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Multi-level lot sizing: A literature survey

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**Multi-Level Lot Sizing:
A Literature Survey**

A. Kimms and A. Drexl



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Multi-Level Lot Sizing: A Literature Survey

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Abstract

Lot sizing certainly belongs to the most established production planning problems. First scientific reports of this subject date from the beginning of the 20th century and at least one chapter about lot sizing can be found in almost every good textbook about production research issues. But, as we will show, some topics of practical importance such as multi-level lot sizing where capacity is scarce and demand is time variant have first been successfully attacked in the recent past. Furthermore, lot size and sequence decisions are usually not integrated as it ought to be for the short-term planning. This paper reviews the history of multi-level lot sizing from the very early work to the state-of-the-art.

1 Scope of the Review

The huge amount of literature can be classified into several categories. We discriminate three important criteria.

First, publications may assume stochastic or deterministic data. The focus here is on deterministic cases. We refer to [14, 39, 76, 81, 108, 194] for an introduction into the problems associated with stochastic data.

Second, authors assume a single- or a multi-level production. Approaches of the former type do not fit for multi-level structures by definition. Hence, we do not give a comprehensive overview of single-level lot sizing here. [7, 16, 17, 64, 66, 73, 119, 177] contain reviews. However, modifications of single-level heuristics may be used to attack multi-level problems, e.g. on a level-by-level basis. Such methods are called improved heuristics. Subsequent sections deal with these.

Last, demand may be assumed to be dynamic or stationary. In a stationary situation, the demand occurs continuously and with a constant rate. Usually, an infinite planning horizon comes along with stationary demand. This is appropriate for mass production for instance. In most cases, the common basis is an economic order quantity (EOQ) policy [1, 93] with no capacity considerations, or the economic lot sizing problem (ELSP) [66] where capacity limits are taken into account. There is a vast amount of literature about multi-level lot sizing for stationary demand. Most of it is based on EOQ-like assumptions. But, these approaches are of minor help in the dynamic case. Hence, it is sufficient to cite some work without going into the details: [10, 11, 18, 26, 37, 38, 43, 46, 55, 56, 68, 69, 71, 70, 75, 78, 82, 83, 87, 90, 91, 100, 102, 101, 103, 104, 105, 109, 131, 136, 133, 137, 138, 139, 140, 153, 154, 158, 159, 160, 161, 170, 171, 172, 175, 173, 174, 176, 192, 193].

In the subsequent sections we should have a closer look at the multi-level lot sizing literature where data are deterministic and demand is dynamic. Though our interest lies in capacitated situations primarily, uncapacitated cases are closely related and shall therefore be reviewed, too.

2 Contributions of General Interest

This is of course not the first review of (multi-level) lot sizing. So, let us start with a look at the history of surveys. A very brief survey was given by Jacobs and Khumawala [106] with seven references only. Research issues for multi-level lot sizing are discussed by Collier in [53]. DeBoth et al. [61] review dynamic lot sizing. The review of Bahl et al. [16] probably is the one that is most often referenced. Fleischmann [73] gives insight how operations research methods were applied to different production planning problems. So he does for lot sizing. Gupta and Keung [85] also provide a review. Goyal and Gunasekaran [79] provide a collection of literature about multi-level production/inventory systems. Recently, Kuik et al. [119] report on different assumptions for lot sizing, among them multi-level structures. Simpson and Erenguc [162] compiled literature related to multi-level lot sizing, too, and conclude: "Clearly, work in this area has only begun." In the textbooks written by Domschke et al. [64] and Tempelmeier [177] we also find notable subsections explaining multi-level lot sizing research.

The most basic problem in material requirements planning (MRP) introduced by multi-level structures is the parts requirement (or explosion) problem which is that of computing the total number of parts needed to fulfill demand. In this context, the low level code (or explosion level) of an item is defined as the number of arcs on the longest path from that particular item to an end item. Especially in the presence of positive component inventory levels, the determination of the so-called net requirement has attracted early researchers. Vazsonyi [183] gives a mathematical statement. Elmaghraby [65] is among the first who present an algorithm. Thompson [181] follows up and coins the notion of a technological order which is a partial order on the basis of the low level codes of the items. Both of them use matrix operations to compute net requirements. Nowadays, this problem does no longer exist since efficient methods using the low level codes of the items are available.

The bill of material (BOM) is a *formal statement of the successor-predecessor relationship of items* and thus defines the gozinto-structure. Special cases of acyclic structures are those where each item has at most one successor (so-called assembly structures), and those where each item has at most one predecessor (so-called divergent, distribution, or arborescence structures, respectively). Structures which belong to the assembly as well as to the distribution type are called serial (or linear). A gozinto-structure that is neither assembly nor divergent is general. Collier [51, 52] is the only who tries to define a measure of complexity for gozinto-structures by making use of the information in the BOM. He introduces the degree of commonality index which reflects the average of the number of successors per component part.

3 Dynamic Demand, Unlimited Capacity

Clark and Scarf [44] provide an early analysis of optimal lot sizing policies in a distribution network. They also invent the notion echelon stock for what is in inventory plus what is in transit but not sold. Veinott [184] formulates lot sizing as minimizing a concave function over the solution set of a Leontief substitution system. His work is a generalization of Zangwill's paper [198]. For serial gozinto-structures Zangwill [199] presents a dynamic programming approach based on a reinterpretation of the constraints as flow constraints in a single-source network. Love [126] uses these results and considers serial structures under certain assumptions about the costs related to different production levels. He presents a dynamic programming algorithm, too, to find so-called nested extreme optimal schedules where the attribute nested means that production for an item in a certain period implies production for all successor items in the same period. Crowston and Wagner [54] extend Love's model and develop dynamic programming and B&B-methods for assembly systems. On the basis of Veinott's and Zangwill's results, Kalymon [112] decomposes problems with divergent gozinto-structures into a series of single-item problems. He uses an enumeration procedure to determine the sequence in which these single-item problems are then solved.

Steinberg and Napier [166] determine optimal solutions for general gozinto-structures with standard MIP-solvers on the basis of a network model. The production quantities may be constrained by an item-specific upper bound. An alternative formulation with a less cumbersome notation is given by McClain et al. [132]. In response, Steinberg and Napier [167] make clear that the alternative formulation is more compact, but that there is no evidence which proves greater computational efficiency.

Rao [147] uses Bender's decomposition to compute optimal solutions for problems with general gozinto-structures.

Krajewski et al. [116] perform a computational study and by using its outcome they determine the parameters of three regression models with least squares estimators. For instance, they specify the average size of unreleased orders for each item as a function of the number of product levels above the item, the total number of components for the item, the number of immediate components, and the general lot size for the system (which is a binary variable indicating small or large lots).

Bitran et al. [30] suggest a planning approach where the gozinto-structure is aggregated into just two levels. Then, the two-level problem is solved and the solution is eventually disaggregated. However, they do not consider setups which makes both, the two-level planning problem as well as the disaggregation, become linear with continuous decision variables only.

Bahl and Ritzman [15] formulate a non-linear mixed-integer program for simultaneous master production scheduling, lot sizing, and capacity requirements planning. They also develop a heuristic. The traditional level-by-level approach

is retained. Furthermore, they assume a lot-for-lot policy for all component parts. In fact, they reduce the multi-level problem to a single-level one under these assumptions.

By means of simulation studies Biggs [22] examines the interaction effect of different lot sizing and sequencing rules when used conjunctively. In a first step, he solves a multi-level lot sizing problem using a single-level lot sizing rule on a level-by-level basis. Note that the same rule is applied to all levels. For each period he computes a sequence of different items with a simple priority rule, afterwards. The test-bed consists of six lot sizing rules combined with five sequencing rules. The performance is analyzed using several criteria, among them the total number of setups for instance. As an outcome of this study, it turns out that interaction effects between lot size and sequence decisions do exist, but, "... a total explanation will have to wait for future research...".

Billington et al. [28] discuss how to reduce the size of problem instances by means of gozinto-structure compression. A similar idea is published by Axsäter and Nuttle [12, 13] where assembly structures are taken into account only. Zangwill [200] analyzes problems with serial gozinto-structures to find out at which levels inventory cannot occur. Though his motivation is to give advice where to invest for setup cost reduction in order to enable just-in-time production and to achieve zero inventories irrespective of demand, the information obtained can also be used to compress the gozinto-structure.

When testing six single-level heuristics on a level-by-level basis using the same lot sizing rule on each level, Benton and Srivastava [19] find out that neither the depth nor the breadth of a gozinto-structure has a significant effect on the performance of lot sizing. Afentakis [3] conducts a computational study which contradicts this result. The reason might be that Benton and Srivastava use gozinto-structures with up to five levels and six items while Afentakis uses structures with up to 45 levels and 200 items. Sum et al. [169] provide a study which confirms Afentakis. They employ 11 lot sizing rules on a level-by-level basis tested on 1980 instances. By the way, a heuristic that is introduced by Bookbinder and Koch [35] performs best. An analysis of variance reveals the impact of the number of items, the number of levels, and the average number of immediate successors per item [51]. It is interesting to note that the ranking of the rules remains stable.

Simplistic applications of single-level lot sizing rules are also described by Berry [20] and Biggs et al. [21, 24, 25]. Another evaluation of the level-by-level technique is done by Veral and LaForge [185]. Choi et al. [42], Collier [49, 50], Ho [99], LaForge [120, 121], and Lambrecht et al. [123] provide computational studies, too.

A study in which single-level lot sizing rules are applied to a multi-level structure in a level-by-level manner is done by Yelle [195]. He uses four different simple lot sizing rules and a two-level test-bed. His basic idea is to apply different rules to different levels. This study is extended by Jacobs and Khumawala [107] who add three more single-level methods plus a multi-level

algorithm proposed by McLaren [135] which in turn bases on the single-level Wagner-Whitin procedure [6, 72, 189, 190]. Tests with a three-level test-bed are done by Choi et al. [41].

Blackburn and Millen [31] apply single-level heuristics level-by-level to assembly structures using the same rule for all levels. Three heuristics are tested this way. Following the ideas of New [141] and McLaren [135], they modify the cost parameters of the items in order to take the multi-level structure indirectly into account. Five types of modifications are tested. Closely related to their lot sizing approach is the one of Rehmani and Steinberg [149] who keep the cost parameters, but, modify the computation of some derived values that guide a single-level heuristic.

Gupta et al. [84] compare the cost modification approach with others. Studies of Wemmerlöv [191] also examine the impact of cost modifications.

Raturi and Hill [148] introduce something similar to capacity constraints into the model of Blackburn and Millen. But, capacity usage is only estimated on the basis of average demand. They heuristically compute shadow prices for the capacities and derive modified setup costs from that. Eventually, the lot sizing problem is solved using the Wagner-Whitin procedure level by level.

An extension of Blackburn and Millen's approach for general gozinto-structures is presented by Dagli and Meral [57]. The combination of lot sizing and capacity planning is treated in [33] where the subject is confined to assembly structures. The impact of a rolling horizon is discussed in [32].

Combining the idea of using different single-level heuristics at different levels with the idea of modifying the cost parameters is evaluated by Blackburn and Millen [34]. Seven algorithms are used in six combinations. In all tests they choose one algorithm for the end item level and another algorithm for the remaining levels.

While the above level-by-level algorithms construct a solution in one pass, Graves [80] presents a multi-pass procedure. Assuming that external demand occurs for end items only, the basic idea is to perform a single pass, and then to modify the cost parameters (how this is done is of no relevance here). Afterwards, he solves the single-item problems for those items again for which external demand exists. If nothing changes for these items the procedure terminates. If there are any changes in the production plan then the whole procedure repeats using the modified cost parameters this time. A proof of convergence is given.

Another multi-pass heuristic is invented by Peng [143] who uses the Wagner-Whitin procedure to generate an initial solution. This solution is then repeatedly reviewed to combine lots until no further cost savings can be achieved.

Moving away from level-by-level lot sizing, Lambrecht et al. [124] compare level-by-level approaches with a period-by-period approach. The latter one computes production quantities for the first period and for all items before the second period is concerned. This is going on period by period. All computational tests are restricted to assembly structures. As a result, the period-by-period approach seems to be competitive. A somehow similar method proposes Afenta-

kis [3]. Starting with solving the lot sizing problem consisting of the first period only, he generates a solution which includes the next period by augmenting the intermediate result. This is done until all periods are covered. While Afentakis uses the term sequential approach instead of level-by-level, he now calls his method a parallel one instead of period-by-period. The proposed method decidedly outperforms all (five) single-level methods that are used level-by-level in his study.

Rosling [152] gives an uncapacitated plant location reformulation of the problem with assembly structures. On the basis of this new MIP-model, he develops an optimal procedure.

Heinrich [96, 95] presents a heuristic for general gozinto-structures which operates in two phases. First, the dynamic multi-level lot sizing problem is reduced into a problem with constant demand (using the average demand for each item for instance). Using modified cost parameters, the resulting problem is then solved with a deterministic search in the set of feasible reorder points. In the second phase, the resulting production amounts are modified to meet the demand of the original problem instance.

Coleman and McKnew [48] keep the idea of level-by-level methods alive and design a new heuristic based on an earlier work [47]. Although their computational study gives promising results, they fail to compete with sophisticated, established methods (e.g. [80]). A study by Simpson and Erenguc [163] indicates that, if this is done, the new heuristic appears rather poor.

Gupta and Brennan [84] evaluate level-by-level approaches if backorders are allowed. Other improved heuristics for models with backlogging are discussed by Vörös and Chand [187].

Afentakis et al. [5] efficiently compute optimal solutions for assembly structures. The proposed method is of the B&B-type. Based on a MIP-reformulation, lower bounds are determined with a Lagrangean relaxation of those constraints which represent the multi-level structure. As a result, a set of single-item, uncapacitated lot sizing problems is to be solved. This is done with a shortest path algorithm. Optimal solutions for general gozinto-structures are determined by Afentakis and Gavish in [4] by transforming complex structures into assembly structures where some items occur more than once. Chiu and Lin [40] also compute optimal solutions for assembly structures with dynamic programming. Their idea is based on a graphical interpretation of Afentakis' reformulation. A level-by-level heuristic with postprocessing operations is described as well.

Kuik and Salomon [117] apply a stochastic local search method, namely simulated annealing, to multi-level lot sizing. This approach can handle general gozinto-structures. A comparison with other approaches is not done.

Salomon [156] suggests a decomposition of the multi-level problem into several single-item problems. Roughly speaking, this is achieved by eliminating inventory variables by substitution, doing a Lagrangean-like relaxation of inventory balances, and adding some new constraints. He then solves the single-item problems with some lot sizing algorithm sequentially, and upon termination

updates the Lagrangean multipliers to repeat the process.

Joneja [110] considers assembly gozinto-structures. By adapting results from lot sizing with stationary demand [153], he develops a so-called cluster algorithm and proves worst case performance bounds. The projection of this work to serial structures closely relates to Zangwill's paper [200]. Roundy [155] presents two cluster algorithms for general structures and worst case bounds for these. In a computational study, one of them turns out to be slightly better than Afentakis' method [3].

McKnew et al. [134] present a linear zero-one-model for multi-level lot sizing. Assembly gozinto-structures are assumed. They claim that the LP-relaxation of the model always yields integer solutions. Rajagopalan [145] argues that they are wrong. But, his counter-example is false.¹

Pochet and Wolsey [144] consider general gozinto-structures and derive cutting planes to ease the computational effort when standard solvers are running.

Atkins [9] suggests a way to compute lower bounds for problems with assembly structures. His basic idea is to convert an assembly structure into a set of serial structures by finding all paths from the lowest level to end items. Some items may now appear in more than one serial structure having their own identity. In such a case the original setup costs are split up and assigned to the (new) items. Eventually, the approach in [126] can be used to solve the resulting instances which, in summary, gives a lower bound.

Simpson and Erenguc [163] invent a neighborhood search procedure and slightly outperform [80], [48], and three other improved heuristics. Lower bounds are obtained with a Lagrangean relaxation.

Richter and Vörös [150] are trying to find a so-called setup cost stability region which is defined as the set of all setup cost values for which a given solution remains optimal. Recently, Vörös [188] analyzes the sensitivity of the setup cost parameters for facilities in series and computes stability regions with a dynamic programming procedure.

Arkin et al. [8] give a classification of the complexity of uncapacitated, multi-level lot sizing problems. Optimization problems with general gozinto-structures and no setup times are proven to be NP-hard.

4 Dynamic Demand, Scarce Capacity

Lambrecht and Vander Eecken [122] consider a serial structure and formulate an LP-model on the basis of a network representation. Setups are not part of their focus which is the reason for having continuous variables only (such approaches are used for master production scheduling). Furthermore, capacity constraints exclusively restrict the production amounts of the end item. Optimal solutions are computed by means of extreme flow considerations. A similar problem is

¹See [145], page 1025: The determinant of the submatrix in the example evaluates to 1 and not to 2 as stated in the paper. Hence, there is no contradiction to total unimodularity.

concerned by Gabbay [74] who has several serial structures being processed in parallel in mind. In contrast to Lambrecht and Vander Eecken he assumes capacity limits on all levels. Zahorik et al. [197] generalize Gabbay's work, but still assume serial structures. Assembly structures are investigated by Afentakis [2] who transforms the problem into a job shop scheduling problem.

A first MIP-model is formulated by Haehling von Lanzener [88]. Remarkably to note that he considers lot sizing and scheduling simultaneously. Another early multi-level lot sizing model is that of Elmaghraby and Ginsberg [67]. An LP-model for master production scheduling in a multi-level system is given in [89].

Gorenstein [77] provides a case description for planning tire production with three levels. He formulates a MIP-model, but does not present any methods. In a short note he points out that LP-based rounding methods could work fine. Vickery and Markland [186] report about experiences in using MIP-model formulations being solved with standard solvers in a real-world situation. Their field of application is a pharmaceutical company.

Zäpfel and Attmann [196] present some kind of a tutorial for multi-level lot sizing. They assume several serial gozinto-structures being processed in parallel. A MIP-model is given on the basis of which a fixed charge problem reformulation and examples are discussed.

Ramsey and Rardin [146] consider serial structures and propose several heuristics. The basic assumption in this work is that items do not share common resources. Two of the heuristics are LP-based methods where a particular choice of the arc weights in a network flow representation makes the solution of the LP-network-problem become a feasible solution. One out of the two LP-based procedures uses the optimal solution of the uncapacitated problem [199] to set the arc weights. Two other heuristics are ad hoc approaches. One makes the multi-level problem collapse into a single-level problem by making additional assumptions. The other one is a greedy heuristic. A computational study shows that the greedy heuristic gives the best results, although the idea of collapsing the gozinto-structure leads to much shorter execution times. Both LP-based approaches are worse with respect to the run-time as well as with respect to the deviation from a lower bound.

Biggs [23] informally discusses the problem of scheduling component parts in order to meet the demand for the end items. In a computational study he uses, among others, the total number of setups to evaluate several rules. As a result, he states that "...the use of the various rules did cause the system to respond differently...".

Harl and Ritzman [92] use modifications of uncapacitated, single-level heuristics to develop a capacity-sensitive multi-level procedure.

Billington et al. [27, 29] assume a general gozinto-structure. Some (potentially all) items share a single common resource with scarce capacity. Beside a MIP-formulation they also present a heuristic to solve the problem. The basic working principle of the heuristic is a B&B-strategy. The B&B-method enu-

merates over the setup variables. At each node lower bounds are computed by relaxing capacity and multi-level constraints. The uncapacitated, single-level subproblem is then heuristically solved being embedded into a Lagrangean-like iteration.

Hechtfisher [94] heuristically solves problems with an assembly structure. Similar to Billington et al. he assumes that (some) items share a single common resource. But additionally, all items which require the resource must have the same low level code. Basically, his approach follows traditional MRP II ideas and operates level-by-level. If production amounts exceed the available capacity, some items are shifted into earlier periods. Guided with a Lagrangean-like penalty expression, the whole procedure iterates until a maximum number of iterations is performed.

Salomon [156] introduces a simulated annealing and a tabu search heuristic for the problem earlier defined by Billington et al. [29]. Furthermore, he discusses LP-based heuristics for assembly structures. Among traditional rounding approaches, he also presents combinations of simulated annealing and tabu search, respectively, with LP-based rounding. Reprints of parts of this outlet are [118] and [157].

Roll and Karni [151] attack the multi-level lot sizing problem with a single resource as well. In contrast to Billington et al. they do not consider setup times. Starting with a first feasible solution, they pass multiple phases making changes to intermediate solutions to gain improvements.

Toklu and Wilson [182] assume assembly gozinto-structures. Furthermore, only end items require a single commonly used scarce resource. They develop a heuristic for this type of problem.

Maes et al. [127, 128, 129] suggest LP-based rounding heuristics for assembly type problems based on the solution of the LP-relaxation of a reformulation as a plant location model [152]. Their ideas cover simple single-pass heuristics fixing binary variables to 1, or fixing them to 1 or to 0, respectively, meanwhile solving a new LP-relaxation in-between. Another suggestion is that of curtailed B&B-procedures.

Mathes [130] extends the MIP-model given by Afentakis et al. [5] and introduces capacity constraints whereby items do not share common resources (so-called dedicated machines). Several valid constraints are derived to improve the run-time performance of standard solvers.

Stadtler [164, 165] compares the impact of different MIP-model formulations on the performance of standard solvers. His assumptions are quite unrestricted: general gozinto-structures, multiple resources which are shared in common, and positive setup times. Due to his assumption that an unlimited amount of overtime per period is allowed, there exists no feasibility problem which would make the development of heuristics really hard.

Helber [97, 98] develops heuristics for the multi-level lot sizing problem. He assumes general gozinto-structures. Items may share common resources, but no item requires more than one resource. Positive setup times are allowed. Among

the proposed procedures are simulated annealing, tabu search, and genetic algorithms which guide a search in the space of setup patterns on the basis of which production plans are derived. Tempelmeier and Helber [180] test a modification of the Dixon–Silver heuristic [63]. Derstroff and Tempelmeier [62, 178, 179] use a Lagrangean relaxation of capacity constraints and inventory balances to provide a lower bound. Within each iteration an upper bound can be computed by solving an uncapacitated, multi-level problem on a level-by-level basis using the objective function including the penalty expressions for evaluation. A postprocessing stage is used to smooth capacity violations.

Clark and Armentano [45] present a heuristic for general gozinto-structures which operates in two phases. First, the Wagner–Whitin procedure is used to solve the uncapacitated problem on a level-by-level basis. Second, production in overloaded periods is shifted into earlier periods to respect the capacity constraints. Interesting to note is that items may require more than just one resource.

Following the successive planning idea, Sum and Hill [168] perform lot sizing and scheduling. They discriminate orders and operations where each order is fulfilled by doing a plenty of operations. They have both, a network (i.e. gozinto-structure in our terminology) of orders and for each order a sub-network of operations. Operations are scheduled with a modified version of a resource constrained project scheduling algorithm [60, 142]. After this is done, they merge and/or break some orders which results in a new network and the whole process repeats until the recent solution is not improved. The key element of lot sizing is the merging and breaking part in this approach. The decision what to merge or to break, respectively, relies on a simple priority rule.

Dauzère-Péres and Lasserre [58, 59, 125] take an integrated approach in lot sizing and scheduling into account. In a first step they make lot sizing decision regarding capacity constraints. Then, they determine a sequence given fixed lot sizes. Afterwards, a new lot sizing problem is solved with new precedence constraints among items which share the same resource. These precedence relations stem from the sequence decisions just made. The whole procedure is repeated until a stopping criterion is met. For solving the lot sizing and the scheduling subproblems, respectively, they employ methods from the literature.

Haase [86] gives a MIP-model for multi-level lot sizing and scheduling with a single bottleneck facility. Solution methods are, however, not provided.

Brüggemann and Jahnke [36] employ simulated annealing to find suboptimal solutions for lot sizing and scheduling problems with batch production. However, their heuristic works only for two-level gozinto-structures. The underlying idea is to proceed level by level again.

Jordan [111] considers batching and scheduling. Due to restrictive assumptions, he is able to compute optimum results for small instances.

Kimms [115] tackles the problem of multi-level lot sizing and scheduling under quite general assumptions. The gozinto-structures may be of the general type and several scarce resources may impose capacity restrictions (single-

machine cases are discussed in [113, 114]). Several heuristics are developed, among them regret based random sampling, a cellular automaton, a genetic algorithm, disjunctive arc based tabu search, and a so-called demand shuffle procedure which, by the way, performs best. The basic idea of demand shuffle is to combine random sampling with a problem specific data structure manipulation. Roughly speaking, this data structure contains the information until when certain (external or internal) demand is to be met. Manipulating this data structure basically equals modifying the deadline of some demand artificially.

5 Intermediate Results

Under unconstrained capacities most authors favorite level-by-level approaches which are easy to implement and which can make use of many single-level heuristics as well as efficient exact methods. However, recent research indicates that other ideas are superior [155, 163]. Due to its complexity [8], optimal solution procedures are unlikely to be able to solve medium- to large-sized problem instances. Hence, heuristics will dominate for practical purposes. Definitely, this holds for problems under capacity constraints.

Reviewing the work on capacitated, multi-level lot sizing reveals that most researchers consider very restrictive cases. Most of them assume a single resource only. Also, a lot of work does not deal with general gozinto-structures. Exceptions can be found in very recent outlets only [62, 97, 98, 178, 179, 180].

Although the importance of considering multi-level lot sizing and scheduling simultaneously was recognized rather early [88], research in this field has just begun. A first step away from pure successive MRP II concepts towards taking interaction effects of lot sizing and scheduling into account is done with iterative procedures [58, 59, 125, 168]. Models and methods which solve the combined problems in one step are presented in [115].

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