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Article

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Cogent Economics & Finance

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Taylor & Francis Group

Suggested Citation: Shin, Inyong (2016) : Change and prediction of income and fertility rates across countries, Cogent Economics & Finance, ISSN 2332-2039, Taylor & Francis, Abingdon, Vol. 4, Iss. 1, pp. 1-34,
https://doi.org/10.1080/23322039.2015.1119367

This Version is available at:
https://hdl.handle.net/10419/147780

Shin, *Cogent Economics & Finance* (2016), 4: 1119367
http://dx.doi.org/10.1080/23322039.2015.1119367

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GENERAL & APPLIED ECONOMICS | RESEARCH ARTICLE

Change and prediction of income and fertility rates across countries

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Received: 26 June 2015
Accepted: 04 November 2015
Published: 01 March 2016

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Reviewing editor:
Caroline Elliott, Huddersfield University, UK

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Abstract: This paper analyzes and predicts the changes of relationship between income and fertility rate of cross-countries using a bivariate mixture model and a latent change score model. This paper has shown that there is a negative relationship between income and fertility rate, which is presented in the form of inverted S-shaped curve which shows the three regimes of demographic transition. Some developed countries have completed their demographic transition in fertility rate, and in developing countries, the demographic transition in fertility rate is still in progress. This paper has also shown that the number of peaks of income distribution has increased in recent years comparing to 1960s and the number won't decrease in the future. However, the number of peaks of fertility rate distribution hasn't changed from 1960s to recent years but due to the shift, finally, the distribution will change to a uni-modal distribution in the future. The income will be applied to the conditional convergence and the fertility rate will be applied to the absolute convergence. The fertility gap among cross-countries will disappear, but the income gap won't. Although the population conditions in developing countries will improve, income inequality in cross-country may not be improved after all.

Subjects: Demography; Economic Forecasting; Economics and Development; Population; Development

Keywords: income distribution; fertility rate; mixture distribution; latent change score model; Bayesian estimation

AMS Subject classifications: J11; J13; O15

ABOUT THE AUTHOR

The author is an economist and statistician. His main research fields are Economic Growth, Computational Economics, Bayesian statistics and Dynamics. His current research area includes demographic transition. Demographic transition has been studied by Economics as well as Demography, Sociology, Statistics, etc. Almost all of previous studies on demographic transition depict the demographic transition due to time flowing. However, he depicts the demographic transition by income per capita level in this research, that is, the demographic transition is analyzed from an aspect of Economics. The problems like gap between rich and poor, and raising population are great economic and political issues of our time to be solved

PUBLIC INTEREST STATEMENT

This research analyzes and predicts the changes of relationship between income and fertility rate of cross countries. Income and fertility rate have a strong interrelationship. The changes of fertility rate itself have been studied long time ago in demography. However, the study on the changes of distribution is pretty new. Furthermore, this research analyzes the changes in the joint distribution of income and fertility rate. This research shows not only the negative relationship between income and fertility rate which is a well-known fact, but also change, convergence, and prediction of their distributions. The fertility gap among cross countries will disappear but the income gap will not. Even though the population conditions in developing countries will improve, income inequality in cross-country may not be improved after all. By playing animations, readers can observe the results visually.

1. Introduction

This research analyzes the changes in income distribution of cross-countries and the changes in fertility rate distribution of cross-countries. There are already plenty of studies which have analyzed each of the changes, that is the changes in income distribution and the changes in fertility rate distribution, individually. But, there are a few empirical researches which analyze the changes in the distribution of the two variables at the same time. This research analyzes the joint distribution of the two variables which are per capita income and fertility rate using cross-country data and aims to predict the future distributions of the two variables.

The changes of birth and death rates itself, not their distribution, have been studied very long time ago in demography. Among the classic literature, Thompson (1929) classified all countries into three types by a combination of birth rates and death rates. Notestein (1945) generalized the Western demographic experience in similar ways and is regarded as the first definition of the transition. Landry (1934) also suggested the existence of three types of demographic regime. The process in which birth and death rates shift from a pre-modern regime of high birth and death rates to a post-modern regime of low birth and death rates through the intermediate regime of high birth rates and low death rates is well known as demographic transition.[1] However, the study on the changes of the distribution is pretty new comparing to the study of the changes of birth and death rates itself. Furthermore, this research analyzes the changes in the joint distribution of income and fertility rate.

Since Quah (1996), many empirical studies on the changes in income distribution of cross-country have been actively carried out. Paapaa and Dijk (1998), Bianchi (1997), Kumar and Russell (2002), etc. analyze the changes in income distribution of cross-country. According to their analyses, the income distribution of cross-country in the 1960s was one peak, but the income distribution in recent years has been changed to twin-peak which is composed of the low-income group and high-income group. Holzmann, Vollmer, and Weisbrod (2007) also analyze the changes in income distribution of cross-country; however, they analyze the distribution using the log GDP per capita, not GDP per capita itself. Income data are often analyzed using the logarithmic scale. The number of modes of the log-income distribution may differ substantially from the number of modes of the income distribution itself due to usage of the different scales. Holzmann et al. (2007) report that the shape of income distribution has changed from the twin-peak in the 1960s to three-peaks in recent years. When the number of peaks is estimated, the results may be different, particularly using level data and using log data as mentioned in the examples above. However, the conclusion that the number of peaks has increased in recent years comparing to 1960s is common.

On the other hand, Strulik and Vollmer (2010) analyze the changes in fertility rate distribution of cross-country. Strulik and Vollmer (2010) show a fertility trap and twin-peaks that are composed by the two groups with high fertility rate and low fertility rate. Strulik and Vollmer (2010) also offer an empirical evidence to the theoretic model of Galor and Weil (1996) which explains the change in fertility using the two periods overlapping generation model and shows that multiple steady states can be found.

In many economic theory models, the fertility rate is decided endogenously and is inevitably linked with income and social fundamentals, etc. It is well known that there is a close relationship between the fertility rate and income, which is the fertility rate decreases as the income increases.[2] For example, Easterlin (1966), Becker (1960), and Nerlove, Assaf, and Efraim (1978) are static studies, and Becker and Barro (1988), Barro and Becker (1989), Lapan and Enders (1990), Benhabib and Nishimura (1989), Becker, Murphy, and Tamura (1990), Kremer (1993), Galor and Weil (1996), Dahan and Tsiddon (1998) and Qi and Kanaya (2010) are dynamic studies. The determinants of birth rate have been sought in the decline of death rate, emphasizing the quality of children, the increase of the opportunity cost of the women, an increase in the status and education of women, urbanization (movement off the farms), social security systems, religious values, social values, etc. Except for religious values and social values, the declining factors in birth rate are deeply related to the economic development. However, as in the examples above, almost all of the empirical studies, on the

distribution of fertility rate and income, analyze only univariate system. In other words, they do not look at the mutual interrelationship. This research takes into account the strong relationship between both variables and analyzes the joint distribution of the two variables, the income and the fertility rate. When both variables are decided with mutual interrelationship simultaneously, the distribution of both variables is also decided with mutual interrelationship simultaneously. This research analyzes the change of joint distribution of the two variables, GDP per capita and fertility rate, using a bivariate mixture model. After that, this research calculates the joint movement of the two variables using a latent change score model and predicts the future joint distribution of the two variables and analyzes the changes in the distribution. This research shows not only the negative relationship between income and fertility rate which is a well-known fact, but also change, convergence, and prediction of their distributions. The mutual interrelationship can make them explainable.[3]

This research analyzes the changes in the two variables using the GDP per capita and fertility rate of 106 countries in 51 years, from 1960 to 2010, and predicts the changes in the two variables up to 2030. Even though we did not find a new determinant of birth rate theoretically, this research yields several intriguing results about the changes and convergences of distribution of income and fertility rate by the econometric analysis: (i) the income distribution in 1960 was a twin-peak, but the income distribution in 2010 has changed to a three-peak. This result is consistent with Holzmann et al. (2007; ii) the distributions of fertility rate in 1960 and 2010 were both twin-peak. It is important to highlight that in 1960, the right peak is higher than the left peak comparing to the ones in 2010, when the left peak is higher than the right peak. This result is consistent with Strulik and Vollmer (2010; iii) according to our forecast until 2030, the distribution of GDP per capita in 2030 will be similar to that in 2010, that is the distribution of income in 2030 will still have a three-peak. However, the distribution of fertility rate in 2030 will be changed to a uni-modal distribution from the twin-peak. The result that the distribution of fertility rate will become a uni-modal distribution is consistent with the population forecasts of the United Nations. Weil (2013) mentions that in making its population forecasts, the United Nations predicts that in all of the countries in the world, the total fertility rate (TFR) will move from its current level toward the replacement fertility over the next 50 years—specifically, that in almost all countries, the TFR would be exactly 2.1 by the year 2050. From Weil (2013), it can be guessed that the TFR distribution of cross-country has a uni-modal distribution with the mean 2.1. For many developing countries, this world population forecast of United Nations will mean a sharp fall in fertility and for many rich countries, it will mean a significant rise in fertility; (iv) growth trend of GDP per capita is clustered into two, but growth trend of the fertility rate is clustered only into one. The result (iv) could be an important cause for the result (iii) which is the distribution of GDP per capita will be still three-peaks in 2030, but the distribution of fertility rate will be changed to uni-modal distribution in 2030; (v) the income per capita will be applied to the conditional convergence and the fertility rate will be applied to the absolute convergence. The fertility gap among cross-countries will disappear, but the income gap will not. Even though the population conditions in developing countries will be improved, income inequality in cross-country may not be improved after all.

This research is organized as follows: Section 2 describes the data used. Section 3 summarizes the analytical methodology on multivariate mixture model and analyzes the results. Section 4 summarizes the analytical methodology on the latent change score model and analyzes the results. Section 5 offers conclusions on this research. Finally, more information on each country as well as animations, which show the changes of variables, can be found in Appendix .

2. Data

The GDP per capita (in PPP terms at 2005 constant prices) and the total fertility rate for each country are used. The data were drawn from the Penn World Table (PWT) and World Development Indicators (WDI) in the World Bank. Table 1 shows the detailed data sources. The PWT and WDI listed about 200 countries. However, among them, only 106 countries were taken into the consideration for the two kinds of data and for 51 years, from 1960 to 2010. Therefore, these 106 countries are analyzed.[4]

Table 1. Data source

Indicators	URL
GDP per capita	https://pwt.sas.upenn.edu/php_site/pwt71/pwt71_form.php
Fertility rate, total (births per woman)	http://data.worldbank.org/indicator/SP.DYN.TFRT.IN

*Accessed on 1 May 2014.

Due to the limitation of the space, only the GDP per capita and the fertility rate in the first year (1960) and the last year (2010) are reported in Table A1 in Appendix.

The logarithm GDP per capita instead of the GDP per capita itself is used in the following analysis. The GDP per capita in this paper means the logarithm GDP per capita instead of the GDP per capita itself hereinafter. Both numbers of digits of GDP per capita and logarithm GDP per capita are so clearly different, so it is easy to make a judgment on whether it is the GDP per capita or the logarithm GDP per capita by the number of digits, even though we make no mention of it each time.

The relationship between both data is plotted in Figure 1. The GDP per capita is plotted on the horizontal axis and the total fertility rate is plotted on the vertical axis. The label of horizontal axis in Figure 1, GDP per capita, means the logarithm GDP per capita as mentioned before. Figure 1 suggests that there is negative relationship between both of them, especially it looks like an inverted S-shaped curve. And, even though Figure 1 is depicted by income level instead of time flowing in the previous studies, Figure 1 still shows the three regimes of the demographic transition well, that is in birth rate, the first regime is the period that shows a gradual change before the demographic transition begins, and the second regime is the period that shows a rapid drop after the first regime. Finally, the third regime is the final period that shows a gradual change again.[5] Table 2 reports the quantile and mean

Figure 1. GDP per capita and fertility rate.

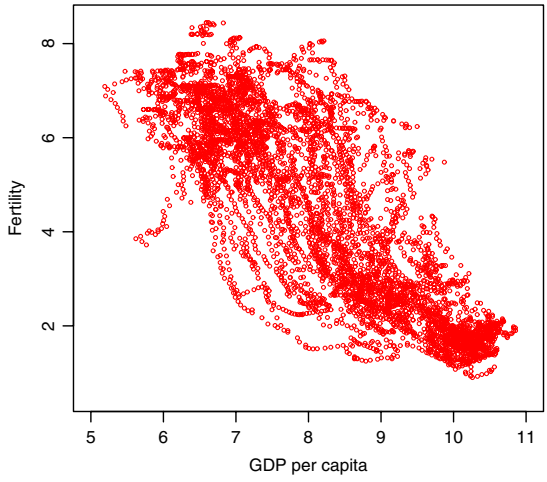

Table 2. Quantile and mean

Variables	Year	Quantile					Mean
		0% (min)	25%	50% (median)	75%	100% (max)	
GDP per capita	1960	5.651	6.693	7.629	8.600	9.953	7.680
	2010	5.483	7.262	8.732	9.879	10.829	8.590
	full sample	5.192	7.021	8.142	9.258	10.855	8.175
Fertility rate	1960	2.001	4.346	6.184	6.790	8.187	5.607
	2010	1.127	1.911	2.584	4.647	7.584	3.182
	full sample	0.901	2.500	4.544	6.259	8.449	4.426

of the GDP per capita and the fertility rate. The mean of each of the distributions is not equal to the median of each of the distributions. It turns out that the distributions of GDP per capita and fertility rate in 1960 and 2010 are asymmetric.

3. Mixture model

As mixture models have been widely used for data clustering, it is proposed a parametric mixture model for data clustering in order to detect clusters generated from arbitrary unknown distributions.

3.1. Method

Although there is no novelty in the method shown in Section 3.1, it is briefly discussed on the estimation methods of mixture distribution for the convenience of readers. This research considers a multivariate Gaussian mixture model which is an effective clustering algorithm in data minding. Mixture models provide an intuitive statistical representation of data structured in groups. Thus, assuming the model as *G*-component normal mixture model (see McLachlan and Peel (2000), Chapter 3, for details). Fraley, Raftery, Murphy, and Scrucca (2012) is also very helpful), besides, density of a random variable is specified as follows:

$$f(\mathbf{x}_j) = \sum_{i=1}^{G} \alpha_i \mathcal{N}_i(\mathbf{x}_j|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i), \tag{1}$$

where $\mathbf{x}_j$, $\boldsymbol{\mu}_i$, $\boldsymbol{\Sigma}_i$, G and α_i are the observed random sample, the component mean, the component covariance matrix, the number of components, and the mixing proportion, respectively. The α_i's are nonnegative quantities that sum to one; that is,

$$\sum_{i=1}^{G} \alpha_i = 1, \quad 0 \le \alpha_i \le 1 \ (i = 1, 2, \cdots, G). \tag{2}$$

The $\mathcal{N}_i(\mathbf{x}_j|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$ denotes the multivariate normal density function with mean $\boldsymbol{\mu}_i$ and covariance $\boldsymbol{\Sigma}_i$; that is,[6]

$$\mathcal{N}_i(\mathbf{x}_j|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i) = \frac{1}{(2\pi)^{\frac{d}{2}}|\boldsymbol{\Sigma}_i|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\mathbf{x}_j - \boldsymbol{\mu}_i)'\boldsymbol{\Sigma}_i^{-1}(\mathbf{x}_j - \boldsymbol{\mu}_i)\right\}, \tag{3}$$

where d is the dimensionality of $\mathbf{x}$. The parameters of the model can be estimated by the method of maximum likelihood. The log likelihood function takes the form

$$\ln L(\mathbf{x}|\boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \sum_{j=1}^{N} \ln\left\{\sum_{i=1}^{G} \alpha_i \mathcal{N}_i(\mathbf{x}_j|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)\right\} \tag{4}$$

where N is the number of sample. The Bayesian information criterion (BIC) is used to decide the number of components (G) in the mixture model. To decide the number of components, BIC decision rule is considered, which decides the number of components by the highest BIC among each of the BIC values which are obtained from one-component model to m-component model, where m is a natural number.

3.2. Results

The results of the distributions of GDP per capita and fertility rate using the *G*-component normal mixture model, which is introduced in Section 3.1, are reported. The results of univariate mixture model and the results of bivariate mixture model are both reported in Sections 3.2.1 and 3.2.2, respectively.

3.2.1. Univariate mixture model

Figure 2 shows the histograms and the distributions of GDP per capita and fertility rate. The densities are estimated by the univariate mixture model. Figure 2 (1), (2), (3), and (4) in the first row shows the histograms of GDP per capita and fertility rate in 1960 and 2010. Figure 2 (5), (6), (7), and (8) in the

second row shows the distributions of GDP per capita and fertility rate in 1960 and 2010. Figure 2 (1), (2), (5), and (6) in the first and second columns shows the histograms and the distributions of GDP per capita in 1960 and 2010. Figure 2 (3), (4), (7), and (8) in the third and fourth columns shows the histograms and the distributions of fertility rate in 1960 and 2010. The characteristic features of the distributions of GDP per capita and fertility rate are also reported in Tables 3 and 4, respectively.

From the analysis of GDP per capita, $G = 2$ in 1960 and $G = 3$ in 2010 are gotten by the BIC decision rule using the univariate mixture model. As seen in Figure 2 (5), the distribution of GDP per capita in 1960 has a twin-peak unlike the distribution of GDP per capita in 2010 that has a three-peak in Figure 2 (6). The number of peaks has increased by one from two to three during 1960–2010 period. It means that the income of each country diverges over time, not converges. Income inequality may not be improved after all. Holzmann et al. (2007) also use the logarithm of GDP per

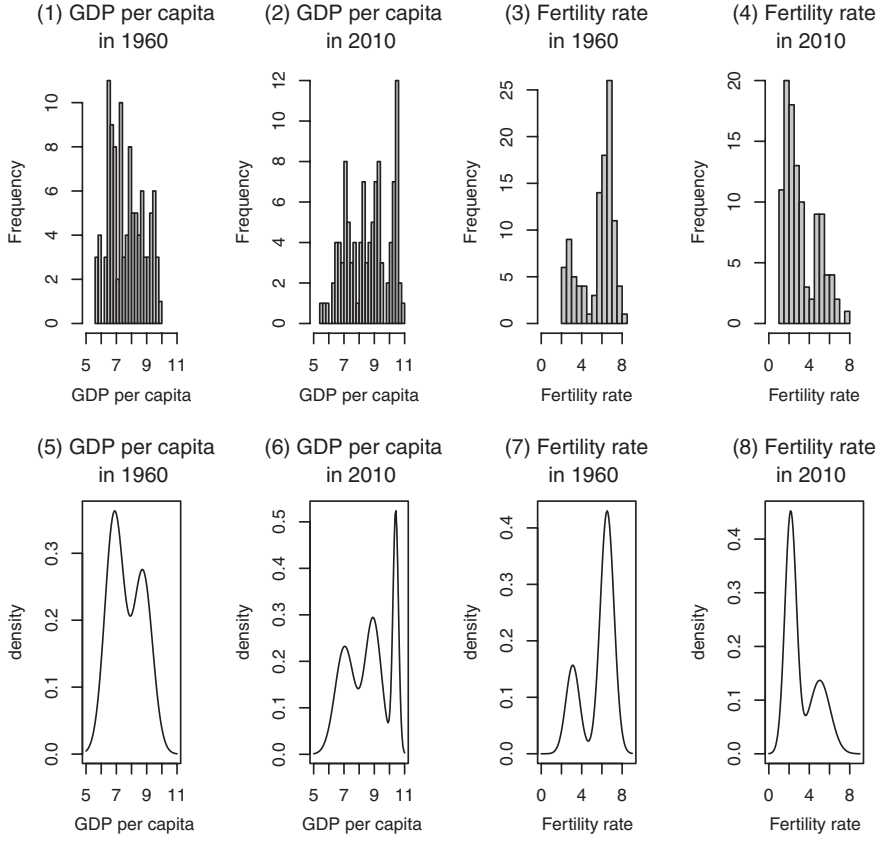

Figure 2. Histograms and densities of the GDP per capita and the fertility rate in 1960 and 2010.

Table 3. The details of the distributions of GDP per capita in 1960 and 2010

Year	1960		2010		
Cluster	**1**	**2**	**1**	**2**	**3**
Means	6.873	8.750	7.044	8.921	10.412
Variances	0.400	0.400	0.395	0.296	0.033
Numbers of sample	61	45	37	43	26
Mixing probabilities	0.570	0.430	0.365	0.398	0.237
log likelihood	−155.346		−162.653		
BIC	−329.346		−362.613		

Note: The number of clusters was decided by the value of BIC.

The order of clusters was decided by the value of mean of distribution (the higher mean, the higher number of cluster).

Table 4. The details of the distributions of fertility rate in 1960 and 2010

Year	1960		2010	
Cluster	**1**	**2**	**1**	**2**
Means	3.117	6.515	2.157	5.032
Variances	0.461	0.461	0.325	1.080
Numbers of sample	29	77	70	36
Mixing probabilities	0.267	0.733	0.643	0.357
log likelihood	−169.030		−172.293	
BIC	−356.715		−367.902	

Notes: The number of clusters was decided by the value of BIC. The order of clusters was decided by the value of mean of distribution (the higher mean, the higher number of cluster).

capita and report that the distribution appears to have only two clusters in 1970–1975, but consists of three clusters—low-, middle-, and high-income groups—from 1976 onwards. Even though the analysis period of this paper is different from Holzmann et al. (2007), the results are the same, that is the number of peaks of income distribution has changed from two to three.[7] Meanwhile, in Kumar and Russell (2002), which analyzes the distribution with GDP per worker, not using logarithm value, the world income distribution in 1960 was one-peak, but the distribution has changed to twin-peaks in 1985. The three researches, Holzmann et al. (2007), Kumar and Russell (2002), and this research, have one key thing in common: that the number of peaks of income distribution increases as time passes, even though the period of analysis, the handling of data, the method of analysis, etc. in each research are different.

From the analysis of fertility rate, the result that the number of components is two ($G = 2$) in both 1960 and 2010 is obtained by the BIC decision rule. Looking at the distributions of fertility rate in Figure 2 (7) and (8), both distributions in 1960 and 2010 have twin peaks. In 1960, the right peak is higher than the left peak, but the height is reversed in 2010, the left peak is higher than the right peak.[8] This result is consistent with Strulik and Vollmer (2010) using data from 1950 to 2005. However, Strulik and Vollmer (2010) assume the two-component model from the very beginning, but this research does not assume the number of components and decides the number of components using the mixture model by the BIC decision rule, which is an ex-post decision-making.

3.2.2. Bivariate mixture model

An attempt of this research, which has not performed in previous studies, is to analyze the distribution of GDP per capita and fertility rate simultaneously instead separately. There are lots of studies using a univariate mixture model which analyzes the distributions of each variable. As far as I know, there are no previous studies using a bivariate mixture model which analyzes the joint distribution of two variables, GDP per capita and fertility rate, in the framework of demographic transition. Figure 3 presents the results by the bivariate mixture model. Figure 3 shows the scatter plot and contour lines. Table 5 reports the characteristic features of the bivariate joint distributions of GDP per capita and fertility rate of each cluster in 1960 and 2010.

In 1960, there were two clusters ($G = 2$), but in 2010, the clusters were divided into four clusters ($G = 4$). The number of clusters is decided by the BIC decision rule as in the univariate mixture model in Section 3.2.1. In this paper, we follow the common rule, the BIC decision rule, everytime to decide the number of clusters. In 1960, there are two clusters of which cluster 1, the low-income and high-fertility group, has the mean of 7.153 and 6.540 and cluster 2, high-income and low-fertility group, has the mean of 9.045 and 3.187. In 2010, there are four clusters which include not only cluster 1 and the cluster 4; during this period, two more clusters have appeared which are marked as cluster 2 and cluster 3. Cluster 1 is the low-income and high-fertility group which has the mean of 6.835 and 5.389. Cluster 4 is high-income and low-fertility group which has the mean of 10.378 and 1.718. Cluster 2 is the lower middle-income and upper middle-fertility group which has the mean of 8.328

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Figure 3. Scatter plots and contour lines.

Note: The range of vertical axis and horizontal axis is different between two graphs.

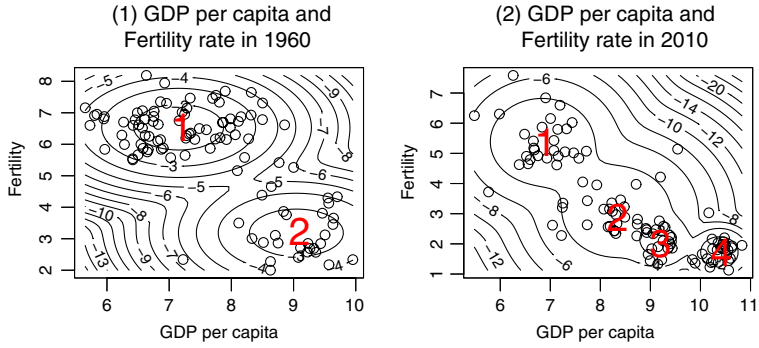

Table 5. The details of the bivariate joint distributions of the GDP per capita and the fertility rate in 1960 and 2010

Year		1960		2010			
Cluster		**1**	**2**	**1**	**2**	**3**	**4**
Means	GDP per capita	7.153	9.045	6.835	8.328	9.173	10.378
	Fertility	6.540	3.187	5.389	3.054	2.085	1.718
Variances	GDP per capita	0.540	0.540	0.314	0.370	0.060	0.047
	Fertility	0.540	0.540	0.602	0.711	0.115	0.009
Numbers of sample		77	29	29	28	22	27
Mixing probabilities		0.722	0.278	0.275	0.290	0.181	0.254
log likelihood		−289.209		−267.055			
BIC		−606.398		−608.725			

Note: The number of clusters was decided by the value of BIC. The order of cluster was decided by the value of mean of GDP per capita (the higher mean, the higher number of cluster).

Table 6. Transition of the countries in the clusters

Cluster (1960/2010)	List of countries (number of the countries)
1/1	Benin, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Comoros, Congo Dem. Rep., Congo Republic of, Cote d Ivorie, Ethiopia, Gambia, Guinea, Guinea-Bissau, Kenya, Madagascar, Malawi, Mali, Mauritania, Mozambique, Niger, Nigeria, Rwanda, Senegal, Tanzania, Togo, Uganda, Zambia, Zimbabwe (29)
1/2	Algeria, Bangladesh, Bolivia, Cape Verde, Ecuador, Egypt, Equatorial Guinea, Fiji, Ghana, Guatemala, Haiti, Honduras, India, Indonesia, Jordan, Lesotho, Morocco, Namibia, Nepal, Nicaragua, Pakistan, Papua New Guinea, Paraguay, Philippines, Sri Lanka, Syria (26)
1/3	Botswana, Brazil, Chile, China, Colombia, Costa Rica, Dominican Republic, El Salvador, Iran, Jamaica, Malaysia, Mauritius, Mexico, Panama, Peru, South Africa, Thailand, Turkey, Venezuela (19)
1/4	Hong Kong, Korea (Republic of), Trinidad & Tobago (3)
2/1	(0)
2/2	Gabon, Israel (2)
2/3	Argentina, Romania, Uruguay (3)
2/4	Australia, Austria, Barbados, Belgium, Canada, Cyprus, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Puerto, Spain, Sweden, Switzerland, United Kingdom, United States (24)

and 3.054. Cluster 3 is upper middle-income and lower middle-fertility group which has the mean of 9.1736 and 2.085.[9] In Table 6, it is reported which country is classified in which cluster in each period between 1960 and 2010. More than half of the countries in cluster 1 in 1960 have shifted to cluster 2 or cluster 3 in 2010 and only three countries, which are Hong Kong, Republic of Korea, and Trinidad & Tobago, have shifted to cluster 4 in 2010. On the other hand, almost all of the countries in cluster

2 in 1960, except for five countries which are Gabon, Israel, Argentina, Romania, and Uruguay, have shifted to cluster 4 in 2010.

4. Bivariate dynamic model

4.1. Model and estimation method

4.1.1. Latent change score model

This research considers a bivariate dynamic model to examine an association between chronological change of GDP per capita and fertility rate simultaneously, using a latent change score model like Equation (5).

$$\begin{aligned} y_{i,t} &= y^l_{i,t} + \epsilon^y_{i,t} \\ f_{i,t} &= f^l_{i,t} + \epsilon^f_{i,t} \end{aligned} \tag{5}$$

In Equation (5), $y_{i,t}$ and $f_{i,t}$ are the observed data which are the GDP per capita and the fertility rate for country i at time t, respectively. $y^l_{i,t}$ and $f^l_{i,t}$ are the latent scores of the variables which are the GDP per capita and the fertility rate for country i at time t, respectively. $\epsilon^y_{i,t}$ and $\epsilon^f_{i,t}$ are the errors in measurement of the variables, the GDP per capita and the fertility rate, for country i at time t, respectively. We have assumed the errors like $\epsilon^y_{i,t} \sim N(0, \sigma^2_y)$, and $\epsilon^f_{i,t} \sim N(0, \sigma^2_f)$. The latent scores have an autoregressive relationship such that the latent score at time t is equal to the latent score at time $t-1$ plus the change that has occurred between the two. This can be written as

$$\begin{aligned} y^l_{i,t} &= y^l_{i,t-1} + \Delta y^l_{i,t} \\ f^l_{i,t} &= f^l_{i,t-1} + \Delta f^l_{i,t} \quad (t \geq 2) \end{aligned} \tag{6}$$

where, $\Delta y^l_{i,t}$ is the latent change score which is the difference between $y^l_{i,t}$ and $y^l_{i,t-1}$. Because we have used logarithm GDP per capita, $\Delta y^l_{i,t}$ shows the growth rate for country i at time t. And, as with $\Delta y^l_{i,t}$, $\Delta f^l_{i,t}$ is the latent change score which is the difference between $f^l_{i,t}$ and $f^l_{i,t-1}$.

We have specified a model for the latent change score as follows:

$$\begin{aligned} \Delta y^l_{i,t} &= \alpha_y + \beta_y y^l_{i,t-1} + \gamma_y f^l_{i,t-1} + \delta_{yf} y^l_{i,t-1} f^l_{i,t-1} + \xi^y_{i,t} \\ \Delta f^l_{i,t} &= \alpha_f + \beta_f f^l_{i,t-1} + \gamma_f y^l_{i,t-1} + \delta_{fy} f^l_{i,t-1} y^l_{i,t-1} + \xi^f_{i,t} \end{aligned} \tag{7}$$

where, α, β, γ, and δ mean constant trend, autoproportional effect, coupling effect, and cross-term effect, respectively. And, ξs are error terms. We assume errors like $\xi^y_{i,t} \sim N(0, \omega^2_y)$, and $\xi^f_{i,t} \sim N(0, \omega^2_f)$.

Substituting Equation (7) into Equation (6), we then obtained:

$$\begin{aligned} y^l_{i,t} &= \alpha_y + (1+\beta_y) y^l_{i,t-1} + \gamma_y f^l_{i,t-1} + \delta_{yf} y^l_{i,t-1} f^l_{i,t-1} + \xi^y_{i,t} \\ f^l_{i,t} &= \alpha_f + (1+\beta_f) f^l_{i,t-1} + \gamma_f y^l_{i,t-1} + \delta_{fy} f^l_{i,t-1} y^l_{i,t-1} + \xi^f_{i,t} \end{aligned} \tag{8}$$

To substitute Equation (8) into Equation (5), we have gotten Equation (9). However, it is difficult to estimate the parameters directly using Equation (9) because Equation (9) has identification problems. Equation (9) has composite error structures which include multiple errors in each equation, that is there are two errors $\xi^y_{i,t}$ and $\epsilon^y_{i,t}$ in the first equation of Equation (9) and there are also two errors $\xi^f_{i,t}$ and $\epsilon^f_{i,t}$ in the second equation of Equation (9).

$$\begin{aligned} y_{i,t} &= \alpha_y + (1+\beta_y) y^l_{i,t-1} + \gamma_y f^l_{i,t-1} + \delta_{yf} y^l_{i,t-1} f^l_{i,t-1} + \underbrace{\xi^y_{i,t} + \epsilon^y_{i,t}}_{\text{errors}} \\ f_{i,t} &= \alpha_f + (1+\beta_f) f^l_{i,t-1} + \gamma_f y^l_{i,t-1} + \delta_{fy} f^l_{i,t-1} y^l_{i,t-1} + \underbrace{\xi^f_{i,t} + \epsilon^f_{i,t}}_{\text{errors}} \end{aligned} \tag{9}$$

4.1.2. Bayesian estimation method

The latent change score model has a hierarchical structure with two levels: the observed data and the latent scores. In case of a hierarchical structure, Bayesian estimation method is very useful and can easily estimate the parameters. So, the hierarchical Bayesian model is widely used to estimate the variables.

When we estimate the parameters in the hierarchical Bayesian model, we generate one conditional distribution after another, sequentially. In case of a hierarchical structure model, Bayes' theorem for probability distribution is often stated as:

$$\text{Posterior} \propto \text{likelihood} \times \text{prior} \times \text{hyperprior}. \tag{10}$$

To calculate Gibbs sampler for our model, posterior distribution is needed. The posterior distribution of our model is derived following the hierarchical modeling structure using conditional distribution. Our posterior density is:

$$\begin{aligned}\text{Posterior} \propto & \underbrace{L(\mathbf{D}|y_{i,t}^{l}, f_{i,t}^{l}, \sigma_y^2, \sigma_f^2)}_{\text{likelihood}} \underbrace{p(y_{i,t}^{l}, f_{i,t}^{l}|\alpha_y, \beta_y, \gamma_y, \delta_y, \alpha_f, \beta_f, \gamma_f, \delta_f, \xi_y^2, \xi_f^2)}_{\text{prior}} \\ & \underbrace{p(\alpha_y|m, s^2)p(\beta_y|m, s^2)p(\gamma_y|m, s^2)p(\delta_y|m, s^2)}_{\text{hyperprior}} \\ & \underbrace{p(\alpha_f|m, s^2)p(\beta_f|m, s^2)p(\gamma_f|m, s^2)p(\delta_f|m, s^2)}_{\text{hyperprior}} \\ & \underbrace{p(\xi_y^2|c, d)p(\xi_f^2|c, d)p(\sigma_y^2|a, b)p(\sigma_f^2|a, b)}_{\text{hyperprior}}\end{aligned} \tag{11}$$

where $\mathbf{D}$ is data. The hyperprior distributions for α, β, γ, and δ are specified to be normal distributions, with parameters m and s^2. Without prior knowledge, these parameters are specified to make the hyperprior vague, $m = 0$ and $s^2 = 10,000$, so that as much as possible the hyperprior should not affect the posterior. And the hyperprior distributions for ξ^2 and σ^2 are specified to be inverse gamma distributions, with parameters a and b, and c and d, respectively. Once again, without prior information, these parameters are specified to make the hyperprior vague, $a = 10,000$, $b = 10,000$, $c = 10,000$, and $d = 10,000$ (see Lynch (2010) for details).

4.2. Estimation results

This research considers two kinds of regression models, which are the pooling model and fixed effect model. In the pooling model, the individual effects of constant terms are not considered, that is α_y and α_f in Equation (8) are considered to be common in all countries. On the other hand, in the fixed effect model, the individual effects affect the intercept of each of the countries, that is α_y and α_f are considered to be $\alpha_{y,i}$ and $\alpha_{f,i}$ where the subscript i means individual occurrences.[10]

4.2.1. Pooling model

Table 7 presents the estimation results of the pooling model which are the posterior mean, median, standard deviation, 95% posterior credible interval, and Geweke's convergence diagnostic for the Bayesian estimation.[11] The Bayesian credible interval is defined as the interval for which the posterior exceeds a given probability, in this case 0.95 (95%). The credible interval in Bayesian statistics is similar to the confidence interval in classical statistics, even though not same.[12] It is needed to check if 95% credible interval includes 0 or not. If not, the parameter can be interpreted as "significant," which is the term used in classical statistics. In Table 7, the 95% credible intervals for all parameters except $f^{ss}{}_2$ do not include 0.[13] The sampling was run with a burn-in of 1,000,000 iterations with 2,000,000 iterations. Based on the results of Geweke's convergence diagnostic, it is considered that this sampling has been converged completely.

The signs of each constant trend (α_y and α_f), each autoproportional effect (β_y and β_f), each coupling effect (γ_y and γ_f), and each cross-term effect (δ_{yf} and δ_{fy}) are opposite, that is α_y is positive and α_f is negative, β_y is negative and β_f is positive, γ_y is negative and γ_f is positive, and δ_{yf} is positive and δ_{fy} is negative. The mean and the standard deviation of σ_y and σ_f are almost 0. It means that both, the observed data and the latent score, are very close.

To examine the changes in both, economic growth rate and fertility rate, due to income change, we have differentiated Equation (7) with respect to $y^l_{i,t-1}$ and have gotten Equation (12).

$$\frac{\partial\left(\Delta y^l_{i,t}\right)}{\partial y^l_{i,t-1}} = \beta_y + \delta_{yf} f^l_{i,t-1}, \quad \frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} = \gamma_f + \delta_{fy} f^l_{i,t-1} \tag{12}$$

From Equation (12),

$$\begin{cases} \text{if } f^l_{i,t-1} > -\frac{\beta_y}{\delta_{yf}}, \text{ then } \frac{\partial\left(\Delta y^l_{i,t}\right)}{\partial y^l_{i,t-1}} > 0 \\ \text{if } f^l_{i,t-1} < -\frac{\beta_y}{\delta_{yf}}, \text{ then } \frac{\partial\left(\Delta y^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0 \end{cases} \text{and,} \begin{cases} \text{if } f^l_{i,t-1} > -\frac{\gamma_f}{\delta_{fy}}, \text{ then } \frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0 \\ \text{if } f^l_{i,t-1} < -\frac{\gamma_f}{\delta_{fy}}, \text{ then } \frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} > 0 \end{cases}. \tag{13}$$

From Table 2, the fertility rate 6.458 falls at about 75% of the whole, and 4.247 falls about 50% of the whole. In other words, based on the sample data, about 75% data satisfy $\frac{\partial\left(\Delta y^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0$. And about 50% data satisfy $\frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0$ and the rest about 50% data satisfy $\frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} > 0$. On the left (the second line) in Equation (13), $\frac{\partial\left(\Delta y^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0$ means that economic growth rate decreases as GDP per capita increases. The result shows the convergence in economic growth theory. On the right (the first line) in Equation (13), $\frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} < 0$ means that the fertility rate decreases rapidly, as GDP per capita increases. Meanwhile, on the right (the second line) in Equation (13), $\frac{\partial\left(\Delta f^l_{i,t}\right)}{\partial y^l_{i,t-1}} > 0$ means that the fertility rate decreases slowly, as GDP per capita increases.

Table 7. Estimation results of the pooling model

Parameters	Mean	Median	Standard deviation	95% HPDI [2.5%, 97.5%]	Geweke's CD
α_y	0.1373	0.1373	0.0178	**[0.1025,0.1721]**	1.2836
β_y	-0.0113	-0.0113	0.0019	**[−0.0150, −0.0076]**	-1.2580
γ_y	-0.0197	-0.0197	0.0032	**[−0.0260, −0.0135]**	-1.4107
δ_{yf}	0.0018	0.0018	0.0004	**[0.0010, 0.0025]**	1.1630
α_f	-0.5554	-0.5554	0.0215	**[-0.5976, -0.5133]**	0.2866
β_f	0.1123	0.1123	0.0038	**[0.1048 , 0.1198]**	-0.8021
γ_f	0.0584	0.0584	0.0023	**[0.0540 , 0.0629]**	-0.6008
δ_{fy}	-0.0138	-0.0138	0.0005	**[−0.0147, −0.0129]**	1.0439
σ_y	0.0000	0.0000	0.0000	**[0.0000, 0.0000]**	0.2976
σ_f	0.0000	0.0000	0.0000	**[0.0000, 0.0000]**	-0.6712
ω_y	0.0035	0.0035	0.0001	**[0.0033, 0.0036]**	-1.3825
ω_f	0.0051	0.0051	0.0001	**[0.0049, 0.0053]**	0.3141
$-\frac{\beta_y}{\delta_{yf}}$	6.4580	6.3420	0.8864	**[5.1060, 8.5030]**	-0.0700
$-\frac{\gamma_f}{\delta_{fy}}$	4.2470	4.2470	0.0939	**[4.0620, 4.4310]**	0.4400

Note: HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic.

The figures highlighted in HPDI represent the cases that the 95% credible intervals do not include 0.

Figure 4. Joint movements for GDP per capita and fertility rate.

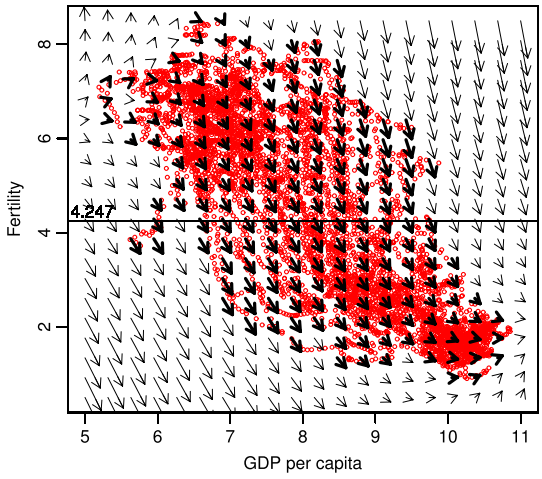

Figure 4 shows the joint movements for GDP per capita and fertility rate in a dynamic vector field. Each arrow shows the general direction of all curves within that specific region of these curves. For any pair of latent scores at time t, the arrow points to where the pair of latent scores is expected to be at time $t + n$. In Figure 4, we have assumed $n = 5$, which means five-year movements. The joint movements were calculated from the coefficients which have been obtained from the pooling model. The bold arrow lines show the joint movements of the field where the data exist. And, the slim arrow lines show the joint movements of the field where the data do not exist. The bold arrow lines demonstrate the change of cross-section data well.

As it can be seen in Figure 4, the fertility rate reduces rapidly above 4.247 when demographic transition occurs from regime 1, where fertility rate is high and stable, to regime 2, where we can notice the rapid drop in fertility rate. By comparison, the fertility rate reduces slowly below 4.247 when demographic transition occurs from regime 2 to regime 3, where fertility rate is low and stable.

4.2.2. Fixed effect model

Table 8 presents the estimation results of the fixed effect model.[14] As it was done in Table 7, we reported the simple summaries about the posterior mean, median, standard deviation, 95% posterior credible interval, and Geweke's convergence diagnostic for the Bayesian estimation. Due to the limitations of the space, we reported the summaries of $\alpha_{y,i}$ and $\alpha_{f,i}$ in Table A2 in Appendix . In Table 8, the 95% credible intervals for all parameters, except f_2^{ss}, do not include 0. The sampling was run with a burn-in of 1,000,000 iterations with 2,000,000 iterations. Based on the results of Geweke's convergence diagnostic, it is considered that this sampling has been converged completely.

As with the pooling model, the signs of each autoproportional effect (β_y and β_f), each coupling effect (γ_y and γ_f), and each cross-term effect (δ_{yf} and δ_{fy}) are opposite, that is β_y is negative and β_f is positive, γ_y is negative and γ_f is positive, and δ_{yf} is positive and δ_{fy} is negative. The mean and the standard deviation of σ_y and σ_f are almost 0. It means that both, the observed data and the latent score, are very close. The results of fixed effect model are very similar to the results of pooling model.

The parameters which are estimated by Bayesian estimation method have their distributions. To promote further analyzing using their distribution is a difficult task; so, it is analyzed using the representative values of the posterior distribution. To put it simply, from now on, this research will use the mean of the posterior distribution of $\alpha_{y,i}$ and $\alpha_{f,i}$ as the representative value of the posterior distribution of $\alpha_{y,i}$ and $\alpha_{f,i}$, instead of the posterior distribution itself. We denote the posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$ as $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$, respectively.

Table 8. Estimation result of the fixed effect model

Parameters	Mean	Median	Standard deviation	95% HPDI [2.5%, 97.5%]	Geweke's CD
β_y	−0.0314	−0.0314	0.0035	**[−0.0383, −0.0246]**	−1.7128
γ_y	−0.0161	−0.0161	0.0053	**[−0.0264, −0.0057]**	−0.4355
δ_{yf}	0.0015	0.0015	0.0007	**[0.0002, 0.0027]**	0.2095
β_f	0.1447	0.1447	0.0063	**[0.1324, 0.1570]**	−0.0208
γ_f	0.0872	0.0871	0.0041	**[0.0792, 0.0952]**	−0.1966
δ_{fy}	−0.0181	−0.0181	0.0008	**[−0.0196, −0.0165]**	−0.2233
σ_y	0.0001	0.0001	0.0000	**[0.0000, 0.0001]**	0.9832
σ_f	0.0000	0.0000	0.0000	**[0.0000, 0.0000]**	0.2377
ω_y	0.0033	0.0033	0.0001	**[0.0031, 0.0034]**	−1.5285
ω_f	0.0045	0.0045	0.0001	**[0.0044, 0.0047]**	0.8035

Note: HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic The figures highlighted in HPDI represent the cases that the 95% credible intervals do not include 0. The posterior means of $\alpha_{y,i}$ and $\alpha_{f,i}$ are reported in Table A2 in Appendix.

Figure 5. The distributions of posterior mean of α_y and α_f.

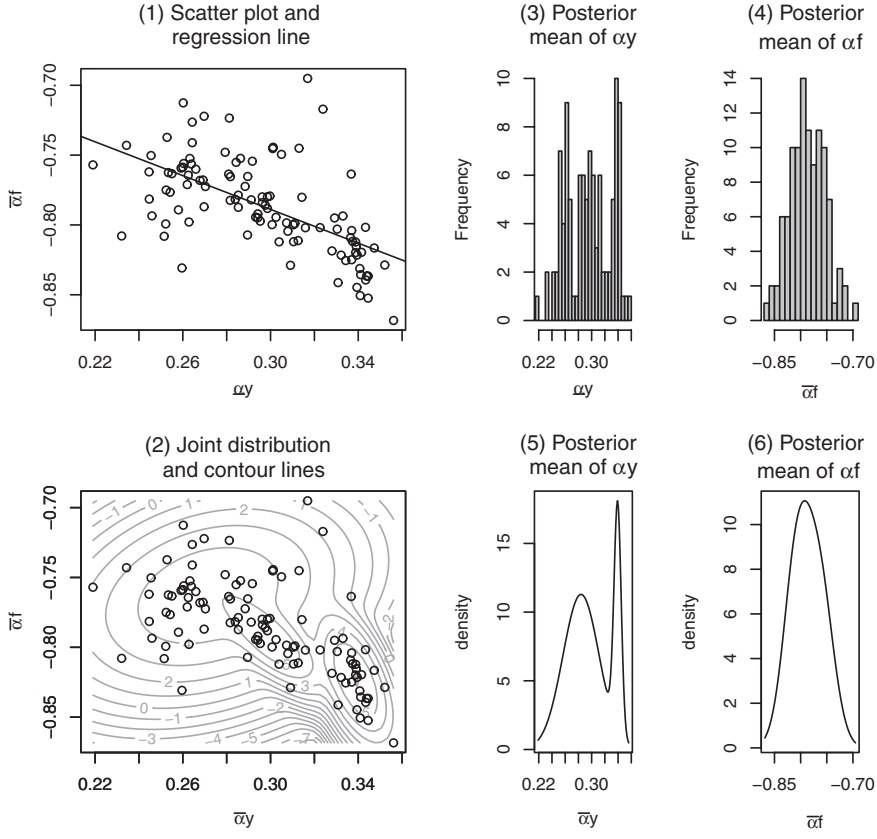

Figure 5 shows the information of the posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$. Figure 5 (1) shows scatter plots and regression line of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$. Figure 5 (2) shows the joint distribution and the contour lines of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$. Figure 5 (3) and (4) shows the histograms of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$, respectively. Figure 5 (5) and (6) shows the univariate distributions of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$, respectively. We determined the number of the clusters in Figure 5 (2), (5), and (6) which is 3, 2, and 1, respectively, by the BIC decision rule, as before. From Figure 5 (1), both variables, $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$, have a negative relationship. From Figure 5 (2), the joint distribution is divided into three clusters. The values of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$ are important factors to determine the convergence destination.

The result, that the joint distribution of $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$ is divided into three clusters, can be taken as a major cause that the number of clusters of joint distribution of GDP per capita and fertility rate has increased in 2010 when compared to 1960. From Figure 5 (5) and (6), the distribution of $\bar{\alpha}_{y,i}$ is clustered into two; however, the distribution of $\bar{\alpha}_{f,i}$ has one peak. It is believed that the distribution of $\bar{\alpha}_{y,i}$, which is clustered into two, is the reason for the previously mentioned that the distribution of fertility rate converges to one peak, but the distribution of GDP per capita does not converge to one peak and the number of peaks has increased. The multiple clusterization of $\bar{\alpha}_{y,i}$ led to the multiple clusterization of convergence destination of the GDP per capita.

4.2.3. Fitness

We have predicted the values of GDP per capita and fertility rate in 2010 from GDP per capita and fertility rate in 1960 using the estimation results. After that, we have compared the prediction values and real data to check how this latent change score model fits with the real data. The panels of Figure 6 show the information of the predicted values. Figure 6 (1) has shown the joint movements for GDP per capita and fertility rate for 50 years, from 1960 to 2010. The red circles are the starting points which are the real data of GDP per capita and fertility rate in 1960. Some developed countries have completed their demographic transition in birth rate, and in developing countries, the demographic transition in birth rate is still in progress. Figure 6 (2) has shown the joint distribution and the contour lines of the predicted values in 2010. The black circles are the predicted values in 2010 which are calculated by the latent change score model. The contour lines are calculated by the bivariate mixture model. Figure 6 (3) and (4) shows the histograms of the predicted values of GDP per capita and fertility rate in 2010, respectively. Figure 6 (5) and (6) shows the distributions of the predicted values of GDP per capita and fertility rate in 2010, respectively. We determined the number of clusters in Figure 6 (2), (5), and (6) which is 3, 3, and 2, respectively, by the BIC decision rule, as before.

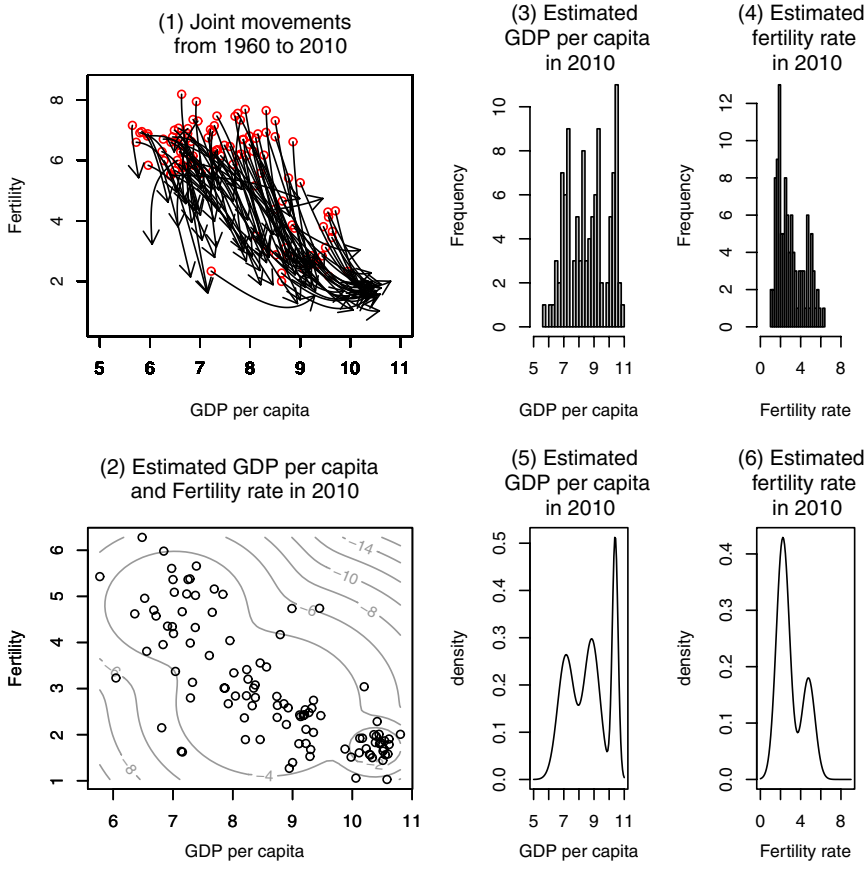

Figure 6. Joint movements for GDP per capita and fertility rate.

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Figure 7. Comparing both distribution of the data and the estimated values.

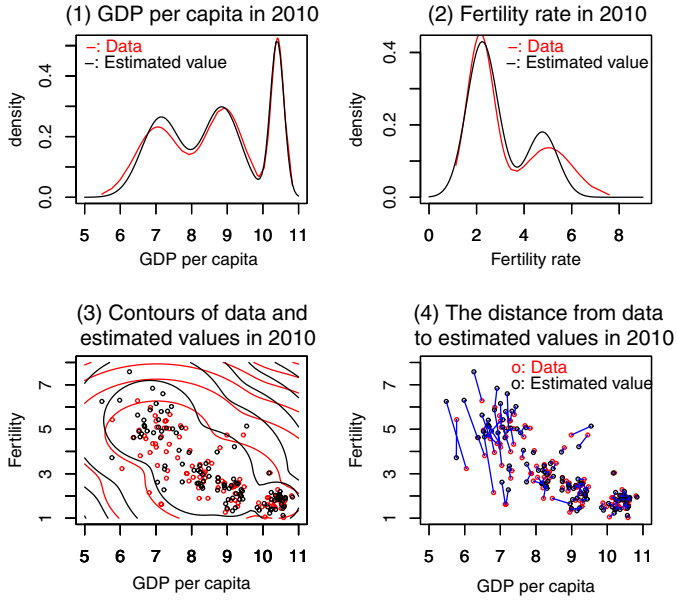


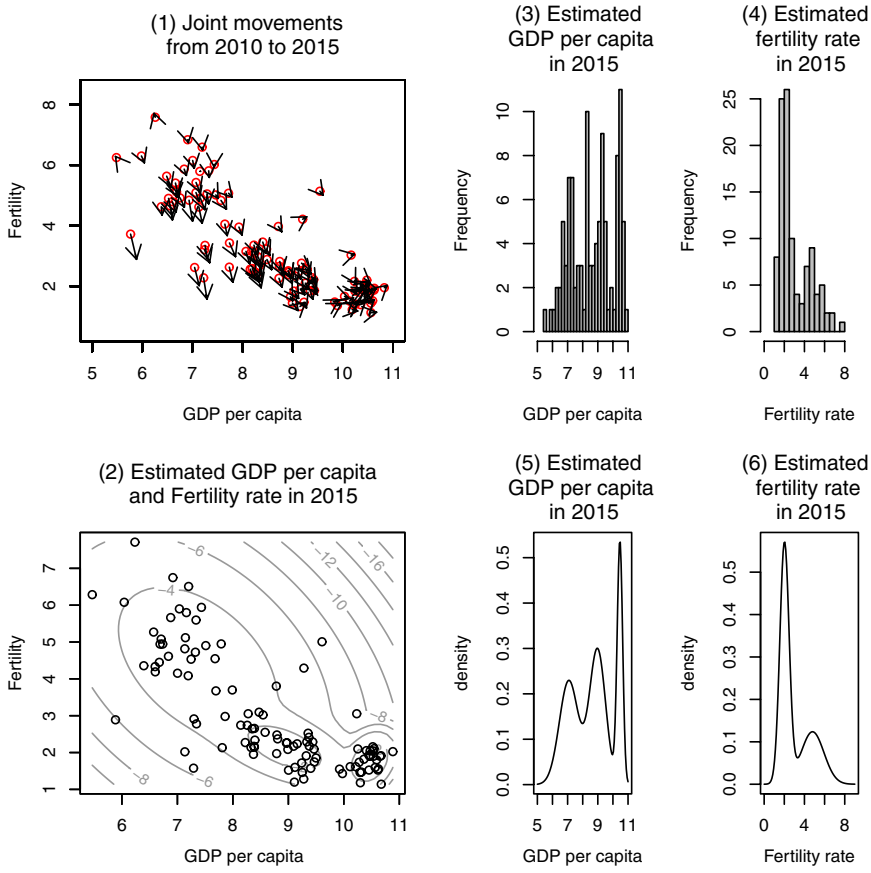
Figure 6 (2) corresponds to Figure 3 (2), and Figure 6 (3) and (4) corresponds to Figure 2 (2) and (4), and Figure 6 (5) and (6) corresponds to Figure 2 (6) and (8).

To compare easily the data and the predicted values by our latent change score model, we have drawn both, the distribution from the data and the distribution from the estimated values, on the same coordinates over one another. Figure 7 (1) shows that Figure 2 (6) overlaid Figure 6 (5). Figure 7 (2) shows that Figure 2 (8) overlaid Figure 6 (6). Figure 7 (3) shows that Figure 3 (2) overlaid Figure 6 (2). Finally, Figure 7 (4) shows the distance from the data to the estimated values in 2010. We have connected both, the data and the estimated values, with lines. If the connected line is short, the estimated value represents real data well. In most of the countries, both the estimated value and the real data are consistent with each other. However, there are some cases in cluster 1 in which the deviation is present. Even though the prediction period of 50 years is quite long, as a whole, the estimated values are quite similar to the real data. This latent score model shows good performance.

4.2.4. Prediction

Because $\alpha_{y,i}$ and $\alpha_{f,i}$ can be considered as constants in the short period time, we have tried to predict the values over the next 5, 10, 15, and 20 years.[15] The panels in Figures 8–11 show joint movements, contour lines, histograms, and distributions which are based on the predicted value over 5 (in 2015), 10 (in 2020), 15 (in 2025), and 20 (in 2030) years, respectively. The predicted values in each year are reported in Table A2 in Appendix . In Figures 8–11, the red circles, which are the starting points, are the real data in 2010. The number of peaks of the distribution of fertility rate in 2030 is one, and the number of peaks of the distribution of GDP per capita is three. As time passes, the distribution of fertility rate converges from two-peaks to one-peak which is the left peak; contrariwise, the distribution of GDP per capita does not converge to one-peak and still remains three-peaks. In Figure 11, as we have seen in the demographic transition, the fertility rate, of the countries whose demographic transition in birth rate is still in progress, shows a sharp drop. However, for countries in the third regime, the fertility rate shows a gradual change again, and there is a lower bound (nonnegative).[16] Therefore, the fertility rate converges to one peak, which means the fertility gap in the cross-country will disappear. The prediction of United Nations about the fertility rate over the next 50 years, that in almost all countries the TFR will be exactly 2.1 by the year 2050, and our result, that the fertility rate converges to one peak, are consistent. On the other hand, in economic growth, conditional convergence can be confirmed worldwide, not absolute convergence. There are many factors which lead to divergence in income like savings, human capital, etc. Countries with similar conditions might

Figure 8. Joint movements for GDP per capita and fertility rate from 2010 to 2015.



converge to the same level of steady state, but countries with different conditions will not automatically converge. Because of the conditional convergence, the number of peak of distribution of GDP per capita will not converge to one and the distribution might have multiple peaks.

4.3. Convergence

4.3.1. Points of convergence

In Figure 3 (2), it has been seen that the joint distribution of GDP per capita and fertility rate in 2010 is divided into four clusters by the bivariate model. This section examines where each cluster converges. The mean of each cluster is used as the representative value of each cluster. The mean of each cluster is calculated as follows: $\alpha_{y,x} = \frac{1}{n_x}\sum_{i\in \text{cluster}x}\alpha_{y,i}$ and $\alpha_{f,x} = \frac{1}{n_x}\sum_{i\in \text{cluster}x}\alpha_{f,i}$, where n_x is the number of element of the cluster x, $(x = 1, 2, 3, 4)$. It is calculated where each cluster of GDP per capita and fertility rate converges, that is where $\alpha_{y,1}$, $\alpha_{y,2}$, $\alpha_{y,3}$, $\alpha_{y,4}$, $\alpha_{f,1}$, $\alpha_{f,2}$, $\alpha_{f,3}$, and $\alpha_{f,4}$ converge. Table 9 reported the mean of GDP per capita and fertility rate of each cluster and the steady state of GDP per capita and fertility rate of each cluster. The GDP per capita and fertility rate at the steady state of each cluster are calculated as Equation (14).

$$y_x^{ss} = \frac{-b_x \pm \sqrt{b_x^2 - 4ac_x}}{2a},\ f_x^{ss} = -\frac{\alpha_{f,x} + \gamma_f y_x^{ss}}{\beta_f + \delta_{fy} y_x^{ss}} \tag{14}$$

where, $a = \delta_{yf}\gamma_f - \beta_y\delta_{fy}$, $b_x = \gamma_y\gamma_f + \alpha_{f,x} - \beta_y\beta_f - \alpha_{y,x}\delta_{fy}$ and $c_x = \gamma_y\alpha_{f,x} - \alpha_{y,x}\beta_f$, $(x = 1, 2, 3, 4)$.

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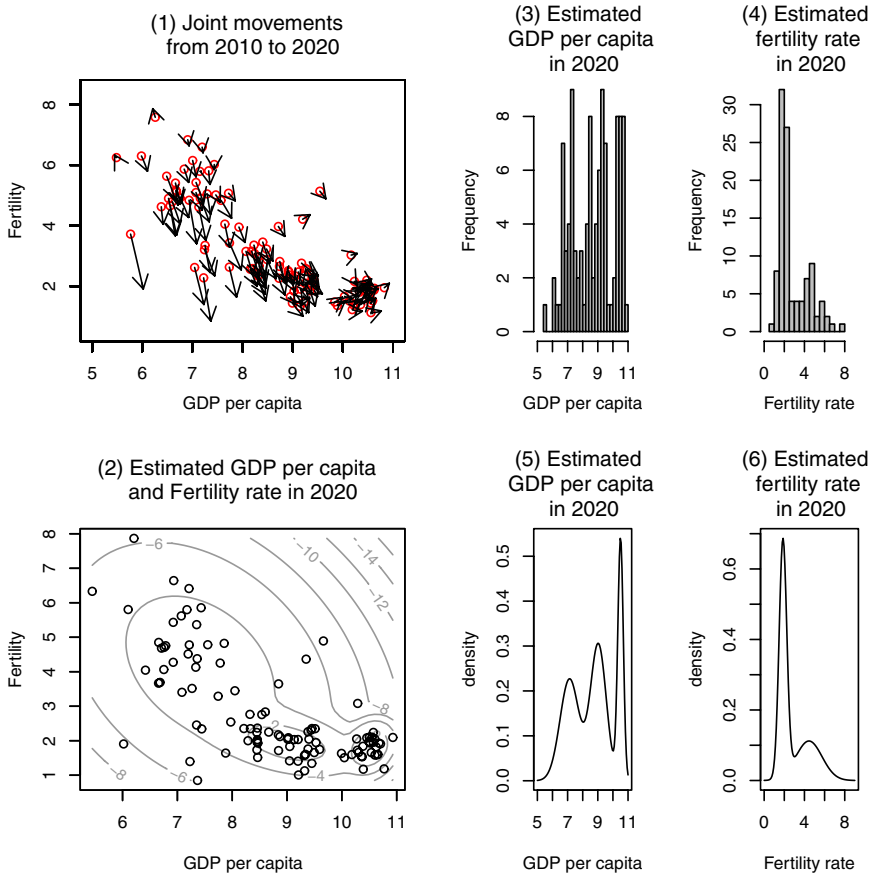
Figure 9. Joint movements for GDP per capita and fertility rate from 2010 to 2020.



In Table 9, the 95% credible intervals for all parameters do not include 0. The sampling was run with a burn-in of 1,000,000 iterations with 2,000,000 iterations. Based on the results of Geweke's convergence diagnostic, it is considered that this sampling has been converged.

For simplification, the means of the posterior distribution of y_x^{ss} and f_x^{ss}, $x = 1, 2, 3, 4$ are used as the representative values of the posterior distribution of y_x^{ss} and f_x^{ss}, $x = 1, 2, 3, 4$. The posterior means of y_x^{ss} and f_x^{ss} are denoted as $\bar{y}_x^{ss}$ and $\bar{f}_x^{ss}$, respectively. Cluster 1 converges at $(\bar{y}_1^{ss}, \bar{f}_1^{ss}) = (8.493, -2.944)$, where the figures in parentheses are the posterior means of y_1^{ss} and f_1^{ss}, and the value of fertility rate at the steady state is negative. Since it is assumed that $\alpha_{y,i}$ and $\alpha_{f,i}$ do not change over time, the unrealistic values have been calculated at the steady state. It can be inferred that the countries that belong to cluster 1 are also going to change their $\alpha_{y,i}$ and $\alpha_{f,i}$ in the future. Clusters 2, 3, and 4 converge to $(\bar{y}_2^{ss}, \bar{f}_2^{ss}) = (9.052, 1.739)$, $(\bar{y}_3^{ss}, \bar{f}_3^{ss}) = (9.690, 0.308)$, and $(\bar{y}_4^{ss}, \bar{f}_4^{ss}) = (10.790, 0.340)$, respectively, where the figures in parentheses are the posterior means of y_x^{ss} and f_x^{ss}, $x = 2, 3, 4$. Comparing the values of GDP per capita and fertility rate at steady state of each cluster, we have gotten $\bar{y}_2^{ss} < \bar{y}_3^{ss} < \bar{y}_4^{ss}$ and $\bar{f}_3^{ss} < \bar{f}_4^{ss} < \bar{f}_2^{ss}$. It is noticed that the fertility rate has been recovering slightly in recent years in developed countries like France and the UK. This may have been reflected in the $\bar{f}_3^{ss} < \bar{f}_4^{ss}$. If the fertility rate converges to a one peak as the prediction of the United Nations, there will be a fall in the birth rate in many developing countries and rise in the birth rate in many developed countries.

4.3.2. Stability at steady state

To examine the stability at the steady state of each cluster, we have used Hessian matrix as Equation (15) and calculated eigenvalues at their steady state $(\bar{y}_x^{ss}, \bar{f}_x^{ss})$, $x = 1, 2, 3, 4$, where $\bar{y}_x^{ss}$ and $\bar{f}_x^{ss}$ are the posterior means of y_x^{ss} and f_x^{ss}.

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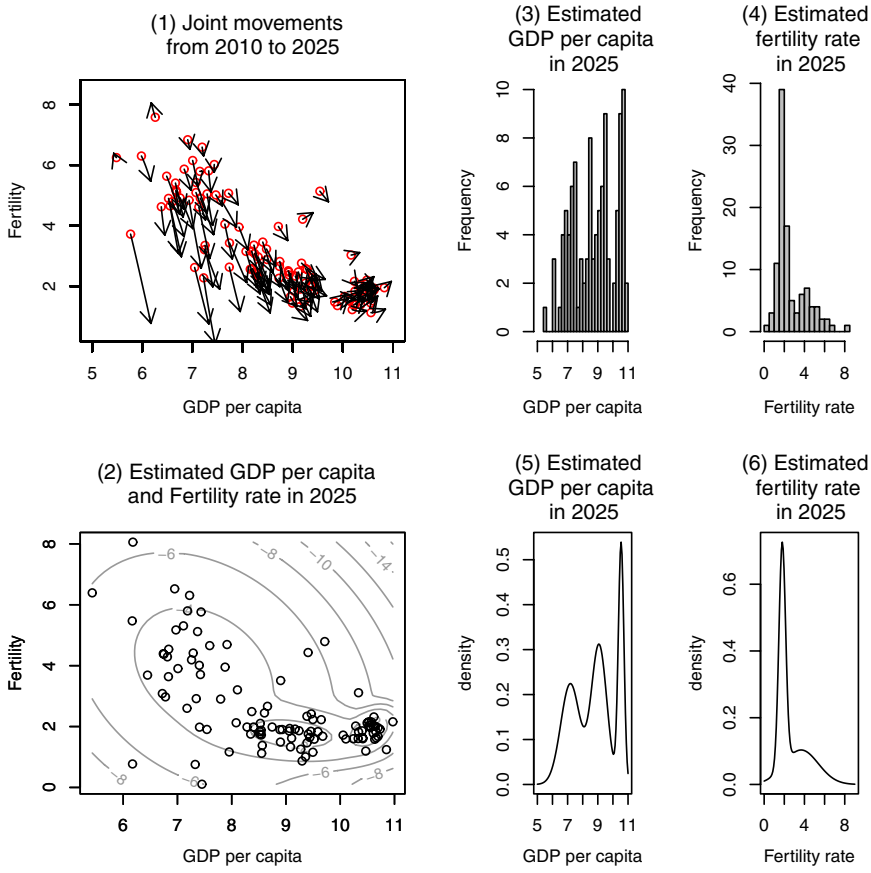
Figure 10. Joint movements for GDP per capita and fertility rate from 2010 to 2025.



$$|D_x| = \begin{vmatrix} \frac{\partial y^l_{i,t}}{\partial y^l_{i,t-1}} & \frac{\partial y^l_{i,t}}{\partial f^l_{i,t-1}} \\ \frac{\partial f^l_{i,t}}{\partial y^l_{i,t-1}} & \frac{\partial f^l_{i,t}}{\partial f^l_{i,t-1}} \end{vmatrix} \text{ at } y_x^{\bar{ss}},\ f_x^{\bar{ss}},\ x = 1,2,3,4. \tag{15}$$

The results are reported in Table 10. The systems have real and complex eigenvalues. Since the real parts are less than one, they converge to their steady state $(y_x^{\bar{ss}}, f_x^{\bar{ss}})$, $x = 1,2,3,4$.

We have presented the results visually in Figure 12 that shows the joint movements of GDP per capita and fertility rate to each steady state. It can be seen that the joint movements have started from the red circles, and have converged to the yellow dot in the end. The each steady state in all of four clusters is stable. The value of fertility rate of cluster 1 in the steady state is negative. This model has assumed α_y and α_f as constant, not variable. It can be considered that the negative fertility rate came from the assumption. Cohen (1992) also has mentioned such points. Cohen (1992) stressed that to estimate the convergence under the assumption, that conditional variable is a constant, is not sufficient for a dynamic model. The results obtained in this research are applicable to short- and medium-term predictions, where the conditional variables do not change. However, the results may be insufficient for long-term prediction where the conditional variables change.

5. Conclusion

This research has analyzed the changes in the distribution of fertility rate and GDP per capita using the cross-section data from 1960 to 2010. Especially, the mutual changes in the two variables, using the latent change score model and the bivariate mixture model, have been analyzed. There are many studies which have analyzed each of the changes in fertility distribution and in GDP per capita distribution, but few studies have analyzed the changes in the distribution of both variables at the

Figure 11. Joint movements for GDP per capita and fertility rate from 2010 to 2030.

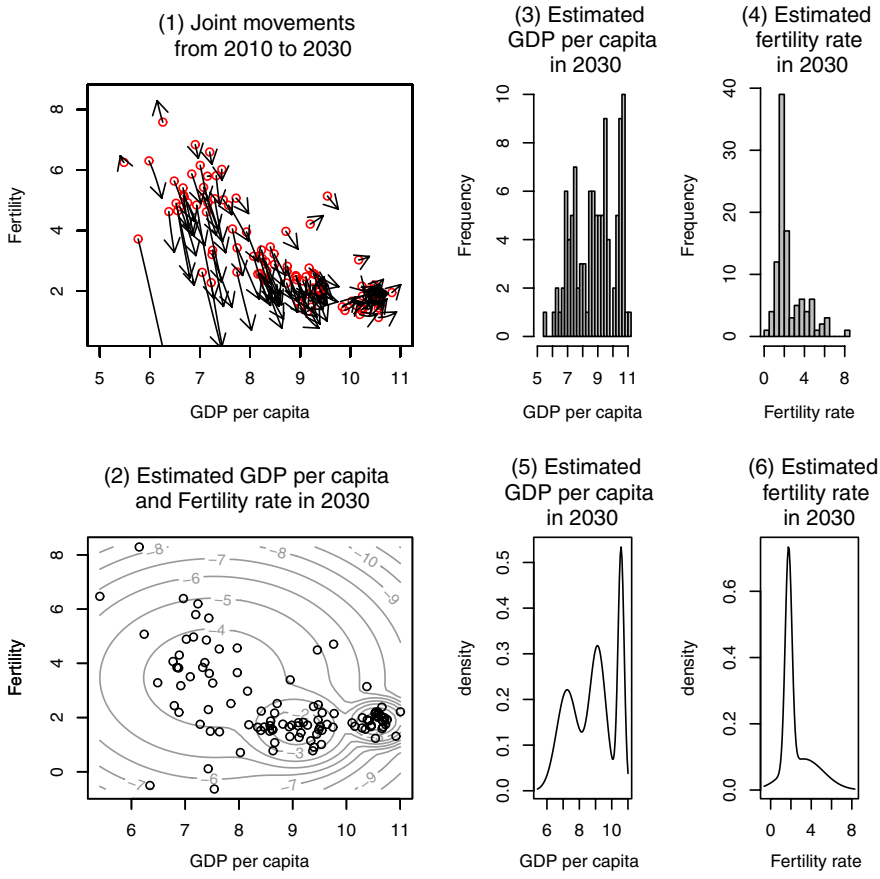


Table 9. Estimation results on the convergence

Parameters	Mean	Median	Standard deviation	95% HPDI [2.5%, 97.5%]	Geweke's CD
$\alpha_{y,1}$	0.2571	0.2571	0.0300	**[0.1983, 0.3159]**	1.5575
$\alpha_{y,2}$	0.2873	0.2873	0.0298	**[0.2288, 0.3460]**	1.6149
$\alpha_{y,3}$	0.3079	0.3079	0.0307	**[0.2476, 0.3683]**	1.6662
$\alpha_{y,4}$	0.3399	0.3398	0.0336	**[0.2738, 0.4060]**	1.7189
$\alpha_{f,1}$	−0.7638	−0.7637	0.0353	**[−0.8330, −0.6947]**	0.4223
$\alpha_{f,2}$	−0.7709	−0.7707	0.0351	**[−0.8400, −0.7022]**	0.4746
$\alpha_{f,3}$	−0.7913	−0.7910	0.0362	**[−0.8624, −0.7205]**	0.4420
$\alpha_{f,4}$	−0.8259	−0.8257	0.0396	**[−0.9036, −0.7486]**	0.3706
y_1^{ss}	8.4930	8.4940	0.0898	**[8.3160, 8.6680]**	−0.1880
y_2^{ss}	9.0520	9.0510	0.1015	**[8.8550, 9.2550]**	−0.6244
y_3^{ss}	9.6900	9.6850	0.1111	**[9.4860, 9.9220]**	−0.8856
y_4^{ss}	10.7900	10.7800	0.1109	**[10.6100, 11.0400]**	-0.9028
f_1^{ss}	−2.9440	−2.6890	1.5560	**[−6.6680, −0.7537]**	1.0197
f_2^{ss}	0.9151	0.9546	0.4345	[−0.0529, 1.6550]	0.2132
f_3^{ss}	1.7390	1.7450	0.2378	**[1.2450, 2.1870]**	−0.3848
f_4^{ss}	2.2810	2.2740	0.1222	**[2.0600, 2.5390]**	−1.0874

Note: HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic. The highlighted gures on HPDI represent the cases that the 95% credible intervals do not include 0. $\alpha_{y,x} = \sum_{i\in \text{cluster } x} \alpha_{y,i} \div$ number of cluster x and $\alpha_{f,x} = \sum_{i\in \text{cluster } x} \alpha_{f,i} \div$ number of cluster x. y_x^{ss} and f_x^{ss} mean the values of GDP per capita and fertility rate at steady state in cluster x, respectively. $\bar{\alpha}_{y,i}$ and $\bar{\alpha}_{f,i}$ are reported in Table A2 in Appendix.

Table 10. Eigenvalues of Hessian matrix at each steady state

cluster	1	2	3	4
eigenvalues	0.9780 + 0.0179*i*	0.9755 + 0.0130*i*	0.9704 + 0.0103*i*	0.9712
	0.9780 – 0.0179*i*	0.9755 – 0.0130*i*	0.9704 – 0.0103*i*	0.9504

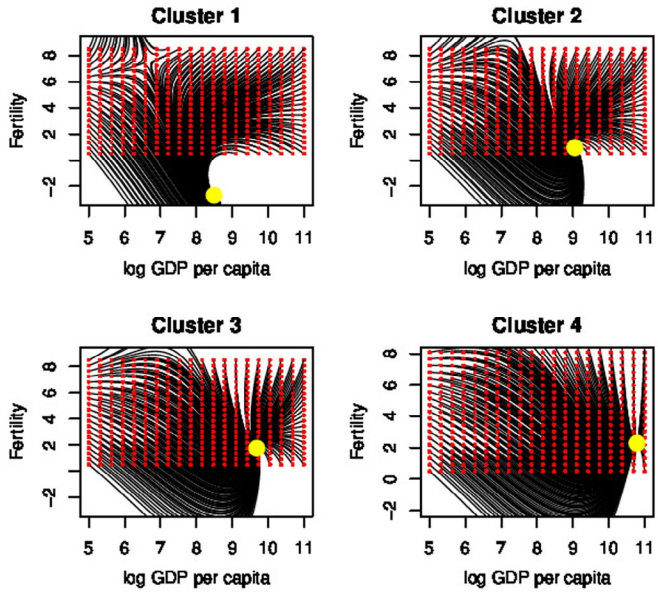


Figure 12. Convergence of the joint movements of each cluster to their steady state.

same time. This is a contribution of this study. The analysis of mutual changes made the change, convergence, and prediction of distributions of the two variables explainable.

The results obtained by the bivariate mixture model in this research are consistent with the results of the previous studies obtained by univariate mixture model, e.g. Holzmann et al. (2007) for GDP per capita and Strulik and Vollmer (2010) for fertility rate as well as the population projections of the United Nations.

The number of peaks of the distribution of GDP per capita has increased from the 1960s to 2010. However, the number of peaks of fertility rate distribution has not changed in two until 2010, but there is a tendency that the right peak is getting smaller and the distribution will converge to the left peak. Additionally, we have calculated the joint movements of GDP per capita and fertility rate using the latent change score model. We have predicted the changes in the distributions of the two variables up to 2030 as follows: there will be no change in the distribution of GDP per capita which will still have three peaks in 2030; however, the distribution of fertility rate will converge to one peak, which means the per capita income will be applied to the conditional convergence and the fertility rate will be applied to the absolute convergence. It can be concluded that the fertility gap among cross-countries will disappear, but the income gap will not. Even though the population conditions in developing countries will be improved, income inequality in cross-country may not be improved after all.

The parameters in the latent change score model used in this paper have been estimated as constant. The value of the parameters may change over time. Introducing time in the estimation equation remains to be seen in our future research.

Supplemental material
Supplemental material for this article can be accessed here http://dx.doi.org/10.1080/23322039.2015.1119367

Acknowledgements
I would like to thank the editor and anonymous referees for their very useful comments and suggestions.

Funding
The author received no direct funding for this research.

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Citation information
Cite this article as: Change and prediction of income and fertility rates across countries, Inyong Shin, *Cogent Economics & Finance* (2016), 4: 1119367.

Notes

1. The three types in Thompson (1929), Landry (1934), and Notestein (1945) are closely parallel to each other. Kirk (1996), Weber (2010), and Galor (2011) surveyed on the demographic transition in detail.
2. Some researches (e.g. Doepke (2005), Murphy (2009), Fernández-Villaverde (2001), etc.) report that an increase in the income increases fertility.
3. This research, I believe, is the first one to consider and analyze the joint distribution of both variables at the same time using a bivariate mixture distribution model and a latent change score model in the framework of demographic transition.
4. In the data, there are unusual countries which are birth control countries, oil-producing countries, and negative growth countries. We have also analyzed the data, exclusive of 10 unusual countries. These countries are China (20), which carries out one-child policy, Iran (51) and Venezuela (104), which are two major oil-producing countries, and Central African Republic (17), Congo Dem. Rep. (23), Guinea (43), Haiti (45), Madagascar (61), Nicaragua (74), and Niger (75), which were poorer in 2010 than in 1960. These countries have lower GDP per capita in 2010 when compared to GDP per capita in 1960. The figures in parentheses are country numbers in Tables A1 and A2.
5. Many previous studies depict the demographic transition due to time flowing; however, Figure 1 is depicted by per capita income level. The horizontal axis shows GDP per capita, instead of time flowing.
6. The investigation of the results using other distributions instead of the normal distribution is left for further study.
7. In this analysis, the three-peaks start to appear from 1972.
8. In this analysis, the left peak starts to be higher than the right peak from 1990.
9. The names of classification in this research—low-income, lower middle-income, upper middle-income and high-income—are just for the sake of convenience. They differ from the classification of World Bank which is defined according to the GNI per capita, calculated using the World Bank Atlas method.
10. With the amendments in individual effects, this research assumes the distributions of priors for $\alpha_{y,i}$ and $\alpha_{f,i}$ as follows: $\alpha_{y,i} \sim N(m, s^2)$ and $\alpha_{f,i} \sim N(m, s^2)$ where $i = 1, \cdots, 106$, $m = 0$ and $s^2 = 10,000$.
11. We post the estimation results using 96 countries in Table A3 in Appendix . Excluding these 10 countries has no significant effect on the results.
12. Credible interval estimates the probability of being in that interval, but confidence interval does not predict that the true value of the parameter has a particular probability.
13. The 0.0000s in Table 7 mean very small positive numbers, not exact 0, because we have rounded to four decimal places.
14. We post the estimation results using 96 countries in Table A4 in Appendix. Except that the 95% credible interval of δ_{yf} includes 0, the results where we have used 96 countries are not so different from the results where we have used 106 countries. Excluding these 10 countries has no significant effect on the results.
15. Predicting $\alpha_{y,i}$ and $\alpha_{f,i}$ over 20 years, the fertility rates of Bangladesh (5) and Zimbabwe (106) are negative values which are unrealistic values. So, predicting over more years has been stopped.
16. It is considered that the result, the arrows at the lower right corner are slightly upward, is due to the recent rising trend in the birth rates in some developed countries, e.g. Sweden, the UK, Spain, Italy, and Finland.

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Appendix

Table A1 shows the information of the cross-country data used in this research. Table A1 includes the GDP per capita, fertility rate, and classification, i.e. which country is classified in what group. There are two and four clusters in 1960 and 2010, respectively.

Table A2 reports the posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$, the data in 2010, and the predicted values in 2010, 2015, 2020, 2025, and 2030 which are estimated using the latent change score model. Due to the limitations of the space, the reports on 95% HPDI and Geweke's CD of $\alpha_{y,i}$ and $\alpha_{f,i}$ are cut. All of the $\alpha_{y,i}$ and $\alpha_{f,i}(i = 1, \cdots, 106)$ fulfill the Geweke's convergence diagnostic; consequently, we can consider that this sampling has been converged.

Tables A3 and A4 are the estimation results of the pooling model and the fixed effect model, respectively, using 96 countries not including 10 unusual countries which are: one birth control country, two oil-producing countries, and seven negative growth countries. Excluding these 10 countries has no significant effect on the results.

Figure A1 shows the animations which have been obtained using the real data from 1960 to 2010. The left-upper panel shows the joint movements of GDP per capita and fertility rate, the right-upper panel shows the bivariate distribution of the GDP per capita and the fertility rate, the left-lower panel shows the distribution of GDP per capita, and the right-lower panel shows the distribution of the fertility rate. Due to the limitation of the space in context, we have shown the panels for only two years, 1960 which is the first year and 2010 which is the last year in Figure 2 and Figure 3. By playing the animations, readers can observe changes in the data of each year starting from 1960 to 2010.

Figure A2 shows the animations which have been obtained using the results of the latent change score model. Figure A2 also shows the changes from 1960 to 2010 as Figure A1. Even though both periods in Figures A1 and A2 are same, they are different in that Figure A1 came from the real data and Figure A2 came from the estimated theoretical values. Due to the limitation of the space in context, we have shown the panels only for 2010 in Figure 6. By playing the animations, readers can observe changes in the estimated values of each year starting from 1960 to 2010.

Lastly, Figure A3 shows the animations which have obtained by the prediction using the latent change score model. Due to the limitation of the space in context, we have shown the panels for every fifth year starting from 2015 up to 2030 in Figures 8–11, respectively. By playing the animations, readers can observe changes in the predicted values of each year starting from 2010 to 2030.

Readers with an interest in playing the animations can also click on the following link paper anime.pdf.

Table A1. GDP per capita, fertility rate, and group

No	Country name	GDP per capita		Fertility rate		Group	
		1960	2010	1960	2010	1960	2010
1	Algeria	4105.0	6263.3	7.65	2.82	1	2
2	Argentina	6043.2	12340.3	3.11	2.22	2	3
3	Australia	15255.0	41113.6	3.45	1.87	2	4
4	Austria	10536.6	38585.5	2.69	1.44	2	4
5	Bangladesh	847.4	1371.0	6.73	2.28	1	2
6	Barbados	16363.2	28088.5	4.33	1.84	2	4
7	Belgium	10131.8	35556.6	2.58	1.84	2	4
8	Benin	745.3	1176.9	6.28	5.10	1	1
9	Bolivia	2616.4	3743.8	6.70	3.36	1	2
10	Botswana	674.1	9675.4	6.62	2.76	1	3
11	Brazil	2483.5	8324.4	6.21	1.84	1	3
12	Burkina Faso	512.5	929.9	6.29	5.87	1	1
13	Burundi	343.3	396.2	6.95	6.30	1	1
14	Cameroon	1415.8	1748.1	5.65	5.02	1	1
15	Canada	12869.3	37103.6	3.81	1.63	2	4
16	Cape Verde	856.2	3916.6	6.89	2.43	1	2
17	Central African Republic	967.7	588.8	5.84	4.63	1	1
18	Chad	793.6	1330.6	6.25	6.60	1	1
19	Chile	3687.3	12524.8	5.58	1.86	1	3
20	China	771.7	7746.1	5.76	1.65	1	3
21	Colombia	2937.6	7536.4	6.81	2.38	1	3
22	Comoros	740.8	856.2	6.79	4.92	1	1
23	Congo Dem. Rep.	696.5	240.5	6.00	6.25	1	1
24	Congo Republic of	996.7	2253.8	5.88	5.07	1	1
25	Costa Rica	4920.5	11500.1	7.31	1.85	1	3
26	Cote d Ivorie	953.5	1283.7	7.35	4.91	1	1
27	Cyprus	3357.4	18755.7	3.50	1.48	2	4
28	Denmark	11581.6	33705.0	2.57	1.87	2	4
29	Dominican Republic	2327.5	10502.9	7.56	2.58	1	3
30	Ecuador	2581.7	6226.8	6.69	2.66	1	2
31	Egypt	919.8	4853.8	6.63	2.88	1	2
32	El Salvador	3338.0	6168.6	6.73	2.26	1	3
33	Equatorial Guinea	610.7	13958.3	5.51	5.14	1	2
34	Ethiopia	386.5	680.4	6.88	4.90	1	1
35	Fiji	1994.5	4151.7	6.46	2.67	1	2
36	Finland	9057.8	32988.8	2.71	1.87	2	4
37	France	10212.2	31299.3	2.85	2.03	2	4
38	Gabon	4878.0	9895.9	4.38	4.21	2	2
39	Gambia	1123.6	1271.5	5.57	5.80	1	1
40	Ghana	1286.1	2094.3	6.75	4.05	1	2

(*Continued*)

Table A1. (*Continued*)

No	Country name	GDP per capita		Fertility rate		Group	
		1960	2010	1960	2010	1960	2010
41	Greece	5587.5	25216.4	2.28	1.51	2	4
42	Guatemala	2930.0	6091.2	6.53	3.97	1	2
43	Guinea	914.0	787.7	6.10	5.17	1	1
44	Guinea-Bissau	784.0	798.4	5.83	5.12	1	1
45	Haiti	1513.5	1410.0	6.32	3.35	1	2
46	Honduras	2203.0	3579.6	7.46	3.15	1	2
47	Hong Kong	3280.4	38684.7	5.16	1.13	1	4
48	Iceland	14075.4	35612.1	4.29	2.20	2	4
49	India	720.4	3477.3	5.87	2.56	1	2
50	Indonesia	665.1	3966.0	5.67	2.43	1	2
51	Iran	4092.4	9432.1	6.93	1.90	1	3
52	Ireland	7223.1	34876.7	3.76	2.07	2	4
53	Israel	6894.4	26034.6	3.87	3.03	2	2
54	Italy	8744.5	28377.5	2.41	1.41	2	4
55	Jamaica	6417.0	8538.6	5.42	2.33	1	3
56	Japan	5586.3	31447.2	2.00	1.39	2	4
57	Jordan	2700.2	4462.9	7.69	3.46	1	2
58	Kenya	1020.0	1246.8	7.95	4.62	1	1
59	Korea Republic of	1655.8	26609.1	6.16	1.23	1	4
60	Lesotho	389.5	1394.7	5.84	3.21	1	2
61	Madagascar	1051.4	702.6	7.30	4.65	1	1
62	Malawi	330.2	655.6	6.91	5.64	1	1
63	Malaysia	1433.8	11956.1	6.19	2.00	1	3
64	Mali	527.1	998.0	6.70	6.84	1	1
65	Mauritania	634.3	1938.6	6.78	4.84	1	1
66	Mauritius	2305.8	10164.1	6.17	1.47	1	3
67	Mexico	4914.3	11939.4	6.78	2.28	1	3
68	Morocco	714.5	3622.4	7.07	2.58	1	2
69	Mozambique	306.9	781.3	6.60	5.41	1	1
70	Namibia	2754.1	4810.4	6.15	3.23	1	2
71	Nepal	534.4	1145.2	5.99	2.62	1	2
72	Netherlands	13437.8	38190.6	3.12	1.79	2	4
73	New Zealand	14263.7	27789.6	4.13	2.16	2	4
74	Nicaragua	2456.9	2289.8	7.34	2.63	1	2
75	Niger	860.9	522.0	7.05	7.58	1	1
76	Nigeria	1552.0	1695.5	6.35	6.02	1	1
77	Norway	12523.3	50487.5	2.85	1.95	2	4
78	Pakistan	613.6	2297.1	6.60	3.43	1	2
79	Panama	2120.3	10857.1	5.87	2.55	1	3
80	Papua New Guinea	1447.9	2774.5	6.28	3.95	1	2
81	Paraguay	1782.6	4069.8	6.50	2.97	1	2
82	Peru	3480.9	7415.0	6.88	2.51	1	3
83	Philippines	1459.0	3193.9	7.15	3.15	1	2

(*Continued*)

Table A1. (*Continued*)

No	Country name	GDP per capita		Fertility rate		Group	
		1960	2010	1960	2010	1960	2010
84	Portugal	4154.0	19782.4	3.01	1.36	2	4
85	Puerto	5665.9	22818.5	4.66	1.66	2	4
86	Romania	1371.4	9377.6	2.34	1.33	2	3
87	Rwanda	760.0	1025.2	8.19	4.84	1	1
88	Senegal	1405.5	1469.3	6.95	5.05	1	1
89	South Africa	3932.5	7513.2	6.17	2.47	1	3
90	Spain	6316.1	27331.5	2.86	1.38	2	4
91	Sri Lanka	609.5	4063.4	5.54	2.34	1	2
92	Sweden	14314.1	36132.4	2.17	1.98	2	4
93	Switzerland	21005.3	39978.0	2.34	1.52	2	4
94	Syria	1536.7	3792.7	7.47	3.08	1	2
95	Tanzania	383.6	1178.5	6.81	5.43	1	1
96	Thailand	954.1	8064.7	6.15	1.44	1	3
97	Togo	713.6	732.9	6.52	4.79	1	1
98	Trinidad & Tobago	8105.8	30749.3	5.26	1.80	1	4
99	Turkey	3176.6	10438.0	6.30	2.10	1	3
100	Uganda	657.4	1101.7	7.00	6.16	1	1
101	United Kingdom	11204.0	34268.0	2.69	1.98	2	4
102	United States	15397.8	41365.0	3.65	1.93	2	4
103	Uruguay	4988.0	11717.7	2.88	2.08	2	3
104	Venezuela	7015.6	9070.6	6.62	2.47	1	3
105	Zambia	1351.0	1517.2	7.02	5.81	1	1
106	Zimbabwe	284.5	319.0	7.16	3.72	1	1

Table A2. The posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$, the data in 2010, and the predicted values in 2010, 2015, 2020, 2025, and 2030

No	Country name	Posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$		Data		Predicted values									
				2010		2010		2015		2020		2025		2030	
		$\tilde{\alpha}_{y,i}$	$\tilde{\alpha}_{f,i}$	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate
1	Algeria	0.294	−0.795	8.742	2.817	8.737	2.830	8.793	2.478	8.842	2.172	8.889	1.899	8.934	1.658
2	Argentina	0.305	−0.750	9.421	2.215	9.322	2.575	9.439	2.287	9.454	2.354	9.466	2.415	9.476	2.472
3	Australia	0.341	−0.831	10.624	1.870	10.540	1.866	10.652	1.900	10.676	1.931	10.697	1.960	10.715	1.987
4	Austria	0.344	−0.837	10.561	1.440	10.553	1.563	10.613	1.524	10.658	1.603	10.696	1.677	10.730	1.743
5	Bangla-desh	0.252	−0.799	7.223	2.277	7.157	1.623	7.291	1.576	7.367	0.849	7.451	0.107	7.542	-0.638
6	Barbados	0.331	−0.841	10.243	1.839	10.287	1.572	10.276	1.736	10.305	1.660	10.331	1.606	10.353	1.569
7	Belgium	0.342	−0.820	10.479	1.840	10.477	1.829	10.530	1.897	10.574	1.954	10.613	2.008	10.645	2.059
8	Benin	0.264	−0.756	7.071	5.095	7.289	3.988	7.133	4.816	7.196	4.514	7.260	4.191	7.325	3.849
9	Bolivia	0.282	−0.765	8.228	3.357	8.233	3.412	8.276	3.054	8.323	2.764	8.370	2.492	8.415	2.237
10	Botswana	0.333	−0.794	9.177	2.761	9.350	2.746	9.355	2.525	9.512	2.348	9.650	2.226	9.771	2.148
11	Brazil	0.308	−0.805	9.027	1.838	9.106	1.803	9.118	1.598	9.200	1.406	9.272	1.256	9.336	1.143
12	Burkina Faso	0.259	−0.759	6.835	5.869	7.008	4.193	6.879	5.663	6.924	5.432	6.972	5.174	7.021	4.890
13	Burundi	0.246	−0.794	5.982	6.304	6.360	4.621	6.041	6.079	6.102	5.803	6.167	5.471	6.237	5.074
14	Camer-oon	0.270	−0.722	7.466	5.017	7.657	4.654	7.511	4.903	7.554	4.783	7.594	4.658	7.633	4.529
15	Canada	0.341	−0.851	10.521	1.627	10.511	1.441	10.565	1.596	10.603	1.583	10.635	1.582	10.663	1.589
16	Cape Verde	0.290	−0.807	8.273	2.427	8.211	1.892	8.371	1.951	8.464	1.514	8.552	1.121	8.634	0.774
17	Central African Republic	0.234	-0.743	6.378	4.626	6.560	3.809	6.394	4.356	6.418	4.046	6.449	3.689	6.488	3.284
18	Chad	0.264	−0.741	7.193	6.596	7.230	5.053	7.202	6.508	7.212	6.412	7.223	6.308	7.236	6.194
19	Chile	0.307	−0.799	9.435	1.858	9.224	1.809	9.467	1.746	9.496	1.656	9.521	1.584	9.544	1.529
20	China	0.296	−0.780	8.955	1.650	8.458	1.894	9.003	1.521	9.046	1.416	9.084	1.332	9.118	1.267
21	Colombia	0.301	−0.800	8.927	2.376	8.897	2.221	8.991	2.085	9.050	1.833	9.104	1.618	9.153	1.438
22	Comoros	0.260	−0.759	6.753	4.919	7.000	5.365	6.839	4.613	6.924	4.274	7.009	3.904	7.094	3.506
23	Congo Dem. Rep.	0.219	−0.757	5.483	6.251	6.045	3.230	5.465	6.286	5.447	6.332	5.429	6.392	5.411	6.471
24	Congo Republic of	0.281	−0.724	7.720	5.072	7.835	5.045	7.790	4.949	7.854	4.823	7.913	4.694	7.968	4.565
25	Costa Rica	0.309	−0.829	9.350	1.847	9.303	1.678	9.403	1.572	9.451	1.343	9.495	1.157	9.535	1.007
26	Cote d Ivoire	0.270	−0.773	7.157	4.910	7.376	5.022	7.246	4.533	7.335	4.131	7.424	3.708	7.512	3.271
27	Cyprus	0.329	−0.795	9.839	1.478	9.877	1.687	9.922	1.550	9.993	1.632	10.053	1.719	10.105	1.805
28	Denmark	0.340	−0.822	10.425	1.870	10.455	1.805	10.475	1.899	10.517	1.933	10.553	1.969	10.585	2.006
29	Do-minican Republic	0.313	−0.811	9.259	2.584	9.119	2.428	9.332	2.290	9.399	2.045	9.459	1.843	9.513	1.681
30	Ecuador	0.298	−0.786	8.737	2.655	8.746	2.636	8.806	2.371	8.871	2.120	8.930	1.903	8.985	1.718

(*Continued*)

Table A2. (*Continued*)

No	Country name	Posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$		Data		Predicted values									
				2010		2010		2015		2020		2025		2030	
		$\bar{\alpha}_{y,i}$	$\bar{\alpha}_{f,i}$	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate
31	Egypt	0.296	−0.784	8.488	2.883	8.378	2.805	8.580	2.549	8.666	2.250	8.745	1.986	8.818	1.757
32	El Salvador	0.294	−0.792	8.727	2.263	8.743	2.374	8.788	1.971	8.845	1.714	8.898	1.490	8.947	1.299
33	Equatorial Guinea	0.324	−0.717	9.544	5.140	8.989	4.737	9.607	5.007	9.663	4.892	9.715	4.792	9.761	4.706
34	Ethiopia	0.252	−0.808	6.523	4.902	6.811	2.149	6.599	4.333	6.684	3.690	6.779	2.975	6.884	2.192
35	Fiji	0.285	−0.779	8.331	2.670	8.325	2.627	8.396	2.338	8.458	2.030	8.517	1.750	8.573	1.498
36	Finland	0.339	−0.815	10.404	1.870	10.385	1.829	10.454	1.922	10.497	1.975	10.534	2.027	10.566	2.075
37	France	0.339	−0.812	10.351	2.030	10.399	1.967	10.406	2.053	10.454	2.083	10.495	2.117	10.531	2.153
38	Gabon	0.317	-0.695	9.200	4.214	9.452	4.741	9.279	4.292	9.348	4.365	9.408	4.432	9.460	4.493
39	Gambia	0.260	−0.713	7.148	5.796	7.251	5.365	7.162	5.798	7.175	5.799	7.186	5.799	7.196	5.799
40	Ghana	0.269	−0.768	7.647	4.052	7.608	3.717	7.694	3.674	7.743	3.290	7.796	2.903	7.850	2.517
41	Greece	0.337	−0.804	10.135	1.510	10.232	1.695	10.214	1.616	10.281	1.723	10.338	1.826	10.387	1.922
42	Guatemala	0.301	−0.744	8.715	3.974	8.794	4.170	8.783	3.802	8.845	3.648	8.902	3.511	8.954	3.391
43	Guinea	0.246	−0.750	6.669	5.174	6.833	3.954	6.687	4.944	6.710	4.684	6.739	4.393	6.773	4.068
44	Guinea-Bissau	0.253	−0.737	6.683	5.115	6.989	4.343	6.735	4.945	6.787	4.753	6.838	4.540	6.890	4.304
45	Haiti	0.255	−0.763	7.251	3.350	7.326	3.135	7.298	2.913	7.350	2.455	7.407	1.981	7.468	1.495
46	Honduras	0.285	−0.787	8.183	3.154	8.255	3.204	8.258	2.740	8.332	2.349	8.403	1.984	8.470	1.650
47	Hong Kong	0.356	−0.868	10.563	1.127	10.580	1.028	10.674	1.141	10.770	1.183	10.851	1.242	10.922	1.311
48	Iceland	0.345	−0.837	10.480	2.200	10.614	1.905	10.545	2.107	10.601	2.047	10.649	2.011	10.691	1.994
49	India	0.281	−0.764	8.154	2.563	7.925	2.671	8.223	2.271	8.288	2.000	8.350	1.751	8.407	1.526
50	Indonesia	0.290	−0.765	8.286	2.434	8.194	2.364	8.380	2.168	8.467	1.931	8.547	1.726	8.618	1.550
51	Iran	0.311	−0.799	9.152	1.904	9.273	2.479	9.240	1.717	9.318	1.572	9.387	1.464	9.448	1.388
52	Ireland	0.343	−0.802	10.460	2.070	10.416	2.285	10.521	2.158	10.574	2.241	10.620	2.317	10.659	2.385
53	Israel	0.337	−0.764	10.167	3.030	10.198	3.040	10.232	3.053	10.288	3.081	10.337	3.112	10.380	3.143
54	Italy	0.337	−0.825	10.253	1.410	10.335	1.500	10.316	1.469	10.370	1.534	10.416	1.599	10.456	1.663
55	Jamaica	0.300	−0.779	9.052	2.334	9.133	2.393	9.094	2.171	9.133	2.033	9.167	1.917	9.199	1.821
56	Japan	0.347	−0.817	10.356	1.390	10.514	1.643	10.453	1.525	10.537	1.657	10.608	1.783	10.669	1.898
57	Jordan	0.291	−0.782	8.404	3.458	8.458	3.555	8.473	3.095	8.540	2.757	8.603	2.447	8.664	2.165
58	Kenya	0.263	−0.798	7.128	4.616	7.154	4.668	7.195	4.084	7.267	3.516	7.346	2.917	7.430	2.294
59	Korea Republic of	0.345	−0.852	10.189	1.226	10.059	1.056	10.297	1.180	10.391	1.173	10.471	1.194	10.540	1.234
60	Lesotho	0.266	−0.760	7.240	3.207	7.292	2.797	7.344	2.779	7.444	2.342	7.543	1.905	7.638	1.474
61	Madagascar	0.245	−0.782	6.555	4.654	6.679	4.702	6.601	4.191	6.657	3.668	6.722	3.085	6.797	2.440
62	Malawi	0.258	−0.789	6.486	5.636	6.716	4.575	6.571	5.270	6.660	4.850	6.751	4.375	6.846	3.847
63	Malaysia	0.323	−0.802	9.389	2.002	9.350	2.048	9.498	1.853	9.594	1.748	9.677	1.681	9.750	1.645

(*Continued*)

Table A2. (*Continued*)

No	Country name	Posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$		Data		Predicted values									
				2010		2010		2015		2020		2025		2030	
		$\bar{\alpha}_{y,i}$	$\bar{\alpha}_{f,i}$	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate
64	Mali	0.260	−0.756	6.906	6.841	6.845	5.982	6.919	6.748	6.933	6.643	6.949	6.524	6.967	6.391
65	Mauritania	0.284	−0.755	7.570	4.837	7.691	5.160	7.678	4.547	7.781	4.251	7.878	3.953	7.969	3.658
66	Mauritius	0.304	−0.812	9.227	1.470	9.000	1.394	9.276	1.282	9.320	1.128	9.360	1.006	9.396	0.910
67	Mexico	0.316	−0.802	9.388	2.281	9.472	2.413	9.462	2.094	9.529	1.946	9.588	1.833	9.640	1.749
68	Morocco	0.295	−0.797	8.195	2.583	8.229	2.844	8.327	2.138	8.450	1.736	8.563	1.381	8.666	1.075
69	Mozambique	0.252	−0.775	6.661	5.409	6.526	4.959	6.707	5.079	6.758	4.706	6.815	4.290	6.877	3.829
70	Namibia	0.292	−0.754	8.479	3.229	8.564	3.469	8.544	3.022	8.604	2.834	8.659	2.666	8.710	2.517
71	Nepal	0.254	−0.777	7.043	2.619	7.139	1.638	7.134	2.020	7.229	1.396	7.327	0.756	7.428	0.111
72	Netherlands	0.341	−0.836	10.550	1.790	10.528	1.689	10.590	1.801	10.625	1.818	10.655	1.840	10.680	1.864
73	New Zealand	0.328	−0.819	10.232	2.160	10.170	1.925	10.252	2.098	10.269	2.052	10.284	2.018	10.298	1.994
74	Nicaragua	0.270	−0.787	7.736	2.631	7.855	3.013	7.808	2.132	7.880	1.642	7.954	1.165	8.027	0.710
75	Niger	0.245	−0.762	6.258	7.584	6.483	6.281	6.234	7.711	6.208	7.867	6.178	8.058	6.145	8.292
76	Nigeria	0.264	−0.726	7.436	6.022	7.391	5.659	7.433	5.941	7.433	5.855	7.435	5.765	7.439	5.669
77	Norway	0.352	−0.829	10.829	1.950	10.804	2.007	10.883	2.025	10.930	2.094	10.970	2.157	11.005	2.212
78	Pakistan	0.284	−0.782	7.739	3.430	7.870	3.008	7.859	2.980	7.973	2.542	8.082	2.124	8.184	1.735
79	Panama	0.314	−0.780	9.293	2.549	9.211	2.540	9.369	2.414	9.437	2.309	9.496	2.231	9.549	2.176
80	Papua New Guinea	0.279	−0.748	7.928	3.954	7.952	4.040	7.990	3.699	8.049	3.449	8.105	3.206	8.160	2.974
81	Paraguay	0.288	−0.772	8.311	2.968	8.349	3.010	8.388	2.655	8.460	2.366	8.527	2.104	8.590	1.869
82	Peru	0.299	−0.788	8.911	2.511	8.856	2.671	8.965	2.261	9.014	2.042	9.060	1.855	9.102	1.695
83	Philippines	0.282	−0.782	8.069	3.151	8.018	3.340	8.142	2.743	8.213	2.353	8.282	1.985	8.348	1.642
84	Portugal	0.330	−0.803	9.893	1.360	9.979	1.513	9.975	1.429	10.045	1.508	10.105	1.592	10.156	1.677
85	Puerto	0.334	-0.826	10.035	1.664	10.116	1.606	10.114	1.617	10.183	1.598	10.242	1.599	10.292	1.615
86	Romania	0.313	−0.745	9.146	1.330	9.291	1.528	9.250	1.460	9.337	1.604	9.411	1.755	9.473	1.906
87	Rwanda	0.260	−0.831	6.933	4.841	7.041	3.372	7.002	4.153	7.084	3.404	7.176	2.600	7.278	1.752
88	Senegal	0.263	−0.764	7.293	5.050	7.373	4.325	7.325	4.724	7.361	4.377	7.403	4.011	7.449	3.628
89	South Africa	0.299	−0.780	8.924	2.467	8.927	2.582	8.976	2.263	9.024	2.087	9.068	1.938	9.108	1.812
90	Spain	0.339	−0.820	10.216	1.380	10.301	1.559	10.292	1.459	10.358	1.544	10.414	1.629	10.463	1.711
91	Sri Lanka	0.286	−0.752	8.310	2.343	8.045	2.838	8.387	2.147	8.457	1.976	8.521	1.828	8.579	1.703
92	Sweden	0.337	−0.812	10.495	1.980	10.447	2.008	10.524	2.045	10.549	2.103	10.571	2.155	10.590	2.200
93	Switzerland	0.339	−0.845	10.596	1.520	10.595	1.580	10.623	1.557	10.645	1.592	10.665	1.626	10.682	1.657
94	Syria	0.293	−0.794	8.241	3.078	8.376	3.078	8.348	2.643	8.449	2.241	8.543	1.876	8.631	1.552
95	Tanzania	0.268	−0.768	7.072	5.428	6.978	5.607	7.144	5.117	7.217	4.779	7.290	4.415	7.365	4.029

(*Continued*)

Table A2. (*Continued*)

No	Country name	Posterior mean of $\alpha_{y,i}$ and $\alpha_{f,i}$		Data		Predicted values									
				2010		2010		2015		2020		2025		2030	
		$\bar{\alpha}_{y,i}$	$\bar{\alpha}_{f,i}$	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate	GDP per captia	Fertility rate
96	Thailand	0.311	−0.812	8.995	1.443	8.946	1.269	9.108	1.197	9.209	1.007	9.297	0.868	9.374	0.773
97	Togo	0.253	−0.763	6.597	4.792	6.909	4.354	6.674	4.449	6.753	4.064	6.834	3.639	6.918	3.175
98	Trinidad & Tobago	0.332	−0.822	10.334	1.802	10.125	1.922	10.361	1.819	10.385	1.839	10.405	1.860	10.422	1.882
99	Turkey	0.310	−0.800	9.253	2.101	9.224	2.115	9.322	1.912	9.383	1.761	9.438	1.643	9.486	1.553
100	Uganda	0.262	−0.771	7.005	6.155	7.022	5.090	7.036	5.900	7.071	5.618	7.111	5.309	7.154	4.971
101	United Kingdom	0.337	−0.809	10.442	1.980	10.360	2.000	10.474	2.045	10.502	2.104	10.526	2.157	10.546	2.203
102	United States	0.343	−0.839	10.630	1.931	10.615	1.786	10.670	1.914	10.704	1.909	10.734	1.913	10.760	1.923
103	Uruguay	0.301	−0.745	9.369	2.080	9.181	2.414	9.376	2.175	9.382	2.261	9.386	2.338	9.389	2.406
104	Venezu-ela	0.303	−0.795	9.113	2.472	9.200	2.431	9.158	2.237	9.201	2.035	9.240	1.863	9.276	1.719
105	Zambia	0.263	−0.752	7.325	5.813	7.284	5.380	7.336	5.597	7.351	5.366	7.371	5.120	7.395	4.858
106	Zimba-bwe	0.232	−0.808	5.765	3.721	5.776	5.429	5.884	2.889	6.019	1.906	6.172	0.771	6.343	-0.506

Table A3. Estimation results of the pooling model

Parameters	Mean	Median	Standard deviation	95% HPDI [2.5%, 97.5%]	Geweke's CD
α_y	0.1344	0.1344	0.0186	**[0.0979, 0.1708]**	−0.2215
β_y	−0.0109	−0.0109	0.0020	**[−0.0147, −0.0070]**	0.1096
γ_y	−0.0172	−0.0172	0.0033	**[−0.0237, −0.0107]**	−0.8885
δ_{yf}	0.0015	0.0015	0.0004	**[0.0007, 0.0023]**	1.1640
α_f	−0.5401	−0.5401	0.0230	**[−0.5853, −0.4951]**	−0.0081
β_f	0.1111	0.1111	0.0041	**[0.1031, 0.1192]**	0.2330
γ_f	0.0571	0.0571	0.0024	**[0.0523, 0.0619]**	0.0737
δ_{fy}	−0.0137	−0.0137	0.0005	**[−0.0147, −0.0128]**	−0.4648
σ_y	0.0000	0.0000	0.0000	**[0.0000, 0.0000]**	0.9343
σ_f	0.0000	0.0000	0.0000	**[0.0000, 0.0000]**	−0.6766
ω_y	0.0033	0.0033	0.0001	**[0.0032, 0.0035]**	−1.6665
ω_f	0.0051	0.0051	0.0001	**[0.0049, 0.0053]**	−0.5282
$-\frac{\beta_y}{\delta_{yf}}$	7.6350	7.3090	3.3940	**[5.6090, 11.4500]**	−0.1229
$-\frac{\gamma_f}{\delta_{fy}}$	4.1550	4.1550	0.1024	**[3.9530, 4.3560]**	−0.5080

Note: HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic. The figures highlighted in HPDI represent the cases that the 95% credible intervals do not include 0.

Table A4. Estimation results of the fixed effect model

Parameters	Mean	Median	Standard deviation	95% HPDI [2.5% , 97.5%]	Geweke's CD
β_y	−0.0320	−0.0320	0.0035	**[−0.0389, −0.0251]**	0.7005
γ_y	−0.0125	−0.0125	0.0055	**[−0.0233, −0.0017]**	0.7338
δ_{yf}	0.0010	0.0010	0.0007	[−0.0004, 0.0023]	−0.5005
β_f	0.1624	0.1624	0.0065	**[0.1496, 0.1752]**	1.2860
γ_f	0.0805	0.0805	0.0042	**[0.0722, 0.0887]**	1.1351
δ_{fy}	−0.0208	-0.0208	0.0008	**[−0.0224, −0.0192]**	−1.2435
σ_y	0.0001	0.0001	0.0000	**[0.0000, 0.0001]**	−0.7488
σ_f	0.0000	0.0000	0.0000	**[0.0000, 0.0001]**	−1.0399
ω_y	0.0032	0.0032	0.0001	**[0.0030, 0.0033]**	1.3175
ω_f	0.0044	0.0044	0.0001	**[0.0042, 0.0046]**	0.6928

Note: HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic. The figures highlighted in HPDI represent the cases that the 95% credible intervals do not include 0.

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Figure A1. The animations based on the data from 1960 to 2010.

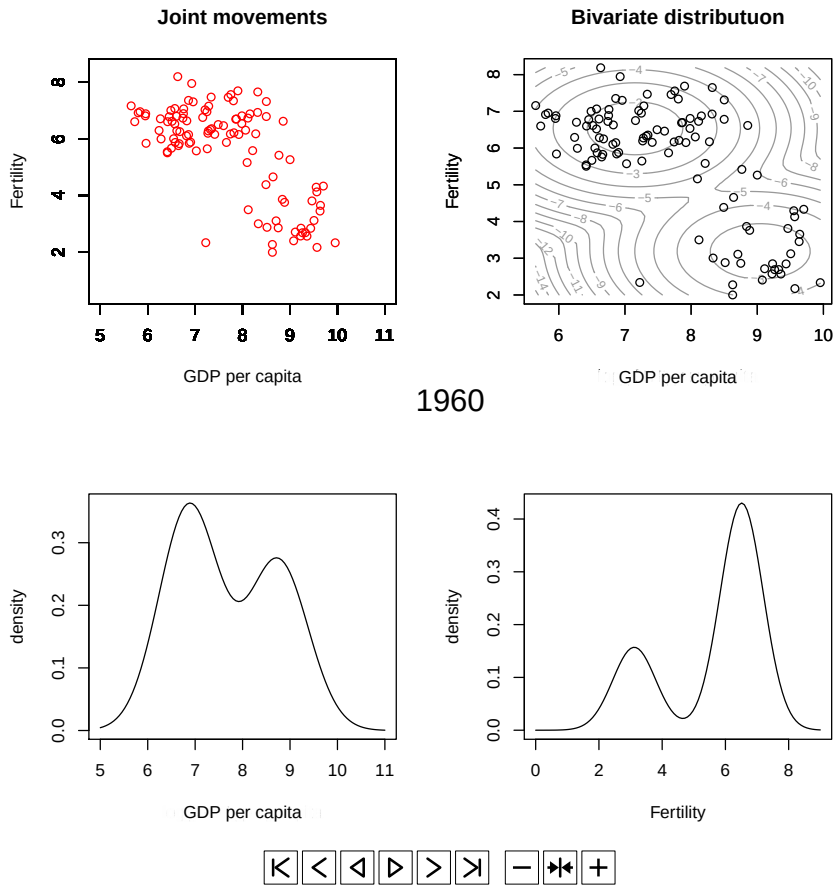

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Figure A2. The animations based on the estimation results from 1960 to 2010.

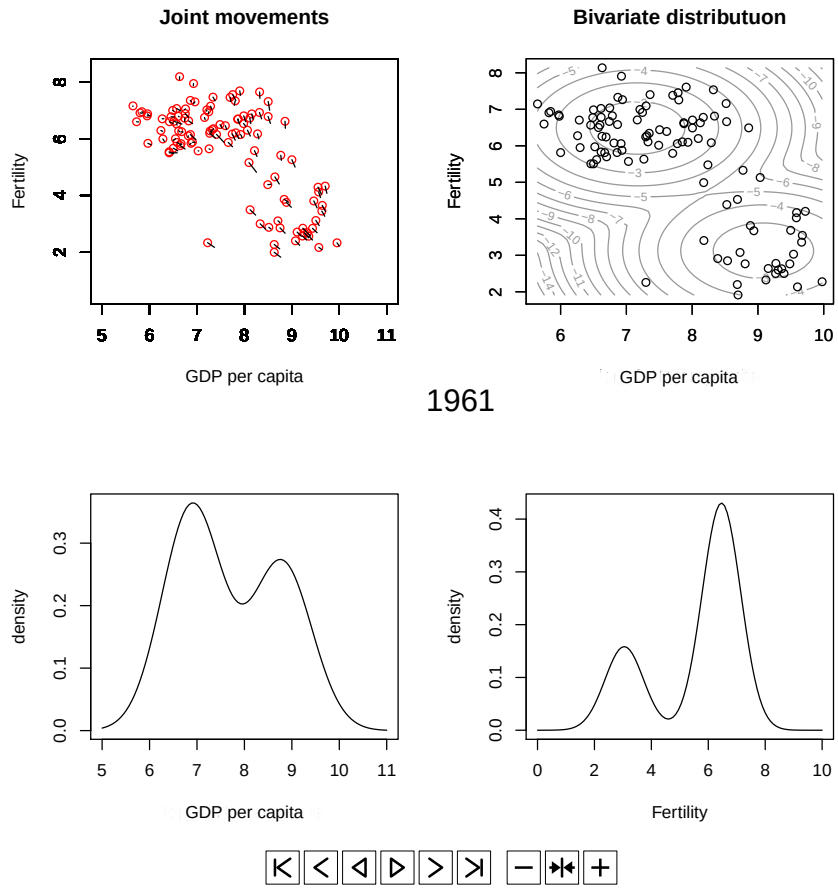

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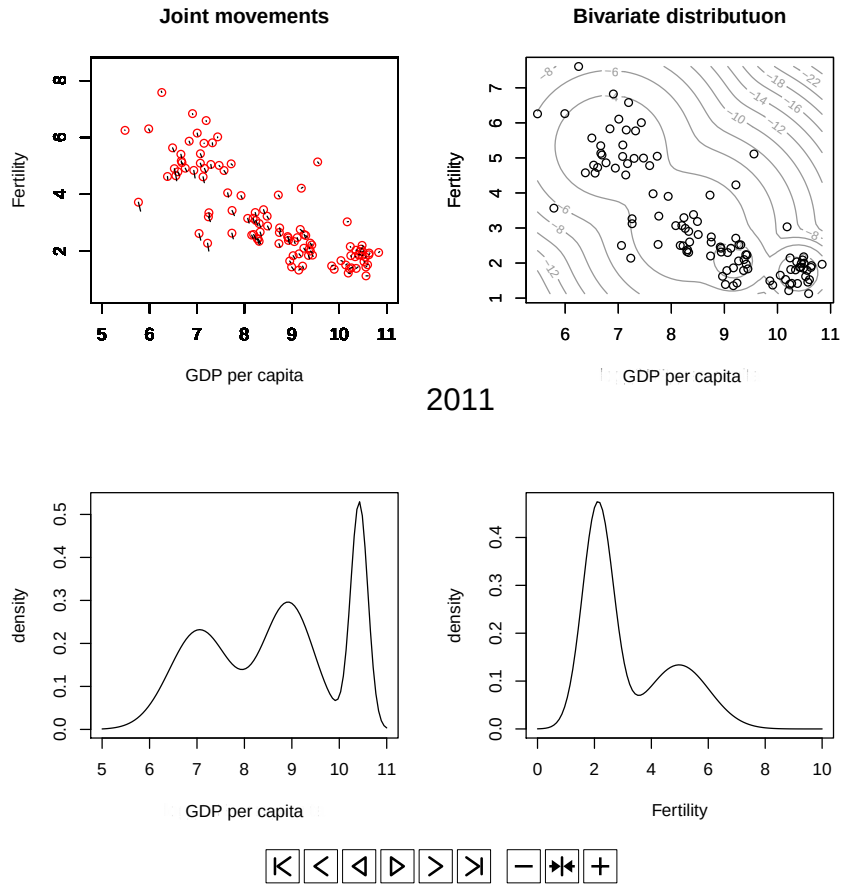
Figure A3. The animations based on the prediction results from 2010 to 2030.



***Cogent Economics & Finance* (ISSN: 2332-2039) is published by Cogent OA, part of Taylor & Francis Group.**