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Summary

Climate Engineering, and in particular Solar Radiation Management (SRM) has become a widely discussed climate policy option to study in recent years. However, its potentially strategic nature and unforeseen side effects provide major policy and scientific challenges. We study the role of the SRM implementation and its strategic dimension in a model with two heterogeneous countries with the notable feature of model misspecification on the impacts from SRM. We find that deep uncertainty leads to a reduction in SRM deployment both under cooperation and strategic behavior, which is a more relevant issue if countries act strategically. Furthermore, we demonstrate that the heterogeneity in impacts from SRM has an asymmetric effect on the optimal policy and could typically lead to unilateral SRM implementation. We also consider heterogeneous degrees of ambiguity aversion, in which case the more confident country only will use SRM.

Keywords: Climate Change, Solar Radiation Management, Uncertainty, Robust Control, Differential Game

JEL Classification: Q53, Q54

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Climate Engineering under Deep Uncertainty and Heterogeneity^{*}

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Abstract

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1 Introduction

Anthropogenic greenhouse gas emissions have increased since the pre-industrial era, driven largely by economic and population growth, and are now higher than ever. This has led to atmospheric concentrations of greenhouse gases (GHGs) that are unprecedented in at least the last 800,000 years. The atmosphere and oceans have warmed, the amounts of snow and ice have diminished, and the sea level has risen (IPCC, 2013).

At the same time, slow progress in international climate negotiations has given rise to skepticism about the prospect of global cooperative action to achieve an ambiguous climate target, such as the 2 or 1.5 degree targets of the Paris agreement. This relatively slow progress has led to the discussion about alternative policy options in order to avoid significant climate change impacts. In particular, different climate engineering methods have been proposed as a means to avoid dangerous climate change. Climate engineering refers to the deliberate intervention in the planetary environment of a nature and scale intended to counteract anthropogenic climate change and its impacts (Shepherd, 2009). One particular technology of climate engineering is Solar Radiation Management (SRM), which involves manipulating directly the sun's incoming radiation. Probably the most popular SRM method suggests injecting sulfur aerosols into the lower stratosphere and reflecting incoming radiation away from the planet back into space (Keith, 2000; Ricke et al., 2008; Shepherd, 2009). This method mimics what occasionally occurs in nature when a volcano erupts. For instance, during the Mount Pinatubo eruption in 1991 huge volumes of sulfur injected into the stratosphere and the aerosols produced in subsequent reactions cooled the planet by about $0.5^{\circ}C$ over the next two years (Randel et al., 1995; Robock, 2000).

One common feature of the different climate engineering options is that they tend to be speculative, in the sense that no large scale experiments have been conducted in order to assess their full potential nor their side-effects and other impacts. Moreover, in particular in the case of SRM, its potential side-effects are largely unknown (Robock et al., 2008; Barrett, 2008) including its effectiveness (Moreno-Cruz and Keith, 2012; Emmerling and Tavoni, 2013). This uncertainty could emerge from sources such as major gaps in knowledge, limited modeling capacity, lack of theories to anticipate thresholds (Heutel et al. (2015a)) and, finally, emergence of surprises and unexpected consequences. This uncertainty must moreover be considered as "deep" uncertainty since even defining the full potential state space and assigning probabilities to events is (almost) impossible. Deep (or Knightian) uncertainty is contrasted to risk (measurable or probabilistic uncertainty) where probabilities can be assigned to events and are summarized by a subjective probability measure or a single Bayesian prior (Vardas and Xepapadeas (2009); Roseta-Palma and Xepapadeas (2004)). The concept of deep uncertainty or ambiguity can be modeled through extensions of the non-expected utility paradigm (e.g., maxmin utility, smooth ambiguity aversion), or through a manipulation of the model the decision maker considers, thus allowing for and considering potentially misspecified models, as in the robust control framework of Hansen and Sargent (2001) and Hansen et al. (2006).

We follow this approach, which seems an appropriate modeling framework for speculative future technologies such as SRM (Goeschl et al., 2013). That is, we consider a decision maker who cannot assign probabilities to events. Therefore, he has limited confidence in his conceptual model and wants to find a good decision over a set or "cloud" of models that surrounds his benchmark model. The cloud of models is obtained by disturbing a benchmark model and introducing a misspecification error, so that the admissible disturbances reflect the set of possible probability measures the decision maker is willing to consider. The more ambiguous the situation is considered by the decision maker, the larger

is the cloud of approximate models that he will consider. The standard expected utility maximizing model can be derived as a special case of the robust control model if the decision maker completely trusts the benchmark model. We apply this methodology to the optimal climate policy based on mitigation and SRM as policy options.

In this paper we combine deep uncertainty in a robust control framework with heterogeneity and strategic interaction (Moreno-Cruz (2015); Ricke et al. (2013); Manoussi and Xepapadeas (2015)) between two regions in a dynamic game of climate change policy in terms of emissions and climate engineering efforts. We formulate the problem in terms of a linear-quadratic (LQ) differential game by extending the standard linear-quadratic model of pollution control, studied by Dockner and Van Long (1993) or in Athanassoglou and Xepapadeas (2012). We analyze the problem in the context of a cooperative and non-cooperative game along with heterogeneity and deep uncertainty. In the cooperative case there is coordination between the two countries for the implementation of climate engineering and the level of emissions in order to maximize the joint global welfare. In the non-cooperative case, each government chooses its own amounts of SRM and emissions independently. The non-cooperative solution is analyzed in terms of Nash equilibrium (NE) strategies.

We first derive the paths for emissions, SRM and global mean temperature change under symmetry and only with pure risk on SRM impacts, for both cooperative and Nash strategies. Adopting the Hansen–Sargent framework, we introduce deep uncertainty into the basic model and study the effect of model misspecification on optimal SRM efforts and mitigation. Specifically, the deep uncertainty is introduced in the underlying diffusion process of marginal SRM damages, reflecting concerns about the benchmark model.

We derive the analytical solutions for the optimal policy mix under symmetry, and then proceed to a numerical simulation of the model in order to explore the effects of heterogeneity and the interaction with ambiguity in the optimal levels of emissions and SRM. The results suggest that heterogeneity in impacts strongly affects the strategic interaction across regions. In the region that is worse off due to the impacts from SRM, we observe no SRM implementation. This leads the other region to undertake a more aggressive policy in terms of high *SRM* implementation to compensate for the high emissions of both regions. We also consider asymmetry in the degree of ambiguity aversion or model misspecification, in this case SRM will be implemented only by the region with a higher degree of confidence. Comparing both the cooperative and non-cooperative solution, we show that for a range of minimum values of the robustness parameter, an optimal policy can be found only at the cooperative solution whereas it breaks down under strategic SRM.

In the next section we introduce the setup of the model of climate policy with SRM under uncertainty. In section three, we solve the cooperative and non-cooperative problem and outline both solution concepts. The results under symmetry are presented in section four and the impact of heterogeneity is discussed in section five. Section six concludes.

2 A model of SRM under model misspecification

We consider a model of optimal climate policy with two heterogeneous countries or regions¹, indexed by i = 1, 2. We develop our model along the lines of the standard linear quadratic model of international pollution control as in Dockner and Van Long (1993). The model is framed in continuous time so we index all variables by t. The relationship between emissions and economic output and consumption is modeled through a reduced form utility function depending on emissions as in Athanassoglou and

¹In the following, we will refer to them as countries for simplicity.

Xepapadeas (2012).² Within our linear-quadratic framework, the utility-emission function is given by the quadratic function

$$U(E_{i}(t)) = A_{1}E_{i}(t) - \frac{1}{2}A_{2}E_{i}(t)^{2}, \qquad (1)$$

where A_1 , A_2 are parameters indicating the intercept and the slope of the private marginal benefits from emissions. That is, A_1 can be regarded as reflecting the level effect on marginal benefits, while A_2 as reflecting the strength of diminishing returns.

The decision maker in each country has two policy options; the reduction of his emissions $E_i(t)$ and the use of SRM denoted by $z_i(t)$. Based on both policy variables, we can describe the state equation for the evolution of the change in global mean temperature as

$$\dot{T(t)} = \lambda \sum_{i=1}^{2} E_i(t) + \phi \sum_{i=1}^{2} z_i(t) - \delta T(t), \qquad (2)$$

where $\delta > 0$ is the heat transfer parameter capturing the effect that a fraction of the energy stored as heat in the atmosphere dissipates (2nd thermodynamic law). In this equation, $\lambda > 0$ is the sensitivity of temperature to emissions, the function $\phi \sum_{i=1}^{2} z_i(t)$ is the reduction in solar radiation and consequently in global mean temperature due to aggregate climate engineering and ϕ is the sensitivity of incoming radiation to climate engineering in reducing the average global temperature. Following evidence indicating that there is a fast and a slow response of global warming to external forcing, with the slow component being relatively small (Held et al., 2010), we assume that the average global temperature T converges fast to a steady state. This quasi steady state for T can be computed as

$$\dot{T}(t) = 0 \Longrightarrow T(t) = \frac{1}{\delta} \left(\lambda \sum_{i=1}^{2} E_i(t) - \phi \sum_{i=1}^{2} z_i(t) \right)$$
(3)

As for the costs of implementing SRM, we assume a simple quadratic cost function for the cost of climate engineering in each country $C(z_i)$ (Goes et al. (2011); Bickel and Agrawal (2013); Gramstad and Tjøtta (2010); Robock et al. (2009)), which is strictly increasing and convex :

$$C(z_{i}(t)) = \frac{1}{2}\beta z_{i}^{2}(t), \ \beta > 0$$
(4)

An important feature of this model are the impacts from temperature increase and SRM implementation. We assume two types of regional damage functions related to climate change, which affect welfare. The first one reflects damages from the increase in the average global surface temperature, represented as usual by a convex, quadratic in our case, function in the degree of global warming since pre-industrial levels (T_0) :

$$D_T(T(t)) = \tau_i (T(t) - T_0)^2,$$
(5)

where τ_i represents the marginal damages in country *i*.

The second damage function represents the impacts or side-effects from the implementation of climate engineering. These impacts potentially include ozone depletion, distorted precipitation patterns, negative effects on biodiversity and many others (see Barrett et al. (2014); Robock et al. (2008)).³

²This function can be considered as a utility function of economic output, which itself is a function of emissions $F(E_i)$, where $F(\cdot)$ is strictly concave with F(0) = 0. Utility, without environmental externalities and policy costs is then given by $U(F(E_i(t))) = U(E_i(t))$.

³We do not consider pure CO_2 concentration impacts such as ocean acidification, which provide a third category of impacts, see Heutel et al. (2015b) for a recent contribution.

We assume that these impacts depend on the total level of SRM implemented and allow for potential heterogeneity. Moreover, since we consider the deep uncertainty, we model the marginal impacts as a stochastic process denoted as $u_i(t)$. The total impacts in country *i* are then given by the equation

$$D_{z}\left(\mathbf{z}\left(t\right), u_{i}\left(t\right)\right) = u_{i}\left(t\right) \cdot \zeta \cdot \sum_{i=1}^{2} z_{i}\left(t\right), \qquad (6)$$

where $\sum_{i=1}^{2} z_i(t)$ represents the aggregate SRM implemented in both regions⁴. Note that the damage function is a priori linear in z_i , but due to the multiplicative term of u_i , marginal impacts are changing over time. Notably, the environmental impacts from the use of *SRM* evolve through time according to the stochastic differential equation:

$$du_{i}(t) = \left[\eta_{i}(1-\gamma)\sum_{i=1}^{2} z_{i}(t) - mu_{i}(t)\right]dt + \sigma d\hat{B}_{i}$$
(7)

Marginal impacts from SRM depend on the amount of SRM implemented at a region-specific marginal rate $\eta_i (1 - \gamma)$. Note that therefore total impacts will have a quadratic shape in the absence of uncertainty and model misspecification. Moreover, we assume that there is an adjustment rate m in marginal impacts. This term can be interpreted as the adaptation to SRM impacts of the socioeconomic or biophysical system.⁵ If $\gamma = 1$ in (7) then the sulfur emitted at t is dispersed, and only the stochastic stock $\sigma d\hat{B}_i$ remains. If $0 < \gamma < 1$, then the remaining stock of sulfur adds a trend to damages (7) along with stochastic shock. Finally, uncertainty is introduced through $\hat{B}_i(t)$, which is a Brownian motion on an underlying probability space (Ω, F, \mathcal{G}) .

Without strategic interactions and the possibility of model misspecification, solving the symmetric problem would be straightforward.⁶ Now we add the strategic interaction and deep uncertainty through model misspecification to the model. Model misspecification is represented by a family of stochastic perturbations to the Brownian motion $\hat{B}_i(t)$, such that the probabilistic structure implied by the stochastic differential equation of marginal impacts from SRM (7) is distorted and the probability measure \mathcal{G} is replaced by another \mathcal{Q} . The perturbed model is obtained by performing a change of measure and replacing $\hat{B}_i(t)$ in (7) by

$$B_i(t) + \int_0^t h_i(s) dt \tag{8}$$

$$d\hat{B}_i = dB_i + h_i(t) dt \tag{9}$$

where $\{B_i(t) : t \ge 0\}$ is a Brownian motion and $\{h_i(t) : t \ge 0\}$ a measurable drift distortion such that $h_i(t) = h_i(u(s) : s \le t)$. Hence, changes to the distribution of $\stackrel{\wedge}{B_i}$ are parameterized as drift distortions to a Brownian motion. The measurable process $h_i(t)$ could correspond to any number of misspecified or omitted dynamic effects such as a miscalculation of climate engineering damages, a miscalculation of the decay rate of sulfur in the stratosphere, or an ignorance of more complex dynamic structures involving irreversibility, feedback or hysteresis effects. The distortion will be zero

⁴For the value and the interpretation of ζ , see Appendix B

 $^{{}^{5}}$ In the numerical part, we set it to a very low value, but its inclusion is necessary to solve for the steady state in this model.

 $^{^{6}}$ Based on a simplified maximization problem as the one given below in equation (13).

when $h_i(t) \equiv 0$ and the two measures \mathcal{G} and \mathcal{Q} coincide.

The dynamics for the environmental impacts from the SRM implementation under model misspecification are given by

$$du_{i}(t) = \left[\eta_{i}(1-\gamma)\sum_{i=1}^{2} z_{i}(t) - mu_{i}(t) + \sigma h_{i}(t)\right]dt + \sigma dB_{i},$$
(10)

where $\{h_i(t) : t \ge 0\}$ is a measurable drift distortion, which can be interpreted as a misspecification error about the future marginal impacts from SRM implementation. The benchmark case of pure risk is defined for $h_i(t) = 0$. As in Hansen and Sargent (2001), the discrepancy between the two measures \mathcal{G} and \mathcal{Q} can be measured through the discounted relative entropy defined as

$$\mathcal{R}\left(\mathcal{Q}\right) = \int_{0}^{\infty} e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}}\left(h_{i}\left(t\right)\right)^{2} dt,$$
(11)

where **E** denotes the expectation operator and ρ the discount rate. To allow for the notion that even when the model is misspecified the benchmark model remains a "good" approximation, the misspecification error is constrained so that we only consider distorted probability measures Q, such that the relative entropy is bounded by a value ξ :

$$\mathcal{R}\left(\mathcal{Q}\right) = \int_{0}^{\infty} e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}}\left(h_{i}\left(t\right)\right)^{2} dt \leq \xi < \infty$$
(12)

By modifying the value of ξ in (12), the decision maker can control the degree of model misspecification (ξ) he is willing to consider. In particular, if the decision maker can use physical principles or statistical analysis to formulate bounds on the relative entropy of plausible probabilistic deviations from his benchmark model, these bounds can be used to calibrate the parameter ξ .⁷

Now we turn to the optimization problem under uncertainty. It is based on a linear-quadratic framework of maximizing expected discounted utility minus costs and impacts in an continuous, infinite time horizon model. In the context of model misspecification, one can derive two robust control problems, which represent equivalent ways of defining the problem. Following Hansen et al. (2006), we define a constraint robust control problem and a multiplier robust control problem.

The constraint robust control problem is given by

$$\max_{E_{i}(t), z_{i}(t)} \min_{h_{i}(t)} \mathbf{E}_{\mathbf{0}} \int_{0}^{\infty} e^{-\rho t} \left[U\left(E_{i}\left(t\right)\right) - C\left(z_{i}\left(t\right)\right) - D_{T}\left(T\left(t\right)\right) + D_{z}\left(z_{i}\left(t\right), u_{i}\left(t\right)\right) \right] dt , i = 1, 2$$
(13)

subject to

$$du_{i}(t) = \left[\eta_{i}(1-\gamma)\sum_{i=1}^{2} z_{i}(t) - mu_{i}(t) + \sigma h_{i}(t)\right] dt + \sigma dB_{i}$$

$$\int_{0}^{+\infty} e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}}(h_{i}(t))^{2} dt \leq \xi, \ u_{0} = u(0)$$
(14)

The multiplier robust control problem on the other hand is given by

⁷See section 4.3.

$$\max_{E_{i}, z_{i}} \min_{h_{i}} \mathbf{E}_{\mathbf{0}} \int_{0}^{\infty} e^{-\rho t} \left[U\left(E_{i}\left(t\right)\right) - C\left(z_{i}\left(t\right)\right) - D_{T}\left(T\left(t\right)\right) + D_{z}\left(z_{i}\left(t\right), u_{i}\left(t\right)\right) + \frac{1}{2}\theta h_{i}\left(t\right)^{2} \right] dt , i = 1, 2$$
(15)

subject to

$$du_{i}(t) = \left[\eta_{i}(1-\gamma)\sum_{i=1}^{2} z_{i}(t) - mu_{i}(t) + \sigma h_{i}(t)\right] dt + \sigma dB_{i}, u_{0} = u(0).$$

To take into account model misspecification, maximization of welfare is subject to the minimizing agent, often referred as malevolent agent or nature, which chooses $h_i(t)$. The parameter $\theta \in \Theta = \{\theta : 0 \leq \theta < \theta \leq +\infty\}$ constrains the minimizing choice of the $h_i(t)$ function, and therefore it can be regarded as the level of deep uncertainty in the model. The lower bound of θ is a so-called breakdown point beyond which it is fruitless to seek more robustness. When $\theta \to \infty$ there are no concerns about model misspecification and the deep uncertainty disappears.

As shown in Hansen et al. (2006), if we assume that there exists a solution (E^*, z^*, h^*) to the robust multiplier problem, then (E^*, z^*) also solves the constraint robust control problem with $\xi = \xi^* = \mathcal{R}(\mathcal{Q}^*)$ and there exists a θ^* such that the robust multiplier and constraint problems have the same solution.

The multiplier robust control problem, with the non-negativity constraint for the SRM implementation, emerging when the decision maker is concerned about model misspecification for each individual county i and now takes the form

$$V_{i}(t) = \max_{E_{i}(t), z_{i}(t)} \min_{h_{i}(t)} \int_{0}^{\infty} e^{-\rho t} \left[U(E_{i}(t)) - C(z_{i}(t)) - D_{T}(T(t)) + D_{z}(z_{i}(t), u_{i}(t)) + \frac{1}{2}\theta h_{i}(t)^{2} \right] dt$$
(16)

subject to

$$(3)$$
 , (10) , $i = 1, 2$

and the non-negativity constraint for SRM

$$z_i\left(t\right) \ge 0 \tag{17}$$

Since this is a constrained optimization problem, we can form the Lagrangian associated with the dynamic programming equation as

$$\mathcal{L}_{i} = U(E_{i}(t)) - C(z_{i}(t)) - (D_{T}(T(t)) + D_{z}(z_{i}(t), u_{i}(t))) + \frac{1}{2}\theta h_{i}(t)^{2} + V_{ui}(t) \left[\eta_{i}(1-\gamma)\sum_{i=1}^{2} z_{i}(t) - mu_{i}(t) + \sigma h_{i}(t)\right] + \frac{1}{2}\sigma^{2}V_{uui}(t) + \omega_{i}(t)z_{i}(t)$$
(18)

where $\omega_i(t)$ denotes the Lagrange multiplier for each country *i* of the non-negativity constraint of the amount of SRM. The complementary slackness conditions are given by

$$\omega_{i}(t) z_{i}(t) \geq 0 \Longrightarrow_{\omega_{i}(t) > 0, if z_{i}(t) > 0}^{\omega_{i}(t) = 0, if z_{i}(t) > 0}$$

We solve the constrained problem both in the cooperative and in the Nash framework in order to avoid negative values of SRM. This condition is particular important in the case of asymmetry, where the best response for the country which is worse off (due to the asymmetry) could be to undertake no SRM. Based on this general model setup, we now define both a cooperative and a non-cooperative solution to the optimal climate policy mix under model-misspecification of the impacts from SRM.

3 Cooperative and Non-cooperative Solutions

3.1 The cooperative solution

First we solve the model of the previous section for the case where a global social planner chooses jointly the optimal policy in both countries. In this case, we can write the Hamilton-Jacobi-Bellman equation (HJB) for the social planner based on (15) as⁸

$$\rho V^{C}(u_{i}) = \max_{E_{i}, z_{i}} \min_{h} \left\{ \sum_{i=1}^{2} \left[A_{1}E_{i} - \frac{1}{2}A_{2}E_{i}^{2} - \frac{1}{2}\beta z_{i}^{2} - \left(\tau_{i}\left(T - T_{0}\right)^{2} + u_{i}\left(t\right)\zeta\sum_{i=1}^{2}z_{i}\left(t\right)\right) + \frac{1}{2}\theta_{i}h^{2} \right] + V_{u_{i}}^{C} \left[\eta_{i}\left(1 - \gamma\right)\sum_{i=1}^{2}z_{i} - mu_{i} + \sigma h \right] + \frac{1}{2}\sigma^{2}V_{uu_{i}}^{C} \right\}.$$

$$(19)$$

In order to solve this problem subject to the non negativity constraints and given the LQ structure of the problem, we start from the quadratic value function

$$V^{C}(u_{i}) = \varepsilon_{0i} + \mu_{1i}u_{i} + \mu_{2i}u_{i}^{2}$$
(20)

with derivatives

$$V_{u_i}^C = \mu_{1i} + 2\mu_{2i}u_i, \ V_{uu_i}^C = 2\mu_{2i}$$

Since it is a constrained optimization problem, we can form the Lagrangian

$$\mathcal{L}^{\mathcal{C}} = \sum_{i=1}^{2} \left[A_{1}E_{i} - \frac{1}{2}A_{2}E_{i}^{2} - \frac{1}{2}\beta z_{i}^{2} - \left(\tau_{i}\left(T - T_{0}\right)^{2} + u_{i}\left(t\right)\zeta\sum_{i=1}^{2}z_{i}\left(t\right)\right) + \frac{1}{2}\theta_{i}h^{2} \right] + V_{u_{i}}^{C} \left[\eta_{i}\left(1 - \gamma\right)\sum_{i=1}^{2}z_{i} - mu_{i} + \sigma h \right] + \frac{1}{2}\sigma^{2}V_{u_{i}u_{i}}^{C} + \sum_{i=1}^{2}\omega_{i}z_{i}.$$
(21)

Minimizing this function first with respect to h^9 (as the malevolent agent), we obtain

$$\frac{\partial \mathcal{L}^C}{\partial h} = 0 \implies h^* = -\frac{\sigma V_{u_i}^C}{\theta_i} = -\frac{\sigma \left(\mu_{1i}\left(\theta_i\right) + 2\mu_{2i}\left(\theta_i\right)\right)}{\theta_i} \tag{22}$$

so that equation (21) can be written as

$$\mathcal{L}^{C} = \sum_{i=1}^{2} \left[A_{1}E_{i} - \frac{1}{2}A_{2}E_{i}^{2} - \frac{1}{2}\beta z_{i}^{2} - \left(\tau_{i}\left(T - T_{0}\right)^{2} + u_{i}\left(t\right)\zeta\sum_{i=1}^{2}z_{i}\left(t\right)\right) \cdot y + \frac{1}{2}\theta_{i}h^{*2} \right] + V_{u}^{C} \left[\eta_{i}\left(1 - \gamma\right)\sum_{i=1}^{2}z_{i} - mu_{i} + \sigma h^{*} \right] + \frac{1}{2}\sigma^{2}V_{uu_{i}}^{C} + \sum_{i=1}^{2}\omega_{i}z_{i}$$

$$(23)$$

 $^{^{8}}$ In the following, we drop the time index to avoid notation clutter.

⁹In the cooperative solution, the policy maker could have different $\theta's$ for each country, but he considers only one distortion of the SRM dynamics and therefore he decides about one h.

Solving the symmetric case where both regions are identical¹⁰, we obtain the optimal emission level E_i^* as

$$E_{i}^{*} = \frac{4\tau\lambda\left(\beta\delta T_{0} + 2\phi\left(2\zeta u - \eta\left(1 - \gamma\right)V_{u}^{C} - \omega\right)\right) + A_{1}\left(\beta\delta^{2} + 8\tau\phi^{2}\right)}{A_{2}\beta\delta^{2} + 8\tau\left(\beta\lambda^{2} + A_{2}\phi^{2}\right)}$$
(24)

and from the Kuhn-Tucker conditions the optimal level of SRM implementation z_i^*

$$z_i^* = \frac{4\tau\phi \left(A_2 T_0 \delta - 2A_1 \lambda\right) + \left(\eta \left(1 - \gamma\right) V_u^C - 2\zeta u + \omega\right) \left(A_2 \delta^2 + 8\tau \lambda^2\right)}{A_2 \beta \delta^2 + 8\tau \left(\beta \lambda^2 + A_2 \phi^2\right)} \tag{25}$$

and due to the non-negativity constraints

$$\omega z_i^* \ge 0 \Longrightarrow_{\omega > 0, if \ z_i^* = 0}^{\omega = 0, if \ z_i^* > 0}$$

This solution provides our benchmark scenario for the remainder of this paper to compare the results with. Note that given the simple utility function of this model, the global level of both emissions and SRM is invariant to heterogeneity across regions, since only the total sum of welfare is considered. Next, we turn to the solution where both countries act strategically in their climate policy decisions.

3.2 The strategic Nash solution

In the non-cooperative strategic solution, we solve the game assuming that each country follows feedback strategies in the level of emissions and climate engineering. Feedback strategies are associated with the concept of Nash equilibrium of the differential game, which provides a time-consistent noncooperative equilibrium. The feedback Nash equilibrium for the linear quadratic climate change game can be obtained as the solution of the dynamic programming representation of the non-cooperative dynamic game¹¹. The Bellman equation for the infinite horizon problem of each country (i = 1, 2) is based on (15) and is given by

$$\rho V_{i}(u_{i}) = \max_{E_{i}, z_{i}} \min_{h_{i}} \left\{ A_{1}E_{i} - \frac{1}{2}A_{2}E_{i}^{2} - \frac{1}{2}\beta z_{i}^{2} - \left(\tau_{i}\left(T - T_{0}\right)^{2} + u_{i}\zeta\sum_{i=1}^{2}z_{i}\right) + \frac{1}{2}\theta_{i}h_{i}^{2} \quad (26) + V_{ui}\left[\eta_{i}\left(1 - \gamma\right)\sum_{i=1}^{2}z_{i} - mu_{i} + \sigma h_{i}\right] + \frac{1}{2}\sigma^{2}V_{uui}\right\}$$

Each country will take as given the emissions and the SRM level of the other country and will solve its own optimal climate policy problem. Given the LQ structure of the problem, we can consider two quadratic value functions for both regions denoted as $V_i(u_i)$, so that for each region we have

$$V_i(u_i) = \varepsilon_{0i} + \mu_{1i}u_i + \mu_{2i}u_i^2, \quad i = 1, 2$$
(27)

with derivatives

$$V_{ui} = \mu_{1i} + 2\mu_{2i}u_i, \ V_{uui} = 2\mu_{2i}.$$

¹⁰ In symmetry is implied that $\tau_1 = \tau_2 = \tau$, $\eta_1 = \eta_2 = \eta$, $\theta_1 = \theta_2 = \theta$, $\omega_1 = \omega_2 = \omega$.

¹¹From now on we will refer to Feedback Nash Equilibrium or solution simply as Nash Equilibrium (NE).

The constrained optimization problem for country i is therefore given by

$$\mathcal{L}_{i} = A_{1}E_{i} - \frac{1}{2}A_{2}E_{i}^{2} - \frac{1}{2}\beta z_{i}^{2} - \left(\tau_{i}\left(T - T_{0}\right)^{2} + u_{i}\zeta\sum_{i=1}^{2}z_{i}\right) + \frac{1}{2}\theta_{i}h_{i}^{2}$$
(28)

+
$$V_{ui} \left[\eta_i (1-\gamma) \sum_{i=1}^2 z_i - mu_i + \sigma h_i \right] + \frac{1}{2} \sigma^2 V_{uui} + \omega_i z_i.$$
 (29)

From the min-max problem we obtain the Kuhn-Tucker conditions. The optimization with respect to E_i and z_i yields the following reaction functions in emissions and SRM implementation

$$E_i^* = \frac{-2\tau_i\lambda\left(-\beta\delta T_0 + \beta\lambda E_j^* + \phi\left(\eta_i\left(1-\gamma\right)V_{ui} + \beta z_j^* - \zeta u_i + \omega_i\right)\right) + A_1\left(\beta\delta^2 + 2\tau_i\phi^2\right)}{A_2\beta\delta^2 + 2\tau_i\left(\beta\lambda^2 + A_2\phi^2\right)} \tag{30}$$

for SRM and for interior solutions

$$z_{i}^{*} = \frac{\left(\eta_{i}\left(1-\gamma\right)V_{ui}-\zeta u_{i}+\omega_{i}\right)\left(A_{2}\delta^{2}+2\tau_{i}\lambda^{2}\right)-2\tau_{i}\phi\left(A_{1}\lambda+A_{2}\left(\lambda E_{j}^{*}-T_{0}\delta+\phi z_{j}^{*}\right)\right)}{A_{2}\beta\delta^{2}+2\tau_{i}\left(\beta\lambda^{2}+A_{2}\phi^{2}\right)}$$
(31)

the complementary slackness conditions

$$\omega_i z_i \ge 0 \Longrightarrow_{\substack{\omega_i \ge 0, \text{ if } z_i \ge 0\\\omega_i \ge 0, \text{ if } z_i = 0}}^{\substack{\omega_i \ge 0, \text{ if } z_i \ge 0}},$$

and moreover for the minimization problem for h_i

$$\frac{\partial \mathcal{L}_i}{\partial h_i} = 0 \Longrightarrow h_i^* = -\frac{\sigma V_{ui}}{\theta_i}$$

Based on the definition of the parameters it is easy to show that $\frac{\partial z_i^*}{\partial z_j^*} < 0$ and $\frac{\partial E_i^*}{\partial E_j^*} < 0$, or that both climate policy options are strategic substitutes between countries. Thus the more mitigation or SRM country 1 does, the less incentive for country 2 to adopt them. Moreover, mitigation and SRM are strategic complements since $\frac{\partial E_i^*}{\partial z_j^*} > 0$. That is, the country with the lower emissions will adopt the SRM in order to compensate for the other country's increased emissions. ¹²

The HJB equation (26) implies that the parameters of the value function and the optimal Nash strategy for each country depend on the parameter θ_i . Thus (26) can be used to determine a symmetric Nash equilibrium under deep uncertainty as specified above. Moreover, for $\theta \to \infty$ the robust NE tends to the NE under the situation of pure risk.

4 Optimal Climate policy under Symmetry

Now we turn to the results about the optimal policy mix of the model including model misspecification (deep uncertainty) and explicitly compute the optimal mitigation and SRM policies when both countries are identical.

 $^{^{12}\}mathrm{Note}$ that mitigation is interpreted as a reduction of emissions E_i^*

The evolution of the uncertain impacts from SRM 4.1

We solve analytically the stochastic process of marginal SRM impacts u_i , by replacing the misspecification error $h_i^*(\theta)$ in equation (10). The value function satisfying (27) under symmetry has a simple quadratic form and respects the usual properties¹³ ($\mu_{1i}(\theta) < 0, \ \mu_{2i}(\theta) < 0$) regarding the curvature of the max-min value function. The marginal damages from SRM under the model misspecification are

$$du_{i}(t) = \left\{ \left[(\eta (1 - \gamma)) \sum_{i=1}^{2} z_{i}^{*}(t) - mu_{i}(t) \right] - \left[\frac{\sigma^{2}}{\theta} \mu_{1i}(\theta) + \frac{2\sigma^{2}}{\theta} \mu_{2i}(\theta) u_{i}(t) \right] \right\} dt + \sigma dB_{i}(t) \quad (32)$$

Consider equation (32), which describes the perceived worst case evolution of the SRM costs, then we can define two different effects of model misspecification for the dynamics of the marginal damages from SRM. The first effect $\left(-\frac{\sigma^2}{\theta}\mu_{1i}(\theta)\right) > 0$ suggests a reduction in the environment's ability of self-adjustment to the exogenous SRM damages. The second effect $\left(\frac{2\sigma^2}{\theta}\mu_{2i}(\theta)\right) < 0$ constitutes a reduction in the rate of adaptation to the SRM impacts.

If additionally we substitute the optimal values for emissions and SRM in the process of $u_i(t)$ (for either the cooperative or strategic Nash solution), we obtain an analytical solution which is an Ornstein–Uhlenbeck diffusion process and satisfies the differential equation

$$du_i(t) = \pi(\theta) \left(\varphi(\theta) - u_i^*(t)\right) dt + \sigma dB_i$$
(33)

The parameter $\varphi(\theta)$ represents the equilibrium or mean value, while $\pi(\theta)$ represents the rate by which the shocks dissipate how "strongly" the SRM impacts react to perturbations. Note that both parameters depend crucially on the robustness parameter θ^{14} . The marginal SRM impacts $u_i^*(t)$ are therefore normally distributed over time.¹⁵

 $^{^{13}}$ For the analytical solution see the Appendix A.1-A.3.

¹⁴For the analytical solutions of $\varphi(\theta)$ and $\pi(\theta)$ see Appendix 6. ¹⁵ $u_i^*(t)$ is distributed normally with mean $\mathbf{E}\left[u_i^*(t)\right] = \varphi(\theta) + (u_0 - \varphi(\theta))e^{-\pi(\theta)t}$ and variance $\mathbf{var}\left[u_i^*(t)\right] = \varphi(\theta) + (u_0 - \varphi(\theta))e^{-\pi(\theta)t}$ $\frac{\sigma^2}{2\pi(\theta)} \left(1 - e^{-2\pi(\theta)t}\right).$



Figure 1: Sample paths of the evolution of the marginal SRM impacts

Figure 1 shows realizations of the marginal impacts from SRM based on both solution concepts for different values of θ .¹⁶ It shows the relation between the evolution of the marginal impacts from SRM and the degree of ambiguity. For higher ambiguity levels (smaller values of θ), the SRM impacts are higher and more volatile.

4.2 Optimal policies at the steady state

We solve the symmetric cooperative and non-cooperative problems under risk based on calibrated values of the model parameters. Values for the parameters A_1, A_2 have been taken from Karp and Zhang (2006) and the default calibration of τ_i yields a standard damage estimate of 5.4% of GDP for a $4.5^{\,0}C$ temperature increase. The values of the parameters used in the calibration of the symmetric cooperative problem are used as the central values for the sensitivity analysis. The parameter ϕ is calibrated such that the SRM implementation will give the regional amount of SO_2 injected (measured in TgS) and the effective forcing per TgS will yield a negative forcing of $2.1W/m^2$. This estimate is based on a best guess estimate Gramstad and Tjøtta (2010) and relates to a range from -0.5 Crutzen (2006) to -2.5 Rasch et al. (2008). We assume a quadratic cost function at a private cost of 10 billion\$/TgS within the range considered in the literature, between 5 Crutzen (2006) and 25 billion USD/TgS. For the calibration of γ , we approximate impacts such that 3% of GDP are lost for a SRM implementation that leads to the offset of an expected global warming of $2.5^{\circ}C$, i.e., of $-3.5W/m^{2.17}$

Using these numerical calibration, we can define the steady-state level of emissions, climate engineering and average global temperature in the symmetric-cooperative and non-cooperative game under standard risk. The values obtained are presented in the following table:¹⁸

 $^{^{16}\}mathrm{Since}$ we focus on the steady state solution, we show the path starting at t=100.

¹⁷Appendix 6 provides the numerical values of all model parameters.

¹⁸The GHG emissions are measured in gigatons of CO_2 equivalents, the level of SRM in teragrams (or megatons) of sulfur, and the temperature as global annual average temperature in degrees centigrade.



Figure 3: Optimal climate policies (Emissions and SRM) for different values of θ .

Cooperation	$E_i^* = 18.6 GtC$	$z_i^* = 0.23TgS$	$T^* = 15.1^0 C$
Nash	$E_i^* = 23.9 GtC$	$z_i^* = 0.45TgS$	$T^* = 16.1^0 C$

First we note that in the strategic Nash solution, global temperature, emissions, and the level of SRM are higher than in cooperation. This is to be expected, since in the non-cooperative case, neither the externality of emissions nor the externality of SRM impacts is taken into account.



Figure 2: Expected optimal climate policies over time for different values of θ

Figure 2 shows the expected optimal levels of emissions and SRM across time for different levels of the robustness parameter. We find that all variables converge fast to their steady state levels. The difference in optimal policies concerning cooperation and Nash is evident. Note that for different level of θ , and as it is decreasing from 1000 to 10 and then to 0.5 (i.e ambiguity increases), both emissions and SRM effort are decreasing.

To investigate more closely the impact of the model misspecification, Figure 3 shows the impact of changing ambiguity intensity, as reflected in changing θ , on the optimal steady states for emissions and SRM for cooperative and the non-cooperative solution. As expected, emissions and climate engineering are higher under the Nash equilibrium relative to cooperation. It is interesting to note however that

as ambiguity increases (or θ decreases), the deviation between cooperative SRM and SRM in Nash equilibrium (see the third panel, which depicts the relative differences) is reduced. For mitigation and thus emissions, on the other hand, the relative difference is approximately constant. This is shown in the third panel of figure 3, which depicts the ratio of the optimal level of emissions and SRM in Cooperation over Nash. Ambiguity aversion thus seems to have a higher impact in the strategic Nash equilibrium case, where SRM in general is used more. The result can be attributed to the fact that here the ambiguity of SRM impacts is related directly to SRM, through its damages, but not directly to emissions.

A way of better understanding the implications from SRM use is through the resulting total impacts from SRM implementation. Figure 4 shows the total SRM damages as the expected percentage reduction of global Gross Domestic Product (GDP)¹⁹ in the symmetric case, both in cooperation and the non-cooperative solution. In both cases we have that as $\theta \longrightarrow \infty$, the damages converge to the case of a pure risk (no model misspecification). In the Nash solution, the damages from SRM are always higher than under cooperation–about 12% of GDP in Nash compared to less than 2% in cooperation. Moreover, as θ decreases, the total impacts from SRM decrease to zero, since SRM is drastically reduced.



Figure 4: Total impacts from SRM $D_{z}(\mathbf{z}(t), u_{i}(t))$ in percent of World GDP.

4.3 Quantifying the ambiguity effect

In order to measure and quantify the effect of the model misspecification, we consider the relative entropy in the model. Relative entropy can be regarded as a measurement of the misspecification error. By the integration of the optimal distortion h_i^* , we determine an entropy measure and we can quantify the degree of the model misspecification that the decision maker is willing to consider. To do so, we compute the solution of the stochastic differential equation (33), which has a unique solution as

$$u_{i}^{*}(t) = u(0) e^{-\pi(\theta)t} + \varphi(\theta) \left(1 - e^{-\pi(\theta)t}\right) + \sigma \int_{0}^{t} e^{-\pi(\theta)(t-s)} dB_{s}.$$

Given the mean and variance of $u_i^*(t)$, we can define the closed-form expression for the relative

 $^{^{19}\}mathrm{We}$ use the calibration based on Karp and Zhang (2006).

entropy of our model as

$$\mathcal{R}\left(\mathcal{Q}\right) = \int_{0}^{\infty} e^{-\rho t} \frac{1}{2} \mathbf{E}_{\mathcal{Q}} \left(h^{*}\left(t\right)\right)^{2} dt$$
(34)

where

$$h^*\left(t\right) = -\frac{\sigma V_u}{\theta} = -\frac{\sigma\left(\mu_1\left(u^*\left(\theta\right)\right) + 2\mu_2\left(u^*\left(\theta\right)\right)\right)}{\theta} \tag{35}$$

so we have that

$$\mathcal{R}\left(\mathcal{Q}\right) = \frac{1}{2}e^{-\rho t} \int_{0}^{\infty} \mathbf{E}_{\mathcal{Q}}\left(-\frac{\sigma\left(\mu_{1}\left(u^{*}\left(\theta\right)\right) + 2\mu_{2}\left(u^{*}\left(\theta\right)\right)\right)}{\theta}\right)^{2} dt.$$
(36)

This measure can be seen as the relative difference between the pure risk and the deep uncertainty case. Figure 5 shows the evolution of the entropy in non-cooperation for different values of θ . For low levels of the robustness parameter θ the misspecification error is high and hence the ambiguity effect on the optimal policy is large. Thus a range of values, in which the ambiguity aversion and the misspecification error are relevant in our model, is the range until a value of around ten.



Figure 5: Relative entropy in the symmetric Nash solution

According to Theorem 6.8.1 from Hansen and Sargent (2008), the problems (14) and (15) have the same solution, which directly associates an entropy bound $\mathcal{R}(\mathcal{Q},\theta)$ with a given value of the ambiguity parameter θ .



Figure 6: Expected global welfare for different values of θ

Figure 6 represents a different metric for the quantification of the ambiguity in terms of welfare²⁰. This welfare level can be computed as the welfare according to (13) at the optimum. The expected welfare level in cooperation is higher than in Nash, while welfare in both cases decreases as ambiguity increases (or θ decreases). It should be noted that the cooperative and the Nash solutions come closer in terms of welfare as ambiguity increases, as it shown in the second panel of figure 6, which shows the welfare in cooperation relative to the Nash solution. The result can be explained by the fact that as ambiguity increases SRM in Nash equilibrium is reduced faster than the cooperative solution from a relatively higher level (see figure 4) and this leads to a faster reduction of welfare differences.

Another way to quantify the ambiguity is to assign different $\theta's$ to different levels of the marginal SRM damages by calculating the proportional deviation of the distorted mean u^* from the benchmark under pure risk u^*_{risk} .



Figure 7: Relative increase in perceived marginal impacts from SRM u^*

Figure 7 shows this relation between the choice of the maximal deviation of the SRM damages that

 $^{^{20}}$ In figure 6 for cooperation and Nash under ambiguity, the welfare is non-monotonic for a very low value of θ . This is the effect of the "breakdown point" we discuss in the next subsection.

the regulator will consider implicitly (Δu) and the value of the robustness parameter θ . For instance, choosing a value of $\theta = 1$ implies that the decision maker considers a perceived increase in marginal damages from SRM around 40% higher than without ambiguity aversion. Another way of interpreting this chart is the following: if according to the existing information about SRM mean damages the regulator is willing to accept a maximum deviation of, say, 40% then he will choose the corresponding θ , which is $\theta = 1$.

4.4 The "breakdown point"

Hansen and Sargent (2008) define the "breakdown point" as the lower bound of θ beyond which it is fruitless to seek more robustness. The minimizing agent (*Nature*) is sufficiently unconstrained and she can push the criterion function to $(-\infty)$ despite the best response of the maximizing agent, the policy maker. Thus there exists a $\underline{\theta}$ above which the value function is well defined and does not lose its concavity. We can easily derive this "breakdown point" in the symmetric case for both the cooperative and the non-cooperative solution.²¹

For the cooperative solution, we find the "breakdown point" to be

$$\underline{\theta}^{C} > \frac{\sigma^{2} \left(\beta + \frac{8\tau A_{2}\phi^{2}}{(A_{2}\delta^{2} + 8\tau\lambda^{2})}\right)}{2 \left(\eta \left(1 - \gamma\right)\right)^{2}} \simeq 0.243$$

while for the non-cooperative Nash solution it equals

$$\underline{\theta}^{N} > \frac{\sigma^{2} \left(\beta + \frac{2\tau A_{2}\phi^{2}}{(A_{2}\delta^{2} + 2\tau\lambda^{2})}\right)}{\left(\eta \left(1 - \gamma\right)\right)^{2}} \simeq 0.473.$$

While overall both values cannot be compared unambiguously, we find that for our numerical calibration²² $\underline{\theta}^C < \underline{\theta}^N$. This means that under cooperation there is a range of robustness parameter values given by $\left[\underline{\theta}^C, \underline{\theta}^N\right]$ in which an optimal policy regulation is possible only in the cooperative case. Within this range, countries acting non-cooperatively don't find an optimal robust policy. The differentiated breakdown point under cooperative and non-cooperative solutions is an interesting and new finding within the robust control framework, and it becomes even more relevant under asymmetry as we show in the next section.

5 Results under Heterogeneity

So far we have considered the case of identical countries. However, the model allows us to introduce heterogeneity, and we will in particular consider three sources of heterogeneity which seem relevant in this context: differences in the impact from climate change on each country (τ_i) , differences in the impacts from climate engineering (η_i) , and differences in the attitude towards ambiguity or concerns about model misspecification (θ_i) . Notably the latter provides a new insight into the behavior of decision makers that have different attitudes towards deep uncertainty in their decision making process. We saw that under cooperation, heterogeneity does not introduce different results in our model, so we focus on the strategic policy solution.

²¹The "breakdown point" can be obtained from the denominator of the coefficient of the value function by preventing the V_i from losing concavity($\mu_{2i} \rightarrow -\infty$), see Appendix 6.

 $^{^{22}\}mathrm{See}$ the calibration values in Appendix 6.

5.1 Heterogeneity in ambiguity aversion

We start by analyzing the third case of heterogeneity in ambiguity aversion (θ_i) between the two countries. Without loss of generality, we look at the case $\theta_1 < \theta_2$. We consider the case in which the fictitious malevolent agent (Nature) in country 1 "pushes" the lower bound of the robust rule to a level where $(\theta_1 < \underline{\theta}^N)$, and thus it is fruitless for the policy maker in this country to seek for more robustness. On the other hand, we assume that in country 2 the policy maker is less ambiguity averse and the regulation is possible $(\theta_2 > \underline{\theta})$.

Proposition 2.

Suppose $\theta_1 \neq \theta_2$ in the cooperative and the Nash framework and consider the problems (19) and (26) respectively. If $\theta_1 < \underline{\theta}$ and $\theta_1 + \theta_2 > \underline{\theta}$, then the formulation of a robust optimal policy is possible in cooperation but not feasible in Nash.

Proof

Cooperation

In the cooperative solution, the policy maker could have different $\theta's$ for each country, but he considers only one distortion of the SRM dynamics (10) and therefore he decides about one h^* . Different concerns about model misspecification are expressed by different $\theta's$. From the minimization of the problem (19), we have that $h^* = -\frac{\sigma V_u^C}{\theta_1 + \theta_2}$. At the cooperative solution $\theta^* = \theta_1 + \theta_2$, so even if θ_1 is below the minimum threshold ($\underline{\theta}$) the sum of the policy maker's robustness parameters might be above the lower bound and regulation in this case is possible at the cooperative solution ($\underline{\theta} < \theta^*$). Thus a policy maker with high concerns about model misspecification ($\theta_1 < \underline{\theta}$) will apply the mix of optimal policies (emissions and SRM) if he cooperates with another policy maker who trusts his model more ($\theta_2 > \underline{\theta}$). If the countries cooperate there is a higher possibility to have a solution of the problem ($\theta_1 + \theta_2 > \underline{\theta}$), even if the policy maker of one of the countries has high concerns about his model. Nash

In the non-cooperative Nash solution the policy maker of each country or region decides independently and Nature chooses a different misspecification error (h_i^*) for each agent. Thus the different $\theta's$ for each agent consider different distortions by Nature to the SRM dynamics (10). From the minimization of the problem (26) we have that $h_i^* = -\frac{\sigma V_{u_i}^N}{\theta_i}$. It is obvious that if θ_1 is below the threshold regulation $(\underline{\theta}_1 < \theta)$ at the Nash equilibrium, the regulation might not be possible $(h_i^* : V_i(t) \longrightarrow -\infty)$.

Now we solve the model for a range of different values for the robustness parameter. Note that even though we consider the non-negativity constraints, as in this case, corner solutions cannot be excluded. We obtain the following picture of SRM implementation between countries shown in Figure 8, when we consider heterogeneity in the degree of ambiguity aversion or confidence in the model in both countries. We keep the value of the robustness parameter θ_1 unchanged for country 1, and we vary the value of the robustness parameter of country 2 in both directions.





Figure 8: SRM Implementation with asymmetry in ambiguity aversion

We observe, as before, that SRM is lower in cooperation in all cases. In the strategic Nash solution, the country with the lower robustness parameter, which is therefore more ambiguity averse about SRM impacts, will refrain from using SRM, while the other, less ambiguity averse country will implement SRM. Note that the only case of both countries applying SRM is the symmetric case where both countries have the same robustness parameter.

5.2 Heterogeneity of impacts from global warming and SRM

Given the focus of this paper is on the impacts from SRM, we now turn to heterogeneity in this dimension. Having obtained the optimal solutions of the model, we examine through numerical simulations the effect of heterogeneity in the SRM impacts and from global warming itself. In the Nash solution, heterogeneity in p_i^{23} and in τ_i refer to the cases of heterogeneity in the impacts from the use of SRM and in the damages from climate change, respectively. In the case of heterogeneity between the two countries we look at the case where country 1 (i = 1) faces the same level of marginal impacts as in the symmetric benchmark case. Then we vary the impacts of country 2 (i = 2), which faces different marginal environmental damages (τ_2) from a temperature increase and different impacts from the use of SRM (p_2). We implement the variation by different percentage increases or decreases from the values of country 1. Based on this setup, we solve the stochastic model either as a joint optimization problem in the cooperative case, or a feedback solution in the Nash case.

Figure 9 shows the optimal policies and climate response for heterogeneity in impacts from SRM (left part) and impacts from global warming (right part). The most striking difference in terms of climate policy is which country implements SRM in both cases. With heterogeneity in SRM impacts, only the country with lower impacts will implement SRM in the Nash solution, which taken into account by the other player, leads him to withdraw from the implementation of *SRM* and allows him a high emission profile. With heterogeneous climate change impacts the situation is reversed: the country with higher impacts from an increase in the global mean temperature will implement all SRM while the other country always does zero SRM. The latter result confirms the "Tuvalu syndrome" (Millard-Ball, 2012) aspect of climate engineering; in that countries with highest impacts from climate changes are most likely to unilaterally engage in geoengineering implementation. The former result

 $^{^{23}}$ Note that for the numerical simulations, we define the heterogeneity in the impacts from the use of SRM as $p_i = \eta_i (1 - \gamma)$

can also be rationalized, lower impacts or side-effects from the implementation of SRM will lead the country to use SRM technology under any circumstances, therefore at equilibrium the other country will refrain from using it.

The global temperature increase is slightly lower if impacts from global warming are increased, but almost invariant to the impacts from SRM, since the total amount of SRM itself is almost unchanged. With regard to GHG emissions, a country with higher impacts from either warming or SRM emits less. That is, the effect on mitigation is symmetric: in the case of higher SRM impacts, the country does not implement SRM and chooses a lower level of emissions since the other country will use SRM and complement it with higher emissions. In the case of climate impacts the effect is much less pronounced, but shows the more easily expected result that a country with higher damages will emit less.



Figure 9: Heterogeneity of impacts from SRM (left) and global warming (right)

6 Conclusion

Climate engineering and in particular SRM is a a controversial technology, but a potentially attractive alternative solution to deal with the consequences of global warming. The discussions about the use of SRM are often focused on potential damages or side-effects. We created a model that explores the range of SRM induced damages in a world with asymmetry and ambiguity in a robust control framework, where countries act cooperatively or strategically.

Our results suggest that deep uncertainty about impacts from SRM can be important, and leads to a reduction in SRM deployment both under cooperation and strategic behavior. Moreover, we show that this effect is a more relevant issue if countries act strategically, supporting the view that the possibility of unilateral climate engineering can be aggravated by the large uncertainty around it. Moreover, we show that the country which is worse off due to the heterogeneity will do zero SRM and the other country will make all the SRM effort in order to compensate for the high emissions from both countries. Our model therefore provides some insights regarding situations where SRM action could be undertaken unilaterally

The climate policy options of mitigation and SRM are strategic substitutes between the countries. Thus the more mitigation or SRM one of the countries does, the less incentives there are for the other to adopt these options. The main question is the extent to which ambiguity affects the results so that they diverge significantly from the pure risk case. We tried to answer this question by quantifying the ambiguity effect in three ways, by computing the relative entropy, the welfare gap and the maximal deviation of the SRM damages. Our results show that in order to have a range of values in which the ambiguity aversion and the welfare gap between risk aversion and ambiguity aversion are significant and affect the optimal policy in our model, the robustness parameter θ has to be lower than ten, which means that the regulator's concerns about model misspecification induce him to consider deviations of marginal SRM damages from the benchmark estimate which exceed 10%. Using a numerical simulation of the analytical model, we show that in Nash equilibrium countries will prefer a mix of policies with high emissions and high levels of SRM to partially compensate global warming damages. This mix of policies will lead to a higher global temperature and damages from SRM than in global cooperation.

When we consider heterogeneous countries with respect to the environmental damages due to a temperature increase, we find that the country with the higher impacts from global warming will implement a higher level of SRM, which when taken into account by the other country, leads the second country to withdraw from the implementation of SRM. The strategic interactions also drive both countries to adopt much higher climate engineering levels in Nash equilibrium compared to the cooperative symmetric solution.

On the contrary, when the source of the heterogeneity is the difference in the impacts from the use of SRM, we find that the country which suffers more from SRM impacts or side-effects will take no climate engineering action, whereas the country with lower impacts will unilaterally undertake all the climate engineering resulting in the same level globally.

When we vary the robustness parameter reflecting the attitude towards ambiguity, a higher degree of ambiguity aversion (lower θ) leads countries to do less SRM both in the Nash equilibrium and with full cooperation. Furthermore the numerical results suggest that as ambiguity aversion increases (θ decreases), the deviation between the cooperative and the Nash equilibrium solutions in terms of level of SRM, welfare and SRM damages as proportions of GDP are reduced.

Ultimately, we consider the interesting case of different ambiguity attitudes in the two countries. Here, we find that all SRM will be implemented by the country with the higher confidence in its model while the other country will not use SRM at all, whereas at the global level the amount of SRM is unchanged. This result suggests that deep uncertainty becomes a more severe problem with heterogeneous countries in a strategic setting, suggesting that the precautionary policy induced by ambiguity aversion can be counteracted, if only one actor is sufficiently confident in his assessment of the (low) impacts or side-effects from climate engineering.

There are a number of extensions of the present model, which would be interesting and policyrelevant in this field: a more complete analysis of SRM under ambiguity would extend the basic model to incorporate non-symmetric games between agents and Nature with heterogeneity in spatial characteristics of agents, such as temperature differences across latitudes or differences in the precipitation patterns across regions. Another interesting extension would be the optimal climate policy in the presence of climate tipping points, and the inclusion of adaptation as third policy option.

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Appendix A

A.1 Properties of the value function

The value function of the symmetric Nash problem $(\tau_i = \tau_j = \tau, \ \eta_i = \eta_j = \eta, \ \theta_i = \theta_j = \theta)$ satisfying (26) has the following simple quadratic form:

$$V_{i}(\theta, u_{i}) = \varepsilon_{0i}(\theta) + \mu_{1i}(\theta) u_{i} + \mu_{2i}(\theta) u_{i}^{2}, i = 1, 2$$
(37)

where the coefficients can be computed as

$$\mu_{1i}(\theta) = -\frac{\theta\left(\zeta - 2p\mu_{2i}\right)\left(\beta z_j^*\left(A_2\delta^2 + 2\tau\lambda^2\right) + 2\tau\phi\left(A_2T_0\delta - \left(A_1 + A_2E_j^*\right)\lambda\right)\right)}{\left(A_2\delta^2 + 2\tau\lambda^2\right)\left(\theta\left(p\left(\zeta - 2p\mu_{2i}\right) + \beta\left(m + \rho\right)\right) + 2\beta\sigma^2\mu_{2i}\right) + 2\tau A_2\phi^2\left(\theta\left(m + \rho\right) + 2\sigma^2\mu_{2i}\right)} \le 0$$

$$\mu_{2i}\left(\theta\right) = \frac{\theta\left(A_2\delta^2 + 2\tau\lambda^2\right)\left(2\zeta p + \beta\left(2m + \rho\right)\right) + 2\tau A_2\phi^2\theta\left(2m + \rho\right) - \sqrt{R^N}}{4\left(A_2\delta^2 + 2\tau\lambda^2\right)\left(p^2\theta - \beta\sigma^2\right) - 8\tau A_2\phi^2\sigma^2} \le 0$$

where

$$R^{N} = \theta \left(A_{2}\delta^{2} + 2\tau\lambda^{2} + 2\tau A_{2}\phi^{2} \right)$$
$$\cdot \left(\left(A_{2}\delta^{2} + 2\tau\lambda^{2} \right) \theta \left(2m + \rho \right) \left(4\zeta p + \beta \left(2m + \rho \right) + 4\zeta^{2}\sigma^{2} \right) + 2\tau A_{2}\phi^{2}\theta \left(2m + \rho \right)^{2} \right)$$

A.2 Evolution of the marginal SRM impacts

The marginal damages from SRM under the model misspecification are

$$du_{i}(t) = \left\{ \left[(\eta (1 - \gamma)) \sum_{i=1}^{2} z_{i}^{*}(t) - mu_{i}(t) \right] - \left[\frac{\sigma^{2}}{\theta} \mu_{1i}(\theta) + \frac{2\sigma^{2}}{\theta} \mu_{2i}(\theta) u_{i}(t) \right] \right\} dt + \sigma dB_{i}(t) \quad (38)$$

where

$$h_{i}^{*}\left(\theta\right) = -\frac{\sigma V_{ui}}{\theta} = -\frac{\sigma \left(\mu_{1i}\left(\theta\right) + \mu_{2i}\left(\theta\right)\right)}{\theta}$$

and

$$z_i^* = \frac{\left(\eta \left(1-\gamma\right) V_{ui} - \zeta u_i + \omega_i\right) \left(A_2 \delta^2 + 2\tau \lambda^2\right) - 2\tau \phi \left(A_1 \lambda + A_2 \left(\lambda E_j^* - T_0 \delta + \phi z_j^*\right)\right)}{A_2 \beta \delta^2 + 2\tau \left(\beta \lambda^2 + A_2 \phi^2\right)}.$$

The Ornstein–Uhlenbeck diffusion of (38) is given by

$$du_{i}(t) = \pi\left(\theta\right)\left(\varphi\left(\theta\right) - u_{i}^{*}\left(t\right)\right)dt + \sigma dB_{i}$$
(39)

where we use the notation

$$p = p_i = p_j = \eta \left(1 - \gamma\right) \tag{40}$$

$$\kappa(\theta) = \frac{2A_2\zeta p\delta^2 + 8\tau\zeta p\lambda^2}{A_2\beta\delta^2 + 4\tau\left(\beta\lambda^2 + A_2\phi^2\right)} + m + \frac{2\sigma^2\mu_2\left(\theta\right)}{\theta}, \ \pi(\theta) = \frac{1}{\kappa(\theta)}$$
(41)

$$\psi(\theta) = \frac{2p(4\tau\lambda(pV_u\lambda - A_1\phi) + A_2\delta(pV_u\delta + 2\tau T_0\phi))}{A_2\beta\delta^2 + 4\tau\left(\beta\lambda^2 + A_2\phi^2\right)} - \frac{\sigma^2\mu_1\left(\theta\right)}{\theta}, \ \varphi(\theta) = \frac{\psi(\theta)}{\kappa(\theta)}$$
(42)

A.3 The "breakdown point"

The coefficients of the value functions under symmetry for cooperation and Nash respectively are

$$\mu_{2}^{C}\left(\theta^{C}\right) = \frac{\theta^{C}\left(A_{2}\delta^{2} + 8\tau\lambda^{2}\right)\left(8\zeta\eta\left(1-\gamma\right) + \beta\left(2m+\rho\right)\right) + 8\tau A_{2}\phi^{2}\theta^{C}\left(2m+\rho\right) - \sqrt{R^{C}}}{4\left(A_{2}\delta^{2} + 8\tau\lambda^{2}\right)\left(2\left(\eta\left(1-\gamma\right)\right)^{2}\theta^{C} - \beta\sigma^{2}\right) - 32\tau A_{2}\phi^{2}\sigma^{2}}$$

and

$$\mu_{2i}^{N}(\theta^{N}) = \frac{\theta^{N}(A_{2}\delta^{2} + 2\tau\lambda^{2})(2\zeta\eta(1-\gamma) + \beta(2m+\rho)) + 2\tau A_{2}\phi^{2}\theta^{N}(2m+\rho) - \sqrt{R^{N}}}{4(A_{2}\delta^{2} + 2\tau\lambda^{2})\left((\eta(1-\gamma))^{2}\theta^{N} - \beta\sigma^{2}\right) - 8\tau A_{2}\phi^{2}\sigma^{2}}$$

In order to avoid $\mu_2^C(\theta^C)$, $\mu_{2i}^N(\theta^N) \longrightarrow \infty$, we need to ensure that

$$4\left(A_2\delta^2 + 8\tau\lambda^2\right)\left(2\left(\eta\left(1-\gamma\right)\right)^2\theta^C - \beta\sigma^2\right) - 32\tau A_2\phi^2\sigma^2 \neq 0$$

 $\quad \text{and} \quad$

$$4\left(A_2\delta^2 + 8\tau\lambda^2\right)\left(2\left(\eta\left(1-\gamma\right)\right)^2\theta^N - \beta\sigma^2\right) - 8\tau A_2\phi^2\sigma^2 \neq 0.$$

This implies for the two breakdown points that they can be computed as

$$\theta^{C} > \frac{\sigma^{2} \left(\beta + \frac{8\tau A_{2} \phi^{2}}{\left(A_{2} \delta^{2} + 8\tau \lambda^{2}\right)}\right)}{2 \left(\eta \left(1 - \gamma\right)\right)^{2}} \text{ and } \theta^{N} > \frac{\sigma^{2} \left(\beta + \frac{2\tau A_{2} \phi^{2}}{\left(A_{2} \delta^{2} + 2\tau \lambda^{2}\right)}\right)}{\left(\eta \left(1 - \gamma\right)\right)^{2}}$$

Appendix B: Numerical values of the parameters

Parameter	Description	Value	Unit
au	slope of social marginal damage cost from a temperature	77.92	10^{9} \$/GtC
	increase*		
A_1	intercept of marginal benefit from emissions	224.26	/tC
A_2	slope of marginal benefit from emissions	1.9212	10^{9} (GtC) ²
ϕ	sensitivity of global mean temperature to SRM	-2.1	$^0C/TgS$
λ	sensitivity of temperature to emissions	0.1	$^{0}C/GtC$
δ	heat transfer parameter	0.18	W/Km^2
ρ	pure rate of time preference	0.01	scalar
σ	standard deviation of $u_i(t)$	0.002	scalar
$p = \eta(1 - \gamma)$	marginal impacts from SRM**	0.085	1/TgS
m	adjustment rate of SRM impacts	1.2	scalar
β	slope of marginal cost from SRM***	40	$10^{9} $ /TgS
ζ	marginal SRM damage parameter**	29185	10^{9} \$/TgS
T_0	world average temperature, 2005	14	^{0}C

 \ast The marginal damage is based the DICE model which uses a marginal damage of 0.00267% of GDP and the gross world product of 2100 of 29185 trillion USD.

** Firstly, p is used to calibrate the steady state of the model, and found to be around 8% of GDP per TgS. Secondly, ζ converts the relative impacts impacts at the calibration point of 1TgS (see Goes et al. (2011)) into total USD values using the GDP value used also for the utility function based on Karp and Zhang (2006),

*** This parameter is calibrated such that at 1TgS, the average estimate of 10 billion USD per teragram of sulfur are obtained, see Bickel and Agrawal (2013); Gramstad and Tjøtta (2010); McClellan et al. (2012); Robock et al. (2009).

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