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Discussion Paper No. 976

**RETAILER'S PRODUCT LINE CHOICE  
WITH MANUFACTURER'S  
MULTICHANNEL MARKETING**

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# Retailer's Product Line Choice with Manufacturer's Multichannel Marketing\*

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## Abstract

This paper studies how a retailer decides the length of product line in a vertically related industry. We study a market with two product varieties. Each retailer decides the number of varieties it procures from an upstream manufacturer. The manufacturer may open an online store and encroach on the resale market. In the case of a monopoly retailer, anticipating the online store's encroachment, the retailer may be willing to shorten its product line, although it can choose a full-length one. In the case of duopoly retailers, on the other hand, retailers may make their product lines completely overlapped, partially overlapped, or non-overlapped. Moreover, the total surplus may decrease due to the efficiency loss in the online channel, although the competition in the resale market becomes more intense.

**Keywords:** channels of distribution; encroachment; buyer power; product line choice

**JEL Classification Numbers:** L14, L22, M11

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# 1 Introduction

From the conventional wisdom, a manufacturer always decides the preferred product line that it wishes to distribute through retailers. However, recent markets have been focusing more on cases in which dominant retailers possess strong channel power to influence the manufacturer to adjust its production line so that it aligns with retailers' orders (Kadiyali et al., 2000). Faced with the loss of power in the traditional wholesale channel, more upstream manufacturers have begun to consider the benefits of "drop-shipping" and adopting online channels through which they gain additional profits (Randall et al., 2006). The emergence of such manufacturer's multichannel marketing has brought new challenges to traditional retailers in form of competition from potential online stores (Tannenbaum, 1995; Dixon and Quinn, 2004), which is sometimes referred to as "franchise encroachment" (Arya et al., 2007; Emerson, 2010). Compared with an online store that can simply put a product variety on the Internet as necessary, a physical retailer is disadvantaged by limited display space and thus has to prudently decide which variety to order from the manufacturer.

Our objective is to identify how an incumbent retailer exercises the channel power of specifying product lines when faced the potential entry of an upstream manufacturer's online store and how this consequences affects social welfare. Although the online store brings the manufacturer additional channel profits, it causes *intra-brand competition* with the incumbent retailer, with adverse effects on profits from the wholesale channel. Realizing this channel tradeoff, the manufacturer strategically chooses its online store's product line. To illustrate, let us assume that the manufacturer carries two varieties. When there is only one variety in the wholesale channel, it is possible that the manufacturer sells only a different variety in the online channel (a partial encroachment). In this manner, since the retailer and online store have a differentiated product line, direct *intra-brand competition* is alleviated. However, when both varieties are available in the wholesale channel, regardless of the type of variety that the online store sells, there is always an overlap. In such a case, *intra-brand competition* is always direct and intense, which cannot be alleviated by a partial encroachment. Given that there are no benefits to be had from a partial encroachment, the manufacturer never

sells only one variety online. Instead, it chooses either selling both varieties online (a full encroachment) or completely shutting down the online channel. That is, the manufacturer uses the online store as a tool to keep a balanced channel distribution of product varieties. It does not expect either variety to be oversupplied.

Anticipating the manufacturer's two encroaching patterns (partial or full), the retailer strategically manages its product line to alleviate the negative effect of the online store's encroachment. When the online retail cost is low enough, encroachment is inevitable. Realizing this, the retailer would rather abandon a part of its product line to trigger the manufacturer's partial encroachment. Although this means losing profits from a smaller range, the resulting partial encroachment is less harmful, which allows the retailer to stay differentiated from the online store and maintain adequate profitability from a short product line.

The above result gains wider significance if we consider an oligopoly downstream market wherein retailers' strategic interactions play an important role. Assume the simplest duopoly retailer case. When online retail cost is low enough, the retailers tend to overlap their product lines in only one variety, as if declaring a relatively higher profitability of the variety whose supply is lower. In this manner, the retailers induce the online store's partial encroachment instead of a full one. Alternatively, when online retail cost is high enough, the retailers may tend to make their product line non-overlapped to ensure a balanced distribution of varieties in the wholesale channel, which then gives the manufacturer an incentive to shut down its online channel.

From the manufacturer's viewpoint, we show that committing not to run an online store can sometimes be beneficial. Although this means sacrificing channel profits from the online store, a manufacturer can benefit from retailers' incentive to enlarge product lines, which enhances channel efficiency when physical retailers are more adapted to resale activities.

Finally, we show that even though full encroachment results in a more intense *intra-brand competition* than a partial one, it may result in lowering social welfare. This phenomenon occurs when the physical retailers are much more adapted to resale activities than the online store and the different varieties are close substitutes. Although a full encroachment is

more pro-competitive than a partial one, it reallocates a greater share to the inefficient online channel, which causes considerable social loss and may even be a dominant effect. This result has an important policy implication—product varieties of online retailing should be well-regulated, and the consequences of overlapping product lines between different channels should be avoided.

Our study has managerial implications for both retailers and manufacturers. First, although we often observe retailers competing with each other in product diversity to attract more diverse consumers, our results imply that the head-to-head competition in enlarging product lines can sometimes be inefficient even when expanding the product line does not incur additional costs. When facing a weak online store, it may be more important for retailers to better coordinate with each other so that the distribution of variety in the wholesale channel can be well-balanced. Second, the manufacturer's ability in online retailing may induce its retailers' passive behaviours and they may become less willing to carry full-length product lines. Hence, even with relatively low online retail costs, it is important for a manufacturer to assure its retailers that they are safe in keeping a full-length product line and that their territories will not be encroached upon.

Now, we discuss the theoretical literature. To the best of our knowledge, this study is the first attempt to discuss retailers' product line choices while considering manufacturer's direct marketing. However, the results and methodologies of several studies are closely related to those of our study. Dukes et al. (2009) consider a similar setting in which two retailers decide their respective product line from a multi-product manufacturer. The longer the product line, the higher is the assortment cost incurred, but this is not reckoned with in our study. They show that one of the retailers may spontaneously cut its product line to induce the rival retailer to carry the full-length line with higher assortment costs. Although we derive similar results that retailers do not choose full-length product lines, the intuitive reasoning behind this is different. In our study, which does not consider assortment costs, retailers shorten their product lines to trigger the online store's partial encroachment. Moner-Colonques et al. (2011) consider a case with two single-product manufacturers and two retailers. The retail-

ers' expansion of their product line means a multi-sourcing pattern. The authors provide us a theoretical explanation of why some retailers choose overlapping product lines. Inderst and Shaffer (2007) considers retailers' incentives for adopting a "single-sourcing" purchasing strategy (cutting the product line) and their cross-border mergers under different bargaining power scenarios. Gabrielsen and Sørsgard (1999) allow a monopoly retailer to decide whether producers should have exclusive dealership under a linear contract. Mills (2015) considers a similar scenario but under a nonlinear contract setting, where contract terms are decided by negotiations between the monopoly retailer and either or both suppliers. Gabrielsen and Sørsgard (2007) consider retailers' incentives for carrying private labels in a setting of vertically differentiated products. Although all these studies focus on buyer power in product line choices, the upstream manufacturer's incentive for direct marketing is not considered. Our study complements this literature by considering the incentive for a manufacturer to strategically recapture channel power.<sup>1</sup>

Arya et al. (2007) may be among the earliest attempts to theoretically discuss the manufacturer's encroaching behavior. A manufacturer encroaches on a retailer's territory if its retail cost is low enough. The encroachment may even benefit the retailer because the manufacturer resets a lower wholesale price to maintain the retailer's demand at an adequate level. This benefit exists only if the retailer decides its quantity before the manufacturer does so that the retailer's output reaches the Stackelberg leader's level.<sup>2</sup> Following the main structure of Arya et al. (2007), we examine a manufacturer's encroachment problem in which a retailer decides its preferred product line, but assume quantities to be simultaneously decided to remove the Stackelberg leader's advantage and to focus on the effect of product line choices. Mizuno (2012) considers a case in which two manufacturers distribute their products to  $n$  retailers by competing for a wholesale market while simultaneously deciding whether to

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<sup>1</sup>For parallel researches, Inderst and Shaffer (2010) discusses how a supplier uses different contract forms (based on the share it receives of a retailer's total purchases or on how much a retailer purchases of its products) when retailers have out-sourcing options. Milliou and Sardonís (2015) examines the relation between multi-product manufacturers' mergers and their incentive for enlarging the product line.

<sup>2</sup>Liao (2014) extends this model by considering a signaling game in which the manufacturer has private information on its own retail cost and shows that in a separating equilibrium of retail competition, the manufacturer signals inefficiency in its retail behavior by setting a lower wholesale price than that under complete information.

encroach upon the resale market. Li et al. (2015) considers a model with  $n$  vertical supply chains and analyzes each manufacturer's incentive to encroach.<sup>3</sup> Considering retailers' channel power in product line choices, we add several new insights to the literature.

The remaining paper is organized as follows. Section 2 introduces a basic model with one manufacturer owned online store and one retailer, from which we see how the retailer employs orders of different variety to affect the manufacturer-owned online store's encroachment. Section 3 extends the basic model to a duopoly retailers case. We will derive retailers' order variety in equilibrium and some related propositions. Section 4 concludes the paper.

## 2 The case of a monopoly retailer

We start with a simplest monopoly retailer case and demonstrate the basic result that the retailer's product line choice affects the online store's encroachment.

### 2.1 The basic model

Let us consider a standard vertically related market comprising one manufacturer  $M$  and one retailer  $R$ . The manufacturer produces imperfectly substitutable products  $X$  and  $Y$ , and the retailer orders either or both varieties from the manufacturer. The products are distributed to the retailer who sells them to consumers through a wholesale channel. In addition, the manufacturer can also open its online store and encroach upon the retailer's territory by directly supplying either or both product varieties to consumers. We call the competition between the retailer and the online store *intra-brand competition*. The retailer's retail cost is normalized to zero, and the online store's cost when the encroachment occurs is  $c > 0$ .<sup>4</sup> The manufac-

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<sup>3</sup>Other parallel studies discuss the manufacturer's direct selling channel in the Salop (1979) setting. In Balasubramanian (1998), the consumers buy products from conventional retailers incurring location-dependent transportation cost and are charged a fixed freight per unit that is irrelevant of location, if they buy from a direct seller. Shulman (2014) considers Balasubramanian's setting in a vertically related market. The direct seller can either buy wholesale products from the authorized retailers who are supplied by the manufacturer, which is referred to as a "gray-market," or buy wholesale products directly from the manufacturer.

<sup>4</sup>This assumption follows Arya et al. (2007). In the real world, online stores usually need to bear higher operating costs than physical stores. Unlike the physical stores, online stores have to bear the risk of returns and redress, as consumers cannot physically inspect a product before ordering. Besides, they have to endeavor to tell consumers sufficient information about products as they browse the Internet (Lieber and Syverson, 2010).



turer's production cost and the fixed cost of introducing both varieties are normalized to zero for simplicity. We assume a representative consumer's utility to be quadratic with the form

$$u(Q_X, Q_Y) = a(Q_X + Q_Y) - \frac{1}{2}(Q_X^2 + 2\gamma Q_X Q_Y + Q_Y^2) + I,$$

where  $Q_n$  denotes the aggregate quantity of variety  $n$  with  $n = X$  or  $Y$ ,  $\gamma \in (0, 1)$  denotes the substitutability between the two varieties, and  $I$  denotes consumer income. Products of the same variety are perfect substitutes whether sold by the retailer or by the online store.<sup>5</sup> Consumer demand is represented by a linear downward sloping inverse demand function  $p_n = a - Q_n - \gamma Q_{-n}$ , where  $n$  and  $-n$  are different varieties.

Let  $B$  denote both varieties, and  $N$  denote none of the varieties.  $N$  cannot be the retailer's strategy because it earns profits only when it orders wholesale products and sells them to consumers, but this is not the case for the manufacturer because it earns profits from the wholesale channel even if it does not open an online store. The game proceeds as follows: In period 1, the retailer orders variety  $X$  or both ( $B$ ).<sup>6</sup> In period 2, the manufacturer decides whether to open the online store ( $N$  if it does not open) or which variety (or varieties) to sell through the online store— $X$ ,  $Y$ , or both ( $B$ ). The manufacturer then sets the corresponding wholesale price(s)  $w_n$ . In period 3, the wholesale products are delivered to the retailer, and the retailer simultaneously competes with the online store in quantity if encroachment occurs; otherwise, the retailer monopolizes the resale market.<sup>7</sup>

The assumption of early ordering by retailers is observed in many industries, especially those in which retailers need to view the product line design as a long-term issue and forecast potential challenges, such as an online store owned by the manufacturer, long before the market forms (e.g., the apparel industry) (Iyer and Bergen, 1997). One explanation is that it is costly for a traditional retailer to face the consequences of an inappropriate order variety.<sup>8</sup>

<sup>5</sup>This assumption is made for analytical simplicity. The main propositions still hold if we allow for imperfect substitutability for the same variety sold by the retailer or the online store.

<sup>6</sup>By symmetry, because it is the same whether the retailer chooses  $X$  or  $Y$ , we can consider either case instead of both.

<sup>7</sup>For simplicity, we assume that the manufacturer cannot reject the retailer's order. If we allow for the possibility of rejection, we have a case of the manufacturer choosing the variety (varieties) for the retailer. If both varieties are chosen for the retailer, it can sell either variety or both. We find our results hold irrespective of whether the manufacturer has the option to reject.

<sup>8</sup>For example, the retailers are less likely to order "very-low-demand" varieties that cannot even cover the

However, given the advantages of inventory, display and product line adjustment, an online store often regards product line design as a short-term issue. Notice that our results hold even if the retailer and manufacturer simultaneously decide their respective product line.

## 2.2 Equilibrium product variety

Let  $K$  be the set of products the retailer orders in period 1, where  $K \subseteq \{X, Y\} \setminus \emptyset$ . Moreover, let  $L$  be the set of products the manufacturer sells through the online store, where  $L \subseteq \{X, Y\}$ . Denote the retailer's variety by  $r \in \{X, B\}$ , and that of the manufacturer by  $m \in \{N, X, Y, B\}$ . Any pair that the retailer and the manufacturer chooses,  $rm$ , defines a product line system. There may be seven different product line systems in period 3:  $rm \in \{XN, XX, XY, XB, BN, BY, BB\}$ .<sup>9</sup> Let  $q_{nR}^{rm}$  and  $q_{nM}^{rm}$  be the retailer and the online store's selling quantities of variety  $n$ , where  $n = X$  or  $Y$ . In period 3, the retailer chooses  $q_{nR}^{rm}$  to maximize its profit which is given by:

$$\pi_R^{rm} = \sum_{n \in K} [p_n(Q_n, Q_{-n}) - w_n^{rm}] q_{nR}^{rm}. \quad (1)$$

If the manufacturer does not open the online store, its profit comes only from the wholesale channel, which is given by

$$\pi_M^{rm} = \sum_{n \in K} q_{nR}^{rm} w_n^{rm}. \quad (2)$$

Otherwise, it competes with the retailer in the resale market and chooses  $q_{nM}^{rm}$  ( $m \neq N$ ) to maximize its profits which is given by

$$\pi_M^{rm} = \sum_{n' \in L} [p_{n'}(Q_{n'}, Q_{-n'}) - c] q_{n'M}^{rm} + \sum_{n \in K} q_{nR}^{rm} w_n^{rm}. \quad (3)$$

Solving the profit maximization problems in period 3, we obtain the equilibrium quantities,  $q_{nR}^{rm}(\mathbf{w})$  and  $q_{n'M}^{rm}(\mathbf{w})$ , where  $\mathbf{w} \in w_X^{rm}$  if  $K = X$ , or  $\mathbf{w} \in (w_X^{rm}, w_Y^{rm})$  if  $K = X, Y$ .

In period 2,  $w_n$  and  $w_{-n}$  are decided by the manufacturer in anticipation of the equilibrium outcomes in period 3. When the manufacturer does not open the online store ( $rm = XN$  or

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fixed cost of storing them. However, this is not the case for the online stores because the demand comes from a larger geographic market (Lieber and Syverson, 2010).

<sup>9</sup>By symmetry,  $BX$  is equivalent to  $BY$ .

$r$	$X$				$B$		
	$N$	$X$	$Y$	$B$	$N$	$Y$	$B$
$w_X^{rm}$	$\frac{a}{2}$	$\frac{5a-c}{10}$	$\frac{(8-4\gamma^2+\gamma^3)a-\gamma^3c}{2(8-3\gamma^2)}$	$\frac{5a-c}{10}$	$\frac{a}{2}$	$\frac{5a-c}{10}$	$\frac{5a-c}{10}$
$w_Y^{rm}$	$\backslash$	$\backslash$	$\backslash$	$\backslash$	$\frac{a}{2}$	$\frac{5a-\gamma c}{10}$	$\frac{5a-c}{10}$
<i>I.S.</i>	$a > 0$	$\frac{c}{a} \leq \frac{5}{7}$	$\frac{c}{a} \leq \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$	$\frac{c}{a} \leq \frac{5}{7+2\gamma}$	$a > 0$	$\frac{c}{a} \in (\frac{5\gamma}{8+8\gamma}, \frac{5}{7}]$	$\frac{c}{a} \leq \frac{5}{7}$

Table 1: Equilibrium wholesale prices

$BN$ ), it solves the following maximization problem:

$$\max_{\mathbf{w}} \sum_{n \in K} q_{nR}^{rm}(\mathbf{w}) w_n^{rm}. \quad (4)$$

When the online store encroaches ( $rm = XX, XY, XB, BY$ , or  $BB$ ), the manufacturer solves the following maximization problem:

$$\max_{\mathbf{w}} \sum_{n' \in L} [p_{n'}(Q_{n'}(\mathbf{w}), Q_{-n'}(\mathbf{w})) - c] q_{n'M}^{rm}(\mathbf{w}) + \sum_{n \in K} q_{nR}^{rm}(\mathbf{w}) w_n^{rm}. \quad (5)$$

The manufacturer decides whether to open the online store as well as the product line for the online store based on the resulting profits. The equilibrium wholesale price,  $w_n$  and  $w_{-n}$ , are denoted in Table 1, where ‘‘I.S.’’ denotes the conditions for interior solutions. For cases in which the manufacturer encroaches,  $c$  must be low enough. Moreover, for the  $BY$  case,  $c$  must not be too low ( $c/a > 5\gamma/(8 + 8\gamma)$ ), otherwise  $w_X^{rm}$  becomes so high that the retailer gives up supplying  $X$ . Thus, the  $BY$  case with  $q_{YR}^{BY} = 0$  becomes the  $XY$  case.

We can easily derive that given  $r = X$  or  $B$ , the wholesale prices decrease with more product varieties sold online. This is summarized in the following proposition:

**Proposition 1** *Given the retailer’s order variety ( $X$  or  $B$ ), the wholesale prices decrease with more product varieties sold online.*

Proposition 1 follows directly from the main result of Arya et al. (2007) that a manufacturer may strategically reduce the wholesale price when it encroaches upon the retailer’s territory. The manufacturer earns profits from both the wholesale and online channels. In our study, the increasing number of product varieties sold online intensifies *intra-brand competition*, which decreases the manufacturer’s profits in the wholesale channel. The manufacturer thus

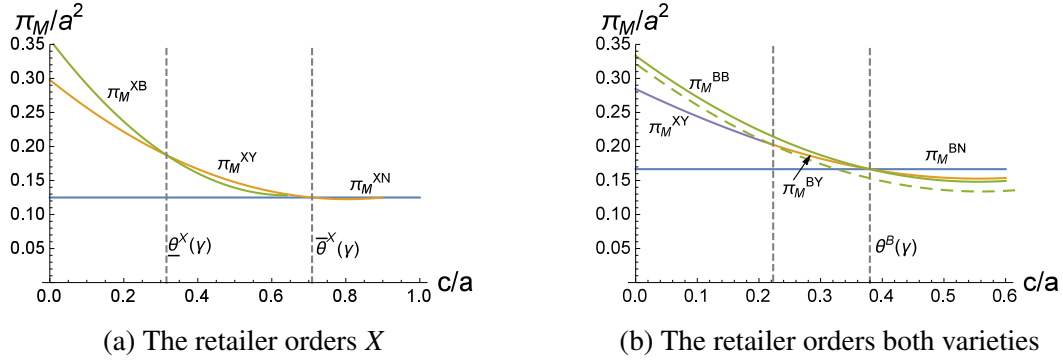


Figure 1: The online store's variety choice in period 2 when  $\gamma = 0.5$

lowers the wholesale price to shift some business back to the retailers to keep the wholesale demand at an adequate level. The wholesale price, conversely, reflects how intensely the presence of an online store leads to *intra-brand competition*. The lower the wholesale price charged, the more severely is the wholesale channel affected, and the stronger is the incentive for the manufacturer to attempt to retrieve the retailer's demands.

For simplicity, we call the manufacturer's encroachment with only one variety a *partial encroachment* and that with both varieties a *full encroachment*. Considering the conditions for interior solutions and making sure that there are no unilateral deviations in each case, we find the equilibrium outcomes in period 2 as follows:

**Lemma 1** (1) When the retailer orders variety X in period 1, there exist  $\underline{\theta}^X(\gamma)$  and  $\bar{\theta}^X(\gamma)$ , with  $\underline{\theta}^X(\gamma) < \bar{\theta}^X(\gamma)$ , so that the online store

- (i) fully encroaches if  $c/a \leq \underline{\theta}^X(\gamma)$ ,
- (ii) partially encroaches with variety Y if  $\underline{\theta}^X(\gamma) < c/a \leq \bar{\theta}^X(\gamma)$ ,
- (iii) is shut down if  $c/a > \bar{\theta}^X(\gamma)$ ;

(2) When the retailer orders both varieties in period 1, there exists  $\theta^B(\gamma)$ , so that the online store

- (i) fully encroaches if  $c/a \leq \theta^B(\gamma)$ ,
  - (ii) is shut down in period 2 if  $c/a > \theta^B(\gamma)$ ,
- where  $\underline{\theta}^X(\gamma) < \theta^B(\gamma) < \bar{\theta}^X(\gamma)$ .

Notice that given  $r = B$ ,  $m = Y$  is weakly dominated by  $m = B$  or  $N$ , because  $\pi_M^{BY} \leq$

$\max\{\pi_M^{BB}, \pi_M^{BN}\}$ . Here, we do not take this weakly dominated strategy as an equilibrium candidate. Figure 1 depicts the threshold values in Lemma 1. For calculations, please see Appendix 5.2.

Notice that given  $r = X$  or  $B$ , the manufacturer's profit curve becomes steeper when it sells more varieties online. This is because the increasing online retail cost leads to more losses when the online store has a wider product range.

In (a) of Figure 1, given that the retailer orders only variety  $X$ , as the online retail cost increases, the online store first stops selling variety  $X$ . The benefits of partial encroachment with a different variety rather than the same as the online one come from two angles. First, the manufacturer obtains a bigger share in the wholesale channels when its online store competes indirectly with the retailer. Second, a higher wholesale price ( $w_X^{XY} > w_X^{XX}$ ) reflects less intense *intra-brand competition*, implying a lower loss of profit in the wholesale channel.

In (b) of Figure 1,  $\pi_M^{BB}$ ,  $\pi_M^{BY}$  and  $\pi_M^{BN}$  intersects at  $\theta^B(\gamma)$ , implying that when the different varieties are distributed evenly in the wholesale channel ( $r = B$ ), selling only one variety online is a weakly dominated strategy for the manufacturer. This follows from the assumption of symmetric online retail costs for both varieties. When the retailer orders both varieties and the manufacturer starts the online channel and sells only variety  $Y$ , it lowers the wholesale prices for both  $X$  and  $Y$  from  $w_X^{BN} = w_Y^{BN}$  to  $w_X^{BY}$  and  $w_Y^{BY}$ , respectively.  $w_Y^{BY}$  is lower than  $w_X^{BY}$  because the online store encroaches on the retailer's share in  $Y$  directly but indirectly in  $X$ . The partial encroachment leads to intense *intra-brand competition* in both varieties. However, if the online store also sells  $X$  ( $m = B$ ), it only further decreases the wholesale price of  $X$  ( $w_Y^{BB} = w_Y^{BY}$ ,  $w_X^{BB} < w_X^{BY}$ ), implying that the additional sale of  $X$  by the online store does not further intensify the *intra-brand competition* in  $Y$ . The additional sale of  $X$  by the online store reduces its sale of  $Y$  through the *cannibalization effect*, which prevents the retailer's sale of  $Y$  from decreasing sharply. The manufacturer then has no further incentive to reduce the wholesale price of  $Y$ . We see that a full encroachment does not seriously cause more of a profit loss in the wholesale channel than a partial one. If selling one variety online is profitable for the manufacturer than selling nothing, it always continues to sell the other

variety. Therefore, selling one variety is a weakly dominated strategy when  $r = B$ .

If we assume that there exists a positive real number  $\tau$  such that selling  $X$  incurs more cost ( $c + \tau$ ) than selling  $Y$ ,  $\pi_M^{BB}$  in (b) of Figure 1 shifts left and downward so that the range over which the online store sells only the more efficient variety  $Y$  becomes wider. All results here still hold true if the cost difference  $\tau$  is so small that the range of selling only  $Y$  online is negligible. We assume symmetric online retail costs for simplicity.

The case that  $r = X$  can be seen as an unbalanced order because only one variety is distributed to the retailer; the case that  $r = B$  can be seen as a balanced order because both varieties are distributed. The manufacturer tends to keep balance in the distribution of varieties—it may sell only the variety that is distributed less to the retailer (when  $r = X$ ) but never sells only one variety when both varieties are evenly distributed (when  $r = B$ ).

Lemma 1 shows how the retailer's product line choice affects the manufacturer's incentive for opening the online store. When the online retail cost is relatively low, the manufacturer considers whether to fully encroach. The threshold values  $\underline{\theta}^X(\gamma) < \theta^B(\gamma)$  show that the manufacturer has a stronger incentive to do so when  $r = B$  than when  $r = X$ . Reducing the number of varieties sold online affects the manufacturer's profits in two ways: First, it alleviates *intra-brand competition* and results in higher wholesale prices, from which the manufacturer's profit in the wholesale channel increases. Second, because of a narrower product line, the manufacturer's profit in the online channel decreases. We first examine the wholesale channel. When the online store sells both varieties,  $w_X^{XB} = w_X^{BB} = w_Y^{BB}$ . If the online store decreases its varieties,  $m$  changes from  $B$  to  $Y$  given that  $r = X$ , but changes from  $B$  to  $N$  given that  $r = B$ . Comparing the subgame outcome under the product line system  $rm = XY$  with that under  $rm = BN$ , the manufacturer sets higher wholesale prices ( $w_X^{BN}$  and  $w_Y^{BN}$ ) when  $r = B$  than that ( $w_X^{XY}$ ) when  $r = X$ . This is because when  $r = B$ , once the manufacturer reduces its varieties, it directly shuts down the online channel, eliminating *intra-brand competition*. Thus, the manufacturer's gain in the wholesale channel is larger when  $r = B$  than when  $r = X$ . However, the manufacturer loses more in the online channel when  $r = B$  than when  $r = X$ , because it loses profits from both varieties in the former case. Since

the online retail cost is so low now that the loss dominates the gain, the manufacturer is more likely to keep full encroachment when  $r = B$  than when  $r = X$ .

**Remark 1**  $\underline{\theta}^X(\gamma) < \theta^B(\gamma)$ .

When the online retail cost is relatively high, the manufacturer considers whether shutting down the online store. The threshold values  $\theta^B(\gamma) < \bar{\theta}^X(\gamma)$  show that the manufacturer has a stronger incentive to do so when  $r = B$  than when  $r = X$ . Opening an online store has two effects on the manufacturer's profits: First, it leads to *intra-brand competition*, from which the manufacturer's profit in the wholesale channel decreases. Second, it brings the manufacturer additional profits in the online channel. We first look at the wholesale channel. When there is only the wholesale channel,  $w_X^{XN} = w_X^{BN} = w_Y^{BN}$ . However, when the manufacturer opens its online store, it sets lower wholesale prices ( $w_X^{BB}$  and  $w_Y^{BB}$ ) when  $r = B$  than that ( $w_X^{XY}$ ) when  $r = X$ . This is because when  $r = B$ , the online store competes directly with the (incumbent) retailer in both varieties; However, when  $r = X$ , the online store can sell a different variety so that decreases in the retailer's share is not that large. Thus, the manufacturer's loss in the wholesale channel is larger when  $r = B$  than when  $r = X$ . However, the manufacturer earns more profits in the online channel when  $r = B$  than when  $r = X$  because it profits from both varieties in the former case. Since online retail cost is now high enough for the loss to dominate the gain, the manufacturer is more likely to shut down its online store when  $r = B$  than when  $r = X$ .

**Remark 2**  $\theta^B(\gamma) < \bar{\theta}^X(\gamma)$ .

### 2.3 The retailer's product line choice in equilibrium

In period 1, the retailer orders either variety  $X$  or both varieties in anticipation of the manufacturer's reaction in period 2. The following proposition indicates the equilibrium variety:

**Proposition 2** *In the case of a monopoly retailer, the equilibrium variety outcome is*

(i) *the retailer orders both varieties in period 1 and the online fully encroaches in period 2 if  $c/a \leq \underline{\theta}^X(\gamma)$  (hereafter the *BB variety outcome*);*

(ii) the retailer orders variety  $X$  in period 1 and the online store partially encroaches with variety  $Y$  in period 2 if  $\underline{\theta}^X(\gamma) < c/a \leq \theta^B(\gamma)$  (hereafter the  $XY$  variety outcome);

(iii) the retailer orders both varieties in period 1 and the online store is shut down in period 2 if  $c/a > \theta^B(\gamma)$  (hereafter the  $BN$  variety outcome).

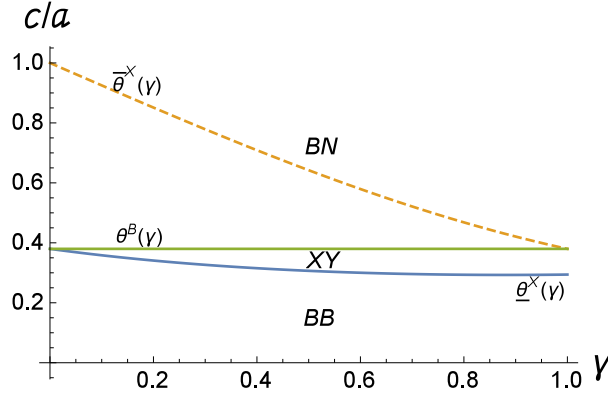


Figure 2: The variety choices  $rm$  in equilibrium

It is important for the retailer to consider two main issues in the ordering process: First, to order as many varieties as possible for a wider product line; Second, to weaken the negative effect of the online store's encroachment. When the retail cost is too low ( $c/a \leq \underline{\theta}^X(\gamma)$ ), whether the retailer orders one variety or both does not change the outcome of the online store encroaching fully. Therefore, the retailer chooses a full-length product line. However, when the online retail cost is not that low ( $\underline{\theta}^X(\gamma) < c/a \leq \theta^B(\gamma)$ ), the retailer can order only  $X$  to give the manufacturer an incentive to encroach with the less-supplied variety  $Y$ . However, if it orders both varieties, the manufacturer follows with a full encroachment. Since the online store now is a relatively efficient one, alleviating the negative effect of the encroachment is more important than enlarging the product line. Therefore, although the retailer loses from one variety, it benefits from inducing the sale of fewer varieties by the online store. When the retail cost is very high ( $c/a \geq \theta^B(\gamma)$ ), the retailer goes for a full-length product line to induce the manufacturer to shut down the online store. Thus, the retailer counters fully the negative effect of the encroachment and establishes a wider product line as well.



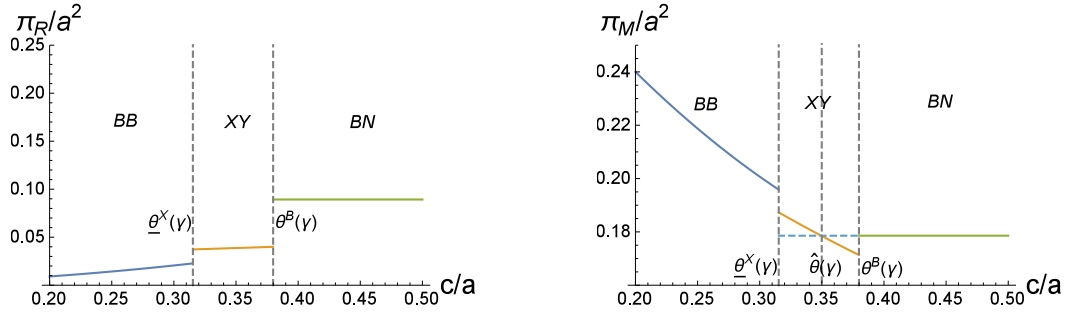


Figure 3: The retailer and the manufacturer's profit function when  $\gamma = 0.3$

With regard to how the change of online retail cost affects the retailer and the manufacturer's profits (see Figure 3), we find that as the online retail cost increases, the retailer's profit always show an increasing tendency. This is because an increasing  $c$  makes the retailer more competitive and gradually removes the manufacturer's incentive to encroach. However, the manufacturer's profit does not always show a decreasing tendency, which is summarized in the following proposition:

**Proposition 3** *As the online retail cost increases, the manufacturer's profit drops at  $\underline{\theta}^X(\gamma)$ , but jumps up at  $\theta^B(\gamma)$ .*

When  $c$  just surpasses  $\underline{\theta}^X(\gamma)$ , the retailer and the online store begins to compete in different varieties, which alleviates *intra-brand competition*. However, because both variety  $Y$  in the wholesale channel and variety  $X$  in the online channel are suddenly removed, the manufacturer bears loss of profits from both channels, which overweighs the positive effect of an alleviation in *intra-brand competition*. When  $c$  just surpasses  $\theta^B(\gamma)$ , the profit from supplying variety  $Y$  through the online channel is suddenly replaced by that through the wholesale channel. This replacement benefits the manufacturer in the following two ways: First, because *intra-brand competition* is eliminated, the manufacturer earns a higher profit by specializing in the wholesale channel. Second, the channel efficiency is improved because  $Y$  is supplied by the efficient retailer instead of the costly online store.

Proposition 3 has further implications for the manufacturer's encroachment decision. Assume that there is a pre-determinate period, period 0. The upward jump feature implies

that it may be better for the manufacturer to commit to the retailer in period 0 that it will not open its online store. This is summarized in the next corollary:

**Corollary 1** *When  $\hat{\theta}(\gamma) < c/a \leq \theta^B(\gamma)$ , it is better for the manufacturer to commit not to open the online store, where  $\hat{\theta}(\gamma)$  is the threshold satisfying  $\pi_M^{XY} = \pi_M^{BN}$ .*

Given the guarantee that the resale market will not be encroached upon, the retailer chooses to carry both varieties. Without such a commitment, at  $\theta^B(\gamma)$ , the encroachment makes the retailer reduce one variety and forces the manufacturer to carry this variety inefficiently, which implies a “lose-lose” consequence. Committing enables the manufacturer to ensure that both varieties are supplied efficiently. Arya et al. (2007) do not consider the possibility that the retailer specifies its product line so that the manufacturer never encroaches if doing so causes profits to fall. Hence, such a “lose-lose” consequence never occurs in that study.

Next we see how the consumer surplus ( $CS$ ) and the total surplus ( $TS$ ) are affected by the retailer and the online store’s choice of variety. Since the social loss only comes from the online retail cost, the total surplus is the representative consumer’s gross utility net of the total online retail cost, which is denoted by  $TS = U(Q_X, Q_Y) - (q_{XM} + q_{YM})c$ . We find that as  $c$  increases, the consumer surplus always decreases, but the total surplus jumps up at  $\theta^B(\gamma)$ . This is summarized in the following proposition:

**Proposition 4** *As  $c$  increases, the consumer surplus always shows an increasing tendency, and the total surplus drops at  $\underline{\theta}^X(\gamma)$ , but jumps up at  $\theta^B(\gamma)$ .*

Figure 4 depicts how  $c$  affects the consumer and the total surplus.

An increasing  $c$  gradually removes the manufacturer’s incentive for online retailing and enhances the (incumbent) retailer’s market power who finally monopolizes the resale market. At  $\underline{\theta}^X(\gamma)$ , both the retailer and the online store reduce one variety and begin to compete indirectly, which results in higher prices. This harms both the consumer and total surpluses. At  $\theta^B(\gamma)$ , the online store is shut down and the retailer becomes a monopolist of both varieties. This increases prices, which further decreases the consumer surplus. However, although the

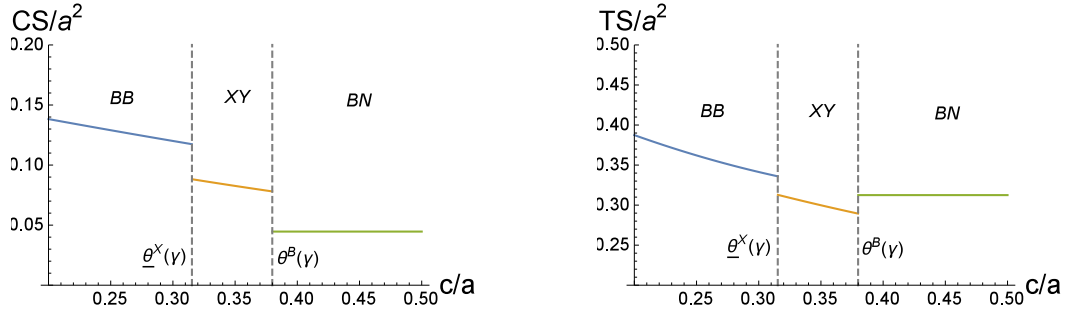


Figure 4: The consumer and the total surplus when  $\gamma = 0.4$

monopolization of the (incumbent) retailer decreases the gross utility  $U(Q_X, Q_Y)$ , shutting down the online channel wipes out the social loss, which results in a higher total surplus.

### 3 Extension: the case with duopoly retailers

Our basic model tells us how a monopoly retailer decides its product line when the manufacturer can strategically recapture its channel power by employing an online channel. This raises a question of how the product line outcomes change if there are additionally strategic interactions between retail competitors. Thus, it is natural to consider a simple extension with duopoly retailers. Assume that there is a monopoly manufacturer,  $M$ , and two retailers,  $R_i$ , with  $i = 1$  or  $2$ .

To remain consistent with the benchmark model, we assume that all players are kept active in the market so that  $N$  (order nothing) cannot be any retailer's choice. Besides, we normalize both retailers' resale costs for both varieties to zero. Let  $K_i$  be the set of products  $R_i$  orders in period 1, where  $K_i \subseteq \{X, Y\} \setminus \emptyset$ . Denote each retailer's variety choice as  $r_i \in \{X, Y, B\}$  and the manufacturer's as  $m \in \{N, X, Y, B\}$ . The game proceeds as in the case of a monopoly retailer except that the retailers decide their respective product line simultaneously and independently in period 1, and the manufacturer discriminates against the retailers by offering different wholesale prices  $w_{ni}^{r_1 r_2 m}$  in period 2. Retailers' order combination  $r_1 r_2$  of period 1 is simplified as follows:  $r_1 r_2 \in \{XX, XY, XB, BB\}$ .<sup>10</sup>

<sup>10</sup>By symmetry,  $YY$  is equivalent to  $XX$ ;  $YX$  is equivalent to  $XY$ ;  $YB, BY$  and  $BY$  is equivalent to  $XB$ .

$r_1 r_2$	$m$	$W_{X1}^{r_1 r_2 m}$	$W_{Y1}^{r_1 r_2 m}$	$W_{X2}^{r_1 r_2 m}$	$W_{Y2}^{r_1 r_2 m}$	I.S.
XX	N	$\frac{a}{2}$	\	$\frac{a}{2}$	\	$a > 0$
	X	$\frac{3a-c}{6}$	\	$\frac{3a-c}{6}$	\	$\frac{c}{a} \leq \frac{3}{5}$
	Y	$\frac{(6-4\gamma^2+\gamma^3)a-\gamma^3c}{6(2-\gamma^2)}$	\	$\frac{(6-4\gamma^2+\gamma^3)a-\gamma^3c}{6(2-\gamma^2)}$	\	$\frac{c}{a} \leq \frac{6-2\gamma-\gamma^2}{6-\gamma^2}$
	B	$\frac{3a-c}{6}$	\	$\frac{3a-c}{6}$	\	$\frac{c}{a} \leq \frac{3}{5+2\gamma}$
XY	N	$\frac{a}{2}$	\	\	$\frac{a}{2}$	$a > 0$
	Y	$\frac{(10-5\gamma^2+\gamma^3)a-2\gamma c}{4(5-2\gamma^2)}$	\	\	$\frac{(10-\gamma-3\gamma^2)a-2c}{4(5-2\gamma^2)}$	$\frac{c}{a} \in \left( \frac{\gamma-\gamma^2}{8-4\gamma^2}, \frac{5-\gamma-\gamma^2}{7-2\gamma^2} \right]$
	B	$\frac{(5+\gamma)a-(1+\gamma)c}{2(5+\gamma)}$	\	\	$\frac{(5+\gamma)a-(1+\gamma)c}{2(5+\gamma)}$	$\frac{c}{a} \leq \frac{5+\gamma}{7+3\gamma}$
XB	N	$\frac{a}{2}$	\	$\frac{a}{2}$	$\frac{a}{2}$	$a > 0$
	X	$\frac{3a-c}{6}$	\	$\frac{3a-c}{6}$	$\frac{3a-\gamma c}{6}$	$\frac{c}{a} \in \left( \frac{3\gamma}{4(1+\gamma)}, \frac{3}{5} \right]$
	Y	$\frac{(15-4\gamma^2+\gamma^3)a-(3\gamma+\gamma^3)c}{6(5-\gamma^2)}$	\	$\frac{(15-4\gamma^2+\gamma^3)a-(3\gamma+\gamma^3)c}{6(5-\gamma^2)}$	$\frac{(15-\gamma-2\gamma^2)a-(3+\gamma^2)c}{6(5-\gamma^2)}$	$\frac{c}{a} \in \left( \frac{\gamma}{2(1+\gamma)}, \frac{15-2\gamma-\gamma^2}{21-\gamma^2} \right]$
	B	$\frac{3a-c}{6}$	\	$\frac{3a-c}{6}$	$\frac{15a-(3+2\gamma)c}{30}$	$\frac{c}{a} \leq \frac{15}{25+4\gamma}$
BB	N	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$	$a > 0$
	Y	$\frac{3a-\gamma c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-\gamma c}{6}$	$\frac{3a-c}{6}$	$\frac{c}{a} \in \left( \frac{\gamma}{2(1+\gamma)}, \frac{3}{5} \right]$
	B	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{c}{a} \leq \frac{3}{5}$

Table 2: Equilibrium wholesale prices

### 3.1 Equilibrium Product Variety

In period 3, the Cournot competition proceeds based on the product line system,  $r_1 r_2 m$ , which are of 14 kinds:  $XXN$ ,  $XXX$ ,  $XXY$ ,  $XXB$ ,  $XYN$ ,  $XYY$ ,  $XYB$ ,  $XBN$ ,  $XBX$ ,  $XBY$ ,  $XBB$ ,  $BBN$ ,  $BBY$ , and  $BBB$ .<sup>11</sup> The optimization system is the same as in the basic model.

In period 2, the manufacturer decides whether to open its online store and which variety (varieties) to sell based on the resulting profits. The equilibrium wholesale price,  $w_{ni}^{r_1 r_2 m}$  and  $w_{-ni}^{r_1 r_2 m}$ , are denoted in Table 2, where ‘‘I.S.’’ denotes the conditions for interior solutions.

The online store encroaches only if its retail cost is low enough. Moreover, for the interior solutions in the  $XBY$  case, the online retail cost must not be too low. When the online store is very efficient in selling variety  $Y$ , even if  $R_2$  orders both varieties, it will be charged an unacceptable  $w_{Y2}^{XBY}$  so that  $R_2$  does not really sell  $Y$ . The  $XBY$  case with a corner solution ( $q_{YR2}^{XBY} = 0$ ) becomes the  $XXY$  case. Thus, we need the online retail cost to be higher than  $\hat{\theta}^{XB}(\gamma) \equiv \gamma/(2+2\gamma)$  in the  $XBY$  case. After several calculations, considering the corner solu-

<sup>11</sup>By symmetry,  $XYX$  is equal to  $XYY$ , and  $BBX$  is equal to  $BBY$ .

tions and checking the manufacturer's incentive to deviate, we find the equilibrium outcomes in period 2 as follows:

**Lemma 2** (1) When both retailers order variety X in period 1, there exist  $\underline{\theta}^{XX}(\gamma)$  and  $\bar{\theta}^{XX}(\gamma)$ , with  $\bar{\theta}^{XX}(\gamma) > \underline{\theta}^{XX}(\gamma)$ , such that the online store

- (i) fully encroaches in period 2 if  $c/a \leq \underline{\theta}^{XX}(\gamma)$ ,
- (ii) partially encroaches with variety Y in period 2 if  $\underline{\theta}^{XX}(\gamma) < c/a \leq \bar{\theta}^{XX}(\gamma)$ ,
- (iii) is shut down in period 2 if  $c/a > \bar{\theta}^{XX}(\gamma)$ ;

(2) When one retailer orders variety X, and the other one orders both varieties in period 1, there exist  $\underline{\theta}_1^{XB}(\gamma)$ ,  $\underline{\theta}_2^{XB}(\gamma)$ ,  $\bar{\theta}_1^{XB}(\gamma)$  and  $\bar{\theta}_2^{XB}(\gamma)$ , such that the online store

(i) fully encroaches in period 2 when  $\gamma \leq 0.690$  if  $c/a \leq \underline{\theta}_1^{XB}(\gamma)$ , as well as when  $\gamma > 0.690$  if  $c/a \leq \underline{\theta}_2^{XB}(\gamma)$ ,

(ii) partially encroaches with variety Y in period 2 when  $\gamma \leq 0.690$  if  $\underline{\theta}_1^{XB}(\gamma) < c/a \leq \bar{\theta}_1^{XB}(\gamma)$ , as well as when  $0.69 \leq \gamma \leq 0.897$  if  $\hat{\theta}^{XB}(\gamma) < c/a \leq \bar{\theta}_1^{XB}(\gamma)$ .

(iii) is shut down in period 2 when  $\gamma \leq 0.897$  if  $c/a > \bar{\theta}_1^{XB}(\gamma)$ , as well as when  $\gamma > 0.897$  if  $c/a > \bar{\theta}_2^{XB}(\gamma)$ ;

(3) When one retailer orders variety X and the other one orders variety Y in period 1, there exist  $\theta^{XY}(\gamma)$ , such that the online store

- (i) fully encroaches in period 2 if  $c/a \leq \theta^{XY}(\gamma)$ ,
- (ii) is shut down in period 2 if  $c/a > \theta^{XY}(\gamma)$ ;

(4) When both retailers order both varieties in period 1, there exist  $\theta^{BB}(\gamma)$ , such that the online store

- (i) fully encroaches in period 2 if  $c/a \leq \theta^{BB}(\gamma)$ ,
- (ii) is shut down in period 2 if  $c/a > \theta^{BB}(\gamma)$ .

Please see Appendix 5.4 for the calculations. The ranges for equilibrium outcomes in period 2 are summarized in Figure 5. For simplicity in notation, we denote the upper bound and lower bound values of the XB case as  $\bar{\theta}^{XB}(\gamma)$  and  $\underline{\theta}^{XB}(\gamma)$  respectively.<sup>12</sup>

<sup>12</sup> $\underline{\theta}_1^{XB}$  and  $\underline{\theta}_2^{XB}(\gamma)$  are connected at  $\gamma = 0.690$ ;  $\bar{\theta}_1^{XB}$  and  $\bar{\theta}_2^{XB}(\gamma)$  are connected at  $\gamma = 0.897$ .

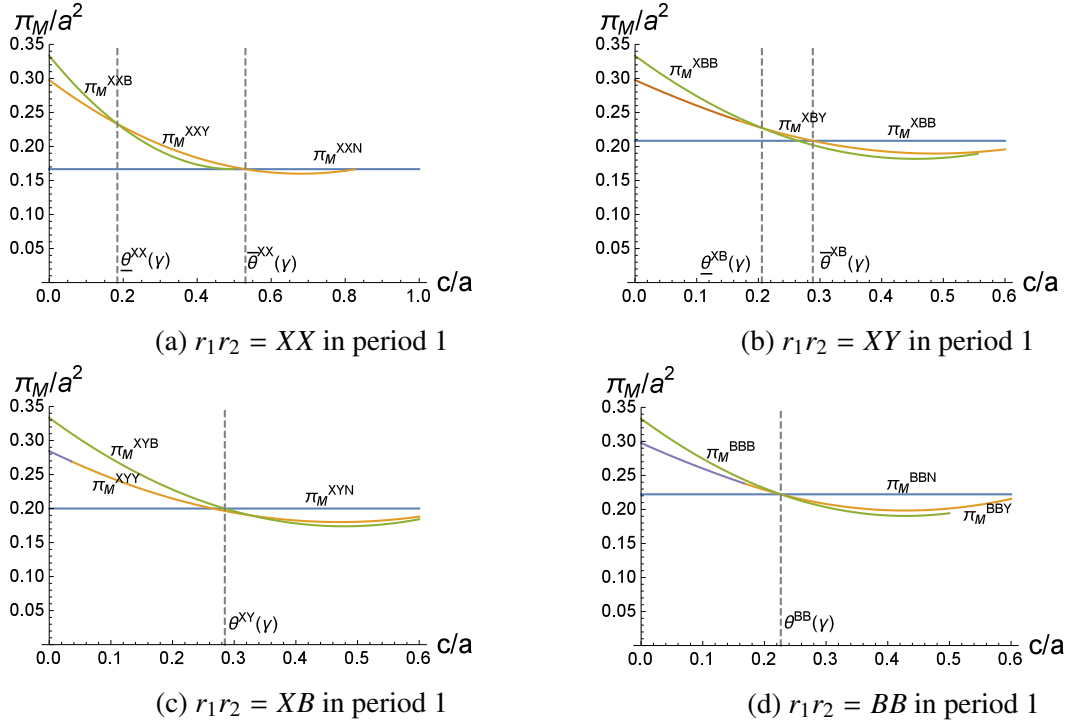


Figure 5: The online store's variety choice in period 2 when  $\gamma = 0.5$

Following the logic of the monopoly retail case, the case that  $r_1 r_2 = XX$  or  $XB$  corresponds to  $r = X$  in Section 2 (see (a) and (b) in Figure 5). These two cases can be seen as the unbalanced orders because variety  $X$  is distributed to both retailers, but  $Y$  is distributed to at most one retailer. Knowing that retailers will supply less of  $Y$  compared with  $X$ , the online store sells only  $Y$  when  $c$  is an intermediate value ( $\underline{\theta}^{XX}(\gamma) < c/a \leq \bar{\theta}^{XX}(\gamma)$  or  $\underline{\theta}^{XB}(\gamma) < c/a \leq \bar{\theta}^{XB}(\gamma)$ ). The threshold values,  $\underline{\theta}^{XX}(\gamma) < \underline{\theta}^{XB}(\gamma) < \bar{\theta}^{XB}(\gamma) < \bar{\theta}^{XX}(\gamma)$ , imply that partial encroachment with  $Y$  is more likely to happen in the  $XX$  case than in the  $XB$  case. This is because the distribution imbalance is more serious in the former case. In the  $XB$  case,  $R_2$ 's share of  $Y$  makes selling  $Y$  online less profitable for the manufacturer than in the  $XX$  case.

The case that  $r_1 r_2 = XY$  or  $BB$  corresponds to  $r = B$  in Section 2 (see (c) and (d) in Figure 5). These two cases can be seen as the balanced orders, because both varieties are evenly distributed to the retailers. To maintain this balance, the manufacturer chooses either to fully encroach or completely shut down the online channel. The threshold values,  $\theta^{BB}(\gamma) < \theta^{XY}(\gamma)$ ,

imply that a full encroachment is more likely to happen in the  $XY$  case than in the  $BB$  case. In the  $XY$  case, the retailers have differentiated product lines so that the resale market before the presence of encroachment is less competitive for online retailing compared with the  $BB$  case where retailers have overlapping product lines and thus compete intensely.

Notice that in the case  $r_1 r_2 = XY$ , a partial encroachment is strictly dominated by a full encroachment or a shutdown of the online channel,<sup>13</sup> which differs from the case  $r_1 r_2 = BB$  in the case of duopoly retailers and  $r = B$  in the basic model. Partially encroaching with  $Y$  forces the manufacturer to reduce the wholesale prices of  $X$  and  $Y$  from  $w_{X1}^{XYN}$  to  $w_{X1}^{XYY}$  and from  $x_{Y2}^{XYN}$  to  $w_{Y2}^{XYY}$ , respectively. If the manufacturer additionally sells  $X$  online, then it would sell at a lower wholesale price of  $X$  ( $w_{X1}^{XYB} < w_{X1}^{XYY}$ ) but at a higher wholesale price of  $Y$  ( $w_{Y2}^{XYB} > w_{Y2}^{XYY}$ ), implying that selling  $X$  additionally alleviates *intra-brand competition* in  $Y$ . Since now each variety is ordered by only one retailer, the demand in the wholesale channel is small, which stimulates the manufacturer's strong incentive for online retailing. When the online store sells only  $Y$ ,  $R_2$ 's share is severely invaded so the manufacturer has to greatly lower the wholesale price of  $Y$ . However, if the manufacturer additionally sells  $X$  online, the *cannibalization effect* diminishes the online sale of  $Y$ , thus alleviating the negative impact on  $R_2$ 's share in  $Y$ . Selling  $X$  additionally not only helps the manufacturer to enlarge the online store's product line, but also enhances the wholesale channel's profit in  $Y$ . If fully encroaching is more profitable than shutting down the online store, a manufacturer would never partially encroach.

### 3.2 Retailers' order varieties at equilibrium

In period 1, the retailers decide which variety (varieties) to order, anticipating the potential online store's encroachment. The equilibrium variety outcome in period 1 is summarized as follows:

**Proposition 5** (i) *The retailers order both varieties, and the online store fully encroaches (hereafter the BBB outcome) if  $c/a \leq \underline{\theta}^{XB}(\gamma)$ ;*

<sup>13</sup>  $\pi_M^{XYB}$  and  $\pi_M^{XYY}$  intersects at a point that is larger than  $\theta^{XY}(\gamma)$ , implying that partial encroachment with either variety is a *strictly dominated strategy* for the manufacturer.

(ii) the retailers order  $X$ , and the online store partially encroaches with  $Y$  in period 2 (hereafter the  $XXY$  outcome) if  $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$ , or if  $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\bar{\theta}^{XB}(\gamma), \hat{\theta}^{XB}(\gamma)\}$ .

(iii) one retailer orders  $X$  and the other retailer orders both varieties, and the online store partially encroaches with  $Y$  (hereafter the  $XYB$  outcome) if  $\max\{\underline{\theta}^{XB}(\gamma), \hat{\theta}^{XB}(\gamma)\} < c/a \leq \theta^{BB}(\gamma)$ ;

(iv) each retailer orders a different variety, and the online store is shut down (hereafter the  $XYN$  outcome) if  $\theta^{XY}(\gamma) < c/a \leq \bar{\theta}^{XB}(\gamma)$ ;

(v) the retailers order both varieties, and the online store is shut down (hereafter the  $BBN$  outcome) if  $c/a > \theta^{BB}(\gamma)$ .

Here, because quantities are decided simultaneously, encroachment always harms retailers' profits. Although charged lower wholesale prices, retailers' losses in market share can never be compensated.<sup>14</sup> The main issue for each retailer is to find a balance between enlarging the product line and weakening the negative impact of the online store's encroachment.

When the online retail cost is too low ( $c/a \leq \underline{\theta}^{XB}(\gamma)$ ), whether  $r_1 r_2 = BB$  or  $XB$ , the online store always fully encroaches. Hence, each retailer chooses a full-length product line (the  $BBB$  outcome).

As the online retail cost gradually increases, retailers' unbalanced order variety may induce the manufacturer's partial encroachment instead of a full one. This is because these unbalanced orders signal to the manufacturer that variety  $Y$  would be supplied less. Observing this, the manufacturer sells only  $Y$  so that the resulting *intra-brand competition* is not that intense. In the range that  $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$  or  $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\bar{\theta}^{XB}(\gamma), \hat{\theta}^{XB}(\gamma)\}$ , only when  $r_1 r_2 = XX$  does the online store partially encroach (the  $XXY$  outcome).<sup>15</sup> Although both retailers give up variety  $Y$ , they benefit from weakening the negative impact from the online store. In the range that  $\max\{\underline{\theta}^{XB}(\gamma), \hat{\theta}^{XB}(\gamma)\} < c/a \leq \theta^{BB}(\gamma)$ , either when  $r_1 r_2 = XX$

<sup>14</sup>If the online store moves after the retailers, it is possible that retailers' share losses caused by the encroachment are compensated for by being charged lower wholesale prices (see Arya et al., 2007).

<sup>15</sup>If  $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$ , when  $r_1 r_2 = XY, XB$  or  $BB$ , the online store sells both varieties. If  $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\bar{\theta}^{XB}(\gamma), \hat{\theta}^{XB}(\gamma)\}$ , when  $r_1 r_2 = XY$  or  $BB$ , the online store sells both varieties. But when  $r_1 r_2 = XB$ , the online store sells only  $Y$ . This because when  $c/a \leq \hat{\theta}^{XB}(\gamma)$ ,  $R_2$  is offered an unacceptable  $w_{Y2}^{XY}$  so that its quantity of  $Y$  becomes zero. The  $r_1 r_2 m = XBY$  case with  $q_{YR2}^{XB} = 0$  becomes the  $XXY$  case.



or  $XB$ , the online store's partial encroachment can be induced. In equilibrium, one retailer is willing to give up one variety and let its retail rival have a full-length product line (the  $XY$  outcome). Although the retailer who cuts product line earns lower profit than its retail rival, it would lose more if it insists on choosing a full-length line because the online store would then fully encroach.

When the online retail cost is relatively high, retailers' balanced order variety may induce the manufacturer's incentive to shut down its online store because running it means a full encroachment, which causes a serious loss of profit in the wholesale channel. In the range that  $\theta^{XY}(\gamma) < c/a \leq \bar{\theta}^{XB}(\gamma)$ , the manufacturer partially encroaches in the face of imbalanced orders but shuts down the online channel when  $r_1 r_2 = XY$  (the  $XYN$  outcome). When one of the retailers orders one variety, the other one would spontaneously order a different variety. Although both of them give up the full-length product line, their non-overlapping product line choices eliminate the online store's partial encroachment and make the competition between them indirect and mitigating. In the range that  $c/a > \theta^{BB}(\gamma)$ , the manufacturer shuts down its online store when  $r_1 r_2 = BB$  (the  $BBN$  outcome). No retailer would unilaterally reduce the number of varieties because this not only gives the retail rival the advantage of diversity but also results in the online store's partial encroachment (when  $c/a \leq \bar{\theta}^{XB}(\gamma)$ ).

Comparing the equilibrium ranges, we find that the following three sets of variety outcomes can coexist: the  $XXY$  and  $BBB$  outcomes,  $XXY$  and  $BBN$  outcomes,  $XYN$  and  $BBN$  outcomes. This implies the possibility of a coordination failure in some ranges, which is summarized in the following corollary:

**Corollary 2** *Retailers' coordination failure may occur in the following ranges, in which each retailer's profit is higher in the former outcome than in the latter one:*

- (i) *If  $\underline{\theta}^{XX}(\gamma) < c \leq \underline{\theta}^{XB}(\gamma)$ , the  $XXY$  and  $BBB$  variety outcomes coexist;*
- (ii) *If  $\theta^{BB}(\gamma) < c \leq \min\{\hat{\theta}^{XB}(\gamma), \bar{\theta}^{XB}(\gamma)\}$ , the  $BBN$  and  $XXY$  variety outcomes coexist;*
- (iii) *If  $\theta^{XY}(\gamma) < c \leq \bar{\theta}^{XB}(\gamma)$ , the  $XYN$  and  $BBN$  variety outcomes coexist.*

Figure 6 depicts the variety outcomes in equilibrium. The shady areas denote the ranges in which coordination failure may exist.

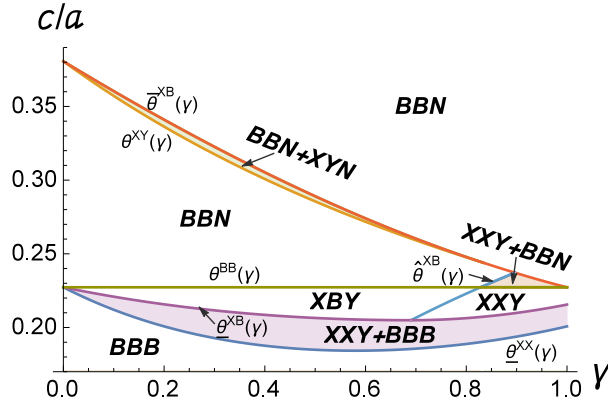


Figure 6: The variety outcomes  $r_1 r_2 m$  in equilibrium

In (i) of Corollary 2, although the  $BBB$  variety outcome earns each retailer profits from a full-length product line, the resulting online store's full encroachment makes the competition too intense. In (ii) of Corollary 2, comparing with the  $BBN$  variety outcome, the  $XXY$  outcome not only removes each retailer's profit from selling  $Y$  but also triggers the online store's partial encroachment. In (iii) of Corollary 2, although both cases,  $r_1 r_2 = BB$  and  $r_1 r_2 = XY$ , make the manufacturer shut down its online store, the former case is clearly less beneficial than the latter one because the competition in the latter case is more mitigating. Despite this, neither retailer is willing to deviate from the less profitable variety outcome.

Finally, we examine the consumer surplus and total surplus in the duopoly retailer case. The consumer surplus here is similar to that in the case of a monopoly retailer. It always shows a decreasing tendency as the online retail cost increases within the ranges in which each variety outcome exists. Moreover, it always jumps down at some threshold points where the number of varieties in the resale market decreases or where the online store is shut down by the manufacturer. However, the total surplus shows a distortion, which is summarized in the following proposition.

**Proposition 6** When  $\gamma > 0.751$ , the total surplus jumps up at  $\underline{\theta}^{XX}(\gamma)$ , wherein the variety outcome changes from  $BBB$  to  $XXY$ .

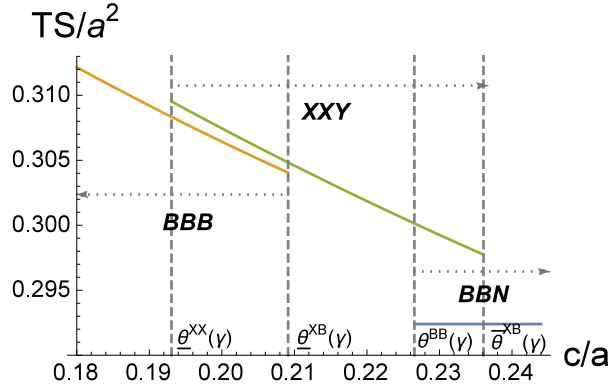


Figure 7: The total surplus when  $\gamma = 0.9$

This proposition is counterintuitive at first glance because now less intense *intra*brand competition may even benefit society (see Figure 7).<sup>16</sup> Recall that the social loss only comes from the online store's sale. We first examine the *XXY* outcome. Assume that  $\gamma$  is almost zero so that the different varieties are almost heterogeneous. Since the online store almost independently monopolizes the market for *Y*, the social loss is very high. As  $\gamma$  grows larger, the markets for *X* and *Y* become more interconnected. Since competition between the retailers and online store intensifies, the online retail cost shifts more business from the online store to the retailers, which decreases the social loss. However, in the *BBB* outcome, because the retailers and the online store always compete in the same variety, the impact of a growing  $\gamma$  on the online store's share is quite limited. When  $\gamma$  is large enough, the online store's share is lower in the *XXY* outcome than in the *BBB* outcome, implying less social loss in the former case. When the variety outcome changes from *BBB* to *XXY*, although the less intense *intra*brand competition harms the consumer's gross utility,  $U(Q_X, Q_Y)$ , the total surplus may still improve by saving on social costs. Notice that in the duopoly retailer case, the shutdown of the online store (when  $c/a = \bar{\theta}^{XB}(\gamma)$ ) does not improve the total surplus as in the case of a monopoly retailer (Proposition 4). This is because partial encroachment causes less social loss here than in the case of a monopoly retailer. Shutting down the online store (removing partial encroachment) is anti-competitive, which cannot be compensated for by the benefit

<sup>16</sup>Each variety is carried by both channels in the *BBB* case but by only one channel in the *XXY* case. Thus, *intra*brand competition changes from a direct one to an indirect one.

of eliminating the social loss.

## 4 Concluding remarks

Our study discusses a market in which a retailer decides its product line from a manufacturer who may encroach upon the resale market through an online store. We show that the manufacturer uses the online channel as a tool to keep the variety in distribution balanced. Specifically, the manufacturer tends to use an online store to sell the variety that is less supplied by the (incumbent) retailers but will not start the online channel if the variety distribution in the wholesale channel is already balanced. As discussed in the case of a monopoly retailer, if the online retail cost is moderately low, a retailer chooses a shorter product line instead of a full-length one to induce the manufacturer's incentive to sell the less-supplied variety through the online store. The retailer benefits from doing so by creating for itself a more mitigating market condition.

For analytical simplicity, we consider the case in which there are two varieties and at most two retailers. It is mathematically difficult to consider a more general case because we already have 27 types of subgames to consider under the two-variety, and two-retailer case. The situation becomes more complicated if we generalize either or both of the two parameters to  $n$ . Examining a continuous product variety offers one way to solve this difficulty. However, our results already provide strong implications of a more general case. Besides the assumption of horizontally differentiated products, it is natural to extend this setting to the one with vertically differentiated products. If consumers show their heterogeneity in evaluating product quality and the (incumbent) retailer(s) can decide products' quality from the manufacturer who also decides the quality of products sold online, the question arises as to what would be the product quality outcomes and how such outcomes would affect social welfare.

Moreover, as noted earlier, the result that the online store never sells one variety when receiving retailers' orders for both varieties follows from the assumption of symmetric retail costs. This is just for analytical simplicity. Our results continues to hold even when there is

a small retail cost gap between the two varieties. How large this retail cost gap can be and still sustain our main results can make for an interesting issue to examine future studies.

## 5 Appendix

### 5.1 The case of a monopoly retailer

We first consider the equilibrium outcomes in period 3. By solving the optimization system of Eq. (1) and (3), the equilibrium quantities denoted by wholesale prices are as follows:

(i) when  $rm = XN$ :

$$q_{XR}^{XN}(\mathbf{w}) = \frac{a - w_X^{XN}}{2};$$

(ii) when  $rm = XX$ :

$$q_{XR}^{XX}(\mathbf{w}) = \frac{a + c - 2w_X^{XX}}{3}, \quad q_{XM}^{XX}(\mathbf{w}) = \frac{a - 2c + w_X^{XX}}{3};$$

(iii) when  $rm = XY$ :

$$q_{XR}^{XY}(\mathbf{w}) = \frac{(2 - \gamma)a + \gamma c - 2w_X^{XY}}{4 - \gamma^2}, \quad q_{YM}^{XY}(\mathbf{w}) = \frac{(2 - \gamma)a - 2c + \gamma w_X^{XY}}{4 - \gamma^2};$$

(iv) when  $rm = XB$ :

$$q_{XR}^{XB}(\mathbf{w}) = \frac{a + c - 2w_X^{XB}}{3}, \quad q_{XM}^{XB}(\mathbf{w}) = \frac{(2 - \gamma)a - (4 + \gamma)c - (2 + 2\gamma)\gamma w_X^{XB}}{6(1 + \gamma)}, \quad q_{YM}^{XB}(\mathbf{w}) = \frac{a - c}{2(1 + \gamma)};$$

(v) when  $rm = BN$ :

$$q_{XR}^{BN}(\mathbf{w}) = \frac{(1 - \gamma)a - w_X^{BN} + \gamma w_Y^{BN}}{2(1 - \gamma^2)}, \quad q_{YR}^{BN}(\mathbf{w}) = \frac{(1 - \gamma)a + \gamma w_X^{BN} - w_Y^{BN}}{2(1 - \gamma^2)};$$

(vi) when  $rm = BY$ :

$$q_{XR}^{BY}(\mathbf{w}) = \frac{(2 - 3\gamma + \gamma^2)a + (2 - 2\gamma^2)c - (4 - \gamma^2)w_X^{BY} + 3\gamma w_Y^{BY}}{6(1 - \gamma^2)},$$

$$q_{YR}^{BY}(\mathbf{w}) = \frac{(1 - \gamma)a + \gamma w_X^{BY} - w_Y^{BY}}{2(1 - \gamma^2)}, \quad q_{XM}^{BY}(\mathbf{w}) = \frac{a - 2c + w_X^{BY}}{3};$$

(vii) when  $rm = BB$ :

$$q_{XR}^{BB}(\mathbf{w}) = \frac{(1 - \gamma)a + (1 - \gamma)c - 2w_X^{BB} + 2\gamma w_Y^{BB}}{3 - 3\gamma^2},$$

$$q_{YR}^{BB}(\mathbf{w}) = \frac{(1-\gamma)a + (1-\gamma)c + 2\gamma w_X^{BB} - 2w_Y^{BB}}{3-3\gamma^2},$$

$$q_{XM}^{BB}(\mathbf{w}) = \frac{(1-\gamma)a - (2-2\gamma)c + w_X^{BB} - \gamma w_Y^{BB}}{3-3\gamma^2},$$

$$q_{YM}^{BB}(\mathbf{w}) = \frac{(1-\gamma)a - (2-2\gamma)c - \gamma w_X^{BB} + w_Y^{BB}}{3-3\gamma^2}.$$

Substituting the equilibrium outcomes in period 3 back to Eq. (4) or (5) and solving the manufacturer's optimization problem, we derive the equilibrium wholesale prices in period 2 as in Table 1. Substituting the equilibrium wholesale prices, we derive the retailer and manufacturer's equilibrium quantities and profits in period 2 as follows:

(i) when  $rm = XN$ :

$$q_{XR}^{XN} = \frac{a}{4}, \pi_R^{XN} = \frac{a^2}{16}, \pi_M^{XN} = \frac{a^2}{8};$$

(ii) when  $rm = XX$ :

$$q_{XR}^{XX} = \frac{5a-7c}{10}, q_{XM}^{XX} = \frac{5a+3c}{10}, \pi_R^{XX} = \frac{4c^2}{25}, \pi_M^{XX} = \frac{5a^2-10ac+9c^2}{20};$$

(iii) when  $rm = XY$ :

$$q_{XR}^{XY} = \frac{2(1-\gamma)a + 2\gamma c}{8-3\gamma^2}, q_{YM}^{XY} = \frac{(8-2\gamma-\gamma^2)a - (8-\gamma^2)c}{2(8-3\gamma^2)},$$

$$\pi_R^{XY} = \frac{4[(1-\gamma)a + \gamma c]^2}{(8-3\gamma^2)^2}, \pi_M^{XY} = \frac{(12-8\gamma+\gamma^2)a^2 - (16-8\gamma+2\gamma^2)ac + (8+\gamma^2)c^2}{4(8-3\gamma^2)};$$

(iv) when  $rm = XB$ :

$$q_{XR}^{XB} = \frac{2c}{5}, q_{XM}^{XB} = \frac{5a-(7+2\gamma)c}{10(1+\gamma)}, q_{YM}^{XB} = \frac{a-c}{2(1+\gamma)},$$

$$\pi_R^{XB} = \frac{4c^2}{25}, \pi_M^{XB} = \frac{5a^2-10ac+(7+2\gamma)c^2}{10(1+\gamma^2)};$$

(v) when  $rm = BN$ :

$$q_{XR}^{BN} = q_{YR}^{BN} = \frac{a}{4(1+\gamma)}, \pi_R^{BN} = \frac{a^2}{8(1+\gamma)}, \pi_M^{BN} = \frac{a^2}{4(1+\gamma)};$$

(vi) when  $rm = BY$ :

$$q_{XR}^{BY} = \frac{-5\gamma a + 8(1+\gamma^2)c}{20(1+\gamma^2)}, q_{YR}^{BY} = \frac{a}{4(1+\gamma)}, q_{XM}^{BY} = \frac{5a-7c}{10},$$

$$\pi_R^{BY} = \frac{25(1-\gamma)a^2 + 64(1+\gamma)c^2}{400(1+\gamma)}, \quad \pi_M^{BY} = \frac{5(3+\gamma)a^2 - 20(1+\gamma)ac + 18(1+\gamma)c^2}{40(1+\gamma)};$$

(vii) when  $rm = BB$ :

$$q_{XR}^{BB} = q_{YR}^{BB} = \frac{2c}{5(1+\gamma)}, \quad q_{XM}^{BB} = q_{YM}^{BB} = \frac{5a-7c}{10(1+\gamma)}, \quad \pi_R^{BB} = \frac{8c^2}{25(1+\gamma)}, \quad \pi_M^{BB} = \frac{5a^2 - 10ac + 9c^2}{10(1+\gamma)}.$$

Making all quantities positive, we obtain the *I.S.* in Table 1.

## 5.2 Proof of Lemma 1

In period 2, we solve for the manufacturer's equilibrium outcome, taking the corner solutions into consideration and checking the incentive for deviation. Simply comparing the threshold values in Table 1 shows that  $\frac{5}{7+2\gamma} < \frac{5}{7} < \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$ . We show that the online store never encroaches with variety X in this case. If  $\frac{5}{7+2\gamma} < \frac{c}{a} \leq \frac{5}{7}$ , we find that  $\pi_M^{XX} < \pi_M^{XY}$ . If  $\frac{c}{a} \leq \frac{5}{7+2\gamma}$ , we find that  $\pi_M^{XX} < \max\{\pi_M^{XY}, \pi_M^{XB}\}$ . Thus, we have proved that encroaching with variety X is a *strictly dominated strategy* in this case. The manufacturer chooses B if  $\frac{c}{a} \leq \frac{5}{7+2\gamma}$  and  $\pi_M^{XB} \geq \max\{\pi_M^{XN}, \pi_M^{XX}, \pi_M^{XY}\}$ , from which we obtain

$$\frac{c}{a} \leq \underline{\theta}^X(\gamma) \equiv \frac{5(8+4\gamma+\gamma^2) - 2\sqrt{40+80\gamma+25\gamma^2-30\gamma^3-15\gamma^4}}{72+64\gamma+17\gamma^2}.$$

The outcome that the manufacturer chooses Y must satisfy the following conditions: if  $\frac{5}{7} < \frac{c}{a} \leq \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$  and  $\pi_M^{XY} \geq \pi_M^{XN}$ , or if  $\frac{5}{7+2\gamma} < \frac{c}{a} \leq \frac{5}{7}$  and  $\pi_M^{XY} \geq \max\{\pi_M^{XN}, \pi_M^{XX}\}$ , or if  $\frac{c}{a} \leq \frac{5}{7+2\gamma}$  and  $\pi_M^{XY} \geq \max\{\pi_M^{XN}, \pi_M^{XX}, \pi_M^{XB}\}$ , from which we obtain

$$\underline{\theta}^X(\gamma) \leq \frac{c}{a} \leq \bar{\theta}^X(\gamma) \equiv \frac{2(8-4\gamma+\gamma^2) - \gamma\sqrt{16-6\gamma^2}}{2(8+\gamma^2)}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \bar{\theta}^X(\gamma).$$

We next see the case in which the retailer orders both varieties in period 1. Notice that for  $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$ ,  $q_{XR}^{BY}$  become zero so that the outcome is the same with that in the YX case. Simply comparing the threshold values in Table 1 shows that  $\frac{5\gamma}{8(1+\gamma)} < \frac{5}{7}$ . We show that the online store never encroaches with either variety except for a single point. If  $\frac{5\gamma}{8(1+\gamma)} < \frac{c}{a} \leq \frac{5}{7}$ , we find

that  $\pi_M^{BY} \geq \max\{\pi_M^{BN}, \pi_M^{BB}\}$  for  $c/a = \frac{10-\sqrt{10}}{18}$ . If  $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$ , we find that  $\pi_M^{YX} < \max\{\pi_M^{BN}, \pi_M^{BB}\}$ . Thus, we have proved that encroaching with either variety is a weakly dominated strategy in this case. The manufacturer chooses  $B$  if  $\frac{5\gamma}{8(1+\gamma)} < \frac{c}{a} < \frac{5}{7}$  and  $\pi_M^{BB} \geq \max\{\pi_M^{BN}, \pi_M^{BY}\}$ , or if  $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$  and  $\pi_M^{BB} \geq \max\{\pi_M^{BN}, \pi_M^{YX}\}$ , from which we obtain

$$\frac{c}{a} \leq \theta^B(\gamma) \equiv \frac{10 - \sqrt{10}}{18}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \theta^B(\gamma).$$

□

### 5.3 The case of duopoly retailers

Let  $q_{nR_i}^{r_1 r_2 m}(\hat{w})$  and  $q_{nM}^{r_1 r_2 m}(\hat{w})$  be the retailer  $i$  and the online store's equilibrium quantities in period 3, where  $\hat{w} \in (w_{X1}^{XXm}, w_{X2}^{XXm})$  if  $r_1 r_2 = XX$ ,  $\hat{w} \in (w_{X1}^{XBm}, w_{X2}^{XBm}, w_{Y2}^{XBm})$  if  $r_1 r_2 = XB$ ,  $\hat{w} \in (w_{X1}^{XYm}, w_{Y2}^{XYm})$  if  $r_1 r_2 = XY$ , or  $\hat{w} \in (w_{X1}^{BBm}, w_{X2}^{BBm}, w_{Y1}^{BBm}, w_{Y2}^{BBm})$  if  $r_1 r_2 = BB$ . The equilibrium outcomes in period 3 are as follows:

(i) when  $r_1 r_2 m = XXN$ :

$$q_{XR1}^{XXN}(\hat{w}) = \frac{a - 2w_{X1}^{XXN} + w_{X2}^{XXN}}{3}, \quad q_{XR2}^{XXN}(\hat{w}) = \frac{a + w_{X1}^{XXN} - 2w_{X2}^{XXN}}{3};$$

(ii) when  $r_1 r_2 m = XXX$ :

$$q_{XR1}^{XXX}(\hat{w}) = \frac{a + c - 3w_{X1}^{XXX} + w_{X2}^{XXX}}{4}, \quad q_{XR2}^{XXX}(\hat{w}) = \frac{a + c + w_{X1}^{XXX} - 3w_{X2}^{XXX}}{4},$$

$$q_{XM}^{XXX}(\hat{w}) = \frac{a - 3c + w_{X1}^{XXX} + w_{X2}^{XXX}}{4};$$

(iii) when  $r_1 r_2 m = XXY$ :

$$q_{XR1}^{XXY}(\hat{w}) = \frac{(2 - \gamma)a + \gamma c - (4 - \gamma^2)w_{X1}^{XXY} + (2 - \gamma^2)w_{X2}^{XXY}}{2(3 - \gamma^2)},$$

$$q_{XR1}^{XXY}(\hat{w}) = \frac{(2 - \gamma)a + \gamma c + (2 - \gamma^2)w_{X1}^{XXY} - (4 - \gamma^2)w_{X2}^{XXY}}{2(3 - \gamma^2)},$$



$$q_{YM}^{XXY}(\hat{w}) = \frac{(3-2\gamma)a - 3c + \gamma w_{X1}^{XXY} + \gamma w_{X2}^{XXY}}{2(3-\gamma^2)};$$

(iv) when  $r_1 r_2 m = XXB$ :

$$q_{XR1}^{XXB}(\hat{w}) = \frac{a+c-3w_{X1}^{XXB}+w_{X2}^{XXB}}{4}, \quad q_{XR2}^{XXB}(\hat{w}) = \frac{a+c+w_{X1}^{XXB}-3w_{X2}^{XXB}}{4},$$

$$q_{XM}^{XXB}(\hat{w}) = \frac{(1-\gamma)a - (3+\gamma)c + (1+\gamma)w_{X1}^{XXB} + (1+\gamma)w_{X2}^{XXB}}{4(1+\gamma)},$$

$$q_{YM}^{XXB}(\hat{w}) = \frac{a-c}{2(1+\gamma)};$$

(v) when  $r_1 r_2 m = XYN$ :

$$q_{XR1}^{XYN}(\hat{w}) = \frac{(2-\gamma)a - 2w_{X1}^{XYN} + \gamma w_{Y2}^{XYN}}{4-\gamma^2},$$

$$q_{YR2}^{XYN}(\hat{w}) = \frac{(2-\gamma)a + \gamma w_{X1}^{XYN} - 2\gamma w_{Y2}^{XYN}}{4-\gamma^2};$$

(vi) when  $r_1 r_2 m = YXX$ :

$$q_{XR1}^{YXX}(\hat{w}) = \frac{(2-\gamma)a + (2-\gamma^2)c - (4-\gamma^2)w_{X1}^{YXX} + \gamma w_{Y2}^{YXX}}{2(3-\gamma^2)},$$

$$q_{YR2}^{YXX}(\hat{w}) = \frac{(3-2\gamma)a + \gamma c + \gamma w_{X1}^{YXX} - 3w_{Y2}^{YXX}}{2(3-\gamma^2)},$$

$$q_{XM}^{YXX}(\hat{w}) = \frac{(2-\gamma)a - (4-\gamma^2)c + (2-\gamma^2)w_{X1}^{YXX} + \gamma w_{Y2}^{YXX}}{2(3-\gamma^2)};$$

(vii) when  $r_1 r_2 m = XYB$ :

$$q_{XR1}^{XYB}(\hat{w}) = \frac{(3-\gamma)a + (3-\gamma)c - 6w_{X1}^{XYB} + 2\gamma w_{Y2}^{XYB}}{9-\gamma^2},$$

$$q_{YR2}^{XYB}(\hat{w}) = \frac{(3-\gamma)a + (3-\gamma)c + 2\gamma w_{X1}^{XYB} - 6w_{Y2}^{XYB}}{9-\gamma^2},$$

$$q_{XM}^{XYB}(\hat{w}) = \frac{(3-\gamma)a + (6+\gamma-\gamma^2)c + 3(1+\gamma)w_{X1}^{XYB} - (\gamma+\gamma^2)w_{Y2}^{XYB}}{9+9\gamma-\gamma^2-\gamma^3},$$

$$q_{YM}^{XYB}(\hat{w}) = \frac{(3-\gamma)a + (6+\gamma-\gamma^2)c - (\gamma+\gamma^2)w_{X1}^{XYB} + 3(1+\gamma)w_{Y2}^{XYB}}{9+9\gamma-\gamma^2-\gamma^3};$$

(viii) when  $r_1 r_2 m = XBN$ :

$$q_{XR1}^{XBN}(\hat{w}) = \frac{a - 2w_{X1}^{XBN} + w_{X2}^{XBN}}{3},$$

$$q_{XR2}^{XBN}(\hat{\mathbf{w}}) = \frac{(2 - 3\gamma + \gamma^2)a + 2(1 - \gamma^2)w_{X1}^{XBN} - (4 - \gamma^2)w_{X2}^{XBN} + 3\gamma w_{Y2}^{XBN}}{6(1 - \gamma^2)},$$

$$q_{XR2}^{XBN}(\hat{\mathbf{w}}) = \frac{(1 - \gamma)a + \gamma w_{X2}^{XBN} - w_{Y2}^{XBN}}{2(1 - \gamma^2)};$$

(ix) when  $r_1 r_2 m = XBY$ :

$$q_{XR1}^{XBY}(\hat{\mathbf{w}}) = \frac{a + c - 3w_{X1}^{XBY} + w_{X2}^{XBY}}{4},$$

$$q_{XR2}^{XBY}(\hat{\mathbf{w}}) = \frac{(2 - 2\gamma + \gamma^2)a + (1 - \gamma^2)c + (1 - \gamma^2)w_{X1}^{XBY} - (3 - \gamma^2)w_{X2}^{XBY} + 2\gamma w_{Y2}^{XBY}}{4(1 - \gamma^2)},$$

$$q_{YR2}^{XBY}(\hat{\mathbf{w}}) = \frac{(1 - \gamma)a + \gamma w_{X2}^{XBY} - w_{Y2}^{XBY}}{2(1 - \gamma^2)},$$

$$q_{XM}^{XBY}(\hat{\mathbf{w}}) = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4};$$

(x) when  $r_1 r_2 m = XBY$ :

$$q_{XR1}^{XBY}(\hat{\mathbf{w}}) = \frac{(3 - \gamma)a + 2\gamma c - 6w_{X1}^{XBY} + 3w_{X2}^{XBY} - \gamma w_{Y2}^{XBY}}{9 - \gamma^2},$$

$$q_{XR2}^{XBY}(\hat{\mathbf{w}}) = \frac{(3 - 4\gamma + \gamma^2)a - (1 - \gamma^2)c + 3(1 - \gamma^2)w_{X1}^{XBY} - 2(3 - \gamma^2)w_{X2}^{XBY} + (5\gamma - \gamma^3)w_{Y2}^{XBY}}{9 - 10\gamma^2 + \gamma^4},$$

$$q_{YR2}^{XBY}(\hat{\mathbf{w}}) = \frac{(3 - 4\gamma + \gamma^2)a + 3(1 - \gamma^2)c - (\gamma - \gamma^3)w_{X1}^{XBY} + (5\gamma - \gamma^3)w_{X2}^{XBY} - 2(3 - \gamma^2)w_{Y2}^{XBY}}{9 - 10\gamma^2 + \gamma^4},$$

$$q_{XM}^{XBY}(\hat{\mathbf{w}}) = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4};$$

(xi) when  $r_1 r_2 m = XBB$ :

$$q_{XR1}^{XBB}(\hat{\mathbf{w}}) = \frac{a + c - 3w_{X1}^{XBB} + w_{X2}^{XBB}}{4},$$

$$q_{XR2}^{XBB}(\hat{\mathbf{w}}) = \frac{(3 - 4\gamma + \gamma^2)a + (3 - 4\gamma + \gamma^2)c + 3(1 - \gamma^2)w_{X1}^{XBB} - (9 - \gamma^2)w_{X2}^{XBB} + 8\gamma w_{Y2}^{XBB}}{12(1 - \gamma^2)},$$

$$q_{YR2}^{XBB}(\hat{\mathbf{w}}) = \frac{(1 - \gamma)a + (1 - \gamma)c + 2\gamma w_{X2}^{XBB} - 2w_{Y2}^{XBB}}{3(1 - \gamma^2)},$$

$$q_{XM}^{XBB}(\hat{\mathbf{w}}) = \frac{(3 - 4\gamma + \gamma^2)a - (9 - 8\gamma - \gamma^2)c + 3(1 - \gamma^2)w_{X1}^{XBB} + (3 + \gamma^2)w_{X2}^{XBB} - 4\gamma w_{Y2}^{XBB}}{12(1 - \gamma^2)},$$

$$q_{YM}^{XBB}(\hat{\mathbf{w}}) = \frac{(1 - \gamma)a - 2(1 - \gamma)c - \gamma w_{X2}^{XBB} + w_{Y2}^{XBB}}{3(1 - \gamma^2)};$$

(xii) when  $r_1 r_2 m = BBN$ :

$$q_{XR1}^{BBN}(\hat{w}) = \frac{(1-\gamma)a - 2w_{X1}^{BBN} + w_{X2}^{BBN} + 2\gamma w_{Y1}^{BBN} - \gamma w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{YR1}^{BBN}(\hat{w}) = \frac{(1-\gamma)a + 2\gamma w_{X1}^{BBN} - \gamma w_{X2}^{BBN} - 2w_{Y1}^{BBN} + w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{XR2}^{BBN}(\hat{w}) = \frac{(1-\gamma)a + w_{X1}^{BBN} - 2w_{X2}^{BBN} - \gamma w_{Y1}^{BBN} + 2\gamma w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{YR2}^{BBN}(\hat{w}) = \frac{(1-\gamma)a - \gamma w_{X1}^{BBN} + 2\gamma w_{X2}^{BBN} + w_{Y1}^{BBN} - 2w_{Y2}^{BBN}}{3(1-\gamma^2)};$$

(xiii) when  $r_1 r_2 m = BBY$ :

$$q_{XR1}^{BBY}(\hat{w}) = \frac{(3-4\gamma+\gamma^2)a + 3(1-\gamma^2)c}{12(1-\gamma^2)} - \frac{(9-\gamma^2)w_{X1}^{BBY} - (3+\gamma^2)w_{X2}^{BBY} - 8\gamma w_{Y1}^{BBY} + 4\gamma w_{Y2}^{BBY}}{12(1-\gamma^2)},$$

$$q_{YR1}^{BBY}(\hat{w}) = \frac{(1-\gamma)a + 2\gamma w_{X1}^{BBY} - \gamma w_{X2}^{BBY} - 2w_{Y1}^{BBY} + w_{Y2}^{BBY}}{3(1-\gamma^2)},$$

$$q_{XR2}^{BBY}(\hat{w}) = \frac{(3-4\gamma+\gamma^2)a + 3(1-\gamma^2)c}{12(1-\gamma^2)} + \frac{(3+\gamma^2)w_{X1}^{BBY} - (9-\gamma^2)w_{X2}^{BBY} - 4\gamma w_{Y1}^{BBY} + 8\gamma w_{Y2}^{BBY}}{12(1-\gamma^2)},$$

$$q_{YR2}^{BBY}(\hat{w}) = \frac{(1-\gamma)a - \gamma w_{X1}^{BBY} + 2\gamma w_{X2}^{BBY} + w_{Y1}^{BBY} - 2w_{Y2}^{BBY}}{3(1-\gamma^2)},$$

$$q_{XM}^{BBY}(\hat{w}) = \frac{a - 3c + w_{X1}^{BBY} + w_{X2}^{BBY}}{4};$$

(xiv) when  $r_1 r_2 m = BBB$ :

$$q_{XR1}^{BBB}(\hat{w}) = \frac{(1-\gamma)a + (1-\gamma)c - 3w_{X1}^{BBB} + w_{X2}^{BBB} + 3\gamma w_{Y1}^{BBB} - \gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YR1}^{BBB}(\hat{w}) = \frac{(1-\gamma)a + (1-\gamma)c + 3\gamma w_{X1}^{BBB} - \gamma w_{X2}^{BBB} - 3w_{Y1}^{BBB} + w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{XR2}^{BBB}(\hat{w}) = \frac{(1-\gamma)a + (1-\gamma)c + w_{X1}^{BBB} - 3w_{X2}^{BBB} - \gamma w_{Y1}^{BBB} + 3\gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YR2}^{BBB}(\hat{w}) = \frac{(1-\gamma)a + (1-\gamma)c - \gamma w_{X1}^{BBB} + 3\gamma w_{X2}^{BBB} + w_{Y1}^{BBB} - 3w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{XM}^{BBB}(\hat{w}) = \frac{(1-\gamma)a - 3(1-\gamma)c + w_{X1}^{BBB} + w_{X2}^{BBB} - \gamma w_{Y1}^{BBB} - \gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YM}^{BBB}(\hat{w}) = \frac{(1-\gamma)a - 3(1-\gamma)c - \gamma w_{X1}^{BBB} - \gamma w_{X2}^{BBB} + w_{Y1}^{BBB} + w_{Y2}^{BBB}}{4(1-\gamma^2)}.$$

By substituting the equilibrium outcomes in period 3 and solving the manufacturer's optimization problem, we derive the equilibrium wholesale prices in period 2 as in Table 2. Substituting the equilibrium wholesale prices, we derive the retailer and manufacturer's equilibrium quantities and profits in period 2 as follows:

(i) when  $r_1 r_2 m = XXN$ :

$$q_{XR1}^{XXN} = q_{XR2}^{XXN} = \frac{a}{6}, \pi_{R1}^{XXN} = \pi_{R2}^{XXN} = \frac{a^2}{36}, \pi_M^{XXN} = \frac{a^2}{6};$$

(ii) when  $r_1 r_2 m = XXX$ :

$$q_{XR1}^{XXX} = q_{XR2}^{XXX} = \frac{c}{3}, q_{XM}^{XXX} = \frac{3a - c}{6}, \pi_{R1}^{XXX} = \pi_{R2}^{XXX} = \frac{c^2}{9}, \pi_M^{XXX} = \frac{3a^2 - 6ac + 7c^2}{12};$$

(iii) when  $r_1 r_2 m = XXY$ :

$$q_{XR1}^{XXY} = q_{XR2}^{XXY} = \frac{(1 - \gamma)a + \gamma c}{3(2 - \gamma^2)}, q_{YM}^{XXY} = \frac{(6 - 2\gamma - \gamma^2)a - (6 - \gamma^2)c}{6(2 - \gamma^2)},$$

$$\pi_{R1}^{XXY} = \pi_{R2}^{XXY} = \frac{[(1 - \gamma)a + \gamma c]^2}{9(2 - \gamma^2)^2}, \pi_M^{XXY} = \frac{(10 - 8\gamma + \gamma^2)a^2 - (12 - 8\gamma + 2\gamma^2)ac + (6 + \gamma^2)c^2}{12(2 - \gamma^2)};$$

(iv) when  $r_1 r_2 m = XXB$ :

$$q_{XR1}^{XXB} = q_{XR2}^{XXB} = \frac{c}{3}, q_{XM}^{XXB} = \frac{3a - (5 + 2\gamma)c}{6(1 + \gamma)}, q_{YM}^{XXB} = \frac{a - c}{2(1 + \gamma)},$$

$$\pi_{R1}^{XXB} = \pi_{R2}^{XXB} = \frac{c^2}{9}, \pi_M^{XXB} = \frac{3a^2 - 6ac + (5 + 2\gamma)c^2}{6(1 + \gamma)};$$

(v) when  $r_1 r_2 m = XYN$ :

$$q_{XR1}^{XYN} = q_{XR2}^{XYN} = \frac{a}{2(2 + \gamma)}, \pi_{R1}^{XYN} = \pi_{R2}^{XYN} = \frac{a^2}{4(2 + \gamma)^2}, \pi_M^{XYN} = \frac{a^2}{2(2 + \gamma)};$$

(vi) when  $r_1 r_2 m = XYX$ :

$$q_{XR1}^{XYX} = \frac{4(2 - \gamma^2)c - (\gamma - \gamma^2)a}{4(5 - 2\gamma^2)}, q_{XR2}^{XYX} = \frac{5(1 - \gamma)a + 4\gamma c}{4(5 - 2\gamma^2)}, q_{XM}^{XYX} = \frac{(5 - \gamma - \gamma^2)a - (7 - 2\gamma^2)c}{2(5 - 2\gamma^2)},$$

$$\pi_{R1}^{XYX} = \frac{[(\gamma - \gamma^2)a - (8 - 4\gamma^2)c]^2}{16(5 - 2\gamma^2)^2}, \pi_{R2}^{XYX} = \frac{[5(1 - \gamma)a + 4\gamma c]^2}{16(5 - 2\gamma^2)^2},$$

$$\pi_M^{XYX} = \frac{(15 - 10\gamma + \gamma^2)a - (20 - 8\gamma)ac + (18 - 4\gamma^2)c^2}{8(5 - 2\gamma^2)};$$

(vii) when  $r_1 r_2 m = XYB$ :

$$q_{XR1}^{XYB} = q_{YR2}^{XYB} = \frac{2c}{5 + \gamma}, \quad q_{XM}^{XYB} = q_{YM}^{XYB} = \frac{(5 + \gamma)a - (7 + 3\gamma)c}{2(5 + 6\gamma + \gamma^2)},$$

$$\pi_{R1}^{XYB} = \pi_{R2}^{XYB} = \frac{4c^2}{(5 + \gamma)^2}, \quad \pi_M^{XYB} = \frac{(5 + \gamma)a^2 - 2(5 + \gamma)ac + (9 + 5\gamma)c^2}{2(5 + 6\gamma + \gamma^2)};$$

(viii) when  $r_1 r_2 m = XBN$ :

$$q_{XR1}^{XBN} = \frac{a}{6}, \quad q_{XR2}^{XBN} = \frac{(2 - \gamma)a}{12(1 + \gamma)}, \quad q_{YR2}^{XBN} = \frac{a}{4(1 + \gamma)},$$

$$\pi_{R1}^{XBN} = \frac{a^2}{36}, \quad \pi_{R2}^{XBN} = \frac{(13 - 5\gamma)a^2}{144(1 + \gamma)}, \quad \pi_M^{XBN} = \frac{(7 + \gamma)a^2}{24(1 + \gamma)};$$

(ix) when  $r_1 r_2 m = XBY$ :

$$q_{XR1}^{XBY} = \frac{c}{3}, \quad q_{XR2}^{XBY} = \frac{4(1 + \gamma)c - 3\gamma a}{12(1 + \gamma)}, \quad q_{YR2}^{XBY} = \frac{a}{4(1 + \gamma)}, \quad q_{XM}^{XBY} = \frac{3a - \gamma c}{6},$$

$$\pi_{R1}^{XBY} = \frac{c^2}{9}, \quad \pi_{R2}^{XBY} = \frac{9(1 - \gamma)a^2 + 16(1 + \gamma)c^2}{144(1 + \gamma)}, \quad \pi_M^{XBY} = \frac{3(3 + \gamma)a^2 - 12(1 + \gamma)ac + 14(1 + \gamma)c^2}{24(1 + \gamma)};$$

(x) when  $r_1 r_2 m = XBY$ :

$$q_{XR1}^{XBY} = \frac{5(1 - \gamma)a + 8\gamma c}{6(5 - \gamma^2)}, \quad q_{XR2}^{XBY} = \frac{(5 + \gamma^2)a - 4(\gamma + \gamma^2)c}{6(5 + 5\gamma - \gamma^2 - \gamma^3)}, \quad q_{YR2}^{XBY} = \frac{(15 - 2\gamma - \gamma^2)a - (21 - \gamma^2)c}{6(5 - \gamma^2)},$$

$$q_{XM}^{XBY} = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4},$$

$$\pi_{R1}^{XBY} = \frac{[5(1 - \gamma)a + 8\gamma c]^2}{36(5 - \gamma^2)^2}, \quad \pi_{R2}^{XBY} = \frac{(25 - 25\gamma + 11\gamma^2 - 11\gamma^3)a^2 - 64(\gamma - \gamma^3)ac + 16(9 + 9\gamma - 5\gamma^2 - 5\gamma^3)c^2}{36(25 + 25\gamma - 10\gamma^2 - 10\gamma^3 + \gamma^4 + \gamma^5)},$$

$$\pi_M^{XBY} = \frac{(25 + 5\gamma - 7\gamma^2)a^2 - 2(15 + 11\gamma - 3\gamma^2 + \gamma^3)ac + (27 + 27\gamma + \gamma^2 + \gamma^3)c^2}{5 + 5\gamma - \gamma^2 - \gamma^3};$$

(xi) when  $r_1 r_2 m = XBB$ :

$$q_{XR1}^{XBB} = \frac{c}{3}, \quad q_{XR2}^{XBB} = \frac{(5 - \gamma)c}{15(1 + \gamma)}, \quad q_{YR2}^{XBB} = \frac{2c}{5(1 + \gamma)}, \quad q_{XM}^{XBB} = \frac{15a - (25 + 4\gamma)c}{30(1 + \gamma)}, \quad q_{YM}^{XBB} = \frac{5a - 7c}{10(1 + \gamma)},$$

$$\pi_{R1}^{XBB} = \frac{c^2}{9}, \quad \pi_{R2}^{XBB} = \frac{(61 - 11\gamma)c^2}{225(1 + \gamma)}, \quad \pi_M^{XBB} = \frac{15a^2 - 30ac + (31 + 4\gamma)c^2}{30(1 + \gamma)};$$

(xii) when  $r_1 r_2 m = BBN$ :

$$q_{XR1}^{BBN} = q_{YR1}^{BBN} = q_{XR2}^{BBN} = q_{YR2}^{BBN} = \frac{a}{6(1 + \gamma)}, \quad \pi_{R1}^{BBN} = \pi_{R2}^{BBN} = \frac{a^2}{18(1 + \gamma)}, \quad \pi_M^{BBN} = \frac{a^2}{3(1 + \gamma)};$$

(xiii) when  $r_1 r_2 m = BBY$ :

$$q_{XR1}^{BBY} = q_{XR2}^{BBY} = \frac{2(1+\gamma)c - \gamma a}{6(1+\gamma)}, \quad q_{YR1}^{BBY} = q_{YR2}^{BBY} = \frac{a}{6(1+\gamma)}, \quad q_{XM}^{BBY} = \frac{3a - 5c}{6},$$

$$\pi_{R1}^{BBY} = \pi_{R2}^{BBY} = \frac{(1-\gamma)a^2 + 4(1+\gamma)c^2}{36(1+\gamma)}, \quad \pi_M^{BBY} = \frac{(5+\gamma)a^2 - 6(1+\gamma)ac + 7(1+\gamma)c^2}{12(1+\gamma)};$$

(xiv) when  $r_1 r_2 m = BBB$ :

$$q_{XR1}^{BBB} = q_{XR2}^{BBB} = q_{YR1}^{BBB} = q_{YR2}^{BBB} = \frac{c}{3(1+\gamma)}, \quad q_{XM}^{BBB} = q_{YM}^{BBB} = \frac{3a - 5c}{6(1+\gamma)},$$

$$\pi_{R1}^{BBB} = \pi_{R2}^{BBB} = \frac{2c^2}{9(1+\gamma)}, \quad \pi_M^{BBB} = \frac{3a^2 - 6ac + 7c^2}{6(1+\gamma)}.$$

Making all quantities positive, we obtain the *I.S.* in Table 2. We will prove that encroaching with variety  $X$  through the online store when receiving the retailer's  $XX$  or  $XB$  type order, and encroaching with either variety through the online store when receiving the retailer's  $XY$  or  $BB$  are dominated strategies later.

## 5.4 Proof of Lemma 2

In period 2, we solve for the manufacturer's equilibrium outcome, taking the corner solutions into consideration and checking the incentive for deviation.

First we see the case in which the manufacturer receives the  $XX$  type order in period 1. Simply comparing the threshold values in Table 2 shows that  $\frac{3}{5+2\gamma} < \frac{3}{5} < \frac{6-2\gamma-\gamma^2}{6-\gamma^2}$ . We show that the online store never encroaches with variety  $X$  in this case. If  $\frac{c}{a} \leq \frac{3}{5}$ , we find that  $\pi_M^{XXX} < \pi_M^{XXB}$ . Thus, we have proved that encroaching with variety  $X$  is a *strictly dominated strategy* in this case. The manufacturer chooses  $B$  if  $\frac{c}{a} \leq \frac{3}{5+2\gamma}$  and  $\pi_M^{XXB} \geq \max\{\pi_M^{XXN}, \pi_M^{XXX}, \pi_M^{XXY}\}$ , from which we obtain

$$\frac{c}{a} \leq \underline{\theta}^{XX}(\gamma) \equiv \frac{(6 + 4\gamma + \gamma^2) - 2\sqrt{2 + 4\gamma + \gamma^2 - 2\gamma^3 - \gamma^4}}{14 + 16\gamma + 5\gamma^2}.$$

The outcome that the manufacturer chooses  $Y$  must satisfy the following conditions: if  $\frac{3}{5} < \frac{c}{a} \leq \frac{6-2\gamma-\gamma^2}{6-\gamma^2}$  and  $\pi_M^{XXY} \geq \pi_M^{XXN}$ , if  $\frac{3}{5+2\gamma} < \frac{c}{a} \leq \frac{3}{5}$  and  $\pi_M^{XXY} \geq \max\{\pi_M^{XXN}, \pi_M^{XXX}\}$ , if  $\frac{c}{a} \leq \frac{3}{5+2\gamma}$  and  $\pi_M^{XXY} \geq \max\{\pi_M^{XXN}, \pi_M^{XXX}, \pi_M^{XXB}\}$ , from which we obtain

$$\underline{\theta}^{XX}(\gamma) \leq \frac{c}{a} \leq \bar{\theta}^{XX}(\gamma) \equiv \frac{(6 - 4\gamma + \gamma^2) - \gamma\sqrt{2(2 - \gamma^2)}}{6 + \gamma^2}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \bar{\theta}^{XX}(\gamma).$$

Second, we see the case in which the manufacturer receives the  $XY$  type order in period

1. Notice that for  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$ ,  $q_{XR1}^{XYX}$  becomes zero so that the outcome is the same with that in the  $NYX$  case. Simply comparing the threshold values in Table 2 shows that  $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{5+\gamma}{7+3\gamma} < \frac{5-\gamma-\gamma^2}{7-2\gamma^2}$ . We show that the online store never encroaches with variety  $X$  in this case. If  $\frac{5+\gamma}{7+3\gamma} < \frac{c}{a} \leq \frac{5-\gamma-\gamma^2}{7-2\gamma^2}$ , we find that  $\pi_M^{XYX} < \pi_M^{XYN}$ . If  $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \leq \frac{5+\gamma}{7+3\gamma}$ , we find that  $\pi_M^{XYX} < \max\{\pi_M^{XYN}, \pi_M^{XYB}\}$ . If  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$ , we find that  $\pi_M^{NYX} < \max\{\pi_M^{XYN}, \pi_M^{XYB}\}$ . Thus, we have proved that encroaching with variety  $X$  is a *strictly dominated strategy* in this case. The outcome that the manufacturer chooses  $B$  must satisfy the following conditions: if  $\frac{1-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} < \frac{5+\gamma}{7+3\gamma}$  and  $\pi_M^{XYB} \geq \max\{\pi_M^{XYN}, \pi_M^{XYX}\}$ , or if  $\frac{c}{a} \leq \frac{1-\gamma^2}{4(2-\gamma^2)}$  and  $\pi_M^{XYB} \geq \max\{\pi_M^{XYN}, \pi_M^{NYX}\}$ , from which we obtain

$$\frac{c}{a} \leq \theta^{XY}(\gamma) \equiv \frac{(10 + 7\gamma + \gamma^2) - \sqrt{10 + 27\gamma + 25\gamma^2 + 9\gamma^3 + \gamma^4}}{18 + 19\gamma + 5\gamma^2}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \theta^{XY}(\gamma).$$

Third, we see the case in which the manufacturer receives the  $XB$  type order in period 1.

Notice that for  $\frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$ ,  $q_{XR2}^{XBY}$  becomes zero so that the outcome is the same with that in the  $XYX$  case; for  $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ ,  $q_{YR2}^{XBY}$  becomes zero so that the outcome is the same with that in the  $XXY$  case; for  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma)}$ ,  $q_{XR1}^{XYX}$  becomes zero so that the outcome is the same with that in the  $NYX$  case. Simply comparing the threshold values in Table 2 shows that  $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{\gamma}{2(1+\gamma)} < \frac{3\gamma}{4(1+\gamma)} < \frac{15}{25+4\gamma} < \frac{3}{5} < \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$ . We show that the online store never encroaches with variety  $X$  in this case. If  $\frac{15}{25+4\gamma} < \frac{c}{a} \leq \frac{3}{5}$  in which the  $XBB$  equilibrium outcome does not exist, we find that  $\pi_M^{XBY} < \max\{\pi_M^{XBN}, \pi_M^{XBY}\}$ . If  $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \leq \frac{15}{25+4\gamma}$ , we find that  $\pi_M^{XBY} < \max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$ . If  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$ , we find that  $\pi_M^{XYX} < \max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$ . If  $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ , we find that  $\pi_M^{XYX} < \max\{\pi_M^{XBN}, \pi_M^{XXY}, \pi_M^{XBB}\}$ . If  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$ , we find that  $\pi_M^{NYX} < \max\{\pi_M^{XBN}, \pi_M^{XXY}, \pi_M^{XBB}\}$ . Thus, we have proved that encroaching with variety  $X$  is a *strictly dominated strategy* in this case. The outcome that the manufacturer chooses  $B$  must

satisfy the following conditions: if  $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \leq \frac{15}{25+4\gamma}$  and  $\pi_M^{XBB} \geq \max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBY}\}$ , or if  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$  and  $\pi_M^{XBB} \geq \max\{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XBY}\}$ , or  $\frac{\gamma-\gamma^2}{4(2-\gamma)} < \frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$  and  $\pi_M^{XBB} \geq \max\{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XXY}\}$ , or  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma)}$  and  $\pi_M^{XBB} \geq \max\{\pi_M^{XBN}, \pi_M^{NYX}, \pi_M^{XXY}\}$ , from which we obtain

$$\text{when } \gamma \leq 0.690, \frac{c}{a} \leq \underline{\theta}_1^{XB}(\gamma) \equiv \frac{5(15 + 4\gamma + \gamma^2) - (5 + 2\gamma)\sqrt{10(5 - \gamma^2)}}{175 + 80\gamma + 13\gamma^2},$$

$$\text{as well as when } \gamma > 0.690, \frac{c}{a} \leq \underline{\theta}_2^{XB}(\gamma) \equiv \frac{5(6 + 4\gamma + \gamma^2) - 2\sqrt{5(-2 + 20\gamma + 5\gamma^2 - 10\gamma^3 - 2\gamma^4)}}{94 + 80\gamma + 13\gamma^2}.$$

The outcome that the manufacturer chooses  $Y$  must satisfy the following conditions: if  $\frac{3}{5} < \frac{c}{a} \leq \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$  and  $\pi_M^{XBY} \geq \pi_M^{XBN}$ , or if  $\frac{15}{25+4\gamma} < \frac{c}{a} \leq \frac{3}{5}$  and  $\pi_M^{XBY} \geq \{\pi_M^{XBN}, \pi_M^{XBY}\}$ , or if  $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \leq \frac{15}{25+4\gamma}$  and  $\pi_M^{XBY} \geq \{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$ , or if  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$  and  $\pi_M^{XBY} \geq \{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XBB}\}$ , from which we obtain

$$\text{when } \gamma \leq 0.690, \underline{\theta}_1^{XB}(\gamma) \leq \frac{c}{a} \leq \bar{\theta}_1^{XB}(\gamma) \equiv \frac{2(15 - 4\gamma + \gamma^2) - (3 + \gamma)\sqrt{2(5 - \gamma^2)}}{2(27 + \gamma^2)},$$

$$\text{as well as when } 0.690 < \gamma \leq 0.897, \hat{\theta}^{XB}(\gamma) \equiv \frac{\gamma}{2(1 + \gamma)} \leq \frac{c}{a} \leq \bar{\theta}_1^{XB}(\gamma).$$

The outcome that the manufacturer does not open its online store must satisfy the following conditions: if  $\frac{c}{a} > \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$ , or if  $\frac{3}{5} < \frac{c}{a} \leq \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$  and  $\pi_M^{XBN} \geq \pi_M^{XBY}$ , or  $\frac{15}{25+4\gamma} \leq \frac{c}{a} < \frac{3}{5}$  and  $\pi_M^{XBN} \geq \max\{\pi_M^{XBY}, \pi_M^{XBY}\}$ , or  $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \leq \frac{15}{25+4\gamma}$  and  $\pi_M^{XBN} \geq \max\{\pi_M^{XBY}, \pi_M^{XBY}, \pi_M^{XBB}\}$ , or  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$  and  $\pi_M^{XBN} \geq \max\{\pi_M^{XYX}, \pi_M^{XBY}, \pi_M^{XBB}\}$ , or  $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$  and  $\pi_M^{XBN} \geq \max\{\pi_M^{XYX}, \pi_M^{XXY}, \pi_M^{XBB}\}$ , or  $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$  and  $\pi_M^{XBN} \geq \max\{\pi_M^{NYX}, \pi_M^{XXY}, \pi_M^{XBB}\}$ , from which we obtain

$$\text{when } \gamma \leq 0.897, \frac{c}{a} > \bar{\theta}_1^{XB}(\gamma),$$

$$\text{as well as when } \gamma > 0.897, \frac{c}{a} > \bar{\theta}_2^{XB}(\gamma)$$

$$\equiv \frac{(1 + \gamma)(12 - 8\gamma + 2\gamma^2) - \sqrt{2(1 + \gamma)(36 - 36\gamma - 4\gamma^2 + 20\gamma^3 - 7\gamma^4 - \gamma^5)}}{2(1 + \gamma)(6 + \gamma^2)}.$$

Fourth, we see the case in which the manufacturer receives the  $BB$  type order in period

1. Notice that for  $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ ,  $q_{XR1}^{BBY}$  and  $q_{XR2}^{BBY}$  becomes zero so that the outcome is the same with that in the  $YYX$  case. Simply comparing the threshold values in Table 2 shows that



$\frac{\gamma}{2(1+\gamma)} < \frac{3}{5}$ . We show that the online store never encroaches with either variety except for a single point in this case. If  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3}{5}$ , we find that  $\pi_M^{BBY} \geq \max\{\pi_M^{BBN}, \pi_M^{BBB}\}$  when  $c/a = \theta^{BB}(\gamma)$ . If  $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ , we find that  $\pi_M^{YYX} < \max\{\pi_M^{BBN}, \pi_M^{BBB}\}$ . Thus, we have proved that encroaching with either variety is a weakly dominated strategy in this case. The outcome that the manufacturer chooses  $B$  must satisfy the following conditions: if  $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3}{5}$  and  $\pi_M^{BBB} \geq \max\{\pi_M^{BBN}, \pi_M^{BBY}\}$ , or if  $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$  and  $\pi_M^{BBB} \geq \max\{\pi_M^{XBN}, \pi_M^{YYX}\}$ , from which we obtain

$$\frac{c}{a} \leq \theta^{BB}(\gamma) \equiv \frac{3 - \sqrt{2}}{7}.$$

The outcome that the manufacturer does not open its online store must satisfy the following conditions:

$$\frac{c}{a} \geq \theta^{BB}(\gamma).$$

□

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