

Boianovsky, Mauro

**Working Paper**

## Modeling Economic Growth: Domar on Moving Equilibrium

CHOPE Working Paper, No. 2015-10

**Provided in Cooperation with:**

Center for the History of Political Economy at Duke University

*Suggested Citation:* Boianovsky, Mauro (2015) : Modeling Economic Growth: Domar on Moving Equilibrium, CHOPE Working Paper, No. 2015-10, Duke University, Center for the History of Political Economy (CHOPE), Durham, NC

This Version is available at:

<https://hdl.handle.net/10419/149737>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**MODELING ECONOMIC GROWTH:  
DOMAR ON MOVING EQUILIBRIUM**

BY

MAURO BOIANOVSKY

CHOPE WORKING PAPER NO. 2015-10

OCTOBER 2015



## ***Modeling economic growth: Domar on moving equilibrium***

Mauro Boianovsky (Universidade de Brasilia)

[mboianovsky@gmail.com](mailto:mboianovsky@gmail.com)

Sixth draft (November 2016)

**Abstract.** The paper makes use of archive material to examine Evsey Domar's role in the introduction of the rate of growth as a variable in economics in the 1940s and 1950s. Domar investigated the nature of what he called the "moving equilibrium" of economic processes with infinite duration. Reactions to Domar's approach at the time brought about methodological assertions on the distinction between models and theories. Domar's model was an open one, in the sense that his growth equation allowed different closures. A main feature of the model was its relatively stable capital-output ratio, which reflected the terms of the debate about A.H. Hansen's stagnation thesis in the 1940s. The real economy was supposed to be stable, although the model itself was not perfectly consistent with that. The estimation of the "residual" by Solow and others led Domar to rethink aspects of his original model.

**Acknowledgements.** I would like to thank Robert Solow, G. C. Harcourt, Danilo Silva, Avi Cohen, Humberto Barreto, Moshe Syrquin, Joanilio Teixeira, Verena Halsmayer, Olav Bjerkholt, Kevin Hoover, Craufurd Goodwin, Romain Plassard, Herrade Igersheim-Chauvet, Matheus Paiva, Mauricio Couri and (other) participants at the 2015 meetings of the History of Economics Society (Michigan State University, June 26-29) and at a seminar at the Center for the History of Political Economy (Duke University, October 30 2015) for helpful comments on an early draft, and Elizabeth Dunn and Leonidas Montes for helping me with the Domar Papers. I've also benefited from helpful reports by two anonymous referees. A research grant from Capes (Brazil) is gratefully acknowledged. I would like to thank Alice Domar and Erica Domar Banderob for permission to quote from the Domar Papers (Economists' Papers Project, David M. Rubenstein Rare Book and Manuscript Library, Duke University).

We start by discarding the idea of equilibrium as a state of rest to which a stable system is expected to return ... Our economy never returns ... Together with static equilibrium must go out the notion that economic processes are finite – that they must ‘eventually’ come to an end. (Domar [1952a] 1957)

### **1. Setting the growth agenda**

Between January and April 1953 the Russian born American economist Evsey Domar (1914-1997) gave a series of lectures on growth economics at the universities of Oxford, Cambridge and Imperial College London. Domar’s (1944, 1946, 1947a, 1948a, 1952a) set of articles – later reprinted in his 1957 collection of essays – together with Roy Harrod’s (1939, 1948) had turned growth into a main field of research, as expressed in the so-called “Harrod-Domar growth model”. Harrod (1939) went unnoticed – probably because of World War II – until Thomas Schelling (1947) and others started to put him in the company of Domar, in the wake of the publication of Domar (1946, 1947a) and Harrod (1948). Domar (1947a, p. 42, n. 11a), who came across Harrod’s essay after writing his first contributions, was the first American economist to refer to Harrod (1939), according to JStor. As Domar ([1952a] 1957, p. 17) observed, Harrod’s “now famous creation of 1939” had to “wait for almost a decade and to be repeated in his [1948] book to receive its deserved recognition” (on the long neglect of Harrod’s essay by American economists, and Harrod’s complaint about it, see Samuelson 2002, p. 221). Nevertheless, Harrod’s more intricate formulation would eventually attract more attention than Domar’s.

For Domar, modeling economic growth involved a new concept of equilibrium in time, as illustrated by the epigraph above. Domar (1953a) called it “moving equilibrium” in the notes he made for his second Oxford lecture, delivered on January 27: “The essence of our approach [is] a moving equilibrium, not a return to a previous position. Emphasize this”. Domar effectively introduced into macroeconomics the “method of Growth Theory” (Hicks 1965, pp. vi, 13-14), based on the notion of the equilibrium of an expanding economy whose component parts grow at the same steady rate and retain some proper relationship to each other, i.e., according to an exponential function (Domar [1952a] 1957, p. 30).

Domar's ([1946] 1957, p. 82) central theme was *the rate of growth*, an analytical instrument that had been overlooked by economists despite some isolated attempts, especially Gustav Cassel's ([1918] 1932; 1935) "uniform progress" (Boianovsky 2009). Although both Harrod and Domar defined economic dynamics in terms of rates of growth (instead of lags as in much of the contemporary literature), they drew different implications from it. Whereas Harrod (1939, p. 21) stressed the instability of dynamic equilibrium, Domar ([1952a] 1957, pp. 32-34) focused on the analysis of the *infinite duration* of dynamic processes in growing economies, and its implications for the interpretation of long-run behavior of variables involving ratios between stocks and flows, such as the rate of profit, capital-output, external debt and foreign investment, the burden of public debt, and the depreciation and replacement of capital.<sup>1</sup>

This paper investigates Domar's role in the introduction of growth modeling into economics, as seen against the background of some theoretical and methodological debates of the post-war period. While Harrod's contributions were extensively debated by macroeconomists between the late 1940s and the 1960s, and by historians of economics ever since, Domar's articles have attracted only scattered attention, usually from the limited perspective of comparisons between aspects of his and Harrod's formulations (Young 1989 pp. 174-77; Asimakopulos 1986; Kregel 1987; Hagemann 2009; Niehans 1990, p. 454; Backhouse 1994 pp. 41-2; Muzhani 2014, pp. 55-68). As far as the history of economics is concerned, Domar has hitherto lived largely in the shadow of Harrod. Most of the commentaries on Domar's growth model are in articles by practitioners (Sunkel 1956; Harrod 1959; Frisch 1961 and Solow 2006), and in books on mathematical economics, macroeconomics, growth and development (Allen 1956; Hirschman 1958; Ackley 1961; Chiang 1967; Hamberg 1971; Wan 1971; Jones 1975).

The complex and sometimes obscure character of Harrod's (1939, 1948) formulation may explain why economists have paid relatively less attention to Domar (Solow 2006, p. 135). Nevertheless, behind the apparent simplicity of Domar's

---

<sup>1</sup> The first complete model of an expanding economy was advanced by John von Neumann in his famous 1937 German article, made available in English in 1946 together with D.G. Champernowne's (1946) "translation" for economists (Boianovsky 2016a). Unlike Domar's, it was a microeconomic general equilibrium model, rooted in the tradition of classical economics. Domar (1965) would briefly mention von Neumann's model only once. Harrod never referred to it.

growth models, there are important theoretical and methodological issues that are explored in some detail here.<sup>2</sup> Material available in the Domar Papers, together with careful reading of his published writings, shed new light on Domar's contributions to dynamics and his own perception of how they fit into the history of growth theory. Interest in growth economics surged in the postwar period not only from the recognition of growth as a condition for full employment in industrialized economies, but also from the international context of the Cold War and the widespread concern with economic development of poor countries.

Economists used to other approaches to growth reacted with skepticism to Domar's investigation of the formal characteristics of the equilibrium growth path. The first survey of the "Economics of Growth", produced by Moses Abramovitz (1952) for the American Economic Association (AEA), left out Harrod and Domar, on the grounds that "these theories, though often referred to as theories of growth, are, properly speaking, theories of the requirements of steady growth at full employment. They make no assertion with respect to the likely development of capital formation over time" (Abramovitz 1952, p. 170, n. 78). In contrast with Abramovitz's approach to growth as the study of the historical forces determining observed growth paths, Hahn & Matthews (1964, p. 779) surveyed exclusively steady state paths and abstract "models of economic growth", taking the "Harrod-Domar model" as their starting point.<sup>3</sup>

It was after the 1940s that the use of models became widespread in economics. That was also the period when growth economics established itself as a new field. These parallel developments in method and theory are tackled in this paper. "Countless craftsmen" had made the "Harrod-Domar model" the "most over-worked tool in economics", Solow wrote to Tobin in March 1959 (Halsmayer 2014, p. 235). Mary Morgan (2012) has studied the emergence of modeling in economics, with no reference to growth economics though. Verena Halsmayer and Kevin Hoover (2016,

---

<sup>2</sup> Whereas the mathematics of Domar's (1946, 1947a) growth models are relatively straightforward, his models with depreciation and replacement of durable capital goods, reproduced in Domar (1957), are mathematically more involved (Muzhani 2014, pp. 60-64). The same applies to his restatement of Fel'dman's 1920s Russian two-sector growth model and the problem of optimal allocation of capital goods, the only new chapter in his 1957 book (Jones 1975, sections 5.6 and 5.7; Boianovsky 2016b).

<sup>3</sup> Abramovitz did not deploy the term "growth path", which only gained currency after Solow & Samuelson (1953) and Solow (1956).

section 3.1) have discussed the rise of the culture of modeling in a growth context in the 1950s, with a focus on Solow. Halmayer (2015, pp. 62-66) has dealt with aspects of Domar's contribution to growth modeling, but, again, her main concern is Solow's (1956) role.

When Domar put forward his models, growth phenomena were studied mainly through historical approaches. The 1951 AEA meetings featured a session on "The theoretical analysis of economic growth", with papers by Domar (1952a) and David Wright (1952), who argued for the traditional historical treatment. Domar ([1952a] 1957, pp. 17-18) contrasted the "historical" approach to the subject (including his Harvard professor Schumpeter) with the "econometric" one, which relied on "highly simplified symbolic models". In a companion paper (Domar 1952b), he argued for modeling as the appropriate economic method.

Domar's approach to method had some similarities with Milton Friedman's (1952, 1953). Domar interacted extensively with Friedman during his stay as assistant professor at Chicago University in 1947-48. As he recalled, "this time my (informal) teacher was Milton Friedman. We argued in every place we met: in the lobby of the Social Science Building, in elevators, at social gatherings, in the street. I doubt that he got much out of these arguments, but I learned a lot. If the devil is already here, why not partake of his wisdom?" (Domar 1992, p. 122). Their dialogue was mainly about method, as there is no evidence of interest by Friedman (or his Chicago colleagues) in growth.<sup>4</sup> Friedman's methodology illuminates aspects of Domar's growth modeling and assumptions, particularly the stable capital-output ratio.

Domar's growth model is encapsulated by the "Domar equation" for the required rate of growth  $r$  in full employment equilibrium:  $r = \alpha\sigma$ , where  $\alpha$  is the marginal (= average) propensity to save and  $\sigma$  is the social average productivity of investment (the inverse of the capital coefficient). As Domar acknowledged, his model did not describe the actual growth path, but only the equilibrium one, with little discussion of whether the economy will follow that path – that is, no stability analysis. This led to the assessment that Domar provided just a "model", not a "theory" of growth. On the other hand, development economists often interpreted

---

<sup>4</sup> In an interview to Colander and Landreth (1996, p. 190), Domar recalled that there was "little interest" in growth theory in Chicago at the time, only in its formal aspects. "I presented a model at a [Chicago] seminar which turned out to be the most unsuccessful presentation I ever made. The formal structure of the model turned them on, but its content – not at all. They never let me finish".

Domar's model as making the prediction that the actual rate of growth is a positive function of the propensity to save. This has been critically called "capital fundamentalism", a proposition that cannot be squared with Domar's (or Harrod's) Keynesian formulation (Boianovsky 2015a). There was no a priori causality in Domar's growth equation, which established the relationship between the three variables  $r$ ,  $\alpha$  and  $\sigma$  necessary for full employment growth. If any two of them are given, the value of the third needed for avoiding running away from macroeconomic equilibrium is determined. Hence, Domar's model was an *open* one, with three possible closures.

In the same vein, commentators have overlooked the relation between Domar's relatively stable output-capital ratio and the secular stagnation problem discussed prominently by his Harvard professor and thesis supervisor Alvin Hansen (1941). Hansen was the main intellectual influence on Domar (Domar 1957, p. vii), whose 1947b PhD thesis – mentioned here for the first time in the literature – comprised four chapters published separately (Domar 1944; 1947a; parts of 1948a; 1948b). Domar's model intended to show that there is a rate of growth of income able to maintain full-capacity utilization of a growing capital stock and, by that, avoid diminishing profit rates and chronic unemployment. The economy can be in equilibrium only if it is growing.

Building economic models was just one element of theorizing, though. Domar (1957, p. 12) pointed out that a "satisfactory theory of growth" cannot be created "from models only", as it "requires a mass of empirical work" revealed by the study of secular facts. He was aware that the facts of economic growth were not entirely borne out by his model. In particular, due to the multiplier effect of investment, the model produced the "paradoxical" result that, given the propensity to save, to eliminate excess capacity, more capital should be built, while, to avoid a shortage of productive capacity, investment should be reduced (Domar [1952a] 1957, p. 31).<sup>5</sup> Consistently with observed data, Domar (1953b) stated that "we should explain why the economy is as stable as it is, rather than the other way around".

Modeling the supply side of the economy was a difficult task, which led Domar ([1952a] 1957, p. 26) to introduce "heroic simplifications" such as the concept

---

<sup>5</sup> Such "Paradox of growth" – as called by Domar 1953b in his Cambridge growth lecture – has some parallels with Harrod's (1948, p. 88) statement of the "paradox" that general overproduction is due to firms producing too little (Kregel 1987, p. 601).



of average productivity of investment, meant to capture the indirect influence of technical progress, labor supply, and other variables. But Domar (1957, pp. 7-8) was never satisfied with the way he handled the production function. Years later, he asserted that “my old growth models notwithstanding, I believe that a country’s most important factor of production is not its physical capital but its human one – the able, educated and trained manpower” (Domar 1989, p. xv). That was part of the “Residual”, as Domar (1961a) dubbed the estimation of the contribution of “technological progress” to growth after the contributions of capital and labor had been accounted for. The empirical dominance of technological progress, as compared to the relatively minor role played by capital, indicated to Domar the limits of his modeling strategy. At the same time, it prompted Domar (1961a) to go back to Leontief’s dynamic input-output system.

Apart from a brief passage in his 1957 preface (p. 8), sometimes interpreted as evidence of Domar’s capitulation to neoclassical growth economics (Easterly 2001, p. 28; Hagemann 2009, p. 80), Domar did not react in print to Solow’s (1956) essay. He did draft a note on what may look like “if some day I will write an article in answer to Solow” (Domar 1961c), discussed below. However, though neoclassical growth theory changed growth modeling, it was the empirical exercise performed by Solow (1957) and others that really caught Domar’s (1961a) attention.

## 2. The mathematics and facts of exponential growth

Domar, who held a degree in mathematical statistics from Michigan University (1939), based his growth modeling on the exponential function: “the logical foundation of these essays is built around a simple growth function – the exponential – and is based on the well known property of its integral to approach a constant multiple of the function itself as time approaches infinity” (Domar 1947c, pp. 2-3). If income grows at the rate  $r$ , we have  $Y = Y_0 e^{rt}$ . If a constant fraction  $\alpha$  of income is invested, then the sum of these investments ( $K$ ) equals  $K = \alpha Y_0 \int_0^t e^{rt} dt = \frac{\alpha Y_0}{r} (e^{rt} - 1)$ . Hence,  $\lim_{t \rightarrow \infty} \frac{K}{Y} = \frac{\alpha}{r}$ . By assigning various magnitudes and modes of behavior to the variables  $\alpha$  and  $r$ , Domar used this apparatus of accounting

relations to investigate dynamics. This is what Domar (1953a) called the “mathematics of growth” in his Oxford lecture of February 10.

The relation between stocks and flows had not been carefully discussed by economists, who had “almost completely neglected the balance sheet as an economic document” (Domar 1955a, p. 250). Cassel’s (1935, p. 24) proposition – that only in the case of exponential growth can the ratio between a stock and a flow be maintained – did not command attention until Domar ([1944] 1957, p. 48, n. 26) discussed it. Domar’s ([1944] 1957) essay on debt burden introduced the analytical implications of infinite economic processes in time. In a steadily growing economy, assuming the annual fiscal deficit is a constant fraction of income, the stock of public debt can grow indefinitely, since the ratio between debt and income converges to a constant given by the quotient between the budget deficit and the rate of growth (Eltis 1998, pp. 27-29).

Domar’s mathematics caught the attention of economists. William Baumol (1951, pp. 220-21) discussed Domar’s models as examples of the use of differential equations in process analysis. Samuelson’s (1948) survey of “dynamic process analysis” showed that any process that grows continuously at a constant rate is described by a differential equation as function of time, whose solution is an exponential expression. Samuelson (pp. 361-62) used Domar’s ([1944, 1946] 1957) debt and growth models to illustrate the application of differential equations to formal features of continuous economic processes, a practice followed by mathematical economics textbooks (Allen 1956; Beach 1957; Chiang 1967).

Domar deployed that apparatus to develop the central message of his PhD thesis, and of his growth research program as a whole.

Technological progress and saving place in our hands the potential power to achieve an expanding economy ... More than that, they make it imperative that such growth be achieved: the alternative is mass unemployment and destruction. But neither technological progress, nor of course saving, guarantees that the rise in income will actually take place. This rise depends on our economic policies. (Domar 1947b, p. iii)

Domar’s model was designed to show how the interaction between aggregate demand and supply over time makes growth a condition of equilibrium. Whereas, according to Domar ([1952a] 1957, p. 19, n. 7), pre-Keynesian economics focused on the capacity side and took adequate demand for granted, Keynesian economists

tended to ignore the problem of capacity altogether, as illustrated by Klein (1950), his colleague at the Cowles Commission in 1947-48. During his period at Cowles, Domar was asked by Jacob Marschak to comment on a draft of Klein's book. Domar (1948c, p. 1) argued that Klein's model had no supply functions of labor and capital and, therefore, no concept of productive capacity. "Evidently, any quantity of goods and services can be produced ... Goods are produced almost by magic. Is this an effect of the 'Keynesian Revolution'?"<sup>6</sup>

Investment appears in Domar's aggregate supply and demand functions, with asymmetrical effects: the former is a function of the (net) *level* of investment, while the latter depends on its *rate of growth*. A higher investment level has a permanent effect on capacity, but a temporary one on income as the multiplier mechanism peters out (Domar [1947a] 1957, p. 98). Hence, in order that sufficient demand is generated and capacity remains fully utilized it is necessary for investment to grow at a certain rate, determined by the equality between aggregate demand and supply.

The demand side of the model ( $Y$ ) is the Keynesian multiplier, with a marginal (= average) propensity to save  $\alpha$ , and investment  $I$  as the "active" or "independent" variable (Domar [1946] 1956, p. 74; [1952a] 1957, p. 20):

$$\frac{dY}{dt} = \frac{dI}{dt} \frac{1}{\alpha}$$

The supply or capacity side was harder to formulate, as "everything is involved here" ([1952a] 1957, p. 22). Simplification was essential in the estimation of changes in capacity.<sup>7</sup> Domar expressed "productive capacity"  $P$  – the level of output  $Y$  under full employment of labor – as a linear function of the capital stock  $K$  and its average productivity  $s$ . Hence,  $P = Ks$  and  $Y \leq P$ . He assumed that the average and marginal (incremental) output-capital ratios were the same.

---

<sup>6</sup> This is an implicit reference to Klein's 1947 book with that title. Domar's discontent had a parallel in Patinkin, who at the time criticized Keynesian economics for disregarding supply. Patinkin would later change his mind about the lack of supply function in Keynes's *General Theory* (Boianovsky 2002). Domar and Patinkin were in close contact during their stay in Chicago in 1947-48 (Patinkin 1981, p. 16).

<sup>7</sup> Domar (1948c, p. 1) remarked that "it is an accepted principle of scientific method that an explanation should contain as few variables as possible. We should be justified in constructing a more complex system only after it has been shown that the simpler one was unsatisfactory." This is an implicit reference to Occam's Razor, a term he did not use.

Domar ([1946, 1947a] 1957) at first distinguished between  $s$  and the “potential social average productivity of investment” for the whole economy  $\sigma$ , which took into account the transfer of labor and other inputs from old plants into new ones, misdirection of investment and the effect of competition and obsolescence on capital values, a process he called “junking” of part of new capacity<sup>8</sup>.  $\sigma$  was not the marginal productivity of capital or a partial-derivative concept, since it indicated the increase in capacity “which *accompanies*, rather than which is caused by” investment (Domar [1947a] 1957, p. 90; see also Hamberg 1971, p. 5, n. 6). The model’s supply-side equation is given by

$$\frac{dP}{dt} = \sigma I$$

The maintenance of full employment over time requires that productive capacity and income grow at the same rate, expressed by Domar’s ([1946] 1957, p. 75) “fundamental equation”

$$\sigma I = \frac{dI}{dt} \frac{1}{\alpha}$$

The solution of this differential equation is  $I = I_0 e^{\alpha\sigma t}$  (Chiang 1967, pp. 419-22).<sup>9</sup> Investment must grow at the exponential rate  $\alpha\sigma$  for continuous maintenance of full-employment income. Since the marginal and average propensities to saving are assumed equal and constant, income must also grow at the rate  $r = \alpha\sigma$  in equilibrium. This is a formula for the *required*, not actual growth rate. In particular, the larger the saving propensity  $\alpha$ , the larger is the increase in income required to avoid excess capacity. Under the assumptions  $P = Ks$  and  $\sigma = s$ , Domar ([1946] 1957, p. 77) used his equation  $\lim_{t \rightarrow \infty} \frac{K}{Y} = \frac{\alpha}{r}$  to show that the “coefficient of utilization”  $\theta$  of capacity is expressed by  $\theta = \lim_{t \rightarrow \infty} \frac{P}{Y} = \frac{r}{\alpha\sigma}$ . For a given actual rate  $r$ , a higher  $\alpha$  brings about excess capacity and unemployment.

Since investment demand and its rate of growth are autonomous constant variables, the “coefficient of utilization”  $\theta = \frac{r}{\alpha\sigma}$  is also constant (Domar’s [1946] 1957, p. 77). For given  $r$ ,  $\alpha$  and  $\sigma$  the disparity between productive capacity and

---

<sup>8</sup> See also Wan 1971, p. 25; Asimakopulos 1986, pp. 284-85; and Solow 2006, p. 135. Usually  $\sigma < s$ , but capital-saving inventions in old plants may cause  $\sigma > s$  (Domar [1947a] 1957, p. 76, n. 11).

<sup>9</sup> Erik Lundberg (1937, p. 185, n. 1) put forward a similar derivation, elaborating on Cassel’s uniform progressive economy (Boianovsky 2009). Domar ([1952a] 1957, p. 17) remarked that Lundberg’s equation, buried in a footnote, went unnoticed.

income when the actual rate of growth is below the required one does not become wider, as the capital stock grows at the  $r$  rate, not at the  $\alpha\sigma$  rate. Domar's model can be, therefore, described as "relatively stable" (Frisch 1960; Rijkeghem 1966). However, if investment is a function of the coefficient of utilization and of the effect of the junking process on asset values, as verbally suggested by Domar ([1946] 1957, pp. 78-79; [1947a] 1957, pp. 99-100) and formalized by Wan (1971, pp. 24-29), the economy may break down. Domar ([1952a] 1957, pp. 31-32) believed this contradicted the observed growth path of industrialized economies. Stabilizers, absent from the formal model, were invoked to explain economic recoveries and steady growth.<sup>10</sup>

Domar used in various ways his demonstration that in the limit the ratio of capital to output will approach the ratio between the fraction saved (and invested) and the rate of growth. He argued that, so long as the propensity to save and the rate of economic growth remain constant in the long run, capital deepening is absent, as illustrated by the US over the last 70 years before World War II (Domar [1946] 1957, p. 77).<sup>11</sup> Citing data from Kuznets (1946), Domar ([1946] 1957, p. 75; [1952a] 1957, p. 26; 1955, p. 257) assumed 12% as the average propensity to save, 3% as the long-term average rate of growth of output and 2% as the rate of growth of labor productivity. Moreover, he expected the rate of profits to remain steady as well (Domar [1952a] 1957, p. 32). The rate of profits tends to equal the share of profits in income multiplied by the ratio between the rate of growth of income and the propensity to save, as indicated by Domar's expression for the capital-output ratio. It may also be expressed as equal to the output-capital ratio times the share of profits,

---

<sup>10</sup> "Fortunately, we can borrow a few stabilizers from the real world, such as changes in the propensity to save, the monetary system, and others ... Technological progress, population movements, new firms, changes in tastes, etc., play an important role ... Technological progress and other changes, which, from a static point of view, appear destabilizing may indeed turn out to be the chief stabilizers of a growing economy" (Domar [1952a] 1957, pp. 31-32).

<sup>11</sup> This is distinct from assuming a given (or relatively stable) output-capital ratio on the grounds of fixed proportions between capital and labor. The distinction between the stability of the capital-output ratio as a feature (implication) of the steady state and (an assumption) of the production function would only become clear after Solow (1956).

which had both remained stable in the US. He estimated 6% as the secular value of the rate of profit (Domar 1945).<sup>12</sup>

Domar ([1952a] 1957, p. 32; 1955, p. 256) claimed that, contrary to the hitherto dominant view, capital deepening and diminishing rate of profits are not attributes of the capital accumulation process. The growth trends mentioned by Domar correspond closely to Nicholas Kaldor's (1961, p. 178) later famous "stylized facts". Again similarly to Kaldor, Domar's secular facts provided a checklist of the kind of abstraction the growth model builder should choose. The contributions by Harrod, Domar and Solow to growth economics shared the common feature of "pragmatism", in the sense of the measurability of structural parameters and the applicability to policy issues. Unlike the trend of growth models in the 1960s, their efforts were never meant to be spent on "purely intellectual exercises" (Wan (1971, p. 10).

The preference of both Domar ([1946] 1957) and Solow (1956) for simple manageable models has been also discussed by Halsmayer (2015, p. 253). She has argued that they differed as much as Solow's use of an aggregate production function paved the way for empirical growth economics, while Domar's model was just a "tool for reasoning". Whereas Solow's (1957) growth accounting did attract Domar's (1961a) attention, it should be noted that Domar's models were empirically grounded, not only because of his discussion of secular (stylized) facts. He used his equation to estimate 3.6% as the US equilibrium growth rate (assuming 12% and 30% as the values of  $\alpha$  and  $\sigma$ ) in 1879-1941, as compared to an observed actual rate of 3.3% over that same period (Domar [1946] 1957, p. 75; Backhouse 1994, p. 41). True enough, he did not attempt to use his growth equation to estimate the sources of economic growth. According to Domar ([1944] 1957, p. 59), American economic growth in the period 1879-1928 was "due to technological improvements, growth of the labor force and the discovery of new resources" – this reflected Hansen's influence and was also reminiscent of Harrod's natural rate of growth (which Domar, however, did not incorporate into his own model).

---

<sup>12</sup> Thomas Piketty (2014, pp. 52 and 166) has called Domar's expressions for the rate of profit and the capital-output ratio the first and second "fundamental laws of capitalism" respectively. Piketty does not refer to Domar in that context. He is mainly interested in the long-term fluctuations of the capital-output (or wealth-income) relation caused by decreased growth.

### 3. Growth models and growth theory

The AEA 1952 *Survey of contemporary economics* included an essay by Richard Ruggles on “Methodological Developments”, with comments by Domar and Friedman. Both Domar (1952b) and Friedman (1952) criticized Ruggles for stating that the purpose of economic research was to “obtain an exhaustive list of all determining variables” (Ruggles 1952, p. 441). Friedman’s (1952) rejection of “descriptive realism” anticipated in a nutshell his 1953 thesis about the (largely) irrelevance of the realism of assumptions in assessing theories or hypotheses, which should be evaluated on the basis of their predictive power (see Mäki 2009). Domar (1952b) did not refer to testing, but to the role of models in the formulation of hypotheses. The essence of economic research

Consists not in copying economic reality, as [Ruggles] seems to imply – this would be both impossible and useless – but in extracting from it a few easily manageable key factors and constructing from them a model, which may be expressed in words or symbols, or even implied, but which for a given purpose can be used as a *substitute for reality itself*. (Domar 1952b, p. 454; italics added)

Domar ([1952a] 1957, p. 22) used the same italicized phrase in his *AER* article, when he claimed that “simplification is the heart of this process”. According to Domar (1952b, p. 454), Ruggles added to the reigning “confusion” about the nature and purpose of the process of abstraction. Contrary to the prevailing view, “such a model is not intended to be a factual statement about real processes, and different and even contradictory models can be legitimately set up regarding the same process.” Domar (pp. 454-55), however, warned that “the solution of a model is a solution of a logical system and nothing else; to endow it with economic significance is a very fine art”. He did not refer on that occasion to growth modeling, but did it extensively in his companion 1952 *AER* piece. Instead of adopting the rather difficult “direct approach” of tackling the determinants of economic growth (technological progress and capital accumulation) and their causes, Domar ([1952a] 1957, pp. 29-30), “like a not-too-honest schoolboy who cannot solve his problem”, preferred to *assume* that the

economy is growing and investigate the conditions that should be satisfied to make such growth possible in equilibrium.

Economic growth had, of course, attracted the attention of economists before the economic modeling era. Domar ([1952a] 1957, p. 17) referred to Adam Smith and Karl Marx, who came close to formulating a growth model “interpreted broadly”. According to Domar (1967 p. 3), “Marx was full of ideas about growth. And, in a way, you can say that his was a grand model of economic development. Nowadays it is somewhat old-fashioned as Marx never used calculus. And, after all, what is a model good for unless it uses symbols and a good deal of mathematics?” In the 1950s Domar came across G.A. Fel’dman’s 1928 two-sector growth model, based on Marx’s expanded reproduction schemes and regarded by Domar as the first formalization of Marxian growth theory. Domar’s (1957, chapter 9) careful restatement and extension of the mathematical properties of Fel’dman’s model brought it to light after decades of oblivion in Russia and elsewhere (Boianovsky 2016b).

Ingvar Svennilson (1958, p. 1007) noticed that Domar’s models were not intended for the interpretation of historical processes of growth. They did not provide a “*theory* of growth” which takes into account the economic behavior behind the determination of parameters in the market economy. Domar’s problem, according to Svennilson, “is not why and how growth takes place”, as he “investigates only the *balance* of growth” between capacity and demand over time. Such distinction between growth models and growth theories may be found in other commentators as well.

In one of the first collections of readings on growth and development, Okun & Richardson (1961, part 2) reproduced Domar (1947a) together with chapters from Smith, J.S. Mill, Marx, Schumpeter, Hansen and others. In their editorial introduction, Okun & Richardson (p. 28) argued that “a theory is designed to explain and perhaps to predict some process or phenomenon”. Whereas most growth theories made pessimistic predictions, “Domar’s ‘model’ cannot be termed pessimistic, if only because it makes no prediction of any kind” (p. 31). The model derived the conditions for equilibrium growth over time, but did not stipulate whether these conditions will be realized. Hence, “the Domar *model* differs from the previous *theories* in two fundamental respects: first, it contains no prediction concerning long-run growth trends; secondly, it is concerned less with explaining the forces that account for



growth than with explaining why the *path* of growth is likely to be strewn with pitfalls.”

In the same vein, Gardner Ackley (1961, p. 517), author of the first macroeconomic textbook to discuss growth, remarked that Domar “did not pretend to provide a theory of growth”, but just “*describe[d] an equilibrium growth path*” with no excess or shortage of capital, with little indication of what might cause the economy to be on that path or not. Ackley (p. 518) distinguished Domar’s model from Harrod’s more “ambitious” aim to provide a “theory” which explained the convergence (or lack of it) to steady growth. Hahn (1958, p. 353) too criticized Domar’s unconcern with “stability” analysis of growth. Hywell Jones (1975, pp. 2-3, 64) repeated the point that the latter provided a model but not a *theory* of growth.

The description of Harrod’s (1939, 1948) theoretical framework as a “growth model” – and its eventual merging with Domar under the guise of the “Harrod-Domar model” – was disputed by Ackley (1961) and Enke (1963), and more recently by Besomi (2001, pp. 86-88) and Halsmayer & Hoover (2016). Whereas Harrod investigated the conditions for equilibrium at a *point of time* – and therefore did not have to assume stability of parameters – Domar’s equilibrium dealt with the rate of growth *over time* (Enke 1963, pp. 175-76; Asimakopulos 1986, pp. 291 and 297). Harrod never protested against the merging of his growth equations with Domar’s, though (Harrod 1959). What he did react against was the notion of a Harrod-Domar growth *model*.

According to Harrod (1960, p. 277; 1968), a model is a formulation that can be statistically verified and empirically tested. He referred to his dynamic equations as “prolegomena to model building” (1960, *ibid*). Harrod (1956, 1968) did not oppose modeling in general (but see Morgan 2012, p. 12 for a different interpretation). A close reading of Harrod’s and Domar’s methodological statements indicate that they were not far apart, as both called for empirical testing as a necessary ingredient of (or complement to) economic models. However, Domar’s concern with steady growth led him to emphasize the measurability of variables, such as “productive capacity” of the economy ([1952a] 1957, p. 19). Domar (1955) saw his accounting formula for the capital-output ratio as a benchmark for investigation of time series. In a way, Domar’s ([1952a] 1957; 1952b) concept of models as a first step into economic research was more restricted than Harrod’s (1968, p. 174), who asked “what is the relation of a model to a hypothesis or to a theory?” According to Harrod (p. 175), the model

becomes a theory when its parameters are statistically estimated and cease to be “adjustable”, which was not the case of his growth equations, particularly in their tautological form.

Domar, together with his contemporaries who dealt with linear programming or dynamic input-output systems, was at the forefront of modeling, and presented some of the arguments for modeling that came up in later discussions. Paul Streeten (1968) discussed the distinction between theories and models, as well as the notion that models are not true/false but useful/useless or valid/invalid, in direct connection with Domar’s growth equations. More recently, Daniel Hausman (1992, 2015) and Axel Leijonhufvud (1997) claimed that, unlike theories, models are not propositions about the real world but conceptual explorations. This is consistent with Domar’s view that the solutions of models are not statements about real processes, but solutions of logical systems.<sup>13</sup> Such elements are discussed further in the next sections.

#### **4. The nature of the assumptions and alternative closures**

Domar’s model did not depend on the assumption of absolute stability of  $\sigma$  and  $\alpha$ . What Domar ([1946] 1957, p. 76 top; [1948a] 1957, p. 111; [1952a] 1957, p. 27) did assume is that appropriate changes in those ratios cannot happen fast enough every time the existence of equilibrium growth is threatened. Should an actual growth rate below the equilibrium one pose the danger of excess capacity, it was only assumed that  $\sigma$  does not abruptly rise and bring the equilibrium rate down to the actual one (Fellner 1951, pp. 117-19; Eisner 1958, p. 710; Hamberg 1971, p. 15). It was also presumed that disequilibrium is not quickly corrected by suitable changes in  $\alpha$  able to produce shifts from a “high investment” to a “high consumption” economy whenever

---

<sup>13</sup> Morgan (2012), on the other hand, argues that creating models is “world-making”, in the sense that models become an “imagined analogous world” (p. 24). Eventually, the model “moves from being the lens that enable economists to interpret the world in the new way to being the things they find and see in the world” (p. 406). Domar’s (1952b, p. 454) assertion that, for a given purpose, the model can be used as a “substitute for reality itself” has some similarity with Morgan’s perspective. Indeed, Hirschman (1958, p. 30) remarked that the best measure of success of Domar’s model is that “today we must pinch ourselves to remember that it is theory rather than a faithful photographic copy of reality”.

investment demand comes down.  $\sigma$  and  $\alpha$  could be made functions of certain variables, but “as a first approximation” Domar preferred to assume constant values for both, which “simplifies the mathematics enormously” ([1952a] 1957, p. 27).

Domar’s argument for a relatively stable  $\sigma$  was not empirical but theoretical. It “must have some stability”, claimed Domar ([1948a] 1957, p. 111), for if it “can be anything” investment opportunities are unlimited and the “problem of capital accumulation” that he tried to solve through his growth model does not exist in the first place. The position Domar criticized was sustained by Henry Simons (1942) – who claimed that the demand function for durable capital goods becomes infinitely elastic at very low interest rates – and Frank Knight (1944). Knight had an inclusive concept of capital as encompassing all production factors that can be accumulated, leaving no fixed factor as source of diminishing returns (see Abramovitz 1952, p. 172; Patinkin 1981, pp. 32-33).<sup>14</sup> In contrast, Domar assumed that the possibilities of capital deepening are limited and, therefore, capital accumulation (at a given income level) is accompanied by diminishing returns and exhaustion of investment opportunities. In its “most definite and explicit form”, this view (which he associated with Hansen, Harrod and Paul Sweezy) was based on the notion that there is a “fairly stable relation” between a given amount of output and the stock of capital needed to produce it (Domar [1948a] 1957, pp. 109-10). Domar’s assumption of a stable capital-output ratio reflected his endorsement of Hansen’s hypothesis of strong diminishing returns as the capital stock increases relative to labor supply and natural resources at a given level of technical knowledge (Backhouse & Boianovsky 2016).

Empirical evidence was not conclusive enough to settle the issue. Domar’s claim was based on the argument that the observed existence of cyclical fluctuations associated with the capital accumulation process provided indirect evidence of the stability of the capital-output ratio. As put by Friedman (1953, section 4), in stating the “crucial assumptions” of a theory one is attempting to assert the essential elements of the abstract model. A model may be described by different sets of postulates that both imply and are implied by the model itself. A “crucial assumption” – such as Domar’s stability of the capital-output ratio – can be used to obtain indirect evidence on the acceptability of the theory, to the extent that it can be regarded as an

---

<sup>14</sup> Knight (1944) has been interpreted as an anticipation of the *AK* model of endogenous growth for not featuring diminishing returns to capital (Barro & Sala-i-Martin 1995, p. 39).

implication or can bring out other testable implications of the theory (Blaug 1992, p. 93). Slow diminishing returns to capital accumulation were not enough to characterize the “problem of capital accumulation” (Domar 1949a, p. 311).

Domar (1955a, p. 250; [1947a] 1957, p. 94) criticized neoclassical theory of the firm for assuming high elasticity of substitution between capital and labor, together with continuously and slowly changing cost curves. These entailed neglect of the relationships of capital and labor to output and of the concept of productive capacity of a plant or of the whole economy. The ideal approach to estimate changes in capacity was the use of a “comprehensive production function” with some degree of substitution between factors and products (Domar ([1952a] 1957, p. 23; 1955, p. 252). Despite previous work by P. Douglas and C. Cobb, an adequate production function of this type “we do not as yet have”. Domar ([1952a] 1957, p. 23) regretted that “no attempt had been made ... to attach it to a demand function” to obtain the required equilibrium rate of economic growth. He refrained from using the Cobb-Douglas function because it seemed “too complicated” (ibid) compared to his supply function  $I\sigma$ . Domar mentioned Tinbergen (1942), but preferred to “express the idea of growth in the simplest possible manner”.<sup>15</sup>

One important feature of Domar’s notion of “productive capacity”, and the capital-output ratio associated with it, was its measurability. Domar (1955a, p. 252) referred to information about ratios between inputs and outputs à la Leontief (1941). Leontief’s input-output system had been met with skepticism because it made no use of “our pet theoretical toys” and employed instead allegedly constant input coefficients not derived from profit maximization (Domar ([1952a] 1957, p. 29). However, the great virtue of Leontief’s model was precisely to get good results without using these concepts and show “their proper places as servants to be called in if and when required” (ibid). Such remarks applied to Domar’s own model.

The calculation of  $\sigma$  was a technical, not behavioral, issue. As Domar (1955, p. 251) observed, Harrod (1939, 1948) approached the same question from the opposite end: the backward relation between a given rise in income and the induced amount of investment, instead of the forward one between investment and the

---

<sup>15</sup> Domar ([1947a] 1957, p. 96) was the first to refer to Tinbergen (1942), who anticipated the neoclassical growth model (Boianovsky & Hoover 2009a, pp. 7 and 10-11). He warned that “the construction of complex models requires so many specific assumptions as to narrow down their applicability” (Domar, ibid).

resulting increase in capacity. It was not just a matter of the order of events, for, in Harrod's treatment, "the ratio between capital and output depends on psychological responses of firms to a given rise in income and it is not readily ascertainable in quantitative terms; it may also differ considerably from the magnitude of the capital coefficient in the more usual sense" (Domar, 1955, p. 251, n. 6).<sup>16</sup>

The distinction between Domar's and Harrod's coefficients has been occasionally acknowledged by commentators (Hirschman 1958, p. 30; Asimakopulos 1986, pp. 293-94). Domar (1953b, p. 3) called attention to differences between Harrod's and his own approach, based on investment as an "independent variable". In correspondence with Hansen of April 14 1965, Domar referred to a "major problem" in Harrod's model: the distinction between the "capital-output ratio (or capital coefficient)" on one hand, and "the reaction of investors to a rise in income (or the accelerator coefficient)" on the other hand (Domar Papers, Box 4).

During his period at Cowles, Domar attempted to model investment decisions (Domar 1947d, 1948b, 1949a; Boianovsky 2016b). Domar did not reproduce those papers in his *Essays*, where he stated that economists had been unable to derive a satisfactory investment function. He chose instead to "admit our ignorance" and treat investment as the model's independent variable (Domar [1952a] 1957, p. 21). The ensuing drawback, from Domar's (ibid) standpoint, was the inability to provide a "mutually determined and self-propelling system like a business cycle model, capable of tracing the path which output will actually take". The model was limited to the determination of the required rates of growth of investment and output to prevent excessive capital accumulation and secular stagnation.

Another significant (and related) feature of the "Domar equation" is that it established an equilibrium relationship between three variables, which could be put various ways round (Robertson 1954, p. 184; Kaldor 1961, p. 181-82). As Domar ([1947a] 1953, p. 97) explained, the original way was the form  $r = \alpha\sigma$ , which

---

<sup>16</sup> Harrod (1959, p. 452) stressed instead the formal equivalence between the equations, based on the notion that both capital coefficients assume that new investment is "properly utilized". If Harrod's warranted rate  $G_w = s/C_r$  is really an equilibrium rate, his accelerator coefficient  $C_r$  must be the technical incremental capital-output ratio, not just a behavioral coefficient. The induced investment must correspond to the amount needed to provide the added capacity to increase income (Hamberg 1971, pp. 10-11).

determined the required equilibrium growth rate for given  $\alpha$  and  $\sigma$ . The equation may be also solved for  $\alpha$  (in terms of  $r$  and  $\sigma$ ) and for  $\sigma$  (in terms of  $r$  and  $\alpha$ ), assuming that  $r$  should be treated as given, “for instance by technological progress”. Hence,  $\alpha = r/\sigma$  is the propensity to save required to bring the economy to its full employment growth path. It may be compared to Harrod’s (1960) equation for the “optimum saving” ratio (see Boianovsky 2015b). Finally, Domar’s equation may be used to determine the capital-output ratio consistent with full employment growth:  $\sigma = r/\alpha$ .<sup>17</sup> With this closure, the capital-output ratio becomes the dependent variable, just as in Solow’s (1956) and Swan’s (1956) formulations. The choice of how to close the model depended on the kind of problem addressed, but empirical evidence also played a role as discussed below.

The comparison between Domar’s growth equations and other formulations is complicated by the fact that he lacked Harrod’s (1939, 1948) “natural rate” as the rate of growth of labor supply in efficiency units (Robinson 1952, p. 44). The concept is implicit in Domar’s ([1944] 1957, p. 57) reference to the growth rate determined by population growth and technological progress as a “ceiling”. That is probably what Domar ([1947a] 1957) had in mind when he loosely described  $r$  as given by “technological progress” in his equations for the equilibrium values of  $\alpha$  and  $\sigma$ . As Harrod (1959, p. 456) remarked, the growth rate in Domar’s equation  $r = \alpha\sigma$  is really Harrod’s natural rate, since the shortfall of  $\sigma$  in respect with (Domar’s)  $s$  is caused by insufficient growth of labor supply, which squeezes potential growth to  $\alpha\sigma$  (see also Jones 1975, p. 64; Kregel 1987, p. 601).

Domar’s versions of his equation are alternative formulations of equilibrium growth, with no discussion of how and whether it is reached. In particular, his Solow-like equation  $\sigma = r/\alpha$  was not accompanied by references to the economic mechanism that may bring the capital-output ratio to such equilibrium value, as that was beyond Domar’s scope. “Why has not the price mechanism been mentioned even once?” asked Domar ([1952a] 1957, p. 25) rhetorically. Domar ([1946] 1957, p. 71; [1948a] 1957, p. 114) assumed given relative prices and price level, and was called to task for that by E.H. Stern (1949, pp. 1162-63) and Hahn (1958, p. 353). Domar (1949b, p. 1171) agreed that relative price changes perform important functions, but refused to

---

<sup>17</sup> It should be noted that this is an equilibrium condition, whereas Domar’s similar expression  $\lim_{t \rightarrow \infty} \frac{K}{Y} = \frac{\alpha}{r}$  is an accounting identity.

accept that their flexibility would ensure equilibrium. Building models to that effect had long been a “favorite pastime” of economists, but recent discussions by O. Lange, D. Patinkin and others had shown that “flexible prices are a remedy which may either cure or intensify the disease”.<sup>18</sup> Hence, that assumption was not seen as particularly restrictive.

## 5. Domar after Solow

Solow recruited Domar to MIT in 1958, soon after his 1957 collection came out. In the foreword, Domar (1957, p. 8; see also n. 10 on p. 23) remarked that Solow (1956) had shown “how a growth model can be enriched by the use of a not very complex but less rigid production function”. That passage has been interpreted as evidence that Domar endorsed Solow’s neoclassical model and gave his own approach up.<sup>19</sup> However, as indicated in his published writings mentioned above and in the note he drafted about Solow’s model, Domar did not see the addition of a neoclassical production function as contradictory with his overall Keynesian approach to growth. The Solow model did not feature investment as an independent variable (or any investment function for that matter). Indeed, Solow (1962, p. 76) would make clear that “the relation between investment and output is two sided, as I think I once heard my friend Evsey Domar say, and I am concerned here with the supply side only”.

Domar’s assumption of constant capital productivity was made mainly for mathematical convenience. As acknowledged in his reaction to Pilvin (1953), constant input coefficients are a simplification that should be “used with care, particularly over longer periods” (Domar 1953d, p. 561). Whereas economists in the past tended to exaggerate the degree of flexibility of the economy, “some of our contemporaries may be in danger of disregarding it altogether” (ibid). Domar (p. 562) conceded to Pilvin (1953) that a production function featuring some substitution

---

<sup>18</sup> On Lange and Patinkin see Backhouse & Boianovsky 2013, chapter 2, and references cited therein. In correspondence of 3 November 1948, Domar asked Patinkin for reprints of his 1948 *AER* article, which Domar wanted to assign to students (Don Patinkin Papers, Box 26).

<sup>19</sup> The same (mistaken) interpretation has been made by the author (Boianovsky & Hoover 2014, p. 206). Kregel’s (1987, pp. 601-02) reading of Domar ([1952a] 1957, p. 23) – as stating that the introduction of a Cobb-Douglas function would lead directly to pre-Keynesian results – is also unwarranted.

between factors was more general than the production function implied by “existing growth models”.

Pilvin (1953) commented on Hamberg’s (1952) claim that Harrod and Domar had not acknowledged the existence of two distinct equilibrium grow paths, one required for full employment of labor and another for full utilization of capital. Pilvin argued that there always exists a common rate of growth of investment that can produce, via the multiplier, sufficient aggregate demand so as to fully utilize both the total stock of capital and total supply of labor, on the assumption that substitution is possible between them. Like Domar’s, his model consisted of two parts: a production function and an “income determining system”. Pilvin intended to generalize Domar’s model, not to challenge it (see Ackley 1961, chapter 19, who used similar diagrams to depict both Domar’s and Pilvin’s models).

Whereas Harrod reacted to the neoclassical growth model by restating his concept of the natural rate of growth in the context of dynamic welfare economics (Boianovsky 2015b), Domar remained largely silent about it. Around 1961, he drafted a couple of pages indicating his intention (never fulfilled) to write a response to Oshima (1959), Eisner (1958) and particularly Solow (1956). In equilibrium, the “demand” sides of Domar’s and Solow’s models were essentially the same, as Domar imputed the multiplier to Solow and assumed that planned and actual investment are the same. “Where we disagree, apparently, is on the supply side, that is, the capacity side” (Domar 1961c).

If a Cobb-Douglas function with capital exponent  $\beta < 1$  is used to describe the “supply” side of Solow’s model, than the effect of higher investment on income capacity is lower than in Domar’s equation, as Domar (1961c) observed.<sup>20</sup> However, the real point of disagreement came down to the full employment assumption. Assuming full employment of labor, the net effect on income capacity will be lower to the extent that it entails taking workers off old projects (that is,  $\sigma < s$  in Domar’s notation). Assuming unemployment, the effect of higher investment on income capacity will be much higher, claimed Domar (ibid).

---

<sup>20</sup> The increase in income will equal the average productivity of capital multiplied by  $\beta$ , which expresses diminishing returns to capital. Solow (1956) actually assumed a general constant-returns-to-scale function, which included the Cobb-Douglas and the fixed coefficient (Leontief) production functions as particular cases.



His notes suggest that the stability issue should be discussed in terms of the dynamic disequilibrium path, when the rate of growth of output differs from the required equilibrium one. It was not a matter of flexibility of the capital-output and capital-labor ratios as in Solow's model, but of entrepreneurs' reaction to excess capacity if output does not grow at the required rate, which introduced "an element of instability". The view that investment is a function of idle capital "is not contradicted by the use of a Cobb-Douglas production function", Domar (1961c) claimed. The same argument applied to the lower tuning point, determined by "Schumpeterian concepts" such as technological progress. "But this again is independent of one formulation or another. Perhaps I should write a paper along these lines". He never did.

What caught Domar's attention was Solow's (1957) growth accounting exercise. He found "striking" the result that the larger share of the increase in income per capita in the US was attributable not to capital formation but to technological progress (Summary record of the debate, Lutz and Hague 1961, p. 339). Empirical work done by Solow, Abramovitz and Kendrick inaugurated a "new act" in the historical play about growth models (Domar 1961a, p. 709). The previous act had featured the "so-called Harrod-Domar models", in which capital, "supported by an invisible chorus of labor, land and technological progress", held the stage. Now, they all appear together on the stage, with the first three "reading from the script while technological progress holds for the rest of the time". Domar did not see Solow (1957) as a test of the validity of the neoclassical vs. the "Harrod-Domar" models, but the empirical findings did affect the way he looked back at those models.

Domar's (1955a, 1961b) calculations of the capital-output ratio indicated its relative stability over time. As seen above, the capital coefficient tends to approach the ratio between the fraction of output invested and the rate of growth of output. Domar warned about the interpretation of such stability, calling attention to the open character of his equation.

This does not necessarily presuppose any specific causal relationship between these three variables. It is possible that the capital coefficient and the fraction of output invested have yielded a certain rate of growth of output. But it is also possible that other factors besides capital have been mainly responsible for the existing rate of growth of output, and the given capital coefficient has simply resulted from the interaction of the other variables. Thus the relative stability

of the capital coefficient is not a sufficient indication of the role of capital formation in economic growth (Domar 1961b, p. 103).

It was an empirical issue. In view of the findings about the contribution of technological progress to growth, the capital coefficient “will emerge as a relatively passive result of the interaction between the propensity to save and the rate of technological progress” (ibid, p. 117). This corresponded to Domar’s third closure  $\sigma = r/\alpha$ . The way “technological progress” was calculated meant that it captured the influence of variables like education, economies of scale, external economies, etc. To emphasize the nature of that concept, Domar (1961a, p. 709) called it the “Residual”.<sup>21</sup>

As Domar (1961a, p. 710, n. 3) observed, Tinbergen (1942) had anticipated Solow (1957). However, Solow introduced a new method to calculate the Residual involving the distinction between shifts of and moves along the aggregate production function that caught Domar’s imagination (Griliches 1996). Given an aggregate linear homogenous production function and assuming  $A$ , a shorthand for any kind of shift in the production function, is (Hicks’s) neutral, Solow wrote

$$Y = A(t)f(L, K)$$

Assuming further that factors are paid their marginal productivities, the rate of growth of output  $\bar{Y}$  is given by the rate of technological progress and the rates of growth of labor and capital as in the expression below, a result “very simple and valuable for us” (Domar 1961a, p. 711):

$$\bar{Y} = \bar{A} + \alpha\bar{L} + \beta\bar{K}$$

so that

$$\bar{A} = \bar{Y} - \alpha\bar{L} - \beta\bar{K}$$

with  $\alpha + \beta = 1$ . The rate of technological progress can therefore be obtained as a Residual. Assuming a specific production function of the Cobb-Douglas kind

$$Y = AL^\alpha K^\beta$$

The expressions for  $\bar{A}$  and  $\bar{Y}$  can then be obtained by taking logarithms and differentiating with respect to time.

---

<sup>21</sup> In editorial correspondence of 25 April 1961 about Domar 1961a, Harrod wrote: “Your article strikes me as most interesting and important” (Domar Papers, Box 3).

Domar next compared Solow's (1957) model with the "so-called Harrod-Domar variety" based on a constant  $K/Y$  ratio or  $\bar{K} = \bar{Y}$ , still under the assumption of a Cobb-Douglas function. Since there is substitution between capital and labor, a constant capital coefficient should be interpreted in a "historical rather than in a technical sense". The expression for  $\bar{A}$  is changed into  $\frac{\bar{A}}{\bar{Y} - \bar{L}} = \alpha$ . Domar's application of Solow's accounting method to the "Harrod-Domar model" showed that the fraction of the rate of growth per capita accounted for by the Residual equals labor's relative share in income, which is rather large. "The authors of constant capital coefficient models did assume *some* technological progress, but they hardly suspected how very specific their assumption could become. The ease with which their secret has been revealed takes my breath away" (Domar (1961a, p. 712).

Domar ([1952a] 1957, pp. 26-27; 1955a, p. 256) had argued that the stability of the capital-output ratio indicated that the law of diminishing returns "had been sufficiently offset in the long run by technological progress". This is the sense in which his models assumed "some" technological progress. Domar ([1947a] 1957, pp. 87-88) was reacting against the hitherto treatment of economic growth as a function of "some abstract technical progress which somehow results in increasing productivity per manhour, and which takes place quite independently of capital formation". He had maintained that labor productivity is a function of "technological progress embodied in capital goods" (Domar [1946] 1957, p. 72). Domar (1961a, p. 712) criticized Solow's (1957) model for assuming that capital does not serve as the instrument for the introduction of technical change into production, but left it at that.<sup>22</sup>

A more significant criticism of Solow's Residual was its aggregative character. Domar (1961a, pp. 726-29) established that Leontief's (1953) disaggregated index of structural change, when proper sectorial weights (known now as "Domar weights") are used, is formally equivalent to the Residual calculated from a Cobb-Douglas function, with the advantage of pointing to the microeconomic character of the process of technical change. This seemed to confirm Domar's

---

<sup>22</sup> Domar (1961a, p. 712, n. 1) referred briefly to Solow's (1960) vintage capital growth model, in which technical progress is embodied in new machines. However, Solow's vintage model does not ensure that a change in the saving ratio will permanently affect the age distribution of capital and the rate of growth (Boianovsky & Hoover 2014, pp. 210-11).

([1952a] 1957, p. 29) expectation that Leontief's dynamic input-output system was the "most interesting and promising piece of research" in economics.

## 6. "Are we back to Adam Smith?"

Domar (1961d) asked that question at a time when growth theory and modeling were taking the economic profession by storm. It did seem like a return to classical political economy's broad emphasis on growth, even though some Smithian themes, such as division of labor and increasing returns, were beyond the modeling strategy and ability of Domar and his contemporaries. Instead, such themes were picked up by development economists, who did not at the time adopt formal modeling (Boianovsky 2015a).

In any event, by the mid 1960s Domar (1965) was pleased about the effects of growth models on economics. Growth had become "an integral part of economic theory", affecting international trade, public finance, monetary economics and other fields. The "particular achievement" of the growth research agenda, from Domar's optimistic viewpoint, was that "static equilibrium under perfect competition has lost its foremost place". The notion of "moving equilibrium" should be applied to the growth of firms as well, which entailed discarding the idea of long-run static equilibrium of the representative firm with optimal-sized plant and zero investment (see also Domar 1948b, p. 34). The development of the theory of growth of the firm, he expected, should "make the whole approach [to economic growth] highly respectable" (1953b, p. 6).<sup>23</sup>

Domar (1992, p. 124) would claim that his "models" introduced growth into the "main body of economic thinking (with the participation of many others and particularly with the help of Simon Kuznets' empirical work)". Surely, Harrod was one of the "many others" who took part in the process. Domar believed his contribution was mainly methodological, that is, the introduction of the method of growth theory through rates of growth extending into infinite time spans, called "moving equilibrium".

---

<sup>23</sup> He referred to work in progress by his Johns Hopkins colleague Edith Penrose (Domar 1957, p. 13, n. 14; see Penrose 1959).

At first, Domar saw his approach to growth as a challenge to economic historians, who, in his view, lacked a theory of growth. Simon Kuznets – Domar’s colleague at Johns Hopkins in the 1950s and a pioneer in empirical growth economics – was one of Domar’s targets. Kuznets (1951, p. 968) argued that business cycle theory meant more than producing models showing the *possibility* of occurrence of certain economic cycles. Its task was to explain common characteristics of business cycles as *observed* in economic history. Domar (1955b) commented on Kuznets’s research on capital formation. According to Domar (pp. 109-10), Kuznets showed at an empirical level that the propensity to save, the capital coefficient and the rate of growth are interrelated, but he did not model the different causations. Kuznets’s empirics lacked an “explicit model” of growth (Domar 1955b, p. 110). Similarly, Domar (1967, p. 4) would describe W.W. Rostow’s well-known historical treatment as a “classification of the various stages of growth”, but “not really a growth theory”.

Interestingly enough, Domar’s research agenda would gradually move to economic history. Learning how history is made was his “lifelong aim” (Domar 1992, p. 118). This did not entail a methodological shift, as economic history should be based on models as much as the rest of economics. His classic article about serfdom and slavery (Domar 1970) illustrated that. The “Domar model of serfdom” (Temin 2014, p. 342) claimed that free land, free labor and an aristocracy could not co-exist in history. Unlike his growth model, factor substitution is prominent in that essay. He regarded testing as a necessary complement to modeling, expressed in the remark that “where I come from, an economic model without empirical testing is equated with a detective story without an end” (Domar 1970, p. 23). Domar had used his growth models to interpret economic data, as illustrated by his estimation of the US equilibrium growth rate for the period 1879-1941. Moreover, growth models should be consistent with observed growth trends, or “stylized facts” as they became known in the literature. Surely, data were not independent of theory, as they were created through measurement procedures, as witnessed by Domar’s (1961a) development of indexes of technological change.

One of the features of the real world indicated by observed data was its relative economic stability, in the sense of the ability to recover from downturns. Domar referred to “stabilizers”, but never incorporated them into the model, which did not trace the actual path of output. The fact that part of the action took place outside the model caught Solow’s critical attention. In correspondence with the

author, Solow (2015) remarked that “I realized only vaguely then, though it is much clearer to me now, that both Harrod and Domar, by mixing together the potential growth story and the Keynesian [capital] utilization story, and by dealing with it off-stage and never in the model itself, had created a bit of a mess. I was already a good Keynesian in 1956, but my instinct even then was that a model should speak without a voice-over” (see also Solow 2006). Solow’s comment points to another phase in growth modeling, inaugurated by his own 1956 model, whose notion of stability is not the same as Domar’s (or Harrod’s). In a lecture delivered at the University of the Philippines, Domar (1974) referred to his growth equations as “simple models not popular anymore”, which had been “replaced by ... mathematical exercises about abstract stability and equilibrium conditions”. Growth modeling had assumed a meaning different from Domar’s original research program.

## References

Abramovitz, M. 1952. Economics of growth. In B.F. Haley (ed), pp. 132-78.

Ackley, G. 1961. *Macroeconomic theory*. New York: Macmillan.

Allen, R.G.D. 1956. *Mathematical economics*. London: Macmillan.

Asimakopulos, A. 1986. Harrod and Domar on dynamic economics. *Banca Nazionale del Lavoro Quarterly Review*. 39: 275-98.

Backhouse, R.E. 1994. *Economists and the economy – the evolution of economic ideas*, 2<sup>nd</sup> edition. Oxford: Basil Blackwell.

Backhouse, R.E. and M. Boianovsky. 2013. *Transforming modern macroeconomics: exploring disequilibrium microfoundations, 1956-2003*. Cambridge: Cambridge University Press.

Backhouse, R.E. and M. Boianovsky. 2016. Secular stagnation: the history of a macroeconomic heresy. *European Journal of the History of Economic Thought*. 23 (5).

Barro, R. and X. Sala-i-Martin. 1995. *Economic growth*. New York: McGraw-Hill.

Baumol, W.J. 1951. *Economic dynamics – an introduction*. New York: Macmillan.

Beach, E.F. 1957. *Economic models: an exposition*. New York: John Wiley.

Besomi, D. 2001. Harrod's dynamics and the theory of growth: the story of a mistaken attribution. *Cambridge Journal of Economics*. 25: 79-96.

Blaug, M. 1992. *The methodology of economics or how economists explain*, 2<sup>nd</sup> edition. Cambridge: Cambridge University Press.

Boianovsky, M. 2002. Patinkin, the Cowles Commission and the theory of unemployment and aggregate supply. *European Journal of the History of Economic Thought*. 9: 226-59.

Boianovsky, M. 2009. Some Swedish stepping–stones to growth economics. In Boianovsky and Hoover 2009b, pp. 47-66.

Boianovsky, M. 2015a. Beyond capital fundamentalism: Harrod, Domar and the history of development economics. *SSRN Working Paper*.

Boianovsky, M. 2015b. Optimum saving and growth: Harrod on dynamic welfare economics. *SSRN Working Paper*.

Boianovsky, M. 2016a. David Gawen Champernowne (1912-2000). In *The Palgrave Companion to Cambridge Economics*, ch. 34. Edited by R. A. Cord. London: Palgrave.

- Boianovsky, M. 2016b. Domar, underconsumption and the Marxian reproduction schemes. Unpublished typescript.
- Boianovsky, M. and K.D. Hoover. 2009a. The neoclassical growth model and twentieth-century economics. In Boianovsky and Hoover 2009b, pp. 1-23.
- Boianovsky, M. and K.D. Hoover (eds). 2009b. *Robert Solow and the development of growth economics*. Annual supplement to vol. 41 of *History of Political Economy*. Durham: Duke University Press.
- Boianovsky, M. and K.D. Hoover. 2014. In the Kingdom of Solovia: the rise of growth economics at MIT, 1956-70. In Weintraub (ed), pp. 198-228.
- Cassel, G. [1918] 1932. *The Theory of Social Economy*. Tr. by S.L. Barron. New York: Harcourt, Brace.
- Cassel, G. 1935. *On quantitative thinking in economics*. Oxford: Clarendon Press.
- Champernowne, D. G. 1946. A note on J. v. Neumann's article on a "Model of economic equilibrium". *Review of Economic Studies*. 13: 10-18.
- Chiang, A. C. 1967. *Fundamental methods of mathematical economics*. Tokyo: McGraw-Hill.
- Colander, D. C. and H. Landreth. 1996. *The coming of Keynesianism to America*. Cheltenham: Elgar.
- Domar, E. [1944] 1957. The 'burden of debt' and the national income. *American Economic Review*. 34: 798-827. As reproduced in Domar (1957), pp. 35-69.



Domar, E. 1945. Public debt and national income. In *Public finance and full employment*, pp. 53-68. New York: Board of Governors of the Federal Reserve System, Postwar Economic Studies # 3.

Domar, E. [1946] 1957. Capital expansion, rate of growth, and employment. *Econometrica*. 14: 137-47. As reproduced in Domar (1957), pp. 70-82.

Domar, E. [1947a] 1957. Expansion and employment. *American Economic Review*. 37: 34-55.

Domar, E. 1947b. *Essays in economics of expansion*. Unpublished PhD thesis. Harvard University.

Domar, E. 1947c. A summary of the thesis *Essays in economics of expansion*. Domar Papers, Box 18.

Domar, E. 1947d. The origin of induced investment. Report to the Atlantic City Meeting of the Econometric Society, January 24-27 1947. *Econometrica*. 15: 163-65.

Domar, E. [1948a] 1957. The problem of capital accumulation. *American Economic Review*. 38: 777-94.

Domar, E. 1948b. Investment, losses, and monopolies. In *Income, employment and public policy – essays in honor of Alvin H. Hansen*, pp. 33-53. New York: Norton.

Domar, E. 1948c. Comments on Lawrence R. Klein's Economic fluctuations in the United States 1921-1941 (Second draft, June 1947). *Cowles Commission Discussion Papers* # 227.

Domar, E. 1949a. Capital accumulation and the end of prosperity. *Econometrica*. 17: 307-14.

Domar, E. 1949b. Rejoinder. *American Economic Review*. 39: 1170-72.

Domar, E. [1952a] 1957. Economic growth: an econometric approach. *American Economic Review*. 42: 479-95. Reproduced as "A theoretical analysis of economic growth" in Domar (1957), pp. 16-34.

Domar, E. 1952b. Comment. In B.F. Haley (ed), pp. 453-55.

Domar, E. 1953a. Problems in the theory of economic growth. Lectures delivered at the University of Oxford during the Hilary Term 1952-53. Domar Papers, Box 18.

Domar, E. 1953b. The problem of economic growth. A lecture delivered before the Marshall Society, Cambridge, on April 23 1953. Domar Papers, Box 18.

Domar, E. 1953c. Recent economic developments in the United States. A lecture given at Imperial College London on March 16, 1953. Domar Papers, Box 18.

Domar, E. 1953d. Further comment. *Quarterly Journal of Economics*. 67: 559-63.

Domar, E. 1955a. The inter-relation between capital and output in the American economy. In *Economic progress*, pp. 249-69, ed. by L.H. Dupriez. Louvain: Institut de Recherches Economiques et Sociales, for the International Economic Association.

Domar, E. 1955b. Comment. In *Capital formation and economic growth*, pp. 107-11, ed. by M. Abramovitz. Princeton: Princeton University Press and NBER.

Domar, E. 1957. *Essays in the theory of economic growth*. New York: Oxford University Press.

Domar, E. 1961a. On the measurement of technological change. *Economic Journal*. 71: 709-29.

Domar, E. 1961b. The capital-output ratio in the United States: its variation and stability. In Lutz and Hague (eds), pp. 95-117.

Domar, E. 1961c. Harrod-Domar vs. Solow. Domar Papers, Box 18.

Domar, E. 1961d. Growth or not to growth. Lecture delivered at the University of Maine. Domar Papers, Box 10.

Domar, E. 1965. Outline of lectures on growth models. Delivered in Bogota, Universidad de los Andes. Domar Papers, Box 1.

Domar, E. 1967. The theory of economic growth: its present state. Lecture delivered at the Economic Development Institute. Domar Papers, Box 9.

Domar, E. 1970. The causes of slavery or serfdom: a hypothesis. *Journal of Economic History*. 30: 18-32.

Domar, E. 1974. Lessons from growth models. Domar Papers, Box 9.

Domar, E. 1989. *Capitalism, socialism and serfdom*. Cambridge: Cambridge University Press.

Domar, E. 1992. How I tried to become an economist. In *Eminent economists – their life philosophies*, pp. 115-27, ed. by M. Szenberg. Cambridge: Cambridge University Press.

Easterly, W. 2001. *The elusive quest for growth*. Cambridge (MA): MIT Press.

Eisner, R. 1958. On growth and the neo-classical resurgence. *Economic Journal*. 68: 707-21.

Eltis, W. 1998. The Harrod-Domar equation from Quesnay to Marx to Harrod and Domar. In *Economic dynamics, trade and growth*, pp. 11-32, ed. by G. Rampa, L. Stella and A. Thirlwall. London: Macmillan.

Enke, S. 1963. *Economics for development*. Englewood Cliffs (N.J.): Prentice-Hall.

Fellner, W. 1951. The capital-output ratio in dynamic economies. In *Money, trade and economic growth – In honor of John Henry Williams*, pp. 105-34. New York: Macmillan.

Friedman, M. 1952. Comment. In B.F. Haley 1952, pp. 455-57.

Friedman, M. 1953, The methodology of positive economics. In *Essays in positive economics*, pp. 3-43. Chicago: University of Chicago Press.

Frisch, R. 1960. A reconsideration of Domar's theory of economic growth. *Econometrica*. 29: 406-13.

Griliches, Z. 1996. The discovery of the Residual: a historical note. *Journal of Economic Literature*. 34: 1324-30.

Hagemann, H. 2009. Solow's 1956 contribution in the context of the Harrod-Domar model. In Boianovsky and Hoover 2009b, pp. 67-87.

Hahn, F.H. 1958. Review of Domar (1957). *Economica*. 25: 352-54.

Hahn, F.H. and R.C.O. Matthews. 1964. The theory of economic growth: a survey. *Economic Journal*. 74: 779-902.

Haley, B.F. (ed). 1952. *A survey of contemporary economics*, vol. II. Homewood (Ill): Richard D. Irwin for the American Economic Association.

Halsmayer, V. 2014. From exploratory modeling to technical expertise: Solow's growth model as a multipurpose design. In Weintraub (ed), pp. 229-51.

Halsmayer, V. 2015. Modeling, measuring and designing economic growth – the neoclassical growth model as a historical artifact: ingredients, construction,

manipulations, uses and interpretation, 1930s-1960s. Unpublished PhD thesis. University of Vienna.

Halsmayer, V. and K.D. Hoover. 2016. Solow's Harrod: transforming macroeconomic dynamics into a model of long-run growth. *European Journal of the History of Economic Thought*. 23: 561-94.

Hamberg, D. 1952. Full capacity vs. full employment growth. *Quarterly Journal of Economics*. 66: 444-49.

Hamberg, D. 1956. *Economic growth and instability*. New York: Norton.

Hamberg, D. 1971. *Models of economic growth*. New York: Harper & Row.

Hansen, A.H. 1941. *Fiscal policy and business cycles*. New York: Norton.

Harrod, R. F. 1939. An essay in dynamic theory. *Economic Journal*. 49: 14-33.

Harrod, R. F. 1948. *Towards a dynamic economics*. London: Macmillan.

Harrod, R.F. 1956. Walras: a re-appraisal. *Economic Journal*. 66: 307-16.

Harrod, R. F. 1959. Domar and dynamic economics. *Economic Journal*. 69: 451-64.

Harrod, R. F. 1960. Second essay in dynamic theory. *Economic Journal*. 70: 277-93.

Harrod, R. F. 1968. What is a model? In *Value, capital and growth: papers in honour of Sir John Hicks*, pp. 173-191. Ed. by J.N. Wolfe. Edinburgh: Edinburgh University Press.

Hausman, D.M. 1992. *The inexact and separate science of economics*. Cambridge: Cambridge University Press.

- Hausman, D.M. 2015. Much ado about models. *Journal of Economic Methodology*. 22: 241-46.
- Hicks, J. 1965. *Capital and growth*. Oxford: Oxford University Press.
- Hirschman, A. 1958. *The strategy of economic development*. New Haven: Yale University Press.
- Jones, H.G. 1975. *An introduction to modern theories of economic growth*. London: Nelson.
- Kaldor, N. 1961. Capital accumulation and economic growth. In Lutz and Hague (eds), pp. 177-222.
- Keynes, J.M. 1936. *The general theory of employment, interest and money*. London: Macmillan.
- Klein, L.R. 1950. *Economic fluctuations in the United States 1921-1941*. New York: Wiley.
- Knight, F.H. 1944. Diminishing returns from investment. *Journal of Political Economy*. 52: 26-47.
- Kregel, J. 1987. Natural and warranted rates of growth. In *The New Palgrave Dictionary of Economics*, vol. 4, pp. 600-02, ed. by J. Eatwell, M. Milgate and P. Newman. London: Macmillan.
- Kuznets, S. 1946. *National product since 1869*. New York: NBER.
- Kuznets, S. 1951. Review of A. Hansen's *Business cycle and national income*. *American Economic Review*. 41: 967-71.
- Leijonhufvud, A. 1997. Models and theories. *Journal of Economic Methodology*. 4: 193-98.

Leontief, W. 1941. *The structure of the American economy, 1919-1939*. Cambridge (Mass.): Harvard University Press.

Leontief, W. *et al* 1953. *Studies in the structure of the American economy*. New York: Oxford University Press.

Lundberg, E. 1937. *Studies in the theory of economic expansion*. London: P. S. King & Son.

Lutz, F.A. and D.C. Hague (eds.) 1961. *The Theory of Capital*. London: Macmillan.

Mäki, U. (ed). 2009. *The Methodology of Positive Economics: Reflections on the Milton Friedman Legacy*. Cambridge: Cambridge University Press.

Morgan, M. 2012. *The world in the model*. Cambridge: Cambridge University Press.

Muzhani, M. 2014. *Mainstream growth economics and capital theorists – a survey*. Montreal: McGill-Queen's University Press.

Myrdal, G. 1968. *Asian drama: an inquiry into the poverty of nations*. New York: Pantheon.

Neumann, J. v. 1946. A model of general economic equilibrium. *Review of Economic Studies*. 13: 1-9.

Niehans, J. 1990. *A history of economic theory: classic contributions 1720-1980*. Baltimore: Johns Hopkins University Press.

Okun, B. and R.W. Richardson (eds). 1961. *Studies in economic development*. New York: Holt, Rinehart and Winston.

- Oshima, H.T. 1959. Income originating in the models of Harrod and Domar. *Economic Journal*. 69: 443-50.
- Patinkin, D. 1981. *Essays on and in the Chicago tradition*. Durham (NC): Duke University Press.
- Penrose, E. 1959. *The theory of the growth of the firm*. Oxford: Blackwell.
- Piketty, T. 2014. *Capital in the twenty-first century*. Tr. by A. Goldhammer. Cambridge (Mass.): The Belknap Press of Harvard University Press.
- Pilvin, H. 1953. Full capacity vs. full employment growth. *Quarterly Journal of Economics*. 67: 545-52.
- Rijckeghem, W. van. 1966. The stability of Domar's model. *Econometrica*. 34: 724-26.
- Robertson, D.H. 1954. Thoughts on meeting some important persons. *Quarterly Journal of Economics*. 68: 181-90.
- Robinson, J. 1952. The model of an expanding economy. *Economic Journal*. 62: 42-53.
- Ruggles, R. 1952. Methodological developments. In B.F. Haley (ed), pp. 408-53.
- Samuelson, P.A. 1948. Dynamic process analysis. In *A survey of contemporary economics*, vol. I, pp. 352-79, ed. by H.S. Ellis. Homewood (Ill): Richard D. Irwin for the American Economic Association.
- Samuelson, P.A. 2002. Reply: complementary innovations by Roy Harrod and Alvin Hansen. *History of Political Economy*. 34: 219-23.



Schelling, T. C. 1947. Capital growth and equilibrium. *American Economic Review*. 37: 864-76.

Simon, H.C. 1942. Hansen on fiscal policy. *Journal of Political Economy*. 50: 161-96.

Solow, R.M. 1956. A contribution to the theory of economic growth. *Quarterly Journal of Economics*. 70: 65-94.

Solow, R.M. 1957. Technical change and the aggregate production function. *Review of Economics and Statistics*. 39: 748-62.

Solow, R.M. 1960. Investment and technical progress. In *Mathematical methods in the social sciences 1959*, pp. 89-104, ed. by K.J. Arrow and S. Karbin. Stanford: Stanford University Press.

Solow, R.M. 1962. Technical progress, capital formation and economic growth. *American Economic Review*. 52: 76-86.

Solow, R.M. 2006. The Domar model. In *The Elgar companion to development studies*, pp. 135-37, ed. by D.A. Clark. Cheltenham: E. Elgar.

Solow, R.M. 2015. Letter to the author, 4 December.

Solow, R.M. and P.A. Samuelson. 1953. Balanced growth under constant returns to scale. *Econometrica*. 21: 412-24.

Stern, E.H. 1949. The problem of capital accumulation. *American Economic Review*. 39: 1160-70.

- Streeten, P. P. 1968. Economic models and their usefulness for planning in South Asia. Appendix 3 to Myrdal, pp. 1941-2004.
- Sunkel, O. 1956. El modelo de crecimiento de Domar [Domar's growth model]. *El Trimestre Económico*. 23: 201-17.
- Svennilson, I. 1958. Review of E. Domar (1957). *American Economic Review*. 48: 1006-1009.
- Swan, T.W. 1956. Economic growth and capital accumulation. *Economic Record*. 32: 334-61.
- Temin, P. 2014. The rise and fall of economic history at MIT. In Weintraub (ed), pp. 337-50.
- Tinbergen, J. 1942. Zur Theorie der langfristigen Wirtschaftsentwicklung. *Weltwirtschaftliches Archiv*. 55: 511-49.
- Wan, H. Y., Jr. 1971. *Economic growth*. New York: Harcourt Brace Jovanovich.
- Weintraub, E.R. (ed). *MIT and the transformation of American economics*. Durham (NC): Duke University Press. Annual supplement to *History of Political Economy*, vol. 46.
- Wright, D. 1952. Economic growth: econometric models in relation to the social setting. *American Economic Review*. 42: 496-502.

Young, W. 1989. *Harrod and his trade cycle group: the origins and development of the growth research programme*. London: Macmillan.