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A new model for the alignment of almost balanced sales territories

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Abstract

The sales territory alignment problem deals with the question of how to align a number of sales coverage units (usually zip-codes or political districts) to sales territories. These sales territories are usually aligned in a way that they are almost balanced relative to one or several attributes like, e.g. potential or work-load. To support this task of the sales manager, a number of models have been proposed. Nevertheless, all of these models suffer from one of the following shortcomings: either, they do not guarantee the values of the chosen balancing attributes to be within certain lower and upper bounds, or the specification of these lower and upper bounds is left to the sales manager. Therefore, we propose a model called EQUALIZER that avoids these shortcomings by determining almost balanced territories without requiring lower and upper bounds from the sales manager and furthermore ensuring contiguous sales territories. In order to motivate EQUALIZER, we compare different models on a small set of instances.

1 Introduction

Companies often divide their salesforce according to regional characteristics. At first, they group small geographic sales coverage units (SCUs), usually zip-codes or political districts, into larger clusters, called sales territories, and then they assign the sales territories exclusively to individual salespersons. Reasons for that exclusive assignment of each SCU to a salesperson are the establishment of a long-term relationship between customers and salespersons, the avoidance of competition among salespersons, the opportunity to evaluate and control the salesperson's performance and the increase of the salesperson's morale and effectiveness (Albers, 1989). Yet, companies still face the problem of how to align these territories because even modest improvements of an alignment can have significant effects on profit. A 5% increase in salesforce productivity from a 100-person salesforce means that only 95 people would be needed to maintain current sales levels (Hess and Samuels, 1971). Since literature has often shown substantial productivity improvements of this magnitude, the problem is of considerable importance for companies (LaForge, Cravens and Young, 1986).

Sales territories are usually aligned in a way that they are almost balanced with respect to one or several attributes of the SCUs (Zoltners and Sinha 1983). The most popular balancing attributes are sales potential or work-load of the salesperson. The underlying idea of establishing territories with equal potential is to provide each salesperson with the same income opportunity and to facilitate an easy evaluation of salespersons' performances. In contrast to that, territories with equal work-load strive for a fair treatment of all salespersons because all salespersons are assumed to have the same work to do (Churchill, Ford and Walker 1993). Thereby, work-load is usually measured by the number of customer calls (Churchill, Ford and Walker 1993). Having several territory alignments being comparable in terms of these balancing attributes, usually the territory alignment which minimizes the required travel time is chosen.

Literature proposes a number of territory alignment models (for an overview see Howick and Pidd, 1990). These models differ in the way in which they try to establish almost balanced territories. The first group takes directly into account that it is almost never possible to have exactly equal territories. Therefore, first the sales manager has to specify lower and upper bounds for each of the balancing attributes and then the model searches for territories which satisfy the restrictions set to these balancing attributes. In contrast, models of the second group are based on mixed-integer formulations and try to establish exactly equal territories in their first step. Therefore, they have to accept that not all SCUs can be assigned exclusively to one

territory. In non-integer solutions some SCUs are "split" among several territories and a solution has to be found by rounding procedures in a second step.

Zoltners and Sinha (1983) have largely discussed the shortcomings of the models in the second group. Thus we only summarize two: first, the rounding procedures in the second step cannot ensure that the resulting territories are almost balanced or within an acceptable range. Second, models of the second group cannot guarantee that sales territories are contiguous. Therefore, the models in the first group appear to be superior. Yet, they suffer from the serious restriction that they ask the sales manager to give values for the upper and lower bounds of the balancing attributes. However, the determination of these bounds is part of the problem the sales manager wants to be solved. As we will show further in this paper, the most popular model of the second group, the model of Zoltners and Sinha, 1983 (Z/S-model), additionally suffers from the restriction that it relies on the assumption of only considering the shortest-path between an SCU and the base unit of the territory to ensure the contiguity of the sales territory.

In this paper, we present a new model called EQUALIZER which determines almost balanced territories without requiring lower and upper bounds from the sales manager and ensuring the contiguity of the sales territories (without relying on the shortest-path assumption). Thus, we organize the remainder of the paper as follows. In Section 2, we present our new model EQUALIZER and a simulated annealing approach in Section 3. In Section 4, we demonstrate via a counter-example why the very popular model of Zoltners and Sinha (1983) suffers from the shortest-path assumption. Computational results demonstrate the solution quality proposed by EQUALIZER in Section 5. The final section 6 summarizes the main conclusions of the paper.

2 Model EQUALIZER

Mathematically, sales territory alignment is a partitioning problem: the model EQUALIZER partitions the set of SCUs into almost balanced territories. Among the different partitions the one which minimizes travel costs is chosen. In EQUALIZER, we consider the case of a single balancing attribute.

Let denote

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i,r = 1,...,R: index of the SCUs,

w_r: value of the balancing attribute in SCU r,

d_{i,r}: distance between SCUs i and r,

E = \{[i,r] \mid i,r \text{ are connected SCUs}\},

j = 1,...,J the index of the sales territories (and salespersons), and
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b_i the base SCU r of territory j (e.g. the home base of the salesperson).

The balancing attribute w_r serves as the (single) criteria to balance the size of the territories j. $d_{i,r}$ is an appropriate distance measure for the problem, for instance driving distances of a road network. The edge set E defines a planar graph E which may represent the road network. Two SCUs are *connected* in E if there is, e.g. a road, suited for the salesperson, between them. Due to the planarity of E, the maximal number of arcs is bounded by 3R-6 for the number of SCUs $R\geq 3$, and therefore, the number of arcs increases only linearly with R (Domschke and Drexl, 1996).

As decision variable we use the binary assignment variable

 $X_{j,r} = 1$ if SCU r is assigned to territory j (and therefore, to salesperson j), and 0 otherwise.

Clearly, not every binary $X_{j,r}$ matrix is feasible. We want the $X_{j,r}$ to define a *contiguous* partition of the SCUs with respect to E. More formally, a partition is given by the constraint

(1)
$$\sum_{i=1}^{J} X_{j,r} = 1$$
 r=1,...,R.

(1) states, that each SCU r is assigned to exactly one territory j. Furthermore, the base SCUs b_j of the territories are preassigned, i.e.

(2)
$$X_{j,b_i} = 1$$
 $j=1,...,J.$

The contiguity of the territories is achieved by the following constraints: let denote

$$U_j = \{ r | r = b_j \text{ or } (X_{j,r} = 1 \text{ and } ([i,r] \in E \text{ and } i \in U_j)) \}$$

where initially $U_i = \{\}$.

 U_j is a (recursively defined) set of SCUs r which is contiguous. Contiguity of U_j is achieved as follows. An SCU is either the base $r = b_j$, or r is connected with U_j by another SCU i: if $X_{j,r}=1$ we must find an i in U_j where [i,r] in E. We can now express the contiguity with

(3)
$$|U_{j}| = \sum_{r=1}^{R} X_{j,r} \qquad j=1,...,J.$$

(3) states that the cardinality of the set of all SCUs which are assigned to sales territory j must be the cardinality of the set U_j . The objective is to generate territories as balanced as possible. Let denote

$$S:=\frac{\sum_{r=1}^{R}w_{r}}{R}$$

the "target size" of the territories, and define

$$W_j(X) = \sum_{r=1}^R w_r \cdot X_{j,r}$$

the size of sales territory j with respect to the partition expressed by X. Furthermore, let denote δ the deviation that gives the tolerance in which the size of the territories may vary. Hence, $(S-\delta)$ and $(S+\delta)$ are the lower and upper bounds for the territories.

The goal is to find a partition with

$$|W_{i}(X) - S| \le \delta$$
 j=1,...,J.

In EQUALIZER we will allow deviations smaller than δ , but will penalize quadratically any deviation that is larger. Hence, the penalty function for a term y is

$$f(\alpha, \delta, y) = \begin{cases} 0 & \text{for } |y| \le \delta \\ \alpha \cdot (|y| - \delta)^2 & \text{for } |y| > \delta \end{cases}.$$

Then, deviations smaller than δ are not penalized, and for larger deviations a quadratic penalty term is chosen. The parameter α is necessary for the algorithm in the next section. Among all partitions where the size of the territories is within the specified range, we consider the minimization of travel costs as objective. The travel costs are expressed by

$$\sum_{i=1}^{J} \sum_{r=1}^{R} w_r \cdot d_{b_j,r} \cdot X_{j,r}$$

The total value of the balancing attribute is $\sum_{r=1}^{R} w_r$ and constant. Like most other models for territory alignment, we do not take into account any tour optimization. Hence, we assume travel costs to be proportional to the distance $d_{b_j,r}$ from the base b_j to territory r, and

proportional to the balancing attribute w_r (as an equivalent for the number of times the salesperson travels to the SCU). Thus, the model EQUALIZER can now be expressed as

(4) Minimize
$$Z_{EQU} = \sum_{j=1}^{J} \sum_{r=1}^{R} w_r \cdot d_{b_j,r} \cdot X_{j,r} + \sum_{r=1}^{R} f \left(\alpha, \delta, \left(\sum_{i=1}^{R} w_r \cdot X_{j,r} - S \right) \right)$$

subject to (1), (2) and (3).

Furthermore, we denote with EQUALIZER(α,δ) the model EQUALIZER, in which fixed values for α,δ have been chosen, and with $Z_{\text{EQU}}(\alpha,\delta)$ the corresponding objective function.

3 Simulated Annealing Approach

The heuristic EQUALIZER_{SA} is a simulated annealing approach for solving the model EQUALIZER. EQUALIZER_{SA} determines δ and the contiguous partition with minimum travel costs. This determination of δ is necessary because the sales manager does not know in advance the range δ , i.e. the lower and upper bounds, in which the size of the territories may vary. Therefore, we solve EQUALIZER(α , δ) and determine a "necessary" deviation δ_0 . EQUALIZER(α , δ) is solved by simulated annealing: we generate a start-solution in which each SCU r is assigned to the "nearest" territory j. More formally, we set (with ties broken arbitrarily)

$$X_{j,r} = 1 \Leftrightarrow d_{b_j,r} = \min_{k=1,\dots,J} \{d_{b_k,r}\} := d_r^{min}$$

It can be shown that this start solution always determines a contiguous partition (Zoltners and Sinha, 1983).

A neighborhood solution is generated by moving one SCU r from one territory j1 to a neighborhood territory j2 in such a way that both territories are still contiguous. That is, we simply set $X_{j1,r}=0$ and $X_{j2,r}=1$ and then check the contiguity of both territories. The simulated annealing algorithm now proceeds as follows: if the new solution leads to a better value $Z_{EQU}(\alpha,\delta)$, it is "accepted", otherwise it is accepted with a certain probability depending on the actual cooling "temperature" (for details about simulated annealing procedures see Van Larhoven and Aarts, 1987).

In order to determine the necessary deviation δ we start solving EQUALIZER($\alpha=\alpha_0$, $\delta=0$) with

$$\alpha_0 = \frac{\displaystyle\sum_{r=1}^R w_r \cdot \displaystyle\sum_{r=1}^R d_r^{min}}{T^2},$$

therefore, there is a penalty for any deviation from the target size S. We then define δ_0 as the maximum deviation in the solution with $\alpha = \alpha_0$, i.e.

$$\delta_0 = \max_{j=1,\dots,J} \left(\left| \sum_{r=1}^R w_r \cdot X_{j,r} - S \right| \right)$$

and solve EQUALIZER($\alpha=\alpha_0$, $\delta=\delta_0$). Solving EQUALIZER(α,δ) lasts only a couple of seconds even for large instances. Therefore, we repeat the search in order to find a good solution. Hence, we solve EQUALIZER(α_k,δ_0) k=1,...,10 times where

$$\alpha_k = k \cdot \frac{\alpha_0}{10}.$$

In each step k, α is incremented by $\alpha_0/10$. The search is stopped if the second term in $Z_{EQU}(\alpha_k, \delta_0)$ vanishes, i.e. there is no penalty term but the objective expresses the travel costs only. Then, a solution is found where the size of the territories is within the tolerated range defined by δ_0 , i.e.

(5)
$$|\sum_{r=1}^{R} w_r \cdot X_{j,r} - S| = |W_j(X) - S| \le \delta_0$$
 $j=1,...,J.$

4 Model from Zoltners and Sinha

The mixed integer programming formulation of the model from Zoltners and Sinha, 1983, (Z/S-model) is as follows (again, we consider only a single balancing attribute):

(6)
$$\sum_{j=1}^{J} \sum_{r=1}^{R} w_r \cdot d_{b_j,r} \cdot X_{j,r} \rightarrow \min!$$

(8)
$$\sum_{j=1}^{J} X_{j,r} = 1$$
 r=1,...,R,

(9)
$$X_{j,r} \in \{0,1\}$$
 $j=1,...,R$,

where LB_r and UB_r are the upper and lower bounds for the balancing attribute. The objective function in (6) is the objective function in EQUALIZER without the penalty term. Constraints (7) guarantee that the values of the balancing attribute of all sales territories are within a specified range. In contrast to EQUALIZER, the sales manager must specify these values as an input to the model. Equations (8) and (9) ensure that each SCU is assigned to exactly one sales territory. Constraints (10) intend to satisfy the contiguity of all sales territories by the use of a hierarchical SCU-adjacency tree structure. The adjacency tree $A_{j,r}$ is the shortest-path tree (based on the distances $d_{i,r}$ in E) whose vertex is the SCU d_j (the base SCU of the territory d_j). A hierarchical SCU-adjacency tree is established by redrawing the SCU-adjacency tree with the base SCU of the territory as the top node (see Figure 1).

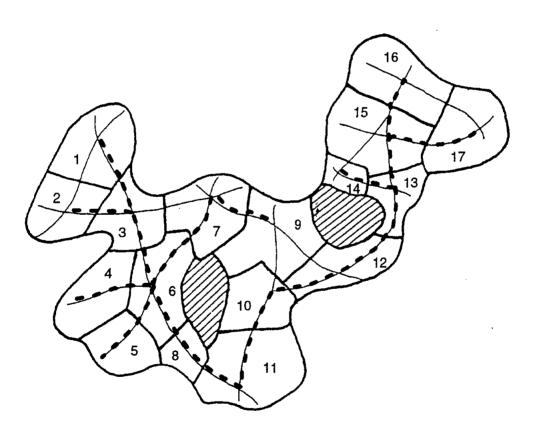


Figure 1: Geographic region including the roads and shortest-paths for the base unit 6 and the rest of the SCUs

Zoltners and Sinha (1983) illustrate the use of such a hierarchical SCU-adjacency tree by the example of a geographic region which consists of 17 SCUs and where SCU 6 is the base SCU of a territory (see Figure 1). The shaded areas in Figure 1 represent two nontraversable

objects (e.g. lakes or mountains). The dotted lines show the available roads between the SCUs and the thicker lines show the roads to take in order to follow the shortest-path (measured in travel time) from the base unit 6 to the rest of the SCUs.

Figure 2 displays the corresponding hierarchical SCU-adjacency tree. The constraints (10) make use of the adjacency levels of the SCUs in figure 2. They require that an SCU at the k-th level of the hierarchical SCU-adjacency tree can only be assigned to the territory if its SCU predecessor at the (k-1)st level is assigned to that territory, too.

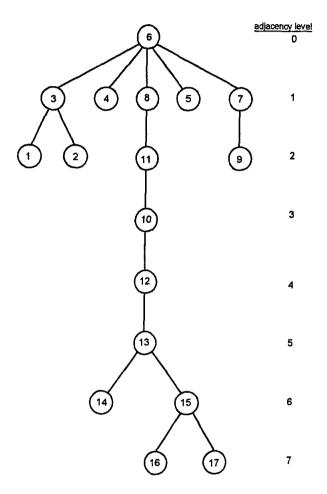


Figure 2: Shortest-path SCU-adjacency tree including adjacency levels

Constraints (10) ensure contiguous territories as long as only shortest-paths are used in $A_{j,r}$. However, Zoltners and Sinha (1983) admit those shortest-paths "establish rigid access and contiguity constraints". Therefore, they allow the inclusion of additional edges in $A_{j,r}$ such that the resulting hierarchical SCU-adjacency tree may contain duplicate nodes (i.e. SCUs) on different branches of the tree, and conclude that the contiguity constraints (10) still hold to be true. Unfortunately, this conclusion is wrong as can easily be demonstrated by the following counterexample. We add (as Zoltners and Sinha have done too) only two additional edges such that it is possible to access SCU 10 via SCU 9 and vice versa. Figure 3 displays the

resulting hierarchical SCU-adjacency tree. Constraints (10) now state that SCU 10 is allowed to be assigned to the territory whenever SCU 9 is assigned and vice versa. Therefore, a feasible solution to constraints (10) is the solution displayed in figure 4 which obviously does not reflect a contiguous sales territory. Therefore, to ensure contiguity of their sales territories, Zoltners and Sinha (1983) have to rely on the shortest-path assumption.

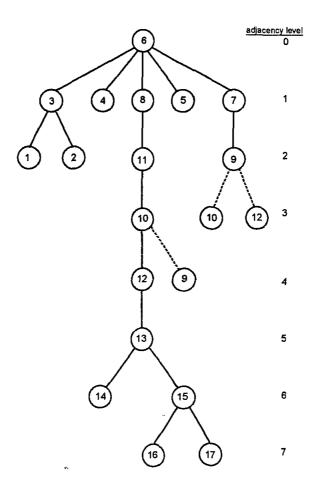


Figure 3: SCU-adjacency tree with shortest paths and additional edges

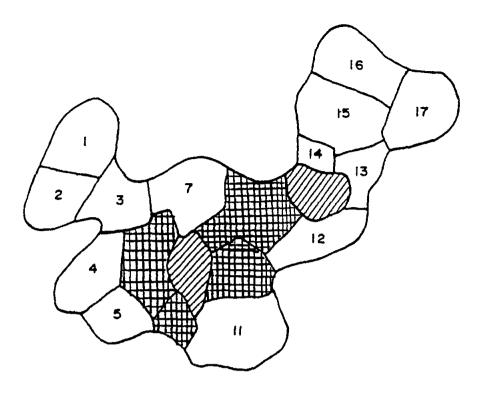


Figure 4: Feasible solution of the model from Zoltners and Sinha which is not contiguous

5 Computational Results

In a computational study we relate the performance of EQUALIZER_{SA} to an optimal procedure SPP and to the model of Zoltners and Sinha (Z/S-model). SPP is a set partitioning procedure which generates all contiguous partitions where constraints (5) hold (i.e. the territory sizes must not have a deviation larger than δ_0). In SPP, the contiguous partition with the smallest objective function value (= travel costs) is optimal. In the algorithm used to solve the Z/S-model, all SPP contiguous partitions are enumerated with the additional constraint that each territory is connected by the shortest-path, i.e. $A_{j,r}$ is the shortest-path SCU adjacency tree. Therefore, we say that the Z/S-model is based on the shortest-path assumption. The enumeration of this (smaller) set of partitions (Z/S-enumeration) leads to the optimal objective for the Z/S-model. Nevertheless, relaxing this assumption may lead to better solutions. A counterexample can easily be constructed where for some δ_0 , there is even no partition with respect to the shortest-path assumption, i.e. Z/S-enumeration does not generate a feasible partition.

Algorithm	∆avg	Δmax	%inf
Z/S-model	0.8%	24.0%	27.0%
EQUALIZER SA	1.0%	23.0%	0.0%

Table 1: Computational results

Table 1 gives the results for the Z/S-model and EQUALIZER on a set of 400 instances. We choose the problem size J=3, 4 and R=14. The 14 SCUs represent Nielsen districts in Germany. The values for the balancing attribute w_r of each SCU have been randomly chosen from the interval [10:50]. This problem size can still be handled by the algorithm SPP. Although the problem size is small, it nevertheless characterizes the territory alignment problem for small companies.

For each instance, an optimal alignment (= partition) is known as the solution from SPP. In a first step, we determine δ_0 for each instance with EQUALIZER and use δ_0 as a parameter for SPP and the Z/S-model. The average values of δ_0 for each of the 200 instances for the alignment of 3 and 4 territories are 4.6% and 9.0%. $\Delta_{\rm avg}$ ($\Delta_{\rm max}$) gives the average (maximum) deviation of the Z/S-model and EQUALIZER from the optimal value $Z_{\rm EQU}(\alpha,\delta_0)$ determined with SPP for the total of the 400 instances. Analysis of variance showed no significant influence of the number of sales territories on these deviations. Recall that $Z_{\rm EQU}(\alpha,\delta_0)$ denotes the travel costs in the partition, the penalty term (depending on α) is equal to zero because

$$|W_{i}(X) - S| \le \delta_{0}$$
 $j=1,...,J.$

%inf denotes the percentage of instances in which no contiguous partition within the deviation δ_0 could be found.

From Table 1 we derive two results: first, even though EQUALIZER_{SA} is a heuristic procedure, it finds on average a solution that deviates only 1.0% from the optimal solution. If an optimal solution is found by Z/S-enumeration for the Z/S-model, this solution deviates 0.8% from the optimal SPP solution. Note that the solution obtained by the heuristic proposed by Zoltners and Sinha deviates by an additional 2% from the optimal solution (Zoltners and Sinha, 1983). Second and more important than the deviation from the optimal solution is the result, that in 27% of all instances a contiguous partition with

$$|W_j(X) - S| \le \delta_0$$
 $r=1,...,R$

cannot be found for the Z/S-model. That number indicates that it is often not possible for the Z/S-model to find almost balanced sales territories.

6 Summary and Conclusions

We have presented a new model for sales territory alignment called EQUALIZER. EQUALIZER determines contiguous, almost balanced sales territories and the required lower and upper bounds in which the size of the territories may vary. Hence, in contrast to a number of other sales territory alignment models, it provides the sales manager with the "most balanced" territory alignment and does not force the sales manager to give the model an input concerning the ranges for the "most balanced" territory alignment.

Our computational study showed that our heuristic to solve EQUALIZER generates solutions that differ on average only 1% from the optimal solutions. Furthermore, we have shown that the contiguity constraints in the mixed-integer program from Zoltners and Sinha (1983) suffer from the serious restriction that they consider only the shortest-path between an SCU and the base unit of a sales territory. Our computational study has shown that due to this restriction, the model from Zoltners and Sinha (1983) was not able to find a solution in 27% of all instances. Therefore, we conclude that this shortest-path assumption is not a reasonable assumption for sales territory alignment models. Further, we reach the conclusion that EQUALIZER is due to its own determination of required upper and lower bounds and the good solution quality a suitable model for the alignment of almost balanced territories.

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