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Labor Market Frictions and Monetary Policy Design

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This paper estimates a New Keynesian DSGE model with search frictions and monetary rules augmented with different labor market indicators. In accordance with a theoretical literature I find that a central bank reacts to a labor market tightness, employment or unemployment. Posterior odds tests speak in favor of models with augmented Taylor rules versus a model with a model with a standard rule. The augmented rules were also shown to be more efficient in terms of welfare.

Keywords: Search frictions, Optimal monetary policy, Bayesian estimation, Taylor rules

JEL classification: E52, E24, C11
1 Introduction

Involuntary unemployment has been long recognized as one of the main negative features of economic downturns and one of the main objectives of a stabilization economic policy. Mortensen-Pissarides (1994) encouraged a literature that used search and matching framework to explain determinants of unemployment and its cyclical movements (see a comprehensive discussion in Gali, 2010). Search and matching framework when incorporated into an otherwise standard general equilibrium model significantly improves its empirical performance (Gertler and Trigari, 2006). However, as Gali points out this class of the literature does not focus on policy analysis.

On the other hand, in New Keynesian (NK) models (see review in Walsh, 2003, Woodford, 2003 or Gali, 2008) that have been widely used for monetary policy analysis the labor market is assumed Walrasian and frictionless. There in no unemployment in these models and thus the role of labor market characteristics for the monetary policy cannot be discussed.

This paper follows the literature that integrates search and matching frictions into a NK model for monetary policy analysis. Thomas (2008), Blanchard and Gali (2009), Faia (2008), Ravenna and Walsh (2009) and Tang(2010), among others, use NK models with search frictions to address a Ramsey optimal policy. They find that an approximated policy loss-function\(^1\) contains an additional term - labor market tightness gap. Moreover, Faia (2008), Thomas (2008) and Tang (2010) found that in a model with labor market frictions a simple monetary rule (Taylor, 1993) that reacts to movements in unemployment or labor market tightness can increase welfare.

The natural question that arises is whether central banks respond to movement in labor market characteristics in reality or whether they can improve welfare by starting to closer monitor a labor market for their policy decisions. If central banks do react to labor market dynamics then economic models should also incorporate this fact into the description of the monetary policy. I argue that a standard Taylor rule in which nominal interest rate solely responds to inflation and output gap is a too simplified description of the real monetary policy design. For example, Curdia, Ferrero, Cee Ng and Tambalotti (2011) empirically estimated fifty five different monetary policy rules and found the

\(^1\)Loss function is obtained by linear-quadratic approximation as in Woodford (2003)
I address these questions in a NK model with an imperfect labor market of Tang (2010). Using Bayesian techniques I estimated different versions of the model with different augmented Taylor rules and conducted a welfare analysis. I use a Tang model to be able to compare the welfare under my estimated rules with the Ramsey optimal policy derived by Tang².

Sala, Seoderstrom and Trigari (2008) also conducted similar estimation. They considered policy rules in which monetary authority responds to unemployment gap instead of output gap and estimated the consequences of the parameter uncertainty. In contrast, in my estimation I put an additional term for labor market into the Taylor rule as prescribed by the theoretical literature. I calculate a welfare for the empirically estimated Taylor rules and compare with the theoretically optimal ones from Faia (2008) and Tang (2010).

My estimation suggests that augmented Taylor rules better describe the actual behavior of central banks. I found a small positive coefficient in front of labor market tightness gap, positive and rather large coefficient for employment and small negative coefficient for unemployment when added separately. My posterior odds ratio test indicate that models with augmented Taylor rules have a better goodness of fit than a model with a standard Taylor rule. The rule with a highest marginal likelihood is the one which targets inflation, employment gap and output gap. Using the Ramsey optimal policy as a point of comparison I showed that models with augmented Taylor rule imply lower welfare losses. The increase in welfare over the standard Taylor rule is up to 0.002% of a steady state consumption.

The rest of the paper is organized as follows. Section 2 describes the model and the market equilibrium. Section 3 discusses social planner solution and optimal monetary policy design. Section 4 presents the result of empirical estimation and welfare analysis. Section 5 concludes.

²The description of the Bayesian estimation can be found in An and Schorfheide (2007), Fernandez-Villaverde, Gueron-Quintana and F. Rubio-Ramirez (2009), Fernandez-Villaverde (2010)
2 Model Description

2.1 Representative household

In this section I briefly describe the model. Economy is populated by an infinitively lived representative household with a continuum of members of measure unity. The household maximizes the utility function (1) subject to a budget constraint and transition equation for labor. Household can change the consumption path by buying and selling nominal government bonds $B^n_t$ or by sending additional household members to work for a real wage $w_t$. The process of finding a job is, however, subject to search frictions on the labor market. All employed and unemployed household members have perfect consumption insurance and ”share the table” within a family. Total labor force is equal to the household size which is 1.

$$W_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u(c_t) - n_t v(h_t) - B_n_t]$$ (1)

c_t is consumption goods, $n_t$ is a number of employed workers in period $t$, $h_t$ is a number of hours worked per person, $v$ is a disutility from working an extra hour and $B$ denotes fixed costs of working.\(^3\) The constraints are

$$n_{t+1} = (1 - d)(n_t + \lambda_t u_t)$$ (2)

$$c_t + \frac{B^n_{t+1}}{P_t R_t} \leq \frac{B^n_t}{P_t} + w_t n_t h_t + u_t b - T_t + D_t$$ (3)

At the beginning of period $t$ unemployed worker finds a jobs with probability $\lambda_t$ and makes a match with a firm in the same period. At the end of the period workers are separated with an exogenous probability $d$. Unemployed members get unemployment benefit $b$ from the government. Household also pays lump-sum tax $T_t$ and gets profit $D_t$ from firms. $P_t$ is a price of consumption goods, $R_t$ is a gross nominal interest rate.

Optimality conditions are the standard Euler equation and a choice for number of employed family members.

$$\frac{u'(c_t)}{P_t} = \beta R_t E_t \frac{u'(c_{t+1})}{P_{t+1}}$$ (4)

$$\Omega_t^n = [w_t h_t - \frac{v(h_t) + B}{u'(c_t)} + E_t \beta_{t,t+1} [(1 - d)\Omega_{t+1}^n] - [b + E_t \beta_{t,t+1} [(1 - d)\lambda_t \Omega_{t+1}^n]$$ (5)

\(^3\)Functional forms used in the utility function are $u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$, $v(h_t) = \frac{h_t^{1+\gamma} - 1}{1+\gamma}$
When the household sends an additional family member to work it receives wage minus disutility of labor and with probability \((1-d)\), expected discounted value of employment in the next period. At the same time the household loses the unemployment benefit and future expected discounted value of potential employment \(\lambda_t n_{t+1}^n\). Here \(\beta_{t,t+1} = \frac{\beta u_{c+1}}{u(c)}\) is a stochastic discount factor and \(\Omega_t^n = \frac{d(\Omega_t)/dn}{u'(c)}\), is a value of employment (where \(\Omega_t^n\) is a Bellman function for the household problem).

### 2.2 Producers

Large number of identical producers of intermediate good operate on a competitive market. They use labor as the only input according to the production function \((6)\).

Firms post vacancies \(v_t\) at fixed costs \(\chi\) in order to find workers. A vacancy meets an unemployed person with probability \(\mu_t\) and the match becomes operative in the same period. At the end of the period share \(d\) of matches dissolves and the rest enters \(t+1\) period according to \(7\).

\[
f(n_t h_t) = A_i(n_t h_t)^{1-\phi}, \quad 0 \leq \phi < 1 \tag{6}
\]

\[
n_{t+1} = (1-d)(n_t + \mu_t v_t) \tag{7}
\]

The optimality conditions are the following:

\[
J_t = p_s^t f'(n_t h_t) h_t - w_t h_t + (1-d) E_t \beta_{t,t+1} J_{t+1} \tag{8}
\]

\[
\chi = (1-d) \mu_t E_t \beta_{t,t+1} J_{t+1} \tag{9}
\]

The first equation states that the value of an additional worker for a firm equals to a marginal product of labor times the real wholesale price \(p_s^t\) minus wage and plus the discounted value of additional worker in the next period \(J_{t+1}\) if he does not lose the job. The second arbitrage condition equalizes the cost of posting a vacancy to its potential benefit - expected value of an additional worker. Combining both optimality conditions one obtains a job-creation equation:

\[
\frac{\chi}{(1-d)\mu_t} = E_t \beta_{t,t+1} [p_s^t f'(n_{t+1} h_{t+1}) h_{t+1} - w_{t+1} h_{t+1} + (1-d) \frac{\chi}{(1-d)\mu_{t+1}}] \tag{10}
\]

\(^4\)Technology follows \(\log(A_{t+1}) = (1 - \rho_A) \log(\xi_A) + \rho_A \log(A_t) + \epsilon_{A,t+1}, \quad |\rho_A| < 1, \epsilon_{A,t+1} \sim N(0, \sigma_A^2)\)
2.3 Labor market

Unemployed workers and unfilled vacancies randomly meet each other on the labor market and matches are formed via the matching function (11) with a constant elasticity of substitution $\epsilon$ and a constant return to scale.

$$m(u_t, v_t) = \overline{m}[\xi u_t^{(\epsilon-1)/\epsilon} + (1 - \xi)v_t^{(\epsilon-1)/\epsilon}]^{\epsilon/(\epsilon-1)} \quad \overline{m} > 0, \ \xi > 0, \ \epsilon < 1 \quad (11)$$

Although all workers are identical matching frictions prevent some workers from finding a job. Search and matching process reflects the idea that worker needs to spend some time on the job search. Thus unemployed workers and unfilled vacancies can coexist on the market. Job-finding rate for a worker is determined as $\lambda_t = m(u_t, v_t)/u_t = m(1, \theta_t)$ and vacancy-filling rate $\mu_t = m(u_t, v_t)/v_t = m(1/\theta_t, 1)$, where $\mu_t = \lambda_t/\theta_t$. Variable $\theta_t = v_t/u_t$ is called a labor market tightness.

The labor market in the model suffers from search externalities. $\lambda(\theta_t)$ is increasing in $\theta_t$ since more vacancies relative to unemployed workers increases the probability of finding a job - thick-market effect. And vice versa $\mu(\theta_t)$ is decreasing in $\theta$. A vacancy is less likely to be filled when there are more vacancies and less unemployed workers on the market - congestion effect. Neither a single firm nor a single worker takes these effects into account. As long as these externalities are not internalized the market equilibrium is inefficient. Optimal policy thus has an incentive to respond to movements of the labor market tightness in order to reduce the welfare losses from the search and matching process.

2.4 Wage and hours bargaining

Wage is determined through a Nash bargaining. The firm and the worker in a match negotiate the wage and hours worked every period to maximize the total match surplus.

$$\max_{w_t, h_t} \{(\Omega^w_t - \Omega^u_t)^\varsigma (J_t - 0)^{1-\varsigma}\}$$

where $\varsigma$ is a worker bargaining power and $1 - \varsigma$ is a bargaining power of the firm. The surplus is then shared according to the worker’s and the firm’s bargaining powers $\varsigma J_t = (1 - \varsigma)\Omega^u_t$. 
All acceptable wage levels lie between the wage that brings a zero surplus for the worker and the wage that creates a zero surplus for the firm. Because of vacancy costs these two wages are not equal and therefore a non-trivial bargaining set exists (Gali, 2010). Nash bargaining solution is one particular wage level from this set which is personally efficient. Neither the worker nor the firm has an incentive to deviate from the Nash wage (Hall, 2005).

Differentiation of the above surplus function with respect to $h_t$ gives

$$p_t^sf'(n_th_t) = \frac{v'(h_t)}{u'(c_t)} \quad (12)$$

The hours worked are chosen such that a marginal rate of substitution between consumption and leisure is equal to the marginal product of labor as in the framework without labor market frictions. In other words the intensive margin is efficient despite the distortions at extensive margin.

Using $\Omega^n_t$, $J_t$ and surplus sharing rule a total wage bill can be expresses as

$$w_th_t = \varsigma[p_t^sf'(n_th_t)h_t + \chi \theta] + (1 - \varsigma) \left[ \frac{v(h_t)}{u'(c_t)} + b \right] \quad (13)$$

The wage splits the total created surplus according to the bargaining power of the sides. The higher is a firm bargaining power $(1 - \varsigma)$ the closer is the wage bill to the alternative costs of working for a household member - unemployment benefit plus saved disutility of working. When the worker has a strong bargaining position a wage bill is close to the firm’s benefit - marginal product of labor and saved vacancy posting costs.

2.5 Retailers

There are two types of retailers in the model. Intermediate retailers are indexed by $j \in [0, 1]$. They buy intermediate goods from producers for price $p_t^s$ and convert them into differentiated intermediate goods indexed by $j$. Intermediate retailers operate under a monopolistic competition and set individual prices $P_j$. Prices are adjusted every period with probability $1 - \alpha$ according to Calvo (1983).

Intermediate retailers sell their differentiated goods to a final goods retailer. He collects a continuum of intermediate goods according to Dixit-Stiglitz (1977) aggregator (14) and creates a final output (consumption basket) $y_t$ which is then sold to the household for
a final price $P_t$. The three-stage-structure of production separates a matching process on the labor market faced by producers from imperfect competition and price setting of intermediate retailers.

Final retailer decides what amount of intermediate goods $j$ to buy by solving a profit maximization problem

$$\max_{y_{jt}} P_t y_t - \int_0^1 P_j y_{jt} dj$$

s.t. $y_t = \left[ \int_0^1 y_{jt}^{(\epsilon_p - 1)/\epsilon_p} dj \right]^{\epsilon_p/(\epsilon_p - 1)}$

which leads to a demand function $y_{jt} = \left[ P_{jt} / P_t \right]^{-\epsilon_p} y_t$ and the final price index $P_t = \left[ \int_0^1 P_{jt}^{1 - \epsilon_p} dj \right]^{1/(1 - \epsilon_p)}$.

An intermediate retailer knows the demand for its goods $y_{jt}$ and decides on the price $P_{jt}$ in order to maximize an expected stream of future profits. One period profit is given by $(1 - \tau) P_{jt} T - P_{Ts} T$ where $\tau$ is a sales-tax.

$$\max_{P_{jt}} \sum_{t=T}^{\infty} \beta_t T^{t+T} \left[ (1 - \tau) \frac{P_{jt}}{P_T} - \frac{P_{Ts}}{P_T} \right] y_{Tj}$$

s.t. $y_{jt} = \left[ P_{jt} / P_t \right]^{-\epsilon_p} y_t$

The condition for the optimal intermediate price $P^*_t$ is then

$$\frac{P^*_t}{P_T} = \frac{E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} u'(c_T) y_T p_T^s (P_T / P_t)^{\epsilon_p}}{E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (1 - \Phi_y) u'(c_T) y_T (P_T / P_t)^{\epsilon_p - 1}}$$

where I used the definition of the stochastic discount factor, the fact that the optimal price is the same for all intermediate retailers $P_{jt} = P^*_t$ and expressed $p_T^s = P_{Ts} / P_T$.

$\Phi_y$ is defined by $1 - \Phi_y = (1 - \tau)(\epsilon_p - 1)/\epsilon_p$ and can be seen as a measure of distortion due to monopolistic competition. $1 - \Phi_y$ is a mark-up of an intermediate retailer adjusted for the sales tax. When prices are flexible and there is no price dispersion the following must hold: $1 - \Phi_y = p^*_t = u(h_t) / u'(c_t) f'(n_{ht})$. In the model setup it is possible if either $\epsilon_p = 1$, meaning that intermediate goods are perfect substitutes and firms have no monopolistic
power, or \( \tau = 1 \) in which case the profits are fully expropriated by the government. Thus the government is able to eliminate inefficiency from monopolistic competition if it is able to impose an appropriate sales tax.

From Calvo pricing it follows that \( (1 - \alpha) \) share of firms change their prices in period \( t \) and set it to \( P^*_t \), while \( \alpha \) of the firms leave the prices unadjusted. As in Calvo (1983) the price index is \( P_t = [(1 - \alpha)P^*_t \epsilon_p + \alpha P_{t-1} \epsilon_p]^{1/(1-\epsilon_p)} \). Using this definition one can obtain a standard Phillips curve

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \hat{p}_t^s
\]

where \( \pi_t = \hat{p}_t - \hat{p}_{t-1} \) and every hat variable denotes a log-deviation from a steady state level \( \hat{x}_t = \log x_t - \log x \). According to the Phillips curve the behavior of current inflation is determined by expectations about future inflation and by real marginal costs of production \( p_t^s \). It is important to note that producers encounter matching frictions during the hiring process. As a result labor market distortions affect the marginal costs and thus enter an aggregate supply dynamics through \( p_t^s \).

### 2.6 Government and Resource Constraint

Government budget is assumed to be balanced \( P_t u_t = P_t + \tau \int_0^1 P_{tj} y_{tj}dj = P_t + \tau P_t y_t \). Government bonds are in zero net supply in equilibrium. These two conditions together with a household budget constraint and definition of the profit define the market clearing condition for final goods (17). Defining a price dispersion measure \( \Delta_t \) the market clearing conditions for intermediate goods market is (18).

\[
y_t = c_t + \chi v_t = c_t + \chi \theta_t (1 - n_t)
\]

\[
f(n_t h_t) = \int_0^1 y_{tj}dj = y_t \int_0^1 \left( \frac{P_{tj}}{P_t} \right)^{-\epsilon_p} dj = y_t \Delta_t
\]

In the baseline model central bank follows a Taylor rule (19), where \( \epsilon_{R,t} \) is a monetary policy shock. I consider different versions of the rule in the estimation later on.

\[
\hat{R}_t = \rho_R R_{t-1} + (1 - \rho_R)[\rho_x \hat{\pi}_t + \rho_y \hat{y}] + \epsilon_{R,t}, \epsilon_{R,t} \sim N(0, \sigma^2_R)
\]

9
3 Social Optimum and Optimal Policy

An efficient equilibrium in the model can be characterized as a social planner solution. Social planner chooses a path for \( \{y_t, n_t, h_t, v_t\}_{t=0}^{\infty} \) to maximizes the utility of the representative household subject to the transition equation for labor, the feasibility constraint on the intermediate goods market, the production technology and the resource constraint for final goods.

\[
\max_{\{c_t, h_t, y_j, v_t, n_t\}} \beta^t \sum_{t=0}^{\infty} \beta^{t-t_0} \left[ \frac{c_t^{1-\sigma-1}}{1-\sigma-1} - n_t \frac{H_t^{1+\gamma}}{1+\gamma} - Bn_t \right]
\]

s.t. \( n_{t+1} = (1-d)(n_t + m(u_t, v_t)) \)

\[
\int_0^1 y_t d\mu_j \leq f(n_t h_t) \quad (\tilde{S}_t)
\]

\[
y_t = c_t + \chi v_t = \left[ \int_0^1 y_t^{\epsilon_p/(\epsilon_p-1)} d\mu_j \right]^{\epsilon_p/(\epsilon_p-1)} \quad (\tilde{\lambda}_t)
\]

which brings the following centralized versions of equilibrium conditions

\[
\int_0^1 y_t d\mu_j = f(n_t h_t) = y_t = y_{jt} \quad (20)
\]

\[
f(n_t h_t) = \frac{v'(h_t)}{u'(c_t)} \quad (21)
\]

\[
\chi = (1-d)(1-\eta_t)\mu_t E_t \beta_{t,t+1} S_{t+1} \quad (22)
\]

\[
S_t = f'(n_t h_t) h_t - \frac{v'(h_t) + B}{u'(c_t)} + (1-d) (1-\eta_t) \lambda_t E_t \beta_{t,t+1} S_{t+1} \quad (23)
\]

where \( S_t = \tilde{S}_t/w'(c_t) \), \( m'_c \) and \( m'_u \) denote derivatives of the matching function with respect to vacancies and unemployment accordingly and \( \eta_t \) is an elasticity of the matching function with respect to unemployment (see Appendix II).

Comparing (18) and (20) one can see that market equilibrium coincides with the social planner solution if price dispersion \( \Delta_t \) is 1 in all periods. In other words prices must be flexible and identical for all firms. (12) is equivalent to (20) if \( p^*_t \) is 1 in all periods. This implies that price of production is equal to the final price. In this case \( \Phi_y = 0 \) and there is no distortions due to monopolistic competition on the intermediate goods market. Conditions (9) and (22) are equivalent if \( J_{t+1} = (1-\eta_t) S_{t+1} \) which would happen if firms would take into account an elasticity of matching function. Using this equality
and substituting the values from (8), (23) and wage equation (13) one can show that the labor market is efficient if unemployment benefit is zero and a worker bargaining power is equal to the elasticity of the matching function with respect to unemployment $\eta = \varsigma$ in all periods (Hosios conditions, 1990). It means that a worker is fully compensated through the wage for positive externalities that he creates for firms. It is also possible to achieve an efficient allocation in a steady state by choosing an appropriate unemployment benefit. From (5) in a steady state

$$\Phi_\theta \equiv b - \frac{\eta - \varsigma}{1 - \varsigma} \left[ f'(nh)h + \chi \theta - \frac{v(h) + B}{w'(c)} \right]$$ (24)

If the government chooses a value of $b$ such that $\Phi_\theta = 0$, then inefficiency due to search externalities can be corrected.

To conclude, a fiscal policy can correct for search and monopolistic competition distortions in the steady state by an appropriate choice of sales tax and unemployment benefit. In is naturally to suggest that monetary policy should focus on the last friction in the model - imperfect price adjustment process. However, as Tang (2010) or Thomas (2008) showed the loss function of the optimal monetary policy takes the form:

$$L_t \approx q_\pi \pi_t^2 + q_y (\bar{y}_t - \bar{y}_t^\ast)^2 + q_\theta (\bar{\theta}_t - \bar{\theta}_t^\ast)^2$$ (25)

where $q_\pi = \frac{\epsilon_p}{\kappa}$ and $\bar{\theta}^\ast = q_{\theta n} \hat{n}_t + q_{\theta A} \hat{A}_t$ is a measure of the labor market tightness gap. Coefficients $q$ are some functions of the structural parameters of the model.

As in a standard New-Keynesian model optimal policy endures losses from inflation fluctuations (which arise from imperfect price adjustment) and losses from a non-zero output gap. In this model the loss-function contains an additional term depending on labor market tightness gap. In means that monetary policy needs to pay attention to a labor market tightness and try to keep it on some efficient level.

Note that search frictions and vacancy costs generate a deviation of output from its efficient level and this is captured by the second term of the function. Additional term of market tightness gap represents the distortions in composition of output (and therefore consumption basket). Whenever labor market tightness gap is not zero a household sends

\textsuperscript{5}If worker is undercompensated in a bargaining process he gets a positive unemployment benefit and vice versa. As a result an economy-wide value of additional employed worker is the same as a value of unemployed person.
inefficient number of its members to search for a job (Ravenna, Walsch, 2009). Labor market frictions create an additional policy trade-off between stabilizing inflation and real activities which is not present when the labor market is efficient. The additional term in the policy function can be seen as a “cost-push” shock which makes “divine coincidence impossible”. Zero inflation is no longer optimal in this set up (see Faia, 2008, Thomas, 2008 and Benigno, Woodford, 2005 among others).

I log-linearized the model (presented in the Appendix I) and simulated it after a positive technology shock to check that the model descriptive statistics are in line with the US data. The model calibration and simulation results are presented in the Appendix III.

4 Simple Rules: Empirical Evidence

One important question that arises after the discussion in the previous chapter is whether a central bank does react to labor market characteristics. John Taylor (1993) was an author of simple monetary policy rules. He showed that the behavior of federal funds rate in the US can be explained by the movements of inflation and output gap. Federal funds rate responds to changes of output with a coefficient close to 0.5, and to the changes in inflation rate with a coefficient larger then one, approximately 1.5. Strong reaction to inflation represents a Taylor principle according to which a central bank should respond to one percent increase in inflation by more than one percent increase in nominal interest rate. As a result the real interest rate which is the difference between the nominal rate and inflation increases and puts a downward pressure on inflation. Original Taylor rule was based solely on empirical evidence. Taylor (1993) simply compared the dynamics of the three described variables using graphs. Following his work a large number of papers estimated this rule with various econometric techniques, Schmitt-Grohe and Uribe (2007), Curdia, Ferrero, Cee Ng and Tambalotti (2011), Judd and Rudebush (1998), Lubik and Schorfheide (2007), Cogley et al (2011) among others. The Taylor rule appears to have a high goodness of fit. Moreover, the linear-quadratic approximation of the policy loss-function (see Woodford, 2003) includes quadratic terms of price inflation and of output gap. From the theoretical prospective a policy faces a trade-off between price stabilization and GDP stabilization. The simple Taylor rule
which includes measures of these two objectives is, therefore, well justified.

In a models with labor market imperfections the situation is different. Normative analysis shows that including labor market variables in monetary policy rule can be welfare improving. For example, Faia (2008) found that a Taylor rule which includes inflation and unemployment gap \( \hat{R}_t = 2.1838\pi_t + 0.15\hat{u}_t \) achieves the highest possible welfare. Tang (2010) investigated simple rules augmented with a labor market tightness. A simple rule with optimized coefficients \( \hat{R}_t = 2.1838\pi_t + 0.00097\hat{\theta}_t \) generates larger volatility of inflation than the strong inflation targeting but lower volatility of labor market tightness, employment and output. It thus outperforms the complete inflation stabilization rule.

I suppose that simple rules are good approximation of actual behavior of central banks. However, it is quite natural to think that there are many other variables that are omitted in this equation. It is natural to think that monetary authority takes many economic characteristics including labor market indicators into account when deciding on the interest rate.

To shed some light on this question I estimate the model with three different versions of a simple monetary rule. The rule was augmented with three labor market variables - employment, unemployment or labor market tightness. I then access the magnitude of the reaction to these labor market indicators. The search and matching inefficiency on the labor market plays a crucial role for the analysis. In a standard new Keynesian model with perfect labor market employment and output move one to one. Thus replacing output gap with an employment gap in the Tylor rule makes a small difference. In the considered model, on contrary, the labor market frictions break the link between output and labor market characteristics. The model also has a labor market tightness explicitly and allows to analyze its significance in the Taylor rule.

I use quarterly seasonally-adjusted data for the US for 1970Q1-2008Q4. My observables are: 1) output - real GDP in 2009 dollars provided by Bureau of Economic Analysis; 2) inflation measured as CPI change, 2005=1, from OECD database; 3) unemployment level data from Bureau of Labor Statistics; 4) Federal Funds Rate from Federal Reserve Bank of St. Louise database. All series (except inflation) were expressed as log deviations from the HP-trend (HP smoothing parameter \( \lambda=1600 \)).
Estimation strategy based on Bayes formula (26), where \( p(\Psi|Y^T) \) is a posterior distribution of parameters given data, \( L(Y^T|\Psi) \) is a likelihood of the data and \( p(\Psi) \) is a prior distribution.

\[
p(\Psi|Y^T) = \frac{L(Y^T|\Psi)p(\Psi)}{\int L(Y^T|\Psi)p(\Psi)\,d\Psi}
\]

The prior believes about parameter distributions are updated based on the observed data. Parameter values which are more likely to lead to the observed data values receive higher weights and vice versa low weights are assigned to unlikely parameters. State variables are estimated with a Kalman filter (Stengel, 1994, for example) and the samples from posterior distributions are obtained with Metropolis-Hastings sampler (Metropolis et al., 1953, Hastings, 1970). I use 5 different chains with 25000 draws each. The scaling parameter in a proposing distribution was adjusted such that an average acceptance ratio is 0.3. Posterior analysis of chain convergence are presented in the Appendix IV.

I fixed some of the parameters at the calibrated values\(^6\). Presumably, parameters that characterize household behavior or price decisions by firms are better identified with microdata which I do not use in the estimation.

For the parameters of the main interest - coefficients in Taylor rules - I specify loose priors centered around standard values as in Lubik and Schorfheide (2007) and Smets and Wouters (2007). For \( \rho_u \) I use a normal prior centered around zero. This prior is motivated by the theoretical evidence. In Faia (2008) an optimized monetary rule \( \hat{R}_t = 3\pi_t + 0.15\hat{u}_t \) reacts positively to unemployment, while positive reaction to labor market tightness in Tang (2010) \( \hat{R}_t = 2.18\pi_t + 0.001\hat{\theta}_t \) implies a negative response to \( \hat{u} \). Sala, Seoderstrom and Trigari (2008) estimated a rule with unemployment gap and found a negative response to unemployment: \( \hat{R}_t = 1.08\hat{R}_{t-1} + 0.2\pi_t - 0.14\hat{u}_t \). The normal prior allows for both positive and negative sign and stay agnostic on whether the coefficient is distinguishable from zero. Priors for \( \rho_n \) and \( \rho_{\theta} \) are the same as for the coefficient in front of the output gap - Gamma distributions - but shifted closer to the origin. I thus allow these coefficient to be very close to zero as in the theoretical literature. Standard errors of the both shocks are assumed to follow inverse gamma distribution with parameters 0.5, 4. All priors and estimation results are presented in the Table (1) and plots for posterior distributions can be found in Appendix IV.

\(^6\)Separation rate and Calvo parameter were adjusted to correspond to a quarterly data
### Table 1: Estimation results

<table>
<thead>
<tr>
<th>Prior</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic dataset</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>Beta (0.5,0.2)</td>
<td>0.82</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.78; 0.85]</td>
<td>[0.80; 0.87]</td>
<td>[0.81;0.89]</td>
</tr>
<tr>
<td>( \rho_\pi )</td>
<td>Gamma (1.5, 0.5)</td>
<td>1.70</td>
<td>1.92</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.30; 2.07]</td>
<td>[1.4; 2.4]</td>
<td>[1.34; 2.14]</td>
</tr>
<tr>
<td>( \rho_y )</td>
<td>Gamma (0.25, 0.13)</td>
<td>0.19</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.08; 0.29]</td>
<td>[0.05; 0.29]</td>
<td>[0.03; 0.24]</td>
</tr>
<tr>
<td>( \rho_\theta )</td>
<td>Gamma (0.15,0.13)</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.0; 0.05]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_n )</td>
<td>Gamma(0.15,0.13)</td>
<td></td>
<td></td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.01; 0.49]</td>
</tr>
<tr>
<td>( \rho_u )</td>
<td>Normal (0, 0.13)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_A )</td>
<td>InvGamma (0.5, 4)</td>
<td>0.51</td>
<td>0.71</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.46; 0.55]</td>
<td>[0.65; 0.78]</td>
<td>[0.56; 0.67]</td>
</tr>
<tr>
<td>( \sigma_R )</td>
<td>InvGamma (0.5, 4)</td>
<td>0.57</td>
<td>0.61</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.52; 0.64]</td>
<td>[0.54; 0.68]</td>
<td>[0.44; 0.60]</td>
</tr>
</tbody>
</table>

*Estimation of different model specification with one of the following Taylor rules: 1) \( \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y}] \) 2) \( \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y} + \rho_\theta \hat{\theta}_t] \) 3) \( \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y} + \rho_n \hat{n}_t] \) 4) \( \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y} + \rho_u \hat{u}_t] \). Means and 90% confidence intervals are presented.

The estimation of the standard simple rule gives the following result

\[
\hat{R}_t = 0.8\hat{R}_{t-1} + (1 - 0.8)[1.7\pi + 0.2\hat{y}]
\]  

(27)

This estimated standard Taylor rule shows that monetary authority stabilizes inflation and "leans against the wind" by raising interest rate when the output is growing faster than a target level. All parameter values are close to their conventional values and the rule respects Taylor principle.
Table 1 presents the results for the estimation of different Taylor rule specifications. Column II shows the coefficients for the rule which additionally includes reaction to deviations of the labor market tightness. The coefficient before inflation is close to the the standard value. The coefficient in front of the output gap becomes smaller because part of the movements of the interest rate is attributed to the reaction to the labor market tightness. The coefficient for $\theta$ is positive and an order of magnitude smaller than the rest of the coefficient in accordance with theoretical analysis. Tang (2010) computed an optimal Taylor rule for the considered model given that the steady state is efficient\(^7\) using a numerical optimization. The resulted Taylor rule $\hat{R}_t = 2.18\pi_t + 0.001\hat{\theta}_t$ is associated with lower welfare losses than a complete inflation stabilization. In order to increase a welfare central bank should react to labor market tightness with a coefficient $10^{-3}$ order of magnitude smaller that the coefficient for inflation. According to my estimated rule the central bank responds to both output and labor market tightness. The coefficient for the labor market tightness gap is one order of magnitude smaller that the one for inflation.

An estimation of the the monetary rule specification with reaction to employment deviations is presented in column III. Coefficient for output gap becomes smaller and coefficient for employment gap is positive and large: 0.26. This rule indicates that monetary authority responds stronger to employment gap than to the output gap. However, the confidence interval for the coefficient in front of employment is rather wide and the coefficient might be very close to zero.

The last column IV shows the coefficients for the specification with unemployment gap. This rule is closer to the strong inflation targeting as the coefficient for inflation increases to 2.5. Coefficient for unemployment is strongly negative: -0.21. Unemployment level can be seen as a separate goal in a policy rule. The results suggest that reaction to unemployment fluctuations is as stronger that to output gap.\(^8\). To sum up, the estimation suggests that monetary authorities target labor market characteristics along the output gap.

\(^7\)All market frictions are neutralized in a steady state by means of sales tax and unemployment benefit

\(^8\)As a robustness check I included the data on vacancy posting index of The Conference Board and recomputed all the results. For the alternative dataset the coefficient in front of unemployment is smaller than the coefficient for the output gap. The rest of the results stays unaffected.
Table (2) presents posterior odds ratios for three different model specifications each with a different augmented Taylor rule. Posterior odds ratio (28) shows a relative probability of a particular model to be a true model. All specifications with augmented Taylor rules are compared to a specification with a standard Taylor rule \( \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_r \pi + \rho_y \hat{y}] \).

\[
\frac{\pi_{M1}}{\pi_{basic}} = \frac{\pi_{M1}^0}{\pi_{basic}^0} \frac{\int \mathcal{L}(Y^T|\Psi, M_1)p(\Psi|M_1)d\Psi}{\int \mathcal{L}(Y^T|\Psi, basic)p(\Psi|basic)d\Psi}
\] (28)

The first multiplier is a prior probability of a particular model to be the right one relatively to a basic model (prior odds ratio). The second term is a ratio of marginal data densities for corresponding models (Bayes factor). Prior odds ratio were set to 1 meaning that both model specifications are a priori equally probable.

In all cases models with augmented rules are more likely than a model with a standard monetary rule (Table 2). The rules with unemployment gap or employment gap are almost 25 times more likely to describe the true behavior of the central bank. Model with the a Taylor rule augmented with labor market tightness is 21 times more likely than a model with a standard rule.

<table>
<thead>
<tr>
<th>Policy rule</th>
<th>Log-Marginal Density</th>
<th>Log Bayes factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{R}<em>t = \rho_r \hat{R}</em>{t-1} + (1 - \rho_r)[\rho_r \pi + \rho_y \hat{y}] )</td>
<td>-217.947</td>
<td>0</td>
</tr>
<tr>
<td>( \hat{R}<em>t = \rho_r \hat{R}</em>{t-1} + (1 - \rho_r)[\rho_r \pi + \rho_y \hat{y} + \rho_\theta \hat{\theta}] )</td>
<td>-196.943</td>
<td>21.00</td>
</tr>
<tr>
<td>( \hat{R}<em>t = \rho_r \hat{R}</em>{t-1} + (1 - \rho_r)[\rho_r \pi + \rho_y \hat{y}] + \rho_n \hat{n}_t )</td>
<td>-192.905</td>
<td>25.04</td>
</tr>
<tr>
<td>( \hat{R}<em>t = \rho_r \hat{R}</em>{t-1} + (1 - \rho_r)[\rho_r \pi + \rho_y \hat{y}] + \rho_u \hat{u}_t )</td>
<td>-192.896</td>
<td>25.05</td>
</tr>
</tbody>
</table>

Model with augmented rules are compared to the model with a standard rule with prior odds ratio equal one.

Based on posterior odds ratio test one can conclude that a central bank indeed reacts to changes in labor market characteristics. In a model with imperfect labor market an augmented monetary rule improves the model’s goodness of fit.

Finally, using the loss function (30) I calculate welfare losses under an optimal policy design and under different alternative monetary rules. Welfare losses are expressed as a percentage of a steady state consumption needed to compensate a household according
to (29).

\[
\sum_{t=0}^{\infty} \beta^t [u((1 + \mathcal{W})c) - u(c)] = \frac{1}{2} yu'(c) \sum_{t=0}^{\infty} \beta^t L_t
\] (29)

where \(c\) and \(y\) are steady state values of consumption and output respectively, \(\mathcal{W}\) is a consumption compensation in \%. Result are shown in Table (3)

Table 3: Welfare loss

<table>
<thead>
<tr>
<th>Rule</th>
<th>(L)</th>
<th>(\mathcal{W}) in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramsey Optimal policy</td>
<td>0.000298</td>
<td>0.000091</td>
</tr>
<tr>
<td>Strong inflation stabilization (\hat{R}_t = 3\pi)</td>
<td>0.00279</td>
<td>0.000856</td>
</tr>
<tr>
<td>Standard Taylor rule (\hat{R}<em>t = 0.8\hat{R}</em>{t-1} + (1 - 0.8)[1.7\pi + 0.2\hat{y}])</td>
<td>0.023</td>
<td>0.0070</td>
</tr>
<tr>
<td>(\hat{R}<em>t = 0.84\hat{R}</em>{t-1} + (1 - 0.84)[1.92\pi + 0.17\hat{y} + 0.025\hat{\theta}])</td>
<td>0.0210</td>
<td>0.0064</td>
</tr>
<tr>
<td>(\hat{R}<em>t = 0.85\hat{R}</em>{t-1} + (1 - 0.85)[1.75\pi + 0.14\hat{y} + 0.26\hat{n}])</td>
<td>0.0172</td>
<td>0.0053</td>
</tr>
<tr>
<td>(\hat{R}<em>t = 0.91\hat{R}</em>{t-1} + (1 - 0.91)[2.5\pi + 0.12\hat{y} - 0.21\hat{u}])</td>
<td>0.0256</td>
<td>0.0079</td>
</tr>
</tbody>
</table>

The rule augmented with an employment gap is the most welfare efficient and requires a compensation 0.0053\% of steady a state consumption. It is more optimal than a standard Taylor rule specification which requires 0.007\% of steady state consumption as a compensation. However, strong inflation stabilization still appears to be much closer to an optimal policy \(^9\).

To conclude, the empirical analyses suggests that central bank’s behavior can be better described by a monetary rule augmented with labor market indicators. Alternative Taylor rules are also more efficient than a standard Taylor rule from a welfare prospective.

As a robustness check I ran Iskrev (2010) identification tests as well as plotted joint draws from priors and posterior distributions. I must admit that identification of the employment and unemployment parameters in the monetary rule is highly dependent

\(^9\)Similar results can be found in Schmitt-Grohe and Uribe (2007) who showed that as far as a central bank strongly responds to inflation fluctuations and slightly to output gap the welfare losses associated with a policy is almost indistinguishable from losses under an optimal policy.
on the calibrated parameter values for the labor market. Linear correlation coefficient between filtered employment and output is 0.7 in the data. Therefore the model has difficulties to identify coefficients on the output and employment simultaneously in a fully estimated model. As a result I had to fix most of the values for the labor market parameters. Non-linear estimation can help to overcome this issue.

5 Conclusion

The goal of this paper is to better understand the effect of labor market frictions on the monetary policy design from theoretical and empirical prospective. I used a NK model with search frictions on the labor market, imperfect price adjustment and imperfect competition on the goods market. Inefficiency on a labor market resulted in an additional trade-off for a policy maker who must choose between inflation stabilization, closing output and stabilizing labor market tightness. Consequently, an optimal policy has an incentive to give up a complete inflation stabilization in order to mitigate the response of labor market variables to productivity shocks.

I study an empirical relevance of this additional trade-off for the central bank. I augmented a monetary policy rule with different labor market indicators and estimated the coefficients for this indicators with Bayesian technique. The estimation of the coefficients in monetary policy rules suggests that monetary authority indeed react to labor market variables. The coefficient for the labor market tightness is small but positive and coefficients for employment and unemployment gaps are the same order of magnitude as the coefficient for the output gap. According to posterior odds ratio tests it is more likely that central bank reacts to labor market indicators and a model with an augmented Taylor rule thus better describes the reality. Such a behavior of a central bank was also shown to be optimal from a welfare point of view.

These findings can be seen as an empirical contribution to the discussion about optimal monetary policy and optimal simple rules. However, the model leaves a room for a richer economic environment. The assumptions about exogenous separation rate or flexible wages can be relaxed. I also abstract from capital accumulation in this paper and assume a fiscal policy being able to eliminate all the distortions in equilibrium apart
from labor market frictions. Studying an extended version of the model with capital accumulation is an interesting task for future research.

Moreover a non-linear estimation of simple monetary rules might help to overcome the problems with poor identification and shed some light on the non-linear dependence between labor market characteristics and the decision of monetary authorities.
6 References


Data sources:


2. OECD, "Main Economic Indicators - complete database", http://dx.doi.org/10.1787/data-00052-en


7 Appendices

7.1 Appendix I Log-linearization of the model

In this Appendix I present log-linearized model. In what follows the variables $\hat{x}_t$ are defined as log deviations from the steady state value, $\hat{x}_t = \log x_t - \log x$.

1) Euler Equation

$$ -\sigma^{-1}(\ln c_t - \ln c) - (\ln P_t - \ln P) \approx (\ln R_t - \ln R) - \sigma^{-1}E_t(\ln c_{t+1} - \ln c) - E_t(\ln P_{t+1} - \ln P) $$

$$ -\sigma^{-1}\hat{c}_t - \pi_t \approx \hat{R}_t - \sigma^{-1}E_t\hat{c}_{t+1} - E_t\pi_t $$

2) Production function

$$ \hat{A}_t + (1 - \phi)(\hat{n}_t + \hat{h}_t) = \hat{\Delta}_t + \hat{y}_t $$

3) Technology process

$$ \hat{A}_{t+1} = \rho \hat{A}_t + \epsilon_{A,t+1} $$

4) Market clearing condition

$$ \hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{\nu}{y}\hat{v}_t \text{ or } \hat{y}_t = s_c\hat{c}_t + s_v\hat{v}_t $$

5) Beverage curve

$$ \hat{n}_t \approx (1 - d)(1 - \lambda)\hat{n}_t + (1 - d)(1 - n)\frac{\lambda}{n}(1 - \eta)\hat{\theta}_t $$

where we define $\eta = \frac{\mu'(\theta)}{\mu} - \text{elasticity of worker-finding rate and therefore } \frac{\lambda'(\theta)}{\lambda} = 1 - \eta$.

From the steady state relationship $n = (1 - d)(n + \lambda u)$, $dn = (1 - d)\lambda u$, $n = \frac{1-d}{d}\lambda(1-n)$

Substituting in the equation above

$$ \hat{n}_t \approx (1 - d)(1 - \lambda)\hat{n}_t + d(1 - \eta)\hat{\theta}_t $$

6) Market tightness

$$ \hat{\theta}_t = \hat{v}_t - \hat{u}_t $$

7) Wage setting

$$ \hat{w}_t = \frac{sf}{wh}[\hat{n}_t - \phi\hat{n}_t + \hat{A}_t + \frac{\chi\theta}{f}\hat{\theta}_t] + \frac{(1 - \varsigma)}{whc^{-\sigma}}[(gh^{\gamma+1} - B - bc^{-\sigma - 1})\hat{h}_t + \sigma^{-1}(gh^{\gamma+1} + B)\hat{c}_t] $$
8) Job creation equations
\[ 0 = \lambda - \theta_t - \sigma^{-1} E_t (\dot{c}_{t+1} - \dot{c}_t) + \dot{J}_{t+1} \]
\[ \dot{J}_t = f \left[ \dot{p}^* + \dot{A}_t - \phi \dot{n}_t + (1 - \phi) \dot{h}_t \right] - \frac{hw}{J} (\dot{h}_t + \dot{w}_t) + (1 - \beta) E_t \left[ (-\sigma^{-1})(\dot{c}_{t+1} - \dot{c}_t) + \dot{J}_{t+1} \right] \]

9) Labor force
\[ \dot{u}_t = -\frac{1}{u} \dot{n}_t \]

10) Phillips curve
\[ \dot{p}^* - \dot{p}_{t-1} = \pi_t + (1 - \alpha \beta) \dot{p}^*_t + \alpha \beta (\dot{p}_{t+1} - \dot{p}_t) \]
\[ \pi_t = (1 - \alpha) (\dot{p}^*_t - \dot{p}_{t-1}) \]
which gives
\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \dot{p}^*_t \]

11) Choice of hours
\[ \dot{p}^*_t + \dot{A}_t - \phi (\dot{n}_t + \dot{h}_t) = \gamma \dot{h}_t + \sigma^{-1} \dot{c}_t \]

12) Matching function
\[ \dot{\lambda}_t \approx \frac{\overline{m}}{\lambda} \theta^{\frac{1}{\sigma}} (1 - \xi) [\xi + (1 - \xi) \theta^{\frac{1}{\sigma}}]^{\frac{1}{1 - \sigma}} \dot{\theta}_t \]
From the steady state relationship \( \lambda^s = \frac{\overline{m}}{\theta^{\frac{1}{\sigma}} (1 - \xi) [\xi + (1 - \xi) \theta^{\frac{1}{\sigma}}]^{\frac{1}{1 - \sigma}}} \) so we can substitute it and get
\[ \dot{\lambda}_t \approx \left( \frac{\overline{m}}{\lambda} \right)^{\frac{1}{1 - \sigma}} (1 - \xi) \dot{\theta}_t \]

13) Price dispersion measure
\[ \dot{\Delta}_t \approx \alpha \Pi^{\rho-1} \Delta_{t-1} + \frac{\Pi^{\rho-1}}{\Delta} \alpha \phi [\Delta - \frac{1}{\Pi} (1 - \alpha \Pi^{\rho-1})^{\frac{1}{1 - \rho}}] \pi_t \]
where as before \( \pi_t = \ln \Pi = \ln \frac{\Pi_t}{\Pi_{t-1}} = 1 - \Pi_t - \Pi \)

7.2 Appendix II Elasticity of matching function

Let us define an elasticity of matching function \( \eta_t = \frac{m'_t u_t}{m(u_t, v_t)} \). Because of a constant return to scale property one can write
\[ m(u_t, v_t) = m'_u u_t + m'_v v_t \]
\[ \frac{m'_v v_t}{m(u_t, v_t)} = 1 - \frac{m'_u}{m(u_t, v_t)} \]
\[ \frac{m'_v v_t}{m(u_t, v_t)} = 1 - \eta_t \]
The derivatives of the matching function can be expressed as

\[ m'_u = \eta_t \frac{m(u_t, v_t)}{u_t} = \eta_t \lambda_t \]

\[ m'_v = (1 - \eta_t) \frac{m(u_t, v_t)}{v_t} = (1 - \eta_t) \mu_t \]

### 7.3 Appendix III Model Calibration and simulation

The model is solved by first-order local approximation around deterministic efficient steady state (see S. Schmitt-Grohe and M. Uribe, 2007). Time is measured in monthly periods. All parameters are summarized in Table 4.

The discount factor is set 0.997 so the annual risk-free interest rate is about 4%. The coefficient of relative risk aversion \( \tilde{\sigma}^{-1} \) is 0.6 which implies a high value for an intertemporal elasticity \( \tilde{\sigma} \approx 6 \). As described in Rotemberg, Woodford (1997) when consumer purchases contain both consumer goods and investment goods they are likely to be more sensitive to interest rate. Since there is no explicit capital accumulation in the model this high value of intertemporal elasticity is appropriate (see also Tang 2010).

Total hours worked are normalized to 1. The value for scaling factor in disutility from labor \( \Gamma \) is then 0.74. A steady state level of unemployment is 6% which corresponds to empirical value for US. Exogenous job-separation rate is \( d=0.028 \) as in Shimer (2005). This brings a value for a job-finding rate in equilibrium 0.45 and implies the average duration of working contract to be 2.9 years. This number is close to empirical evidence that jobs last for two and a half years (Thomas, 2008). Inverse Frish elasticity is a little bit controversial parameter because macroeconomic literature usually uses a higher value than microeconomic evidence suggests. In the business cycles analysis this value varies significantly as well. While Trigari (2009) and Gali (2010) set it to be equal to 5, for example, Rotemberg and Woodford (1997) found the value 9.5 in their empirical studies and Tang (2010) obtained 11.9 from a moment matching procedure. I choose a value of 11.96 for comparability reasons. Implied labor supply elasticity \( 1/\gamma \) is then 0.08 which is on the lower bound of the interval proposed by microlevel evidence (Card, 1994, or Altonji, 1986). At the same time, lower elasticity of labor (higher Frish elasticity) will

\[ ^{10} \text{Monthly data is supposed to better capture the employment dynamics and central banks are more likely to use monthly data to develop monetary policy set-up (Thomas, 2008).} \]

\[ ^{11} \text{All non-participating workers are counted as employed in the model.} \]
lessen the reaction of labor to exogenous shocks which otherwise may become excessively large in the absence of capital adjustment (Tang, 2010).

Labor output share \((1 - \phi)\) is 0.7 which is pretty standard. Unemployment benefit \(b\) and sales tax \(\tau\) are chosen such that in equilibrium both \(\Phi_y\) and \(\Phi_\theta\) are equal to zero so that monopolistic and labor market distortions are eliminated in a deterministic steady state. \(p^*=1\) accordingly. Elasticity of substitution on production technology \(\epsilon_p\) is equal to 11 and the steady state mark-up is approximately 10% which is close to empirical findings. Calvo parameter \(\alpha=0.88\) meaning that the probability of price adjustment is 12% and average price duration is 8 month as in Nakamura and Steinsson (2008) and Basu and Gottschalk (2009).

According to Hagedorn, Manovskii (2008) low worker bargaining power is needed to account for only moderate procyclical movements of wages. Small reaction of wages and low vacancy posting costs will create a strong response of firms to productivity shocks and make labor market tightness more volatile than output. As in their studies and Shimer (2009) I set worker bargaining power \(\varsigma=0.0532\) and vacancy posting costs to 0.0045 of the quarterly wage. GDP share of vacancy posting costs is therefore \(s_v=0.0014\). I also set steady state value for \(\theta\) equal to 0.634 as in these studies. As a result the replacement ratio \(\frac{b}{wh}\) is relatively high. Hagedorn, Manovskii argue that workers include other compensations apart from unemployment benefits in the value of non-market activities. Fixed costs of working \(B\) are adjusted accordingly.

Finally, for the matching function \(\xi = \frac{1}{2}\) which is consistent with an efficient steady state and is often assumed in the literature (see Gali 2010). \(\bar{m} = (1/2)^{\epsilon/(1-\epsilon)}\) as in Den Haan et al (2000) and \(\epsilon=0.435\).

Table 5 presents the comparison of the empirical and simulated moments after a one standard deviation positive technology shock.

The model reproduces the main important features of the data. Vacancies and labor market tightness are more volatile than output, while employment, hours and wages are less volatile. Employment and vacancies are positively correlated with an output while unemployment has a negative correlation. Wage correlation with an output is stronger than in the data due to the flexible wage assumption. Finally, the model is able to capture a negative correlation between \(u\) and \(v\) - Beverage curve.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.997</td>
</tr>
<tr>
<td>$\tilde{\sigma}^{-1}$</td>
<td>Relative risk aversion</td>
<td>0.6</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Scaling factor in disutility of labor</td>
<td>0.74</td>
</tr>
<tr>
<td>$d$</td>
<td>Job destruction</td>
<td>0.028</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Frisch elasticity of labor supply</td>
<td>11.96</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Sales tax</td>
<td>-0.1</td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefit</td>
<td>0.12</td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>Worker bargaining power</td>
<td>0.052</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution in matching</td>
<td>0.435</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Elasticity of a worker-finding rate</td>
<td>0.356</td>
</tr>
<tr>
<td>$s_v$</td>
<td>GDP share of vacancy costs</td>
<td>1.4%</td>
</tr>
<tr>
<td>$b/w_h$</td>
<td>Replacement ratio</td>
<td>0.17</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Elasticity of substitution in production</td>
<td>11</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Calvo parameter</td>
<td>0.88</td>
</tr>
<tr>
<td>$B$</td>
<td>Fixed costs of working</td>
<td>0.53</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Vacancy posting costs</td>
<td>0.35</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>Steady state value of hours</td>
<td>1</td>
</tr>
<tr>
<td>$u$</td>
<td>Steady state level of unemployment</td>
<td>6%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Steady state market tightness</td>
<td>0.634</td>
</tr>
<tr>
<td>$A$</td>
<td>Steady state technology level</td>
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<tr>
<td>$y$</td>
<td>Steady state level of GDP</td>
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</tr>
<tr>
<td>$\lambda$</td>
<td>Job-finding rate</td>
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</tr>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>------------------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Standard deviation (in %)</td>
<td></td>
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<tr>
<td>y</td>
<td>1.53</td>
<td>0.91</td>
</tr>
<tr>
<td>v</td>
<td>12.90</td>
<td>6.48</td>
</tr>
<tr>
<td>u</td>
<td>11.25</td>
<td>3.88</td>
</tr>
<tr>
<td>θ</td>
<td>23.73</td>
<td>8.02</td>
</tr>
<tr>
<td>n</td>
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<td>0.25</td>
</tr>
<tr>
<td>h</td>
<td>0.42</td>
<td>0.03</td>
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<tr>
<td>w</td>
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<td>0.55</td>
</tr>
<tr>
<td>Correlation with output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>0.90</td>
<td>0.78</td>
</tr>
<tr>
<td>u</td>
<td>-0.88</td>
<td>-0.73</td>
</tr>
<tr>
<td>θ</td>
<td>0.91</td>
<td>0.98</td>
</tr>
<tr>
<td>n</td>
<td>0.89</td>
<td>0.96</td>
</tr>
<tr>
<td>h</td>
<td>0.68</td>
<td>0.73</td>
</tr>
<tr>
<td>w</td>
<td>0.25</td>
<td>0.99</td>
</tr>
<tr>
<td>Correlation between u and v</td>
<td>-0.93</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

Summary statistics calculated with US data for 1964-2006 (source: J-H Tang, 2010). All series were logged and HP-filtered with $\lambda = 1600$. 
The model was simulated after a one-standard deviation positive technology shock (Figure 1).

When worker productivity increases firms increase the demand for labor through both extensive and intensive margin. Workers in turn increase the labor supply since the wage also rises. Initial increase in labor market tightness slowly declines as firms find workers though a matching process. Monetary authority tries to keep inflation at zero level and increase a nominal interest rate.

Using the calibrated parameter values one can explicitly compute the parameters in the loss function (30). It states that the largest losses occur due to inflation fluctuations. As a result an optimal monetary policy focuses on the inflation stabilization. Modest weigh is also assigned to an output gap and a small coefficient to a labor market tightness gap. Woodford (2005) showed than if the coefficient before inflation and output in the loss function are positive than a zero inflation policy minimizes the losses regardless of shocks. In the current setup it is no longer the case. Additional channel for the transmission of shocks via labor market creates an incentive for monetary policy to
deviate from complete inflation stabilization.

\[ L_t = 656.129 \pi_t^2 + 18.118 (\gamma_t^c - \gamma_t^c)^2 + 0.012 (\theta_t - \theta_t^*)^2 \]  

(30)

Figure 2: Comparison of Different Policy Regimes. Impulse responses to a positive one standard deviation technology shock under: zero-inflation equilibrium (dashed line), social planner solution (dotted line) and optimal policy (solid line)

Figure (2) presents the comparison of market equilibrium (solid line), social planner solution (doted) and optimal monetary policy (dashed line). Social planner would keep all the variables at the steady state levels and let an output to absorb the positive effect of a technology shock. Under flexible price equilibrium deviations of employment, vacancies and consequently labor market tightness are rather large. It can be explained by a low bargaining power of workers. Since they are undercompensated for their participation in labor market the level of vacancies relative to unemployed people is inefficiently high. Not surprisingly, optimal policy, lies in between. It allows inflation do deviate from a zero level in order to be able to stabilize employment, market tightness and output and
to bring them closer to the efficient level. Complete inflation stabilization is no longer a best possible solution as in the standard NK model 12.

7.4 Appendix IV Posterior analysis

Figure 3: Posterior distributions for model with a simple rule $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi_t + \rho_\theta \theta_t + \rho_y y_t]$. Priors are plotted with gray lines and posteriors with black solid lines. Dashed green lines indicate the posterior mode.

12Policymaker would also allow for a high positive output gap in the first period and would then keep it moderately negative in subsequent periods. The reason is that the labor market tightness response to future output gaps is larger that to a current output gap. It is more beneficial to absorb the whole effect of a technology shock in the first period in order to keep the future output gaps closer to zero.
Figure 4: MCMC univariate convergence diagnostic for $R_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_{\pi} \hat{\pi} + \rho_{\theta} \hat{\theta}]$. 

$SE_{EOBS_u \_obs \ (Interval)}$ $SE_{EOBS_u \_obs \ (m2)}$ $SE_{EOBS_u \_obs \ (m3)}$

$RHOR \ (Interval)$ $RHOR \ (m2)$ $RHOR \ (m3)$

$RY \ (Interval)$ $RY \ (m2)$ $RY \ (m3)$
Figure 5: MCMC univariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) [\rho_{\pi} \pi + \rho_y \hat{y} + \rho_{\theta} \hat{\theta}]$.

Figure 6: Multivariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) [\rho_{\pi} \pi + \rho_y \hat{y} + \rho_{\theta} \hat{\theta}]$. 
Figure 7: Posterior distributions for model with a simple rule $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_y \hat{y} + \rho_n \hat{n}]$. Priors are plotted with gray lines and posteriors with black solid lines. Dashed green lines indicate the posterior mode.
Figure 8: MCMC univariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y} + \rho_n \hat{n}]$.

Figure 9: MCMC univariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \hat{y} + \rho_n \hat{n}]$. 

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Figure 10: Multivariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \hat{\pi} + \rho_y \hat{y} + \rho_n \hat{n}]$. 
Figure 11: Posterior distributions for model with a simple rule $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_x \pi + \rho_y \hat{y} + \rho_u \hat{u}]$. Priors are plotted with gray lines and posteriors with black solid lines. Dashed green lines indicate the posterior mode.
Figure 12: MCMC univariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \bar{y} + \rho_u \bar{u}]$.

Figure 13: MCMC univariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_\pi \pi + \rho_y \bar{y} + \rho_u \bar{u}]$. 

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Figure 14: Multivariate convergence diagnostic for $\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r)[\rho_{\pi} \pi + \rho_y \hat{y} + \rho_u \hat{u}]$. 

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