

Drexl, Andreas; Haase, Knut

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Fast approximation methods for sales force deployment

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**Fast Approximation Methods for
Sales Force Deployment**

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October 1996

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Andreas Drexl, Knut Haase
Lehrstuhl für Produktion und Logistik, Institut für Betriebswirtschaftslehre
Christian-Albrechts-Universität zu Kiel, Olshausenstraße 40, 24118 Kiel, Germany
eMail: Drexl@bwl.uni-kiel.de, Haase@bwl.uni-kiel.de
URL: <ftp://www.wiso.uni-kiel.de/>

Abstract: Sales force deployment involves the concurrent resolution of four interrelated subproblems: sizing the sales force, salesman location, sales territory alignment, and sales resource allocation. The first subproblem addresses the topic of selecting the appropriate number of salesman. The salesman location aspect of the problem involves determining the location of each salesman in one sales coverage unit. Sales territory alignment may be viewed as the problem of grouping sales coverage units into larger geographic clusters called sales territories. Sales resource allocation refers to the problem of allocating scarce salesman time to the aligned sales coverage units.

All four subproblems have to be resolved in order to maximize profit of the selling organization. In this paper a novel nonlinear mixed-integer programming model is formulated which covers all four subproblems simultaneously. For the solution of the model we present approximation methods which serve to solve large-scale problem instances arising in practise.

The methods which provide lower bounds for the optimal objective function value are benchmarked against upper bounds. On the average the solution gap, i.e. difference between upper and lower bound, is roughly 3%. Furthermore, it is shown, how the methods can be used to analyze various problem settings which are of highly practical relevance. Hence, the methods presented in this paper are effective and efficient and will be very helpful for marketing management.

Keywords: Marketing models, sales force sizing, salesman location, sales territory alignment, sales resource allocation

1 Introduction

In many selling organizations, sales force deployment is a key means by which sales management can improve profit. In general, sales force deployment is complicated and has attracted much analytical study. It involves the concurrent resolution of four interrelated subproblems: sizing the sales force, salesman¹ location, sales territory alignment, and sales resource allocation. *Sizing the sales force* advocates selecting the appropriate number of salesmen. The *salesman location* aspect of the problem involves determining the location of each salesman in one of the available sales coverage units (SCUs). *Sales territory alignment* may be viewed as the problem of grouping SCUs into larger geographic clusters called sales territories. *Sales resource allocation* refers to the problem of allocating salesman time to the assigned SCUs. Research has yielded some models and methods that can be helpful to sales managers.

The choice of the SCUs depends upon the specific application. SCUs are usually defined in terms of a meaningful sales force planning unit for which the required data can be obtained. Counties, zip codes, and company trading areas are some examples of SCUs (cp. e.g. ZOLTNER AND SINHA 1983 and CHURCHILL et al. 1993). Note, it is more meaningful to work with aggregated sales response functions on the level of SCUs rather than with individual accounts because then substantially less response functions have to be estimated and the model size does not explode (cp. e.g. SKIERA AND ALBERS 1996).

In literature, a large variety of different approaches are labeled with general terms like 'territory design', 'resource allocation' or 'distribution of effort'. Frequently, from a modelling point of view the multiple-choice knapsack problem is the matter of concern. This knapsack model covers several important practical settings and – what has been a driving source for its repeated use – can be solved very efficiently (cp. e.g. SINHA AND ZOLTNER 1979). As already mentioned 'resource allocation' addresses the question: How much of the available time should each salesman allocate to the SCUs which are assigned to him?

Early work in this area has been published by e.g. LAYTON (1968), HESS AND SAMUELS (1971), PARASURAMAN AND DAY (1977), and RYANS AND WEINBERG (1979), respectively. WAID et al. (1956) present a case study where the allocation of sales effort in the lamp division of General Electric is investigated. FLEISCHMANN AND PARASCHIS (1988) study the

¹ Note, we avoid the term 'salesperson' in order to make the 'his/her' distinction superfluous.

case of a German manufacturer of consumer goods. For the solution of the case problem they employ a classical location-allocation approach.

Sales resource allocation models consist of several basic components, i.e. sales resources, sales entities, and sales response functions, respectively. As discussed in, e.g., ZOLTNERS AND SINHA (1980) and ALBERS (1989), specific definitions for these components render numerous specific sales resource allocation models. BESWICK AND CRAVENS (1977) discuss a multistage decision model which treats the sales force decision area (allocating sales effort to customers, designing sales territories, managing sales force, etc.) as an aggregate decision process consisting of a series of interrelated stages.

The sales force sizing subproblem has been addressed by e.g. BESWICK AND CRAVENS (1977) and LODISH (1980). The sales resource allocation subproblem has been analyzed, among others, by LODISH (1971), MONTGOMERY et al. (1971), BESWICK (1977) and ZOLTNERS et al. (1979). TAPIERO AND FARLEY (1975) study temporal effects of alternative procedures for controlling sales force effort. LAFORGE AND CRAVENS (1985) discuss empirical and judgement-based models for resource allocation. Allocation of selling effort via contingency analysis is investigated by LAFORGE et al. (1986). The impact of resource allocation rules on marketing investment-decisions is studied by MANTRALA et al. (1992).

Among the four interrelated subproblems, so far the alignment subproblem has attracted the most attention. For it, several approaches appeared in the literature. These approaches can be divided between those which depend upon heuristics and those which utilize a mathematical programming model. Heuristics have been proposed, among others, by EASINGWOOD (1973), and HESCHEL (1977). Two types of mathematical programming approaches have been developed. SHANKER et al. (1975) formulated a set-partitioning model. Alternatively, the models of LODISH (1975), HESS AND SAMUELS (1971), SEGAL AND WEINBERGER (1977), ZOLTNERS (1976), and ZOLTNERS AND SINHA (1983) are SCU-assignment models. For an overview see HOWICK AND PIDD (1990).

Some of the papers published so far on the alignment subproblem aimed at aligning sales territories in a way almost balancing with respect to one or several attributes. The most popular balancing attributes are sales potential or workload of the salesmen. A detailed discussion of the shortcomings of the balancing approaches can be found in SKIERA AND ALBERS (1996) and SKIERA (1996).

Glaze and Weinberg (1979) address the three subproblems of locating the salesmen, aligning accounts and allocating calling time. More specific, they present the procedure TAPS which seeks to maximize sales for a given salesforce size while attempting to achieve equal workload between salespersons also and in addition minimizing total travel time.

Recently, SKIERA AND ALBERS (1994), (1996) and SKIERA (1996) formulated a conceptual model which addresses both the sales territory alignment and the sales resource allocation problems simultaneously. Conceptual means that the sales territory connectivity requirement is formulated verbally, but not in terms of a mathematical programming formulation. For the solution of their model they propose a simulated annealing heuristic. The objective of their model is to align SCUs and to allocate resources in such a way that sales are maximized.

The remainder of the paper is structured as follows: In §2 the problem setting under consideration is described as a nonlinear mixed-integer programming model. A fast method for solving large-scale problem instances approximately is presented in §3. The results of an in-depth experimental study are covered by §4. §5 discusses insights for marketing management. A summary and conclusions are given in §6.

2 Nonlinear Mixed-integer Programming Model

The larger the size of the sales force the more customers can be visited which in turn has a positive impact on sales. On the other hand increasing the sales force size tends to increase the operational costs per period. In addition, the number of possible calls to customers, the operational costs and the salesmen's resource (time) which might be allocated to customers

is affected by the location of the salesmen, too. To make things even more complicated, the alignment decision is very important for all these issues as well. Clearly, we have to take care of all the mutual interactions of the different factors affecting the quality of the overall sales force deployment. The aim of what follows is to provide a formal model which relates all the issues to each other.

Let us assume that the overall sales territory has already been partitioned in a set of J SCUs. The SCUs have to be grouped into pairwise disjoint sales territories (clusters) in such a way that each SCU $j \in J$ is assigned to exactly one cluster and that the SCUs of each cluster are connected. In each cluster a salesman has to be located in one of the assigned SCUs, called sales territory center. Note, connected means that we can 'walk' from a location to each assigned SCU without crossing another sales territory. $I \subseteq J$ denotes the subset of SCUs which are potential sales territory centers. To simplify notation $i \in I$ denotes both the sales territory center i and the salesman located in SCU i .

In practice, selling time consists of both the calling time and the travel time. For notational purposes let denote $z_{i,j}$ the calling time per period which is spent by salesman i to visit customers in SCU j . Further, assume $b_j \in [0, 1]$ to denote the calling time elasticity of SCU j and $g_j > 0$ a scaling parameter. Then

$$S_{i,j} = g_j (z_{i,j})^{b_j} \quad (1)$$

defines expected sales $S_{i,j}$, $i \in I, j \in J$, as a function of the time to visit customers. More precisely, equation (1) relates $z_{i,j}$ and $S_{i,j}$ for all sales territories $i \in I$ and SCUs $j \in J$. Hence, via b_j it is possible to take care of the fact that firm's competitive edge might be different in different SCUs. Note, expected sales are defined via concave rather than s-shaped functions, as is assumed to be the case with individual accounts (cp. Mantrala et al. 1992).

Let denote $t_{i,j}$ the selling time of salesman $i \in I$ in SCU $j \in J$. Note, $t_{i,j}$ includes the time to travel from SCU i to SCU j , the time to travel to customers in SCU j and the customer calling time, respectively. Then, $p_{i,j} = z_{i,j}/t_{i,j}$ relates the calling time $z_{i,j}$ to the selling $t_{i,j}$. Substituting $z_{i,j}$ in equation (1) by $p_{i,j} t_{i,j}$ yields

$$S_{i,j} = g_j (p_{i,j} t_{i,j})^{b_j} = c_{i,j} (t_{i,j})^{b_j} \quad (2)$$

Note, equation (2) has first been proposed by SKIERA AND ALBERS (1994). In equation (2) the parameter

$$c_{i,j} = g_j (p_{i,j})^{b_j} \quad (3)$$

is introduced. $c_{i,j}$ measures the sales contribution when SCU j is part of sales territory i where $c_{i,j}$ is a function of $p_{i,j}$. This is best illustrated as follows: Suppose that for salesman i the travel times to customers in SCUs j and k are different. Then in general $p_{i,j}$ and $p_{i,k}$ will be different also. Clearly, this produces different parameters $c_{i,j}$ and $c_{i,k}$ – and puts emphasis on the location decision.

Now we are ready to state the model formally. We summarize the model parameters

J	set of SCUs, indexed by j
I	set of SCUs ($I \subseteq J$) for locating salesmen, indexed by i
\mathcal{N}_j	set of SCUs which are adjacent to SCU j
f_i	per period fixed cost for locating a salesman in SCU $i \in I$
$c_{i,j}$	expected sales if SCU $j \in J$ is covered by the salesman located in $i \in I$
b_j	calling time elasticity of SCU $j \in J$
T_i	total selling time available per period for salesman $i \in I$

introduce the decision variables

$x_{i,j}$	=1, if SCU $j \in J$ is assigned to the salesman located in SCU $i \in I$ ($x_{i,j} = 0$, otherwise)
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$t_{i,j}$ selling time allocated by the salesman located in SCU $i \in I$ to SCU $j \in J$ ($t_{i,j} \geq 0$)

and then formulate an integrated model for sales force sizing, salesman location, sales territory alignment, and sales resource allocation as follows:

$$\text{maximize } \sum_{i \in I} \sum_{j \in J} c_{i,j} (t_{i,j})^{b_j} - \sum_{i \in I} f_i x_{i,i} \quad (4)$$

subject to

$$t_{i,j} \leq T_i x_{i,j} \quad (i \in I, j \in J) \quad (5)$$

$$\sum_{j \in J} t_{i,j} \leq T_i x_{i,i} \quad (i \in I) \quad (6)$$

$$\sum_{i \in I} x_{i,j} = 1 \quad (j \in J) \quad (7)$$

$$\sum_{j \in \bigcup_{v \in V} \mathcal{N}_v - V} x_{i,j} - \sum_{k \in V} x_{i,k} \geq 1 - |V| \quad (i \in I, V \subseteq J - \mathcal{N}_i - i) \quad (8)$$

$$x_{i,j} \in \{0, 1\} \quad (i \in I, j \in J) \quad (9)$$

$$t_{i,j} \geq 0 \quad (i \in I, j \in J) \quad (10)$$

Objective (4) maximizes sales while taking fixed cost of the salesman locations into account – and hence maximizes profit contribution or profit for short. The salesman i is allowed to allocate selling time to SCU j only when SCU j is assigned to him (cp. equation (5)). Equation (6) guarantees that the maximum workload per period (consisting of travel and call time) and salesman i is regarded. Equation (7) assigns each SCU to exactly one of the salesmen. Equation (8) guarantees that all the SCUs assigned to one sales territory are connected with each other. Note that these equations work similar to constraints destroying short cycles in traveling salesman model formulations (an example can be found in HAASE 1996). Clearly, it would be sufficient to take care of connected subsets $V \subseteq J - \mathcal{N}_i - i, i \in I$, of SCUs only. Equations (9) and (10) define the decision variables appropriately.

So far we did not mention the following assumption which is covered by our model: $x_{i,i} = 1$ means that SCU i is assigned to the salesman located in sales territory i . In other words, $x_{i,i} = 1$ does not only tell us where to locate salesman i , it also defines how to align SCU i . This assumption is justified with respect to practice. Moreover, we assume by definition of the binary alignment variables $x_{i,j} \in \{0, 1\}$ that accounts are exclusively assigned to individual salesman. Note, this is an assumption in marketing science and marketing management because of several appealing reasons.

The model (4) to (10) has linear constraints, but a nonlinear objective. Furthermore, we have continuous and binary decision variables. Therefore, there is no chance to solve this model with standard solvers. In HAASE AND DREXL (1996) it is shown how the objective function can be linearized in order to make the model accessible to mixed-integer programming solvers (cp. BRADLEY et al. 1977 also where it is shown how to approximate nonlinear functions by piece-wise linear ones). This makes it possible to compute upper bounds for medium-sized problem instances which in turn facilitates to evaluate the performance of the heuristics.

Clearly, all the parameters of the sales response function (1) have to be estimated. This can be done as follows if a sales territory alignment already exists since several periods, i.e. if our concern is to rearrange an already existing sales territory alignment. Then information for each SCU about the sales, the time to travel to customers as well as the time to visit the customers is already available. Usually, these informations can be extracted from sales reports. In this situation b_j and g_j can be estimated as follows. Transform equation (1) to equation (11)

$$\ln(S_{i,j}) = \ln(g_j) + b_j \ln(z_{i,j}) \quad (11)$$

and then calculate estimates of b_j and g_j via linear regression. Finally, for the computation of $c_{i,j}$ we need estimates of $p_{i,j}$. In this regard the time to travel from SCU i to SCU j and the time to visit the customers within SCU j are required. If salesman i has already covered SCU j in the past we just have to look at his sales reports. Otherwise, we assume that the time to travel within an SCU is independent from the salesman. Then the only information required for a salesman $k \neq i$ is the time to travel from k to i . This is easily available e.g. from commercial databases or simply by assuming that the travel time is proportional to the travel distance. In the case where the sales territory has to be designed from scratch, more efforts are necessary. Unfortunately, going into details is beyond the scope of this paper.

The four interrelated subproblems are addressed in our model by the decision variables $x_{i,j}$ and $t_{i,j}$. Let denote $x_{i,j}^*$ and $t_{i,j}^*$ an optimal solution for a given problem instance:

- Apparently, the optimal size of the sales force $|\mathcal{I}|$ which corresponds to the optimal number of sales territories (clusters) is given by the cardinality of the set $\mathcal{I} = \{i \mid x_{i,i}^* = 1\}$.
- For each of the sales territories in the set \mathcal{I} the SCU i with $x_{i,i}^* = 1$ is the optimal location of the salesman, i.e. the optimal sales territory center.
- For each sales territory $i \in \mathcal{I}$ the optimal set \mathcal{J}_i of aligned sales territories or SCUs is given by $\mathcal{J}_i = \{j \mid x_{i,j}^* = 1\}$.
- Finally, $t_{i,j}^* > 0$ is the optimal sales resource allocation for $i \in \mathcal{I}$ and $j \in \mathcal{J}_i$.

This interpretation of an optimal solution $x_{i,j}^*$ and $t_{i,j}^*$ illustrates that the model is ‘scarce’ in the sense that two types of decision variables cover all the four subproblems of interest. This suggests that in fact that the model is a suitable representation of the overall decision problem. Moreover, it comprises the first step towards a solution of the problem.

The aim of the following section is to present heuristic methods which balance computational tractability with optimality.

3 Approximation Methods

This section discusses a solution approach which has been developed specifically for the model. Two reasons led to this development. First, standard methods of mixed-integer programming seem to lend themselves to solving the linearized version of the model. However, even for modestly sized problems the formulation translates into very large mixed-integer programs which in turn result in prohibitive running times (for details see HAASE AND DREXL 1996). In fact it is conjectured that – except for smaller problem sizes – no exact algorithm will generally produce optimal solutions in a reasonable amount of time. Second, apart from exact methods, so far no heuristic is available for solving the model. The simulated annealing procedure of SKIERA AND ALBERS (1994), (1996) and SKIERA (1996) solves two of our subproblems, i.e. the sales territory alignment and the sales resource allocation. Unfortunately, it does not tackle the sales force sizing and the salesman location subproblems. In addition, although dealing only with two of the four subproblems in general the running times of the simulated annealing procedure do not allow to solve large-scale problem instances in a reasonable amount of time.

Our heuristic may be characterized as a construction and improvement approach. It consists of the Procedure Construct and the Procedure Improve.

- The **Procedure Construct** determines the sales force size and hence the number of salesman. In addition, it calls two other procedures: The **Procedure Locate** which computes the SCU in which each salesman has to be located and the **Procedure Align** which aligns the SCUs to the already existing sales territory centers.
- The **Procedure Improve** systematically interchanges adjacent SCUs of two different clusters. This way it improves the feasible solution which is the outcome of the Procedure Construct.

Note that the sales resource allocation subproblem can be solved as soon as all sales territories are aligned by equation (13) or equation (14). Now, first we describe the procedures designed to generate feasible solutions followed by the description of equations (13) and (14). Then the improvement procedure will be presented.

3.1 Compute Feasible Solution

Recall J to denote the set of SCUs, $I = \{i_1, \dots, i_{|I|}\}$ to be the set of SCUs which are potential locations, and \mathcal{N}_j to denote the set of SCUs which are adjacent to SCU j , respectively. In addition, let denote

\underline{S}	the minimum number of sales territory centers which might be established ($\underline{S} \geq 0$)
\overline{S}	the maximum number of sales territory centers which might be established ($\overline{S} \leq I $)
s	the 'current' number of sales territory centers ($\underline{S} \leq s \leq \overline{S}$)
I_1	the set of selected locations ($ I_1 = s, I_1 = \{i_1, \dots, i_s\}$)
I_0	the set of non-selected locations ($I_0 \cap I_1 = \emptyset, I_0 \cup I_1 = I$)
$L(I_1)$	the locations (i.e. SCUs) of the sales territory centers $i \in I_1$
$j(i)$	the SCU j where sales territory center $i \in I_1$ is located in
$i(j)$	sales territory center i to which SCU j is assigned to
J_0	the set of SCUs which are not yet aligned (initially $J_0 = J \setminus \cup_{i \in I_1} j(i)$)
J_i	the set of SCUs which are aligned to sales territory center $i \in I_1$
V_i	sales territory (cluster) of salesman $i \in I_1$
C_i	sum of sales contributions of location $i \in I_1$ ($C_i = \sum_{j \in J_i} c_{i,j}$)
LB	a lower bound on the optimal objective function value

Based on these definitions the set \mathcal{A}_i of SCUs which might be aligned to sales territory center i may be formalized according to equation (12).

$$\mathcal{A}_i = \cup_{k \in V_i} \mathcal{N}_k \cap J_0 \quad (12)$$

Note that the number of sales territory centers equals the number of salesmen (i.e. the sales force size) which in turn equals the number of locations. Therefore, some of the newly introduced parameters are superfluous, but this redundancy will be helpful for the description of the procedures.

In the sequel Z will denote the objective function value of a feasible solution at hand. Clearly, Z is a function of the decision variables $x_{i,j}$ and $t_{i,j}$. The algorithms do not operate on the set of $x_{i,j}$ variables, only the $t_{i,j}$ variables will be used directly. In what follows it is more convenient to express the $x_{i,j}$ decisions partly also in terms of the number of salesman s , and in terms of $L(I_1)$, respectively. Redundancy will simplify the formal description and ease understanding substantially. With respect to this redundancy $Z(\dots)$ will be used in different variants, but from the local context it will be evident what it stands for.

We introduce a global variable $lose[h, i]$, $h \in I, i \in I$, which is used for locating the salesmen in the set of potential locations. The variable $lose[h, i]$ is a means for selecting some elements of a probably large set I quickly. The meaning of $lose[h, i]$ will be explained below in more detail.

An overall description of the Procedure Construct is given in Table 1. Some comments shall be given as follows: The Procedure Construct just consists of an overall loop which updates the current number s of salesmen under consideration. Then it passes calls to Procedure Locate and to Procedure Align and afterwards evaluates the resource allocation by equation (13) or (14). Finally, the objective function values $Z(s, x, t)$ and $Z(s + 1, x, t)$ are compared with the best known lower bound LB which is updated whenever possible. Note, the number of salesman s for which search is performed is – without loss of generality – restricted to the interval $\underline{S} \leq \overline{S}$.

Table 1. Procedure Construct

```

Initialize  $\underline{s} = \underline{S}, \bar{s} = \bar{S}, LB = -\infty, lose[h, i] = 0, h \in I, i \in I$ 
WHILE  $\underline{s} \leq \bar{s}$  DO
     $s = \lfloor \frac{\bar{s} + \underline{s}}{2} \rfloor$ 
    call Procedure Locate ( $s$ )
    call Procedure Align ( $L(I_1)$ )
    evaluate resources allocation by equation (13) or (14)
    IF  $Z(s + 1, x, t) > Z(s, x, t)$  THEN
         $\underline{s} = s + 2$ 
    ELSE
         $\bar{s} = s - 1$ 
    ENDIF
    IF  $Z(s + 1, x, t) > LB$  THEN
         $LB = Z(s + 1, x, t)$ 
         $|I| = s + 1$ 
    ENDIF
    IF  $Z(s, x, t) > LB$  THEN
         $LB = Z(s, x, t)$ 
         $|I| = s$ 
    ENDIF
ENDWHILE

```

When a call to Procedure Locate is passed we start with $|I_1| = s, I_0 = I \setminus I_1$ which implies $I_1 \cap I_0 = \emptyset$ and $I_1 \cup I_0 = I$ and initialize $L(I_1)$. Note that the Procedure Locate uses as calling parameter only the current number s of locations. A description of the Procedure Locate is given in Table 2.

Table 2. Procedure Locate (s)

```

Initialize  $|I_1| = s, I_0 = I \setminus I_1, L(I_1), improve = \text{TRUE}$ 
WHILE improve DO
    improve = FALSE
    FOR  $k = 1$  TO  $|I_1|$  DO
         $h = \min\{i \in I_0 \mid lose[i_k, i] \leq lose[i_k, g] \forall g \in I_0\}$ 
         $I_1 = I_1 \setminus i_k \cup h$ 
        update  $I_0$  and  $L(I_1)$ 
        IF  $Z(L(I_1)) > LB$  THEN
            improve = TRUE
             $LB = Z(L(I_1))$ 
        ELSE
             $I_1 = I_1 \cup i_k \setminus h$ 
             $I_0 = I_0 \setminus i_k \cup h$ 
             $lose[i_k, h] = lose[i_k, h] + 1$ 
        ENDIF
    ENDFOR
ENDWHILE

```

In the Procedure Locate the for-loop tells us that as starting locations $L(I_1)$ the ‘first’ $|I_1|$ elements of the set I of potential locations are chosen. The procedure stops when within the for-loop no further improvement of the set of locations can be found. As an outcome we know the locations $L(I_1)$ of the current number s of salesman.

Capitalizing on the definitions given above a compact description of the Procedure Align is given in Table 3. Within the while-loop one of the not yet aligned SCUs is chosen and aligned to one of the already existing sales territory centers. The criterion for choosing SCU h and sales territory center i is motivated below.

Table 3. Procedure Align ($L(I_1)$)

```

Initialize  $J_0, V_i, C_i$  and  $\mathcal{A}_i$ 
WHILE  $J_0 \neq \emptyset$  DO
  compute  $(h, i)$  such that  $c_{h,i}/C_i \geq c_{k,j}/C_j \forall i \in I_1, \forall j \in I_1, \forall h \in \mathcal{A}_i, \forall k \in \mathcal{A}_j$ 
   $J_0 = J_0 \setminus h$ 
   $V_i = V_i \cup h$ 
   $C_i = C_i + c_{h,i}$ 
  update  $\mathcal{A}_i$ 
ENDWHILE

```

Apparently, as a final step of the overall Procedure Construct the sales resource allocation subproblem has to be solved. This is done by evaluating equations (13) or (14), where $a = 1/(1 - b)$ (cp. EINBU 1981) in the case of $b_j = b \forall j \in J$.

$$t_{i,j} = \frac{(c_{i,j} x_{i,j})^a}{\sum_{h \in J, (c_{i,h})^a} \cdot T_i} \quad (i \in I, j \in J) \quad (13)$$

In the general case where $b_h \neq b_j, h \in J, j \in J, h \neq j$, allocation is done by equation (14). $a_j = 1/(1 - b_j)$ and $\alpha_i = 1/(1 - \beta_i)$ are used for short, where β_i is the the 'average' elasticity which has to be calculated by bisection search. Note, it is beyond the scope of this paper to show how equation (14) can be derived and the reader is referred to SKIERA AND ALBERS (1994).

$$t_{i,j} = \left(\frac{(c_{i,j} b_j)^{\alpha_i}}{\sum_{h \in J, (c_{i,h} b_h)^{\alpha_h}} \cdot T_i} \right)^{(a_j/\alpha_i)} \quad (i \in I, j \in J) \quad (14)$$

3.2 Improve Feasible Solution

In general feasible solutions at hand can easily be improved by the following simple Procedure Improve. For a compact description of the procedure we define two boolean parameters:

$$\begin{aligned}
 add(V_i, j) &= \begin{cases} \text{TRUE} & \text{if } V_i \cup j \text{ is connected} \\ \text{FALSE} & \text{otherwise} \end{cases} \\
 drop(V_i, j) &= \begin{cases} \text{TRUE} & \text{if } V_i \setminus j \text{ is connected and } j \neq j(i) \\ \text{FALSE} & \text{otherwise} \end{cases}
 \end{aligned}$$

The function $add(V_i, j)$ defines only those alignments to be feasible where we add SCU j to the sales territory V_i such that the newly derived sales territory consists of connected SCUs only. Similarly, the function $drop(V_i, j)$ admits only alignments to be feasible where we $drop$ SCU $j \neq j(i)$ from sales territory V_i without running into disconnectedness. In other words: Both functions define those moves of an SCU j to/from a sales territory V_i to be feasible where the outcome does not violate the connectivity requirement. As a consequence, only those SCUs are suspected move candidates which are located on the border of each of the sales territories. In this respect the functions $add(V_i, j)$ and $drop(V_i, j)$ are complementary. As a consequence the Procedure Improve might be characterized as an interchange method, too.

Note that ‘add’ and ‘drop’ are used in discrete location theory also (cp. e.g. MIRCHANDANI AND FRANCIS 1990 and FRANCIS et al. 1992). Clearly, the resource allocation $t_{i,j}$ has to be updated with respect to each move by evaluating equation (13) or (14).

A formal description of the Procedure Improve is given in Table 4. For the sake of compactness, the calling parameter $V = (V_1, \dots, V_s)$ denotes the vector of sales territory alignments currently under investigation and $Z(V)$ the corresponding objective function value, respectively. $Z(V | V_{i(j)} \setminus j, V_i \cup j)$ tells us that the objective function value has to be computed with respect to the current alignment under investigation where SCU j is subtracted from sales territory $V_{i(j)}$ while sales territory V_i is augmented by SCU j . Clearly, the computation of the objective function requires an update of the resource allocation $t_{i,j}$ via equation (13) or (14) also.

Table 4. Procedure Improve (V)

```

Initialize improve = TRUE, LB =  $Z(V)$ 
WHILE improve DO
    improve = FALSE
    FOR  $j = 1$  TO  $|I|$  DO
        FOR  $i = 1$  TO  $|I_1|$  DO
            IF  $add(V_i, j) \wedge drop(V_i, j) \wedge LB < Z(V | V_{i(j)} \setminus j, V_i \cup j)$  THEN
                 $LB = Z(V | V_{i(j)} \setminus j, V_i \cup j)$ 
                 $V_{i(j)} = V_{i(j)} \setminus j$ 
                 $V_i = V_i \cup j$ 
                improve = TRUE
            ENDIF
        ENDFOR
    ENDFOR
ENDWHILE

```

Finally, we shall explain in more detail how the different procedures work and further motivate why they are constructed the way they are:

- First, without any formal treatment we start off with the observation that – for ‘reasonable’ parameters $c_{i,j}$ and f_i – the objective function is concave with respect to the sales force size, i.e. the number of salesman. Therefore, ‘gradient search’ within the interval $\underline{S} \leq s \leq \bar{S}$ is implemented in the Procedure Construct.
- Second, the global variable $lose[h, i]$ is used in the Procedure Locate like in tournament selection. The tournament is finished when the ‘best’ player h (i.e. the one which so far has lost the least number of games) does not win against any other player $k \in I_1$. As already mentioned above this is an effective means for selecting some elements of a probably large set quickly.
- Third, the Procedure Align is greedy in the sense that the steepest ascent of the objective function is used as criterion for the choice of the next SCU to be aligned. More precisely, the choice depends on the ratios $c_{h,i}/C_i$, i.e. the rational is to take care of the relative weights of the expected sales contributions.
- Fourth, the Procedure Improve belongs to the variety of local search methods (for a survey of advanced local search methods cp. e.g. PESCH 1994). In order to keep our explanations as simple as possible we distract our attention from the resource allocation $t_{i,j}$. Starting with an incumbent sales territory alignment $x = (x_{i,j})$ we search all its neighbors $\hat{x} \in \mathcal{H}(x)$, where $\mathcal{H}(x)$ equals the set of feasible solutions which are properly defined by the functions $add(V_i, j)$ and $drop(V_i, j)$. $\mathcal{H}(x)$ is called neighborhood of x . Searching over all neighbors $\hat{x} \in \mathcal{H}(x)$ in a steepest ascent manner may be characterized

as a ‘best fit strategy’. By contrast, a ‘first fit strategy’ might by less time consuming while presumably producing inferior results.

- Fifth, the Procedures Construct and Improve comprise deterministic methods. In the next section we will show that these simple deterministic methods produce already very promising results. Therefore, there is no necessity to make the methods more sophisticated (and more complicated) by incorporating either self-adaptive randomization concepts (cp. e.g. KOLISCH AND DREXL 1996) or procedure parameter control techniques adopted from sequential analysis (cp. e.g. DREXL AND HAASE 1996). Furthermore, if desired it is straightforward to incorporate simulated annealing randomization schemes (for a comprehensive introduction into the theory and techniques of simulated annealing cp. e.g. JOHNSON et al. 1989, 1991).
- Finally, when solving difficult combinatorial optimization problems one is likely going to be trapped in local optima when searching greedily in a steepest ascent manner only. Therefore, numerous researchers have devised (less greedy) steepest ascent/mildest descent procedures which provide the ability to escape from local optima while avoiding cycling through setting some moves ‘tabu’ (for a comprehensive introduction into the theory and techniques of tabu search see e.g. GLOVER 1989, 1990). While, clearly, there might be some potential for improvement there seems to be no necessity in this respect to incorporate tabu search techniques.

4 Experimental Evaluation

The outline of this section is as follows: First, we elaborate on the instances which are used in our computational study. Second, we describe how to compute benchmark solutions in order to judge the performance of the methods presented in the preceding section. Third, numerical results will be presented.

Even in current literature, the systematic generation of test instances does not receive much attention. Generally, two possible approaches can be found adopted in literature when having to come up with test instances. First, practical cases. Their strength is their high practical relevance while the obvious drawback is the absence of any systematic structure allowing to infer any general properties. Thus, even if an algorithm performs good on some practice cases, it is not guaranteed that it will continue to do so on other instances as well. Second, artificial instances. Since they are generated randomly according to predefined specifications, their plus lies in the fact that fitting them to certain requirements such as given probability distributions poses no problems. (A detailed such procedure for generating project scheduling instances has been recently proposed by KOLISCH et al. 1995). However, they may reflect situations with little or no resemblance to any problem setting of practical interest. Hence, an algorithm performing well on several such artificial instances may or may not perform satisfactorily in practice. Therefore, we decided to devise a combination of both approaches, thereby attempting to keep the strengths of both approaches while avoiding their drawbacks.

4.1 Practical Case

First, we used the data of a case study which have been compiled by SKIERA (1996) in order to evaluate his simulated annealing procedure. This instances are roughly characterized as follows: The company is located in the northern part of Germany. The sales region covers the whole area of Germany. The sales territory is partitioned into 95 SCUs (two-digit postal areas). The number of salesman employed is ten where the location of each salesman, i.e. the sales territory center is assumed to be fixed. Then the sales force sizing and the salesman location subproblems are (presumed to be) of no relevance. For the remaining two subproblems the solutions currently used by the company and the solution computed by SKIERA (1996) are available as a point of reference for our procedure. While, clearly, all the data

available are of great practical interest we refrain, however, from the tedious task of citing all the respective details.²

4.2 Generation of Instances

Second, we generated instances at random. We assumed that only two instance-related factors do have a major impact on the performance of the algorithms, viz. the cardinality of the set I of potential sales territory centers and the cardinality of the set J of SCUs, respectively. Both factors relate to the ‘size’ of a problem, hence (I, J) denotes the size of an instance.

When generating instances at random a critical part is the specification of a connected sales territory. In order to do so we employ the Procedure Generate which is able to generate a wide range of potential sales territories while preserving connectivity. The basic idea is to define a set $K = \{1, \dots, 2 \cdot Q\} \times \{1, \dots, 2 \cdot Q\}$ with $K \geq J$ of unit squares located on a grid. For every unit square $(\alpha, \beta) \in K$ the set of adjacent unit squares $\mathcal{N}_{(\alpha, \beta)}$ or neighbours is defined as follows:

$$\mathcal{N}_{(\alpha, \beta)} = \left\{ \{(\tilde{\alpha}, \tilde{\beta}) \in K \mid |\tilde{\alpha} - \alpha| \leq 1, |\tilde{\beta} - \beta| \leq 1\} \setminus (\alpha, \beta) \right\}$$

The Procedure Generate is formally described in Table 5. As calling parameters the set of sales territory centers I and the set of SCUs J are used. Note that – starting with the ‘central’ unit square $\mathcal{M} = \{(Q, Q)\}$ – the set \mathcal{M} is incremented until it equals the set of SCUs J which have to be generated while preserving connectivity of the sales territory. Similar to the Procedure Align \mathcal{A} denotes the set of those unit squares of the grid which are candidates to be aligned to the already generated sales territory. In a last step the set of sales territories I is chosen at random.

Table 5. Procedure Generate (I, J)

```

Initialize  $\mathcal{M} = \{(Q, Q)\}$  and  $\mathcal{A} = \mathcal{N}_{(Q, Q)}$ 
WHILE  $|\mathcal{M}| \leq |J|$  DO
    chose  $(\alpha, \beta) \in \mathcal{A}$  at random
     $\mathcal{M} = \mathcal{M} \cup (\alpha, \beta)$ 
     $\mathcal{A} = \mathcal{A} \cup \mathcal{N}_{(\alpha, \beta)} \cap \mathcal{M}$ 
ENDWHILE
 $\mathcal{N}_{(\alpha, \beta)} = \mathcal{N}_{(\alpha, \beta)} \cap \mathcal{A} \forall (\alpha, \beta) \in \mathcal{M}$ 
chose  $I \subseteq J$  sales territories at random

```

It is easy to verify that the Procedure Generate is capable to produce a large range of quite different shaped sales territories. Nevertheless, the question is whether this construction process which basically relies on unit squares and hence on SCUs of equal size does produce instances which are meaningful for the methods to be evaluated? The answer is ‘yes’ because the grouping, i.e. building of larger units is just what the Procedure Align does.

Summarizing the instances treated in the computational study are characterized as follows:

- The set of SCUs J is given by $\{50, 100, 250, 500\}$.
- The set of potential sales territory centers I is given by $\{10, 25, 50\}$.
- The scaling parameter g_j is chosen at random out of the interval $[10, 210]$.
- The expected sales $c_{i,j}$ equal $g_j (d_{i,j})^b$ where the distances $d_{i,j}$ are computed as follows:

$$d_{i,j} = \max \left\{ 0, 0.4 - \frac{|\alpha_i - \alpha_j| + |\beta_i - \beta_j|}{100} \right\}$$

² All the instances used in this study are available on our ftp-site under the path /pub/operations-research/salesforce via anonymous ftp.

b was set to 0.3 with respect to empirical findings of ALBERS AND KRAFFT (1992). As a consequence because of travel times being proportional to travel distances $d_{i,j}$ expected sales $c_{i,j}$ decrease the longer the distance between i and j is and vice versa.

- The fixed cost f_i of sales territory centers are drawn at random out of the interval $[750, 1,250]$.
- The maximum workload T_i per period and salesman is set to 1,300 for all $i \in I$. This is an estimate of the annual average time salesman in Germany have to work (cp. SKIERA AND ALBERS 1994).
- The lower bound \underline{S} for the number of sales territory centers is set to 0 while the upper bound \bar{S} equals $|I|$.

Note, to calculate the scaling parameter g_j at random as described above might not be the best choice whenever the data are spatially autocorrelated. While, certainly, it is not that difficult to generalize the generator such that autocorrelation is covered also we do not follow these lines here because of the following reason: The practical case described in Subsection 4.1 has spatially autocorrelated data. Solving the practical case with our procedures is by no means more difficult than solving the artificial instances (details are provided below). Hence, we refrain from introducing some more parameters in order to get a more ‘realistic’ instance generator.

Clearly, only ‘reasonable’ combinations of J and I are taken into account (details are provided below). In addition, due to the computational effort required to attempt all the sizes only ten instances were considered in the experiment for each instance class (J, I) .

4.3 Computation of Benchmarks

Unfortunately, it is not possible to solve the nonlinear mixed-integer programming (NLP-) model (4) to (10) by the use of a ‘standard’ solver. Hence, even for small-sized problem instances there is no ‘direct’ way to get benchmarks. Consequently, in a companion paper (cp. HAASE AND DREXL 1996) the model (4) to (10) has been reformulated as a mixed-integer linear programming (MIP-) model. In order to do so one has to replace the nonlinear objective by a piecewise-linear one such that an optimal solution of the MIP-model provides a lower bound for the NLP-model. Clearly, solving the LP-relaxation of the MIP-model yields an upper bound of the optimal objective of the NLP-model and, hence, benchmarks.

The LP-relaxation of the MIP-model can be solved directly by the use of one of the commercially available LP-solvers. This way it is possible to compute upper bounds for problems having up to $J = 500$ SCUs and $I = 50$ sales territories in a reasonable amount of time. A more efficient approach uses the MIP-model within a set partitioning/column generation framework. Going into details is beyond the scope of this paper and the interested reader is referred to HAASE AND DREXL (1996).

4.4 Computational Results

The algorithms have been coded in C and implemented on a 133 Mhz Pentium machine under the operating system Linux. The parameter K of the Procedure Generate is defined to be $K = FAC \times |J|$ where $FAC = 1.5$ has been used. Note, $FAC > 1$ serves to generate sales territories where not all units form part of the overall sales region, i.e. lakes and other ‘non-selling’ regions can be included also.

Table 6 provides a comparison of lower and upper bounds. Columns 1 and 2 characterize the instance class, i.e. problem size under consideration in each row in terms of $|J|$ and $|I|$, respectively. Columns 3 and 4 report the results which have been obtained using the LP-solver of CPLEX (cp. CPLEX 1995). More specific, column 3 provides the average upper bound UB which has been obtained by solving the LP-relaxation of the linearized version. Column 4 shows the average CPU-time in sec required to compute UB . Recall that averages over ten instances for each row, i.e. instance class (J, I) , are provided. Columns 5 to 7 with the header

Table 6. Comparison of Lower and Upper Bounds

		CPLEX		CONIMP		
$ J $	$ I $	UB	CPU	LB	CPU	GAP
50	10	12,043.39	3.20	11,508.89	$\ll 1$	4.46
	25	12,793.16	14.50	12,217.26	0.90	4.50
	50	13,271.83	49.20	12,736.17	2.60	4.04
100	10	26,119.68	10.30	25,610.99	0.50	1.95
	25	28,332.57	47.10	27,619.47	4.10	2.54
	50	29,583.04	172.60	28,464.76	13.50	3.73
250	50	71,185.04	720.68	69,774.65	142.50	1.99
500	50	133,702.41	3,424.34	130,962.26	626.70	2.05

CONIMP present the results of the Procedures Construct and Improve. More specific, LB cites the average best feasible solution, i.e. lower bound computed. CPU denotes the average CPU-time in sec required by the algorithms to compute LB . $\ll 1$ denotes that the average is only an ϵ above zero sec. Finally, $GAP = \frac{(UB-LB)}{UP} \cdot 100$ measures the average percentage deviation between upper and lower bound, i.e. the solution gap. Note, GAP covers both the tightness of the LP-relaxation and the deviations of the lower bounds obtained from the (unknown) optimal objective function values. On the average, the solution gap roughly equals 3%. Hence, the feasible solution computed indeed must be very close to the optimal one.

Table 7. Comparison of the Procedures Construct and Improve

		CON		IMP		
$ J $	$ I $	LB	CPU	LB	CPU	API
50	10	11,454.97	$\ll 1$	11,508.89	$\ll 1$	0.47
	25	12,190.75	0.90	12,217.26	$\ll 1$	0.22
	50	12,724.97	2.60	12,736.17	$\ll 1$	0.09
100	10	25,173.43	$\ll 1$	25,610.99	0.50	1.73
	25	27,528.56	4.00	27,619.47	0.10	0.33
	50	28,416.35	13.40	28,464.76	0.10	0.17
250	50	69,454.64	132.50	69,774.65	10.00	0.46
500	50	129,746.85	544.70	130,962.26	82.00	0.94

Now the question shall be answered which of the Procedures Construct or Improve contributes to which extent to the fact that the lower bounds are very close to the optimum. Table 7 gives an answer. The header CON groups the information provided with respect to the Procedure Construct while the header IMP does so for the Procedure Improve. In the former case LB denotes the lower bound obtained while in the latter one it shows the additional improvement. In both cases CPU denotes the required CPU-time in sec. API provides the average percentage improvement.

It has already been mentioned that our model is more general than the one of SKIERA AND ALBERS (1996) because it covers the sales force sizing and the salesman location subproblems

also. Consequently, our methods cover the more general case, too. Surprisingly, although being more general, our methods are more efficient than the simulated annealing method of SKIERA AND ALBERS. While our algorithms solve the practical case close to optimality in a CPU-time \ll 1 sec, the simulated annealing method requires up to ten minutes on a 80486 DX-33 machine to do so. Moreover, the solution computed by our algorithms is slightly better than the one found by the simulated annealing method. Note, suboptimality means that profit increase is about 5% compared with the alignment used by the company so far. Clearly, the run-times of the simulated annealing algorithm will become prohibitive when applied to large-scale problem instances.

Regarding the results reported in Tables 6 and 7 some important facts should be emphasized:

- Roughly speaking, the solution gap decreases from 4% to 2% while the size of the instance increases, because of two reasons. First, relaxing the connectivity requirements makes the LP-bounds for small problem instances weak compared to large ones. Second, the quality of the piece-wise linear approximation increases with increasing problem size and hence makes the LP-bounds more tight.
- The larger the cardinality of the set I the more time has to be spent in evaluating the size and the location of the sales force. Clearly, this takes the more CPU-time the larger I is in relation to J . From another point of view, if there is no degree of freedom with respect to the size of the sales force and the location of the salesmen, i.e. $\underline{S} = \bar{S}$, then the alignment and the allocation subproblems are solved very effective and very efficient by our algorithms also. This decidedly underlines the superiority of our approach compared to the one of SKIERA AND ALBERS (1996).
- In general, the quality of the solutions computed by the procedure Construct is already that good that only minor improvements can be obtained subsequently. In other words, exploiting the degree of freedom on the level of the sizing and the locating decisions appropriately already gives an overall sales force deployment which is hardly to improve by realigning some of the SCUs.

The scope of the experiment conducted so far was to show how good our algorithms work. Seriously this can only be done with respect to the optimal objective function or at least an upper bound. Therefore, the experiment was limited to include only instances of the size for which the LP-relaxation of the MIP-model can be solved in reasonable time. Clearly, there is no obstacle for using the algorithms on larger instances which might become relevant e.g. in a global marketing context. The CPU-times required by our procedure show that for really huge instances comprising thousands of SCUs it is possible to compute near-optimal solutions within some hours of computation. Summarizing there is no obstacle for using the algorithms even on very large instances.

5 Insights for Marketing Management

In what follows we will discuss managerial implications of our findings. More precisely, we will state some insights and subsequently assess their validity on basis of experiments.

Insight 1: The results are robust with respect to wrong estimates of parameters.

In order to evaluate insight 1 we took one of the instances with $|J| = 250$ SCUs and $|I| = 50$ potential locations. Now assume that $b_j = 0.3$ and the $c_{i,j}$ which are generated along the lines described in Subsection 4.2 $\forall j \in J$ and $i \in I$ are the (unknown, but) ‘true’ values of the parameters of the sales response function. The parameters \hat{b} and $\hat{c}_{i,j}$ which are used in the experiment are then generated via data perturbation as follows: Calculate $\hat{b} = b + \Delta b$ and choose $\hat{c}_{i,j} \in [c_{i,j}(1 - \Delta c), c_{i,j}(1 + c_{i,j})]$ at random where Δc and Δb are perturbation control parameters.

Table 8 presents the results of this study. Across rows and columns we provide the percentage decrease $DEC = \frac{(OPT-ACT)}{OPT} \cdot 100$ of profit where OPT denotes the ‘optimal’ objective value which has been calculated based upon the ‘true’ parameter values while ACT is the one which has been computed with respect to the perturbed parameters. The results show that even in the case when the parameters are estimated very ‘bad’ (i.e. all of them are under- or overestimated drastically) the percentage decrease of profit does hardly exceed 3%.

Table 8. Robustness of the Model

\hat{b}	0.20	0.25	0.30	0.35	0.40
Δb	-0.10	-0.05	0.00	0.05	0.10
$\Delta c = 0.00$	1.89	1.67	0.00	0.17	0.25
$\Delta c = 0.05$	2.25	1.92	0.36	0.87	1.25
$\Delta c = 0.10$	2.87	3.06	1.58	1.97	2.07

Insight 2: Profit is not that sensitive with respect to sales force size.

In order to evaluate insight 2 once more we took the instance with $|J| = 250$ SCUs and $|I| = 50$ potential locations. Then, the size of the sales force was set to the levels 29, 30, 31, 32, 33 by fixing $\underline{S} = s = \bar{S}$ accordingly. Table 9 provides the results of this experiment. $OFV(s)$ denotes the objective function value (normalized to the interval $[0, 1]$) which has been computed by our methods with respect to the size s . The results which are typical for various other experiments not documented here support insight 2 which means that the objective function is fairly flat near the optimum number of salesmen. Hence, the ‘flat maximum principle’ (cp. Chintagunta 1993) is valid also in this context.

Table 9. Profit as Function of Sales Force Size

s	$OFV(s)$
29	0.99577
30	0.99919
31	1.00000
32	0.99996
33	0.99941

Insight 3: Profit is sensitive with respect to the location of the salesmen.

Once more we relate to the instance already used twice. Table 10 provides part of the protocol of a run. More precisely, the outcome of some typical iterations of the Procedure Locate where potential locations are evaluated systematically is given in terms of normalized objective function values $OFV(s)$. Similar to Table 9 the size of the sales force is fixed in each row. Clearly, the process converges to the best found objective function value (hence, $OFV(s) = 1$ in column seven), but the values go up and down depending on the specific old and new locations under investigation. Hence, the ‘flat maximum principle’ is not valid with respect to the location of the salesmen.

Table 10. Profit as Function of Location of Salesmen

s	$OFV(s)$ - Selected Iterations					
29	0.934	0.960	0.947	0.973	0.986	1.000
30	0.938	0.927	0.957	0.934	0.977	1.000
31	0.928	0.921	0.952	0.950	0.949	1.000
32	0.933	0.924	0.965	0.967	0.962	1.000
33	0.917	0.945	0.933	0.948	0.978	1.000

The insights evaluated in Tables 8, 9 and 10 can be summarized as follows:

- For reasonable problem parameters the size of the sales force does not affect firm's profit that much.
- The location of the salesmen in general will affect firm's profit drastically. Consequently, existing alternatives must be evaluated.
- Fortunately, the model is very robust with respect to the estimation of the parameters of the sales response function. Even in the case when there is a systematic estimation bias (over- or underestimation of *all* the parameters) the decision is not that bad in terms of firm's profit. Usually, there is no systematic bias, hence, the sales force deployment evaluated by the algorithms will be superb.

6 Summary and Conclusions

In this paper it is shown how four interrelated sales force deployment subproblems can be modelled and solved simultaneously. These subproblems are: sizing the sales force, salesman location, sales territory alignment, and sales resource allocation. More specific an integrated nonlinear mixed-integer programming model is formulated. For the solution of the model we present a newly developed effective and efficient approximation method.

The methods are evaluated on two sets of instances. The first one stems from a case study while the second one is based on the systematic generation of a representative set of problem instances covering all problem parameters at hand. The results show that the method allows to solve large-scale instances close to optimality very fast.

The methods which provide lower bounds for the optimal objective function value are benchmarked against upper bounds. On the average the solution gap, i.e. difference between upper and lower bound, is roughly 3%. Furthermore, it is shown, how the methods can be used to analyze various problem settings which are of highly practical relevance. Hence, the methods presented in this paper are effective and efficient and will be very helpful for marketing management.

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