mirrored" in the money market. From there it was a short step to the conclusion that speculation (in the broadest sense) largely governs the behavior of real output, employment and capital accumulation (1952: p. 185).

To formalize this twofold effect, he elaborated a macroeconomic model in which the level of income and its fluctuations across time were presented as the consequences of the

⁵ In fairness to Keynes, the exclusion of the propensity to consume was suggested by its formalization. Since it depended on realized income, it could not be considered as a cause of fluctuations. And, regarding liquidity preference, Keynes maintained that its modifications would necessarily be the consequence of a previous variation of entrepreneurs' long-term expectations (1936: p. 316). Accordingly, this component could not be viewed as a cause of fluctuations: "Liquidity preference [...] does not increase until after the collapse in the marginal efficiency of capital" (1936: p. 316).

interaction between the stocks and the flows of capital assets. As depicted in figure 1, he assumed first that the rate of interest (v) was determined when entrepreneurs wanted to hold the whole stock of existing assets ($C=K$); second, that the level of gross investment (k) was fixed when the flow of new capital assets was such that the supply price equaled the rate of interest (1952: p.76); and third, that the equilibrium was stationary when the flow of gross investment equaled the flow of depreciation (d). Clower graphically portrayed this framework by distinguishing the stock part from the flow part of the model (1952: p.75). The two were interconnected in dynamical analyses in order to describe the processes of the rise and fall of the stock of capital, once the flow of gross investment did not match the flow of depreciation.

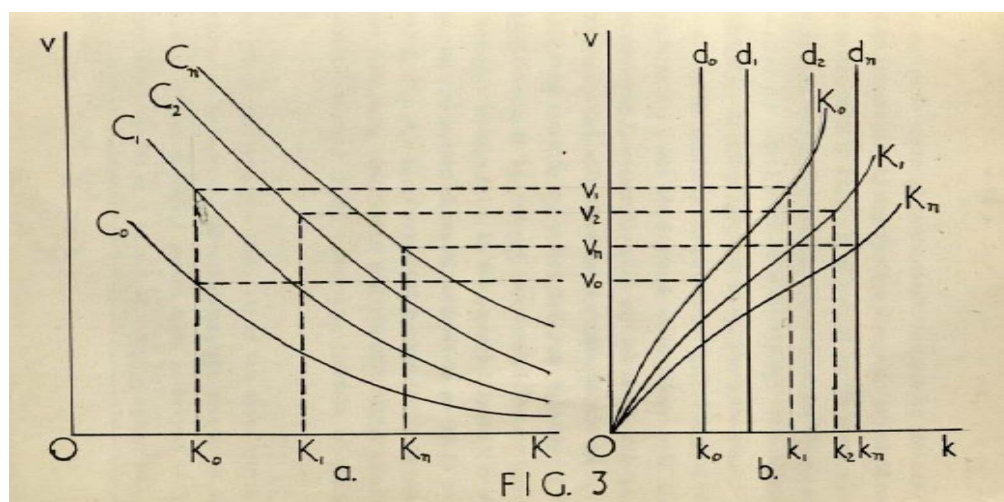


Figure 1 Clower's (1952: p. 75) diagram showing the market for capital assets

To explain fluctuations on this basis, Clower claimed to follow the intuitions expressed by Keynes in chapter 5 ("Expectation as determining output and employment") of the *General Theory*. There, Keynes put forward the distinction between short-run and long-run expectations as well as a proposal to explain the trade cycle.⁶ He maintained that a

⁶ In the short-run, entrepreneurs had to anticipate the price at which output would be sold as well as the capacity of absorption of the economy during a given market period; in the long-run, they had to anticipate the future proceeds of an additional unity of capital taking into account the potential modifications of the taste of consumers, of the effective demand and of the variations of nominal wages.

disturbance of entrepreneurs' long-term expectations led to the emergence of a new stationary position, and that the process of transition to this position could be used to explain fluctuations. Like Keynes, Clower considered "the process of transition to the (new stationary) position" to address the trade cycle (1952: p. 74). In figure 1, the economy was initially in a situation of stationary equilibrium. Entrepreneurs' stock-demand for capital assets (C_o) set the rate of interest at the level (v_o) at which the flow of gross investment (k_o) equaled the flow of depreciation (d_o). Here, the process of transition towards a new stationary equilibrium (after a positive disturbance of entrepreneurs' long-term expectations) was expressed via the transition from K_o to K_n . According to Clower, this process was stable in the absence of uncertainty: entrepreneurs' absolute confidence on the returns on investments led them to increase their stock-demand for physical assets until the stationary position was reached. Yet, in the context of uncertainty considered by Keynes (1936), assets demand would have been subject to violent and repeated changes so that the economy would have never reached the new stationary position. Because of this instability of the stationary equilibrium, the stock of capital assets would have varied continuously thus explaining the trade cycle (1952: p. 88-92).

1.2.2 The articulation of stocks and flows: the essence of capital accumulation processes

Clower repeatedly stressed that the 'stock-flow' architecture was not specific to his interpretation of "Keynes' views on the trade cycle" (1952: p. 11). In the introduction of his dissertation, the Keynesian models in the tradition of Harrod (1939) were reduced to this structure:

On a fairly rigorous but highly restrictive *mechanical* level of analysis, capital accumulation is considered as one aspect of the more fundamental distinction between stocks and flows (i.e., the acceleration principle) (1952: p. 2).

Following the seminal work of Harrod (1939), the ambition of Keynesians was to account for capital accumulation as an endogenous process resulting from the interaction of the multiplier and the accelerator.⁷ Since the accelerator was a function linking the flow of output with the stock of capital assets, Clower considered that this approach deduced cyclical dynamics from the interactions of stocks and flows. Because of this common structure, Clower concluded that he had found a way to unify Keynesian theories of capital accumulation:

The argument in previous chapters has been devoted primarily to demonstrating the unity of recent theories of capital accumulation. In retrospect, it appears that the thread which links together various theories – a thread that is hidden by difference in method and content – is to be found in the distinction between the *using* and the *holding* of assets [reference to Keynes (1936)]. This distinction obviously implies but it is not implied by the distinction between stocks and flows [reference to the models following Harrod (1939) and Hicks (1950)] (1952:p. 184).

In this quotation, Clower suggested that his own macromodel could serve as a basis to elaborate the general theory of the trade cycle since the ‘stock-flow’ architecture referred back to individual decisions and so, was deeply grounded. Thanks to Keynes’ liquidity preference theory, Clower linked the articulation of stocks and flows to entrepreneurs’ decisions to hold (precautionary and speculative motives) and to use (transaction motive) assets. This way of justifying the general nature of his theory might reveal a methodological argument. Clower might have been seduced by Paul A. Samuelson’s (1947) methodology, inspired by Eliakim H. Moore’s principle of generalization by abstraction.⁸ His “general theory” seemed to stem from the application of the assertion that “the existence of analogies between central features

⁷ Clower indicated that “the models of Mr. Harrod and Professor Hicks are only two of many possible mechanical theories of capital accumulation [references to Metzler (1941) and Samuelson (1944)], but since the results and shortcomings of these two models are broadly characteristic of mechanical analyses, generally, we need not consider other theories” (1952: p. 53).

⁸ In the dissertation, Clower referred to the *Foundations* but not directly to Moore. Yet the principle of generalization by abstraction later became a clear reference, used to justify his second line of research, developed in the 1950s (see “On the existence of a general theory of price determination” (1954?: p. 49) Box 4).

of various theories implies the existence of a general theory which underlies the particular theories and unifies them with respect to those central features” (Samuelson, 1947: p. 3). Since the articulation of stocks and flows (the analogy between Keynesian business cycle models) was literally at the heart of his macromodel, Clower thought he had found a “general theory of capital accumulation”.

1.3 Microfoundations of the “general theory of capital accumulation”

Clower sought to develop his own theory of choices, the “producer-consumer” theory of the firm, to ground the ‘stock-flow’ structure.⁹ To demonstrate the relevance of this microfoundation, Clower undertook to justify the central features of Keynes and Keynesian theories of the trade cycle. Yet a general theory of markets was missing. Therefore, the connections with macroeconomics remained mainly informal and incomplete.

1.3.1 The “producer-consumer” theory of the firm

Entrepreneurs were the central figure in Clower’s “general theory of capital accumulation”. The choices they made concerning the holding and the using of physical assets underlined the ‘stock-flow’ architecture and explained the fluctuations (in a context of uncertainty). Yet according to Clower, these aspects of entrepreneurs’ behaviors were not captured by the standard theory of the firm. There was no distinction between the holding and the using of assets, and no psychological dimension to account for entrepreneurs’ appreciation of the business climate (1952: p. 71). Accordingly, he proposed modifications. This resulted

⁹ Clower also modified the standard theory of the consumer to ground his business cycle model. He started with James S. Duesenberry’s (1949) idea that the preferences were interdependent. This meant that in addition with absolute income, the relative position in the society mattered in patterns of consumption. Typically, individuals would increase their consumption expenditures with increasing consumption expenditures in their social network. This microfoundation was intended to endogenize the trend and the “floor” of his theory of the trade cycle. The maintenance of the consumption, to keep up with the Joneses, would underpin the minimum limit of investment at which the economy would rebound. And since this “floor” was supposed to depend on the stock of capital assets accumulated and that this stock was likely to increase from one depression to another (1952: p. 43), it would grow over time thus describing a rising trend.

in the “producer-consumer” theory of the firm (1952: p.71; p.187), a micromodel inspired by the works of Leonid Hurwicz (1946) and Johannes V. de Graaff (1950).¹⁰ The first modification consisted in introducing asset holding in entrepreneurs’ optimization programs. To do so, Clower proposed to account for the evolution of the wealth of the firm (x_i') in the calculation of profits (π). The second modification consisted in substituting an entrepreneurial “decision function” (ϕ) for the traditional technical one – the function developed by Clower was supposed to establish both a technical and a subjective link between the quantity produced, consumed, and held at the end of the market period. To do so, Clower introduced the quantity of assets that entrepreneurs sought to hold at the end of the market period (D) in the traditional production function. There would have been uncertainty since (D) ultimately depended on entrepreneurs’ expectation of assets’ prices (and so of the value of their wealth), at the reopening of markets (1952: p. 194).¹¹ The resulting function would have remained technical because it described how entrepreneurs made their output decisions (x_i) so as to hold the quantity of wealth desired at the end of the market period. Formally, entrepreneurs’ maximization program was defined as follows:

$$\left\{ \begin{array}{l} \text{Max } \pi = - \sum_{i=1}^n p_i(x_i + x_i') \\ \text{s. t. } \phi(x_1, x_2, \dots, x_n; D_1, D_2, \dots, D_n) = 0 \end{array} \right.$$

¹⁰ These two economists were the main references of Clower, both in his dissertation and in the paper that he devoted to the “producer-consumer” theory of the firm (1952a). Yet, it is important to note that in the early 1950s, there was a general concern for the economic effects of the interactions between stocks and flows. The proposals of Hurwicz, De Graaff and Clower were part of a broader reflection on the incorporation of wealth (i.e., assets and debts) in standard microeconomics, in order to explain the influences of stocks on economic behaviors and vice versa (See L. Klein’s paper “Assets, Debt and Economic Behavior” (1951) for a review). In another way, these preoccupations underlined the proposals of M.A. Copeland (1949) to broaden social accounting to monetary flows.

¹¹ Clower did not formalize explicitly the expectations. He thought that the introduction of D would be sufficient to account for entrepreneurs’ degree of uncertainty. Thus, unlike Hicks (1939), he did not resort to intertemporal optimization and expectations to address decision making in a context of uncertainty. This may be explained by Hicks (1939)’s own difficulty to elaborate a theory of expectations rooted in individuals choices.

With x_i , the quantity used in the production less the quantity produced; x_i' , the quantity that entrepreneurs decided to hold at the end of the market period less the quantity held from the outset; $\phi(x_1, x_2, \dots, x_n; D_1, D_2, \dots, D_n)$, the “decision function”; and D_i the quantity held at the end of the market period.

1.3.2 The connections with macroeconomics

The construction of aggregates was mentioned in chapter II. Clower explained that he used the “composite-commodity theorem” (1952: p. 18). This theorem proposed by Hicks (1939) defined conditions to treat the aggregate as an individual (Hoover, 2012: p. 36). A representative consumer and a representative firm replicate the behaviors of all the individuals, and the commodities whose prices vary in the same direction and almost in the same proportions are represented by a single commodity. Clower neither formally explicated the conditions for the application of this theorem nor did he address its applicability to the real world. It was as if the issues raised by aggregation were considered as secondary. This is surprising to say the least since Clower was engaged not only in a conceptual analysis but also in an empirical study of capital accumulation processes.

On the other hand, Clower was concerned with the justification of some macroeconomic features through individual behaviors. In chapter III, he focused on the connection between the “producer-consumer” theory of the firm and the accelerator. This mechanism was supposed to work only if inputs were not fully utilized (1952: p. 45). He argued that, in contrast with standard microeconomics, the “producer-consumer” theory of the firm could account for the underutilization of capital assets. The distinction between the holding and the using of assets, coupled with an entrepreneurial decision function, would open up the possibility to introduce precautionary behaviors in the theory of the firm:

Professor Hicks' formulation of the acceleration principle is not, as a rule, consistent with the usual theory of the firm. If the formal theory of the firm is modified to conform to common sense views (the existence of desired excess capacity), the difficulties considered (deduction of the accelerator) do not arise (1952: p. 47).

Entrepreneurs would decide capacity utilization depending on their appreciation of the business climate. Clower inferred that the "producer-consumer" theory of the firm was an adequate foundation for the accelerator (1952: p. 47). However, this conclusion remains subject to caution since nothing explains how to account for the tensions on output decisions when entrepreneurs are pessimistic. One avenue would have been to specify the entrepreneurial "decision function" but it was not explored by Clower.

In chapter IV, Clower focused on the connection between the "producer-consumer" theory of the firm and Keynes's theory of investment. This was undertaken in the course of a general reflection on the deduction of key Keynesian functions (1952: pp. 60-66). Clower's procedure was to deduce individual supply and demand functions from optimization programs and then, by simple summations, to obtain their aggregated version. The supply and demand functions resulting from the resolution of the "producer-consumer" program differed from the standard ones (1952: p. 61):

$$\begin{cases} C^s = C^s(p, w, v, K_0) \\ L^d = L^d(p, w, v, K_0) \\ K = K(p, w, v, K_0) \\ k = k(p, w, v, K_0) \end{cases}$$

All the functions depended on prices (p), wages (w), the interest rate (v) and, what was new, on the existing stock of capital assets, K_0 . In addition with the usual functions C^s and L^d characterizing respectively the supply of consumer goods and the labor demand, Clower deduced K representing the demand for capital assets and k , the supply of *new* capital assets (1952: p. 61). On this basis, Clower claimed to deduce Keynes' theory of investment. He

considered that the asset demand curve and the supply of new capital assets could be used to replace Keynes' relations. In his theory, the marginal efficiency of capital was the discount rate which equalized the value of expected net returns of the capital with the supply price of a marginal unit of capital. The level of investment was such that the interest rate equalized this discount rate. In Clower's model, the supply of new capital goods characterized the quantity of new capital assets that the marginal producer decided to produce given the rate of interest, and the asset demand curve included entrepreneurs' calculation concerning the expected net return of holding real assets. In this context, Clower contended that the level of investment was determined, in equilibrium, at the point of the supply curve which equaled the demand price in the market for existing assets. This would have been only an "elaborate way of stating the equilibrium condition mentioned [by Keynes]" (1952: p. 62).

In the dissertation, the main shortcoming of Clower's project to microfound Keynesian macroeconomics was the absence of a general theory of markets. In a program *à la* Hicks (1939), this was seen as a crucial step to demonstrate that macroeconomics could be deduced from the theory of choices.¹² Without market models, Clower could neither account for the logical properties of his "general theory of capital accumulation" nor offer formal proofs of the consistency between economic behaviors and aggregates. It was partly because of this gap that Clower failed to obtain his Oxford D. Phil. His examiners, Ian M.D. Little and Charles M. Kennedy, acknowledged that the microeconomics expounded in appendices presented "some undoubted contributions to economic theory". But at the same time, they stressed that there was "no very substantial connection with the main theme of the dissertation". Besides, they found that "there [was not] any new contribution" to macroeconomics and that "the

¹² Because of the absence of market models, it is also difficult to understand the kind of theory of the trade cycle contemplated by Clower. Did he have in mind an equilibrium model of the business cycle? Or did he consider that the trade cycle had to be thought by means of derivations with regards to equilibria, like in the models of Harrod (1939) and Hicks (1950)? Whether markets were supposed to clear or not in various stages of the trade cycle modified fundamentally the understanding of this phenomenon.

statistics [study] was not of the highest quality”. Accordingly, they decided that the thesis justified only an Oxford B. Litt.¹³ Retrospectively, Clower recognized that his thesis “was not in a form fit for publication” and “did not produce what he had hoped”.¹⁴ This would have led him to “develop healthier motivations”, staying “six months at home not only with *Value and Capital* but also with Pareto and Walras”. The first outgrowths of these investigations appeared in “Business Investment and the theory of prices” (1953). Clower put forward the basic structure of a ‘stock-flow’ market theory, the missing element of his doctoral dissertation. At that time, he taught at the Washington State University. There, he met Don Bushaw, a mathematician specialized in dynamics.¹⁵ With his help, Clower proposed the first formal analyses of the ‘stock-flow’ price theory, in a series of three papers published in 1954.¹⁶ This collaboration culminated in the publication of *Introduction to Mathematical Economics* (1957).

2. The ‘stock-flow’ market theory: statics and dynamics

Let us focus on the main statics and dynamics properties of the ‘stock-flow’ market theory.

2.1 ‘Stock-flow’ market models: statics

The ‘stock-flow’ market models portrayed economies in which the typical commodity was consumed, produced, and held by individuals. This resulted in the formulation of two sets of functions. The first one accounted for the flow dimension of the market theory. The functions characterized the rate at which commodities were newly produced (s_i) and newly

¹³ The quotations are taken from the jury’s report (08/15/1952). Oxford University Archives: FA4/18/3/1, SS.R (52)16.

¹⁴ The quotations are taken from a resume written by Clower in 1964. R. W Clower Papers, Box 1-2001-0088, Rubenstein Rare Book and Manuscript Library.

¹⁵ Bushaw did his PhD in Mathematics at the Washington State University. Solomon Lefschetz, a mathematician specialized in topology was his supervisor (see website of WSU).

¹⁶ In the mathematical appendix of “An Investigation into the dynamics of investment” (1954), it is indicated that “this note was prepared by R.W. Clower and D.W. Bushaw, who is instructor in mathematics at the State College of Washington, Pullman” (1954: p. 78).

consumed (d_i) during the market period – the excess-flow-demand $[(d_i) - (s_i)]$ is here expressed by the variable ED_F . The second set of functions accounted for the stock dimension of the market theory. The functions characterized the stock of commodities that individuals inherited from the past, at the beginning of the market period $(S_i^t = S_i^0 + \int_{t_0}^t (d_i - s_i) dt)$ ¹⁷ and the stock of commodities that they wanted to hold at the end of the market period (D) – the excess-stock demand $[(D) - S]$ is here expressed by the variable ED_S . With the exception of the stock-supply, all the functions were supposed to depend on current market prices.¹⁸

A ‘stock-flow’ market theory explained the determination of prices by taking into account consumption, production, and the resulting variations of the stocks of commodities in presence in the economy. In this context, the key variable was the stock of commodities held by individuals. The flows of new consumptions or productions were supposed to adjust the stocks at a quantity desired when, at the beginning of a market period, some individuals considered that the stock inherited from the past was no longer adapted given current market prices. In view of this, two types of equilibria were distinguished. The first one was called “temporary”¹⁹ since the stocks of commodities showed tendency either to rise or to fall. The second equilibrium, called “stationary”, characterized situations in which the stock of commodities was constant from market periods to market periods. Formally, the “temporary” equilibrium was a situation represented by the following system:

$$\begin{cases} ED_F + ED_S = 0 \\ ED_F \neq 0 \end{cases}$$

¹⁷ This is the mathematical expression offered by Bushaw and Clower (1954b: p. 328). They considered a continuous-time model. A discrete-time model required using a sum instead of an integral.

¹⁸ (S) was a vertical line which indicated that at a given moment of time, the quantity held by individuals could not change and was independent of current market prices.

¹⁹ From 1953 to 1957, the terminology changed. The expression “non-stationary equilibrium” was substituted to those of “temporary equilibrium” in *Introduction to Mathematical Economy*.

The first line of the system characterized the “market excess-demand”. That represented the total quantity purchased during the market period less the total quantity offered during the market period. Put simply, when individuals desired to alter their stock of commodities, this entailed differences between the rates of consumption and production in the economic system. The condition was that at the end of the market period, all the individuals satisfied their optimizing programs (i.e., they held the quantity of stocks desired, given current market prices) and so, that the markets cleared. If the equilibrium was “temporary”, then at the reopening of the markets, because of the new quantity of stocks in presence in the economy, the set of prices would be different thus leading to other adjustments of consumptions and productions. If individuals did not desire to alter their stock of commodities at the reopening of markets, then the economy was supposed to have reached the stationary equilibrium. Formally, this situation is represented by the following system:

$$\begin{cases} ED_F + ED_S = 0 \\ ED_F = 0 \end{cases}$$

2.2 ‘Stock-flow’ market models: dynamics

In ‘stock-flow’ models, price determination processes were affected by current activities as well as the resulting variations of the stocks of commodities in the economy. So, changes in prices were supposed to be patterned by the excess-flow-demands and the excess-stock-demands (1954b: p. 329; 1957: p. 118):

$$\frac{\partial p_i}{\partial t} = f_i(ED_F; ED_S)$$

This resulted in formal differences between characteristic polynomials (1954b: p. 338-340) so that stable coefficient matrices in pure stock and pure flow models could be unstable in ‘stock-flow’ models. According to Bushaw and Clower, this result had an empirical content

since the data of the economy were constantly changing (1957: p.80). This was used to legitimize ‘stock-flow’ models:

From the very outset, there is a presumption in favor of a stock-flow theory over a pure stock theory or a pure flow theory. Whether or not this presumption is decisive, however, depends on the extent to which the logically simpler pure stock and pure flow theories provide an adequate basis for the interpretation of empirical phenomena – on the extent to which stock-flow relationships can be ignored or else dealt with implicitly in terms of the simpler models. The preceding argument seems to shed some light on this question. If it were the case that stock-flow relationships could be safely ignored, our discussion might have been expected to support this surmise by indicating that the intersection of stock-flow relationships into a model would affect no essential changes in its character. But our analysis leads in fact to precisely the contrary conclusion. [...] We are thus led to the inference that an effective general model for the study of price determination and kindred questions, whatever other characteristics it may have, should be a genuine stock-flow model (1954b: p. 343).

Consideration of pure stock or pure flow models could lead to ignore a potential problem of instability. Those approximations were not neutral. Because of that, it would be better to use ‘stock-flow’ market models.

3. Decoding the ‘stock-flow’ market analyses

The program of microfoundations opened in Clower’s doctoral dissertation was in the background of the statics and dynamics analyses of ‘stock-flow’ market models. A careful study of the early papers (1954; 1954a) reveals that Clower intended to demonstrate that his market models could be an adequate interface with Keynes’ theory of investment and the liquidity preference theory. Then, the link between the “producer-consumer” theory and Keynesian business cycle models can be established by combining some arguments contained in the micro and macro chapters of *Introduction to Mathematical Economics*. Finally, in the “Keynesian appendix” of this book, Bushaw and Clower undertook the derivation of the

standard IS/LM models from their ‘stock-flow’ price theory. All of this shows that the micro-macro relation was still a focal point. Accordingly, why did macroeconomics remain in the background? There was a methodological reason for that. Clower considered that macroeconomics should not be the priority until the logical properties of disaggregated systems were fully known. Otherwise, the simplifications required to build aggregates would be a source of error that could have damaging consequences for policy recommendations.

3.1 The Keynesian connections

In 1954, Clower developed two ‘stock-flow’ models in partial equilibrium to address the theory of investment and the theory of interest rate. More or less explicitly, statics and dynamics properties were used to connect Keynes’ *General Theory*.

Following the lines of his doctoral dissertation, Clower (1954) explained the determination of the level of investment and claimed that “a curve K_r which Keynes would call schedule of marginal efficiency of capital” (1954: p. 76) could be deduced from his ‘stock-flow’ market theory. The level of investment was set by distinguishing two logical steps. The first one was the determination of the price of capital goods, at the intersection between the stock-supply (S) and the stock-demand (D).²⁰ The second step was the determination of gross investment and depreciation. Clower considered that the level of gross investment was fixed when the rate of production of new capital assets during the market period (s) was such that the supply price equaled the price of capital assets (1954: p. 67). Likewise, depreciation was determined when the rate of consumption of capital goods during the market period (d) was such that the demand price equaled the price of capital assets (1954: p. 69). Net investment was considered as a residual, the difference between gross

²⁰ Note the modification of the formalization of the “temporary” equilibrium. There is no inconsistency with the general case expounded in 2.1. Here, Clower assumed that the price at which individuals wanted to hold stocks was independent of the variations of stocks (cf. the mathematical appendix (1954: p. 78)).

investment and depreciation given the current price of capital goods. On this basis, Clower proposed to deduce Keynes' theory of investment. He assumed different levels of the rate of interest. *Ceteris paribus*, for each level, the demand for existing capital assets would be different since entrepreneurs' appraisals of their discounted value would be modified. This resulted in different price of capital goods and so, different levels of net investment. According to Clower, the relation associating the different levels of interest rate and the different levels of net investment could be viewed as an approximation of Keynes' marginal efficiency of capital schedule (1954: p. 76).

Clower (1954a) was less explicit when he turned to the connections between the liquidity preference theory and the 'stock-flow' market theory. The analysis of an unpublished manuscript (probably a first version of his 1954 paper) is necessary to support our viewpoint. In this manuscript, Clower maintained that the opposition between the liquidity preference and the loanable funds theories of interest was an opposition between short-run and long-run analyses. In the short-run, saving and investment could be considered to be negligible because of existing large stocks of assets. Therefore, the dynamic path of the rate of interest would be mainly explained by speculative behaviors. However, in the long-run, saving and investment would be the main forces underlying the course of the rate of interest. Because of the nature of this opposition, Clower maintained that it was possible to conciliate the two existing theories in one framework. He proposed the 'stock-flow' price theory:

Changes in the rate of interest will be speculative in nature since changes in "productivity and thrift" during any short space of time can have little direct effect upon holder demands or upon existing stocks of bonds. In the long run, however, "productivity" will largely govern the quantity of outstanding bonds, while "thrift" will have a definite influence on the level of holder demands for debt. Thus, a "stock" theory of interest is preferable to a "flow" theory if attention is centered upon short-run problem. And if one is concerned mainly with traditional long-run economic questions, the use of a "flow" theory is indicated. However, to deal

adequately with both kinds of problems, one must have recourse to a stock-flow theory of interest such as that just outlined – a theory which reconciles the two existing approaches.²¹

The distinction between short-run and long-run analyses was ultimately related to the elasticity of the excess-flow-demand curve. According to Clower, “a specific ‘stock-flow’ relationship” (1954a: p. 114) was that the slope of the excess-flow-demand curve depended on the length of the market period. The shorter the market period, the less the quantities of bonds newly issued and currently bought would have affected price determination because of existing large stocks. In economic terms: on the financial markets, saving and investment (represented by the excess-demand for bonds) would have almost no effect on the dynamics of the rate of interest in the short-run. Instead, it would be linked to speculative behaviors (represented by the excess-stock-demand for existing bonds), which would correspond to Keynes’ theory of interest.²²

3.2 From micro to macro

Thanks to a combination of arguments contained in the micro and macro chapters of *Introduction to Mathematical Economics*, it appears that Clower continued the microfoundational program sketched in his doctoral dissertation. In chapter VI, “Microeconomics II”, Bushaw and Clower deduced the excess-stock-demand and excess-flow-demand functions of their price-theory from the “producer-consumer” theory of the firm:

The function $\phi(x_1, x_2, \dots, x_n; D_1, D_2, \dots, D_n)$ is appropriately called a *decision function*; for when allowance is made for the holding of assets (one of which may be money), the essentially subjective character of the function is apparent. [...] Finally, the entrepreneur is assumed to

²¹ This quotation is from “A Suggestion for Generalizing the Pure Theory of Production” (1954?). R. W Clower Papers, Box 4, Rubenstein Rare Book and Manuscript Library.

²² This “short-term” dynamic feature of ‘stock-flow’ models did not imply that the stationary equilibrium was unstable. Rather, Clower demonstrated that if the excess-flow-demand curve was both flat and had the same sign of the excess-stock-demand curve, the stationary equilibrium was stable (1954a: p. 113). Nonetheless, this would pave the way for such a result once uncertainty would be taken into account (1954a: p. 114).

want to hold a combination of assets, to use a combination of input flows, and to produce a combination of output flows such that the quantity $\pi = -\sum_{i=1}^n p_i(x_i + x_i')$ is a maximum, provided such a *stock-flow plan* exists. [...] Then if the first-order conditions represent a determinate statical system, the equilibrium values of the variables $x_1, x_2, \dots, x_n, D_1, D_2, \dots, D_n$ may be expressed in terms of the parameters p_1, p_2, \dots, p_n to obtain the n business *excess demand functions* $[ED_F]$ and the n *stock demand functions* $D_i = D_i(p_1, p_2, \dots, p_n)$. The later functions, taken in conjunction with the given values S_1, S_2, \dots, S_n then determine corresponding equilibrium values for the investment demand variable x'_i $[ED_S]$ (1957: p. 172).

Then, in chapter III, “Macroeconomic Dynamics I”, Bushaw and Clower maintained that the adjustment processes displayed in their ‘stock-flow’ price theory could be used to ground the business cycles models based on the accelerator:

From a formal standpoint, it is interesting to note that model IV is identical with the basic model which underlies elementary discussions of multiplier and accelerator phenomena; and there is clearly more than a similarity involved. In effect, the present model provides a market (price adjustment) basis for aggregative models of the multiplier-accelerator variety (1957: p. 75).

Clower still pursued the elaboration of the market interface contemplated in his doctoral dissertation. Yet there was no proposal to formally deduce the “general theory of capital accumulation” from the ‘stock-flow’ price theory. Instead, in the “Keynesian appendix”, Bushaw and Clower undertook the derivation of the standard IS/LM model:

A more significant difficulty is that of going from a supposedly satisfactory general model to a specialized aggregative model that is logically and empirically consistent with it. Generally speaking, most aggregative models developed in the past have been formulated independently of, and without a careful examination of their consistency with, any acceptable general theory of price determination. To the extent that one has faith in the essential correctness of a particular general theory, however, it is important to reconcile any proposed aggregative model

with it. [...] We shall proceed by discussing the derivation of what is undoubtedly the most influential of existing aggregative models, the so-called Keynesian system (1957: p. 43).

The starting point was a system of simultaneous equations describing respectively the equilibrium on capital market (a), securities market (b), consumer goods market (c) and labor market (l). Each market was formalized depending on the nature of the commodity considered. Consumer goods and labor were viewed as flow commodities, securities were stock commodities, and capital goods were “stock-flow” commodities (1957: p. 44):

$$\begin{cases} ED_F^a(p_a; p_b; p_c; p_l) + ED_S^a(p_a; p_b; p_c; p_l) = 0^{23} \\ ED_F^b(p_a; p_b; p_c; p_l) = 0 \\ ED_F^c(p_a; p_b; p_c; p_l) = 0 \\ ED_F^l(p_a; p_b; p_c; p_l) = 0 \end{cases}$$

On the basis of this disaggregated system, Bushaw and Clower made few manipulations (e.g., presentation of the system in the form of national accounting and quantities-prices substitutions) to deduce the “fundamental building block of the Keynesian system” (1957: p. 46) and Keynes’ standard functions (consumption, investment, liquidity preference, and labor supply). What is proposed is frustrating for us. Bushaw and Clower argued that it was entirely possible to go back to macroeconomics. Yet, the macromodel was not the one contemplated in the dissertation. It was still an open question to know whether or not an aggregate business cycle model could be formally deduced from the ‘stock-flow’ price theory. Besides, Bushaw and Clower did not demonstrate that a “stock-flow” model could be used to clarify the properties of the Keynesian theory. Unemployment, the liquidity trap or the effects of a fall in nominal wages were outside the scope. The Keynesian appendix only claimed that the ‘stock-flow’ price theory was compatible with a Keynesian model.

²³There is only one equation instead of two, to express the equilibrium on the capital market. This is not consistent with the standard treatment of stock-flow markets. Yet, Bushaw and Clower (1957) argued that this reflected an assumption made by Keynes in the *General Theory*, namely that the variations of the stock of capital assets were not taken into account in the determination of equilibrium prices (1957: p. 44).

3.3 Macroeconomics was not a priority

A methodological argument can be raised to explain this lack of interest for macroeconomic issues. Clower might have considered that a clarification of the logical properties of fully disaggregated systems was an essential step before addressing specific macroeconomic problems. This interpretation is supported by the “Keynesian appendix” of *Introduction to Mathematical Economics*. There, Bushaw and Clower patronized macroeconomics. This level of analysis was considered as approximate because of the assumptions required to construct aggregates. Aggregative models were viewed as specializations of their general theory of markets. Their appendix aimed to show that it would be always easy to back to macroeconomics:

For our purpose it is enough to have illustrated some of the steps which must be taken moving from a general, detailed system to a more specialized system and from this in turn to a highly rarified model like that afforded by the Keynesian system (1957: p. 68).

To be brief, macroeconomics was not the priority. The reason was that the logical properties of “stock-flow” models were still not clear. This could be problematic when policy recommendations were formulated since the kind of simplifications required to build aggregates would be a potential source of error. In the absence of a clear knowledge of the static and dynamic features of general models, this may not be controlled:

The first difficulty encountered in following this approach (construction of aggregated models) is that of defining aggregative variables and relations which are capable of yielding meaningful interpretations of reality; for it is only too easy to define aggregative quantities in such a way as to embody precisely those obscurities which occur explicitly in more detailed models (1957: p. 59 underlined by the authors).

Following this methodological principle, macroeconomics would have had to resurface in a second step. But that was not the case. How does one explain that?

4. The ‘stock-flow’ market theory: a blind alley

Under the assumptions adopted by Bushaw and Clower, the ‘stock-flow’ market theory could hardly be a relevant interface with Keynesian macroeconomics. This may explain why macroeconomics never resurfaced. An implicit recognition of this inadequacy already appeared in *Introduction to Mathematical Economics* and was confirmed by the fact that Clower stopped developing these models at the end of the 1950s. At that time, disequilibrium economics probably appeared to him as a more fruitful avenue of research. Then, the difficulties faced by the very few economists who tried to connect Keynesian macroeconomics with the ‘stock-flow’ market models also proved their inadequacy. Emphasis is given to the contributions of Lloyd (1960) and Baumol (1962), two economists who explicitly tried to use Clower’s price theory to ground Keynes’ economics. I show that Lloyd’s proposal did not stand up to an argument raised by Don Patinkin (1958), and that Baumol’s proposal was incompatible with the assumptions adopted by Bushaw and Clower.

4.1 A project shelved

The reasons why Clower stopped investigating the logical properties of the ‘stock-flow’ market theory at the end of the 1950s are uncertain. All that is clear is that at that time, Clower reconsidered the Keynesian theory from a disequilibrium perspective.²⁴ It is hard to determine whether or not this was due to the recognition that ‘stock-flow’ models could hardly be used to ground Keynesian macroeconomics. Nonetheless, Clower retrospectively indicated in a research project that while writing *Introduction to Mathematical Economics*, he had realized that his hopes to microfound Keynesian macroeconomics had been unduly optimistic:

²⁴ On the intellectual path followed by Clower from his PhD thesis to his famous article “The Keynesian Counter-Revolution: A Theoretical Appraisal” (1965), see my working paper titled: “Clower’s *about-face* regarding the Keynesian Revolution” (2015a).

Virtually all of my theoretical research during the past twelve years has been concerned directly or indirectly with the task of reformulating traditional demand-and-supply analysis to provide a coherent microeconomic foundation for the theory of income, employment and money. At the outset I conjectured that the key to a satisfactory solution of this problem might lie in the generalization of established price theory to deal explicitly with trading on capital as well as current account. This conjecture motivated my early articles on stock-flow analysis [...]. As early as 1957, however, it became clear that my initial conjecture was unduly optimistic – that the integration of value theory and income analysis would require much more than the statement of an improved theory of asset prices. The essential ingredients for a satisfactory resolution of the problem were finally suggested by work that I had been pursuing rather casually for a number of years involving disequilibrium models.²⁵

Following Clower's viewpoint, the goal here is to question the adequacy between the 'stock-flow' market theory and Keynesian macroeconomics. The problem of the integration of Keynes' income analysis and the 'stock-flow' value theory, mentioned by Clower in the preceding quotation, was not addressed in *Introduction to Mathematical Economics*.²⁶ Yet, in this book, Bushaw and Clower (1957) questioned the role of money in their price theory – an issue raised in the preceding quotation. They explicitly recognized that money played no role in price determination processes. This was proved following the lines set out by Patinkin (1949). They stressed the indetermination of monetary prices, due to the "invalid dichotomy":²⁷

The homogeneity properties which follow from our analysis of consumer and business behavior lead to a macroeconomic model of a *barter* economy, not to a model of a money

²⁵ This quotation is taken from a research proposal probably written in 1965. The "Counter-Revolution" paper was about to be published in Franck H. Hahn and Franck H.P. Brechling's book *The Theory of Interest Rates: Proceedings of a Conference held by the International Economic Association* (1965a). R. W Clower Papers, Box 5, Rubenstein Rare Book and Manuscript Library.

²⁶ As evidenced of that, note that neither the concept of involuntary unemployment nor those of unemployment are listed in the index of *Introduction to Mathematical Economics*.

²⁷ On Clower's position in the debate over monetary and value theory opened by Patinkin, see my working paper: "Disequilibrium as the Origin and Originality of Clower's (1967) Microfoundations of Monetary Theory" (2015b).

economy. Money does not influence the price determination process in any way whatever
(1957: p. 242 underlined by the authors).

Since the ‘stock-flow’ models portrayed barter economies instead of money economies, Bushaw and Clower were aware that under the assumptions adopted in *Introduction to Mathematical Economics*, their price-theory could hardly ground Keynesian macroeconomics.

4.2 Very few followers, no solid connection with the Keynesian theory

Lloyd (1960) and Baumol (1962) explicitly tried to use the ‘stock-flow’ market models to connect Keynesian macroeconomics. The liquidity preference theory was the target.²⁸ According to Lloyd, the double equilibrium condition could be used to support Keynes’ position. Yet his thesis did not stand up to the static analysis of Patinkin (1958). On his part, Baumol suggested that in dynamics, ‘stock-flow’ models might be used to do so. Yet I show that this required rejecting the assumptions made by Bushaw and Clower.

4.2.1 Lloyd vs. Patinkin

The ‘stock-flow’ market theory emerged in the context of the controversy over liquidity preference versus loanable fund theories of interest, reopened by Hicks’ demonstration of their formal equivalence, in *Value and Capital* (1939: pp. 158-162).²⁹ Hicks argued that by virtue of Walras’ law, one could omit one equation of the general equilibrium system to set equilibrium prices. Whether this equation was the excess-demand-for-money or the excess-demand-for-bonds did not change anything. Therefore, the two existing theories of interest would have been equivalent.

²⁸ George Horwitch (1957) also proposed to use the ‘stock-flow’ price theory to analyze the dynamics of the rate of interest under various scenarios (open-market policy, disturbance of saving or investment...). But his analysis is here omitted since he was not really concerned with the derivation of Keynes’ theory of interest from the ‘stock-flow’ market models. Instead, he was involved in an assessment of the existing positions regarding the determinant factors of the rate of interest.

²⁹ For a review of these debates, and in particular of the role played by the distinction between stocks and flows in this context, see Harry G. Johnson (1962).

Lloyd tried to challenge this thesis using the ‘stock-flow’ market models developed in *Introduction to Mathematical Economics* (1960: p. 206). He pointed out that two independent equations characterized the equilibrium conditions if bonds and money were assumed to be purchased, issued, and held by individuals (1960: p. 208). Accordingly, if one of these two markets had to be omitted, it would have been necessary to get rid of two equations instead of one. Yet, by virtue of Walras’ law, only one independent equation could be eliminated. Hence the question: how could one eliminate two equations? According to Lloyd, the solution was to connect one equation of the system to another in order to make it redundant. Having this logic in mind, he assumed that the two remaining equations were the excess-flow-demand-for-money and the excess-flow-demand-for-bonds (i.e., the Hicksian configuration). Then he argued that the demonstration of the equivalence between the liquidity preference and the loanable funds theories of interest required the formulation of additional assumptions, e.g., money was created and destroyed only by bankers through purchases and sells of bonds (1960: p. 208). Lloyd inferred that in general, the two theories of interest might not be equivalent:

In this case [stock-flow commodities] certain limiting conditions must be met before Hicks’ proof is valid. By Walras’ law we are able to eliminate one of our excess-demand equations, but only one. In order to eliminate a stock-flow good from our equation system we must eliminate not one but two equations. One way we might do this is to make certain assumptions concerning the institutional make-up of the model we are working with, thus causing some of the equations in our system to be redundant by connecting them to other equations. In particular, it is possible to link the excess flow demand function for money to the excess flow demand function for bonds in such a way as to make them equivalent (Lloyd, 1960: p. 208).

Although Lloyd’s argument may be admitted, it was not exempt from Patinkin’s (1958) criticism. In an attempt to close the debate over liquidity preference vs. loanable funds theories of interest, Patinkin (1958) contended that the distinction between the two theories

was a non sense in a general equilibrium model. By virtue of Walras' law, the interest rate was determined by the general system of equations (1958: p. 301). Accordingly, it did not make sense to isolate one market rather than another to explain its determination.

4.2.2 Baumol's intuition

In a dynamical context, Baumol suggested an avenue to connect Keynes' theory of interest with the 'stock-flow' models (1962: p. 50). His original intuition was that the speeds at which markets moved back to balance might be used to explain what would be the relevant theory of interest. He assumed that if one market was faster than another to return to equilibrium, then it was the primary determinant of the interest rate (1962: p. 52). Considering that bond markets were highly organized, Baumol maintained that the stock equilibrium would be established in no more than a few minutes while the real sector would still be unbalanced. Therefore, speculative behaviors would be the primary determinant of the interest rate, in the short-run. Real sector would become a determinant but over longer periods of time (1962: p. 52-53). According to him, 'stock-flow' models could be useful to support this position:³⁰

For the argument only states that, in the very short-run, interest rate determination will satisfy the stock but *not* the flow equilibrium condition for the bond market (1962: p. 52).

Yet, in the formalization of the 'stock-flow' models proposed by Bushaw and Clower (1954b; 1957), the "market-excess-demand" was always nil ($ED_F + ED_S = 0$), even in a dynamic analysis:

Even though it is possible to distinguish situations of 'apparent equilibrium' (stock *or* flow, but not stock *and* flow 'equilibrium') involving the satisfaction of one of the sets of equations $X_i=0$

³⁰ This approach was closed to that expressed by Clower (1954a), in a partial-equilibrium framework (see 3.1).

$[ED_F = 0]$, $X'_{i=0}$ $[ED_S = 0]$, but not both, it is not possible to attach any significance to such situations in a dynamical system of the kind considered here (1954b: p.331).

Accordingly, there was no room for Baumol's intuition. It is ironic that a potential road to the *General Theory*, though complicated, was closed by those who first tried to connect Keynesian macroeconomics.

5. Conclusion: the indeterminate fate of Clower's 'stock-flow' general-equilibrium program

My paper aimed at demonstrating that a project to microfound Keynesian macroeconomics was hidden behind the 'stock-flow' market models developed by Clower in the 1950s. This appeared clearly in light of his doctoral dissertation, an unpublished manuscript found at Duke University.

Following in Hicks's (1939) footsteps, Clower originally aimed to elaborate the microfoundations of a "general theory of the trade cycle". This theory, inspired by Keynes (1936), was supposed to include the business cycle models in the tradition of Harrod (1939) and Hicks (1950). The reason was that its structure, the articulation of stocks and flows, was considered as the essence of capital accumulation processes. From there, the 'stock-flow' general-equilibrium program was born. Since the relation of stocks and flows had no room in standard microeconomics, Clower set about revising the theory of choices. His proposal, the "producer-consumer" theory of the firm, was the foundation stone of the 'stock-flow' market models. It is argued that behind their statics and dynamics analyses, there were attempts to prove that Keynesian macroeconomics could be deduced from this market structure. Yet, considering that macroeconomics should not have been addressed without a complete knowledge of the logical properties of fully disaggregated systems, Bushaw and Clower preferred postponing their reflections on macroeconomic issues. But macroeconomics never

resurfaced. The reason was that Clower shelved the project and that under the assumptions adopted notably in *Introduction to Mathematical Economics*, the ‘stock-flow’ models could hardly be a relevant interface with Keynesian macroeconomics.

In spite of this, it is not possible to discard the ‘stock-flow’ general equilibrium program. From its very origins, what mattered was the ‘stock-flow’ architecture, perceived as fundamental to understand the dynamics of the business cycle. In a way, the theoretical picture superimposed on the framework was secondary. This left the door open for reutilizations of ‘stock-flow’ market models, under alternative assumptions. In 1968, whilst keeping the same basic structure, Clower proposed to introduce disequilibrium transactions in a ‘stock-flow’ general equilibrium system. According to him, the dynamics of the monetary economy pictured by Keynes (1936) required formulating such a framework:

If trading processes are not synchronized, we move from the barter economy of ‘classical’ economics to the money economy of John Maynard Keynes; from a world where supply creates its own demand to a world where demands are directly constrained by current accruals of cash and cash substitutes and where supplies are directly constrained by current levels of factor unemployment. To investigate the dynamic properties of such systems clearly requires the use of stock-flow analysis (1968: p. 277).

Clower never formalized the disequilibrium ‘stock-flow’ model that he had in mind, thus leaving open an avenue to explore. At the same time, he kept advocating for the introduction of stocks and flows in the general equilibrium theory, until the end of his career. In a paper co-written with Robert L. Sexton, Philip E. Graves and Dwight R. Lee, “Incorporating inventories into supply and demand analysis” (1992), Clower insisted on the need to formulate a ‘stock-flow’ framework to understand the logical properties of models which addressed simultaneously the trade cycle and economic growth:

However, in order to gain a full understanding of business cycles, the distinction between stocks and flows in supply and demand is essential. Explicit analysis of saving, investment, and growth processes is possible only in the context of stock-flow model (1992: p. 41).

The ‘stock-flow’ market theory was here presented as the only possible framework to fully capture the dynamic properties of the economic system. Therefore, the basic message of the authors was that in one way or another, the ‘stock-flow’ general equilibrium program should be further developed.

Recent economics has chosen to incorporate the relations of stocks and flows by ignoring the market dimension. The ‘stock-flow’ analysis was either built into a microfoundational form (through dynamic optimization programs as used in new classical DSGE models) or into an aggregate form (through an accounting framework such as those used in “stock-flow consistent models”). In view of this, the relevant question is no longer whether or not the relations of stocks and flows deserve to be incorporated in economics, but what is the best modeling strategy to do so.

References

- Backhouse, R., and Mauro, Boianovsky. (2013). *Transforming Modern Macroeconomics: Exploring Disequilibrium Microfoundations 1956-2003*. Cambridge: Cambridge University Press.
- Baumol, W.J. (1959). Introduction to Mathematical Economics by D. W. Bushaw; R. W. Clower, *Econometrica*, 27(3), pp. 512-513.
- Baumol, W.J. (1962). Stocks, Flows and Monetary Theory, *The Quarterly Journal of Economics*, 76(1), pp. 46-56.

Bushaw, D.W., and Robert W. Clower. (1954b). Price Determination in a Stock-Flow Economy, *Econometrica*, 22(3), pp. 328-343.

Bushaw, D.W., and Robert W. Clower. (1957). *Introduction to Mathematical Economics*, Homewood: The Irwin Series in Economics.

Chase, Sam B. Jr. (1963). *Asset Prices in Economic Analysis*. Berkeley: University of California Press.

Clower, R.W. (1952). *Theories of capital accumulation with special reference to their ability to explain the experience of the U.S since 1870*, Thesis submitted for the Degree of Doctor of Philosophy, Robert Clower papers, Box 1-2001-0088, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

Clower, R.W. (1953). Business Investment and the Theory of Prices, *Proceedings of the Twenty-Eighth Annual Conference of the Western Economic Association*, pp. 22-24

Clower, R.W. (1954). An Investigation into the Dynamics of Investment, *The American Economic Review*, 44(1), pp. 64-81.

Clower, R.W. (1954a). Productivity, Thrift and the Rate of Interest, *The Economic Journal*, 64(253), pp. 107-115.

Clower, R.W. (1954?). On the Existence of a General Theory of Price Determination. Manuscript. Robert Clower papers, Box 4, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

Clower, R.W. (1954i?). A Suggestion for Generalizing the Pure Theory of Production. Manuscript. Robert Clower papers, Box 4, David M. Rubenstein Rare Book and Manuscript Library, Duke University.

- Clower, R.W. (1965). The Keynesian Counter-Revolution: A Theoretical Appraisal, In Walker [1984], pp. 34-58.
- Clower, R.W. (1968). Stock-Flow Analysis, *International Encyclopedia of the Social Sciences*, pp. 273-277.
- Clower, R.W., Graves, P.E., Lee, D.R., Sexton, R.L. (1992). Incorporating inventories into supply and demand analysis, *Atlantic Economic Journal*, 20(4), pp. 41-45.
- Copeland, M.A. (1949). Social Accounting for Monetary Flows. *American Accounting Association*, 24(3), pp. 254-264.
- Dos Santos, C. and G. Zezza. (2004). The Role of Monetary Policy in Post-Keynesian Consistent Macroeconomic Growth Models: Preliminary Results, In M. Lavoie (Ed.), *Central Banking in the Modern World: Alternative Perspectives*. Cheltenham: Edward Volume.
- Duarte P.G., and Gilberto T, Lima. (2012). *Microfoundations Reconsidered: The Relationship of Micro and Macroeconomics in Historical Perspective*. Cheltenham: Edward Elgar Publishing Limited.
- Duesenberry, J.S. (1949). *Income, Saving and the Theory of Consumer Behavior*. Cambridge, MA: Harvard University Press.
- Gondolfo, G (2010). *Economic Dynamics*, 4th edition. Cambridge, MA: Harvard University Press. New-York: Springer.
- Graaff, J. DeV. (1950). Income Effect and the theory of the firm. *Review of Economic Studies*, 23(46), pp. 79-86.
- Hadar, J. (1965). A Note on Stock-Flow Models of Consumer Behaviors. *Quarterly Journal of Economics*, 74, pp. 304-309.

- Hansen A.H. (1954). *Business Cycle and National Income*. New York: Norton.
- Hahn, F.H, and F.P.R., Brechling. (1965a). *The Theory of Interest Rate. Proceedings of a Conference held by the International Economic Association*, London: Mac Millan and Co.
- Harrison, G.W. (2008). Stocks and Flows. *The New Palgrave Dictionary of Economics, Second Edition*: Steven N. Durlauf and Lawrence E. Blume.
- Harrod, R.F. (1939). An Essay in Dynamic Theory, *Economic Journal*, 49(193), pp. 14-33.
- Hicks, J. R. ([1939] 1946). *Value and Capital, an inquiry into some Fundamental Principles of Economic Theory*. 2nd ed. Oxford: Clarendon Press.
- Hicks, J. R. (1950). *A Contribution to the Theory of the Trade Cycle*. Oxford: Clarendon Press.
- Higgins, R.S. (1972). Microeconomic Foundations of Investment and Stock-Flow Analysis. *Western Economic Journal*, 10(1), pp.19-32.
- Hoover, K. D. (2012). Microfoundational Programs, In Duarte and Lima (2012), pp. 19-61.
- Horwitch, G.H. (1957). Money, Prices, and the Theory of Interest Determination, *The Economic Journal*, 67(268), pp. 625-643.
- Hurwicz, L. (1946). Theory of the Firm and of Investment. *Econometrica*, 14, pp. 109-136.
- Johnson, H.G. (1962). Monetary Theory and Policy, *The American Economic Review*, 52(3), pp. 335-384.
- Keynes, J. M. ([1936] 2007) *The General Theory of Employment, Interest and Money*. New York: Palgrave Macmillan.

Klein, L.R. (1951). Assets, Debt and Economic Behavior. In *Studies in Income and Wealth from the National Bureau of Economic Research*, 14, pp. 195-228.

Lange, O. (1944). *Price Flexibility and Employment*. Cowles Commission for Research in Economics Monograph no. 8. San Antonio: Principia Press Trinity University.

Lloyd, C.L (1960). The Equivalence of the Liquidity Preference and Loanable Funds Theories and the New Stock-Flow Analysis, *The Review of Economic Studies*, 27(3), pp. 206-209.

Nordin, J.A. (1958). Introduction to Mathematical Economics by D. W. Bushaw; R. W. Clower, *Journal of Farm Economics*, 40(2), pp. 496-498.

Patinkin, D. (1949). The Indeterminacy of Absolute Prices in Classical Economic Theory, *Economica*, 25, pp. 300-318.

Patinkin, D. (1956). *Money, Interest and Prices*. Evanston: Row Peterson.

Patinkin, D. (1958). Liquidity Preference and Loanable Funds: Stocks and Flow Analysis, *Econometrica*, 59, pp. 1-27.

Plassard, R. (2015a). Clower's *about-face* regarding the Keynesian Revolution. Working-paper.

Plassard, R. (2015b). Disequilibrium as the Origin and Originality of Clower's (1967) Microfoundations of Monetary Theory. Working-paper.

Samuelson, P.A. ([1947] 1979). *Foundations of Economic Analysis*. New York: Atheneum.

Smith, V.E. (1958). Introduction to Mathematical Economics by D. W. Bushaw; R. W. Clower, *Journal of the American Statistical Association*. 53 (281), pp.224-226.

Smith, V.E. (1961). *Investment and Production*. Harvard Economic Studies, Vol. (117). Cambridge, Mass: Harvard University Press; Oxford University Press.

Spivey, A. (1958). Introduction to Mathematical Economics by D. W Bushaw; R. W. Clower, *Southern Economic Journal*, 25(1), pp. 105-106.

Walker, D.A. (1984). *Money and Markets: Essays in Honor of Robert W. Clower*. Cambridge: Cambridge University Press.

Weintraub, E.R. (1979). *Microfoundations: the compatibility of Microeconomics and Macroeconomics*. Cambridge: Cambridge University Press.

Weintraub, E.R. (1991). *Stabilizing Dynamics: Constructing Economic Knowledge*, Cambridge: Cambridge University Press.