Iftikhar, Zainab; Zaharieva, Anna

Working Paper
General equilibrium effects of immigration in Germany: Search and matching approach

Center for Mathematical Economics Working Papers, No. 568

Provided in Cooperation with:
Center for Mathematical Economics (IMW), Bielefeld University

Suggested Citation: Iftikhar, Zainab; Zaharieva, Anna (2016) : General equilibrium effects of immigration in Germany: Search and matching approach, Center for Mathematical Economics Working Papers, No. 568, Bielefeld University, Center for Mathematical Economics (IMW), Bielefeld, http://nbn-resolving.de/urn:nbn:de:hbz:361-290599984

This Version is available at:
http://hdl.handle.net/10419/149025

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licenses), you may exercise further usage rights as specified in the indicated licence.
General Equilibrium Effects of Immigration in Germany: Search and Matching Approach

Zainab Iftikhar and Anna Zaharieva
General Equilibrium Effects of Immigration in Germany: Search and Matching Approach

Zainab Iftikhar,* Anna Zaharieva†

September 22, 2016

Abstract

In this study we develop and calibrate a search and matching model of the German labour market and analyze the impact of recent immigration. Our model has two production sectors (manufacturing and services), two skill groups and two ethnic groups of workers (natives and immigrants). Moreover, we allow for the possibility of self-employment, endogenous price and wage setting and fiscal redistribution policy. We find that search frictions are less important for wages of the low skilled, especially in manufacturing, whereas wages of the high skilled are more sensitive to their outside opportunities. Furthermore, employment chances of immigrant workers are up to four times lower than employment chances of native workers, especially in the high skill segment. Our results show that recent immigration to Germany, including refugees, has a moderate negative effect on the welfare of low skill workers in manufacturing (-0.6%), but all other worker groups are gaining from immigration, with high skill service employees gaining the most (+4.3%). This is because the productivity of high (low) skill workers is increasing (decreasing) and there is a higher demand for services. The overall effect of recent immigration is estimated at +1.6%. Finally, we observe that productive capacities of immigrant workers are underutilized in Germany and a policy implementing equal employment opportunities can generate a welfare gain equal to +0.9% with all worker groups (weakly) gaining due to the redistribution.

Keywords: search frictions, immigration, general equilibrium, redistribution, welfare

JEL Classification: J23, J31, J38, J64

* IRES, Université Catholique de Louvain, Department of Economics, 3, Place Montesquieu, B-1348 Louvain-la-Neuve, Belgium. Email: zainab.kkhan@gmail.com
† Center for Mathematical Economics, Bielefeld University, 33501 Bielefeld, Germany. Email: azaharieva@wiwi.uni-bielefeld.de
1 Introduction

This study investigates the effect of immigration on the German labour market in a search and matching framework. In particular, we estimate the long-term impact of immigrants who entered Germany in the period 2008-2016. Existing literature is focused on estimating wage effects of immigration and is mostly based on the assumption of a competitive labour market with the marginal pricing of labour\(^1\). This approach doesn’t take into account the bargaining nature of wage-setting in Germany and low mobility of workers, implying that frictional unemployment may be an important transmission channel when analyzing the effect of immigration, which is not captured by the neo-classical competitive market framework. Moreover, there remain open questions concerning the general equilibrium effects and welfare changes in response to immigration. Our study addresses these issues by developing a search and matching model with endogenous job-creation, bargained wages, self-employment by native and immigrant workers, endogenous output prices and fiscal adjustments. Hence we are able to decompose the total effect of immigration into the separate contributions of a rise in labour supply, endogenous response of the labour demand, higher aggregate demand for goods and services and the corresponding adjustment in output prices, unemployment benefits and public expenditures. To the best of our knowledge there is no other study for Germany that identifies and estimates these effects in a unified labour market framework with search frictions. To calibrate the model we use data from the German Socio-Economic Panel and aggregate macroeconomic indicators.

More specifically, we consider a labour market with two production sectors: tradable (manufacturing) and non-tradable (services). In both sectors output is produced by means of high and low skill labour combined together by means of a CES production function\(^2\). Thus there are four separate submarkets in our model depending on the sector and the skill level of workers. Job creation (vacancies) in every submarket is modeled by means of a search and matching model following Pissarides (2000). Every worker can be a native or an immigrant, so there are eight distinct groups in our model depending on the ethnic origin, skill level of the worker and the production sector. Native and immigrant workers apply for the same vacancies in each of the four submarkets, however, their hiring chances, productivities and wages can be different. Wages are set by means of Nash-bargaining and reflect differences in productivities and outside opportunities of workers.

Every worker can be employed, unemployed or self-employed. Our motivation to include the state of self-employment into the model is due to the fact that in Germany immigrant workers are overrepresented in self-employment compared to regular paid employment (Metzger (2016)). Thus it is possible that some immigrant workers enter self-employment out of necessity if their chances of finding a regular paid job are too low. For example, Constant and Zimmermann (2006) report that immigrant workers in Germany are more likely to enter self-employment if they feel discriminated in the regular labour market. This means that immigration effects produced in the absence of self-employment can lead to biased estimates since such a model would overpredict the rise in unemployment in response to the new immigration inflow.

Further, workers have homothetic preferences with a CES utility function and decide how

---

\(^1\)Exceptions include Chassamboulli and Palivos (2014) and Battisti, Felbermayr, Peri and Poutvaara (2014)

\(^2\)In our benchmark calibration we use the Cobb-Douglas form as a limiting case of the CES, but in the extensions we also consider the elasticity of substitution between high and low skill workers larger than 1.
to spend their income (wage, unemployment benefit or profits from self-employment) between
consumption of goods and services. From an aggregate perspective, we set a fixed price for
tradables (manufactured goods), but the price for services is endogenously determined, so in
the equilibrium the demand for services is equal to the supply of services. Finally, we assume
a balanced budget of the planner, who collects income taxes, pays out unemployment benefits
and distributes the rest as a lump-sum transfer to all workers.

We find that the number of immigrant workers in Germany remained virtually unchanged in
the years 2008-2010, but increased thereafter by almost 20%. So having calibrated our model by
using GSOEP data from 2008 we analyzed the implications of a 20% increase in immigration.
At the calibration stage we find that wages of low skill workers, especially, in manufacturing
are closer to the marginal productivities of these workers. The situation is different for high
skill workers, whose wages are closer to their reservation wages and reflect to a large extent
their outside opportunities in terms of finding jobs. This means that search frictions are less
important for the low skilled, whose wages can even be approximated by the marginal product,
but are more important for the high skilled. Intuitively, this could be due to the fact that low
skilled workers in Germany are largely covered by labour unions, whereas high skill workers are
more likely to negotiate the wage individually.

In order to calibrate the matching function for Germany we used aggregate data of the Federal
Employment Agency (BA) for the period 2001-2013. We find that the average vacancy duration
increased by 72% over this period, reflecting a rising shortage of the workforce. Our estimate of
the elasticity of the matching function (job-filling rate) is equal to 0.44 and is not far from 0.5,
which is often assumed in the search and matching literature. Finally, at the calibration stage we
observe a large gap in unemployment rates between native and immigrant workers. Given that
vacancies are publicly posted in our model, differences in unemployment rates are reflected in the
probability of being hired conditional on matching. We find that the hiring chances of immigrant
workers are lower compared to native workers, especially, for high skill workers. For example,
immigrant worker are three times less likely to get a job in manufacturing occupations and four
times less likely to get a job in services. The situation is less dramatic for low skill workers
in manufacturing, where the hiring chances of both immigrants and natives are approximately
equal. Intuitively, this could be due to the larger importance of language skills in high skill jobs,
more uncertainty about the quality of foreign education and/or stronger hiring discrimination.

Having performed the calibration we analyzed the implications of a 20% increase in immigra-
tion observed in Germany in the last eight years. The benchmark scenario in our model is skill
and sector-neutral immigration. In this case we find that immigration leads to 0.55% higher
aggregate welfare in Germany, but this estimate is prior to the fiscal adjustment of the budget.
Further we find that immigration leads to a positive surplus of the public budget since taxes
paid by new immigrant employees dominate additional expenses on benefits paid to the new
unemployed migrants. Allowing for higher lump-sum transfers to restore the balance of the bud-
get we report a total increase in welfare equal to 1.65%. This positive effect of immigration on
public finances in Germany is not completely unexpected. For example, in his empirical research
Bonin (2014) reports that a netto contribution of an average immigrant living in Germany in

\[^3\]In addition, we also consider a scenario with only low skill immigration. Since detailed information on the
skill composition of recent immigrants in Germany is not yet available, our two scenarios provide a lower and an
upper bound for the real effect of immigration.
2012 was about €3300 with the total contribution of all immigrants equal to €22 Bln.

Considering the effect on different worker groups, we find that productivity of low skill workers falls, whereas the productivity of high skill workers is increasing. Since manufacturing is a low skill intensive sector, and low skill immigrants are the majority, output in manufacturing increases stronger than the output in services, which is a high skill intensive sector. This leads to the overproportional increase in the internal demand for services and raises the price of services. So there is an asymmetric effect on the two sectors. Whereas manufacturing workers are worse off due to the fact that services become more expensive, workers employed in services are gaining since higher prices stimulate job creation in their sector. Overall, low skill workers employed in manufacturing face lower wages and higher unemployment. Combined with a higher price of services this leads to the change in welfare of these workers equal to $-1.7\%$ prior to the budget adjustment and $-0.6\%$ afterwards. High skill workers employed in manufacturing gain in terms of productivity, which leads to higher wages and lower unemployment, but this positive effect is mitigated by higher prices, so the change in their welfare is equal to $-0.6\%$ prior to the budget adjustment and $+0.4\%$ afterwards. Low skill workers employed in services gain higher wages and lower unemployment, which is due to the positive effect of the higher price. So the change in their welfare is equal to $+1.6\%$ prior to the budget adjustment and $+2.8\%$ afterwards. Finally, high skill workers employed in services gain most from immigration. Not only their productivity increases, but also the price for their services is higher. This is associated with a change in welfare equal to $+3.3\%$ prior to the budget adjustment and $+4.3\%$ afterwards.

Given the lower hiring chances of immigrant workers observed in Germany, it is possible that their skills are underutilized. Thus there is a question about the change in welfare associated with improving the hiring chances of immigrant workers. In order to address this question we have generated a counterfactual economy with equal hiring chances between native and immigrant workers. In reality, this step could take the form of an easier access to language and integration courses, easier application process for recognizing foreign diplomas and/or anti-discrimination policies. We find that welfare gains of immigrant workers, especially high skilled, are substantial and dominate small welfare losses of native workers stemming from the fall in marginal productivity. So the average welfare gain of all workers prior to the budget adjustment is equal to $0.5\%$. Furthermore, higher wages of immigrant workers, resulting from better outside opportunities, mean higher tax income and a moderate surplus of the public budget. Taking into account better financing of public goods, which in our model takes the form of lump-sum transfers to the population, we find that the overall welfare gain increases to $0.9\%$. Thus we conclude that due to the redistributive tax policy even native workers gain welfare from an improvement in the hiring chances of immigrants.

Further, we have done several extensions and robustness checks. First, we have extended the model to account for proportional unemployment benefits. So in this extension we keep a constant replacement rate, but the unemployment benefit is moving in the same direction as the wage. We find that proportional unemployment benefits amplify the positive effect of immigration. This is because increasing wages of all worker groups, except low skill workers in manufacturing, raise their unemployment benefits and put an additional upward pressure on wages. So the overall gain in worker’s welfare is equal to $2.0\%$ with proportional unemployment benefits, which is larger than $1.6\%$ in the benchmark case. Next we have increased the elasticity
of substitution between high and low skill workers from 1 (Cobb-Douglas function in the benchmark case) to $1/0.34 \approx 2.94$ following the estimation by D’Amuri, Ottaviano and Peri (2010). We find that stronger complementarity is beneficial for the low skilled and costly for the high skilled. For example, the rise in wages of all high skill workers is mitigated, whereas the rise in wages of the low skilled in services is amplified. This, however, doesn’t change the overall welfare increase from skill- and sector-neutral immigration, which remains at the level of 1.6%.

Using a search and matching framework to address the impact of immigration is a relatively recent tendency in the literature. For example, Chassamboulli and Palivos (2014) have incorporated a nested CES production function with capital, skilled and unskilled labour into the frictional labour market and calibrated this model to the U.S. economy, which experienced a high-skill biased immigration in the period 2000-2009. They conclude that the overall effect of immigration in the U.S. in the considered period should be estimated as positive. Second closely related study is by Battisti, Felbermayr, Peri and Poutvaara (2014). These authors calibrated their search and matching model to 20 different OECD countries and arrived at a general conclusion that immigration in the period 2000–2011 has increased the welfare of native workers in almost all these countries. Our model has a number of similarities with Battisti et al. (2014) e.g. the wage gap between natives and immigrants, redistributive fiscal policy and perfect substitution between these groups in the CES production function. However, there are also notable deviations, for example, in our model we consider two production sectors – manufacturing and services – with an endogenous price setting in services and self-employment of native and immigrant workers. We find that the model with two production sectors is better in capturing the asymmetric effects of immigration on high and low skill workers, who are not equally distributed between these sectors. In particular, our model takes into account that low skill workers, who are overrepresented in manufacturing, are more protected against changes in the domestic output demand since manufactured goods are largely traded on the international market. Another departure from Battisti et al. (2014) is that in our model differences in the unemployment rates between natives and immigrants are captured by lower hiring chances of immigrant workers, whereas it is captured by higher job destruction rates in their study.

Another study by Ortega (2000) also deals with the issue of immigration in a frictional labour market. He considers a two–country model and shows that both equilibria with and without immigration between the two countries are possible, but the equilibrium with immigration Pareto dominates the other one. Finally, there are two other studies combining immigration and search frictions by Moreno-Galbis and Tritah (2016) and Nanos and Schluter (2014). In the former paper immigrant workers earn lower wages which leads to higher profits of firms. So they show theoretically and empirically that an inflow of immigrant workers intensifies job creation due to the higher expected profits of firms, which benefits natives and incumbent immigrants. This effect is also present in our study, however, in our model this positive effect is reduced due to the fact that not all matches between firms and immigrant workers lead to employment. Hence an inflow of immigrant workers reduces the job-filling rate for open vacancies and may even lead to lower job creation. We show that this negative effect disappears if hiring chances of native and immigrant workers are equalized. To the best of our knowledge our study is the first one which formalized this effect.

The study by Nanos and Schluter (2014) is also close to our research because it uses empirical
data for Germany and estimates a "migrant effect", which is a wage difference between native and migrant workers stemming from the fact that migrant workers receive lower wage offers conditional on the same productivity. This effect is also a part of our model, because worse employment opportunities of immigrant workers lead to lower reservation wages and generate a migrant wage gap. One important advantage of Nanos and Schlueer (2014) over our work is that they explicitly allow for on-the-job search in their model, however, it comes along with the assumption that labour markets of immigrant workers are completely separated from those of natives, which is different in our model and allows us to quantify the general equilibrium effects of migrants on native workers.

Next we turn to the characterization of the large literature on the effect of immigration in a competitive labour market framework without frictions. Two early studies by Borjas (1999) and BenGad (2004) consider a setting with homogeneous workers and show that the response of wages strongly depends on the mobility of capital stock. If capital supply is fixed and inelastic, wages will be expected to fall while the return on capital will rise in response to an exogenous increase of labour supply. As a result native workers are penalized by immigration whereas the position of capital owners is improved. Moreover, the gain of capital holders is found to dominate the former wage effect and the overall welfare in the host country is increased (positive immigration surplus). In contrast, the immigration surplus is zero and wages remain unchanged if capital is perfectly mobile and the capital market is competitive. Schmidt, Stilz and Zimmermann (1994) and BenGad (2008) extend this framework to the case of heterogeneous skill groups. Even though both skilled and unskilled wages are generally expected to fall with a skill-balanced immigration, the elasticity of labour demand and the skill direction of immigration are found to be crucial in this setting. Empirical evidence suggests that the demand for skilled labour is less elastic than the demand for unskilled labour. Hence according to these studies the immigration surplus is maximized when the inflow of immigrants is exclusively skilled. To the best of our knowledge Schmidt et al. (1994) is the only immigration study emphasizing the importance of incorporating labour unions and wage bargaining into the model, so they consider a right-to-manage model with a monopoly union, however, it is not calibrated to any specific economy.

Altonji and Card (1991) identify one of the positive general equilibrium effects of immigration. Specifically, in their model immigrants are likely to increase the overall demand for goods and services in the receiving country. This demand effect will tend to increase the overall demand for native labour, thus raising native wages and employment. Among more recent studies this idea is emphasized by Moretti (2010a,b), who writes that "Every time a local economy generates a new job..., additional jobs might also be created, mainly through increased demand for local goods and services" (multiplier effect). Even though Moretti (2010a,b) doesn’t consider immigration he finds that for each additional job in manufacturing, 1.6 jobs are created in the non–tradable sector. Moreover, one additional skilled job in the tradable sector generates 2.5 job in local goods and services. It is a similar multiplier effect in response to immigration that we quantify in our study for Germany.

Recent studies investigating the effect of immigration started with the seminal approach by Borjas (2003) who observed that the rise in the U.S. immigration was not uniformly distributed across education/experience cells. This novel approach was further extended in the research of Manacorda, Manning and Wadsworth (2012) for the United Kingdom and Damuri, Ottaviano,
Peri (2010) for Germany. These studies allow for the imperfect substitution of native and immigrant workers and show that the additional inflow of foreign workers to UK and Germany has a large negative effect on wages of the existing immigrants in the country leaving native wages virtually unaffected. This conclusion is supported by Felbermayr, Geis and Kohler (2010) who include the possibility of unemployment into the model and show that a new immigration inflow of workers from the east-ward enlargement of the European Union had a strong positive effect on the unemployment of incumbent foreign workers in Germany. On the contrary, the effect on native employment is very modest.

Dustmann and Preston (2012) have criticized the education-experience cell approach by observing that immigrant workers downgrade at arrival, which implies accepting jobs with a lower education/experience requirement than comparable native workers. Hence immigrants may compete with native workers at parts of the skill distribution which is different to where they should be assigned based on their observable characteristics. In response to this critique Dustmann, Frattini and Preston (2013) develop a new estimation strategy and study a continuous effect of immigration on workers situated at different quantiles of the earnings distribution. Their findings suggest that immigration reduced wages of natives below the 20th percentile of the wage distribution in the UK, but slightly increased wages in the upper part of the distribution.

Overall, all of these studies are focused on the effect of immigration on wages rather than (un)employment. Only D’Amuri et al. (2010) and Felbermayr et al. (2010) follow a reduced form approach and assume a negative relation between employment and wages, however, this assumption is not micro-founded and there is no economic mechanism in their model leading to this relationship. This is the primary difference between these studies and our approach.

The outline of the paper is as follows. Section 2 presents notation and the economic environment. Section 3 describes the model, while section 4 presents the calibration of the model to the German labour market. Section 5 contains our main results for the benchmark model. Sections 6 and 7 include extensions, counterfactual evidence and some alternative calibrations. Finally, section 8 concludes the paper.

2 Economic environment

There are four demographic groups of workers: low skill immigrants \(g^L_I\), low skill natives \((g^L_N)\), high skill immigrants \(g^H_I\) and high skill natives \(g^H_N\). The workers have homothetic preferences with a CES utility function and discount future utility flows at rate \(r\). The total labour force is normalized to 1, so we get:

\[
g^L_I + g^L_N + g^H_I + g^H_N = 1
\]  

(1)

We use the subscript \(n\) to denote the origin of the worker \(\{I, N\}\) and the superscript \(i\) to indicate the skill level of the worker \(\{L, H\}\). Every worker belongs to one of the two production sectors: (1) manufacturing or (2) services. Subscript \(j\) corresponds to the production sector \(\{1, 2\}\), so that \(g^j_n\) is the number of workers with origin \(n\), skill level \(i\) belonging to the production sector \(j\). With this notation we have: \(g^j_n = g^j_{1n} + g^j_{2n}\). We assume that skills are specific to a given sector and not transferable, thus it is not possible for workers to change the sector. Every
worker can be employed in a firm, unemployed or self-employed, so that: 
\[ g'_{jn} = e'_{jn} + u'_{jn} + s'_{jn}, \]
where \( e'_{jn} \) is the number of employed workers, \( u'_{jn} \) - the number of unemployed workers and \( s'_{jn} \) - the number of self-employed workers in a given origin-skill-sector group. There is a possibility of self-employment or entrepreneurship for both immigrant and native workers of any skill type. However, we are agnostic about the reasons for self-employment since it is not clear ex-ante whether push or pull factors are dominating for different demographic groups. Thus in our model transitions into self-employment take place at an exogenous arrival rate \( \alpha'_{jn} \). Note that we allow this rate to be heterogeneous across worker groups, this way we try to capture possible differences in the regulation and access to self-employment, e.g. mandatory licenses, in different occupations. At rate \( \gamma \) self-employed workers are not successful in their business and return to the unemployment pool.

There are four labour markets in the economy depending on the production sector and the skill level of workers. Let \( v_j \) denote the number of vacancies in each of these markets. Both native and immigrant workers are applying to the same vacancies in a given labour market. The unemployed workers are matched with vacancies through a matching function given by \( m(u_{ij}, v_j) \), where \( u_{ij} \) is the total number of searching native and immigrant workers: \( u_{ij} = u_{ij N} + u_{ij I} \). The matching function is homogeneous of degree 1, concave and increasing in both its arguments. Later we shall discuss the specific form we use for our empirical analysis. The rate at which a worker is matched with a vacant position in a specific labour market is \( m(u_{ij}, v_j) / u_{ij} = \lambda(\theta_j) \), where \( \theta_j = v_j / u_j \) reflects the tightness of the labour market for skill group \( i \) in sector \( j \). Even though the probability of matching with vacancies \( \lambda(\theta_j) \) is identical for native and immigrant workers, their hiring chances are not. This difference is captured by an additional parameter \( h_{jI} \leq h_{jN} = 1 \), which is specific to the given labour market and reflects a lower hiring probability for immigrant workers compared to natives. Intuitively, it captures a lower language proficiency of immigrant workers and hiring discrimination by firms\(^4\). Note that \( h_{jN} = 1 \) for all sub-markets. Thus the job-finding rate for immigrant workers becomes \( h_{jI} \lambda(\theta_j) \leq \lambda(\theta_j) \). Next the rate at which a firm fills a vacant position is \( m(u_{ij}, v_j) / v_j = q(\theta_j) \). Thus \( q(\theta_j) \) is the probability of filling a vacancy. Again this rate is lower and equal to \( h_{jI} q(\theta_j) \) if the firm is matched with an immigrant worker. Since the matching function is increasing in both arguments we have: \( \lambda'(\theta_j) > 0 \) while \( q'(\theta_j) < 0 \). Every job can be destroyed at rate \( \delta \) due to some adverse exogenous shock. We assume that this rate is identical for all worker groups.

In terms of production we follow the approach of Acemoglu (2001) and assume that workers with different skills produce different intermediate goods which are used in the production of the final good in each of the two production sectors. The total input of low skilled intermediate goods in sector \( j = 1, 2 \) is equal to \( n_{jL} = e_{jL} + \psi_{jN} + e_{jN} + \varphi_{jI} + s_{jI} + \varphi_{jN} s_{jN} \) as it is produced by low skilled workers in paid employment and self-employed agents who can be native or immigrant. Even though we assume perfect substitution between native and immigrant workers (infinite elasticity of substitution), we do allow for differences in the effective productivity of these groups reflected in parameters \( \psi_{jN} \) for paid workers and \( \varphi_{jN} \) for the self-employed. This assumption is motivated by the findings in empirical studies, reporting finite but large elasticities of substitution between native and immigrant workers in the range between 20 and 30\(^5\).

---


\(^5\)See the debate in Ottaviano and Peri (2012)
intermediate goods in sector \(j\) is equal to \(n_j^H = e_j^H + \psi_j^H e_j^H + \varphi_j N_j^H + \varphi_j^N x_j^N\). The total output \(Y_j\) of the final good in sector \(j = 1, 2\) is then given by the CES technology:

\[ Y_j = \left( \chi_j (n_j^I)^{\frac{1}{\rho}} + (1 - \chi_j) (n_j^H)^{\frac{1}{\rho}} \right)^{\rho - 1} \]

where \(\rho\) is the elasticity of substitution between high and low skill labour. In the limiting case when \(\rho \to 1\), this production function converges to the Cobb-Douglas technology with the power parameter \(\chi_j\). Intermediate goods are sold in a competitive market, thus intermediate goods producers are paid the marginal productivity of their goods, that is \(p_j y_{jn}^i\), where \(p_j\) is the price per unit of the final good and \(y_{jn}^i = \partial Y_j / \partial e_{jn}^i\). In a similar way the output of self-employed entrepreneurs is given by \(p_j \nu_{jn}^i\), where \(\nu_{jn}^i = \partial Y_j / \partial s_{jn}^i\).

Intermediate goods producers can open vacancies in each of the four labour markets with the corresponding up-front cost \(K_j^i\) per vacancy. Intuitively, this corresponds to the cost of capital and equipment necessary for the production. Afterwards, when the position is filled, there are no other costs than gross wages \(w_{jn}^i\). In the equilibrium, the number of vacancies \(v_j^i\) is determined at the point where the present value of expected profits is equal to the entry cost \(K_j^i\). We assume that self-employed operating in the same labour market are not able to pay the full cost of equipment, so their capital is rented. Thus self-employed entrepreneurs are incurring a flow cost of capital \(rK_j^i\). The principle advantage of being self-employed is the possibility to avoid the cost of labour as self-employed entrepreneurs are not hiring further workers. Finally, unemployed workers receive benefits \(b_{jn}^i\) which are financed from the public budget. The positive surplus of the budget remaining after the unemployment benefits are paid out is distributed as a lump-sum payment \(T\) to all agents (employed, unemployed and self-employed).

Let \(t\) denote the income tax, so that paid workers receive the flow income \(w_{jn}^i (1 - t)\) from their employers and \(T\) from the public budget. Further, let \(\pi_{jn}^i\) denote the flow profits of self-employed. These profits are classified as income for the purpose of taxation, so the net income of self-employed is then \(\pi_{jn}^i (1 - t) + T\). Unemployment benefits are not taxed. Given the total disposable income workers maximize their utility \(\Omega(c_1, c_2)\) by choosing the optimal consumption bundle \((c_1, c_2)\) subject to the budget constraint. Finally, we assume that manufactured goods are traded on international markets and their price \(p_1\) is fixed. On the contrary, service sector is non-tradable, and it’s price is endogenous in the model. In the equilibrium \(p_2\) is determined at the point where the aggregate demand for services is equal to the aggregate supply of services.

3 The Model

As discussed in the preceding section, variables \(p_j y_{jn}^i\) and \(p_j \nu_{jn}^i\) represent the nominal marginal productivity of a worker in a given origin-skill-sector group and they are equal to the price of the respective intermediate good. Given the CES production technology, the marginal productivities \(y_{jjl}^I\) and \(y_{jjh}^H\) can be found as:

\[
\begin{align*}
y_{jjl}^I &= \frac{\partial Y_j}{\partial e_{jjl}} = \left( \chi_j (n_j^I)^{\frac{1}{\rho}} + (1 - \chi_j) (n_j^H)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho - 1}} \chi_j (n_j^I)^{\frac{1}{\rho}} \\
y_{jjh}^H &= \frac{\partial Y_j}{\partial e_{jjh}} = \left( \chi_j (n_j^I)^{\frac{1}{\rho}} + (1 - \chi_j) (n_j^H)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho - 1}} (1 - \chi_j) (n_j^H)^{\frac{1}{\rho}}
\end{align*}
\]
All the remaining marginal productivities can be found as functions of \( y_{jL}^N \) or \( y_{jH}^N \). The marginal products of different worker groups are then given by:

\[
y_{jN}^L = \frac{\partial Y_j}{\partial e_{jN}^L} = \psi_{jN}^L y_{jI}^L \quad y_{jL}^N = \frac{\partial Y_j}{\partial e_{jL}^N} = \psi_{jL}^N y_{jI}^L
\]

(2)

\[
\nu_{jI}^L = \frac{\partial Y_j}{\partial s_{jI}^L} = \phi_{jI}^L y_{jI}^L \quad \nu_{jN}^L = \frac{\partial Y_j}{\partial s_{jN}^L} = \phi_{jN}^L y_{jI}^L
\]

(3)

\[
\nu_{jI}^H = \frac{\partial Y_j}{\partial s_{jI}^H} = \phi_{jI}^H y_{jI}^H \quad \nu_{jN}^H = \frac{\partial Y_j}{\partial s_{jN}^H} = \phi_{jN}^H y_{jI}^H
\]

(4)

### 3.1 Demand Side of the Economy

The pool of consumers is given by employees, unemployed agents and the self-employed. There is no possibility of saving income or transferring it over time. Let \( M_{jn}^i \) denote the net disposable income of a given consumer which can be \( w_{jn}^i (1 - t) + T, \pi_{jn}^i (1 - t) + T \) or \( b_{jn}^i + T \) depending on the labour market status of a given worker. The CES utility function \( \Omega(c_1, c_2) \) is given by:

\[
\Omega(c_1, c_2) = \left[ a_1 c_1^{\frac{\sigma - 1}{\sigma}} + a_2 c_2^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}
\]

(5)

where \( c_1 \) is consumption of good 1 (manufactured good) and \( c_2 \) is consumption of good 2 (services). Moreover, \( a_1 + a_2 = 1 \) and \( \sigma > 0 \) is the elasticity of substitution between the two goods. If \( \sigma \to \infty \) the two goods are perfect substitutes, in contrast, when \( \sigma \to 0 \), the two goods are perfect complements. Note that there are no other differences among consumers apart from their income. The utility maximization problem determines the quantity of goods demanded:

\[
\max_{c_1, c_2} \Omega(c_1, c_2) \quad \text{s.t.} \quad p_1 c_1 + p_2 c_2 = M_{jn}^i
\]

The utility maximization problem gives us the following solution:

\[
c_j(p_1, p_2, M_{jn}^i) = \frac{M_{jn}^i \left( \frac{p_2}{a_2} \right)^{-\sigma}}{p_1 \left( \frac{p_1}{a_1} \right)^{-\sigma} + p_2 \left( \frac{p_2}{a_2} \right)^{-\sigma}}
\]

(6)

It is straightforward to see that consumption \( c_j(p_1, p_2, M_{jn}^i) \) is increasing in the income \( M_{jn}^i \) but decreasing in its price \( p_j \).

The aggregate income in the economy is equal to \( p_1 Y_1 + p_2 Y_2 \) and it can be used for consumption of manufactured goods or services. Thus we get the following aggregate demand equation for services:

\[
\frac{(p_1 Y_1 + p_2 Y_2) \left( \frac{p_2}{a_2} \right)^{-\sigma}}{p_1 \left( \frac{p_1}{a_1} \right)^{-\sigma} + p_2 \left( \frac{p_2}{a_2} \right)^{-\sigma}} = Y_2 \quad \Rightarrow \quad p_2 = p_1 \left( \frac{a_2}{a_1} \right) \left( \frac{Y_2}{Y_1} \right)^{-\frac{1}{\sigma}}
\]

where the left-hand side is the aggregate demand for services and the right-hand side is the output. Since the price \( p_1 \) is fixed, we can see that a higher output of good 2 (services) has a negative effect on the price of services \( p_2 \). However, there is an opposite effect of output \( Y_1 \). This is because higher production of manufactured goods implies higher income of workers in manufacturing and higher demand for services.
In addition, note that the indirect utility of consumers \( \Omega^i_{jn} \equiv \Omega(c_1(p_1, p_2, M^i_{jn}), c_2(p_1, p_2, M^i_{jn})) \) is given by:

\[
\Omega(p_1, p_2, M^i_{jn}) = \frac{M^i_{jn}}{P} \quad \text{where} \quad P = \left[p_1 \left(\frac{p_1}{a_1}\right)^{-\sigma_1} + p_2 \left(\frac{p_2}{a_2}\right)^{-\sigma_2}\right]^{\frac{1}{1-\sigma}}
\]

Here \( P \) is the aggregate price index and it is increasing in each of the prices \( p_1 \) and \( p_2 \). Thus higher prices are associated with lower consumers’ utility \( \Omega(p_1, p_2, M^i_{jn}) \).

### 3.2 Present value equations

Let \( V^j_i \) be the present value of an open vacancy for firms operating in submarket \( i, j \). Further, \( J^i_{jN} \) is the present value of profits associated with hiring a native worker in this submarket. In a similar way, let \( J^i_{jI} \) be the present value of profits associated with hiring an immigrant worker. The present value of a given vacancy is then given by:

\[
rV^i_j = q(\theta^i_j)\left[\frac{u^i_{jN}}{u^i_j}(J^i_{jN, j} - V^i_j)h^i_{jN} + \frac{u^i_{jI}}{u^i_j}(J^i_{jI, j} - V^i_j)h^i_{jI}\right]
\]  

(7)

where \( u^i_{jI}/u^i_j \) is a probability that the firm will be matched with an unemployed immigrant worker and \( u^i_{jN}/u^i_j \) is a probability that the firm will be matched with an unemployed native worker, since \( u^i_j = u^i_{jI} + u^i_{jN} \). Also note that \( h^i_{jI} \leq 1 \) is the probability of hiring conditional on a match for the immigrant worker in submarket \( i, j \), while \( h^i_{jN} = 1 \). The present value of a filled job is given by the following equation:

\[
rJ^i_{jn} = p_jy^i_{jn} - w^i_{jn} - \delta[J^i_{jn} - V^i_j] - \alpha^i_{jn}[J^i_{jn} - V^i_j]
\]  

(8)

where \( p_jy^i_{jn} \) is the value of output produced by the worker, \( w^i_{jn} \) is the nominal gross wage reflecting the cost of labour. In addition, at rate \( \delta \) the job can be destroyed. Alternatively, at rate \( \alpha^i_{jn} \) the worker quits the job and enters the state of self-employment. The net present value of profits \( J^i_{jn} - V^i_j \) is then expressed as:

\[
J^i_{jn} - V^i_j = \frac{p_jy^i_{jn} - w^i_{jn} - rV^i_j}{r + \delta + \alpha^i_{jn}}
\]  

(9)

Recall that firms have to pay an up-front cost \( K^i_j \) of capital and equipment for entering the submarket \( i, j \). Due to the free-entry, in the equilibrium we have that \( K^i_j = V^i_j \), which is equivalent to:

\[
rK^i_j = q(\theta^i_j)\left[h^i_{jN} \frac{u^i_{jN}}{u^i_j} \left[p_jy^i_{jn} - w^i_{jn} - rK^i_j\right] + h^i_{jI} \frac{u^i_{jI}}{u^i_j} \left[p_jy^i_{jn} - w^i_{jn} - rK^i_j\right]\right]
\]  

(10)

The present value of unemployment for immigrants is given by the following equation:

\[
rU^i_{jn} = \frac{b^i_{jn} + T}{P} + h^i_{jn} \lambda(\theta^i_j)[W^i_{jn} - U^i_{jn}] + \alpha^i_{jn}[E^i_{jn} - U^i_{jn}]
\]  

(11)
where \( \Omega(p_1, p_2, b_{jn}) = (b_{jn} + T)/P \) since unemployment benefits are not taxed. \( h_{jn} \lambda(\theta_{jn}) \) is the probability of being matched and hired by the firm. \( W_{jn}^i - U_{jn}^i \) is the rent associated with the job and \( E_{jn}^i - U_{jn}^i \) is the rent associated with entering self-employment at rate \( \alpha_{jn} \). Note that this equation is indirectly based on the assumption that unemployed workers search for jobs in their primary production sector and do not switch the sector. Even though this assumption is restrictive, it is supported by our empirical findings of relatively low cross-sectional labour mobility in Germany presented in a later section of this paper. Moreover, other studies such as Nanos and Schluter (2014) also report evidence on strong occupational immobility in Germany.

The present value of paid-employment is given by the following equation

\[
rW_{jn}^i = \frac{w_{jn}^i(1-t) + T}{P} - \delta[W_{jn}^i - U_{jn}^i] + \alpha_{jn}[E_{jn}^i - W_{jn}^i] (12)
\]

where the last term is again the rent from becoming self-employed. Note that this possibility is available in both states: employment and unemployment. Moreover, value changes \([E_{jn}^i - U_{jn}^i]\) and \([E_{jn}^i - W_{jn}^i]\) are not restricted to be positive or negative. Rather we let empirical data reveal which worker groups gain by entering self-employment because the pool factors are dominating or lose value because of the dominating push factors. Some real life examples of the former case include situations when workers with innovative entrepreneurial ideas obtain funding for establishing a new start-up, which is the case of self-employment out of opportunity. Whereas examples of the latter case include cases when temporary contracts of employees or regular unemployment benefits of the unemployed expire, which leads to the self-employment out of necessity. The worker’s rent from having a job can then be expressed as:

\[
W_{jn}^i - U_{jn}^i = \frac{[w_{jn}^i(1-t) + T]/P - rU_{jn}^i + \alpha_{jn}(E_{jn}^i - W_{jn}^i)}{r + \delta + \alpha_{jn}}
\]

The present value of self-employment is given by the following equation

\[
rE_{jn}^i = \frac{\pi_{jn}^i(1-t) + T}{P} - \gamma[E_{jn}^i - U_{jn}^i] (13)
\]

where \( \pi_{jn}^i = p_j\nu_{jn}^i - rK_j^i \) is the nominal profit/income of self-employed workers and \( \gamma \) is the rate at which their business turns unsuccessful. Note that \( rK_j^i \) is the cost of renting capital for self-employed agents and \( \nu_{jn}^i \) is their productivity in terms of production units, so that \( p_j\nu_{jn}^i \) is the nominal value of output.

Finally, we can write down the government budget constraint:

\[
\sum_{n=N, l, j=1, 2, i=L, H} t(w_{jn}^i e_{jn}^i + \pi_{jn}^i u_{jn}^i) - \sum_{n=N, l, j=1, 2, i=L, H} \sum b_{jn}^i u_{jn}^i = T (14)
\]

where the first term on the left-hand side is the total revenue from income taxes. The second term corresponds to the total expenditures on unemployment benefits, while \( T \) is the remaining budget surplus which is distributed as a lump-sum payment to all economic agents. Recall that the total population size is normalized to 1.
3.3 Steady state values of unemployment and self-employment

Changes in the stocks of employed, unemployed and self-employed workers are given by:

\[
\begin{align*}
\dot{u}_{jn}^i &= \delta e_{jn}^i + \gamma s_{jn}^i - (h_{jn}^i \lambda_j^i(\theta_j^i) + \alpha_{jn}^i)u_{jn}^i = 0 \\
\dot{s}_{jn}^i &= \alpha_{jn}^i(e_{jn}^i + u_{jn}^i) - \gamma s_{jn}^i = 0 \\
\dot{e}_{jn}^i &= h_{jn}^i \lambda_j^i(\theta_j^i)u_{jn}^i - (\delta + \alpha_{jn}^i)e_{jn}^i = 0
\end{align*}
\]

where \( g_{jn}^i = u_{jn}^i + e_{jn}^i + s_{jn}^i \). Each of these equations implies that the change in the stock of workers in a given state is given by the inflow minus the outflow of workers from this state.

The focus of our research is on the steady state distribution, that is \( \dot{u}_{jn}^i = 0 \) and \( \dot{s}_{jn}^i = 0 \) which implies the following steady state distribution of workers across states:

\[
\begin{align*}
s_{jn}^i &= \frac{\alpha_{jn}^i g_{jn}^i}{\alpha_{jn}^i + \gamma} \\
u_{jn}^i &= \frac{(\delta + \alpha_{jn}^i)\gamma g_{jn}^i}{(\delta + h_{jn}^i \lambda_j^i(\theta_j^i) + \alpha_{jn}^i)(\alpha_{jn}^i + \gamma)} \\
e_{jn}^i &= g_{jn}^i - s_{jn}^i - u_{jn}^i \quad (15)
\end{align*}
\]

These equations show, for example, that a higher job-finding rate \( h_{jn}^i \lambda_j^i(\theta_j^i) \) is associated with a lower unemployment rate \( u_{jn}^i/g_{jn}^i \).

3.4 Wage determination through Nash Bargaining

Once matched, workers and firms bargain over the wage, where the net gain of the worker is given by \( W_{jn}^i - U_{jn}^i \) and the net gain of the firm is \( J_{jn}^i - V_{jn}^i \). Let the bargaining power of workers in submarket \( i, j \) be given by \( \beta_j^i \). So the bargaining problem is given by:

\[
\left( \left[ w_{jn}^i(1-t) + T \right]/P - r U_{jn}^i + \alpha_{jn}^i(E_{jn}^i - U_{jn}^i) \right)/\left( r + \delta + \alpha_{jn}^i \right) \beta_j^i \left( \frac{p_j y_{jn}^i - w_{jn}^i - r V_{jn}^i}{r + \delta + \alpha_{jn}^i} \right)^{(1-\beta_j^i)} \rightarrow \max w_{jn}^i
\]

Thus we get the following first order condition:

\[
\beta(1-t) \left( \frac{p_j y_{jn}^i - w_{jn}^i - r V_{jn}^i}{r + \delta + \alpha_{jn}^i} \right) = (1 - \beta) \left( \frac{w_{jn}^i(1-t) - b_{jn}^i}{r + \delta + \alpha_{jn}^i + h_{jn}^i \lambda_j^i(\theta_j^i)} \right)
\]

where we have inserted the flow value of unemployment \( r U_{jn}^i \). Note that the option to become self-employed doesn’t influence the outcome of bargaining since the possibility to become self-employed is not lost after entering paid employment. In a similar way, lump-sum transfers \( T \) are not distortive as all individuals are eligible for receiving these transfers from the budget in every labour market state. Taking this into account, we get the the following equilibrium wage equation\(^6\)

\[
w_{jn}^i = \frac{\beta_j^i(p_j y_{jn}^i - r V_{jn}^i)(r + \delta + h_{jn}^i \lambda_j^i(\theta_j^i) + \alpha_{jn}^i) + (1 - \beta_j^i)(r + \delta + \alpha_{jn}^i)b_{jn}^i/(1-t)}{r + \delta + \beta_j^i h_{jn}^i \lambda_j^i(\theta_j^i) + \alpha_{jn}^i} \quad (16)
\]

This equation shows that the wage is a weighted average between the net productivity \( p_j y_{jn}^i - r V_{jn}^i \) and the gross unemployment benefit \( b_{jn}^i/(1-t) \). Recall, that in the equilibrium with free-entry we have that \( K_j^i = V_j^i \), so that higher start-up costs increase the required present value of an

\(^6\)See appendix A for proof
open vacancy \( V_j^i \). However, this is only possible if the present value of profits is higher, which can be achieved with a lower wage.

### 3.5 The impact of immigration on job creation

In this subsection we explain the channels through which additional immigration influences the economy in our model. Specifically, our attention is dedicated to the reaction of the market tightness \( \theta_j^i \) in response to the rise in immigration \( g_{jI}^i \). Existing search and matching models\(^7\) emphasize the point that immigrant workers are less likely to be eligible for unemployment benefits, which weakens their bargaining position as compared to natives and leads to lower wages. Thus an inflow of new immigrant workers reduces the average wage in the host country and has a positive effect on job creation (market tightness). This effect is also present in our model. To see this consider a simplified version of the model with \( \lambda_j^i = 1 \) and let for simplicity \( p_j y_j^iN - w_j^iN = p_j y_j^iI - w_j^iI = \Pi \). The free-entry condition then becomes:

\[
\frac{r K_j^i}{q(\theta_j^i)} = \left(1 - h_{jI}^i\right)\frac{u_j^i I}{u_j^i} \frac{\Pi - r K_j^i}{r + \delta + \alpha_j^i} = \left(1 - h_{jI}^i\right)\left(\frac{\Pi - r K_j^i}{r + \delta + \alpha_j^i}\right)
\]

In a hypothetical situation when native and immigrant workers are hired with the same probability, that is \( h_{jI}^i = 1 \), the probability for a firm to be matched with an immigrant worker \( u_j^i I/u_j^i \) is equal to the population fraction of immigrants \( g_{jI}^i/(g_{jI}^i + g_{jN}^i) \). In this situation the response of the market tightness \( \theta_j^i \) to higher \( g_{jI}^i \) solely depends on the amount of firm profits generated by native and immigrant workers: \( p_j y_j^iN - w_j^iN \) vs. \( p_j y_j^iI - w_j^iI \). For example, Moreno-Galbis and Tritah (2016) show that higher immigration has a positive effect on the market-tightness \( \theta_j^i \) if immigrants are equally productive as natives but receive lower wages: \( p_j y_j^iN - w_j^iN < p_j y_j^iI - w_j^iI \). Note however that this result strongly depends on the assumption of equal employment opportunities between natives and immigrants. To see this consider the case \( h_{jI}^i < 1 \) and let for simplicity \( p_j y_j^iN - w_j^iN = p_j y_j^iI - w_j^iI = \Pi \). The free-entry condition then becomes:

\[
\frac{r K_j^i}{q(\theta_j^i)} = \left(1 - h_{jI}^i\right)\frac{u_j^i I}{u_j^i} \frac{\Pi - r K_j^i}{r + \delta + \alpha_j^i} = \left(1 - h_{jI}^i\right)\left(\frac{\Pi - r K_j^i}{r + \delta + \alpha_j^i}\right)
\]

The right-hand side of this equation represents the expected present value of profits after the match with the applicant, let it be denoted \( J(\theta_j^i, g_{jI}^i) \). It is a decreasing function of \( \theta_j^i \). This is because the unemployment rate of native workers is more sensitive to changes in the market tightness. Thus the unemployment rate of native workers falls stronger in response to a higher \( \theta_j^i \) than the unemployment rate of immigrant workers. This means that firms are more likely to be matched with immigrant workers if \( \theta_j^i \) is relatively high (formally \( u_j^i I/u_j^i \) is increasing in \( \theta_j^i \)). Since

\(^7\)Chassambouli and Palivos (2014), Battisti et al. (2014), Moreno-Galbis and Tritah (2016)
not every match with an immigrant worker leads to employment, expected profits of firms (on the right hand side) are falling in $\theta_j$. The left-hand side of this equation corresponds to the expected cost of an open vacancy. Figure 1 shows the equilibrium value of the market tightness obtained at the intersection between the decreasing expected profit curve and the increasing expected cost curve. Next consider the implications of an immigration inflow, that is a higher $g_{ji}$. Given a higher absolute number of immigrant workers, firms are even more likely to be matched with an immigrant than before, which has a negative effect on expected profits $\bar{J}(\theta_j, g_{ji})$ because a lower number of actual matches ends up in employment. If expected profits of firms fall then firms open less vacancies per unemployed worker (lower $\theta_j$), which hurts employment prospects of all worker groups. Note that we consider a hypothetical case when native and immigrant workers generate the same profits $\Pi$, thus lower employment chances of immigrants can only stem from hiring discrimination.

Figure 1: The impact of $g_{ji}$ on $\theta_j$ for identical profits from native and immigrant workers

To sum up, this section shows that an immigration inflow is likely to have a positive effect on natives’ employment chances if profits generated by immigrant workers are higher than firms’ profits associated with native workers (for example, due to lower wages of immigrant workers). On the other hand, this effect can be reversed if both worker groups generate equal profits, but immigrant workers suffer from hiring discrimination. Thus this subsection shows that hiring discrimination is likely to reduce the positive impact of immigration on native workers. To the best of our knowledge this is a new effect which was not identified in the previous search and matching literature on immigration. In the next sections we continue our analysis in a quantitative framework and allow for the endogenous responses of prices $p_j$ and productivities $y_{jn}$ which were assumed fixed throughout this subsection.

4 Calibration

To calibrate our model we use data from the German Socio-Economic Panel, wave 2008, which is based on individual surveys. Even though in 2008 financial crisis had already reached the German financial system, it did not yet spread to the German labour market. The unemployment rate fell from 9.0% down to 7.8% between the years 2007 and 2008 and went slightly up to 8.1% in
2009, thus our sample from 2008 can be seen as the last pre-crisis year for the German labour market\textsuperscript{8}. Overall, however, the labour market in Germany was largely unaffected by the financial crisis and unemployment was falling from 11.7\% in 2005 down to 6.4\% in 2015, which is partially due to the labour market reform in 2004-2005 (Hartz reforms).

Table 26 in the appendix gives the basic profile of the sample we use for our analysis. We have a sample of 11075 workers. These are the workers who are actively participating in the German labor market either as paid workers, self-employed or registered unemployed. Retired people and non-participants are excluded from our sample. There are 9854 native and 1221 immigrant workers, which corresponds to the immigration rate equal to 11\%. We use a broad definition of immigration which includes foreign citizens and those, who changed their citizenship for the German one at some point in their life. Exact definitions of all variables are presented in the appendix. High skill workers are defined as those who completed more than 12 years of schooling, which includes individuals with university and college degrees. In our sample, 28\% of immigrants and 39\% of natives are high skilled, which shows that there is a larger proportion of low skilled among immigrant workers compared to natives. The fraction of self-employed is equal to 10.8\%.

The unemployment rate in our sample is equal to 10.7\%, which is somewhat higher than the official level of 7.8\%. This is probably due to the fact that we have excluded workers in community services, marginally employed, those with zero working hours and workers on vocational training from the sample. The unemployment rate of immigrant workers is higher than the average and equal to 16.3\%. This number is close to the official statistics in 2008, according to which the unemployment rate of foreign workers in Germany was equal to 18.1\%\textsuperscript{9} Even though both unemployment rates decreased over the last years the unemployment rate of immigrant workers (14.6\%) remains more than two times higher than the average unemployment rate (6.4\%) in 2015.

Sector 1 includes such industries as agriculture, energy, mining, manufacturing, construction and trade, whereas Sector 2 includes transportation, banking, insurance and other services. 47\% of workers in our sample are assigned to Sector 1 versus 53\% of workers in Sector 2. 75\% of workers in Sector 1 are low skilled, while only 50\% are low skilled in Sector 2, which reflects the fact that mining, manufacturing and construction are low skill intensive sectors, while service occupations are balanced in terms of skills. Table 27 in the appendix shows mobility of German employees in the period 2007-2008 by sector. This transition matrix includes intermediate unemployment spells between the jobs as well as direct transitions from employment in one sector to the other. In particular this table shows that only 2.4\% of employees in sector 1 changed their sector within one year. The number is even lower for sector 2, where only 1.7\% of employees have changed their sector. Even though these numbers are not negligible they are small and indicate low cross-sectional mobility of German workers. So this evidence justifies to some extent our simplifying assumption that unemployed workers search for jobs in the same production sector as their prior employment.

\textsuperscript{8}Statistik der Bundesagentur fuer Arbeit. Arbeitslosigkeit im Zeitverlauf, March 2016
4.1 Transition rates

In this subsection we identify a vector of parameters: \(\{r, \gamma, \delta, \alpha_j^i, h_j^i, \eta, \lambda_0\}\). First, we set the annual discount rate equal to 4\%, which means \(r = 0.01\) on quarterly basis. The average job duration for employed workers is equal to 12.25 years or 49 quarters\(^{10}\). There is no significant difference for the self-employed. Thus we set \(\delta = \gamma = 1/49 \approx 0.02\). With these estimates we can calculate the values for \(\alpha_j^i, \lambda^i(\theta_j)\) and \(h_j^i\) based on equations (15). Tables 1 and 2 give the summary of the results for Manufacturing (sector 1) and Services (sector 2) respectively.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natives (g_{1N}^L/g_1 = 0.6381)</td>
<td></td>
<td></td>
<td>Natives (g_{1N}^H/g_1 = 0.2220)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_{1N}^L/g_{1N}^L)</td>
<td>0.0953</td>
<td>(\alpha_{1N}^L = 0.0021)</td>
<td>(s_{1N}^H/g_{1N}^H)</td>
<td>0.1296</td>
<td>(\alpha_{1N}^H = 0.0030)</td>
</tr>
<tr>
<td>(u_{1N}^L/g_{1N}^L)</td>
<td>0.1368</td>
<td>(\lambda(\theta_{1}^L) = 0.1241)</td>
<td>(u_{1N}^H/g_{1N}^H)</td>
<td>0.0688</td>
<td>(\lambda(\theta_{1}^H) = 0.2677)</td>
</tr>
<tr>
<td>Immigrants (g_{1I}^L/g_1 = 0.1121)</td>
<td></td>
<td></td>
<td>Immigrants (g_{1I}^H/g_1 = 0.0276)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_{1I}^L/g_{1I}^L)</td>
<td>0.0504</td>
<td>(\alpha_{1I}^L = 0.0011)</td>
<td>(s_{1I}^H/g_{1I}^H)</td>
<td>0.1414</td>
<td>(\alpha_{1I}^H = 0.0033)</td>
</tr>
<tr>
<td>(u_{1I}^L/g_{1I}^L)</td>
<td>0.1490</td>
<td>(h_{1I}^L = 0.9120)</td>
<td>(u_{1I}^H/g_{1I}^H)</td>
<td>0.1855</td>
<td>(h_{1I}^H = 0.3157)</td>
</tr>
</tbody>
</table>

Table 1: Calibration of labour turnover rates in sector 1: \(g_1 = 0.47, \delta = \gamma = 0.02\)

We can see that self-employment is more frequent for high skill workers, especially, in services. High skill natives in services are most likely to become self-employed (\(s_{2N}^H/g_{2N}^H = 0.2039\)), whereas low skill immigrants in services are least likely to become self-employed (\(s_{2I}^L/g_{2I}^L = 0.0324\)). The job-finding rate is lower in the low skill submarket compared to the high skill submarket, which is intuitive since education typically improves the chances of finding a job.

Concerning employment chances of immigrant workers, we find that the hiring probability in the low skill segment (\(h_{1I}^L = 0.9120\)) is relatively close to 1, which is the normalized value for native workers. But the situation is different in the high skill submarket, where the employment chances of immigrant workers are almost three times worse (\(h_{1I}^H = 0.3157\)) than the employment chances of native workers. The situation is even worse in sector 2, where \(h_{2I}^H = 0.2518\) indicating that immigrant workers are about four times less likely to be hired compared to native workers. Intuitively, this could be due to the larger importance of language skills in high skill jobs, more uncertainty about the quality of foreign education and/or stronger hiring discrimination.

The next step includes estimation of the matching function \(m(u_j^i, v_j^i)\). We assume a Cobb-

---

\(^{10}\) The job duration of immigrant workers is below the average and is equal to 10.25 years or 41 quarters, which corresponds to the quarterly job destruction rate of 0.024. Even though the difference between this number and 0.02 for native workers is significantly different from 0, it is small from the economic perspective as it only explains a small portion of the observed differences in unemployment rates between native and migrant workers. Thus we continue by assuming identical separation rates \(\delta\) for both ethnic groups.
Low skill workers $g^L_2/g_2 = 0.5049$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^L_{2N}/g^L_{2N}$</td>
<td>0.0730</td>
<td>$\alpha^L_{2N} = 0.0016$</td>
<td>$s^H_{2N}/g^H_{2N}$</td>
<td>0.2039</td>
<td>$\alpha^H_{2N} = 0.0051$</td>
</tr>
<tr>
<td>$u^L_{2N}/g^L_{2N}$</td>
<td>0.1315</td>
<td>$\lambda(\theta^L_2) = 0.1305$</td>
<td>$u^H_{2N}/g^H_{2N}$</td>
<td>0.0360</td>
<td>$\lambda(\theta^H_2) = 0.5304$</td>
</tr>
</tbody>
</table>

High skill workers $g^H_2/g_2 = 0.4951$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
<th>Variable</th>
<th>Target</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^L_{21}/g^L_{21}$</td>
<td>0.0324</td>
<td>$\alpha^L_{21} = 0.0007$</td>
<td>$s^H_{21}/g^H_{21}$</td>
<td>0.159</td>
<td>$\alpha^H_{21} = 0.0038$</td>
</tr>
<tr>
<td>$u^L_{21}/g^L_{21}$</td>
<td>0.2077</td>
<td>$h^L_{21} = 0.5794$</td>
<td>$u^H_{21}/g^H_{21}$</td>
<td>0.1271</td>
<td>$h^H_{21} = 0.2518$</td>
</tr>
</tbody>
</table>

Table 2: Calibration of labour turnover rates in sector 2: $g_2 = 0.53$, $\delta = \gamma = 0.02$

Douglas form for the matching function given as follows:

$$m(u^i_j, v^i_j) = \lambda_0 (u^i_j)^\eta (v^i_j)^{1-\eta}$$  \hspace{1cm} (19)

and since $q(\theta^i_j) = \frac{m(u^i_j, v^i_j)}{v^i_j}$ and $\theta^i_j = \frac{v^i_j}{u^i_j}$ we need the values of $q(\theta^i_j)$and $\theta^i_j$ to carry out our calibration. These values are later used to determine parameters $\lambda_0$ and $\eta$. For estimating these two parameters we use statistical information of the German Employment Office (2014) (Bundesagentur fuer Arbeit) for the period 2001-2013. This report has information on the absolute numbers of unemployed individuals (table 7.1), open vacancies (table 6.4) and the average duration on vacancies. Using this data we find the job-filling rate $q(\theta)$ as the inverse of the vacancy duration (expressed in quarters) and estimate the following regression:

$$\ln q(\theta_t) = \ln \lambda_0 - \eta \ln \theta_t + \epsilon_t$$  \hspace{1cm} (20)

Our data is summarized in table 28 in the appendix. It shows that the average vacancy duration in Germany increased from 46 weeks in 2001 to 79 weeks in 2013 indicating the rising shortage of qualified workers. Due to the lack of sector- and skill-specific data on vacancies and unemployment we use aggregate data for Germany to estimate the values of $\theta^i_{jn}$. This regression gives us values for $\lambda_0$ and $\eta$ equal to 0.5832 and 0.4379 respectively. In addition, figure 2 shows actual and fitted values of the regression ($R^2 = 0.7$). We can then estimate the values of $\theta^i_j$ for the four different submarkets as:

$$\theta^i_j = \left( \frac{h^i_{jn} \lambda(\theta^i_j)}{\lambda_0} \right)^{\frac{1}{1-\eta}}$$  \hspace{1cm} (21)

Given data on unemployment rates and estimated market tightness values we can also uncover the underlying values of vacancies per worker $v^i_j/g^i_j$ in every submarket. This data is summarized in the following table:
Figure 2: Actual and fitted values for the job-filling rate based on regression (20)

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low skill</td>
<td>High skill</td>
</tr>
<tr>
<td>$\theta^i_j$</td>
<td>0.0638</td>
<td>0.2501</td>
</tr>
<tr>
<td>$v^i_j / g^i_j$</td>
<td>0.0088</td>
<td>0.0204</td>
</tr>
</tbody>
</table>

Table 3: Estimated values of endogenous variables $\theta^i_j$ and $v^i_j / g^i_j$

This table shows that market tightness values for low skill workers are similar in the two production sectors, but there are large sectoral differences faced by high skill workers, with the market tightness in services being more than three times larger ($0.8512 > 0.2501$).

4.2 Wages and productivities

In this subsection we estimate a vector of parameters $\{\beta^i_j, K^i_j, b^i_{jn}, t\}$. In order to do so we run the following regression on our sample data to get the wage relationships of different groups of paid-workers.

$$\ln w_l = \phi + \phi_1 H_l + \phi_2 N_l + \phi_3 H_l \cdot N_l + \phi_4 M_l + \phi_5 H_l \cdot M_l + \phi_6 N_l \cdot M_l + \phi_7 H_l \cdot N_l \cdot M_l + \varepsilon_l$$

where $H$ is the dummy for higher education, $N$ for native, $M$ for sector 1 and $\varepsilon$ is the error term. The reference group includes low skill immigrant workers employed in services (sector 2) and earning a wage $w^L_{2I}$. Remember this regression excludes the self-employed workers. We use the coefficients obtained from this regression and get the following relations

$$w^H_{2I} = \hat{\phi}_1 w^L_{2I}$$
$$w^H_{1I} = \hat{\phi}_1 + \hat{\phi}_4 + \hat{\phi}_5) w^L_{2I}$$
$$w^L_{2N} = \hat{\phi}_2 w^L_{2I}$$
$$w^L_{1N} = \hat{\phi}_2 + \hat{\phi}_4 + \hat{\phi}_6) w^L_{2I}$$
$$w^H_{2N} = \hat{\phi}_3 w^L_{2I}$$
$$w^H_{1N} = \hat{\phi}_3 + \hat{\phi}_4 + \hat{\phi}_5 + \hat{\phi}_6 + \hat{\phi}_7) w^L_{2I}$$
We run a similar regression to estimate the flow of income among self-employed workers of different skills and origins.

\[
\ln \pi_l = \kappa + \kappa_1 H_l + \kappa_2 N_l + \kappa_3 H_l \cdot N_l + \kappa_4 M_l + \kappa_5 H_l \cdot M_l + \kappa_6 N_l \cdot M_l + \kappa_7 H_l \cdot N_l \cdot M_l + \xi_l
\]

We have the following relations from the above regression:

\[
\begin{align*}
\pi_{2l}^I &= (1 + \hat{\kappa} - \hat{\phi} + \hat{\kappa}_1)w_{2l}^I \\
\pi_{2l}^H &= (1 + \hat{\kappa} - \hat{\phi} + \hat{\kappa}_1)w_{2l}^H \\
\pi_{2N}^L &= (1 + \hat{\kappa} - \hat{\phi} + \hat{\kappa}_2)w_{2N}^L \\
\pi_{2N}^H &= (1 + \hat{\kappa} - \hat{\phi} + \hat{\kappa}_1 + \hat{\kappa}_2 + \hat{\kappa}_3)w_{2N}^H
\end{align*}
\]

\[
\begin{array}{cccc}
\text{Variable} & \text{Paid Employment} & \text{Self-employment} \\
\hline
\phi & 7.3188^{**} & 5.4306^{**} & 6.8037^{**} & 1.6244^{**} \\
\phi' & +0.4108^{**} & +0.3481^{**} & -0.9945^{*} & +1.1058^{**} \\
\hat{\kappa} & +0.2082^{**} & +0.1092^{**} & +0.6259 & +0.6105 \\
\hat{\kappa}' & -0.0030 & +0.0520 & -0.5143 & -0.6678 \\
\text{Constant} & -0.3211^{**} & +0.0713 & +1.1351 & +0.9720^{*} \\
H \cdot N & -0.2692^{**} & -0.0985 & -1.1984^{*} & -1.1288^{*} \\
\text{Manufacturing} (M) & -0.2314^{**} & -0.0438 & -0.9208^{*} & -1.0429^{*} \\
N \cdot M & -0.2160^{*} & +0.0434 & +0.8515 & +1.0327 \\
\text{Male} & +0.4761^{**} & & +0.8256^{**} & \\
\text{Age} & +0.0592^{**} & & +0.1909^{**} & \\
\text{Age squared} & -0.0007^{**} & & -0.0020^{**} & \\
\text{Tenure} & +0.0113^{**} & & +0.0296^{**} & \\
\text{West} & +0.1597^{**} & & +0.2843^{**} & \\
\text{Middle firm} (20-200 employees) & +0.2199^{**} & & +0.1995 & \\
\text{Large firm} (200-2000 employees) & +0.3383^{**} & & -0.0168 & \\
\text{Very large firm} (>2000 employees) & +0.3612^{**} & & -0.3260 & \\
R^2 & 0.0875 & 0.4094 & 0.0321 & 0.3033 \\
\text{No. of observations} & 7443 & 7350 & 848 & 803 \\
\hline
\end{array}
\]

Table 4: Regression results, GSOEP 2008. Reference categories include Low skill, Immigrant, Female workers employed in the East, in Services and Very small firms (less than 20 employees)

Parameter estimates from these regressions are given in table 4, where we use GSOEP information on gross wages and gross income of the self-employed. The first data column of this table contains estimated parameters \( \hat{\phi} \) without controlling for individual characteristics. The average skill premium is then about 41%, moreover the average wage discount of immigrant workers is about 21%. The overall log wage gap of 0.21 is in line with reports in the literature for Germany (it is 0.22 in Nanos and Schluter (2014) and 0.23 in Dustmann et al. (2010)). In addition, workers employed in manufacturing earn 32% more compared to service workers. Not surprising, these numbers are smaller when we control for individual heterogeneity of workers. For example, the sector of production (parameter \( \phi_4 \)) turns insignificant after we control for the worker’s gender. This is because male workers are overrepresented in manufacturing jobs (66%) and underrepresented in service jobs (only 42%). Thus our results indicate that there
are no significant wage differences between workers with identical characteristics (e.g. two men or two women) employed in manufacturing and services, but because of the gender wage gap (48%)\footnote{The estimate of the gender wage gap is rather large in our regression due to the fact we combined full- and part-time employees into one group for the purpose of calibration. Given that relatively many female workers are part-time employed, the gender wage gap gets smaller but remains large and significant if we condition on full-time employees.} and due to the strong selection of male workers into manufacturing jobs and female workers into service jobs there are large wage differences between the two sectors (32%). Given that the purpose of our calibration is to capture average wage differences between worker groups and production sectors rather than wage differences between observationally identical workers in these sectors, in the following we use estimates $\hat{\phi}$ and $\hat{\kappa}$ for our calibration.

Having obtained information on wages, we aim to uncover the underlying productivities of workers $p_j y_{jn}^i$. To do this we make use of the aggregate data on wages and GDP for Germany in the year 2008\footnote{The data source is DESTATIS, Statistisches Bundesamt, 2015: Fachserie 18 Reihe 1.2: Volkswirtschaftliche Gesamtrechnungen}. This data implies that the ratio between the average gross wage and GDP in manufacturing is equal to 0.4894 whereas in services it is lower and equal to 0.3913. This implies the following:

$$\frac{g_i^1 N g_i^1 w_{1N}^i}{g_i^1} + \frac{g_i^1 I g_i^1 w_{1I}^i}{g_i^1} = 0.4894 \left( \frac{g_i^1 N p_1 y_{1N}^i}{g_i^1} + \frac{g_i^1 I p_1 y_{1I}^i}{g_i^1} \right)$$

\hspace{10cm} (22)

$$\frac{g_i^2 N g_i^2 w_{2N}^i}{g_i^2} + \frac{g_i^2 I g_i^2 w_{2I}^i}{g_i^2} = 0.3913 \left( \frac{g_i^2 N p_2 y_{2N}^i}{g_i^2} + \frac{g_i^2 I p_2 y_{2I}^i}{g_i^2} \right)$$

\hspace{10cm} (23)

To estimate the remaining parameters $\beta_j^i$ and $rK_j^i$ we need data on the tax rate and the unemployment replacement rate in Germany. The average gross wage in our sample is equal to 2505 EUR, whereas the net wage after taxes and social contributions is equal to 1619 EUR. Hence we set $t = 1 - 1619/2505 = 0.35$ for all submarkets. Furthermore, we choose the net unemployment benefit replacement rate equal to 0.445, so that $b_j^1 = 0.445 w_{jn}^i (1 - t)$\footnote{DICE Database (2013), "Unemployment Benefit Replacement Rates, 1961 - 2011", Ifo Institute, Munich.}. Consider submarket $i,j$. In every submarket we have two equations for wages of immigrant and native workers given by (16), one ratio between the average wage and the GDP given above (depending on the sector and skill level of the submarket) and one free-entry condition. With these four equations we can calibrate the two parameters $\beta_j^i$ and $K_j^i$ in every submarket and two endogenous variables $p_j y_{jl}$ and $p_j y_{jN}^i$. We normalize the marginal product of low-skilled immigrant workers in services to 1, that is $p_2 y_{2l}^L = 1$. The values of parameters $\beta_j^i$ and $K_j^i$ are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low skill</td>
<td>High skill</td>
</tr>
<tr>
<td>$\beta_j^i$</td>
<td>0.8706</td>
<td>0.6480</td>
</tr>
<tr>
<td>$rK_j^i$</td>
<td>0.4678</td>
<td>0.5758</td>
</tr>
</tbody>
</table>

Table 5: Estimated parameter values for $\beta_j^i$ and $K_j^i$

Having estimated these parameters we can find values of productivities for paid workers $p_j y_{jn}^i$, gross wages $w_{jn}^i$ and gross income of the self-employed agents $p_j \pi_{jn}^i$.\footnote{DICE Database (2013), "Unemployment Benefit Replacement Rates, 1961 - 2011", Ifo Institute, Munich.}
Table 6: Estimated values of endogenous variables $p_{j}y_{jn}^{i}$, $w_{jn}^{i}$, $p_{j}\pi_{jn}^{i}$.

From the Nash bargaining solution we get the following equation for wages:

$$w_{jn}^{i}(1 - t) = \beta_{jn}^{i}(1 - t)(p_{j}y_{jn}^{i} - rV_{i}^{j}) + (1 - \beta_{jn}^{i})P(rU_{jn}^{i} - \alpha_{jn}^{i}(E_{jn}^{i} - U_{jn}^{i}))$$

From this equation one can see that $\beta_{jn}^{i}$ is the relative weight on the net marginal productivity $(1 - t)(p_{j}y_{jn}^{i} - rV_{i}^{j})$, while $1 - \beta_{jn}^{i}$ is a weight on the reservation wage of the worker $P(rU_{jn}^{i} - \alpha_{jn}^{i}(E_{jn}^{i} - U_{jn}^{i}))$ with the main component given by $rU_{jn}^{i}$. We find that $\beta_{jn}^{i}$ is relatively high for low skill workers (see table 5), but it is low for high skill workers. Intuitively, this means that outside opportunities are relatively unimportant for wages of low skill workers, but are relevant for wages of the high skilled. So far there is little empirical evidence on the skill-specific bargaining power. One exception is a study by Cahuc, Postel-Vinay and Robin (2006), who find no significant power for intermediate and low skill workers and modestly positive bargaining power for high skill workers in France. So there seem to be a contradiction between this study and our findings. To reconcile our findings we want to note the following. Even though it is intuitive to think that high skill workers possess more individual bargaining power than low skill workers, one can not neglect the fact that low skill workers are more frequently represented by trade unions, especially in Germany. So our results rather indicate the fact that collective bargaining power is larger for low skill workers and so their wages are relatively close to their marginal productivities, while individual outside opportunities are less important for low skill workers. With respect to manufacturing one can even say that search frictions are irrelevant for wages of low skill workers and can be well approximated by their marginal productivities. On the contrary, high skill workers have lower collective bargaining power since their jobs are less unionized, but they possess over moderate individual bargaining power, so that outside opportunities in terms of finding jobs are stronger reflected in their wages.

### 4.3 Production function and preference parameters

As a benchmark production function we first consider the limiting case $\rho \to 1$, so the aggregate production function takes the Cobb-Douglas form:

$$Y_{j} = \left(e_{jI}^{L} + \psi_{j}^{L}e_{jN}^{L} + \phi_{j}^{L}s_{jI}^{L} + \phi_{j}^{L}s_{jN}^{L}\right)^{\chi_{j}}\left(e_{jI}^{H} + \psi_{j}^{H}e_{jN}^{H} + \phi_{j}^{H}s_{jI}^{H} + \phi_{j}^{H}s_{jN}^{H}\right)^{1-\chi_{j}}$$

A more general case with $\rho > 1$ is treated further in section 7. Let $\Gamma_{j}$ be the ratio between the high skill intermediate input and the low skill, that is:

$$\Gamma_{j} = \frac{e_{jI}^{H} + \psi_{j}^{H}e_{jN}^{H} + \phi_{j}^{H}s_{jI}^{H} + \phi_{j}^{H}s_{jN}^{H}}{e_{jI}^{L} + \psi_{j}^{L}e_{jN}^{L} + \phi_{j}^{L}s_{jI}^{L} + \phi_{j}^{L}s_{jN}^{L}}$$

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing Low skill</th>
<th>Manufacturing High skill</th>
<th>Services Low skill</th>
<th>Services High skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{j}y_{jn}^{i}$</td>
<td>0.9390</td>
<td>0.9302</td>
<td>1.1314</td>
<td>1.1733</td>
</tr>
<tr>
<td>$w_{jn}^{i}$</td>
<td>0.4629</td>
<td>0.4548</td>
<td>0.5120</td>
<td>0.5784</td>
</tr>
<tr>
<td>$p_{j}\pi_{jn}^{i}$</td>
<td>0.5694</td>
<td>0.4662</td>
<td>0.4974</td>
<td>0.5122</td>
</tr>
</tbody>
</table>
The marginal products of different worker groups are then given by:

\[
y^L_{jI} = \chi_j (\Gamma_j)^{1-\chi_j} \\
y^H_{jI} = (1 - \chi_j)(\Gamma_j)^{-\chi_j} \\
y^L_{jN} = \chi_j \psi^L_{jN} (\Gamma_j)^{1-\chi_j} \\
y^H_{jN} = (1 - \chi_j)\psi^H_{jN} (\Gamma_j)^{-\chi_j} \\
v^L_{jn} = \chi_j \phi^L_{jn} (\Gamma_j)^{1-\chi_j} \\
v^H_{jn} = (1 - \chi_j)\phi^H_{jn} (\Gamma_j)^{-\chi_j}
\]  

(24)

(25)

(26)

In this subsection we aim to estimate a vector of remaining parameters \( \{\chi_j, \varphi^i_{jn}, \psi^i_{jN}, a_1, \sigma\} \). We start by obtaining parameters of the production function. This goal can be achieved by using equations (24)-(26) for the marginal productivities of paid workers \( p_j y^i_{jn} \) and self-employed agents \( p_j \nu^i_{jn} \). Recall that self-employed workers pay the flow cost \( rK^i_j \) depending on the submarket. Thus their gross income (before taxes) can be written as \( \pi^i_{jn} = p_j \nu^i_{jn} - rK^i_j \), which allows us to find their marginal productivity \( p_j \nu^i_{jn} \). Then we can solve the system of 16 equations (24)-(26) to identify 14 parameters \( \{\chi_j, \varphi^i_{jn}, \psi^i_{jN}\} \) and two prices \( p_1 \) and \( p_2 \). This gives us \( p_1 = 1.8395 \), \( p_2 = 2.4558 \), \( \chi_1 = 0.6973 \) and \( \chi_2 = 0.4054 \). The remaining 12 parameters are summarized in the table below:

<table>
<thead>
<tr>
<th>( \psi^i_{jN} )</th>
<th>( \varphi^i_{jn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1046</td>
<td>0.9946</td>
</tr>
<tr>
<td>0.9908</td>
<td>1.0371</td>
</tr>
</tbody>
</table>

Table 7: Estimated parameter values \( \psi^i_{jN} \) and \( \varphi^i_{jn} \).

Based on the estimated parameters of the production function and the numbers of employed workers we can find the model-implied fraction of services in the total GDP \( p_2 Y_2 / (p_1 Y_1 + p_2 Y_2) \), which is equal to 0.5949. The elasticity of substitution between manufactured goods and services \( \sigma \) we set equal to 0.3 as in Pissarides and Ngai (2004). With these parameters and the aggregate equilibrium condition \( C_2 = Y_2 \) we get:

\[
0.5949 = \frac{p_2 Y_2}{p_1 Y_1 + p_2 Y_2} = \frac{p_2 (\frac{p_2}{a_2})^{-\sigma}}{p_1 (\frac{p_2}{a_1})^{-\sigma} + p_2 (\frac{p_2}{a_2})^{-\sigma}} \quad \Rightarrow \quad a_1 = 0.3517
\]

Finally, we find that the lump-sum transfer from the budget, guaranteeing a balanced budget constraint, is equal to \( T = 0.1387 \). Table 29 in the appendix contains a full list of all parameter values that we use in the calibration and their explanations.

5 Results

5.1 Immigration inflow

In this section we analyze the effect of recent immigration on the German labour market. Figure 3 shows that the absolute number of immigrant workers was equal to 15566 thousands in 2008 which corresponds to the fraction 19\% \textsuperscript{14}. It remained virtually unchanged in the years 2009

\textsuperscript{14}This number includes foreign workers, individuals who acquired German citizenship at some point of their life as well as their children. So it is higher than the fraction of immigrants in our sample since individuals having
and 2010 but decreased sharply in 2011 reaching the level 14853 thousands. This is due to the fact that some foreign workers came to Germany in the times of financial crisis but then returned to their home countries. Since 2011 the number of immigrant workers was gradually rising and reached the level 16386 thousands in 2014. Due to the large inflow of refugees, the net inflow of immigrant workers in 2015 is equal to 1140 thousands\textsuperscript{15}. Thus comparing the lowest level in 2011 and the forecasted number for 2015 we get the rise in immigration equal to $(16386 + 1140 - 14853)/14853 = 18\%$. Given that the net immigration in 2016 is expected to be positive, we consider a 20\% increase in immigration in our analysis to include refugees and other immigrant workers coming to Germany in 2016.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Absolute numbers of immigrant workers (left scale) and their fraction as \% of the total population (right scale). Source: DESTATIS, Bevoelkerung mit Migrationshintergrund – Ergebnisse des Mikrozensus, Fachserie 1, Reihe 2.2}
\end{figure}

In terms of skills we will consider two scenarios – low skill immigration and a skill-neutral inflow of immigrants. There are two reasons why we use these two scenarios. On the one hand, reliable data on the skill composition of recent immigrants and refugees in Germany is not available yet. On the other hand, many high skill immigrants are downgraded upon arrival and apply for low skill jobs. Given this evidence (see Dustmann et al. (2013)) our two scenarios serve as a lower and an upper bound on the real effect of immigration. In both scenarios the total increase in the number of immigrant workers is set at 20\%. Given that the absolute number of immigrant workers is 1221 in our sample, in each of the two scenarios we add 244 immigrant workers to the market. In the first scenario we perform a sector-neutral increase in immigration and consider the situation when all new immigrant workers are low skilled. On the contrary, in the second scenario we perform a 20\% increase in immigration which is sector- and skill-neutral. For every scenario, we will decompose the total effect on wages and unemployment in the following way:

\textsuperscript{15}DESTATIS, Pressemitteilung Nr. 105 vom 21.03.2016

German citizenship from birth are classified as natives in our sample even if their parents had other nationality but they are classified as individuals with immigration background in the official statistics.
The effect of labour supply \( \{ u, y, v, p \} \). In this step we keep constant all productivities \( y_{jn}^i \), vacancies \( v_{jn}^i \), prices \( p_j \) and the lump-sum transfer \( T \). Thus the only effect of immigration on unemployment in this step is due to the rise in the number of searching workers. Thus we can evaluate, what will be the rise in unemployment of incumbent workers if they are facing more competition in searching for jobs. If we increase the number of immigrants with a skill level \( i \), then by construction there is no effect on workers of the opposite skill level at this step. But it is likely to worsen the outside opportunities of unemployed workers with a skill level \( i \). So we expect a negative effect on their wages.

The effect of productivity \( \{ u, y|v, p \} \). In this step we allow for a change in the marginal productivities of workers \( y_{jn}^i \), as well as wages and unemployment but we keep constant vacancies \( v_{jn}^i \), prices \( p_j \) and the lump-sum transfer \( T \). We expect that a larger number of workers with skill level \( i \) will reduce the marginal productivity of incumbent workers with this skill level and increase the marginal productivity of workers with the opposite skill level. Changes in productivities are translated into changes in wages. However, by construction, there is no additional effect on unemployment at this step since the number of vacancies remains unchanged.

The effect of labour demand \( \{ u, y, v|p \} \). At this step we allow for endogenous vacancies \( v_{jn}^i \), productivities \( y_{jn}^i \), wages and unemployment rates but we keep constant output prices \( p_j \) and the lump-sum transfer \( T \). Consider a change in the number of type \( i \) immigrants. On the one hand, if there is an increase in the number of workers, hiring becomes easier for firms, which should lead to more vacancies. This is an outward shift of the Beveridge curve. On the other hand, if employee’s productivity is falling due to immigration, this will have a negative effect on profits, the number of vacancies and the market tightness. In addition, there can be a positive spill-over from the labour market for workers of the opposite skill level since their productivity is predicted to rise. So the overall effect on the market tightness for type \( i \) workers is ambiguous.

Aggregate supply and demand \( \{ u, y, v, p \} \). The only variable which remains fixed at this step is the lump-sum transfer from the budget \( T \). All other variables change endogenously in response to an inflow of immigrant workers. On the one hand, many of the immigrant workers will find jobs which is associated with a rise in total output supply. However, this change is unlikely to be symmetric between the two sectors as low skill workers are a dominant production factor in manufacturing, whereas high skill workers are relatively more important in services. On the other hand, immigrant workers will have a positive effect on the demand of both goods, which should have a positive effect on the endogenous price \( p_2 \).

5.2 Welfare calculation

Next we calculate welfare changes associated with a new wave of immigrants. Let \( \Sigma_{jn}^i \) denote welfare of a person \( i, n \) in sector \( j \). Each of the individuals can be either employed (with probability \( e_{jn}^i / g_{jn}^i \)), unemployed (with probability \( u_{jn}^i / g_{jn}^i \)) or self-employed (with probability \( s_{jn}^i / g_{jn}^i \)). The corresponding utilities are then \( [w_{jn}^i(1 - t) + T]/P \), \([b_{jn}^i + T]/P \) and \([π_{jn}^i(1 - t)]/P \).
Hence the average welfare can be written as:

$$\Sigma_{jn} = e_{jn} [w_{jn}^i (1 - t) + T] g_{jn}^i P + \frac{u_{jn}^i [b_{jn}^i + T]}{g_{jn}^i P} + \frac{s_{jn}^i [\pi_{jn}(1 - t) + T]}{g_{jn}^i P}$$

To measure aggregate welfare effects of immigration we define two new variables $\Sigma$ and $\tilde{\Sigma}$. $\Sigma$ refers to ex-post welfare of a new representative worker based on the new distribution $g_{jn}^i$. Whereas $\tilde{\Sigma}$ refers to ex-post welfare of a representative incumbent worker keeping the distribution of workers across demographic groups $g_{jn}^i$ fixed. Given these definitions we get:

$$\Sigma = \Sigma_1^L g_{11}^L + \Sigma_1^L g_{1N}^L + \Sigma_1^N g_{11}^L + \Sigma_1^N g_{12}^H + \Sigma_2^L g_{21}^L + \Sigma_2^L g_{22}^L + \Sigma_2^N g_{21}^H + \Sigma_2^N g_{22}^H$$

$$\tilde{\Sigma} = \Sigma_1^L g_{11}^L + \Sigma_1^L g_{1N}^L + \Sigma_1^N g_{11}^L + \Sigma_1^N g_{12}^H + \Sigma_2^L g_{21}^L + \Sigma_2^L g_{22}^L + \Sigma_2^N g_{21}^H + \Sigma_2^N g_{22}^H$$

If $\Sigma^0$ denotes ex-ante welfare of a representative worker before the new wave of immigration, then the change in welfare can be calculated as $(\Sigma - \Sigma^0)/\Sigma^0$ for the new representative worker and $(\tilde{\Sigma} - \Sigma_0)/\Sigma_0$ for the representative incumbent worker. Note that these percentage changes in welfare are identical to the equivalent variation ($EV$) measure expressed as a percentage of ex-ante income. This is because indirect utility can be expressed as real income, so that:

$$\Omega(P_0, M_{jn}^{0i}(1 + EV)) = \frac{M_{jn}^{0i}(1 + EV)}{g_{jn}^i P_0} = \frac{M_{jn}^{1i}}{P_1} = \Omega(P_1, M_{jn}^{1i}) \Rightarrow EV = \frac{M_{jn}^{1i}}{M_{jn}^{0i}/P_0} - 1$$

where $M_{jn}^{0i}/P_0$ is the expected real income of worker $\{i, j, n\}$ before the change and $M_{jn}^{1i}/P_1$ is the expected real income of this worker after the change.

Next we calculate the welfare of firms in every submarket $i, j$. Here we follow the approach of Acemoglu and Shimer (1999), who calculate the welfare of firms as a total number of new hires (per period) multiplied by the present value of profits and minus the cost of capital:

$$\lambda(\theta_j)^i u_{jn}^i \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right]_{\text{PV of profits by natives}} + \lambda(\theta_j)^i h_{jn}^i \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right]_{\text{PV of profits by immigrants}} - r K_j^i v_j^i$$

The first term in the above equation is a total number of hired native workers (per period) multiplied by the present value of profits generated by these workers. The second term is a total number of hired immigrant workers (per period) multiplied with their present value of profits. Note that each of the two present values includes the capital cost $-r K_j^i$. This is intuitive, since in our model firms pay an upfront cost $K_j^i$ upon the entry, but this is equivalent to paying a flow cost $-r K_j^i$ in every period of time and in all states. Thus the flow cost $-r K_j^i$ is paid by filled jobs and open vacancies, which is the last term in the above expression. Rearranging this expression we get:

$$v_j^i \left( \frac{\lambda(\theta_j)^i u_{jn}^i}{v_j^i} \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right] + \lambda(\theta_j)^i h_{jn}^i \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right] - r K_j^i \right)$$

$$= v_j^i \left( q(\theta_j)^i u_{jn}^i \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right] + q(\theta_j)^i h_{jn}^i \left[ \frac{p_j g_j^i w_{jn}^i - w_{jn}^i - r K_j^i}{r + \delta + \alpha_j^i} \right] - r K_j^i \right) = 0$$
The free-entry condition (10) implies that firms enter submarket \( \{i, j\} \) as long as expected profits \( V_{ij} \) are higher than the cost \( K_{ij} \), thus in the equilibrium the welfare of firms producing intermediate goods is equal to zero. Finally, note that the profits of final goods producers are also zero, because they pay the marginal product for all intermediate goods.

5.3 First scenario: low skill immigration

In this subsection we analyze the implications of a 20% rise in immigration, where all 244 new immigrant workers are low skilled. To keep the rise in immigration sector-neutral we add 163 workers to the manufacturing sector and 81 workers to services, so the ratio remains \((585 + 163)/(293 + 81) \approx 2\). We conduct our analysis according to the aforementioned steps.

- In this step we analyze the implications of a change in labour supply, that is \( \{u|y,v,p\} \).
  The corresponding rows of table 9 show that there is a moderate rise in unemployment of low skill workers since they are facing more competition for a fixed number of vacancies. Immigrant low skill workers suffer a stronger increase in unemployment than native workers especially in services, where the unemployment of low skill immigrant workers in increasing from 20.73% to 21.47%. As expected worse outside opportunities are translated into lower wages of low skill workers, but the effect is negligibly small. This is due to the high bargaining power of low skill workers, which implies a much higher weight on the marginal productivity rather than outside options.

- In this step we estimate the response of productivities, that is \( \{u,y|v,p\} \). First, note that there is no direct effect on unemployment since vacancies are still fixed and the change in labour supply was already accounted for. As expected, low skill immigration leads to lower productivity of low skill workers and a higher productivity of high skill workers. These changes are translated into wages. Specifically, we find that high skill natives employed in manufacturing gain most from low skill immigration at this step. Their wages are expected to rise by 0.0266, which is approximately 4.6%.

- In the next step we allow for an endogenous response in vacancies, that is \( \{u,y,v|p\} \). Due to the higher productivity of high skill workers, firms expect higher profits and create more vacancies, this is the job-creation effect of immigration. So the unemployment of high skill workers falls, for example, the unemployment rate of high skill immigrants in manufacturing is reduced from 18.56% down to 17.85%. Due to this effect, there is also a positive spillover to the labour market for low skill workers: since there are more employed high skill workers, the drop in the productivity of the low skilled is less pronounced. So both worker types gain from a more intensive job-creation in the high skill sub-market. For example, the rise in the unemployment rate of low skill immigrant workers in manufacturing is reduced from +0.0060 in the first step down to only +0.0030 with the positive effect of job creation. The effects on wages are rather small.

- In this step we analyze the implications of aggregate supply and demand in services, that is \( \{u,y,v,p\} \). On the one hand, a higher absolute number of employees leads to more output in both production sectors. But at the same time, there is more internal demand for services since low skill workers are the majority group and their wages rise in response
to the high skill immigration. Taking both effects together we find that the price $p_2$ is increasing up to the level 2.5541. This means that the rise in total income is stronger than the rise in service-output. Higher price $p_2$ implies higher nominal productivity of workers, which amplifies the positive job creation effect from the previous step. Unemployment of all worker groups employed in services is falling compared to the pre-immigration level and their nominal wages are increasing.

Finally, we also look at the profits of self-employed workers in response to a rise in $p_2$. As expected the profits of self-employed workers in services rise with natives benefiting more than the immigrants. In manufacturing on the other hand low-skilled workers lose profits with immigrants losing about 1.9% while natives losing about 2% of their profits. High skilled workers in manufacturing seem to be gaining profits at this stage. The reason is the fact that it is mainly the low skilled workers in manufacturing who’s productivity is negatively affected by the influx of low-skilled immigrants. In services the rise in price and demand makes up for the drop in productivity but in manufacturing the price is determined on the international markets and remains unchanged, so the low-skilled workers observe a decline in profits.

<table>
<thead>
<tr>
<th>$u, y, v, p, T$</th>
<th>$p_2$</th>
<th>$\theta^L_1$</th>
<th>$\theta^H_1$</th>
<th>$\theta^L_2$</th>
<th>$\theta^H_2$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p$</td>
<td>2.554</td>
<td>0.0611</td>
<td>0.2729</td>
<td>0.0776</td>
<td>1.0502</td>
<td>0.1467</td>
</tr>
</tbody>
</table>

Table 8: Sector-specific variables, first scenario: low skill immigration

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$w^L_1I$</th>
<th>$w^L_1N$</th>
<th>$w^H_1I$</th>
<th>$w^H_1N$</th>
<th>$w^L_2I$</th>
<th>$w^L_2N$</th>
<th>$w^H_2I$</th>
<th>$w^H_2N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p$</td>
<td>0.4629</td>
<td>0.4547</td>
<td>0.5120</td>
<td>0.5784</td>
<td>0.3510</td>
<td>0.4239</td>
<td>0.4962</td>
<td>0.5684</td>
</tr>
</tbody>
</table>

Table 9: First scenario: low skill immigration. This table shows absolute changes of endogenous variables with the initial levels of each variable given in the first row.

The last rows of table 9 show changes in $\Sigma^L_{jn}$ for all demographic groups. Low skill workers in manufacturing lose welfare, since their unemployment rates are higher and their nominal wages fall. In addition, higher price $p_2$ reduces their level of consumption. Specifically, the drop in
welfare of low skill workers is equal to 3.6 – 3.7%. All other demographic groups are gaining from low skill immigration, even low skill workers employed in services because their income is positively affected by the price $p_2$. Based on the equations from section 5.2, we find that the welfare rise of a representative incumbent worker is equal to 1.13% (see table 25). However, taking into account that the share of low skill workers is increased after the change, the welfare rise of a representative new worker is equal to 0.84%. However, these numbers are not the final estimates of welfare. Indeed, there is a public surplus associated with higher wages of high skill workers and their tax contributions. In order to balance the new public budget, we increase the lump-sum transfer $T$ up to the level 0.1467. This leads to higher welfare of all demographic groups, so the average rise in welfare of incumbents is increased to 2.89% and the average welfare of a new representative worker is equal to 2.60%. This information is summarized below:

<table>
<thead>
<tr>
<th>Welfare change $(\Sigma - \Sigma^0)/\Sigma^0$</th>
<th>Unbalanced budget $(T \cdot \text{fixed})$</th>
<th>Balanced budget $(\text{new } T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(representative incumbent worker, $g_{jn}^i \cdot \text{fixed}$)</td>
<td>+1.13%</td>
<td>+2.89%</td>
</tr>
<tr>
<td>Welfare change $(\tilde{\Sigma} - \Sigma^0)/\Sigma^0$</td>
<td>+0.84%</td>
<td>+2.60%</td>
</tr>
<tr>
<td>(representative new worker, new $g_{jn}^i$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Changes in aggregate welfare, first scenario: low skill immigration

5.4 Second scenario: Skill-neutral immigration

In this subsection we analyze the implications of a skill-neutral and a sector-neutral rise in immigration. Thus we add 117 low skill immigrant workers to manufacturing, 29 high skill immigrant workers to manufacturing, 58 low skill immigrant workers to services and 40 high skill immigrant workers to services. The total rise in immigration is equal to 244 new workers which guarantees the same distribution of immigrants across the four demographic groups. Table 10 shows the results for this experiment.

- First we analyze the response of the labour market to a labour supply shock $\{u|y, v, p\}$. An increase in labour supply leads to higher unemployment and lower wages for all workers as expected. Both for unemployment and wages immigrant workers lose more than natives as a result of labour supply shock. High skilled immigrant workers in manufacturing observe the highest rise in unemployment from 0.1856 to 0.1937 while high skilled immigrant workers in services observe the highest decline in wages. The fact that wages of incumbent immigrants are adversely affected by the new inflow of immigrant workers in Germany is inline with the results of D’Amuri et al. (2010).

- Now we look at the response of productivities, $\{u, y|v, p\}$. We find that productivities of low skill workers fall, whereas the productivities of high skill workers increase, both effects being stronger in manufacturing, due to the fact that high skill workers are scarce in manufacturing. This implies that wages of low skill workers fall further. Concerning high skill workers, the drop in wages of high skill workers in services is mitigated, but the first effect of higher labour supply is still dominating at this step. The situation is different
for high skill workers in manufacturing, whose rise in productivity dominates the negative effect of labour supply, so their wages rise. High skill native workers in manufacturing gain the most as their wage increases by 0.0074 (+1.3%) while low skilled immigrant workers in this sector lose the most in terms of wages as their wage falls by 0.0031 (−0.72%). Again since we keep the number of vacancies fixed, no direct effect on unemployment is observed at this stage.

• Next we analyze the situation with an endogenous response of vacancies \( \{u, y, v|p\} \). As explained in the previous step, the productivity of high skill workers rises, which stimulates job-creation for high skill workers, especially in manufacturing. This leads to higher wages of high skill workers, moreover, the initial rise in their unemployment is almost completely neutralized at this step. At the same time, higher employment of high skill workers raises the productivity of low skill workers, so their productivity almost returns to the pre-immigration level. Thus the rise in unemployment of the low-skilled associated with larger labour supply is also almost neutralized at this step by the endogenous response in vacancies.

• Next we investigate the consequences of a price change in services \( \{u, y, v, p\} \). Job-creation leads to higher output but at the same time higher demand is also observed as the number of consumers increases due to higher number of immigrants in the economy. The higher demand clearly dominates the higher output effect and leads to an increase in the price \( p_2 \). The new price in services is \( p_2 = 2.5166 \) which is about 2.5% higher than the pre-immigration level. A higher price implies a higher nominal productivity of workers in services and this is also reflected in the wages and unemployment of workers in this sector. The wages in this sector rise while unemployment is falling.

The profits of all self-employed workers rise at this stage except low skilled workers in manufacturing. Self-employed workers in services gain from the rise in the price \( p_2 \), which increases the profits. But in manufacturing the decline in the marginal productivity of low skill workers reduces their profits from self-employment.

<table>
<thead>
<tr>
<th>( u, y, v, p )</th>
<th>( p_2 )</th>
<th>( \theta_1^L )</th>
<th>( \theta_1^H )</th>
<th>( \theta_2^L )</th>
<th>( \theta_2^H )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5166</td>
<td>0.0631</td>
<td>0.2530</td>
<td>0.0764</td>
<td>0.9526</td>
<td>0.1437</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Sector-specific variables, second scenario: skill-neutral immigration

Finally, we consider welfare changes with the unbalanced budget (that is, \( T = 0.1388 \) which is the pre-immigration level). We observe that workers in services gain from skill-neutral immigration because of the higher price \( p_2 \). Workers in manufacturing on the other hand face losses. Overall, there is a small welfare gain from 0.1021 to 0.1027 across groups with unbalanced budget, which is +0.55%. When we balance the budget then the new lump-sum transfer is equal to 0.1437. This has a positive effect on the welfare of all worker groups as all groups experience a gain in the welfare, while low skilled workers in manufacturing witness a decline in their loss compared to the unbalanced budget. Both native and immigrant workers move from −0.0017 with unbalanced budget to −0.0006 welfare loss with balanced budget. For the overall welfare with a balanced budget we observe that it is increasing up to +1.65% for new representative worker and by +1.83% for an incumbent worker.
5.5 Self-employment: Necessity or Opportunity?

In this subsection we analyze value changes of workers associated with self-employment: \( E_{jn}^i - U_{jn}^i \) and \( W_{jn}^i - E_{jn}^i \). Initially we didn’t put any restrictions on the signs of these variables, so they can be positive or negative. For example, if we find that \( W_{jn}^i - E_{jn}^i < 0 \), this indicates that workers are forced to enter entrepreneurship since their former state is not available anymore. Alternatively, if \( W_{jn}^i - E_{jn}^i > 0 \), this indicates a profitable opportunity, which workers use in order to increase their income. Value changes \( W_{jn}^i - U_{jn}^i \) and \( E_{jn}^i - U_{jn}^i \) can be found as follows:

\[
W_{jn}^i - U_{jn}^i = \frac{w_{jn}^i (1 - t) - b_{jn}^i}{P(r + \delta + h_{jn}^i \lambda(\theta_j^i) + \alpha_{jn}^i)} - E_{jn}^i - U_{jn}^i = \frac{\pi_{jn}^i (1 - t) - h_{jn}^i}{P(r + \gamma + \alpha_{jn}^i)} - h_{jn}^i \lambda(\theta_j^i) (W_{jn}^i - U_{jn}^i)
\]

Resulting present values \( U_{jn}^i \) (blue), \( E_{jn}^i \) (red) and \( W_{jn}^i \) (green) are illustrated on figure 4. First note that \( W_{jn}^i > U_{jn}^i \) for all worker groups, which means that employed workers have higher present values than unemployed workers. Consider first sector 1 (manufacturing). We find that low skill workers gain value from starting their own business, especially immigrant workers (\( E_{11}^L > W_{11}^L > U_{11}^L \)). For high skill workers the situation is somewhat different since
This means that high skill workers are generally better off in paid employment but would prefer self-employment as an alternative to unemployment. Overall, one can see that there is small heterogeneity in the present values of workers in sector 1. Next consider sector 2 (services). Here we find much stronger heterogeneity in present values, with low skill immigrants being the category with lowest values. Native workers in services prefer to be regularly employed but self-employment is better for them than unemployment. The situation is different for immigrant workers with high skilled preferring self-employment to any other state and low skilled being worst off in self-employment. This latter result, however, should be taken with care since the number of low skilled immigrant workers in services is very small in our sample, which could bias the result. Overall, one can conclude that high skill immigrant workers gain from self-employment in service occupations, while low skill immigrant workers gain from self-employment if manufacturing.

Figure 4: Present values $U^i_{jn}$, $E^i_{jn}$ and $W^i_{jn}$ for all worker groups

6 Extensions and counterfactual evidence

6.1 Equal employment opportunities

In this subsection we construct a counterfactual economy where both native and immigrant workers have equal hiring chances, formally we set $h^i_{jn} = 1$ in all submarkets. This case is important because it is possible that immigrant workers are underutilized in Germany in terms of their productive potential. So this case will allow us to estimate the effect of hiring equality on wages, prices and welfare of workers across different groups. In reality, this step could take the form of an easier access to language and integration courses, easier application process for recognizing foreign diplomas and/or anti-discrimination policies (see Kaas and Manger (2012) for an experiment on hiring discrimination in Germany). Further, we shall only consider the
model with existing immigrants and not extend it by introducing immigration shocks as the idea of this exercise is to see the implications of equal employment opportunities among natives and immigrants on labour market and welfare rather than effects of immigration shocks.

Our results with $h_{jn}^i = 1$ in all submarkets are shown in Table 15. To understand the results, note that $h_L^1 = 0.9120$, which is very close to 1, whereas all other $h_{jn}^i$ variables are rather small. This means that low skill immigrant workers have similar employment chances as native workers in manufacturing, but their chances are much lower in the other three submarkets. Hence increasing $h_{jn}^i$ up to 1 has a strong positive effect on wages of immigrant workers but a negative effect on wages of native workers in all submarkets except low skill manufacturing jobs. This is because more equal employment chances give more effective bargaining power to immigrant workers which improves their wages. But on the other hand, higher employment in a given worker group reduces the productivity of all workers in this group which leads to lower wages of native workers. The situation is different in the low skill manufacturing submarket. The productivity of these workers rises due to a higher employment of high skill workers. Thus both native and immigrant low skill workers in manufacturing gain from this policy.

Even though the effects on wages have different signs, we find that all worker groups gain in terms of lower unemployment. Unemployment of immigrant workers falls down to the same level as unemployment of native workers. But the unemployment rate of native workers also falls. This is because easier hiring is beneficial for firms and intensifies the process of job creation, so there are more vacancies, which has a positive indirect effect on the employment chances of native workers. Concerning self-employed, the corresponding row for profits shows that only low skilled self-employed workers in manufacturing gain from the considered policy regardless of their origin. All other groups of self-employed workers observe moderate profit losses.

<table>
<thead>
<tr>
<th>$p_2$</th>
<th>$\theta_l^I$</th>
<th>$\theta_H^I$</th>
<th>$\theta_L^2$</th>
<th>$\theta_H^2$</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p, T$</td>
<td>2.4469</td>
<td>0.0650</td>
<td>0.2616</td>
<td>0.0705</td>
<td>0.9052</td>
</tr>
</tbody>
</table>

Table 14: Equal employment opportunities

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$w_{1I}^L$</th>
<th>$w_{1N}^L$</th>
<th>$w_{1I}^H$</th>
<th>$w_{1N}^H$</th>
<th>$w_{2I}^L$</th>
<th>$w_{2N}^L$</th>
<th>$w_{2I}^H$</th>
<th>$w_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4629</td>
<td>0.4547</td>
<td>0.5119</td>
<td>0.5783</td>
<td>0.3510</td>
<td>0.4239</td>
<td>0.4973</td>
<td>0.5697</td>
<td></td>
</tr>
<tr>
<td>+0.0040</td>
<td>+0.0034</td>
<td>+0.0165</td>
<td>-0.0088</td>
<td>+0.0004</td>
<td>-0.0043</td>
<td>+0.0819</td>
<td>-0.0028</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$\pi_{1I}^L$</th>
<th>$\pi_{1N}^L$</th>
<th>$\pi_{1I}^H$</th>
<th>$\pi_{1N}^H$</th>
<th>$\pi_{2I}^L$</th>
<th>$\pi_{2N}^L$</th>
<th>$\pi_{2I}^H$</th>
<th>$\pi_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1490</td>
<td>0.1368</td>
<td>0.1856</td>
<td>0.0688</td>
<td>0.2073</td>
<td>0.1313</td>
<td>0.1264</td>
<td>0.0358</td>
<td></td>
</tr>
<tr>
<td>-0.0125</td>
<td>-0.0013</td>
<td>-0.1184</td>
<td>-0.0016</td>
<td>-0.0758</td>
<td>-0.0005</td>
<td>-0.0916</td>
<td>-0.0011</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$\Sigma_{1I}^L$</th>
<th>$\Sigma_{1N}^L$</th>
<th>$\Sigma_{1I}^H$</th>
<th>$\Sigma_{1N}^H$</th>
<th>$\Sigma_{2I}^L$</th>
<th>$\Sigma_{2N}^L$</th>
<th>$\Sigma_{2I}^H$</th>
<th>$\Sigma_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5694</td>
<td>0.4660</td>
<td>0.4973</td>
<td>0.5121</td>
<td>0.3916</td>
<td>0.3908</td>
<td>0.5236</td>
<td>0.5628</td>
<td></td>
</tr>
<tr>
<td>+0.0038</td>
<td>+0.0035</td>
<td>-0.0090</td>
<td>-0.0091</td>
<td>-0.0034</td>
<td>-0.0044</td>
<td>-0.0043</td>
<td>-0.0045</td>
<td></td>
</tr>
</tbody>
</table>

| $u, y, v, p | T$ | $\Sigma_{1I}^L$ | $\Sigma_{1N}^L$ | $\Sigma_{1I}^H$ | $\Sigma_{1N}^H$ | $\Sigma_{2I}^L$ | $\Sigma_{2N}^L$ | $\Sigma_{2I}^H$ | $\Sigma_{2N}^H$ |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| +0.0012 | +0.0007 | +0.0071 | -0.0009 | +0.0025 | -0.0004 | +0.0140 | -0.0001 |
| +0.0017 | +0.0012 | +0.0076 | -0.0005 | +0.0029 | 0.0000 | +0.0145 | -0.0003 |

Table 15: Equal employment opportunities. This table shows absolute changes of endogenous variables with the initial levels of each variable given in the first row.

Finally, we analyze the impact of equal opportunities policy with both unbalanced and
balanced budget. The new value for lump-sum transfers with $h_{jn}^i = 1$ is equal to $T = 0.1407$. But we shall first analyze the impact with unbalanced budget. The row for welfare with unbalanced budget $\{u, y, v, p, T\}$ shows that all native workers except low skilled in manufacturing observe very small welfare losses due to lower wages while all immigrant workers and low skilled natives in manufacturing gain in terms of welfare. The overall welfare gain prior to the fiscal adjustment is equal to $+0.5\%$. Once we allow for a balanced budget $\{u, y, v, p, T\}$ the losses that native workers observe with unbalanced budget either disappear or get lower. Low skilled natives in services, for example, observe no impact on their welfare whereas it was negative with an unbalanced budget. Similarly we notice that the welfare loss of high-skilled natives in manufacturing is reduced from $-0.0009$ to $-0.0005$. All immigrant workers, on the other hand, experience a further gain in welfare from the redistribution. The overall welfare gain with a balanced budget is equal to $+0.9\%$. This shows that productive abilities of immigrant workers are not fully utilized in Germany and there is room for a moderate welfare improvement due to the equal opportunities policy.

### 6.2 Unemployment benefits

In the benchmark model presented above we assumed that nominal unemployment benefits $b_{jn}^i$ remained fixed and unchanged after the influx of immigrants. This could be the case for long term unemployed receiving unemployment assistance, but it is not a realistic assumption for the short term unemployed, since their benefits are closely linked to their wages prior to unemployment. In this subsection we analyze the opposite extreme case, when unemployment benefits are proportional to former wages. In order to achieve this goal we set a fixed replacement rate for all worker groups equal to $z$, that is: $b_{jn}^i = zw_{jn}^i(1-t)$. With this modification equation (16) can be rewritten as:

$$w_{jn}^i = \beta_i^j(\theta_i^L - rV_i^L)(r + \delta + h_{jn}^i\lambda(\theta_i^j) + \alpha_{jn}^i)(r + \delta + \alpha_{jn}^i(1 - (1 - \beta)z) + \beta_i^jh_{jn}^i\lambda(\theta_i^j))$$

Using these equations and $z = 0.445$ produces the same benchmark equilibrium as before. However, the new equilibrium after the inflow of immigrant workers will be different, since nominal unemployment benefits are now endogenous. We focus on the case of a skill- and sector-neutral rise in immigration equal to $20\%$ to guarantee that our results are comparable with the second experiment above. These results are summarized in tables 16-17 below:

<table>
<thead>
<tr>
<th>$u, y, v, p, T$</th>
<th>$p_2$</th>
<th>$\theta_1^L$</th>
<th>$\theta_1^H$</th>
<th>$\theta_2^L$</th>
<th>$\theta_2^H$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5291</td>
<td>0.0633</td>
<td>0.2508</td>
<td>0.0743</td>
<td>0.9137</td>
<td>0.1442</td>
<td></td>
</tr>
</tbody>
</table>

Table 16: Sector-specific variables, second scenario, proportional unemployment benefits

Proportional unemployment benefits amplify any initial change in wages. For example, one can see from table 17 that all worker groups with the exception of low skill workers in sector 1, get higher nominal wages after the inflow of immigrants. This is because any increase in wages stemming from higher productivity leads to higher unemployment benefits, which improves the bargaining position of workers and reinforces the rise in wages. Higher wages are beneficial for workers but not for firms. Specifically, the market tightness of high skill workers in sector 2 ($\theta_2^H$)
Table 17: Worker-specific variables, second scenario, proportional unemployment benefits. This table shows absolute changes of endogenous variables with the initial levels of each variable given in the first row.

<table>
<thead>
<tr>
<th></th>
<th>$w^I_{1I}$</th>
<th>$w^I_{1N}$</th>
<th>$w^H_{1I}$</th>
<th>$w^H_{1N}$</th>
<th>$w^I_{2I}$</th>
<th>$w^I_{2N}$</th>
<th>$w^H_{2I}$</th>
<th>$w^H_{2N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p$</td>
<td>0.4629</td>
<td>0.4547</td>
<td>0.5120</td>
<td>0.5784</td>
<td>0.3510</td>
<td>0.4239</td>
<td>0.4962</td>
<td>0.5684</td>
</tr>
<tr>
<td></td>
<td>-0.0028</td>
<td>-0.0028</td>
<td>+0.0072</td>
<td>+0.0078</td>
<td>+0.0246</td>
<td>+0.0265</td>
<td>+0.0408</td>
<td>+0.0450</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$u^I_{1I}$</th>
<th>$u^I_{1N}$</th>
<th>$u^H_{1I}$</th>
<th>$u^H_{1N}$</th>
<th>$u^I_{2I}$</th>
<th>$u^I_{2N}$</th>
<th>$u^H_{2I}$</th>
<th>$u^H_{2N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p$</td>
<td>0.1489</td>
<td>0.1367</td>
<td>0.1856</td>
<td>0.0688</td>
<td>0.2073</td>
<td>0.1312</td>
<td>0.1264</td>
<td>0.0358</td>
</tr>
<tr>
<td></td>
<td>+0.0005</td>
<td>+0.0005</td>
<td>+0.0002</td>
<td>-0.0001</td>
<td>-0.0039</td>
<td>-0.0056</td>
<td>-0.0042</td>
<td>-0.0013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Sigma^I_{1I}$</th>
<th>$\Sigma^I_{1N}$</th>
<th>$\Sigma^H_{1I}$</th>
<th>$\Sigma^H_{1N}$</th>
<th>$\Sigma^I_{2I}$</th>
<th>$\Sigma^I_{2N}$</th>
<th>$\Sigma^H_{2I}$</th>
<th>$\Sigma^H_{2N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, y, v, p</td>
<td>T$</td>
<td>0.0977</td>
<td>0.0964</td>
<td>0.1019</td>
<td>0.1156</td>
<td>0.0787</td>
<td>0.0918</td>
<td>0.1032</td>
</tr>
<tr>
<td>$u, y, v, p, T$</td>
<td>-0.0021</td>
<td>-0.0021</td>
<td>-0.0008</td>
<td>-0.0009</td>
<td>+0.0020</td>
<td>+0.0022</td>
<td>+0.0041</td>
<td>+0.0046</td>
</tr>
<tr>
<td>&amp;u, y, v, p</td>
<td>$T$</td>
<td>-0.0009</td>
<td>-0.0008</td>
<td>+0.0005</td>
<td>+0.0004</td>
<td>+0.0033</td>
<td>+0.0035</td>
<td>+0.0054</td>
</tr>
</tbody>
</table>

Table 18: Changes in aggregate welfare, second scenario: proportional unemployment benefits

Next we consider the implications of immigration for the public budget. On the one hand, unemployment benefits rise in response to higher wages, which is an additional burden for the budget, but on the other hand, higher wages, especially in service occupations, lead to higher tax contributions and reduce the pressure on public finances. Overall, we find that both effects neutralize each other and public expenditures increase to the level $T = 0.1442$, which is only slightly more than 0.1436 in the setting with fixed unemployment benefits. Taking into account this rise in public expenditures, the overall effect from immigration is increased to 2.16% for the average incumbent worker and 1.98% for the new representative worker. Comparing 1.98% with 1.63% in the case of fixed benefits, we conclude that proportional unemployment benefits amplify the positive effect of skill- and sector-neutral immigration reported in the previous section.
7 Alternative calibrations

7.1 CES production technology

In this subsection we consider a different calibration of our model with a more general CES production function as described in section 2. Specifically, we take the elasticity of substitution between different skill groups of labour in Germany from the recent study by D’Amuri et al. (2010). They find that $1/\rho = 0.34 \ (\rho = 2.94)$, thus the Cobb-Douglas production function implies a lower elasticity of substitution between high and low skill labour compared to the one observed empirically for Germany. In order to address this issue and obtain more precise estimates for the effect of immigration, we deviate from the Cobb-Douglas specification and perform a new calibration of our model with $\rho = 2.94$. All other transition parameters $\{\gamma, \alpha, \eta, \lambda\}$, frictional parameters $\{\beta_i^j, K_i^j, b_i^j, t\}$ as well as nominal wages, productivities, profits from self-employment and public expenditures $\{w_i^j, p_j^i, y_i^j, \pi_i^j, T\}$ remain the same as in the benchmark calibration. In addition, since variables $\psi_{iN}^j$ and $\phi_{iN}^j$ reflect ratios between productivities of respective worker groups and productivities of immigrant workers in these groups, these parameters also remain unchanged. The only change in terms of model calibration refers to the two parameters of the production function $\chi_1$ and $\chi_2$, the corresponding fixed price $p_1$, as well as the share of good 1 in total consumption $a_1$. Setting, $\rho = 2.94$ and using the new productivity equations from section 3 we find the following:

<table>
<thead>
<tr>
<th>$\chi_1$</th>
<th>$\chi_2$</th>
<th>$p_1$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5389</td>
<td>0.4071</td>
<td>1.9853</td>
<td>0.3123</td>
</tr>
</tbody>
</table>

Table 19: New calibrated parameters with the CES production function

Next we set these parameter values and document the implications of a skill-neutral and sector-neutral 20% rise in immigration (second scenario) with a CES production function. Our results are documented in tables 20-21.

<table>
<thead>
<tr>
<th>$u, y, v, p, T$</th>
<th>$p_2$</th>
<th>$\theta_L^1$</th>
<th>$\theta_L^{H1}$</th>
<th>$\theta_L^{H2}$</th>
<th>$\theta_H^2$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5154</td>
<td>0.0634</td>
<td>0.2495</td>
<td>0.0776</td>
<td>0.9407</td>
<td>0.1436</td>
<td></td>
</tr>
</tbody>
</table>

Table 20: Sector-specific variables, second scenario, CES production function

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$w_{1I}^L$</th>
<th>$w_{1N}^L$</th>
<th>$w_{1I}^H$</th>
<th>$w_{1N}^H$</th>
<th>$w_{2I}^L$</th>
<th>$w_{2N}^L$</th>
<th>$w_{2I}^H$</th>
<th>$w_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4618</td>
<td>0.4546</td>
<td>0.5121</td>
<td>0.5786</td>
<td>0.3514</td>
<td>0.4245</td>
<td>0.4964</td>
<td>0.5688</td>
<td></td>
</tr>
<tr>
<td>-0.0010</td>
<td>-0.0010</td>
<td>+0.0024</td>
<td>+0.0027</td>
<td>+0.0216</td>
<td>+0.0235</td>
<td>+0.0295</td>
<td>+0.0345</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$w_{1I}^L$</th>
<th>$w_{1N}^L$</th>
<th>$w_{1I}^H$</th>
<th>$w_{1N}^H$</th>
<th>$w_{2I}^L$</th>
<th>$w_{2N}^L$</th>
<th>$w_{2I}^H$</th>
<th>$w_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1489</td>
<td>0.1367</td>
<td>0.1856</td>
<td>0.0688</td>
<td>0.2071</td>
<td>0.1311</td>
<td>0.1266</td>
<td>0.0358</td>
<td></td>
</tr>
<tr>
<td>+0.0003</td>
<td>+0.0003</td>
<td>+0.0003</td>
<td>+0.0001</td>
<td>-0.0091</td>
<td>-0.0062</td>
<td>-0.0059</td>
<td>-0.0019</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u, y, v, p, T$</th>
<th>$\Sigma_{1I}^L$</th>
<th>$\Sigma_{1N}^L$</th>
<th>$\Sigma_{1I}^H$</th>
<th>$\Sigma_{1N}^H$</th>
<th>$\Sigma_{2I}^L$</th>
<th>$\Sigma_{2N}^L$</th>
<th>$\Sigma_{2I}^H$</th>
<th>$\Sigma_{2N}^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0953</td>
<td>0.0940</td>
<td>0.0993</td>
<td>0.1128</td>
<td>0.0768</td>
<td>0.0895</td>
<td>0.1005</td>
<td>0.1140</td>
<td></td>
</tr>
<tr>
<td>-0.0015</td>
<td>-0.0015</td>
<td>-0.0011</td>
<td>-0.0013</td>
<td>+0.0017</td>
<td>+0.0019</td>
<td>+0.0027</td>
<td>+0.0033</td>
<td></td>
</tr>
<tr>
<td>-0.0004</td>
<td>-0.0004</td>
<td>-0.0000</td>
<td>-0.0002</td>
<td>+0.0028</td>
<td>+0.0030</td>
<td>+0.0038</td>
<td>+0.0045</td>
<td></td>
</tr>
</tbody>
</table>

Table 21: Worker-specific variables, second scenario, CES production function. This table shows absolute changes of endogenous variables with the initial levels of each variable given in the first row.
These tables reveal the following. If the elasticity of substitution between high and low skill workers is larger ($\rho = 2.94 > 1$) it is easier to substitute workers with different skills for each other. Table 21 shows that this change is beneficial for the low skilled and costly for the high skilled. For example, the rise in wages of all high skill workers is mitigated, whereas the rise in wages of the low skilled in services is amplified. As before, the only group of workers losing from immigration are the low skill workers employed in sector 1 (manufacturing), however the drop in their wages is weaker with a higher elasticity of substitution. When considering welfare, we can see that the increase in the price of services $p_2$ is similar to the case of a Cobb-Douglas production function. This rise in prices eliminates the welfare gain of high skill workers in manufacturing, so only workers supplying services gain from immigration in terms of welfare. Nevertheless, the overall welfare gain from immigration is positive and equal to 0.71% and 0.53% for the incumbent and new representative worker respectively when public expenditure $T$ is fixed. Allowing for the adjustment in the public budget we find that $T$ increases to the level 0.1436 due to higher taxes. This raises the total welfare gain from immigration to the level 1.82% and 1.64% depending on the reference group. Thus the overall welfare gain remains the same with the CES production technology.

<table>
<thead>
<tr>
<th>Welfare change $(\bar{\Sigma} - \Sigma^0)/\Sigma^0$</th>
<th>Unbalanced budget $(T$ - fixed)</th>
<th>Balanced budget $(new T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(representative incumbent worker; $g^i_{jn}$ - fixed)</td>
<td>+0.71%</td>
<td>+1.82%</td>
</tr>
<tr>
<td>Welfare change $(\tilde{\Sigma} - \Sigma^0/\Sigma^0)$</td>
<td>+0.53%</td>
<td>+1.64%</td>
</tr>
<tr>
<td>(representative new worker, new $g^j_{jn}$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 22: Changes in aggregate welfare, second scenario: CES production function

### 7.2 Robustness with respect to $\sigma$

In this subsection we consider another calibration of our model with $\sigma = 0.1$. Elasticity of substitution $\sigma$ is a preference parameter and so it is difficult to estimate this parameter empirically. Ngai and Pissarides (2004) conclude that [0.1...0.3] is the most realistic range for this parameter. Since $\sigma = 0.3$ is our benchmark parameter value, in this subsection we consider the lower limit and set $\sigma = 0.1$. Changing the elasticity of substitution, implies a lower value of $a_1 = 0.22$ but there are no other changes in the parameters. Table 24 shows our new results for the skill- and sector-neutral 20% increase in immigration. A rise in immigration leads to higher output on the one hand, while on the other hand a rise in demand is also observed. This leads to a stronger rise in the price for services, $p_2 = 2.5651$ as higher output does not seem to compensate for high demand so the market for services clears at a higher price. This is intuitive because $1/\sigma$ is the elasticity of the relative price $p_2/p_1$ with respect to the relative output $Y_2/Y_1$. With $\sigma = 0.1$, this elasticity is increased to 10, which means that the relative price $p_2/p_1$ becomes more sensitive to changes in the relative output $Y_2/Y_1$. A stronger increase in the price of services implies that wages of workers employed in services rise stronger compared to the benchmark case with $\sigma = 0.3$. Also unemployment of these workers falls stronger. But wages and unemployment of workers employed in manufacturing remain the same as in the benchmark case since they are
not directly affected by the price $p_2$.

\[
\begin{array}{ccccccccc}
& p_2 & \theta^L_1 & \theta^H_1 & \theta^L_2 & \theta^H_2 & T \\
u, y, v, p, T & 2.5651 & 0.0631 & 0.2530 & 0.0830 & 1.0337 & 0.1482 \\
\end{array}
\]

Table 23: Alternative calibration with $\sigma = 0.1$

<table>
<thead>
<tr>
<th>$u, y, v, p$</th>
<th>$w^L_1$</th>
<th>$w^L_2$</th>
<th>$w^H_1$</th>
<th>$w^H_2$</th>
<th>$w^L_1$</th>
<th>$w^L_2$</th>
<th>$w^H_1$</th>
<th>$w^H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0026</td>
<td>-0.0025</td>
<td>+0.0064</td>
<td>-0.0071</td>
<td>+0.0345</td>
<td>+0.0375</td>
<td>+0.0558</td>
<td>+0.0650</td>
<td></td>
</tr>
<tr>
<td>+0.0080</td>
<td>+0.0007</td>
<td>-0.0010</td>
<td>-0.0004</td>
<td>-0.0143</td>
<td>-0.0101</td>
<td>-0.0107</td>
<td>-0.0034</td>
<td></td>
</tr>
<tr>
<td>0.1490</td>
<td>0.1368</td>
<td>0.1856</td>
<td>0.0688</td>
<td>0.2069</td>
<td>0.1310</td>
<td>0.1260</td>
<td>0.0357</td>
<td></td>
</tr>
<tr>
<td>0.5694</td>
<td>0.4660</td>
<td>0.4973</td>
<td>0.5121</td>
<td>0.1732</td>
<td>0.3927</td>
<td>0.5255</td>
<td>0.5648</td>
<td></td>
</tr>
</tbody>
</table>

Table 24: Alternative calibration with $\sigma = 0.1$. This table shows absolute changes of endogenous variables with the initial levels of each variable given in the first row.

Next we analyze the effect of skill-neutral immigration on welfare in the new setting with $\sigma = 0.1$. The new value of $T$ is equal to 0.1391. The change in welfare with an unbalanced budget is given in row $\{u, y, v, p|T\}$. It appears that all workers in manufacturing lose welfare as a result of higher price $p_2$ while all workers in services gain in terms of welfare. The average welfare gain at this step is equal to +1.86%. However the increase in immigration will also change the lump-sum transfer made to each worker in the economy to balance the new budget. Workers employed in services pay higher taxes which leads to the surplus of the public budget, so that $T = 0.1482$. With this transfer all the workers experience a rise in welfare except the low skilled working in manufacturing who still observe a loss in welfare. But this loss is considerably reduced compared to the one with unbalanced budget. The high skilled natives in services benefit most as their welfare rise is given by 7.5%. The overall average gain in welfare is about +3.9% for new representative worker while +4.1% for an incumbent worker with balanced budget. This experiment shows that more sensitive prices of services produce larger welfare benefits for workers employed in this sector. So the overall welfare rise is stronger with more sensitive prices. Thus our benchmark case with a total welfare gain equal to +1.65% for a new representative worker is a conservative estimate of the skill-neutral immigration in Germany.

8 Conclusion

In this study we explored the effects of immigration on the German labour market using a search and matching framework. Within this framework we analyzed the welfare effects of immigration on consumers in Germany. We developed a model with two sectors of production – manufacturing and services – and workers heterogeneous in their skills (low or high), origin (native or immigrant)
Unbalanced budget \( (T - \text{fixed}) \) & Balanced budget \( \text{new } T \) \\
Welfare change \( (\Sigma - \Sigma^0)/\Sigma^0 \) & +2.1\% & +4.1\% \\
(representative incumbent worker, \( g_{jn}^I \) - fixed) & & \\
Welfare change \( (\bar{\Sigma} - \Sigma^0)/\Sigma^0 \) & +1.86\% & +3.9\% \\
(representative new worker, new \( g_{jn}^I \)) & & \\

Table 25: Changes in aggregate welfare, second scenario: \( \sigma = 0.1 \)

and the production sector. There are four submarkets where four types of intermediate goods are produced. In each final good sector the low and high skilled intermediate goods are combined to produce the final good using CES production technology. Our model differs from the existing literature as it includes self-employment as an alternative occupational possibility for workers. Our model also considers redistribution policy to capture the impact of immigration on the public budget. We assume that manufactured goods are traded on international markets and keep the price in manufacturing fixed. In contrast, the price in services is endogenously determined within the model since services are consumed domestically to a large extent.

For the calibration of parameters we used GSOEP data from 2008 as well as aggregate macroeconomic indicators. We simulated the model to analyze the implications of a 20% increase in immigration observed in Germany in the period 2008-2016. Our results show that the overall welfare effect of immigration is positive and estimated at +1.6\%. Considering separate groups, only low skill workers in manufacturing have a slight drop in welfare (−0.6\%). All other worker groups, especially, high skill workers in services gain from immigration. This effect is driven by higher internal demand for services, which leads to higher prices of services. The gains of workers employed in service jobs are partially redistributed through the tax system and reduce welfare losses of low skill workers in manufacturing.

We also performed several extensions of our model to analyze the effects of proportional unemployment benefits and equal employment opportunities between natives and immigrants. Our results show that unemployment benefits proportional to wages amplify the positive effect of immigration. Similarly, equal employment opportunities also enhance the positive effects of immigration by allowing to fully utilize productive abilities of immigrants. In both extensions we observe an increase in welfare compared to the benchmark case. One policy recommendation that emerges from our study is that creating equal employment opportunities among natives and immigrants could generate a welfare gain.

9 Acknowledgements

We are thankful to Christiane Clemens, David de la Croix, Herbert Dawid, Frederic Docquier and Bruno van der Linden as well as seminar participants at the University of Bielefeld and Universite Catholique de Louvain and session participants at the Belgian Day for Labour Economists and the Istanbul Search and Matching workshop for their useful comments and suggestions.
10 References


DESTATIS, Bevoelkerung mit Migrationshintergrund – Ergebnisse des Mikrozensus, Fachserie 1, Reihe 2.2.


11 Appendix

**Proof for the Nash Bargaining Solution**

The first order condition of the Nash maximization problem is given by:

\[ \beta_j^i (1 - t)(J_j^i - V_j^i) = (1 - \beta_j^i)P(W_j^i - U_j^i) \]

Inserting \( rU_j^i \) into the equation for \( W_j^i - U_j^i \) we get:

\[ W_j^i - U_j^i = \frac{[w_j^i(1 - t) - b_j^i]/P}{r + \alpha_j^i + \delta + h_j^i \lambda(\theta_j)} \]

Inserting this rent into the first order condition we get:

\[ \beta_j^i (1 - t)(p_jy_j^i - V_j^i)(r + \delta + \alpha_j^i + \delta + h_j^i \lambda(\theta_j)) = (1 - \beta_j^i) \frac{[w_j^i(1 - t) - b_j^i]/P}{r + \alpha_j^i + \delta + h_j^i \lambda(\theta_j)} \]

Rewriting this equation and expressing \( w_j^i \) gives us the final equation:

\[ w_j^i = \frac{\beta_j^i (p_jy_j^i - rV_j^i)(r + \delta + h_j^i \lambda(\theta_j)) + \alpha_j^i + (1 - \beta_j^i)(r + \delta + h_j^i \lambda(\theta_j))b_j^i}{r + \delta + \beta_j^i h_j^i \lambda(\theta_j) + \alpha_j^i} \]

**Definition of variables and data description**

Our definition of the immigrant status is based on variables \( yp137 \) (German nationality) and \( yp139 \) (German nationality since birth). Only if both variables take value 1 we classify the individual as a native. Otherwise the person is assigned an immigrant status. Thus our definition of immigrants includes individuals with foreign citizenship (1144) and those who changed their citizenship for the German one at
some later stage in their life (921). The definition of self-employment is based on variables $yp4602a$ (Self-employed farmer), $yp4602b$ (Free-Lance professional), $yp4602c$ (Other self-employed), $yp4602d$ (Family member working for relative). If one of these variables has a positive answer, the person is assigned to self-employment (1194). The definition of unemployment is based on variable $yp15$ (Registered unemployed). The individual is classified as unemployed if $yp15$ takes value 1 (1183). Our definition of employment is based on variable $yp19$ (Gainfully employed). Thus workers are classified as employed if they are full-time employed or regularly part-time employed and haven’t been classified as self-employed (8699).

Thus we exclude workers in military or community services, workers on vocational training and those with zero working hours near retirement. The skill level of workers is based on variable $ybilzeit$, which takes values between 7 and 18. Low skill workers are those with schooling less or equal to 12 years (12788), workers with more schooling years are classified as high skilled. The sector of production is based on variable $e1110608$. Sector 1 includes Agriculture, Energy, Mining, Manufacturing, Construction and Trade, whereas Sector 2 includes Transportation, Banking, Insurance and Other services. Sector information for unemployed workers is recovered from their jobs in the previous 5 years (2003-2008) based on variables $e1110607, e1110606, e1110605, e1110604, e1110603$. To avoid a possible bias we don’t delete observations with missing sector information, rather we apply the fractions of workers employed in sectors 1 and 2 respectively to all workers classified as employed, so their total number remains the same. We apply the same procedure to unemployed and self-employed workers.

<table>
<thead>
<tr>
<th>Obs</th>
<th>Low skill</th>
<th>High skill</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s^L_{jn}$</td>
<td>$e^L_{jn}$</td>
<td>$u^L_{jn}$</td>
</tr>
<tr>
<td>Native</td>
<td>317</td>
<td>2558</td>
<td>456</td>
</tr>
<tr>
<td>Immigrant</td>
<td>30</td>
<td>469</td>
<td>87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture 1</td>
<td>129</td>
<td>0</td>
</tr>
<tr>
<td>Energy 2</td>
<td>0</td>
<td>102</td>
</tr>
<tr>
<td>Mining 3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Manufacturing 4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Construction 5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Trade 6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Transport 7</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Bank, Insurance 8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other services 9</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 26: Sample Profile: Source Socio-Economic Panel 2008, Germany

Table 27: Industry transition matrix in Germany, 2007-2008
<table>
<thead>
<tr>
<th>Year</th>
<th>Unemployed workers</th>
<th>Open vacancies</th>
<th>Unemployment duration in weeks</th>
<th>Employed workers</th>
<th>Vacancy duration (in days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>3852564</td>
<td>434037</td>
<td>38.3</td>
<td>39485000</td>
<td>46</td>
</tr>
<tr>
<td>2002</td>
<td>4061345</td>
<td>374963</td>
<td>37.6</td>
<td>39257000</td>
<td>56</td>
</tr>
<tr>
<td>2003</td>
<td>4376795</td>
<td>269836</td>
<td>37.4</td>
<td>38918000</td>
<td>51</td>
</tr>
<tr>
<td>2004</td>
<td>4381281</td>
<td>206850</td>
<td>38.1</td>
<td>39034000</td>
<td>41</td>
</tr>
<tr>
<td>2005</td>
<td>4493000</td>
<td>255758</td>
<td>38.4</td>
<td>39760000</td>
<td>43</td>
</tr>
<tr>
<td>2006</td>
<td>4106697</td>
<td>354288</td>
<td>40.1</td>
<td>39192000</td>
<td>53</td>
</tr>
<tr>
<td>2007</td>
<td>3760076</td>
<td>423440</td>
<td>45.6</td>
<td>39587000</td>
<td>65</td>
</tr>
<tr>
<td>2008</td>
<td>3258453</td>
<td>389048</td>
<td>42.1</td>
<td>40348000</td>
<td>64</td>
</tr>
<tr>
<td>2009</td>
<td>3414531</td>
<td>300641</td>
<td>36.9</td>
<td>40372000</td>
<td>61</td>
</tr>
<tr>
<td>2010</td>
<td>3238421</td>
<td>359349</td>
<td>37.6</td>
<td>40587000</td>
<td>55</td>
</tr>
<tr>
<td>2011</td>
<td>2975836</td>
<td>466288</td>
<td>36.9</td>
<td>41152000</td>
<td>64</td>
</tr>
<tr>
<td>2012</td>
<td>2896985</td>
<td>477528</td>
<td>36.6</td>
<td>41080000</td>
<td>77</td>
</tr>
<tr>
<td>2013</td>
<td>2950250</td>
<td>434353</td>
<td>36.9</td>
<td>41841000</td>
<td>79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Vacancy duration (in quarters)</th>
<th>$q(\theta_t)$</th>
<th>$\theta_t = \frac{u_t}{v_t}$</th>
<th>$\ln \theta_t$</th>
<th>$\ln q(\theta_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>.5</td>
<td>2</td>
<td>.1126619</td>
<td>-2.183364</td>
<td>.6931472</td>
</tr>
<tr>
<td>2002</td>
<td>.6086956</td>
<td>1.642857</td>
<td>.0923248</td>
<td>-2.382442</td>
<td>.4964369</td>
</tr>
<tr>
<td>2003</td>
<td>.5543478</td>
<td>1.803922</td>
<td>.0616515</td>
<td>-2.786258</td>
<td>.589963</td>
</tr>
<tr>
<td>2004</td>
<td>.4456522</td>
<td>2.243902</td>
<td>.0472122</td>
<td>-3.053102</td>
<td>.8082165</td>
</tr>
<tr>
<td>2005</td>
<td>.4673913</td>
<td>2.139535</td>
<td>.0569237</td>
<td>-2.860444</td>
<td>.7605885</td>
</tr>
<tr>
<td>2006</td>
<td>.5760869</td>
<td>1.735849</td>
<td>.0862708</td>
<td>-2.450264</td>
<td>.5514967</td>
</tr>
<tr>
<td>2007</td>
<td>.7065217</td>
<td>1.415385</td>
<td>.1126147</td>
<td>-2.183783</td>
<td>.3474013</td>
</tr>
<tr>
<td>2008</td>
<td>.6956522</td>
<td>1.4375</td>
<td>.1193965</td>
<td>-2.125305</td>
<td>.3629055</td>
</tr>
<tr>
<td>2009</td>
<td>.6630435</td>
<td>1.508197</td>
<td>.0880475</td>
<td>-2.429878</td>
<td>.4109147</td>
</tr>
<tr>
<td>2010</td>
<td>.5978261</td>
<td>1.672727</td>
<td>.1109643</td>
<td>-2.198547</td>
<td>.5144554</td>
</tr>
<tr>
<td>2011</td>
<td>.6956522</td>
<td>1.4375</td>
<td>.1566914</td>
<td>-1.853477</td>
<td>.3629055</td>
</tr>
<tr>
<td>2012</td>
<td>.8369565</td>
<td>1.194805</td>
<td>.1648362</td>
<td>-1.802803</td>
<td>.1778832</td>
</tr>
<tr>
<td>2013</td>
<td>.8586956</td>
<td>1.164557</td>
<td>.1472258</td>
<td>-1.915788</td>
<td>.1523407</td>
</tr>
</tbody>
</table>

Table 29: Overview of all parameter values

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>Parameter explanation and source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Annual discount rate equal to 4% (assumption)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Average tenure of self-employed equal to 12.23 years (GSOEP)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Average tenure of employees equal to 12.25 years (GSOEP)</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Constant from the regression of $\ln q(\theta_i)$ on $\ln \theta_i$ (BA data)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Slope from the regression of $\ln q(\theta_i)$ on $\ln \theta_i$ (BA data)</td>
</tr>
<tr>
<td>$t$</td>
<td>1-ratio between the net (1619 EUR) and gross salary (2505 EUR)</td>
</tr>
<tr>
<td>$z$</td>
<td>Unemployment of immigrants</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>From equation (24) and wage $w_{1H}^I$ = 0.5119 (via $p_1y_{1H}^I$)</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>From equation (24) and wage $w_{2H}^I$ = 0.4962 (via $p_1y_{2H}^I$)</td>
</tr>
<tr>
<td>$p_1$</td>
<td>From equation (24) and normalisation $p_1y_{2H}^I = 1$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Net unemployment benefit replacement rate in 2008 (CESifo DICE)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>From Ngai and Pissarides (2004), (we also use $\sigma = 0.1$)</td>
</tr>
<tr>
<td>$a_{1N}$</td>
<td>Self-employment rates $\frac{s_{1N}}{g_{1N}} = 0.0953$ and $\frac{s_{2N}}{g_{1N}} = 0.1296$ (GSOEP)</td>
</tr>
<tr>
<td>$a_{1I}$</td>
<td>Self-employment rates $\frac{s_{1I}}{g_{1I}} = 0.0504$ and $\frac{s_{2I}}{g_{1I}} = 0.1414$ (GSOEP)</td>
</tr>
<tr>
<td>$a_{2N}$</td>
<td>Self-employment rates $\frac{s_{1N}}{g_{2N}} = 0.0730$ and $\frac{s_{2N}}{g_{2N}} = 0.2039$ (GSOEP)</td>
</tr>
<tr>
<td>$a_{2I}$</td>
<td>Self-employment rates $\frac{s_{1I}}{g_{2I}} = 0.0324$ and $\frac{s_{2I}}{g_{2I}} = 0.1590$ (GSOEP)</td>
</tr>
<tr>
<td>$h_{1I}$</td>
<td>Unemployment of immigrants $\frac{w_{1I}}{g_{1I}} = 0.1490$ and $\frac{w_{2I}}{g_{2I}} = 0.1855$ (GSOEP)</td>
</tr>
<tr>
<td>$h_{2I}$</td>
<td>Unemployment of immigrants $\frac{w_{2I}}{g_{2I}} = 0.2077$ and $\frac{w_{2I}}{g_{2I}} = 0.1271$ (GSOEP)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Gross wage income/output=0.4894 in Sector 1 (DESTATIS 2008)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Gross wage income/output=0.3913 in Sector 2 (DESTATIS 2008)</td>
</tr>
<tr>
<td>$rK_1^I$</td>
<td>Free-entry conditions (10) for $V_{1I}^L$ and $V_{1H}^I$</td>
</tr>
<tr>
<td>$rK_2^I$</td>
<td>Free-entry conditions (10) for $V_{2I}^L$ and $V_{2H}^I$</td>
</tr>
<tr>
<td>$\psi_{1N}$</td>
<td>From (25), $w_{1N}^I = 0.4547$ and $w_{1N}^H = 0.5784$ (via $p_1y_{1N}^I$ and $p_1y_{1N}^H$)</td>
</tr>
<tr>
<td>$\psi_{2N}$</td>
<td>From (25), $w_{2N}^I = 0.4239$ and $w_{2N}^H = 0.5684$ (via $p_2y_{2N}^I$ and $p_2y_{2N}^H$)</td>
</tr>
<tr>
<td>$\phi_{1N}$</td>
<td>From (26), $\pi_{1N}^I = 0.5727$ and $\pi_{1N}^H = 0.5012$ (via $p_1y_{1N}^I$ and $p_1y_{1N}^H$)</td>
</tr>
<tr>
<td>$\phi_{2N}$</td>
<td>From (26), $\pi_{2N}^I = 0.4693$ and $\pi_{2N}^H = 0.5160$ (via $p_1y_{2N}^I$ and $p_1y_{2N}^H$)</td>
</tr>
<tr>
<td>$\phi_{1I}$</td>
<td>From (26), $\pi_{1I}^I = 0.1751$ and $\pi_{2I}^H = 0.5234$ (via $p_2y_{2I}^I$ and $p_2y_{2I}^H$)</td>
</tr>
<tr>
<td>$\phi_{2I}$</td>
<td>From (26), $\pi_{2I}^I = 0.3943$ and $\pi_{2I}^H = 0.5625$ (via $p_2y_{2I}^I$ and $p_2y_{2I}^H$)</td>
</tr>
</tbody>
</table>

Table 30: Wage to GDP ratio for Germany in 2008. Source: DESTATIS, Volkswirtschaftliche Gesamtrechnungen, Fachserie 18, 1.2