

Ho, Anson T. Y.; Zhou, Jie

**Working Paper**

## Housing and tax-deferred retirement accounts

Bank of Canada Staff Working Paper, No. 2016-24

**Provided in Cooperation with:**

Bank of Canada, Ottawa

*Suggested Citation:* Ho, Anson T. Y.; Zhou, Jie (2016) : Housing and tax-deferred retirement accounts, Bank of Canada Staff Working Paper, No. 2016-24, Bank of Canada, Ottawa

This Version is available at:

<http://hdl.handle.net/10419/148131>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Staff Working Paper/Document de travail du personnel 2016-24

# Housing and Tax-Deferred Retirement Accounts



by Anson T. Y. Ho and Jie Zhou

Bank of Canada staff working papers provide a forum for staff to publish work-in-progress research independently from the Bank's Governing Council. This research may support or challenge prevailing policy orthodoxy. Therefore, the views expressed in this paper are solely those of the authors and may differ from official Bank of Canada views. No responsibility for them should be attributed to the Bank.

Bank of Canada Staff Working Paper 2016-24

May 2016

## **Housing and Tax-Deferred Retirement Accounts**

by

**Anson T. Y. Ho<sup>1</sup> and Jie Zhou<sup>2</sup>**

<sup>1</sup>Department of Economics  
Kansas State University  
Manhattan, KS 66506-4000, USA  
atyho@ksu.edu

<sup>2</sup>Financial Stability Department  
Bank of Canada  
Ottawa, Ontario, Canada K1A 0G9  
jzhou@bankofcanada.ca

## **Acknowledgements**

We are grateful for comments from Jason Allen, Julia Le Blanc, John Y. Campbell, Shutao Cao, Steven P. Cassou, Kaiji Chen, Kim P. Huynh, Igor Livshits, Jim MacGee, Miguel Molico, Makoto Nakajima, Yaz Terajima, seminar participants at Emory University, Lakehead University, the University of Winnipeg, the Bank of Canada, and conference participants at the 2015 CEA meeting, the ECB's 4th conference on household finance and consumption in 2015, the 2015 Midwest Macro meeting, and the 2015 QSPS workshop. We thank Boyan Bejanov for his excellent assistance with the Bank of Canada's EDITH High Performance Cluster, where all the computations were performed. All errors are our own.

## Abstract

Assets in tax-deferred retirement accounts (TDA) and housing are two major components of household portfolios. In this paper, we develop a life-cycle model to examine the interaction between households' use of TDA and their housing decisions. The model generates life-cycle patterns of home ownership and the composition of net worth that are broadly consistent with the data from the Survey of Consumer Finances. We find that TDA promotes home ownership, as households take advantage of the preferential tax treatments for *both* TDA and home ownership. They substitute TDA assets for home equity by accumulating wealth in TDA and making smaller down payments (taking out bigger mortgages); consequently, they become homeowners earlier in their lives. On the other hand, housing-related policies, such as a minimum down payment requirement and mortgage interest deductibility, affect households' housing decisions more than their use of TDA.

*JEL classification: C61, D14, D91, E21, H24, R21*

*Bank classification: Economic models; Housing*

## Résumé

Les comptes de retraite à impôt différé et l'immobilier sont deux composantes importantes du portefeuille d'actifs des ménages. Dans notre étude, nous élaborons un modèle de cycle de vie qui permet d'examiner l'interaction entre l'utilisation des comptes de retraite à impôt différé et les décisions en matière de logement. Le modèle génère des profils d'accession à la propriété durant le cycle de vie et une composition de la valeur nette des ménages qui sont conformes, dans l'ensemble, aux données provenant de la *Survey of Consumer Finances* (enquête sur les finances des consommateurs). Nos résultats indiquent que les comptes de retraite à impôt différé favorisent l'accession à la propriété, car les ménages tirent avantage du traitement fiscal préférentiel que leur procurent *à la fois* ce type de compte et la possession d'une résidence. À l'avoir propre foncier, les ménages substituent les actifs détenus dans des comptes de retraite à impôt différé. En effet, ils accumulent leur épargne dans ces comptes et réduisent leurs mises de fonds (ils contractent donc des prêts hypothécaires plus élevés); par conséquent, ils deviennent propriétaires plus tôt dans leur vie. Cela dit, les politiques relatives au logement, comme l'exigence de mise de fonds minimale et la déductibilité des intérêts hypothécaires, influencent davantage les décisions des ménages concernant le logement que l'utilisation des comptes de retraite à impôt différé.

*Classification JEL : C61, D14, D91, E21, H24, R21*

*Classification de la Banque : Modèles économiques; Logement*

## Non-Technical Summary

Assets in tax-deferred accounts (TDA) and housing are two major components of household portfolios. These assets share similarities in terms of favorable tax treatments and illiquidity, which makes them competing assets in household portfolios. While TDA provides high return for savings, it restricts households' abilities to make down payments for home purchases. On the other hand, home ownership reduces households' cost of housing service, but it involves substantial investment in home equity that limits the amount of funds available for TDA savings. To what extent does households' use of TDA affect their housing decisions? How does the existence of TDA influence the composition of household net worth regarding home equity, TDA wealth and other wealth held in regular taxable accounts (TA) over the life cycle? This paper develops a quantitative life-cycle model to examine the interaction between households' use of TDA and their housing decisions. The model generates life-cycle patterns of home ownership and households' net worth composition that are broadly consistent with the data observed from the Survey of Consumer Finances (SCF) for households with employer-sponsored defined contribution (DC) plans.

We find that TDA not only affects retirement savings but also significantly increases home ownership. In the presence of TDA, households choose to save in TDA *and* take out bigger mortgages (i.e., make smaller down payments) to buy a house. In doing so, they can enjoy tax benefits for both TDA (income tax deferral) and home ownership (mortgage interest deductibility). As their wealth increases (due to the tax benefits) and down payments are smaller, households become homeowners earlier in their lives, leading to a large increase in home ownership. On the other hand, the effects of further raising the TDA contribution limit are small, which suggests that the impacts of TDA contributions are nonlinear. Experiments on housing-related factors (e.g., a minimum down payment requirement and mortgage interest deductibility) show that they have only moderate effects on households' use of TDA. The housing-related factors affect households' housing decisions more than their use of TDA.

This paper sheds light on households' joint decisions of retirement savings and housing as well as their wealth accumulation over the life cycle, through a model with detailed institutional features of both TDA and housing. Our results also have important implications for retirement preparedness and suggest an explanation to findings in recent studies that holdings in 401(k) plans for a substantial share of U.S. households remained low. The TDA share of net worth is small over the most part of a household's life cycle, because housing investment is also an attractive savings vehicle with consumption value.

# 1 Introduction

Assets in tax-deferred retirement accounts (TDA) and housing are two major components of household portfolios. In the United States, approximately one-half of households hold assets in TDA.<sup>1</sup> The total amount of assets in TDA stood at \$12.9 trillion (77% of GDP) in 2013.<sup>2</sup> Meanwhile, about two-thirds of U.S. households are homeowners, and housing is the single most important asset for the majority of U.S. households.

Assets in TDA and housing share similarities in terms of favorable tax treatments and illiquidity. TDA provides tax benefits through its tax-deferral feature,<sup>3</sup> but early withdrawals of TDA assets before retirement age are typically subject to a 10% penalty in addition to the income taxes incurred from asset withdrawals. Home ownership provides a cheaper way to obtain housing service, because mortgage interest is income tax deductible and the service flow from owner-occupied housing is untaxed. However, mortgage down payment imposes a significant liquidity constraint on households, and high transaction cost is incurred in buying and selling a house. These similarities between TDA and housing make TDA assets and home equity (via home ownership) competing assets in household portfolios. While TDA provides high return for savings, it restricts households' abilities to make down payments for home purchases. On the other hand, home ownership reduces households' cost of housing service, but it involves substantial investment in home equity that limits the amount of funds available for TDA savings.

This interesting interaction between households' use of TDA and their housing decisions is also illustrated in the data from the Survey of Consumer Finances (SCF). There are several distinct differences between households with and without TDA, after controlling for age, income and other household characteristics.<sup>4</sup> First, households with TDA have much higher home ownership rates. Second, home equity constitutes a smaller fraction of net worth for TDA holders. Third, from the life-cycle perspective, the share of net worth in home

---

<sup>1</sup>Common types of TDA in the United States include Individual Retirement Accounts (IRA) and employer-sponsored defined contribution (DC) pension plans such as 401(k) plans. The objective of TDA is to encourage retirement savings.

<sup>2</sup>See the [Board of Governors of the Federal Reserve System \(2014\)](#).

<sup>3</sup>Contributions to TDA (up to a limit) are income tax deductible and investment income earned in TDA is tax exempt. Subsequent asset withdrawals from TDA upon retirement are taxed as ordinary income. The tax benefits of TDA are explained in detail in [Munnell, Quinby, and Webb \(2012\)](#). In short, TDA contributions and subsequent investment returns grow at a faster (untaxed) rate. Since most households have lower marginal tax rates after retirement, deferring the tax incidence from working ages to retirement enables households to "smooth" their income streams and reduces their lifetime income tax payments.

<sup>4</sup>Detailed description of the SCF data is provided in [Section 2](#).

equity decreases faster with age for TDA holders. These stylized facts seem to suggest that TDA increases households' wealth and their demand for housing services. Home ownership that provides cheaper housing service flow becomes more attractive in the presence of TDA. Meanwhile, the tax benefits for TDA lead to the substitution of TDA wealth for home equity in household portfolios. As assets in TDA increase with age, home equity represents a smaller fraction of net worth.

We therefore ask: To what extent does households' use of TDA affect their housing decisions? How does the existence of TDA influence the composition of household net worth regarding home equity, TDA wealth and other wealth held in regular taxable accounts (TA) over the life cycle? Contrarily, do housing considerations impose a significant liquidity constraint that limits households' contributions to TDA? As TDA contributions and home ownership are endogenous decisions, we answer these questions via a life-cycle model that focuses on the tax benefits and illiquid nature of TDA and housing assets. In the model, households have access to TDA with features similar to the U.S. 401(k) plans, and they can also save in regular TA. Housing decisions entail both choices on tenure (renting or owning) and house size. Buying a house requires a long-term mortgage with a minimum down payment. There is a progressive income tax system that mimics the U.S. tax code with favorable tax treatments on TDA and home ownership. Households make endogenous decisions on housing and TDA contributions under uninsurable earnings and housing price risks. The benchmark model generates life-cycle patterns of home ownership and households' net worth composition that are broadly consistent with the data observed from the SCF for households with employer-sponsored defined contribution (DC) plans.<sup>5</sup>

Counterfactual experiments are conducted to evaluate the impacts of TDA-related and housing-related factors. We find that TDA not only affects retirement savings but also significantly increases home ownership. In the presence of TDA, households choose to save in TDA *and* take out bigger mortgages (i.e., make smaller down payments) to buy a house. In doing so, they can enjoy tax benefits for both TDA (income tax deferral) and home ownership (mortgage interest deductibility). As their wealth increases (due to the tax benefits) and down payments are smaller, households become homeowners earlier in their lives, leading to a large increase in home ownership. On the other hand, further raising the TDA contribution limit slightly increases the TDA share of household net worth, and it has insignificant impacts on home ownership and wealth accumulation. These nonlinear impacts

---

<sup>5</sup>Home ownership increases with age. The fraction of net worth in home equity decreases with age, while the TDA share of net worth increases with age. The model is also consistent with the data in that renters' income is much lower than that of homeowners.



of the TDA contribution limit are due to the fact that most households are not bounded by the contribution limit set in the benchmark, and therefore further raising the limit is irrelevant to them.

Experiments on housing-related factors show that they have only moderate effects on households' use of TDA. An increase in the minimum down payment requirement lowers home ownership as households, particularly the young ones, become credit-constrained. They defer their home purchases and reduce their TDA savings only slightly for making down payments. When mortgage interest and property taxes are not income tax deductible, households do not increase their TDA contributions to shelter their income from tax. They instead increase their home equity to reduce their mortgage interest payments. Overall, housing-related factors affect households' housing decisions more than their use of TDA, because housing is an asset that provides untaxed service flow.

This paper is related to two separate strands of the literature on TDA and housing. The literature studies these two items separately, and little attention has been paid to the interaction between TDA and households' housing decisions. The TDA literature explores the effects of TDA on households' retirement savings decisions ([Engen, Gale, and Scholz, 1996](#); [Poterba, Venti, and Wise, 1996](#); [Benjamin, 2003](#); [Engelhardt and Kumar, 2007](#), among others), the macroeconomic impacts of TDA ([Imrohoroglu, Imrohoroglu, and Joines, 1998](#); [Nishiyama, 2011](#)), and its implications for wealth distribution ([Chernozhukov and Hansen, 2004](#)). Other studies on TDA have examined households' asset allocation decisions between stocks and bonds (how much of each asset to hold) and asset location decisions (where to hold assets, in TA or TDA) in the presence of TDA ([Amromin, 2003](#); [Dammon, Spatt, and Zhang, 2004](#); [Huang, 2008](#); [Gomes, Michaelides, and Polkovnichenko, 2009](#); [Zhou, 2009](#)).

In the housing literature, previous studies focused on the preferential tax treatment of housing ([Gervais, 2002](#); [Díaz and Luengo-Prado, 2008](#); [Chambers, Garriga, and Schlagenhaut, 2009b](#)), home ownership over the life cycle and over time ([Chambers, Garriga, and Schlagenhaut, 2009a](#); [Chen, 2010](#); [Bajari, Chan, Krueger, and Miller, 2013](#); [Sommer, Sullivan, and Verbrugge, 2013](#); [Halket and Vasudev, 2014](#)), and the interaction between housing and non-housing consumption ([Campbell and Cocco, 2007](#); [Li and Yao, 2007](#); [Yang, 2009](#)). Moreover, a number of papers have studied households' portfolio choices between stocks and bonds under the influence of housing/mortgage debt ([Cocco, 2005](#); [Yao and Zhang, 2005](#); [Becker and Shabani, 2010](#)).

We are aware of only two papers that study household decisions in the presence of both TDA and housing. [Amromin, Huang, and Sialm \(2007\)](#) show empirically that many house-

holds that prepay their mortgages could be better off by contributing their prepayments to TDA. This is because the pre-tax returns households earn in TDA may be higher than the after-tax mortgage rates they pay for their mortgages. [Marekwica, Schaefer, and Sebastian \(2013\)](#) develop a model to study households' asset allocation between stocks and bonds in the presence of TDA and housing. They assume a fixed TDA contribution rate in the model and that households always choose the maximum possible mortgages over the life cycle. While these studies shed light on households' decisions on portfolio choice, they do not consider that TDA contributions, home ownership, and the level of home equity are all related endogenous decisions.

This paper offers additional insights on households' joint decisions of retirement savings and housing over the life cycle. Through a model with detailed institutional features of both TDA and housing, we show that TDA is not only a retirement savings vehicle but also has a significant impact on households' housing decisions. In particular, the existence of TDA increases home ownership significantly, as TDA induces households to make smaller down payments and become homeowners earlier. Our results also have important implications for retirement preparedness and suggest an explanation to findings in recent studies that holdings in 401(k) plans for a substantial share of U.S. households remained low ([Munnell, Golub-Sass, and Muldoon, 2009](#); [Munnell, 2012](#); [Poterba, 2014](#)). The TDA share of net worth is small over the most part of a household's life cycle because housing investment is also an attractive savings vehicle with consumption value. This finding complements [Scholz, Seshadri, and Khitatrakun \(2006\)](#) in understanding households' savings for retirement and sheds light on households' life-cycle portfolios in wealth accumulation.

The rest of the paper is organized as follows. Section [2](#) presents some stylized facts on households' use of TDA and their housing decisions. Section [3](#) describes the model in detail. Section [4](#) outlines the parameterization of the model. Section [5](#) reports the results of our benchmark model. Section [6](#) evaluates the effects of changes in TDA-related and housing-related factors. Section [7](#) concludes.

## 2 Stylized Facts

In this section, we provide some stylized facts about households' use of TDA and their housing decisions from the SCF. The SCF is a triennial cross-sectional survey conducted by the Board of Governors of the Federal Reserve System. It provides the most complete data on household balance sheets in the United States and also contains data on earnings and

other demographic information.

The data summarized below are from the 2001, 2004 and 2007 SCFs. As these SCFs span many years, the average values from these three waves minimize the influence of business cycles. We focus on households with heads aged between 25 and 64 in the paper.<sup>6</sup> All statistics use population weights.

Empirically, we define TDA as retirement accounts whose owners make pre-tax contributions and their own investment decisions. These accounts include Individual Retirement Accounts (IRA) and most of the employer-sponsored DC pension plans (such as 401(k)/403(b)/457/SRA and Thrift Savings plans). For home ownership, we exclude a principal residence being a mobile home or on a farm/ranch, both because the number of these households is very small and they are unlikely to be covered by employer-sponsored DC pension plans.

## 2.1 Comparison Between Households with and without DC Plans

For households with heads aged 25–64, approximately 40% of them report ownership of employer-sponsored DC plans in each of the three waves of SCF. If we also consider IRA, slightly more than 50% of households in the SCFs have both DC plans and IRA or one of them. Thus, a significant portion of U.S. households hold assets in TDA.

Table 1 provides summary statistics for households with heads aged 25–64 in the 2007 SCF, divided into households with employer-sponsored DC plans and those without. A number of observations are noteworthy. Households with DC plans are mostly married, and they have more years of education than those without DC plans. More importantly, the nonfinancial income and the home ownership rate are substantially higher for households with DC plans. For example, the home ownership rate is close to 80% for households with DC plans, while it is about 55% for those without DC plans.

Table 2 compares households with and without DC plans by income and age. The upper panel shows that across income quartiles, households with DC plans have higher home ownership rates than those without DC plans, although the home ownership rate is very close for these two groups of households in the top income quartile. For homeowners, the ratio of home equity to net worth is much higher for households without DC plans than that for households with DC plans. This is mainly due to the fact that households with DC plans also accumulate wealth in their TDA (see more details in the next sub-section). The

---

<sup>6</sup>This is because the paper studies households' joint decisions of TDA and housing. The sample size of households with heads aged 65 and above having employer-sponsored DC plans is very small.

lower panel looks at different age groups. Again, one can see that the home ownership rate is significantly higher for households with DC plans across all age groups, while the ratio of home equity to net worth is much higher for households without DC plans.

To gain a better understanding of the impact of DC plans, we further estimate a probit model of home ownership on DC plan status by controlling age, education, marital status, family size and nonfinancial income.<sup>7</sup> The marginal effects of the probit model show that having DC plans has a large and significant impact on the probability of home ownership. The probability of being a homeowner for households with DC plans is 13.6 percentage points higher than that for households without DC plans. The effect is significant at the 1% level.

The results so far are from the 2007 SCF. Similar patterns can be observed in the 2001 and 2004 SCFs. Next we focus exclusively on households with employer-sponsored DC plans using the three waves of SCF.

## 2.2 Households with DC Plans

We first document the home ownership rate for households with employer-sponsored DC plans. Home ownership can be viewed as the extensive margin of savings through home equity. Figure 1 plots the life-cycle profile of home ownership for these households from the three waves of SCF. The overall home ownership rate for households with DC plans is 78% at ages 25–64.<sup>8</sup> The ownership rate increases with age, rising from 62% at age group 25–34 to 89% at age group 55–64. Comparing homeowners and renters, we find that the nonfinancial income of renters is much lower than that of homeowners.<sup>9</sup>

For households that have DC plans and that are also homeowners, housing and assets in TDA are the two most important components of their wealth. Figure 2 reports their net worth composition regarding home equity, TDA wealth and TA wealth over the life cycle. Home equity is the difference between the value of principal residence and the mortgage balance. TDA wealth is the balance in TDA, net of loans against main job pensions. TA wealth is defined as financial assets in TA, net of debts associated with TA. More details on the data are provided in Appendix A. Given the skewness of the wealth distribution in the data, we therefore choose to report the median values.<sup>10</sup>

---

<sup>7</sup>Results of the probit model are available upon request.

<sup>8</sup>In contrast, it is 55% for households with heads aged 25–64 but without DC plans in these SCFs.

<sup>9</sup>For the age group 25–64 in the 2007 SCF, the median nonfinancial income of homeowners was 1.8 times of that of renters.

<sup>10</sup>We obtain similar patterns regarding the composition of net worth if we compute the average of the middle 10% households by net worth.

Home equity dominates households' net worth for young homeowners (a median ratio of 61% at age group 25–34). As households move to later stages of their life cycle, the median home equity to net worth ratio drops to about 43% at age group 55–64. On the other hand, the TDA wealth to net worth ratio increases with age. The median ratio is about 23% at age group 25–34, and it increases to 34% at age group 55–64. The median TA wealth to net worth ratio remains low throughout households' life cycle, implying that households primarily hold their financial wealth in TDA to take advantage of its preferential tax treatment. In terms of dollar value, total net worth increases with age as households build up their assets in all TA, TDA and home equity. These results suggest that TDA assets become a more important component of net worth as households age.

To summarize, our key finding from the SCF data is that households with DC plans are associated with a higher home ownership rate than those without DC plans. Furthermore, for households that have DC plans and that are also homeowners, our stylized facts suggest that the composition of net worth regarding home equity, TDA wealth and TA wealth has clear life-cycle patterns. It appears that households are using the two savings vehicles (i.e., TDA and housing) jointly. As both the TDA contributions and home ownership decisions are endogenous, in the next section we develop a model to better understand how housing decisions interact with households' use of TDA.

### 3 Model

The life-cycle model used for our analysis is described in this section. Model households have access to both TDA and TA. They make housing decisions and savings decisions in TDA and TA under uninsurable earnings and house price risks. Housing service can be acquired through either renting or owning. Buying a home requires a long-term fixed-rate mortgage with a minimum down payment. The model economy also features a progressive income tax system that mimics the U.S. tax code with favorable tax treatments on TDA contributions and owner-occupied housing.

#### 3.1 Demographics and Preferences

Households enter the model at age 25, work until age 64, and live at most up to age 95. In modeling terms, they work in the first  $R = 40$  periods and at most live for  $J = 71$  periods. They have stochastic lifetimes and  $s_j$  denotes the survival probability in period  $j$  conditional on being alive in period  $j - 1$ . In any period  $j$ , households derive utility from consumption

of non-durable goods,  $c_j$ , and housing service,  $h_j$ . Housing service should be interpreted broadly as reflecting both the physical size of a house and its quality. The utility function is time-separable with discount factor  $\beta$ . The instantaneous utility function is

$$u(c_j, h_j) = \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma}, \quad (1)$$

where  $\gamma$  is the coefficient of relative risk aversion and  $\omega$  measures the preference for housing relative to consumption of non-durable goods.<sup>11</sup>

Households also gain utility from leaving an estate after death. Let  $W_j$  be the estate left behind in period  $j$ . For simplicity, we assume that the utility function applied to the estate is the same as the utility function applied to housing service and consumption when alive:

$$u(W_j) = \frac{(W_j)^{1-\gamma}}{1-\gamma}. \quad (2)$$

### 3.2 Income Process

Households supply labor inelastically to work in the first  $R$  periods of life. Specifically, household  $i$  of age  $j$  receives stochastic labor income  $Y_{ij}$ , against which the household cannot borrow. Let  $y_{ij} = \ln(Y_{ij})$  denote the log of income, which is defined as

$$y_{ij} = f_{ij} + \eta_j + \varepsilon_{ij}, \quad (3)$$

where  $f_{ij}$  is a deterministic age-earnings profile,  $\eta_j$  is an aggregate income shock, and  $\varepsilon_{ij}$  is an idiosyncratic persistent shock.<sup>12</sup> Deterministic age-earnings profile  $f_{ij}$  is a function of household age and other characteristics (e.g. education level), and it is estimated to capture the hump-shape life-cycle earnings pattern. The aggregate income shock,  $\eta_j$ , is common among all households and follows an AR(1) process:

$$\eta_{j+1} = \rho_\eta \eta_j + \xi_{j+1}^\eta. \quad (4)$$

Similarly, the idiosyncratic persistent shock,  $\varepsilon_{ij}$ , also follows an AR(1) process:

$$\varepsilon_{ij+1} = \rho_\varepsilon \varepsilon_{ij} + \xi_{j+1}^\varepsilon. \quad (5)$$

---

<sup>11</sup>Although we do not consider family size, there is literature focusing on the influence of the life-cycle profile of family size on households' consumption and housing decisions. See [Browning and Lusardi \(1996\)](#), [Attanasio, Banks, Meghir, and Weber \(1999\)](#), [Gourinchas and Parker \(2002\)](#), and [Halket and Vasudev \(2014\)](#).

<sup>12</sup>An aggregate income shock is included to correlate a household's income with house price shocks.

We assume aggregate shocks and idiosyncratic shocks are uncorrelated, where  $\xi_j^\eta$  and  $\xi_j^\varepsilon$  are independent and identically distributed (i.i.d.) random variables normally distributed with mean zero and variance  $\sigma_\eta^2$  and  $\sigma_\varepsilon^2$ , respectively.

Households retire after  $R$  periods and receive social security benefits, determined as a constant fraction,  $\lambda$ , of the last working period's deterministic earnings and idiosyncratic shock.<sup>13</sup> That is,

$$y_{ij} = \ln(\lambda) + f_{iR} + \varepsilon_{iR}. \quad (6)$$

This specification simplifies the solution for the model, as it retains heterogeneity in retirement income without keeping track of households' entire income histories.<sup>14</sup> To simplify our notations, subscript  $i$  expressing household-specific variables is dropped in the rest of the paper.

### 3.3 Housing

The size of housing services available ( $H$ ) in the model is discretized into five levels with  $H = \{H_1, H_2, H_3, H_4, H_5\}$ , where  $H_1$  and  $H_5$  correspond to the minimum and maximum house sizes, respectively. Housing prices follow the process in Cocco (2005). Let  $P_j$  be the price per unit of housing in period  $j$ , measured in terms of non-durable consumption goods (the numeraire). Hence, a house of size  $h \in H$  is valued at  $P_j h$ . Let  $p_j = \log(P_j)$  be the period  $j$  log price of one unit of housing and  $\tilde{p}_j = p_j - g(j-1)$  be the detrended log price of housing, where  $g$  is a constant growth rate of house price over time. As in Cocco (2005), we assume that fluctuations in house prices are perfectly positively correlated with aggregate labor income shocks, and uncorrelated with households' idiosyncratic income shocks.<sup>15</sup>

Housing services can be obtained either by renting or owning. Let  $DR_j \in \{0, 1\}$  denote a household's housing tenure choice in period  $j$ , with  $DR_j = 1$  indicating renting a house and  $DR_j = 0$  indicating owning a house. During their working lives ( $j \leq R$ ), households can choose to be renters or owners. They also make decisions on house size. We assume that rental housing is generally in smaller units than owner-occupied housing, an idea similar to that in Gervais (2002) and Amior and Halket (2014). Let  $h_j$  denote the house size choice in

---

<sup>13</sup>We allow  $\lambda$  to differ by education group.

<sup>14</sup>Similar set-up is used in the literature for studying the effect of TDA on savings. See Imrohoroglu, Imrohoroglu, and Joines (1998) and Laibson, Repetto, and Tobacman (1998), among others.

<sup>15</sup>Cocco (2005) uses Panel Study of Income Dynamics (PSID) data and estimates the correlation between aggregate income shock and house price shock to be 0.553. Setting the correlation to 1 simplifies the solution for the problem. Indeed, the value for aggregate income shock in this model is small (about 2%), and the higher correlation should not significantly affect household decisions.

period  $j$ , such that

$$h_j = \begin{cases} \in \{H_1, H_2, H_3\} & \text{if } DR = 1 \text{ (renter)} \\ \in \{H_2, H_3, H_4, H_5\} & \text{if } DR = 0 \text{ (owner)} \end{cases}. \quad (7)$$

After retirement ( $j > R$ ), we assume that renters are not allowed to purchase houses and they make decisions only on the rental house size.<sup>16</sup> Retired homeowners can choose to be owners or become renters. If they continue to be owners, they can choose to stay in their own houses or downsize to a smaller unit.

Renters pay a fraction  $\phi$  of the house value as the periodic rental cost. Households can choose to purchase a house through a traditional  $N$ -period mortgage with a fixed mortgage interest rate at  $r_m$ .<sup>17</sup> Let  $n$  denote the period in which the current house is purchased. Households pay a fraction  $\theta_n^D$  of the house value as down payment in period  $n$ . Working-age households can choose their down payment ratio from five choices ranging from 10% to 100%. Hence, there is a minimum down payment requirement. Retired homeowners who choose to downsize are required to pay for their new homes in full. Down payment choice for home buyers is formulated as

$$\theta_n^D = \begin{cases} \in \{0.1, 0.2, 0.5, 0.75, 1\} & \text{if } n \leq R \\ 1 & \text{if } n > R \end{cases}. \quad (8)$$

The initial housing value is captured by three parameters: the house size ( $h$ ), the period of house purchase ( $n$ ), and the housing price shock in that period ( $\tilde{p}_n$ ). The initial loan principal, denoted by  $L$ , for a house of size  $h$  before any mortgage payment is given by

$$L = \begin{cases} (1 - \theta_n^D)e^{\rho(n-1)+\tilde{p}_n}h & \text{if } n \in [1, R] \\ 0 & \text{otherwise} \end{cases}. \quad (9)$$

This mortgage contract is characterized by a constant mortgage payment over the length of the mortgage, which results in an increasing amortization schedule of the principal and a decreasing schedule for interest payments. Mortgage payment in period  $j$  is defined as

$$M_j = \begin{cases} \frac{r_m L(1+r_m)^N}{(1+r_m)^N - 1} & \text{if } n \in [1, R] \text{ and } n \leq j \leq (n + N - 1) \\ 0 & \text{otherwise} \end{cases}. \quad (10)$$

---

<sup>16</sup>This simplifies the computation problem. A similar set-up is used in [Nakajima and Telyukova \(2012\)](#), in which they restrict renters during retirement from buying houses and argue that only a very small proportion of renters buy houses late in life.

<sup>17</sup>A 30-year fixed-rate mortgage is common in the United States. For a more complicated mortgage choice problem, see [Chambers, Garriga, and Schlagenauf \(2009c\)](#). To make the model tractable, we do not allow households to choose to default or refinance their mortgages. For mortgage default and refinancing, see [Agarwal, Driscoll, and Laibson \(2013\)](#), [Chen, Michaux, and Roussanov \(2013\)](#), [Khandani, Lo, and Merton \(2013\)](#) and [Campbell and Cocco \(2015\)](#).



Mortgage payment  $M_j$  can further be decomposed into principal payment  $E_j$  and mortgage interest payment  $I_j$ , with  $M_j = E_j + I_j$ . The principal payment and interest payment in period  $j$  are given as

$$E_j = \begin{cases} \frac{r_m L (1+r_m)^{j-n}}{(1+r_m)^{N-1}} & \text{if } n \in [1, R] \text{ and } n \leq j \leq (n + N - 1) \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

and

$$I_j = \begin{cases} \frac{r_m L [(1+r_m)^N - (1+r_m)^{j-n}]}{(1+r_m)^{N-1}} & \text{if } n \in [1, R] \text{ and } n \leq j \leq (n + N - 1) \\ 0 & \text{otherwise} \end{cases} . \quad (12)$$

The remaining loan principal,  $LL_j$ , after mortgage payment in period  $j$  is

$$LL_j = \begin{cases} \frac{L [(1+r_m)^N - (1+r_m)^{j-n+1}]}{(1+r_m)^{N-1}} & \text{if } n \in [1, R] \text{ and } n \leq j \leq (n + N - 1) \\ 0 & \text{otherwise} \end{cases} . \quad (13)$$

Housing transaction is endogenous in the model. To capture realtor fees and other costs associated with housing transaction, selling and buying a house are associated with transaction costs that equal to a fraction, denoted as  $\theta^S$  and  $\theta^B$ , respectively, of the house value. If homeowners desire to own a house of different size, their existing house must be sold and the full mortgage balance becomes due upon the sale.<sup>18</sup> Homeowners also pay a fraction  $\delta$  of the house value as annual maintenance costs and property tax at rate  $\tau$ .

Housing expenditure depends on a household's tenure choice. Let  $x_j$  denote a household's expenditure on housing in period  $j$ . There are five different situations regarding the household's housing tenure status: (1) a household is a renter in both the last period and the current period ( $DR_{j-1} = DR_j = 1$ ); (2) a household was a homeowner in the last period and becomes a renter in the current period ( $DR_{j-1} = 0, DR_j = 1$ ); (3) a household was a renter in the last period and is an owner in the current period ( $DR_{j-1} = 1, DR_j = 0$ ); (4) a household is an owner in both periods and stays in the same house ( $DR_{j-1} = DR_j = 0$  and  $h_{j-1} = h_j$ ); and (5) a household is an owner in both periods but changed house size in the current period ( $DR_{j-1} = DR_j = 0$  and  $h_{j-1} \neq h_j$ ). Hence, the household expenditure on housing (net of the equity on the previously owned house when there is a house sale in the

---

<sup>18</sup>We assume there is no tax on housing capital gains.

current period) is given by

$$x_j = \begin{cases} \phi P_j h & \text{if } DR_{j-1} = DR_j = 1 \\ \phi P_j h_j + LL_{j-1} - (1 - \theta^S) P_j h_{j-1} & \text{if } DR_{j-1} = 0 \text{ and } DR_j = 1 \\ M_j + (\theta^B + \theta_j^D + \tau + \delta) P_j h_j & \text{if } DR_{j-1} = 1 \text{ and } DR_j = 0 \\ M_j + (\tau + \delta) P_j h_j & \text{if } DR_{j-1} = DR_j = 0 \text{ and } h_j = h_{j-1} \\ M_j + (\theta^B + \theta_n^D + \tau + \delta) P_j h_j & \\ + LL_{j-1} - (1 - \theta^S) P_j h_{j-1} & \text{if } DR_{j-1} = DR_j = 0 \text{ and } h_j \neq h_{j-1} \end{cases}, \quad (14)$$

where  $M_j$  is the mortgage payment in period  $j$  as defined in equation (10), and  $LL_{j-1}$  is the remaining loan principal in period  $j - 1$  defined in equation (13).

### 3.4 Tax-deferred Account and Taxable Account

All households have access to two types of accounts: a TDA and a TA. In each working period, households can contribute their pre-tax labor income to TDA, up to a fraction  $\bar{q}$  of their income.<sup>19</sup> Assets withdrawn from TDA prior to age 60 ( $j < R - 4$ ) are subject to an early withdrawal penalty at rate  $pen \in (0, 1)$ , in addition to the ordinary income tax incurred. Households are not allowed to contribute to TDA after retirement, and they decide the amount of withdrawals from TDA and pay tax on the withdrawals at the ordinary income tax rate.<sup>20</sup> There is a minimum required distribution after age 70 ( $j > R + 6$ ).

The amount of financial assets in TDA at the beginning of period  $j$  is denoted by  $a_j^D$ . Let  $q_j$  denote a household's contributions to TDA in period  $j$ , with  $q_j < 0$  implying asset withdrawals. Thus,

$$q_j \in \begin{cases} [-a_j^D, \bar{q} * Y_j] & \text{if } j \leq R \\ [-a_j^D, 0] & \text{if } j \geq R + 1 \text{ and } j \leq R + 6 \\ [-a_j^D, -\frac{1}{J-j+1} a_j^D] & \text{if } j > R + 6 \end{cases}, \quad (15)$$

where  $\frac{1}{J-j+1}$  is the minimum withdrawal rate after age 70.

<sup>19</sup>Although there is a statutory dollar limit on the maximum TDA contributions, in reality employees may have a lower contribution limit because the Internal Revenue Service (IRS) requires companies to pass an annual nondiscrimination test based on company-specific situations. A contribution limit as a percentage of income is also used in Dammon, Spatt, and Zhang (2004), Gomes, Michaelides, and Polkovnichenko (2009), Zhou (2009), and Marekwica, Schaefer, and Sebastian (2013).

<sup>20</sup>Amromin and Smith (2003) look at the liquidity risk associated with TDA and conclude that early withdrawals mainly come from liquidity-constrained households. We do not model penalty-free early withdrawals from TDA because in reality the magnitude is very small.

Employers also match a fraction,  $\tilde{q}$ , of employees' contributions. However, employer matching applies only to an employee's contributions up to 6% of the employee's labor income. Therefore, the employer's contributions ( $q_j^E$ ) are

$$q_j^E = \begin{cases} \min(\tilde{q} * q_j, \tilde{q} * 0.06 * Y_j) & \text{if } j \in [1, R] \text{ and } q_j > 0 \\ 0 & \text{otherwise} \end{cases} . \quad (16)$$

We do not consider the household's investment decision between stocks and bonds in either account and assume assets earn a constant rate of return,  $r$ , in both TDA and TA. The law of motion of assets in TDA is

$$a_{j+1}^D = \begin{cases} (1+r)(a_j^D + q_j + q_j^E) & \text{if } j \leq R \\ (1+r)(a_j^D + q_j) & \text{if } j > R \end{cases} . \quad (17)$$

Let  $a_j^T$  be the financial assets in TA at the beginning of period  $j$ . The law of motion of assets in TA is

$$a_{j+1}^T = (1+r) [a_j^T + Y_j - q_j - x_j - \Gamma_j - c_j] , \quad (18)$$

where  $Y_j$  is labor income,  $q_j$  is TDA contributions,  $x_j$  is the household's expenditure on housing defined in equation (14),  $\Gamma_j$  is the total tax liabilities (discussed in section 3.5), and  $c_j$  is consumption of non-durable goods. Both TDA and TA are subject to zero borrowing constraint such that

$$a_j^T \geq 0 \text{ and } a_j^D \geq 0 \text{ for all } j. \quad (19)$$

When households are born in the model, they are endowed with random idiosyncratic initial wealth  $a_1^T$ . After death, a household may leave an estate. For simplicity, we abstract from the estate tax. The estate left by a household in period  $j$  is

$$W_j = \begin{cases} a_j^T + a_j^D + (1 - \theta^S)P_j h_{j-1} - LL_{j-1} & \text{if } DR_{j-1} = 0 \\ a_j^T + a_j^D & \text{if } DR_{j-1} = 1 \end{cases} . \quad (20)$$

### 3.5 Taxes

A household's tax liability consists of three parts. First, household income is taxed through a piece-wise linear progressive income tax system,  $T(\cdot)$ . Total income is defined as the sum of interest income in TA, the household's labor income, and funds withdrawn from TDA. Income contributed to TDA is tax deductible. For homeowners, mortgage interest payments

and property taxes are also deductible. Adjusted gross income,  $AGI$ , subject to income tax is defined as total income minus total amount of deductions, such that

$$AGI_j = \begin{cases} \frac{r}{1+r}a_j^T + Y_j - q_j - I_j - \tau P_j h_j & \text{if } DR_j = 0 \\ \frac{r}{1+r}a_j^T + Y_j - q_j & \text{if } DR_j = 1 \end{cases}, \quad (21)$$

where  $q_j$  is the contributions to (withdrawals from) TDA,  $I_j$  is mortgage interest payments defined in equation (12), and  $\tau P_j h_j$  is the property tax. The marginal income tax rates depend on the  $AGI$ . Let  $IC = \{IC_1, IC_2, IC_3, IC_4, IC_5\}$  be the cut-off points of the tax brackets. Let  $\tau_1, \tau_2, \tau_3, \tau_4$ , and  $\tau_5$  denote corresponding marginal tax rates. Suppose  $AGI_j \in (IC_3, IC_4]$ . Then, income tax payment  $T(AGI_j) = \tau_1(IC_2 - IC_1) + \tau_2(IC_3 - IC_2) + \tau_3(AGI_j - IC_3)$ .

Second, households also pay payroll taxes. Let  $\tau_{ss}$  be the payroll tax rate and  $Y_{ss}$  be the maximum earnings that are subject to payroll tax. Third, the early withdrawal penalty for households who withdraw funds from TDA before age  $R - 4$  should also be included in the tax payments. Hence, the total tax liability of a household in period  $j$  is defined as

$$\Gamma_j = \begin{cases} T(AGI_j) + \min(\tau_{ss} * Y_j, \tau_{ss} * Y_{ss}) - pen * q_j & \text{if } q_j < 0 \text{ and } j < R - 4 \\ T(AGI_j) + \min(\tau_{ss} * Y_j, \tau_{ss} * Y_{ss}) & \text{otherwise} \end{cases}. \quad (22)$$

### 3.6 Household Problem

In each period  $j$ , households choose their consumption ( $c_j$ ), contributions to TDA ( $q_j$ ), housing tenure choice ( $DR_j$ ), house size ( $h_j$ ), and the down payment ratio ( $\theta_n^D$ ) if they buy a new house. The decisions are based on the following state variables: the aggregate income shock ( $\eta_j$ ), the idiosyncratic income shock ( $\varepsilon_{ij}$ ), financial wealth in TA ( $a_j^T$ ) and TDA ( $a_j^D$ ) at the beginning of the period, housing tenure choice last period ( $DR_{j-1}$ ), house size last period ( $h_{j-1}$ ), the period in which the household bought the current house ( $n$ ) and the house price shock in that period ( $\tilde{p}_n$ ), and the down payment ratio at the time of purchase ( $\theta_n^D$ ). The household's decision problem in recursive form is written as

$$\begin{aligned} & V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, DR_{j-1}, h_{j-1}, n, \tilde{p}_n, \theta_n^D) \\ = & \max_{c_j, q_j, DR_j, h_j, \theta_n^D} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} \\ & + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, DR_j, h_j, n, \tilde{p}_n, \theta_n^D)] \\ & + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}, \end{aligned} \quad (23)$$

subject to income processes (3) to (6) and constraints (7) to (22), in addition to the non-negativity constraint on consumption. A more detailed explanation about the decision problem conditional on a household’s tenure choice is relegated to Appendix B.

## 4 Parameterization

In this section, we outline our choices of benchmark parameters. All nominal values are adjusted to 2007 dollars by the Consumer Price Index. Table 3 summarizes our benchmark parameter values.

### 4.1 Demographics and Preferences

Households enter the model at age 25, work until age 64, begin to receive retirement benefits at age 65, and live at most up to age 95. A model period is set to one year, thus  $J = 71$  and  $R = 40$ . We use the year 2000 life table of the National Center for Health Statistics to parameterize the conditional survival probabilities.

The annual discount factor  $\beta$  is 0.96. The coefficient of relative risk aversion  $\gamma$  is 2, which falls in the range of 1–3 widely used in the macroeconomic literature. Households’ preference for housing relative to non-durable consumption goods,  $\omega$ , is set at 0.2, following [Li and Yao \(2007\)](#) and [Yao and Zhang \(2005\)](#).

### 4.2 Income Process

Households have different labor income profiles conditional on their education levels. For households with assets in TDA in the 2007 SCF, 64% of household heads have grades of 14 years or more, 30% have grades of 12–13 years, and 6% have grades of less than 12 years. As the proportion of households with less than high school education is small, we assume that households with college and high school education account for 64% and 36% of total households in the model economy, respectively. The corresponding parameter values for the age-earnings profiles of these two groups are taken from [Cocco, Gomes, and Maenhout \(2005\)](#). The median income of period 1 households is set at \$38,000 and normalized to 1 in our model. Households’ initial wealth  $a_1^T$  is conditional on their education levels. The distribution for initial wealth, from which households’ initial financial assets in TA are randomly drawn, is estimated from the 2001–2007 SCFs data for households with heads aged 23–24.

The remaining parameters of the labor income process in working periods are  $\rho_\eta$ ,  $\sigma_\eta$ ,

$\rho_\varepsilon$ , and  $\sigma_\varepsilon$ . For aggregate income shock, we set  $\rho_\eta = 0.748$ , and the standard deviation of aggregate income shock  $\sigma_\eta = 0.019$ . These values are taken from [Cocco \(2005\)](#) who estimates these parameters using the Panel Study of Income Dynamics (PSID). For the idiosyncratic persistent income shock, we set  $\rho_\varepsilon = 0.973$  and  $\sigma_\varepsilon = 0.133$ . These values are from [Heathcote, Storesletten, and Violante \(2010\)](#).<sup>21</sup> We discretize the income shocks using the Tauchen method outlined in [Adda and Cooper \(2003\)](#).<sup>22</sup>

During retirement, the social security replacement rate for high school graduate ( $\lambda_{\text{HS}}$ ) and college graduate ( $\lambda_{\text{COL}}$ ) is 0.6 and 0.4, respectively ([Díaz and Luengo-Prado, 2008](#); [Munnell and Soto, 2005](#)).

### 4.3 Housing

In order to parameterize the levels of house size, we look at households with heads aged 25–64 in the 2007 SCF and compute the ratio of reported house value to the median labor income of period 1 households. The ratio at the bottom 10 percentile is 2.2. Hence, we set the minimum house size in the model at 2 times of the median labor income of period 1 households. We use 5 points to approximate the levels of house size corresponding to 2, 4, 6, 8, and 10 times of median income in model period 1.<sup>23</sup> Following [Cocco \(2005\)](#), the annual growth rate of house price  $g$  is set at 1% and the standard deviation of house prices is 6.2%.

House maintenance cost parameter  $\delta$  is set at 1.5% as in [Yao and Zhang \(2005\)](#). For housing transaction costs,  $\theta^S$  is set at 6% for sellers and  $\theta^B$  at 1.5% for buyers. The property tax rate  $\tau$  is set at 1%. We set the rental costs,  $\phi$ , at 6.5% of the value of the house, which falls in the range 6.0–7.5% used in [Yao and Zhang \(2005\)](#) and [Li and Yao \(2007\)](#).

The length of a traditional fixed-rate mortgage contract is  $N = 30$  years, following [Chambers, Garriga, and Schlagenhauf \(2009a\)](#). Mortgage interest premium is 2.7%, according to the average mortgage interest premium over the rate on certificate of deposit from 1998 to 2007 ([IMF, 2010](#)). The rate of return for savings in both TA and TDA is set at 2%. Thus, the mortgage interest rate  $r_m$  is 4.7%. The minimum down payment ratio  $\theta^D$  is set at 10% as in [Yang \(2009\)](#).

---

<sup>21</sup>The transitory shock in their estimated income process is not included to simplify the computation problem.

<sup>22</sup>The aggregate income shock is approximated by a three-state Markov process and the idiosyncratic persistent income shock by a five-state Markov process.

<sup>23</sup>The median house size in the data is 6.03.

## 4.4 Tax-deferred Accounts

Households can make contributions to TDA in working periods. [Joulfaian and Richardson \(2001\)](#) find that for households with employer-sponsored DC plans, the average employee contribution rate is 5.9%. [Smith, Johnson, and Muller \(2004\)](#) examine a large, nationally representative sample of workers. They find that for those with DC plans, the average contribution rate between 1990 and 2001 was 5.4%, and the median was about 6% of earnings. Thus, we set the contribution limit,  $\bar{q}$ , to 8% of annual labor earnings in the benchmark model. Sensitivity analysis on increasing the contribution limit is conducted in a latter section. The employer matching rate,  $\tilde{q}$ , is set at 33.3% of an employee's contributions (up to 6% of the employee's labor income).<sup>24</sup> Adding employee contributions and employer matching together, a maximum of 10% of an employee's labor income can be contributed to TDA each period in the benchmark. According to the U.S. regulations, early withdrawal penalty,  $pen$ , is 10%, penalty-free withdrawal starts at age 60 ( $R - 4$ ) and mandatory TDA withdrawal starts after age 70 ( $R + 6$ ).

## 4.5 Income and Payroll Taxes

The income tax system in the model mimics the federal income tax code in the United States, prevailing in 1993–2000. We use the tax code in year 2000.<sup>25</sup> To be consistent with other monetary variables, taxable income thresholds are converted to 2007 dollars using the consumer price index. Table 4 describes the cut-off points of the tax brackets and the marginal tax rates. There are five tax brackets, with marginal tax rates of 15%, 28%, 31%, 36%, and 39.6% corresponding to taxable income thresholds at \$52,800, \$127,600, \$194,400, and \$347,200, respectively.

Payroll tax rates and maximum taxable earnings for payroll tax are taken from the Old-Age, Survivors, and Disability Insurance (OASDI) program in 1968–2007. We use the average tax rate for employees (5.60%) and the average real maximum taxable earnings (\$74,160) to compute the payroll taxes.

---

<sup>24</sup>Many employers have a higher matching rate, for example, 50% of the employee's contributions (up to 6% of the employee's earnings). However, not all employers provide matching. [Even and Macpherson \(2005\)](#) and [Mitchell, Utkus, and Yang \(2005\)](#) find that about 80% of employers provide matching. Hence, we use a relatively low matching rate in the paper. A slightly higher matching rate in the model does not change our main results.

<sup>25</sup>Personal exemptions in income tax are also considered in our calculation. We take the case of a household composed of a couple filing jointly and set total personal exemptions to \$8,850.

## 5 Benchmark Results

We solved the model numerically.<sup>26</sup> Data and results from model simulations are reported in Table 5 and graphically presented in Figure 3 and Figure 4. Given that our parameter values are strongly restricted to those in the existing literature, the benchmark model performs reasonably well as the simulation results closely resemble the facts observed from the SCFs.

The model generates an overall home ownership rate similar to that in the data for households with DC plans and with heads aged 25–64 (73.2% in the model versus 78.7% in the data). The life-cycle pattern of home ownership rate is also broadly consistent with the data. The only notable difference is that the ownership rate for the youngest age group (ages 25–34) is lower in the model than in the data (43% vs. 62%). A few factors absent from the model may contribute to this difference. Some households in reality purchase their houses through sub-prime mortgages and make much smaller down payments than required in the model. Also, the distribution of initial wealth in the model is parameterized by using the 2007 SCF data on net worth, net of any outstanding debt including student loans that are amortized with favorable interest rates. Thus, young households in the model have stricter credit constraints, which reduce their ability to make down payments.

The model also delivers reasonable results in the life-cycle patterns of net worth composition for *homeowners*. It generates a decreasing fraction of net worth in home equity, an increasing fraction in TDA wealth, and a relatively small fraction in TA wealth over the life cycle (before retirement). In terms of levels, the model delivers the composition of net worth similar to the data, with two exceptions. First, the home equity to net worth ratio for the youngest age group in the model (73.9%) is higher than that in the data (60.7%), which is likely due to the fact that some households in reality pay smaller down payments. Second, the TDA to net worth ratio in the model is higher than that in the data for the oldest group, and we attribute this deviation to the history of TDA. Since TDA such as IRA and 401(k) only started to become popular in early 1980s, not all households between ages 55–64 in the SCF have TDA throughout their work lives and are able to utilize TDA to the full extent.<sup>27</sup> In this sense, our model predicts that TDA wealth for future generations will be more important as a longer period of their work lives is covered by TDA. Nevertheless, the TDA share of net worth is relatively low over the most part of a household’s life cycle and the majority of household wealth consists of home equity. Our model also shows that

---

<sup>26</sup>See Appendix C for a description of the computation strategy.

<sup>27</sup>To keep this already high-dimensional model tractable, we do not incorporate stochastic TDA eligibility. For papers with stochastic eligibility, see Zhou (2012).



homeowners have higher income than renters. The median income for homeowners is 1.9 times of that for renters in the model, which is close to 1.8 times observed in the 2007 SCF.<sup>28</sup>

Households' use of TDA in the benchmark model is reported in Table 6. The median ratio of households' contributions to household income is 6%, as most households just contribute enough to capture all the benefits from the employer's match. The median contribution rate matches that in Smith, Johnson, and Muller (2004) and Vanguard (2015). The TDA participation rate, defined as the fraction of households making positive TDA contributions, is about 78% in the benchmark. It is very close to that reported in Munnell (2014), who finds that about 21% of eligible workers do not participate in 401(k) plans in the 2004–2013 SCFs (i.e., the participation rate is 79%). The overall match between the model predictions and the data makes the model a good laboratory for examining the impacts of TDA-related and housing-related factors on household decisions.

## 6 Experiments

In this section, we consider two sets of experiments on TDA-related and housing-related factors based on the benchmark. These experiments are chosen as they reflect different institutional features of TDA and housing. Comparative statics analysis is conducted to investigate the impacts of these factors on households' housing decisions and the composition of net worth. We look at *all* model households unconditional on their housing tenure choices because home ownership is endogenous in the model. When home equity is of concern, only homeowners are considered.

### 6.1 TDA-related Experiments

Three experiments related to the institutional settings of TDA are considered. First, the impacts of TDA on households' decisions are assessed by eliminating TDA completely. Second, the TDA contribution limit is increased to evaluate whether households' use of TDA is restricted by the TDA contribution limit. Third, the employer's match of TDA contributions is removed to study its importance on households' use of TDA. Results from the experiments are reported in Table 7 and Table 8. Values for the benchmark model are normalized to 1, so that all the experiment results reported are levels relative to the benchmark model.

---

<sup>28</sup>For net worth composition, renters in the model, as in the data, accumulate more wealth in TDA than in TA.

### 6.1.1 Eliminating TDA

Since TDA provides attractive tax benefits to households, the first and foremost question is to what extent TDA affects households' life-cycle decisions on housing and retirement savings. To answer this question, we conduct an experiment in which households cannot contribute to TDA ( $\bar{q} = 0$ ), losing both the tax benefits of TDA and the employer's match of TDA contributions. Results are reported in the upper panel of Table 7.

Eliminating TDA leads to significant changes in household portfolios. The median net worth for households aged 25–64 decreases significantly by 18.2% due to the absence of TDA tax benefits and employer matching. Assets that would have been saved in TDA are shifted to TA, leading to a substantial increase in the TA share of net worth. The overall home ownership rate plummets by 31.5% and the median income of homeowners is 11.4% higher, suggesting that homeowners are further concentrated in higher-income households. Homeowners allocate much bigger shares of their wealth to home equity for reducing mortgage interest payments; the fraction of home equity in net worth and in home value increases by 36.3% and 76.5%, respectively.

Changes in life-cycle patterns provide insights on the mechanism at work. The impacts are generally stronger on younger age groups. The decrease in net worth is most pronounced for households in the 25–34 age group (53.2%), who have the longest investment horizon and benefit the most from TDA. These households substantially increase their TA wealth. They are building up financial assets in TA so that they can make bigger down payments and increase their home equity when they buy a house. The substitution of home equity for TA assets for homeowners is motivated by the existence of a mortgage premium ( $r_m > r$ ). Households are induced by cost-saving motives to reduce their mortgage interest payments by having smaller mortgages (making bigger down payments). This effect shows up in the characteristics of homeowners: young homeowners are households with higher incomes who can afford to have a much higher home equity to home value ratio.

Households' decisions to invest more in home equity when they buy a house also affect the timing of home purchases. The stronger impact on home ownership for young households is due to a significant portion of them deferring their home purchases. For instance, households aged 25–34 have the biggest drop in home ownership (62%), while home ownership for households aged 55–64 only decreases by 10%. Young households decide to make bigger down payments and they need more time to acquire sufficient financial assets. Nonetheless, home ownership is lower for all age groups compared with those in the benchmark. This is because (1) households lose the tax benefits from TDA (negative wealth effect) and (2) their

preferred down payments are larger in the economy without TDA, and hence households are more constrained compared with the benchmark.

The findings from this experiment suggest that TDA promotes home ownership by encouraging households to contribute to TDA, make smaller down payments and become homeowners earlier in their lives. TDA enable households to accumulate more wealth, thus households demand more housing services. By holding assets in TDA and taking out bigger mortgages (making smaller down payments), households can enjoy tax benefits for both TDA (income tax deferral) and home ownership (mortgage interest deductibility). This decision also implies that the benefits of TDA outweigh the burden of higher mortgage interest payments. As a result, households are less restricted by their liquidity constraint on down payments in the presence of TDA and are able to become homeowners earlier in their lives.

### 6.1.2 Higher TDA Contribution Limit

Since TDA have significant impacts on households' life-cycle decisions, we further investigate whether the impacts are affected by the TDA contribution limit. In this experiment, TDA contribution limit ( $\bar{q}$ ) is increased to 12% of household annual income from 8% in the benchmark. The set-up for employer's match remains the same as in the benchmark, so that the experiment is focused on households' contributions to TDA. Results are shown in the lower panel of Table 7.

Raising the TDA contribution limit has little impact on households' wealth accumulation. The median net worth of households aged 25–64 increases only by 0.9%. The increase in net worth is mainly due to the additional tax benefits captured by shifting assets to TDA. In terms of net worth composition, the share of net worth in TDA rises by 8.7% and that in TA drops by 24.8% for households aged 25–64. These changes imply that a fraction of the new savings in TDA is shifted from assets previously held in TA. Households' housing decisions are virtually unaffected. Home ownership and the median income of homeowners are the same as in the benchmark model. There is a slight drop in home equity share of net worth (1.5%) as the overall net worth increases.

In contrast with eliminating TDA, these results suggest that the effects of the TDA contribution limit are very nonlinear. A higher contribution limit has insignificant effects because the majority of households are not bounded by the contribution limit set in the benchmark. The median household TDA contribution rate remains at 6.0% when the contribution limit is increased. Only a small fraction of households who used to max out their contributions in the benchmark benefits from a higher TDA contribution limit. For the

majority of households, further raising the contribution limit is irrelevant to them.

The effect of the contribution limit is made more evident by the life-cycle patterns. Older households generally have higher income than younger households. They are less liquidity-constrained and can afford to contribute more to TDA. Hence, they are more likely to max out their TDA contributions in the benchmark and can potentially benefit from a raise in the limit. This shows up in the decreasing trend of the TA share of net worth (and TA wealth) over the life cycle. Due to the increase in TDA savings for age groups 45–54 and 55–64, they exhibit a more significant decrease in the TA share of net worth (21.4% and 36.6%, respectively) and lower home equity share of net worth (2.2% and 4.1%, respectively).

### 6.1.3 No Employer Matching

The benefits of TDA consist of both income tax deferral and employer’s match of households’ contributions to TDA. Employer matching is an important consideration because it is an immediate return on employees’ contributions. As shown in Section 6.1.1 that TDA has significant impacts on housing decisions, we further evaluate the extent to which employer matching affects households’ decisions by removing the employer’s match from the benchmark model, i.e.,  $\tilde{q} = 0$ . Experiment results are reported in Table 8.

Removing the employer’s match creates asset reallocation in household portfolios. Since the return on TDA contributions is lower, households have less incentive to save in TDA. The median TDA contribution rate drops to 3.0% from 6.0% in the benchmark, and the share of net worth in TDA decreases by 25.8%. Households instead acquire more housing assets and reduce their mortgages; home ownership rises by 1.2% and the median home equity to home value ratio increases by 2.6% compared with those in the benchmark. As a result, the home equity share of net worth rises by 12.7%. Since households increase their TA wealth to prepare for bigger down payments in the future, TA share of net worth also rises by 19.4%.

These results are driven by a combination of income effect and substitution effect. Without the employer’s match, households accumulate less wealth (median net worth drops by 5.4%) and reduce their demand of housing services. Thus, home ownership becomes less attractive to them due to a negative income effect. Nonetheless, home equity and TDA wealth are competing assets for households’ wealth accumulation. A lower return on TDA contributions also means that home equity becomes relatively more attractive. This substitution effect induces households to invest in home equity (via home ownership) and reduce their TDA wealth. The combination of these effects leads to a very small increase in home ownership, a significant drop in TDA wealth and an increase in home equity share of net

worth compared with those in the benchmark.

Life-cycle savings patterns further indicate that households defer their TDA contributions to later stages of their lives. Without the employer’s match, the opportunity cost of *not* contributing to TDA when young is lowered. Households prioritize their home purchases to get longer periods of untaxed service flow. For households in the 25–34 age group, they have the largest drop in TDA wealth and the TDA share of net worth, indicating that their TDA contributions are significantly smaller than those in the benchmark. Meanwhile, they also have the biggest increase in home ownership and the home equity share of net worth. As households age, they make bigger contributions to TDA to build up their retirement savings, as reflected in the higher TDA wealth and TDA share of net worth in older age groups.

## 6.2 Housing-related Experiments

As shown in the TDA-related experiments that households’ decisions are heavily influenced by the cost of housing service and their abilities to make down payments, the effects of housing tax benefits and down payment requirement are further explored in this section. Experiment results are reported in Table 9. As before, all the results reported are levels relative to the benchmark model.

### 6.2.1 Increase in Down Payment Requirement

The down payment requirement represents a significant barrier for home ownership, particularly for young households as it takes time for them to accumulate financial assets. Here we investigate the impact of down payment requirement by increasing the minimum down payment ratio to 20% from 10% in the benchmark.

As down payment becomes a bigger hurdle, the home ownership rate for households aged 25–64 reduces by 6.8%. Bigger down payments also mean more wealth accumulated in home equity for homeowners, and the median home equity to home value ratio increases by 6.2%. Since households need to accumulate more assets in TA to fulfill the heightened down payment requirement, the fraction of net worth in TA increases by 15.7% and that in TDA drops by 1.9%. Overall, raising the minimum down payment requirement does not affect households’ wealth accumulation much. The median net worth for households aged 25–64 decreases by only 0.2%.

The above results are mainly driven by the impacts on the youngest age group. For households aged 25–34, home ownership decreases by 27.2%. This is because young households need more time to save up for down payments and they defer their home purchases.

Only higher-income young households can fulfill the heightened down payment requirement, and their home equity to home value ratio increases substantially by 33.9%. As fewer households become homeowners and invest in home equity, financial assets generally become more important in net worth. The fractions of net worth allocated to TA and TDA are higher, with significantly more assets allocated to TA for future down payments.

The effects of higher down payment quickly fade as age goes up. For households aged 35 and above, the decrease in home ownership is much smaller. While the median TDA contribution rate in this experiment is the same as in the benchmark (6.0%), there is a drop in TDA wealth for the 35–44 age group. It indicates that some households who used to make high TDA contributions reduce their contributions in order to make bigger down payments. These results suggest that the minimum down payment requirement significantly affects young households' home ownership decisions, but it only slightly reduces households' retirement savings in TDA. Households mainly respond by deferring their home purchases.

### 6.2.2 No Tax Benefits for Home Ownership

The income tax deductibility of mortgage interest payments and property taxes significantly reduces a household's cost of owning a home. Consequently, home ownership provides significant cost advantage over renting for obtaining housing services. This makes home equity and TDA savings competing assets in households' asset portfolios. We investigate the role of tax benefits for home ownership on households' decisions. In this experiment, both mortgage interest payments and property taxes are not income tax deductible.

Compared with the benchmark model, higher cost of home ownership reduces the overall home ownership rate by 19%.<sup>29</sup> Homeowners reduce their mortgage interest costs by increasing home equity. The median ratio of home equity to home value increases by 45%. TDA share of net worth drops by 4.4%, while home equity share of net worth rises for homeowners. Homeowners substitute home equity for TDA assets because the cost savings on mortgage interest become higher and more attractive than the benefits from TDA. As fewer households buy houses and accumulate wealth in housing (through down payments and capital gains), household net worth drops. Although the median TDA contribution rate in this experiment (6.0%) is the same as that in the benchmark, many households reduce their TDA contributions, resulting in a lower TDA wealth and an increase in TA wealth.

The life-cycle patterns also show that young households are influenced the most by the tax benefits on home ownership. To make bigger down payments and reduce mortgage

---

<sup>29</sup>A drop in home ownership is qualitatively consistent with [Gervais \(2002\)](#), in which TDA is not considered.

interest payments, many young households defer their home purchases to middle age and accumulate more assets in TA. This is reflected in a big drop in home ownership for the 25–34 age group and a hike in home equity ratios for the 35–44 and 45–54 age groups.

Our results suggest that tax benefits of home ownership is an important factor that affects home ownership and the composition of net worth. Households in general do not increase their use of TDA when mortgage interest payments and property taxes are not income tax deductible. Instead, households accumulate more wealth in TA, and then use their assets in TA to make bigger down payments when they buy houses. In doing so, they can reduce their mortgage interest payments.

## 7 Conclusion

This paper studies the interaction between households’ use of TDA and their housing decisions. We develop a life-cycle model in which the tax benefits and illiquid nature of TDA and housing are highlighted, and households make endogenous TDA contributions and housing decisions. With commonly used parameter values, the model generates life-cycle patterns of home ownership and the composition of household net worth that are broadly consistent with the data.

We find that the existence of TDA has significant impacts on households’ housing decisions. In particular, TDA promotes home ownership as households substitute TDA savings for home equity to take advantage of TDA tax benefits. They are willing to make smaller down payments and become homeowners earlier in their lives. The effects of the TDA contribution limit on households’ decisions are nonlinear; further raising the TDA contribution limit only moderately increases the TDA share of net worth and has little effect on home ownership. We also find that the employer’s match of TDA contributions is an important factor, without which households significantly alter their life-cycle patterns of TDA savings and the composition of net worth. On the other hand, housing-related factors affect households’ housing decisions more than their use of TDA. It suggests that TDA is an attractive vehicle for retirement savings, and households’ use of it is not hindered by housing-related policy changes.

These findings illustrate the important interaction between TDA and housing in household portfolios. As households leverage their asset portfolios by accumulating savings in TDA and taking out bigger mortgage loans, their non-housing consumption could be more sensitive to house price shocks than previously suggested ([Li and Yao, 2007](#)). While our work

focuses on understanding households' life-cycle decisions, this serves as an important step for better understanding the macroeconomic impacts of retirement-related and housing-related policies. The existing framework can be extended to investigate the macroeconomic impacts of retirement-related policies such as social security reform in the presence of housing ([Chen, 2010](#)), and the welfare impacts of eliminating mortgage interest deductibility ([Gervais, 2002](#)). We leave these for future research.



## Appendix A: The SCF Data

The Survey of Consumer Finances (SCF) provides the most complete data on household balance sheets in the United States. We use the 2001, 2004 and 2007 SCFs to construct the home ownership rate and the composition of net worth for households that have employer-sponsored defined contribution (DC) pension plans and that are also homeowners.

Financial assets in regular taxable accounts (TA) include checking accounts, savings accounts, certificates of deposit, money market accounts, mutual funds, bonds, directly held publicly traded stocks, brokerage accounts, trusts and managed investment accounts. TA wealth is defined as financial assets in TA net of debt associated with TA, which includes credit cards, education loans, borrowing in brokerage accounts and other consumer loans.

TDA wealth is the sum of balances in Individual Retirement Accounts (IRA) and employer-sponsored DC pension plans from one's current main job (such as 401(k)/403(b)/457/SRA and Thrift Savings plans) net of loans against main job pensions. Note that TDA wealth includes holdings in IRA, because balances in IRA consist mostly of rollovers from 401(k) plans. Home equity is the difference between the value of principal residence and the mortgage balance on principal residence.

Finally, a household's net worth is the sum of home equity, TDA wealth and TA wealth. It excludes social security wealth and future earnings. Once we find the net worth, we compute the composition of net worth for households in each survey and then take the average.

## Appendix B: Household Problem

The recursive formulation of a household's problem specified in equation (23) depends on the household's endogenous tenure choice. We specify five different scenarios with respect to a household's home ownership status in the last period and current tenure choice. Technically, households that do not own a house have state variables  $n = 0$ ,  $\tilde{p}_n = 0$ , and  $\theta_n^D = 0$ .

1. Consider a household that rents in both periods  $j - 1$  and  $j$  (i.e.,  $DR_{j-1} = DR_j = 1$ ).

The Bellman equation for this situation is given by

$$\begin{aligned}
 & V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, 1, h_{j-1}, 0, 0, 0) \\
 = & \max_{c_j, q_j, h_j} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, 1, h_j, 0, 0, 0)] \\
 & + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}. \tag{24}
 \end{aligned}$$

2. Consider a household that owns a house in period  $(j - 1)$  and rents in period  $j$  (i.e.,  $DR_{j-1} = 0$  and  $DR_j = 1$ ). The Bellman equation for this situation is given by

$$\begin{aligned}
& V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, 0, h_{j-1}, n, \tilde{p}_n, \theta_n^D) \\
= & \max_{c_j, q_j, h_j} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, 1, h_j, 0, 0, 0)] \\
& + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}. \tag{25}
\end{aligned}$$

3. Consider a household that rents in period  $j - 1$  and chooses to buy a house in period  $j$  (i.e.,  $DR_{j-1} = 1$  and  $DR_j = 0$ ). This requires the household to make an additional decision on down payment  $(\theta_j^D)$ . The Bellman equation for this situation is given by

$$\begin{aligned}
& V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, 1, h_{j-1}, 0, 0, 0) \\
= & \max_{c_j, q_j, h_j, \theta_j^D} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, 0, h_j, j, \tilde{p}_j, \theta_j^D)] \\
& + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}. \tag{26}
\end{aligned}$$

4. Consider a homeowner who continues to own the same house as in the last period (i.e.,  $DR_{j-1} = DR_j = 0$ , and  $h_{j-1} = h_j$ ). The Bellman equation for this situation is given by

$$\begin{aligned}
& V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, 0, h_{j-1}, n, \tilde{p}_n, \theta_n^D) \\
= & \max_{c_j, q_j, h_j} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, 0, h_j, n, \tilde{p}_n, \theta_n^D)] \\
& + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}. \tag{27}
\end{aligned}$$

5. Consider a homeowner who decides to change the house size (i.e.,  $DR_{j-1} = DR_j = 0$  and  $h_{j-1} \neq h_j$ ). The down payment ratio  $(\theta_n^D)$  is a state variable for the existing house, but a choice variable  $(\theta_j^D)$  for the new house bought in the current period. The Bellman equation for this situation is given by

$$\begin{aligned}
& V(j, \eta_j, \varepsilon_j, a_j^T, a_j^D, 0, h_{j-1}, n, \tilde{p}_n, \theta_n^D) \\
= & \max_{c_j, q_j, h_j, \theta_j^D} \frac{(c_j^{1-\omega} h_j^\omega)^{1-\gamma}}{1-\gamma} + \beta s_{j+1} E_j [V(j+1, \eta_{j+1}, \varepsilon_{j+1}, a_{j+1}^T, a_{j+1}^D, 0, h_j, j, \tilde{p}_j, \theta_j^D)] \\
& + \beta(1 - s_{j+1}) \frac{(W_{j+1})^{1-\gamma}}{1-\gamma}. \tag{28}
\end{aligned}$$

## Appendix C: Numerical Solution

We use numerical dynamic programming techniques to approximate the decision rules as well as the value function. In each period, we need to solve for five control variables: consumption ( $c_j$ ), contributions to TDA ( $q_j$ ), housing tenure choice ( $DR_j$ ), house size ( $h_j$ ), and the down payment ratio ( $\theta_n^D$ ) if a household buys a new house. These decisions are based on the following state variables: the aggregate income shock ( $\eta_j$ ), the idiosyncratic income shock ( $\varepsilon_{ij}$ ), financial wealth in TA ( $a_j^T$ ) and TDA ( $a_j^D$ ) at the beginning of the period, housing tenure choice last period ( $DR_{j-1}$ ), house size last period ( $h_{j-1}$ ), the period in which the household bought the current house ( $n$ ) and the house price shock in that period ( $\tilde{p}_n$ ), and the down payment ratio at the time of purchase ( $\theta_n^D$ ).

We discretize the state-space along the two continuous state variables,  $a_j^T$  and  $a_j^D$ .<sup>30</sup> The model is solved using backward induction. We use Brent-Dekker algorithm (Brent, 1971) as the basic optimization routine. In the last period ( $j = J$ ), the policy functions are determined by the motive to leave an estate. Using these decision rules, we obtain this period's value function. In periods prior to  $J$ , we calculate optimal decision rules for each possible combination of nodes, using stored information about the subsequent period's decision rules and value function. For points that do not lie on the state-space grids, we evaluate the value function using a linear interpolation along the two wealth dimensions. After computing the values of all the alternatives, we pick the maximum, thus obtaining the decision rules for the current period. This process is iterated until  $j = 1$ .

Once we determine the optimal decision rules for all possible nodes in each period, we conduct simulations. For each simulation, we first generate a series of aggregate income shock and house price shock. Then we simulate the income history of 71 cohorts of households. Each cohort consists of 500 households. Cohort 1 corresponds to the period 1 households (the youngest) in the model. Their income history includes only one period. These households do not own houses at the beginning of period 1 and their initial financial wealth in TA is drawn from an initial wealth distribution by education group. Cohort 2 corresponds to the period 2 households in the model. Their income history includes two periods. These households are subject to the realization of house price shocks in the recent two periods. The same approach is applied to other cohorts. Cohort 71 corresponds to the period 71 households (the oldest) in the model. The income history of cohort 71 includes 71 periods. These households are subject to the whole series of house price shocks. Finally, for households before retirement (age 25–64), we compute the home ownership rate and the composition

---

<sup>30</sup>The grids are unequally spaced. They are finer for lower values of wealth.

of net worth regarding home equity, TDA wealth and wealth held in TA. We then compare the average of 100 simulations with the real data. Because a large amount of computation time is required to solve the model, all programs are parallelized via a combination of MPI and OpenMP, and they are executed on the Bank of Canada's EDITH High Performance Cluster.

## References

- ADDA, J., AND R. COOPER (2003): *Dynamic Economics: Quantitative Methods and Applications*. The MIT Press, Cambridge, 2nd ed. edn.
- AGARWAL, S., J. C. DRISCOLL, AND D. I. LAIBSON (2013): “Optimal Mortgage Refinancing: A Closed-Form Solution,” *Journal of Money, Credit, and Banking*, 45(4), 591–622.
- AMIOR, M., AND J. HALKET (2014): “Do Households Use Home-ownership to Insure Themselves? Evidence across US Cities,” *Quantitative Economics*, 5(3), 631–674.
- AMROMIN, G. (2003): “Household Portfolio Choices in Taxable and Tax-Deferred Accounts: Another Puzzle?,” *European Finance Review*, 7(3), 547–582.
- AMROMIN, G., J. HUANG, AND C. SIALM (2007): “The tradeoff between mortgage prepayments and tax-deferred retirement savings,” *Journal of Public Economics*, 91(10), 2014–2040.
- AMROMIN, G., AND P. SMITH (2003): “What explains early withdrawals from retirement accounts? Evidence from a panel of taxpayers,” *National Tax Journal*, pp. 595–612.
- ATTANASIO, O. P., J. BANKS, C. MEGHIR, AND G. WEBER (1999): “Humps and bumps in lifetime consumption,” *Journal of Business & Economic Statistics*, 17(1), 22–35.
- BAJARI, P., P. CHAN, D. KRUEGER, AND D. MILLER (2013): “A Dynamic Model Of Housing Demand: Estimation And Policy Implications,” *International Economic Review*, 54(2), 409–442.
- BECKER, T., AND R. SHABANI (2010): “Outstanding Debt and the Household Portfolio,” *Review of Financial Study*, 23(7), 2900–2934.
- BENJAMIN, D. (2003): “Do 401(k)s Increase Saving? Evidence from Propensity Score Subclassification,” *Journal of Public Economics*, 87(5-6), 1259–1290.
- BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM (2014): *Financial Accounts of the United States: Flow of Funds, Balance Sheets, and Integrated Macroeconomic Accounts*. Federal Reserve Statistical Release.
- BRENT, R. P. (1971): “An Algorithm with Guaranteed Convergence for Finding A Zero of A Function,” *The Computer Journal*, 14(4), 422–425.

- BROWNING, M., AND A. LUSARDI (1996): “Household Saving: Micro Theories and Micro Facts,” *Journal of Economic Literature*, 34(4), 1797–1855.
- CAMPBELL, J., AND J. COCCO (2007): “How Do House Prices Affect Consumption? Evidence from Micro Data,” *Journal of Monetary Economics*, 54(3), 591–621.
- (2015): “A Model of Mortgage Default,” *Journal of Finance*, 70(4), 1495–1554.
- CHAMBERS, M., C. GARRIGA, AND D. E. SCHLAGENHAUF (2009a): “Accounting For Changes in the Homeownership Rate,” *International Economic Review*, 50(3), 677–726.
- (2009b): “Housing Policy and the Progressivity of Income Taxation,” *Journal of Monetary Economics*, 56(8), 1116–1134.
- (2009c): “The Loan Structure and Housing Tenure Decisions in An Equilibrium Model of Mortgage Choice,” *Review of Economic Dynamics*, 12(3), 444–468.
- CHEN, H., M. MICHAUX, AND N. ROUSSANOV (2013): “Houses as ATMs? Mortgage Refinancing and Macroeconomic Uncertainty,” NBER Working Papers 19421, National Bureau of Economic Research.
- CHEN, K. (2010): “A Life-Cycle Analysis of Social Security with Housing,” *Review of Economic Dynamics*, 13(3), 597–615.
- CHERNOZHUKOV, V., AND C. HANSEN (2004): “The Impact of 401(k) Participation on the Wealth Distribution: An Instrumental Quantile Regression Analysis,” *Review of Economics and Statistics*, 86(3), 735–751.
- COCCO, J. F. (2005): “Portfolio Choice in the Presence of Housing,” *Review of Financial Studies*, 18(2), 535–567.
- COCCO, J. F., F. J. GOMES, AND P. J. MAENHOUT (2005): “Consumption and Portfolio Choice over the Life Cycle,” *Review of Financial Studies*, 18(2), 491–533.
- DAMMON, R., C. SPATT, AND H. ZHANG (2004): “Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing,” *Journal of Finance*, 59(3), 999–1037.
- DÍAZ, A., AND M. J. LUENGO-PRADO (2008): “On the User Cost and Homeownership,” *Review of Economic Dynamics*, 11(3), 584–613.

- ENGELHARDT, G., AND A. KUMAR (2007): “Employer Matching and 401(k) Saving: Evidence from the Health and Retirement Study,” *Journal of Public Economics*, 91(10), 1920–1943.
- ENGEN, E., W. GALE, AND J. SCHOLZ (1996): “The Illusory Effects of Saving Incentives on Saving,” *The Journal of Economic Perspectives*, 10(4), 113–138.
- EVEN, W., AND D. MACPHERSON (2005): “The Effects of Employer Matching in 401(k) Plans,” *Industrial Relations*, 44(3), 525–549.
- GERVAIS, M. (2002): “Housing Taxation and Capital Accumulation,” *Journal of Monetary Economics*, 49(7), 1461–1489.
- GOMES, F., A. MICHAELIDES, AND V. POLKOVNICHENKO (2009): “Optimal Savings with Taxable and Tax-Deferred Accounts,” *Review of Economic Dynamics*, 12(4), 718–735.
- GOURINCHAS, P.-O., AND J. A. PARKER (2002): “Consumption Over the Life Cycle,” *Econometrica*, 70(1), 47–89.
- HALKET, J., AND S. VASUDEV (2014): “Saving up or settling down: Home ownership over the life cycle,” *Review of Economic Dynamics*, 17(2), 345 – 366.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2010): “The Macroeconomic Implications of Rising Wage Inequality in the United States,” *Journal of Political Economy*, 118(4), 681–722.
- HUANG, J. (2008): “Taxable and Tax-Deferred Investing: A Tax-Arbitrage Approach,” *Review of Financial Studies*, 21(5), 2173–2207.
- IMF (2010): *International Financial Statistics Yearbook, 2010*, International Financial Statistics Series. International Monetary Fund.
- IMROHOROGLU, A., S. IMROHOROGLU, AND D. H. JOINES (1998): “The Effect of Tax-Favored Retirement Accounts on Capital Accumulation,” *American Economic Review*, 88(4), 749–68.
- JOULFAIAN, D., AND D. P. RICHARDSON (2001): “Who Takes Advantage of Tax-Deferred Saving Programs? Evidence from Federal Income Tax Data,” *National Tax Journal*, 54(3), 669–688.

- KHANDANI, A., A. LO, AND R. MERTON (2013): “Systemic Risk and the Refinancing Ratchet Effect,” *Journal of Financial Economics*, 108(1), 29–45.
- LAIBSON, D. I., A. REPETTO, AND J. TOBACMAN (1998): “Self-Control and Saving for Retirement,” *Brookings Papers on Economic Activity*, (1), 91–196.
- LI, W., AND R. YAO (2007): “The Life-Cycle Effects of House Price Changes,” *Journal of Money, Credit and Banking*, 39(6), 1375–1409.
- MAREKWICA, M., A. SCHAEFER, AND S. SEBASTIAN (2013): “Life Cycle Asset Allocation in the Presence of Housing and Tax-deferred Investing,” *Journal of Economic Dynamics and Control*, 37(6), 1110 – 1125.
- MITCHELL, O., S. UTKUS, AND T. YANG (2005): “Turning Workers into Savers? Incentive, Liquidity, and Choice in 401(k) Plan Design,” *NBER Working Paper No. 11726*.
- MUNNELL, A. H. (2012): “401(k) Plans in 2010: An Update from the SCF,” *Issue in Brief 12-13, Center for Retirement Research at Boston College*.
- (2014): “401(k)/IRA Holdings in 2013: An Update from the SCF,” *Issue in Brief 14-15, Center for Retirement Research at Boston College*.
- MUNNELL, A. H., F. GOLUB-SASS, AND D. MULDOON (2009): “An Update on 401(k) Plans: Insights From the 2007 SCF,” *Issue in Brief 9-5, Center for Retirement Research at Boston College*.
- MUNNELL, A. H., L. QUINBY, AND A. WEBB (2012): “What’s the Tax Advantage of 401(k)s?,” *Issue in Brief 12-4, Center for Retirement Research at Boston College*.
- MUNNELL, A. H., AND M. SOTO (2005): “What Replacement Rates Do Households Actually Experience in Retirement?,” *Working paper No. 2005-10, Center for Retirement Research at Boston College*.
- NAKAJIMA, M., AND I. A. TELYUKOVA (2012): “Home Equity in Retirement,” Working paper, Federal Reserve Bank of Philadelphia.
- NISHIYAMA, S. (2011): “The Budgetary and Welfare Effects of Tax-Deferred Retirement Saving Accounts,” *Journal of Public Economics*, 95(11), 1561–1578.
- POTERBA, J. M. (2014): “Retirement Security in an Aging Population,” *American Economic Review*, 104(5), 1–30.



- POTERBA, J. M., S. F. VENTI, AND D. A. WISE (1996): “How Retirement Saving Programs Increase Saving,” *Journal of Economic Perspectives*, 10(4), 91–112.
- SCHOLZ, J. K., A. SESHADRI, AND S. KHITATRAKUN (2006): “Are Americans Saving “Optimally” for Retirement?,” *Journal of Political Economy*, 114(4), 607–643.
- SMITH, K. E., R. W. JOHNSON, AND L. A. MULLER (2004): “Deferring Income in Employer Sponsored Retirement Plans: The Dynamics of Participant Contributions,” *National Tax Journal*, 57(3), 639–670.
- SOMMER, K., P. SULLIVAN, AND R. VERBRUGGE (2013): “The Equilibrium Effect of Fundamentals on House Prices and Rents,” *Journal of Monetary Economics*, 60(7), 854–870.
- VANGUARD (2015): “How America Saves 2015: A Report on Vanguard 2014 Defined Contribution Plan Data,” *Vanguard Institutional Investor Group*.
- YANG, F. (2009): “Consumption over the Life Cycle: How Different is Housing?,” *Review of Economic Dynamics*, 12(3), 423–443.
- YAO, R., AND H. H. ZHANG (2005): “Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints,” *Review of Financial Studies*, 18(1), 197–239.
- ZHOU, J. (2009): “The Asset Location Puzzle: Taxes Matter,” *Journal of Economic Dynamics and Control*, 33(4), 955–969.
- (2012): “Life-cycle Stock Market Participation in Taxable and Tax-deferred Accounts,” *Journal of Economic Dynamics and Control*, 36(11), 1814–1829.

Table 1: Sample statistics in the 2007 SCF

	Households with DC plans	Other households
Head age (mean / median)	44.5 / 45	44.9 / 45
Years of school (mean / median)	14.3 / 14	13.1 / 13
Marital status: married (%)	66.8	47.0
Nonfinancial income (\$, mean / median)	105,507.3 / 77,000	57,687.6 / 37,100
Home ownership rate (%)	79.8	55.2

Note: This table provides summary statistics for households with heads aged 25–64 in the 2007 Survey of Consumer Finances (SCF). Survey households are divided into two groups: households with employer-sponsored defined contribution (DC) plans and those without DC plans.

Table 2: Comparison between households with and without DC plans in 2007 SCF

	Income Quartile				Overall
	Q1	Q2	Q3	Q4	
Home ownership rate					
Households with DC plans	0.441	0.605	0.826	0.926	0.798
Households without DC plans	0.335	0.525	0.720	0.917	0.552
Home equity/net worth					
Households with DC plans	0.728	0.660	0.571	0.477	0.543
Households without DC plans	0.976	0.960	0.861	0.720	0.897

	Age Group				Overall
	25–34	35–44	45–54	55–64	
Home ownership rate					
Households with DC plans	0.581	0.807	0.856	0.908	0.798
Households without DC plans	0.386	0.497	0.600	0.706	0.552
Home equity/net worth					
Households with DC plans	0.652	0.548	0.562	0.460	0.543
Households without DC plans	0.941	0.915	0.914	0.850	0.897

Note: Income is defined as nonfinancial income. Overall refers to all households with heads aged 25–64 in the 2007 SCF. The home equity to net worth ratio refers to the median in each age and income group. Home equity refers to that of households' principal residence.

Table 3: Summary of parameter values

Parameters	Description	Values	Target / Data Source
Demographics			
$J$	Lifespan	71	Real age 25–95
$R$	Last working period	40	Work until age 64
$s$	Survival probability	see text	Life table in year 2000
Preferences			
$\gamma$	Relative risk aversion	2	
$\beta$	Discount factor	0.96	
$\omega$	Preferences on housing	0.2	Li and Yao (2007), Yao and Zhang (2005)
Income			
$f$	Age earnings profile	see text	Cocco, Gomes, and Maenhout (2005)
$\rho_\eta$	Persistence of aggregate shock	0.748	Cocco (2005)
$\sigma_\eta$	s.d. aggregate shock	0.019	Cocco (2005)
$\rho_\varepsilon$	Persistence of idiosyncratic shock	0.973	Heathcote, Storesletten, and Violante (2010)
$\sigma_\varepsilon$	s.d. idiosyncratic income shock	0.133	Heathcote, Storesletten, and Violante (2010)
$\lambda_{\text{COL}}$	SS replacement rate for COL	0.4	Díaz and Luengo-Prado (2008)
$\lambda_{\text{HS}}$	SS replacement rate for HS	0.6	Díaz and Luengo-Prado (2008)
Housing			
$N$	Mortgage length	30	Chambers, Garriga, and Schlagenhauf (2009a)
$r_m$	Mortgage interest rate	4.7%	
$\theta^D$	Down payment ratios	see text	
$H$	House size	see text	
$g$	House price growth rate	1%	Cocco (2005)
$\sigma_{\bar{p}}$	s.d. house prices	6.2%	Cocco (2005)
$\theta^S$	Transaction cost for seller	6%	
$\theta^B$	Transaction cost for buyer	1.5%	
$\tau$	Property tax rate	1%	
$\delta$	Housing maintenance cost	1.5%	Yao and Zhang (2005)
$\phi$	Rental cost of housing	6.5%	
Savings			
$r$	Return on saving	2%	
$\bar{q}$	TDA contributions limit	8%	See text
$pen$	TDA penalty rate	10%	See text
$\tilde{q}$	Employer’s matching rate	33.3%	See text
Tax code			
$IC_{1,\dots,5}$	Income cut-off points	see text	Tax code in 2000
$\tau_{1,\dots,5}$	Marginal tax rates	see text	Tax code in 2000
$\tau_{ss}$	Payroll tax rate	5.6%	OASDI tax rate on employees
$Y_{ss}$	Earnings limit for payroll	1.952	Maximum taxable earnings

Table 4: Cut-off points of tax brackets and marginal tax rate

Taxable Income	Normalized Income	Marginal Tax Rate
(\$0, \$52,800]	(0, 1.389]	15%
(\$52,800, \$127,600]	(1.398, 3.357]	28%
(\$127,600, \$194,400]	(3.357, 5.116]	31%
(\$194,400, \$347,200]	(5.116, 9.137]	36%
> \$347,200	9.137 +	39.60%

Notes: We normalize \$38,000 as 1 in the model.

Table 5: Home ownership and net worth composition for homeowners: data vs. model

	Age Group				Overall
	25–34	35–44	45–54	55–64	
Home ownership rate					
Model	0.434	0.761	0.864	0.890	0.732
Data	0.620	0.788	0.846	0.891	0.787
TDA/net worth					
Model	0.190	0.329	0.384	0.440	0.385
Data	0.229	0.297	0.311	0.336	0.298
TA/net worth					
Model	0.023	0.074	0.095	0.065	0.100
Data	0.063	0.069	0.073	0.081	0.074
Home equity/net worth					
Model	0.739	0.572	0.506	0.479	0.538
Data	0.607	0.549	0.495	0.427	0.518

Notes: Data refers to households with employer-sponsored defined contribution plans in the 2001, 2004 and 2007 Survey of Consumer Finances (SCF). Net worth composition is calculated for homeowners. We calculate the *median* ratios in each SCF and report the average of the median values across all years. Since the ratios are computed separately, they are not referred to the same household and thus the sum of the ratios does not necessarily add up to 1. Overall is defined as all households aged 25–64.

Table 6: Households' use of TDA in the benchmark model

	Age Group				Overall
	25–34	35–44	45–54	55–64	
TDA contribution rate	0.060	0.060	0.060	0.064	0.060
TDA participation rate	0.715	0.798	0.814	0.774	0.775

Notes: Tax-deferred account (TDA) contribution rate is the median ratio of TDA contributions to household income. TDA participation rate refers to the percentage of households making positive TDA contributions. Overall is defined as all households aged 25–64.

Table 7: Experiments on TDA contribution limit

	Age Group				Overall
	25–34	35–44	45–54	55–64	
Eliminating TDA					
Net worth	0.468	0.758	0.878	0.872	0.818
TDA wealth	–	–	–	–	–
TA wealth	4.094	3.854	3.212	4.419	4.111
TDA/net worth	–	–	–	–	–
TA/net worth	7.981	8.355	3.270	4.969	7.781
Home ownership rate	0.380	0.564	0.746	0.900	0.685
Median income of owner	1.122	1.261	1.085	1.020	1.114
Home equity/net worth	1.169	1.376	1.478	1.410	1.363
Home equity/home value	1.504	2.414	1.520	1.053	1.765
Higher TDA contribution limit					
Net worth	1.016	1.009	1.020	1.026	1.009
TDA wealth	1.198	1.034	1.135	1.150	1.064
TA wealth	0.856	0.817	0.784	0.659	0.784
TDA/net worth	1.103	1.045	1.093	1.113	1.087
TA/net worth	0.835	0.838	0.786	0.634	0.752
Home ownership rate	0.999	1.001	1.004	0.999	1.001
Median income of owner	0.999	0.999	1.000	1.000	1.000
Home equity/net worth	0.996	1.001	0.978	0.959	0.985
Home equity/home value	1.006	1.003	0.994	0.981	0.996

Note: All results, except the home ownership rate, are median values with respect to different age groups. Values for the benchmark model are normalized to 1, such that the experiment results reported are levels relative to the benchmark model. The home equity to net worth and home equity to home value ratios are calculated for *homeowners* only. Overall is defined as all households aged 25–64.

Table 8: Experiment without employer’s match of TDA contributions

	Age Group				Overall
	25–34	35–44	45–54	55–64	
Net worth	0.921	0.926	0.936	0.934	0.946
TDA wealth	0.352	0.632	0.733	0.759	0.703
TA wealth	1.920	1.106	1.114	1.161	1.087
TDA/net worth	0.426	0.755	0.770	0.814	0.742
TA/net worth	1.950	1.014	1.105	1.226	1.194
Home ownership rate	1.031	1.010	1.010	1.006	1.012
Median income of owner	0.993	0.996	0.990	0.999	0.995
Home equity/net worth	1.151	1.132	1.131	1.119	1.127
Home equity/home value	1.023	1.016	1.031	1.039	1.026

Note: All results, except the home ownership rate, are median values with respect to different age groups. Values for the benchmark model are normalized to 1, such that the experiment results reported are levels relative to the benchmark model. The home equity to net worth and home equity to home value ratios are calculated for *homeowners* only. Overall is defined as all households aged 25–64.

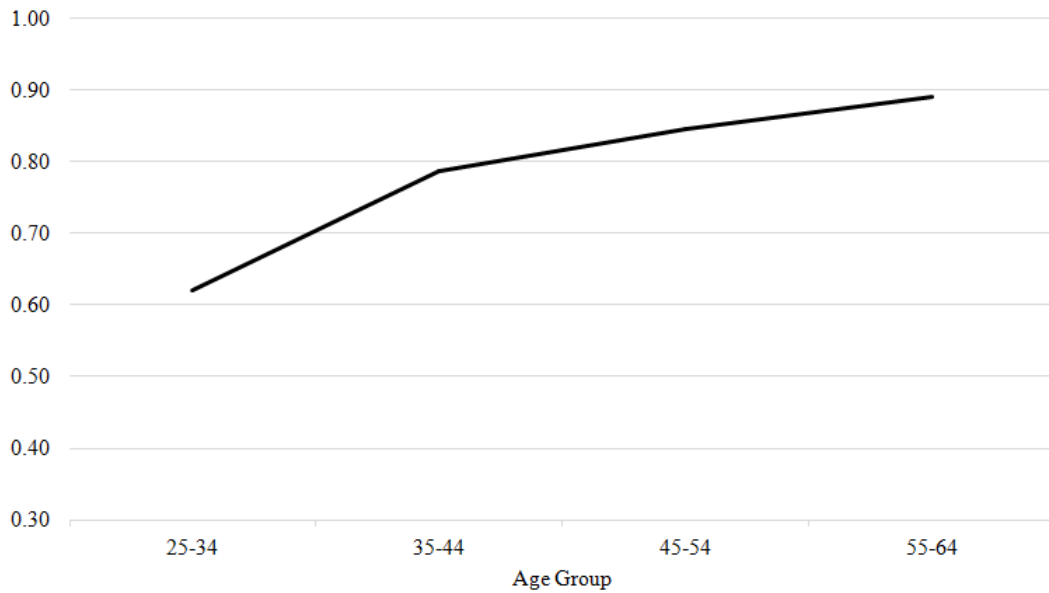
Table 9: Experiments on housing-related factors

	Age Group				Overall
	25–34	35–44	45–54	55–64	
Increase min. down payment to 20%					
Net worth	0.885	0.997	0.995	0.995	0.998
TDA wealth	1.077	0.904	0.964	0.990	0.943
TA wealth	1.473	1.254	1.102	1.078	1.205
TDA/net worth	1.117	0.938	0.967	0.985	0.981
TA/net worth	1.378	1.212	1.081	1.073	1.157
Home ownership rate	0.728	0.924	0.982	0.998	0.932
Median income of owner	1.070	1.032	1.009	1.000	1.023
Home equity/net worth	1.044	1.037	1.011	1.002	1.007
Home equity/home value	1.339	1.094	0.997	0.957	1.062
No tax benefits on home ownership					
Net worth	0.728	0.892	0.946	0.971	0.922
TDA wealth	1.035	0.770	0.803	0.899	0.921
TA wealth	1.290	1.079	1.101	1.262	1.113
TDA/net worth	1.175	0.863	0.901	0.957	0.956
TA/net worth	1.488	1.304	0.990	1.230	1.191
Home ownership rate	0.600	0.756	0.864	0.918	0.810
Median income of owner	1.006	1.020	1.028	1.014	1.031
Home equity/net worth	1.054	1.131	1.107	1.040	1.065
Home equity/home value	1.138	1.248	1.394	1.053	1.450

Note: All results, except the home ownership rate, are median values with respect to different age groups. Values for the benchmark model are normalized to 1, such that the experiment results reported are levels relative to the benchmark model. The home equity to net worth and home equity to home value ratios are calculated for *homeowners* only. Overall is defined as all households aged 25–64.

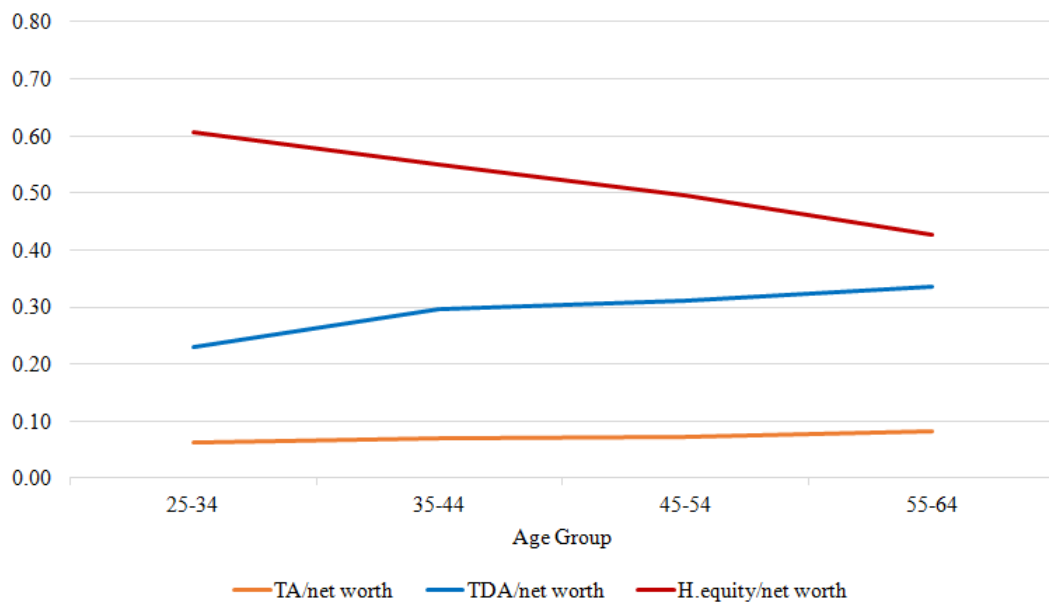


Figure 1: Home ownership rate by age group for households with DC plans



Note: Home ownership rate for households with defined contribution (DC) plans is the average of the 2001–2007 Survey of Consumer Finances.

Figure 2: Net worth composition by age group for households with DC plans that are also homeowners: median



Note: This figure shows the average of median ratios for households with defined contribution (DC) plans in the 2001–2007 Survey of Consumer Finances.

Figure 3: Home ownership rate by age group for households with DC plans

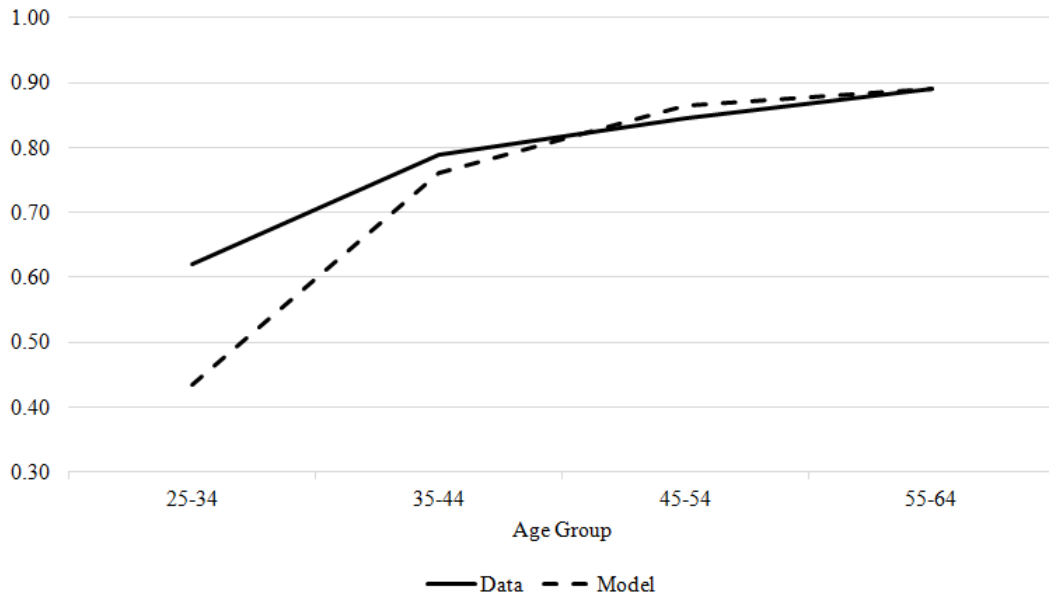


Figure 4: Net worth composition by age group for households with DC plans that are also homeowners: median

