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# **Household Debt and Macrodynamics - How do Income Distribution and Insolvency Regulations interact?**

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# Household Debt and Macrodynamics - How do Income Distribution and Insolvency Regulations interact?\*

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## Abstract

Using agent-based simulation methods we explore the interplay between income distribution, personal insolvency regulations and household borrowing focusing on the effects on macroeconomic dynamics. In order to capture the empirically observed distribution of income and wealth, we model them by means of a Generalised Pareto Distribution. In the presence of social comparison effects, the insolvency regime decides on lower income households' incentives to expose themselves to possibly unsustainable levels of debt. Our main findings can be summarised as follows: for both, creditor friendly and debtor friendly regimes, higher skewness in the distribution of income and wealth leads to an increase in the number of defaults and to lower levels of GDP. Comparing the two regimes, we observe a higher number of defaults and higher aggregate debt under pro-debtor laws given the same starting values for the distribution of income. While debt-financed consumption leads to higher levels of GDP under pro-debtor policies, over-borrowing low-income households put a downward pressure on economic growth. The opposite is true for pro-creditor policies, where we observe positive GDP growth rates, as they prevent households from taking up unsustainable levels of debt ex ante.

**Keywords:** Optimal Insolvency Regulation; Income and Wealth distribution; Agent-based Model; Computational Simulation;

**JEL classification:** D10, D31, E02, E21.

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# 1 Introduction

Insolvency laws regulate the comparative rights of the defaulting borrower and the various lenders. In doing so their impact is not restricted to solving issues turning around the distribution of what has been left by the defaulting borrower (*ex post effects*) but in addition also concerns both borrowers' incentives to borrow more than would be wise in light of future earning prospects, and lenders' incentives to establish appropriate financial constraints (*ex ante effects*). Academic research so far has overwhelmingly evaluated the optimality of an insolvency law according to its capability to avoid moral hazard on the part of the borrower (see for instance [White, 1998](#); [Berkovich and Israel, 1999](#); [Adler et al., 1999](#)).

While significant research concentrates on the incentive structure of insolvency laws ([Povel, 1999](#); [Bebchuk, 2002](#)) much less attention is devoted to the macroeconomic implications of varying insolvency laws. Moreover, not least since the recent financial crisis is it widely acknowledged that credit matters for both macroeconomic stability as well as macroeconomic growth. This applies both to the access to credit as well as to defaults occurring in significant numbers. However both is highly dependent on the prevailing insolvency law. By implication its integration into macroeconomic models with a focus on the role of credit appears overdue. The following paper takes these considerations into account. In doing so, however, we focus our attention to household debt which has been rising continuously for some decades, now raising political concerns not only about its sustainability but also about implied macroeconomic stability.

Surprisingly, the problem is mostly analysed without taking into account the impact of income and wealth distribution as an additional driver for household indebtedness. Resulting from the assumption of the representative agent in conventional macroeconomic models, economic and social interaction used to be rather neglected and income distribution did not play any role. However, as soon as access to credit for everyone is encouraged by established policies, income and wealth distribution truly gain significance. In this context, US pro-debtor policies are often blamed for favouring the extension of credit to low income households over fiscal redistribution (see for instance [Rajan, 2010](#)). There is a growing literature showing that a country's income distribution indeed matters for household indebtedness. Households in the lower part of the income distribution often tend to overborrow in order to maintain a certain standard of living (see for instance [Barba and Pivetti, 2009](#); [Atkinson and Morelli, 2010](#)). The desire of poor households to keep up with their wealthier neighbours might not only be driven by status consumption, but

also by the fear of social exclusion in a world where certain standards are expected to be fulfilled. A recent experimental study by [Carbone and Duffy \(2014\)](#) confirms that individual consumption is strongly affected by average consumption. Following this strand of literature, we argue that an optimal insolvency law should very well take the skewness of income and wealth distribution into account, in particular with respect to the finding that prevailing regulations have a strong impact on borrowers' incentives to take up debt and possibly even encourage strategic default. However, and contrary to the current discussion, we incorporate macroeconomic stability as an additional variable determining the optimality of alternative insolvency regulations.

We build on an agent-based model of relationships between households and banks developed in [König and Grössl \(2014\)](#). The model is perfectly suitable for the analysis as it shows theoretically that income distribution and household over-indebtedness are closely related. We enrich the model by accounting for the empirically observed skewness of income and wealth distribution and model the latter explicitly by means of a Generalised Pareto distribution. The framework also allows for household insolvencies and is built around the assumption that consumption and therefore loan demand are driven by social phenomena. A further novelty implied by the focus on household defaults, are the varying durations of the insolvency restructuring periods depending on whether an economy is creditor or debtor friendly. Being well aware that insolvency regulations are far more complicated in reality, the variation in the length until an unfortunate debtor is released from remaining debt, already enables the model to reproduce stylised phenomena.

The remainder of the paper is organised as follows. The following [Section 2](#) gives an overview on related literature. In [Section 3](#) we briefly present the model. [Section 4](#) describes the simulation procedure and presents the results. We first describe the results for the creditor friendly economy, followed by results on a debtor friendly economy. Finally, [Section 5](#) concludes.

## **2 Literature Review**

There is a large literature dealing with household debt and household insolvencies. This paper combines two strands of previous research. The first includes literature investigating the link between income distribution and household debt. The second includes literature on household insolvencies in general and optimal insolvency regulations in particular.

A key motivation for this paper originates from the sharp increase in household debt overhang in most industrialised countries and the role of income and wealth distribution therein. In this respect, [Iacoviello \(2008\)](#) provides evidence that while the skewness of income distribution and

the size of household debt in the US were stable from 1967 until 1980, a rise in both variables has been observed thereafter. Income inequality increased strongly in the 1990s and household debt followed in the 2000s. During the same period, an increasing number of private insolvencies was observed ([Athreya, 2008](#)). There are two opposing interpretations of this development, accusing either supply or demand side factors. For the case of the US, [Rajan \(2010\)](#) claims that prior to the recent crisis, the supply of credit had been increasingly extended to low income groups resulting from political motivations to conceal the increased income inequality. Political scientists often argue that pro-debtor policies favour the extension of credit to low income households over fiscal redistribution in order to avoid losing potential voters ([Ahlquist and Ansell, 2014](#)). US monetary policy is prone to support credit by setting interest rates accordingly, and laws have been designed to provide an insurance for private individuals against over-indebtedness ([Dobbie and Song, 2014](#)).<sup>1</sup>

There is also a growing literature pointing to the importance of demand side factors, accusing an increase in inequality as the main reason for soaring household debt. Households in the lower part of the income distribution often tend to overborrow in order to maintain a certain standard of living (see for instance [Barba and Pivetti, 2009](#); [Atkinson and Morelli, 2010](#)). There are some recent studies accounting for relative consumption preferences ([Drechsel-Grau and Schmid, 2013](#); [Fischer, 2013](#)). In this regard, [Cynamon and Fazzari \(2008\)](#) argue that changing consumption norms since the 1990s have played a decisive role.

Given the origin of the recent crisis, inequality is mostly discussed for the case of the US ([Piketty and Saez, 2003](#); [vanTreeck, 2014](#)), yet several studies also report increasing income inequality in Europe since the 1980s ([Brandolini, 2007](#); [Franzini, 2009](#); [Fredriksen, 2012](#)). In this respect, [Franzini \(2009\)](#) deplores the increase in the top income shares in the face of stagnating low incomes. He also argues that the financial crisis further exacerbated differences in income. Apart from studying inequality separately in the individual European countries, these studies all argue that inequality in Europe should be considered from a “one-country” perspective. After controlling for different currencies and adjusting purchasing power parities, they find that overall income inequality has increased as well. As opposed to the US, this generalisation should be treated with care though, as the individual countries differ in many respects such as their institutional, in particular legal environment. We argue instead that a country’s institutional background is crucial and should be very well taken into account.

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<sup>1</sup>Changing credit environments as a determinant for the increase in household insolvencies in general are discussed by [White \(2007b\)](#) and [Livshits et al. \(2010\)](#).

A further motivation for this paper originates from heterogeneity of household overindebtedness across countries and the role of personal insolvency regulations therein. Most literature approaches personal insolvency regulations from the viewpoint of conflicting interest between debtors' opportunity to cope with unfortunate events inducing insolvency in the absence of an adequate insurance system on the one hand and strategic default on the other.

A relatively large literature indeed reports a positive correlation between private credit demand and debtor friendly insolvency laws (Livshits et al., 2007; Chatterjee and Gordon, 2012). Similarly, Jappelli et al. (2008) relate pro-debtor reforms to an increase in the number of personal insolvencies in several countries. In this regard, debtor friendly insolvency regulations are often evaluated critically, as the option of having one's debt discharged might render filing for bankruptcy beneficial thus generating moral hazard (White, 1998; Wang and White, 2000). That households might even default strategically has been empirically confirmed by Fay et al. (2002). They find that for creditor friendly insolvency regulations, where benefits from filing are rare, insolvency rates tend to be substantially smaller. The literature often distinguishes between debtor friendly Anglo-Saxon insolvency regulations which are considered to be extremely generous as individuals can get immediately discharged from pre-bankruptcy debt ("fresh start"), and rather creditor friendly continental European countries (see for instance Niemi-Kiesilainen, 1999; Gerhard, 2009).<sup>2</sup> While distorted incentives in debtor friendly environments represent one side, the insurance character in otherwise often rather poorly developed social systems reflects the other. In creditor friendly regimes on the contrary households mostly have to undergo a long and demanding debt restructuring process until unpaid debts might finally get charged off. Niemi-Kiesilainen (1999) distinguishes three factors characterising most continental European insolvency regulations. First, restricted access to debt restructuring, second, a compulsory repayment plan which is pre-conditional for the discharge of residual debt and third, mandatory debt counselling services to deal with defaulting households. Given this, Niemi-Kiesilainen (1999) points out that insolvencies in continental Europe are also linked to moral values, as the main rationale behind the extensive restructuring procedure is to ensure that there is not an easy solution to the problem of overwhelming debt. Hence, insolvency laws in continental Europe they tend to be less lenient with respect to debt relief compared to Anglo-Saxon economies, which may be largely ascribed to the prevention of distorting incentives for debtors. Overall, insolvency laws are extremely heterogeneous, even within European countries. For an overview

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<sup>2</sup>Heuer (2014) provides a more refined distinction, classifying insolvency laws according to the 'market model', the 'restrictions model', the 'liability model' and the 'mercy model'.

on consumer insolvency laws in selected countries see [Gerhard \(2009\)](#) and [Heuer \(2014\)](#). For a more detailed evaluation of US personal bankruptcy laws specifically see [Porter \(2011\)](#).

A number of studies conduct comparative analysis evaluating different insolvency regulations with the objective to identify their respective merits. For instance [Livshits et al. \(2007\)](#) employ a life-cycle model comparing a “Fresh Start” System with a “No Fresh Start” system. Calibrating their model to US and German data, they find that due to higher income and expense uncertainty in the US, the “Fresh Start” system is welfare enhancing while the opposite is true for the “No Fresh Start” system, which performs better with German data. They argue that the performance of an insolvency regulation depends on the underlying social system. Put differently, prevailing institutions play a crucial role and insolvency regulations should be adjusted to the respective social and economic environment (in line with [Niemi-Kiesilainen, 1999](#)). In a similar analysis, [Chatterjee and Gordon \(2012\)](#) compare the current US law on consumer bankruptcy with an alternative regime without debt relief tools. Contrary to [Livshits et al. \(2007\)](#) they focus on optimal garnishment rates, arguing that household insolvency exists also in the absence of discharge options highlighting the importance of garnishment. In their model, the commitment to repay debt leads to a reduction in interest rates and hence, facilitates poor households’ access to credit. Overall welfare increases if garnishment laws are strict enough, enabling less wealthy households to smooth consumption and more wealthy households to benefit from lower borrowing rates. Their criticism of the current US bankruptcy law results from the sharp increase in consumer insolvencies in the aftermath of the sub-prime crisis. They report that the outstanding volume of consumer debt has declined though not because overall borrowing declined, but rather because many overindebted households made use of their option to default. Non-performing loans resulting from a massive debt relief were removed from banks’ balance sheets. In this debate about the merits of different insolvency laws, strategic default is indeed the main argument put forward against a “Fresh start” system ([White, 1998](#); [Athreya, 2006](#)). On the other hand though, the opportunity of having one’s debt charged off provides an insurance for individuals against adverse shocks (“bad luck”) such as job loss or divorce by offering them a fresh start ([Dobbie and Song, 2014](#)). In this regard, the findings by [Livshits et al. \(2007\)](#), who argue that a country’s insolvency law should be considered in the context of its underlying system plays an essential role.

Regardless of the insolvency regulation and possible discharge of residual debt, research on post-bankruptcy predominantly agrees that households are not better off after filing. In this



respect, [Cohen-Cole et al. \(2009\)](#) find that debt relief does not benefit debtors as they not only have difficulties to get external finance afterwards but that they also struggle to repay potential debt as opposed to prior receiving insolvency protection. And [Jagtiani and Li \(2014\)](#) report that their access to credit is constrained even long after the discharge date. That previously defaulted households are charged significantly higher interest rates compared to non-filers has been found by [Han and Li \(2011\)](#). They also show that filers are more prone to face repayment difficulties after bankruptcy and accumulate less wealth. Hence, in spite of the insurance character of bankruptcy in rather debtor friendly economies, individual welfare may still be punished in the sense, that access to external finance may be more difficult post-bankruptcy.

Optimal insolvency laws and their relation to different economic and social systems has been discussed vividly in the literature. However, the role of income distribution in light of rising household debt has not received sufficient attention so far.

The following section presents the model and describes how we model income distribution and varying insolvency procedures.

## **3 The Model**

### **3.1 Overview**

The paper builds on an agent-based model developed in [König and Grössl \(2014\)](#). The model is suitable to study household bankruptcy and their feedback effects on macroeconomic dynamics. It contains features such as household-bank relationships, and consumption preferences which are partially driven by social phenomena inducing a number of households to live above their means, which in turn may force them to default on their debt. Macroeconomic phenomena emerge from the bottom-up resulting from interaction between agents on the micro level ([Kirman, 1995](#); [Tsefatson, 2006](#); [Delli Gatti et al., 2011](#)). As the present analysis focuses on the role of income distribution and insolvency laws, we model income and wealth distribution explicitly according to a Generalised Pareto distribution (Section [3.2](#)) and enrich the model by accounting for different insolvency procedures (Section [3.3](#)). This leads to varying consumption patterns for insolvent households (Section [3.4](#)). Moreover, banks receive different loan repayments depending on the insolvency regulation (Section [3.5](#)). Further elements of the original model are only briefly outlined. For a detailed description of the model see [König and Grössl \(2014\)](#) or the Appendix.

The model economy is composed of  $h$  households ( $h = 1, 2, \dots, H$ ), a representative commercial bank, a central bank and a government. Agents follow simple behavioural rules and heterogeneity enters the model through different channels. The most important source of heterogeneity is households' income. In addition to that, households hold either varying amounts of deposits and cash, where they earn additional income through interest rate payments on deposits, or loans on which they have to pay interest. For simplicity it is assumed that households can either save or borrow, but never both and loan contracts have a duration of one period. Moreover, loan demand may not always be fully satisfied. It can be rejected in two cases: First, if loan demand exceeds household specific credit lines or second, if a household recently filed on its debt and is still subject to an insolvency procedure.

The crucial element of the model is a household's decision about desired consumption. Households follow a consumption norm (Cynamon and Fazzari, 2008), according to which they care about their relative position in the economy. Choosing a reference standard against which individuals compare themselves has a long tradition in the economic literature beginning with the famous contribution by Duesenberry (1949). In a very recent experimental study, Carbone and Duffy (2014) confirmed that individuals indeed consider consumption decisions made by others. Desired consumption in our model is

$$C_{ht}^* = \gamma_1 Y_{ht-1} + \gamma_2 \bar{C}_{t-1} - \gamma_3 i_{Dt} - \gamma_4 i_{Lt}. \quad (1)$$

The median economy-wide consumption of the last period is

$$\bar{C}_{t-1} = \frac{1}{2}(C_{\frac{H}{2}, t-1} + C_{\frac{H}{2}+1, t-1}). \quad (2)$$

$Y_{ht}$  refers to a household's disposable income, and  $i_{Dt}$  and  $i_{Lt}$  denote interest rates on deposits and loans, respectively.  $\gamma_1$  is the marginal propensity to consume out of earnings,  $\gamma_2$  the parameter for the consumption norm and  $\gamma_3$  and  $\gamma_4$  are reaction coefficients for the lending and borrowing rate,  $\gamma_j \in (0, 1)$  for  $j = 1, 2, 3, 4$ . If households' desired consumption exceeds disposable income, they require external finance to reach their desired consumption level. As can be seen from equation (1), mostly households in the lower part of the income distribution require a loan. Above average households require external finance only if they suffer from a negative wage shock. Loans provide poor households with the opportunity to keep up with their wealthier

neighbours or to insure against unforeseen contingencies.

As opposed to König and Grössl (2014), where we assumed that all households compare themselves to the average, as a novelty of this paper we model upward-looking consumption preferences with median income as the decisive variable determining a household's consumption preferences.

$$\gamma_2 = \begin{cases} \gamma_2 & \text{if } \frac{Y_{ht-1}}{\frac{1}{2}(Y_{\frac{h}{2},t-1} + Y_{\frac{h}{2}+1,t-1})} \leq 1 \\ 0 & \text{if } \frac{Y_{ht-1}}{\frac{1}{2}(Y_{\frac{h}{2},t-1} + Y_{\frac{h}{2}+1,t-1})} > 1 \end{cases} \quad (3)$$

Accordingly, below median income households have a higher propensity to consume and hence either save less out of their incomes than above median income households, or even take up debt to satisfy desired consumption. Recent research with a similar approach to relative consumption assumes that all households compare themselves to households with higher levels of consumption splitting them into different income groups (i.e. Drechsel-Grau and Schmid, 2014; Belabed et al., 2013; Frank et al., 2014; Cardaci, 2014). We choose to take the median consumption of the last period as a reference standard with upward-looking comparisons for two main reasons. First, empirical evidence shows that it is mostly poor income households (or households in bad financial shape) that require external finance (see for instance Flynn, 1999; Atkinson and Morelli, 2010). Splitting households in percentiles and modelling upward-looking behaviour would render unrealistically many households as debtors. Second, with the present modelling choice, assuming a Generalised Pareto distribution, the median tends to be a better proxy than the mean. If a distribution is heavily skewed, i.e. if few households possess a very high proportion of income and wealth, the mean is extremely high. Hence, taking the latter as a reference point would lead to unrealistically high values of loan demand and induce excessive debt levels for the majority of households. Moreover, only a very small fraction of very rich households would accumulate savings. Dynan et al. (2004) points out that mostly high income households can afford to save a larger fraction of their income. Thus, we suppose that median consumption is a good proxy to balance loan and saving decisions of households. I.e. a household's position in the income distribution decides about its classification as either a saver or a borrower.

Regarding households' income dynamics, it is further important to note, that due to adverse shocks, wage income at the micro level can be relatively volatile and hence, above median income

households might eventually fall below the median in the proceeding period. That notwithstanding, the majority of defaulting households possess rather low income and wealth. This mechanism is described in Section 3.3. The following section describes how income distribution is modelled.

### 3.2 Income Distribution

There is a huge debate in the literature about the appropriate distribution function for the size of incomes. Despite this discussion, income distribution is to the best of our knowledge only rarely modelled explicitly. There are several distributions which are deemed to fit the distribution of incomes (see for instance McDonald, 1984). Given this, there is empirical evidence that income and wealth distribution are heavily skewed and follow a power law (see for instance Levy and Solomon, 1997).

Accounting for this insights, we model the distribution of income explicitly, drawn from a Generalised Pareto distribution which was first introduced by Picklands (1975). It is particularly suitable to model long right tails, stating that a very large proportion of income and/or wealth is owned by a very small proportion of people. It contains three parameters. A location parameter,  $\mu$ , a scale parameter,  $\sigma$ , and a shape parameter,  $\zeta$ . The cumulative distribution function of a random variable  $N$  is

$$P(N \leq \eta) = \begin{cases} 1 - \left(1 + \frac{\zeta(\eta - \mu)}{\sigma}\right)^{-1/\zeta} & \text{for } \zeta \neq 0 \\ 1 - \exp\left(-\frac{\eta - \mu}{\sigma}\right) & \text{for } \zeta = 0 \end{cases} \quad (4)$$

with  $\sigma > 0$  and  $\eta - \mu \geq 0$  when  $\zeta \geq 0$  and  $\eta \leq \mu - \sigma\zeta$  when  $\zeta < 0$  and  $\mu = 0$ . It is generalised as it contains several special cases. When  $\zeta > 0$  and  $\mu = \frac{\sigma}{\zeta}$ , one gets the Pareto distribution with  $a = 1/\zeta$  and  $K = \sigma/\zeta$ .<sup>3</sup> In the context of our analysis, the shape parameter  $\zeta$  plays the key role as increasing values for  $\zeta$  represent increasing skewness of the income distribution, i.e. the frequency of poor income households increases. Put differently, fewer rich households get richer and more poor households get poorer.  $\mu$  relates to the average household income and  $\sigma$  to the standard deviation thereof.

At the beginning of the simulation, each household is endowed with an income randomly drawn

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<sup>3</sup>The corresponding cumulative distribution function for a Pareto distributed random variable would then be

$$F(x) = \begin{cases} 1 - \left(\frac{K}{x}\right)^a & \text{for } x \geq K \\ 0 & \text{for } x < K \end{cases} \quad (5)$$

where  $K$  denotes the scale and  $a$  the shape parameter.

from the above described Generalised Pareto probability distribution.

$$Y_{ht} = \eta_{ht} Y_t \quad (6)$$

$\eta_{ht}$  denotes the parameter of income distribution and adds up to one,  $\sum \eta_{ht} = 1$ .  $Y_{ht}$  stands for a households disposable income and  $Y_t$  denotes the GDP of the model economy.

### 3.3 Insolvency Regulations

In reality insolvency procedures are very complex, and respective laws differ substantially across countries, ranging from the opening of proceedings over the filing of claims and verification to reorganisation plans (EU Note 2011) which again vary in length and practical operation.

To keep the model and the analysis as simple as possible, we assume that households enter an insolvency restructuring period,  $j = 1, 2, \dots, J$ , once they default on their debt. During the insolvency period, households are punished in two ways. They are excluded from the credit market and creditors can seize a share of their income each period during the restructuring process. For a single household this means that it falls back to a minimum income after filing for bankruptcy.

$$Y_{ht,j}^{np} = \theta Y_{ht} \quad (7)$$

where  $Y_{ht,j}^{np}$  denotes “non-pledgeable” income and  $\theta$  the exemption rate. Once a household enters an insolvency process, it remains in this state until it reaches the end of the process  $J$

$$Y_{ht+1} = Y_{ht+1,j}^{np} \quad \text{if} \quad Y_{ht} = Y_{ht,j}^{np} \quad \& \quad j < J \quad (8)$$

At the end of the insolvency period,  $J$ , residual debt is discharged and the household can apply for credit anew. In the context of the model, the duration of an insolvency period determines the degree of creditor or rather debtor friendliness of a regulation:  $J^{DF} < J^{CF}$ . Well knowing that punishment of default is more severe in a creditor friendly economy, it is assumed that households take this into account and adjust consumption preferences accordingly. This translates into a higher willingness to take up debt in order to finance consumption expenditure under pro-debtor laws. This idea is based on research by Grössl and Fritsche (2007b), who show that households borrow more with a default option in place as they face limited liability. There is also empirical evidence reporting that households are more prone to strategic default under rather pro-debtor regulations (i.e. Wang and White, 2000; Fay et al., 2002). Based on these insights, we model

lower incentives to overborrow in a creditor friendly economy as expressed through

$$\gamma_2^{CF} < \gamma_2^{DF}. \quad (9)$$

The following section outlines consumption dynamics of insolvent households.

### 3.4 Consumption Dynamics of Insolvent Households

Insolvency procedures change aggregate consumption dynamics inasmuch as insolvent households cannot consume more than a minimum income. In a world where debtors are discharged immediately such as assumed in König and Grössl (2014), three different types of consumers can be identified: borrowers ( $L_{ht-1} > 0; D_{ht-1} = BG_{ht-1} = 0$ ), savers ( $L_{ht-1} = 0; D_{ht-1} > 0; BG_{ht-1} > 0$ ), and those who neither save nor borrow, ( $L_{ht-1} = D_{ht-1} = BG_{ht-1} = 0$ ).<sup>4</sup> With enduring insolvency periods however, the process turns somewhat more complex and a fourth type enters the dynamics, namely households who defaulted in an earlier period and who are subject to an insolvency process:  $Y_{ht-1} = Y_{ht-1,j}^{np}$ . The possibility of new debt is excluded as insolvent households are not allowed to participate at credit markets. In very rare cases though, for instance after profiting from a positive wage shock, it may theoretically be able to accumulate savings:  $D_{ht-1} \geq 0$  and hence  $BG_{ht-1} \geq 0$ , which are also seized by the lender. The income of an insolvent household is then

$$Y_{ht,j}^{np} = \theta Y_{ht} = \theta[(1 - \tau)Y_{ht}^W + i_{Dt-1}D_{ht-1}]. \quad (10)$$

Resulting in the following consumption pattern

$$C_{ht} = \begin{cases} C_{ht}^* & \text{if } C_{ht}^* \leq Y_{ht,j}^{np} + D_{ht-1} \\ Y_{ht,j}^{np} & \text{if } C_{ht}^* > Y_{ht,j}^{np} + D_{ht-1}. \end{cases} \quad (11)$$

In case a household holds deposits, they amount to

$$D_{ht}(1 + \kappa) = C_{ht}^* - Y_{ht,j}^{np} - D_{ht-1}(1 + \kappa) \geq 0. \quad (12)$$

As mentioned above, bankrupt households are not allowed to take up new debt and are only discharged from their liabilities at the end of the insolvency period. Consumption dynamics change insofar that, with an enduring duration of the insolvency period,  $J$ , relatively more

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<sup>4</sup>See König and Grössl (2014) for a detailed description.

insolvent households consume exactly what they earn, i.e. their propensity to consume is one,  $c = 1$ . Aggregate consumption is composed of the four different subtypes of consumers as will become clear when we turn to the aggregation procedure.

### 3.5 Banks

The banking sector is only outlined briefly in this paper. For a detailed description see [König and Grössl \(2014\)](#) or the Appendix. It is assumed that lenders set credit lines based on a household's income of the previous period. While insolvent households are not allowed to participate at credit markets, they also face extremely tight credit constraints after leaving the insolvency process ([Cohen-Cole et al., 2009](#); [Han and Li, 2011](#)). Lenders also account for the exemption rate in place as default is more attractive for households when exemption rates are high (see for instance [Athreya, 2006](#)). For the sake of simplicity though, we abstract from varying exemption rates for the moment in our analysis. Banks set credit lines according to

$$L_{ht}^{\max} = \lambda \frac{(Y_{ht-1} - \theta Y_{ht-1})}{1 + i_{Lt}} \quad (13)$$

where

$$\lambda = \begin{cases} \lambda_1 & \text{if } \frac{Y_{ht-1}}{\text{median} \sum Y_{ht-1}} < 1 \\ \lambda_2 > \lambda_1 & \text{if } \frac{Y_{ht-1}}{\text{median} \sum Y_{ht-1}} \geq 1. \end{cases} \quad (14)$$

We assume that the bank is cautious in the sense that lending behaviour is more restrictive for households whose income is below the median level.

With enduring insolvency periods, households' loan repayment behaviour,  $x_{ht}$ , is composed of loans plus interest rates from solvent households and the pledgeable share of income of those households who are either unable to repay their current debt or who are already subject to an insolvency process

$$x_{ht} = \begin{cases} L_{ht-1} (1 + i_{Lt-1}) & \text{if } (1 - \theta) Y_{ht-1} \geq L_{ht-1} (1 + i_{Lt-1}) \\ (1 - \theta) Y_{ht-1} & \text{if } (1 - \theta) Y_{ht-1} < L_{ht-1} (1 + i_{Lt-1}) \\ (1 - \theta) Y_{ht-1} & \text{if } Y_{ht-1} = Y_{ht-1,j}^{np} \quad \& \quad j < J. \end{cases} \quad (15)$$

The longer the duration of a debt restructuring period,  $J$ , the smaller the losses for the lender as it seizes relatively more income, and hence macroeconomic write-offs. From the perspective of a

bank, non-repayment of credit reduces its cashflow and might even become negative. If the latter is the case, the central bank acts as a lender of last resort and absorbs any losses to maintain financial sector stability. The duration of the insolvency period affects not only a banks cash flow but via household consumption also macroeconomic variables.<sup>5</sup> For the moment, the analysis concentrates rather on the demand side. Future research may include a more sophisticated banking sector, where loan supply should take insolvent households into account.

### 3.6 Aggregation

We have seen that on the micro level, households and banks cope with complexity by means of simple behavioural rules. For the aggregation process we proceed as is usual in agent-based computational economics, namely by simulating the model. Aggregate time series emerge through the interaction of agents at the micro level (see for instance [Delli Gatti et al., 2008, 2011](#)). As the focus lies on the relationship between households and banks, we assume that supply is driven by aggregate demand and firms adjust output accordingly. Deviations follow solely from exogenous shocks.

$$Y_t = Y_t^d + \rho_t \quad (16)$$

where  $\rho_t$  is a uniformly distributed temporary macro stochastic supply shock with support  $[\underline{\rho}, \bar{\rho}]$ . Aggregate demand comprises aggregate consumption and government expenditures,  $G$ , which we assume to be exogenous

$$Y_t^d = \sum_{h=1}^H C_{ht} + \bar{G}. \quad (17)$$

Aggregate household consumption is composed of the four different subtypes of consumers,  $H = H_1, H_2, H_3, H_4$ , depending on their individual desire to consume and potential related constraints. The aggregate consumption function is

$$\sum_{h=1}^H C_{ht} = \sum_{h=1}^{H_1} Y_{ht,j}^{np} + \sum_{h=H_1+1}^{H_2} (Y_{ht} + L_{ht}^{max}) + \sum_{h=H_2+1}^{H_3} (Y_{ht} + L_{ht}^d) + \sum_{h=H_3+1}^{H_4} (\gamma_1 Y_{ht} + \gamma_2 \bar{C}_{t-1} - \gamma_3 i_{Dt} - \gamma_4 i_{Lt}). \quad (18)$$

Type  $H_1$  is insolvent, Type  $H_2$  is a borrower who is (partially) credit constrained, Type  $H_3$  is a borrower who is not credit constrained and Type  $H_4$  can satisfy desired consumption without

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<sup>5</sup>Stock-flow-consistency of the model is ensured as we assume that the commercial bank is owned by the social planner and that all positive cash flows are directly transferred. In case of negative cashflows, the commercial bank requires central bank loans. For a detailed description of the financial sector see [König and Grössl \(2014\)](#) or the Appendix.



relying on external funds. The time path for the aggregate dynamics is then given by

$$Y_t = \sum_{h=1}^H C_{ht} + \bar{G} + \rho_t \quad (19)$$

In the following section, we describe the simulation procedure, the computational experiments and resulting outcomes.

## 4 Simulation

### 4.1 Calibration and Computational experiments

Being interested in the effects of varying income distributions under different insolvency laws on aggregate debt and macroeconomic stability, we conduct several computational experiments. We oppose a creditor friendly economy to a debtor friendly economy, where the degree of creditor or debtor friendliness is assumed to be determined by the length of period until a defaulting household is discharged from residual debt. Following the literature on insolvency laws and strategic default (see for instance [White, 1998, 2007a](#); [Grössl and Fritsche, 2007a](#)), we presume that households in a debtor friendly economy have stronger preferences to take up larger amounts of debt, while the opposite is true for a creditor friendly economy. For all insolvency regimes we simulate three different scenarios to test the effects of increasing income inequality. We begin by analysing different levels of skewness related to the distribution of incomes in a creditor friendly economy and conduct the same exercise for a debtor friendly economy thereafter. We should expect increasing inequality to come along with higher aggregate debt and a higher number of insolvencies as relatively more households concentrate at the lower part of the income distribution where the desire to keep up with other households is stronger. Using the same starting values for the creditor friendly and debtor friendly economy, we should, however, expect lower aggregate debt and fewer insolvencies for creditor friendly insolvency regimes.

Table 1 shows the initial parameter values for the simulation. As described in Section 3, a household's desire to keep up with other households' consumption,  $\gamma_2$ , differs across insolvency regimes as households' behaviour is influenced by the underlying institutional setting. Note that we model upward-looking preferences, i.e. only households whose incomes are below the median care about others' consumption.  $\lambda_{1,2}$  is the credit line parameter and can be interpreted as expectations about future economic development; it is kept constant across scenarios as the role of credit lines has already been studied in previous work ([König and Grössl, 2014](#)). The

parameter which determines the minimum income,  $\theta$ , is also kept constant as we assume that it is based on socio-political motives rather than being subject to optimal insolvency laws. According to the laws of several EU countries (i.e. German law (§§ 832, 835 ZPO); French law (§ L312-2, Code de la consommation); Dutch law (FW § 295 and Wetboek van Burgerlijke Rechtsvordering, § 475d)), the share of seizable income depends on various variables like income, household members, etc.  $\theta$  is an approximative factor, based on averages from these laws.

For internal validation of the model, we run 300 independent simulations for each scenario, each one with a different random draw. Simulated data as reported in Tables 2 and 3 and the related Figures 3 and 4, refer to the mean value across simulations.

The model is simulated over 2000 periods plus a burn in of 100 periods for the initialisation. We assume that one time period refers to a month time as households receive wages each period.

Table 1: Initial parameter Values for the Two Model Economies

Time, $T$	2000
Number of Banks, $B$	1
Number of Households, $H$	200
$\gamma_1$ - “Standard consumption”	0.6 / 0.6
$\gamma_2$ - “Impact of others consumption”	0.4 / 0.3
$\gamma_3$ - Impact of the lending rate on consumption	0.05
$\gamma_4$ - Impact of the borrowing rate on consumption	0.05
$\theta$ - Parameter for pledgeable income	0.4
$\lambda_1$ - Credit line parameter for the richer half	2
$\lambda_2$ - Credit line parameter for the poorer half	1
$\mu$ - Location parameter of the distribution	$\bar{Y}$
$\sigma$ - Scale parameter of the distribution	Std. Dev. $Y$
$\zeta$ - Shape parameter of the distribution	0.4 / 0.5 / 0.6
$\bar{G}$ - Government expenditure	100
$\tau$ - Tax rate	0.1

## 4.2 Results

### 4.2.1 The Role of Income Distribution in a Creditor Friendly Economy

In this section we present the simulation results for the creditor friendly economy. We assume that debt relief follows after an eight-year insolvency period. Three different scenarios of varying income distributions are tested. Initial income distribution is displayed in Figure 1 and its corresponding kernel density<sup>6</sup> of the distribution variable,  $\eta$ , in Figure 2.<sup>7</sup> As described in Section 3.2, the distribution of income is modelled by means of a Generalised Pareto distribution,

<sup>6</sup>Kernel density estimates the probability distribution of a random variable.

<sup>7</sup>Although the analysis is based only on small differences in the skewness, substantial effects can already be observed.

Figure 1: Income at  $t=1$ , Cumulative Generalised Pareto Distribution

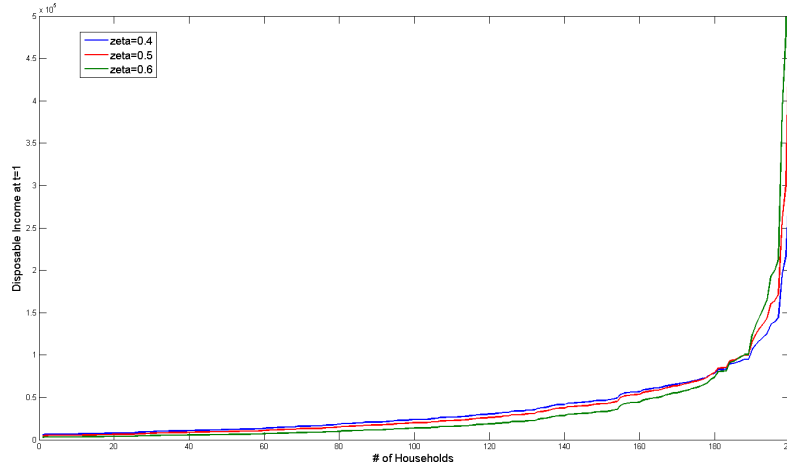
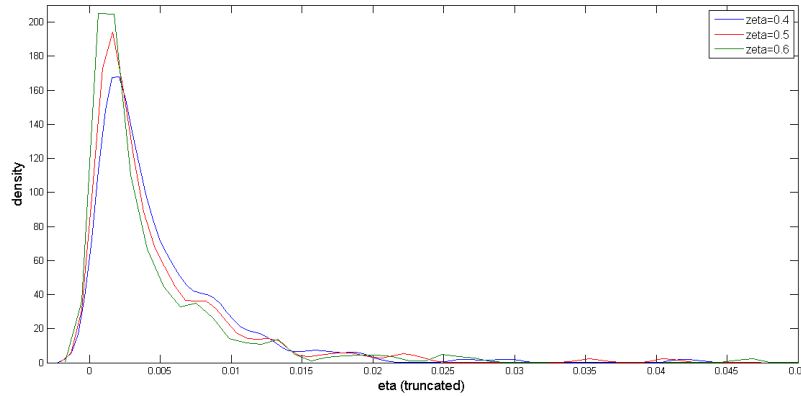


Figure 2: Kernel density of the distribution variable  $\eta$



i.e. with increasing skewness few rich households get richer, while the overall number of poor households increases. Higher  $\zeta$ 's represent higher inequality. Households are subject to wage shocks, which can change the distribution of income over time. However, as we assume that the magnitude of shocks is the same across scenarios, their impact on the distribution of incomes weakens for higher income inequality.

From Table 2, we can see that GDP decreases with rising inequality. Explanations follow from the interplay of households social orientation and the prevailing insolvency regulation. With increasing income inequality, other households' consumption gains importance as more households earn lower incomes. Households at the bottom of the income distribution increasingly seek to keep up with their richer peers through debt-financed consumption. However, increasing inequality renders fewer households creditworthy and they hence face difficulties to repay their debt. This is evidenced by a sharp rise in the number of insolvencies for stronger skewness. A higher number of bankrupt households has to live on a minimum income during the insolvency

period, which has a negative effect on the level of aggregate consumption, as expressed in equation (18). This again puts a downward pressure on the level of GDP for higher inequality. The effect on GDP volatility, as measured by the variance of time series, is less intuitive. It decreases with increasing inequality. Within the framework of the model, volatility of macroeconomic times series such as consumption and hence GDP, is predominantly caused either by household specific wage shocks which influence consumption and hence loan demand, and/or macroeconomic supply shocks. Yet, both are kept constant across scenarios. The explanation therefore derives from the changing income distribution which affects borrowing behaviour through changing consumption preferences, thereby strongly influencing the composition of consumer types as well as the size of loans required to finance desired consumption. Given this, a major source of volatility is repeated loan taking by households. The duration of loan contracts of only one period and repeated repayment behaviour directly affects borrowers consumption patterns. In this respect, the higher number of insolvent households that are excluded from credit markets as well as slightly tighter credit constraints in higher inequality scenarios suggests a stabilising effect on the aggregate level.

Surprisingly, aggregate debt declines for rising inequality. Three determinants can be identified in this regard: demand-side, supply-side and institutional factors. On the demand side, an explanation follows from decreasing median income when inequality is high. Although more households earn relatively lower incomes, the amount of loans they require to satisfy desired consumption declines as the macroeconomic part of equation (1) is lower for higher inequality. As a supply-side factor, tightening credit conditions reduce aggregate debt. As can be seen from equation (13), lenders take a households income into account when deciding about loan supply. With relatively more households earning lower incomes, credit constraint tighten (as also confirmed in Table 2). Moreover, equation (14) shows that below median income households face even tighter borrowing constraints than above median income households (as expressed through the parameter  $\lambda$ ). Another reason is to be found in the institutional environment, namely the comparatively long duration of the insolvency period in the creditor friendly regime: Defaulting households remain excluded from credit markets during the restructuring process and are only discharged from residual debt after eight years. The rising number of insolvencies for higher inequality regimes hence points to a lower number of households that participate at credit markets.

Aggregate savings increase with increasing skewness as the rich get richer and accordingly save more. This is also confirmed by higher wealth-to-GDP ratios.

Table 2: Simulation Results: Creditor Friendly, Debt relief after 8 years ( $\gamma_2 = 0.3$ )

Scenario	1a	1b	1c
Shape parameter of the Income Distribution	$\zeta=0.4$	$\zeta=0.5$	$\zeta=0.6$
GDP (mean), in mio.	4,369	4,329	4,288
$\sigma^2$ GDP (cyclical), in mio.	30,913	29,525	29,396
GDP Growth Rate	0,00016	0,00009	0,00006
Aggregate Debt, $\sum L$ , in mio.	61,64	58,79	53,91
Aggregate Savings, $\sum D$ , in bn.	4,905	4,985	5,066
Max. income	365420	469030	603450
Median Income	18308	16438	14501
Number of insolvencies	36	55	112
Credit constraints in %	49,99	49,99	50,01
Debt-to-GDP ratio	14,10	13,58	12,57
Wealth-to-GDP ratio	1122,44	1151,49	1181,55

The table reports mean values for the 300 iterations.

Figure 3: GDP - Creditor friendly regime (Moving averages), Generalised Pareto Distribution,  $\gamma_2 = 0, 3$

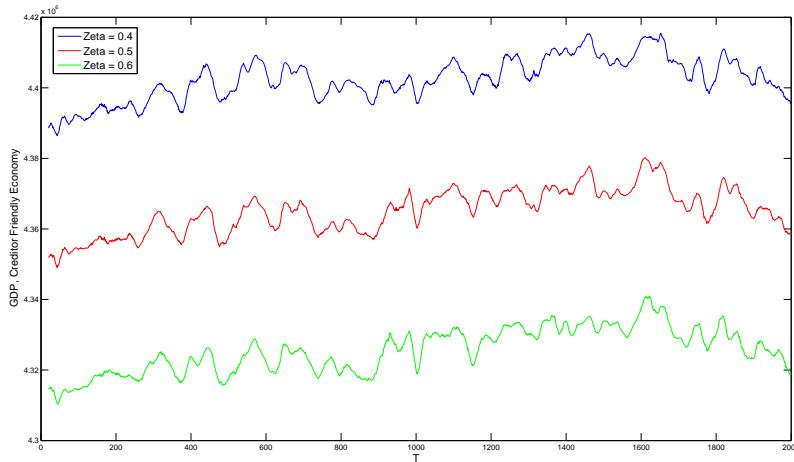


Figure 3 shows the course of GDP development across all inequality scenarios for the creditor friendly economy. One can also see, that in spite of the decrease in the level of GDP for higher inequality, GDP time series still show a positive growth rate under pro-creditor laws.

#### 4.2.2 The Role of Income Distribution in a Debtor Friendly Economy

We now turn to a debtor friendly regime where residual debt of defaulting households is discharged after three years.<sup>8</sup> Again we simulate three different scenarios for varying income distributions. We now assume that households incorporate the pro-debtor insolvency law in

<sup>8</sup>Despite the existence of more debtor friendly insolvency regulations, such as the US or UK law, we decided for the three year insolvency period as it represents the most debtor friendly law in Continental Europe.

their consumption decision. As described in Section 3.3, households have stronger incentives to take up higher amounts of debt, as they are aware that discharge from debt follows after a relatively short time period (see also Grössl and Fritsche, 2007b). As opposed to previous scenarios, “keeping up with the Joneses” behaviour is therefore more pronounced which results in an overall higher propensity to consume.

In line with the creditor friendly economy, we observe lower levels of GDP with increasing inequality (see Table 3 and also Figure 4). Again aggregate debt decreases and the number of insolvencies increases for stronger skewness.

As households at the lower scale of the income distribution have an overall higher propensity to consume as opposed to the creditor friendly regime, aggregate debt is much higher. Yet, despite the increasing relevance of other households’ consumption, higher inequality leads to a reduction in aggregate debt as observed for the creditor friendly case. While the underlying causes are again the decrease in median income leading to both lower demand for credit as well as a tightening supply, the influence of the prevailing debtor friendly insolvency regulation on aggregate debt is comparatively weaker: Shorter insolvency periods imply that insolvent households are released from remaining debt earlier, and hence regain access to credit markets faster. Although earlier access to debt contributes to slightly higher aggregate consumption, and hence higher levels of GDP, it also leads to very high levels of debt and repeated insolvencies.<sup>9</sup>

As in the creditor friendly economy, GDP volatility is largest if inequality is lowest, because the higher propensity to consume of low income households as opposed to high income households exposes them to a higher risk of default. Hence, the number of insolvencies is already relatively high for the scenario with lowest inequality. Volatility of time series decreases for the higher inequality scenarios. First, because relatively more insolvent households are excluded from credit markets, and second, due to tightening credit on the side of the lenders. Interestingly, volatility is lowest for medium inequality, and slightly higher for the highest inequality scenario. An explanation follows from reduced loan demand due to lower median income and therefore lower aggregate debt: Although, the largest number of defaults is documented for the high inequality scenario, the rise of defaults from the medium to the high inequality scenario

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<sup>9</sup>From a creditors perspective, debtor friendly insolvency laws imply that they can seize less of a insolvent households wealth. As the bank is always bailed out in our model, there are no repercussions on GDP dynamics.

Table 3: Simulation Results: Debtor Friendly, Debt relief after 3 years ( $\gamma_2 = 0.4$ )

Scenario	2a	2b	2c
Shape parameter of the Income Distribution	$\zeta=0.4$	$\zeta=0.5$	$\zeta=0.6$
GDP (mean), in mio.	4,489	4,438	4,385
$\sigma^2$ GDP (cyclical), in mio.	191,555	147,411	154,519
GDP Growth Rate	-0,00044	-0,00021	-0,00027
Aggregate Debt, $\sum L$ , in mio.	179,469	163,647	144,677
Aggregate Savings, $\sum D$ , in bn.	4,801	4,894	4,987
Max. income	365420	469030	603450
Median Income	18304	16435	14498
Number of insolvencies	153	303	489
Credit constraints in %	50,00	50,05	50,08
Debt-to-GDP ratio	39,98	36,88	32,99
Wealth-to-GDP ratio	1069,43	1102,73	1137,37

The table reports mean values for the 300 iterations.

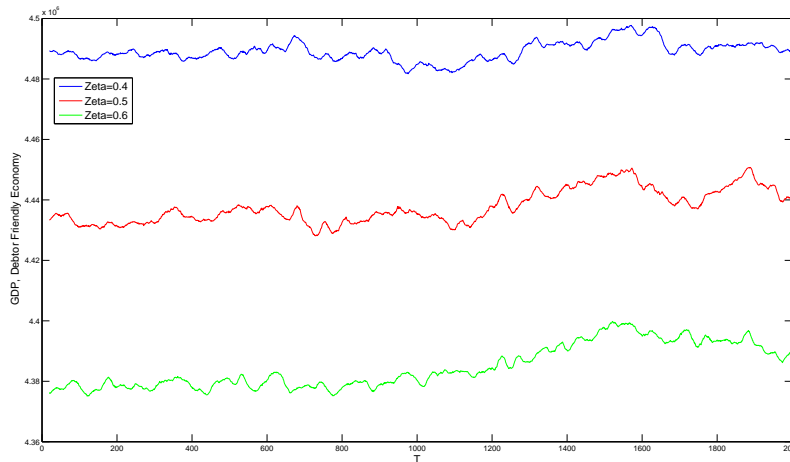
(approx. 1,61 times as many defaults) is lower than from the low to the medium inequality scenario (where almost twice as much defaults are documented). We presume that the effect of excluded households from credit markets lowers aggregate volatility for the medium inequality scenario, whereas in the high inequality scenario, the size of loan demand is relatively lower and credit constraints are relatively higher enabling relatively more debtors to repay their obligations. In this regard, insolvency laws act as a device to reduce volatility for medium inequality.

Contrary to the creditor friendly regime, from Figure 4 and Table 3, one can see that GDP time series show a slightly negative growth rate. The debtor friendly regime with stronger incentives to overborrow has an overall negative effect on GDP development. We can further see from Table 3, that like in the creditor friendly regime, savings increase for higher inequality as the range of the income distribution is expanded. Under the debtor friendly regulation however, the overall amount of savings is lower across all levels of skewness as low median income households have a higher propensity to consume and hence fewer households save at all.

#### 4.2.3 Debtor versus Creditor Friendly Economies

Opposing the debtor friendly to the creditor friendly insolvency regime, several results stand out. First, in both model economies, inequality has a negative effect on the level of GDP. Second, given that the need for external finance rises with increasing inequality, insolvency laws can have

Figure 4: GDP - Debtor friendly regime with moral hazard (Moving averages), Generalised Pareto Distribution,  $\gamma_2 = 0,4$



a stabilising effect on GDP volatility as they reduce credit market activity. The stabilising effect is substantially larger for the creditor friendly regime.

Third, the level of GDP, GDP volatility, aggregate debt and the number of insolvencies are considerably higher in the debtor friendly economy, while savings are slightly lower. Because credit demand is remarkably larger due to low income households' higher propensity to consume, not only debt-financed consumption as an essential element of GDP is higher, but also aggregate debt leading to both higher aggregate volatility and a higher number of insolvencies. Forth, from Figures 3 and 4, we can see that the creditor friendly economy exhibits positive growth rates, while we observe slightly negative growth for the debtor friendly economy over the course of the simulation. Put differently, while debt-financed consumption increases the level of the GDP in the first place it can have negative effects on economic growth. The latter is particularly pronounced under moral hazard behaviour.

To sum up, from a microeconomic perspective, debtor friendly insolvency laws promote higher consumption in the favour of households preferences. Moreover, households profit from earlier discharge of residual debt ex post insolvency. Creditor friendly economies on the contrary prevent them ex ante from taking up unsustainable levels of debt not defaulting in the first place. On the macroeconomic level however, debtor friendly insolvency laws impede economic growth and negatively affect macroeconomic stability while the opposite is true for creditor friendly laws.



## 5 Summary and Conclusion

Insolvency laws vary strongly across countries. An optimal insolvency law, that balances the conflicting interests of creditors and debtors, is still the subject of controversy among both, scholars and policymakers. The debate is essentially based on the large number of filings in the United States where an extremely pro-debtor consumer insolvency law is often deemed to promote strategic default (see for instance [White, 2007a](#)). In Europe, where insolvency laws are very heterogeneous, yet rather classified to be creditor friendly, lawmakers are currently discussing about uniform insolvency laws for the EU to prevent debtors from using legal loopholes such as defaulting under the law of a more pro-debtor EU country. This process is however, not only complicated because of the difficulty of finding an optimal insolvency law, but also because of the strong heterogeneity of European countries with respect to institutional, economical and social factors. Our results have therefore important implications as we analysed heterogeneous income distributions under different insolvency laws.

In this paper we have explored the effects of increasing inequality under varying consumer insolvency regulations. In doing so, we applied an agent-based model of household-bank relationships where increasing inequality leads to higher credit demand. We further assumed that pro-debtor policies are more likely to promote moral hazard as punishment in case of default is less severe. Opposing a creditor friendly with a debtor friendly economy we find that aggregate debt, aggregate consumption and the levels of GDP are higher under pro-debtor regulations while volatility and economic growth perform better under pro-creditor regulations. In fact, the tendency of low income households to overborrow under pro-debtor laws puts a downward pressure on economic growth, while GDP growth rates are positive in the creditor friendly regime, where debt-financed consumption is more sustainable. In both model economies however, insolvency laws have a stabilising effect for higher inequality, while the effect is more pronounced under creditor friendly laws. From a microeconomic perspective, insolvent households are better off under debtor friendly regimes in the sense that the insolvency duration is shorter, while creditor friendly insolvency regimes prevent households from overborrowing in the first place.

## References

- Adler, B., B. Polak, and A. Schwartz (1999, November). Regulating Consumer Bankruptcy: A Theoretical Inquiry. Yale School of Management Working Papers ysm128, Yale School of Management.
- Ahlquist, J. S. and B. Ansell (2014). Does inequality induce more borrowing? electoral institutions and responses to economic polarization.
- Athreya, K. (2006, November). Fresh start or head start? Uniform bankruptcy exemptions and welfare. *Journal of Economic Dynamics and Control* 30(11), 2051–2079.
- Athreya, K. B. (2008, May). Default, insurance, and debt over the life-cycle. *Journal of Monetary Economics* 55(4), 752–774.
- Atkinson, A. and S. Morelli (2010). Inequality and banking crises: A first look. In *Paper prepared for the Global Labour Forum in Turin organised by the International Labor Organization*.
- Barba, A. and M. Pivetti (2009). Rising household debt: Its causes and macroeconomic implications - a long-period analysis. *Cambridge Journal of Economics* 33(1), 113–137.
- Bebchuk, L. A. (2002). The ex ante costs of violating absolute priority in bankruptcy. *Journal of Finance* 57, 445–460.
- Belabed, C., T. Theobald, and T. van Treeck (2013, November). Income Distribution and Current Account Imbalances. INET Research Notes 36, Institute for New Economic Thinking (INET).
- Berkovich, E. and R. Israel (1999). Optimal bankruptcy laws across different economic systems. *The Review of Financial Studies* 12(2), 347–377.
- Brandolini, A. (2007, April). Measurement of income distribution in supranational entities the case of the european union. *Banca D'Italia, temi di discussione del Servizio Studi* (No. 623).
- Carbone, E. and J. Duffy (2014). Lifecycle consumption plans, social learning and external habits: Experimental evidence. *Journal of Economic Behavior & Organization* 106, 413–427.
- Cardaci, A. (2014). Inequality, household debt and financial instability: an agent-based perspective.

- Chatterjee, S. and G. Gordon (2012). Dealing with consumer default: Bankruptcy vs garnishment. *Journal of Monetary Economics* 59, Supplement(0), S1 – S16. Supplement issue:October 15-16 2010 Research Conference on 'Directions for Macroeconomics: What did we Learn from the Economic Crises' Sponsored by the Swiss National Bank (<http://www.snb.ch>).
- Cohen-Cole, E., B. Duygan-Bump, and J. Montoriol-Garriga (2009). Forgive and forget: Who get credit after bankruptcy and why? *Boston Reserve Bank, Working Paper No. QAU09-2*.
- Cynamon, B. Z. and S. M. Fazzari (2008, October). Household debt in the consumer age: Source of growth–risk of collapse. *Capitalism and Society* 3(2), 1–32.
- Delli Gatti, D., S. Desiderio, E. Gaffeo, P. Cirillo, and M. Gallegati (2011). *Macroeconomics from the bottom-up*. Springer Milan.
- Delli Gatti, D., E. Gaffeo, M. Gallegati, G. Giulioni, and A. Palestrini (2008). *Emergent Macroeconomics – An Agent-Based Approach to Business Fluctuations*. Berlin, Milan, Heidelberg, New York: Springer.
- Dobbie, W. and J. Song (2014). Debt relief and debtor outcomes: Measuring the effects of consumer bankruptcy protection. *American Economic Review* (forthcoming).
- Drechsel-Grau, M. and K. D. Schmid (2013). Saving behavior under upward looking comparisons in germany, 1995-2011. *IMK Working Paper - Macroeconomic Policy Institute* (118).
- Drechsel-Grau, M. and K. D. Schmid (2014). Consumption and savings decisions under upward-looking comparisons. *Journal of Economic Behavior & Organization* 106(0), 254–268.
- Duesenberry, J. (1949). *Income, Saving and the Theory of Consumer Behavior*. Harvard University Press, Cambridge, Massachusetts.
- Dynan, K. E., J. Skinner, and S. P. Zeldes (2004, April). Do the Rich Save More? *Journal of Political Economy* 112(2), 397–444.
- Fay, S., E. Hurst, and M. J. White (2002, June). The household bankruptcy decision. *American Economic Review* 92(3), 706–718.
- Fischer, T. (2013). Inequality and financial markets - a simulation approach in a heterogeneous agent model. In *Managing Market Complexity - The Approach of Artificial Economics*. Springer.

- Flynn, E. (1999). Bankruptcy by the numbers. *Bankruptcy Institute Journal* 18(1526).
- Frank, R. H., A. S. Levine, and O. Dijk (2014, January). Expenditure Cascades. *Review of Behavioral Economics* 1(1-2), 55–73.
- Franzini, M. (2009, November/December). Why europe needs a policy on inequality. *Intereconomics* 44(6).
- Fredriksen, K. B. (2012). Income inequality in the european union. *OECD Economics Department Working Papers* (No. 952).
- Gerhard, M. (2009). Consumer bankruptcy regimes and credit default in the us and europe - a comparative study. *CEPS Working Document* 218.
- Grössl, I. and U. Fritsche (2007a). Default option, risk-aversion and household borrowing behaviour. *DEP Discussion Papers Macroeconomics and Finance Series* 5.
- Grössl, I. and U. Fritsche (2007b). The store-of-value-function of money as a component of household risk management. *DIW Berlin Discussion Papers* 660.
- Han, S. and W. Li (2011, March-April). Household borrowing after personal bankruptcy. *Journal of Money, Credit and Banking* 43(No.2-3).
- Heuer, J.-O. (2014). *Rules and Norms of Consumer Insolvency and Debt Relief - A Comparison and Classification of Personal Bankruptcy Systems in 15 Economically Advanced Countries*. Ph. D. thesis, Bremen International Graduate School of Social Sciences.
- Iacoviello, M. (2008). Household debt and income inequality: 1963-2003. *Journal of Money, Credit and Banking* 40(5), 929–965.
- Jagtiani, J. and W. Li (2014). Credit access after consumer bankruptcy filing: New evidence. *Federal Reserve Bank of Philadelphia, Supersedes Working Paper No. 13-24*.
- Jappelli, T., M. Pagano, and M. di Maggio (2008, October). Households' indebtedness and financial fragility. CSEF Working Papers 208, Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy.
- Kirman, A. P. (1995, October). Economies with interacting agents. Discussion Paper Serie A 500, University of Bonn, Germany.

- König, N. and I. Grössl (2014, June). Catching up with the Joneses and Borrowing Constraints: An Agent-based Analysis of Household Debt. *Macroeconomics and Finance Series 201404*, Hamburg University, DEP (Socioeconomics) Discussion Papers, Macroeconomics and Finance Series.
- Levy, M. and S. Solomon (1997). New evidence for the power-law distribution of wealth. *Physica A* 242, 90–94.
- Livshits, I., J. MacGee, and M. Tertilt (2007, March). Consumer Bankruptcy: A Fresh Start. *American Economic Review* 97(1), 402–418.
- Livshits, I., J. MacGee, and M. Tertilt (2010). Accounting for the rise in consumer bankruptcies. *American Economic Journal: Macroeconomics* 2(2), 165–93.
- McDonald, J. (1984). Some generalized functions for the size distribution of income. *Econometrica* 52(3), 647–63.
- Niemi-Kiesilainen, J. (1999). Consumer bankruptcy in comparison: Do we cure a market failure or a social problem. *Osgoode Hall Law Journal* 37(1/2).
- Picklands, J. (1975). Statistical inference using extreme order statistics. *The Annals of Statistics* 3, 119–131.
- Piketty, T. and E. Saez (2003). Income inequality in the united states. *Quarterly Journal of Economics* Vol. CXVIII(1).
- Porter, K. (2011). The pretend solution: An empirical study of bankruptcy outcomes. *Texas Law Review* 90:103, 103–162.
- Povel, P. (1999). Optimal “soft” or “tough” bankruptcy procedures. *Journal of Law, Economics, and Organization* 15, 659–684.
- Rajan, R. (2010). *Fault Lines: How Hidden Fractures Still Threaten The World Economy*. Princeton University Press.
- Tesfatsion (2006). *ACE: A Constructive Approach to Economic Theory*, Chapter 16, pp. 831–880. *Handbook of Computational Economics: Agent-Based Computational Economics*.
- vanTreeck, T. (2014). Did inequality cause the U.S. financial crisis? *Journal of Economic Surveys* 28(3), 421–448.

Wang, H. and M. White (2000). An optimal personal bankruptcy system and proposed reforms. *Journal of Legal Studies XXXI(1)*, 255–286.

White, M. J. (1998). Why it pays to file for bankruptcy: A critical look at the incentives under u.s. personal bankruptcy law and a proposal for change. *The University of Chicago Law Review* 65(3), 685–732.

White, M. J. (2007a). Abuse or protection? consumer bankruptcy reform under bapcpa. *Economie publique, Public economics*.

White, M. J. (2007b). Bankruptcy reform and credit cards. *National Bureau of Economic Research (NBER)* (Working Paper 13265).

## Appendix: The Complete Model

### Households

Households,  $h = \{1, \dots, H\}$ , receive income  $Y_{ht}$ , composed of wage income  $Y_{ht}^W$ , non-wage income  $Y_t^{NW}$  and taxes  $T_{ht}$ .

$$Y_{ht} = Y_{ht}^W + Y_t^{NW} - T_{ht} \quad (20)$$

Wage income is exogenously given at  $t = 0$ .

$$\begin{aligned} Y_{ht}^W &= \eta_{ht} Y_t \\ \eta_{ht} &= \eta_{ht-1} + u_{ht} \end{aligned} \quad (21)$$

with  $Y_t = \text{GDP}$ ;  $\eta_{ht}$  = parameter of income distribution with  $\sum \eta_{ht} = 1$  and an idiosyncratic shock uniformly distributed on the support  $(0, h_u)$ . Income is distributed according to a Generalised Pareto distribution, where the cumulative distribution function of a random variable  $N$  is:

$$P(N \leq \eta) = \begin{cases} 1 - \left(1 + \frac{\zeta(\eta - \mu)}{\sigma}\right)^{-1/\zeta} & \text{for } \zeta \neq 0 \\ 1 - \exp\left(-\frac{\eta - \mu}{\sigma}\right) & \text{for } \zeta = 0 \end{cases} \quad (22)$$

with  $\mu$  = location parameter,  $\sigma$  = scale parameter,  $\zeta$  = shape parameter.

Non-wage income:  $Y_{ht}^{NW} = i_{Dt-1} D_{ht-1}$  or  $Y_{ht}^{NW} = -i_{Lt-1} L_{ht-1}$ , depending on whether the household saves or dissaves.

Taxes are  $T_{ht} = \tau(Y_{ht}^W + i_{Dt-1} D_{ht-1})$

$D_h =$  Deposits

$L_h =$  Loans

Desired consumption  $C_t^*$  is:

$$C_{ht}^* = \gamma_1 Y_{ht-1} + \gamma_2 \bar{C}_{t-1} - \gamma_3 i_{Dt} - \gamma_4 i_{Lt} \quad (23)$$

$\gamma_1 =$  marginal propensity to consume out of disposable income

$\gamma_2 =$  parameter of the Joneses effect

$\gamma_3, \gamma_4 =$  reaction coefficients for the lending and borrowing rate, respectively.

$\gamma_j \in (0, 1)$  for  $j = 1, 2, 3, 4$ .

With the median economy-wide consumption of the last period

$$\bar{C}_{t-1} = \frac{1}{2}(C_{\frac{h}{2}, t-1} + C_{\frac{h}{2}+1, t-1}). \quad (24)$$

and upward-looking consumption preferences

$$\gamma_2 = \begin{cases} \gamma_2 & \text{if } \frac{Y_{ht-1}}{\frac{1}{2}(Y_{\frac{h}{2}, t-1} + Y_{\frac{h}{2}+1, t-1})} \leq 1 \\ 0 & \text{if } \frac{Y_{ht-1}}{\frac{1}{2}(Y_{\frac{h}{2}, t-1} + Y_{\frac{h}{2}+1, t-1})} > 1 \end{cases} \quad (25)$$

We have four types of consumers. They either save, borrow or neither or are bankrupt. Loans and deposits have a duration of one period.

- **Type 1:**  $D_{ht-1} = L_{ht-1} = BG_{ht} = 0$  with  $BG_h =$  cash;  $Y_{ht} > Y_{ht,j}^{np}$

$$Y_{ht} = (1 - \tau)Y_{ht}^W. \quad (26)$$

$$C_{ht} = \begin{cases} C_{ht}^* & \text{if } Y_{ht} \geq C_{ht}^* \\ C_{ht}^* & \text{if } C_{ht}^* > Y_{ht} \text{ \& } L_{ht}^d \leq L_{ht}^{\max} \\ Y_{ht} + L_{ht}^{\max} & \text{if } L_{ht}^d > L_{ht}^{\max}. \end{cases} \quad (27)$$

with

$$L_{ht}^d = C_{ht}^* - Y_{ht} \geq 0, \quad (28)$$

$$L_{ht} = \min(L_{ht}^d, L_{ht}^{\max}). \quad (29)$$

$$D_{ht} + BG_{ht} = Y_{ht} - C_{ht}^* \text{ if } Y_{ht} - C_{ht}^* > 0 \quad (30)$$

Cash holdings are a fixed proportion,  $\kappa < 1$ , of savings

$$BG_{ht} = \kappa D_{ht} \quad (31)$$

For deposits (??), this implies

$$D_{ht} = \frac{Y_{ht} - C_{ht}^*}{1 + \kappa}$$

- **Type 2:**  $D_{ht-1} > 0$  and hence  $BG_{ht-1} > 0$ , and  $L_{ht-1} = 0$ ; and  $Y_{ht} > Y_{ht,j}^{np} \rightarrow$  (“SAVER”).

$$Y_{ht} = (1 - \tau)(Y_{ht}^W + i_{D_{t-1}} D_{ht-1}). \quad (32)$$

$$C_{ht} = \begin{cases} C_{ht}^* & \text{if } C_{ht}^* \leq Y_{ht} + D_{ht-1}(1 + \kappa) \\ C_{ht}^* & \text{if } C_{ht}^* > Y_{ht} + D_{ht-1}(1 + \kappa) \ \& \ L_{ht}^d \leq L_{ht}^{\max} \\ Y_{ht} + D_{ht-1}(1 + \kappa) + L_{ht}^{\max} & \text{if } L_{ht}^d > L_{ht}^{\max} \end{cases} \quad (33)$$

with

$$L_{ht}^d = C_{ht}^* - Y_{ht} - D_{ht-1} \geq 0. \quad (34)$$

$$L_{ht} = \min(L_{ht}^d, L_{ht}^{\max}) \quad (35)$$

Accumulation of deposits if

$$D_{ht} + BG_{ht} = Y_{ht} - C_{ht}^* - D_{ht-1} - BG_{ht-1} > 0 \quad (36)$$

- **Type 3:**  $D_{ht-1} = BG_{ht-1} = 0$  and  $L_{ht-1} > 0$ ; and  $Y_{ht} > Y_{ht,j}^{np} \rightarrow$  (“BORROWER”).

$$Y_{ht} = (1 - \tau)Y_{ht}^W - L_{ht-1}i_{L_{t-1}}. \quad (37)$$



$$C_{ht} = \begin{cases} C_{ht}^* & \text{if } C_{ht}^* \leq Y_{ht} - L_{ht-1} \\ Y_{ht} - L_{ht-1} & \text{if } C_{ht}^* > Y_{ht} - L_{ht-1} \geq Y_{ht,j}^{np} \\ Y_{ht,j}^{np} & \text{if } C_{ht}^* > Y_{ht} - L_{ht-1} < Y_{ht,j}^{np} \end{cases} \quad (38)$$

with non-pledgeable income

$$Y_{ht,j}^{np} = \theta Y_{ht} \quad \text{with } 0 < \theta < 1 \quad (39)$$

$$D_{ht}(1 + \kappa) = Y_{ht} - L_{ht-1} - C_{ht}^* \geq 0 \quad (40)$$

Bankrupt households are excluded from credit markets:

$$L_{ht} = 0 \quad \text{if } Y_{ht} - L_{ht-1} \leq Y_{ht,j}^{np} \quad (41)$$

- **Type 4:**  $D_{ht-1} \geq 0$  and hence  $BG_{ht-1} \geq 0$ , and  $L_{ht-1,j} = L_{ht-1}$ ; and  $Y_{ht} = Y_{ht,j}^{np} \rightarrow$  (“BANKRUPT”). Although bankrupt households can only accumulate deposits in very rare cases, for instance after profiting from a positive wage shock, this rather unlikely, but theoretically possible option is included in the formalisation. Income of an insolvent household is

$$Y_{ht} = \theta Y_{ht,j} = \theta[(1 - \tau)Y_{ht}^W + i_{Dt-1}D_{ht-1}]. \quad (42)$$

$$C_{ht} = \begin{cases} C_{ht}^* & \text{if } C_{ht}^* \leq Y_{ht,j}^{np} + D_{ht-1} \\ Y_{ht,j}^{np} & \text{if } C_{ht}^* < Y_{ht,j}^{np} + D_{ht-1} \end{cases} \quad (43)$$

Deposits amount to

$$D_{ht}(1 + \kappa) = C_{ht}^* - Y_{ht,j}^{np} - D_{ht-1}(1 + \kappa) \geq 0 \quad (44)$$

Households’ insolvency period,  $j = 1, \dots, J$ , continues until  $j = J$ .

$$Y_{ht+1} = \begin{cases} Y_{ht+1,j}^{np} & \text{if } Y_{ht} = Y_{ht,j}^{np} \quad \& \quad j < J \\ Y_{ht+1} & \text{if } Y_{ht} = Y_{ht,j}^{np} \quad \& \quad j = J \end{cases} \quad (45)$$

Consumption preferences differ in debtor and creditor friendly economies.

$$\gamma_2 = \begin{cases} \gamma_2^{DF} & \text{if } J^{DF} \\ \gamma_2^{CF} < \gamma_2^{DF} & \text{if } J^{CF} \end{cases} \quad (46)$$

## The Financial Sector

The financial sector consists of commercial banks and the central bank. We refer to commercial banks as “the bank”, which is representative for a consolidated banking sector.

The balance sheet constraint of the commercial bank is

$$L_t^s + B_t^s = D_t^d + F_t^d \quad (47)$$

with

$L_t^s$  = loan supply to  $H$

$B_t^s$  = loan supply to the government

$D_t^d$  = deposit demand of the bank

$F_t^d$  = demand for central bank money

Interest rates on loans,  $i_{Lt}$ , and deposits,  $i_{Dt}$ , are set by the bank following the policy rate

$$i_{Ft} = i_{Ft-1} + \phi\epsilon_t \quad (48)$$

set by the central bank, with  $\epsilon_t = \epsilon_{t-1} + \nu_t$  and  $\nu_t$  drawn from a normal distribution with support  $\nu \sim N(0, \sigma_\nu^2)$ . The interest rate pattern is

$$\begin{aligned} i_{Lt} &= \beta i_{Ft} & \beta &\geq 1 \\ i_{Dt} &= \Psi i_{Ft} & \Psi &\leq 1 \\ i_{Bt} &= i_{Ft}. \end{aligned} \quad (49)$$

$i_{Bt}$  = interest rates on government bonds (no mark-up)

$B_t^s$  = loan supply to the government

$D_t^d$  = deposit demand of the bank

$F_t^d$  = demand for central bank money

The bank accepts all deposits from households.

$$D_t^d = \sum_{h=1}^H D_{ht}. \quad (50)$$

Public debt bears no risk, hence government bonds are always granted

$$B_t^s = B_t. \quad (51)$$

This does not apply for private debt. Loans are considered to be risky if

$$L_{ht}(1 + i_{Lt}) > Y_{ht+1}$$

Because the bank can only seize a share of defaulted households income,  $Y_{ht} - Y_{ht,j}^{np}$ , losses are even higher

$$\mathfrak{L}_{ht} = L_{ht}(1 + i_{Lt}) - (Y_{ht+1} - Y_{ht+1,j}^{np})$$

$\Leftrightarrow$

$$L_{ht}(1 + i_{Lt}) \leq (Y_{ht+1} - Y_{ht+1,j}^{np}).$$

As the bank only knows the household's previous income, it takes the latter as a proxy. Thus, a bank that wants to avoid any loss, imposes the following credit constraint

$$L_{ht}^{\max} \leq \frac{(Y_{ht-1} - Y_{ht-1,j}^{np})}{1 + i_{Lt}}. \quad (52)$$

The bank additionally distinguishes between below median and above median income households, following  $\lambda$  which is set by the central bank and can be interpreted a parameter that mirrors current economic conditions.

$$L_{ht}^{\max} = \lambda \frac{(Y_{ht-1} - Y_{ht-1,j}^{np})}{1 + i_{Lt}} \quad (53)$$

where

$$\lambda = \begin{cases} \lambda_1 & \text{if } \frac{Y_{ht-1}}{\text{median} \sum Y_{ht-1}} < 1 \\ \lambda_2 > \lambda_1 & \text{if } \frac{Y_{ht-1}}{\text{median} \sum Y_{ht-1}} \geq 1 \end{cases} \quad (54)$$

Lending behaviour towards private households is

$$L_t^s = \min \left( L_{ht}^d, L_{ht}^{\max} \right) \quad (55)$$

Central bank loans are required whenever deposits are not sufficient to cover lending. The bank's cashflow at the beginning of each period is

$$CF_t^B = x_{Ht} + B_{t-1} (1 + i_{Ft-1}) - \sum D_{ht-1} (1 + i_{Dt-1}) - F_{t-1} (1 + i_{Ft-1}) \quad (56)$$

$$x_{Ht} = \sum_{h=1}^H x_{ht} \quad (57)$$

where  $x_{ht}$  represents actual loan repayments of households, and  $x_{Ht}$  aggregate repayments

$$x_{ht} = \begin{cases} L_{ht-1} (1 + i_{Lt-1}) & \text{if } (1 - \theta) Y_{ht-1} \geq L_{ht-1} (1 + i_{Lt-1}) \\ (1 - \theta) Y_{ht-1} & \text{if } (1 - \theta) Y_{ht-1} < L_{ht-1} (1 + i_{Lt-1}) \\ (1 - \theta) Y_{ht-1} & \text{if } Y_{ht-1} = Y_{ht-1,j}^{\text{np}} \quad \& \quad j < J. \end{cases} \quad (58)$$

If the bank's cashflow becomes negative,  $CF_t^-$ , the central bank acts as a lender of last resort and absorbs any losses to maintain financial sector stability.

The central bank's balance sheet is

$$BG_t^s - BG_{t-1}^s = \left( F_t^d - F_{t-1}^d \right) + CF_t^- \quad (59)$$

The supply of cash in the economy changes whenever the bank has a negative cashflow,  $CF_t^-$ , resulting from unsustainable lending. Positive bank cashflows,  $CF_t^+$ , act as an additional revenue for the government.

## The Government

The government balance sheet is

$$B_t - B_{t-1} = G_t + B_{t-1} i_{Ft-1} - \left( T_{Ht} + CF_t^+ \right) \quad (60)$$

$CF_t^+$  denotes a positive bank cashflow and

$$T_{Ht} = \sum_{h=1}^H T_{ht} \quad (61)$$

Government expenditure are

$$G_t = G \quad (62)$$

rendering public borrowing the residual.

### Macroeconomics

It is assumed that firms seek to adjust current production flexibly to current aggregate demand, and that deviations follow exclusively from random shocks

$$Y_t = Y_t^d + \rho_t \quad (63)$$

$\rho_t$  is a temporary stochastic shock drawn from a uniform distribution,  $[\underline{\rho}, \bar{\rho}]$ , and with:  $\sum \rho_t = 0$ .

Aggregate demand is

$$Y_t^d = \sum_{h=1}^H C_{ht} + \bar{G} \quad (64)$$

where aggregate household consumption consists of four different subtypes of consumers

$$\sum_{h=1}^H C_{ht} = \sum_{h=1}^{H_1} Y_{ht,j}^{np} + \sum_{h=H_1+1}^{H_2} (Y_{ht} + L_{ht}^{max}) + \sum_{h=H_2+1}^{H_3} (Y_{ht} + L_{ht}^d) + \sum_{h=H_3+1}^{H_4} (\gamma_1 Y_{ht} + \gamma_2 \bar{C}_{t-1} - \gamma_3 i_{Dt} - \gamma_4 i_{Lt}). \quad (65)$$

Taking equation (63) and equation (64) together renders as the time path for aggregate production

$$Y_t = \sum_{h=1}^H C_{ht} + \bar{G} + \rho_t \quad (66)$$