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Article

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Cogent Economics & Finance

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Suggested Citation: Corso, Eduardo Ariel (2014) : Ambiguity, ambiguity aversion and stores of value: The case of Argentina, Cogent Economics & Finance, ISSN 2332-2039, Taylor & Francis, Abingdon, Vol. 2, Iss. 1, pp. 2-12,
<https://doi.org/10.1080/23322039.2014.947001>

This Version is available at:

<https://hdl.handle.net/10419/147731>

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LETTER

Ambiguity, ambiguity aversion and stores of value: The case of Argentina

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Cogent Economics & Finance (2014), 2: 947001



Received: 23 January 2014
Accepted: 15 July 2014
Published: 22 August 2014

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Reviewing editor:
Sergi Jimenez, Universitat Pompeu Fabra, Spain

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LETTER

Ambiguity, ambiguity aversion and stores of value: The case of Argentina

Eduardo Ariel Corso^{1*}

Abstract: We study the household portfolio allocation in an economy with a history of nominal anchor volatility. Applying smooth ambiguity preferences to a static portfolio choice problem, we rationalize two facts about the Argentine experience of the last 20 years: the dollarization of household financial assets and its bias towards investment real estate as a means of preserving the real value of wealth. We find that ambiguity explains portfolio dollarization. In addition, ambiguity aversion reduces the demand for assets denominated in US dollars and increases the demand for investment real estate.

Keywords: ambiguity, ambiguity aversion, Argentina, dollarization, investment real estate, stores of value

JEL classifications: G10, G11, D1

1. Introduction

In 1921, Frank Knight proposed the classic distinction between risk and uncertainty. Knight's concept of risk refers to a situation in which agents can assign probability values univocally, said values being determined either objectively or subjectively. The notion of uncertainty in his analysis is equivalent to the later concept of ambiguity and refers to a situation in which agents do not have enough information to assign univocally determined probability values to the realization of stochastic variables. The experimental relevance of the distinction between risk and ambiguity was first highlighted by Ellsberg (1961). His findings have generated the development of new representation of

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Eduardo Ariel Corso is a senior analyst of Economic Research at the Central Bank of Argentina. He is also a professor of finance and monetary theory at the Department of Economics, University of Buenos Aires, Argentina. His current research focuses on households' portfolio selections in uncertain environments. His research issues include financial intermediation and macroeconomic volatility.



PUBLIC INTEREST STATEMENT

At times, agents must make decisions in contexts without enough information to assign univocally determined probability values to the realization of relevant variables. These so-called ambiguous contexts are common to economies experiencing recurrent changes in their monetary and exchange rate regimes. The Argentine economy is an interesting case study in this regard given its history of macroeconomic and nominal anchor volatility over the past 70 years.

In this paper, the smooth ambiguity preferences of Klibanoff, Marinacci and Mukerji are applied to study Argentine households' demands for stores of value. Specifically, the paper explores asset dollarization, as well as the demand for investment real estate, as defensive mechanisms to preserve the real value of wealth. The main finding shows that ambiguity and ambiguity aversion prove to be relevant factors explaining such defensive mechanisms.

preferences over acts in the presence of ambiguity: the maxmin expected utility of Gilboa and Schmeidler (1989), the multiplier preferences of Hansen and Sargent (2001) and Strzalecki (2011), the smooth preferences of Klibanoff, Marinacci, and Mukerji (2005) and the variational preferences of Maccheroni, Marinacci, and Rustichini (2006), among others. Over the years, extensive literature applied these approaches to financial topics. Complete surveys were conducted by Epstein and Schneider (2010) and Guidolin and Rinaldi (2013). Despite the great extent of these applications, one issue remains unexplored: the effects of ambiguity and ambiguity aversion on the demand for stores of value. This article contributes to this branch of the literature.

2. Ambiguity, ambiguity aversion and portfolio allocation

2.1. A formal statement of portfolio choice under ambiguity

Let S be the set of states of nature and $E \subset S$ the set of events, i.e. the realization of the gross real returns vector r of financial assets in $t+1$ (r_{t+1}) to which an agent assigns a non-zero probability. Let Z be the set of results/payments defined as the possible realizations of portfolio return r_p in $t+1$. Let F be the set of actions whose elements, i.e. vectors v_t of the fractions of wealth allocated to the assets, are the choice variables. The agent has an initial wealth w that is arbitrarily indexed to one. He determines the optimal asset allocation between n instruments that are considered relevant stores of value. The gross real portfolio return in $t+1$ is then defined as $r_{p,t+1} = v_t \cdot r_{t+1}'$.

The agent faces ambiguity about the probability measure over the set E . This ambiguity will be represented by the set $M = \{\mu_1, \dots, \mu_j\}$ of feasible subjective probability distributions μ . The agent also has a subjective probability distribution $\Pi = \{\pi(\mu_1), \dots, \pi(\mu_j)\}$ defined on the elements of set M . We use $\pi(\mu_i) = \pi_i$ interchangeably. Elements of set Π are priors that represent the agent's belief about the feasibility that any μ_i is the probability distributions that effectively determine the realizations $r_{t+1} \in E$.

Following Klibanoff et al. (2005), we assume that the agent exhibits smooth preferences for ambiguity. Letting u and ϕ be the utility and ambiguity functions, respectively, the vector of optimal asset allocation v_t^* can be written as:

$$v_t^* \in \arg \max_v \sum_{i=1}^j \pi(\mu_i) \cdot \phi \left(\sum_{r \in E} u(w_t \cdot v_t \cdot r_{t+1}') \cdot \mu_i(r_{t+1}) \right) \quad (1)$$

or

$$v_t^* \in \arg \max_v \sum_{i=1}^j \pi_i \cdot \phi(E_{\mu_i} u)$$

We denote $E_{\mu_i} u(v) = E_{\mu_i} u$ interchangeably.

The concavity of the utility function determines the agent's degree of risk aversion. A concave function ϕ implies ambiguity aversion.

2.2. The case of Argentina: some stylized facts

The combination of financial repression and rising inflation that characterized Argentina's economy from the mid-1940s to the mid-1970s discouraged the demand for stores of value denominated in local currency. This process was exacerbated after the crisis that followed the financial liberalization in the late 1970s. As a consequence, the economy experienced a secular trend towards disintermediation, falling to a minimum during the hyperinflationary experiences of 1989 and 1990 (see Figure 1).

In this context, the agents developed defensive mechanisms to preserve the purchasing power of their wealth. During the 1960s and 1970s, as the economy remained relatively closed, investment real estate evolved as a non-financial option to preserve the real value of wealth. Figure 2 shows the

Figure 1. M3 (private, bi-monetary)/GDP ratio (1945–2012).

Source: Central Bank of Argentina.

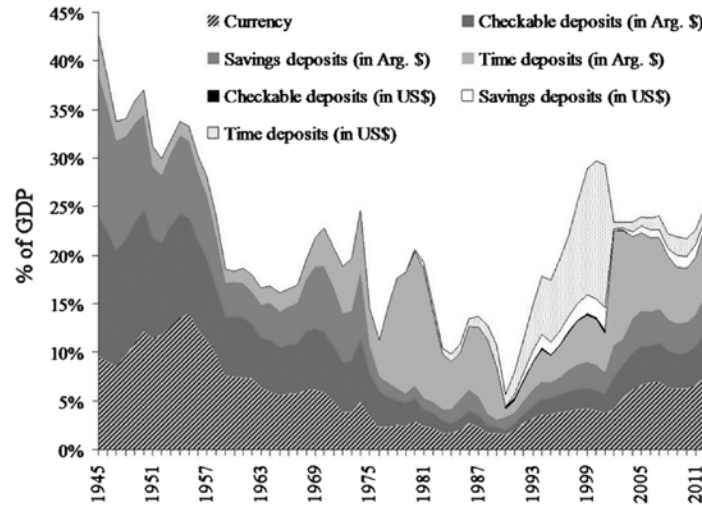
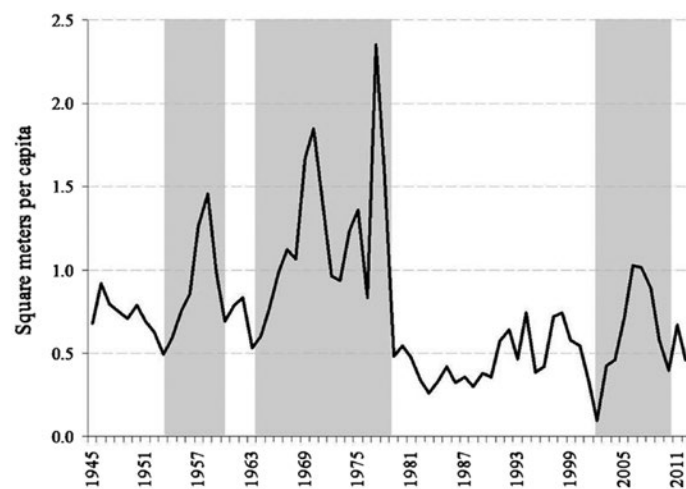


Figure 2. Floor area built of new construction and expansion. Square metres per capita. City of Buenos Aires (1945–2012).

Source: Department of Statistics. City of Buenos Aires.



main stages of the expansion in real estate investment between 1945 and 2012 in the country's main urban conglomerate.

Although agents increasingly perceived external assets as an insurance against episodes of devaluations during that period, it was not until the experience of openness and financial liberalization in the late 1970s that the holdings of foreign assets were consolidated as a choice of stores of value.

The mega-devaluation events and financial crises that took place during the 1980s and the consolidation of high inflation implied the de facto dollarization of numerous contractual structures of the economy. This process peaked during the hyperinflationary experiences.

The convertibility regime established in the 1990s validated dollarization and gave rise to dollar-denominated contracts in the local financial system. As a result, financial re-intermediation during the 1990s showed a high share of deposits denominated in US dollars. The convertibility crisis produced a new disincentive for asset demand in the local financial system.

Between 2003 and 2012, Argentina's macroeconomic conditions were much more favourable than in the past. However, external assets and investment real estate maintained their position as

Table 1. Composition of the non-financial private sector portfolio (main assets). December 2012

Main assets	Billions of dollars	Total assets considered (%)
Currency	40,538	5.38
Checkable deposits (in Arg. \$)	20,329	2.70
Savings deposits (in Arg. \$)	19,561	2.60
Time deposits (in Arg. \$)	33,103	4.40
Checkable deposits (in US\$)	2	.00
Savings deposits (in US\$)	2,931	.39
Time deposits (in US\$)	4,303	.57
Treasury securities	14,496	1.93
Central Bank securities	1,995	.26
External assets	202,561	26.90
Local corporate equities	34,255	4.55
Real estate	378,815	50.31
Total	752,889	100.00

Source: Central Bank of Argentina, Ministry of Economy and Public Finance and National Institute of Statistics and Census.

Table 2. Main four stores of value in the non-financial private sector portfolio. December 2012

Main stores of value	Billions of dollars	Total assets considered (%)
Time deposits (in US\$)	33,103	5.10
External assets	202,561	31.22
Real estate	378,815	58.39
Local corporate equities	34,255	5.28
Total	648,734	100.00

Source: Central Bank of Argentina, Ministry of Economy and Public Finance, and National Institute of Statistics and Census.

preferential stores of value in the portfolio of the Argentine households. Owing to a lack of information, it is not possible to differentiate between the holdings of households and firms in the Argentine case. To estimate the asset holdings of households, Table 1 presents the gross asset holdings of the non-financial private sector, considering real estate as the only component of fixed capital. Note that equity holdings are also gross, so that they are not offset by the supply made by firms. Table 2 shows the holdings of a portfolio that only considers the four main stores of value demanded by the sector.

2.3. Modelling the demand for stores of value in Argentina

There are many explanatory factors behind the relative portfolio holdings in Table 2. This article tries to show that some defensive mechanisms that agents developed during recent decades can be rationalized as a consequence of ambiguity and ambiguity aversion. To this end, we calibrate the optimization problem 1 to explore two responses in terms of household asset allocation. The first response is the dollarization that characterized the re-intermediation process when the currency board was in effect following the 1989/1990 hyperinflations (Figure 1). The second response is households' demand for investment real estate and external assets as stores of value during the period 2003–2012 (Figure 2 and Table 2).

These facts can be rationalized by assuming only two feasible distributions in set M . The first, μ_1 , represents the behaviour of real returns in the context in which the agent makes the portfolio decision. Calibration 1 analyses the re-intermediation process during the 1990s. In this case, μ_1 represents the behaviour of real returns from 1993 to December 1998. The period was selected to capture the behaviour of real returns during the years in which the convertibility regime was considered

sustainable. In Calibration 2, μ_1 represents the behaviour of real returns from January 2003 to December 2012 since we study portfolio decisions during that period. In contrast, the second feasible subjective distribution $\mu_2 \in M$ is the same for both Calibrations, corresponding to the behaviour of real returns in a representative period of currency crisis. In the case of Argentina, the currency crisis was a recurrent shock and constituted a critical element to understand the private sector portfolio composition. Given its macroeconomic impact, we selected the period January 1981 to December 1983, characterized by recurrent mega-devaluation episodes. The choice of this period is based on the hypothesis that the mega devaluations are still tangible in agents' memory, affecting asset allocation decisions. In sum, the two assumed feasible subjective distributions represent the current process in which the agent makes the portfolio decision and the memory of a critical event, respectively.

A central element of this approach consists of identifying those events or processes that have had a significant impact on the private agents' financial behaviour. This is critical to applying this approach to other case studies. A natural extension of this study could be to deepen the analysis of alternative criteria to identify potential feasible subjective distributions $\mu_i \in M$.

Regarding the functional form of the two feasible subjective distributions, we assume for simplicity that the agent forms expectations based on the empirical distribution of real returns for the periods considered.

We assume the subjective priors $\pi(\mu_i)$ as given. We sustain that if the agent assigns priors (no matter what process generates it) to the feasibility of subjective distributions μ_p , some values of these priors can have significant effects on the relative asset holdings. In this sense, the aim of the proposed calibration exercises is to show that different values of priors, as well as degrees of ambiguity aversion, can explain specific aspects of asset allocation in Argentina.

While a detailed study goes beyond the scope of this letter, an additional extension would be to study the factors underlying these subjective priors. The first approximation would be to assume that priors are functions of the current proportion of working-age people that were employed during the critical episodes of the 1980s. However, this would assume that there is no intergenerational transmission of critical episodes, which at first appears to be a strong assumption at least for Argentina. Another possibility, which would broaden the applicability of the approach to other case studies, consists of assuming that priors $\pi(\mu_i)$ depend on the evolution of specific variables, indicative of the sustainability of the current macroeconomic regime (that is, that real returns in $t+1$ will be generated by distribution μ_i). Examples of such variables are the current account balance, the real exchange rate, and the monetary policy bias.

In addition to Argentina, another particularly interesting case study is Peru (an economy whose monetary history reflects many similarities with Argentina). In that case, as was mentioned, assuming that priors depend on variables taken as indicators of the sustainability of the current macroeconomic regime could be a relevant strategy to explain the de-dollarization process observed in recent years. For example, priors may depend on the bias of monetary policy, which proved to be remarkably successful in generating positive real returns for assets denominated in local currency over long periods of time.

Under the assumptions considered above, Expression 1 becomes:

$$v_t^* \in \arg \max_v \pi(\mu_1) \cdot \phi(E_{\mu_1} u(v)) + \pi(\mu_2) \cdot \phi(E_{\mu_2} u(v)) \quad (2)$$

$$\text{s.t. } \sum_{i=1}^n v_i = 1 \text{ and } 0 \leq v_i \leq 1$$

with the following first-order condition (f.o.c.):

$$\pi(\mu_1) \cdot \phi'(E_{\mu_1} u) \cdot E_{\mu_1} u'_v + \pi(\mu_2) \cdot \phi'(E_{\mu_2} u) \cdot E_{\mu_2} u'_v = 0 \tag{3}$$

Multiplying and dividing the left-hand side of (3) by $\varphi = \sum_{i=1}^2 \pi(\mu_i) \cdot \phi'(E_{\mu_i} u)$ yields:

$$\varphi \cdot \left[\pi(\mu_1) \cdot \xi_1 \cdot E_{\mu_1} u'_v + \pi(\mu_2) \cdot \xi_2 \cdot E_{\mu_2} u'_v \right] = 0$$

or

$$\varphi \cdot \left[\pi^*(\mu_1) \cdot E_{\mu_1} u'_v + \pi^*(\mu_2) \cdot E_{\mu_2} u'_v \right] = 0$$

where the distortion variable $\xi_i = \frac{\phi'(E_{\mu_i} u)}{\varphi} = d\Pi^*/d\Pi$ is the Radon–Nikodym derivative of the probability measure Π^* with respect to Π . With $\varphi \neq 0$, the f.o.c. results in:

$$\frac{\pi^*(\mu_1)}{\pi^*(\mu_2)} = \frac{\xi_1 \cdot \pi(\mu_1)}{\xi_2 \cdot \pi(\mu_2)} = - \frac{E_{\mu_2} u'_v}{E_{\mu_1} u'_v} \tag{4}$$

According to Equation 4, the agent will choose the optimal portfolio holdings such that the expected marginal utilities ratio equals the ratio of subjective priors adjusted by their subjective assessments in terms of ambiguity, i.e. ξ_1/ξ_2 .

The theorems that follow allow us to obtain an accurate interpretation of Equation 4.

THEOREM 1 *The distortion variables ξ_i increase the subjective priors of those feasible probability distributions whose expected utilities are lower than the weighted average of the expected utilities.*

See proof in Appendix 1.

THEOREM 2 *The distortion variables ξ_i of those feasible probability distributions, whose expected utilities are lower than the weighted average of the expected utilities, increase with the degree of ambiguity aversion.*

See proof in Appendix 2.

The proposition that follows is derived from Theorem 2 and the first-order condition 4, and refers to the conditions to be met by the optimal asset allocation vector when ambiguity aversion changes.

PROPOSITION 1. *Suppose that $E_{\mu_1} u(v^*) < \sum_{i=1}^2 E_{\mu_i} u(v^*)$. A rise in ambiguity aversion changes the optimal asset allocation vector from v^* to v^{**} , implying that $\xi_1^*(v^{**})/\xi_2^*(v^{**}) > \xi_1^*(v^*)/\xi_2^*(v^*)$. Then, $\left\langle -E_{\mu_2} u'_v/E_{\mu_1} u'_v \right\rangle_{\phi^*} > -E_{\mu_2} u'_v/E_{\mu_1} u'_v$ such that $\left[\xi_1^*(v^{**}) \cdot \pi(\mu_1) \right] / \left[\xi_2^*(v^{**}) \cdot \pi(\mu_2) \right] = \left\langle -E_{\mu_2} u'_v/E_{\mu_1} u'_v \right\rangle_{\phi^*}$.*

Where $\langle \rangle_{\phi^*}$ means “given the more concave ambiguity function ϕ^{**} ”.

3. Calibrations

The objective of this letter is not to argue that agents have faced changes in their degree of ambiguity aversion. The main argument is that the presence of ambiguity and ambiguity aversion may be relevant elements to explain some stylized facts of assets allocation in Argentina. Calibrations are performed for different values of ambiguity aversion simply to differentiate between the effects of ambiguity and ambiguity aversion.

In Calibrations 1 and 2, the assumptions of the above proposition are satisfied. In both cases, we assume that the agent determines the optimal asset allocation for four instruments: a term

Table 3. Descriptive statistics of annual real returns (calculated using a monthly time series)

	Term deposits (local currency) (a) (%)	External assets (US dollars) (b), (c) (%)	Investment real estate (%)	Equities (%)
<i>January 1981/December 1983 (prototypical currency crisis)</i>				
Mean	-12.23	89.54	.14	-10.03
Median	-1.87	109.83	-7.89	-16.93
Standard deviation	19.07	79.83	31.72	46.86
<i>January 1993/December 1998 (currency board)</i>				
Mean	6.16	2.78	-2.06	-3.59
Median	6.13	4.84	-2.13	2.37
Standard deviation	2.18	4.44	3.23	28.47
<i>January 2003/December 2012 (current period)</i>				
Mean	-5.48	-3.45	2.01	2.33
Median	-6.67	-5.40	-1.19	6.14
Standard deviation	7.86	16.00	18.18	32.92

Source: Central Bank of Argentina and Federal Reserve System.

Notes: (a) Domestic term deposits in local currency. 30/59 days.

(b) Market yield on US Treasury securities at 1 year constant maturity.

(c) From January 1993 to December 1998, we considered domestic term deposits in US Dollars. 30/59 days.

deposit in the domestic financial system denominated in local currency, external assets denominated in US dollars (in Calibration 1, we use local term deposits denominated in US dollars instead of external assets), investment real estate and equities. In addition, we assume that the agent forms expectations using the empirical distributions of returns. Table 3 shows the descriptive statistics of the real returns for the periods considered to calculate the empirical distributions μ_1 and μ_2 .

3.1. Calibration 1

In the first calibration, we study the effects of ambiguity on asset dollarization when the currency board was in effect. We assume that set M has two elements. The first (μ_1) corresponds to the empirical distribution of real returns calculated from January 1993 to December 1998. The second (μ_2) corresponds to the empirical distribution of the currency crisis period from January 1981 to

Figure 3. Optimal demands for term deposits (in Arg. \$).

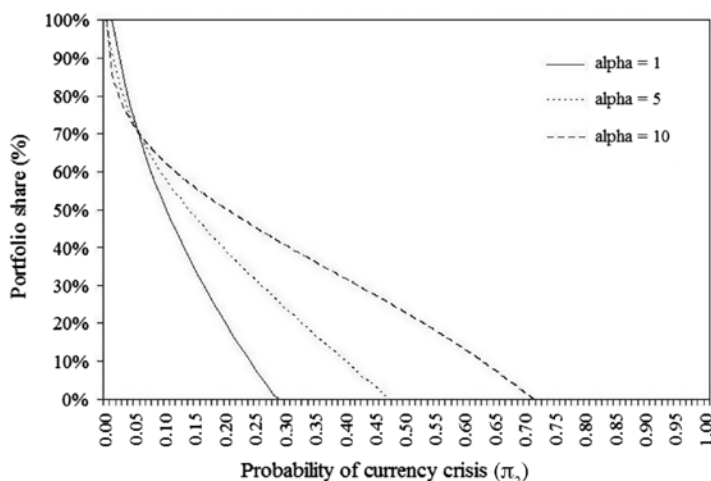
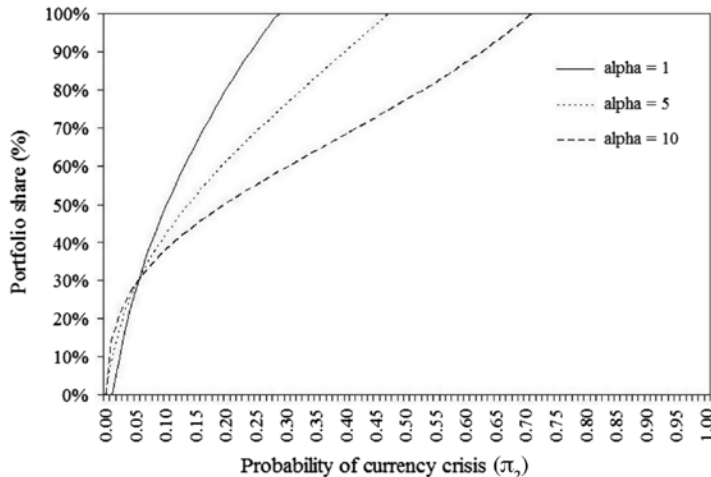


Figure 4. Optimal demands for term deposits (in US\$).



December 1983. Regarding preferences, we assume a constant relative risk aversion (CRRA) utility function $u(w_{t+1}) = (1/(1-\delta)) \cdot (w_{t+1})^{1-\delta}$ and a constant absolute ambiguity aversion (CAAA) ambiguity function $\phi(E_{\mu_i} u) = -(1/\alpha) \cdot \exp(-\alpha \cdot E_{\mu_i} u)$. Transaction costs and short sales are not considered. The exercise is calibrated for values of the subjective prior π_2 of the currency crisis between zero and one, a CRRA coefficient $\delta=3$ and values for the CAAA coefficient $\alpha=1, 5$ and 10 . Figures 3 and 4 show optimal asset allocations corresponding to term deposits denominated in local currency and US dollars, respectively. The optimal demands for investment real estate and equities are zero for each value of π_2 considered.

3.2. Calibration 2

In this case, μ_1 is the empirical distribution of real returns from January 2003 to December 2012. As in Calibration 1, μ_2 is the empirical distribution from January 1981 to December 1983. The utility function and the ambiguity function have the same form as in Calibration 1, and the CRRA and CAAA coefficient values are also the same as those of Calibration 1. Figures 5–7 show the optimal demand for external assets, investment real estate and equities for values of π_2 between zero and one. The optimal demand for term deposits denominated in local currency is zero for every value of priors considered.

Figure 5. Optimal demand for external assets (in US\$).

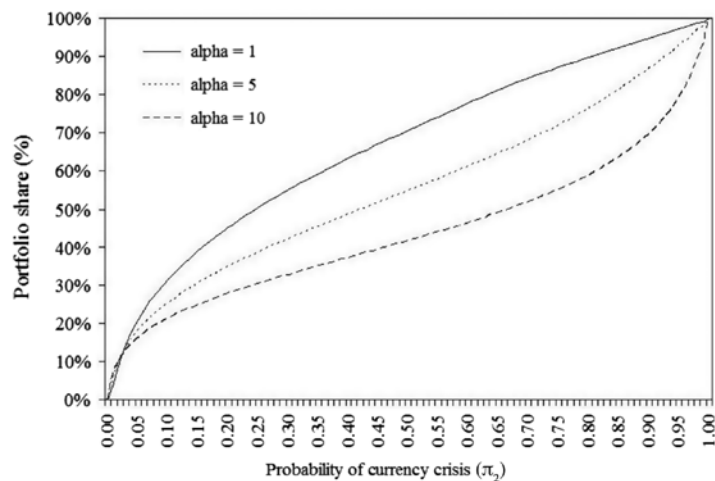


Figure 6. Optimal demand for investment real estate.

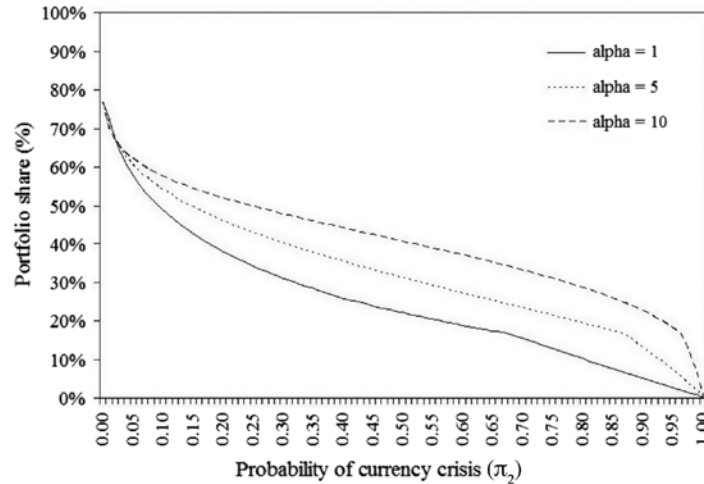
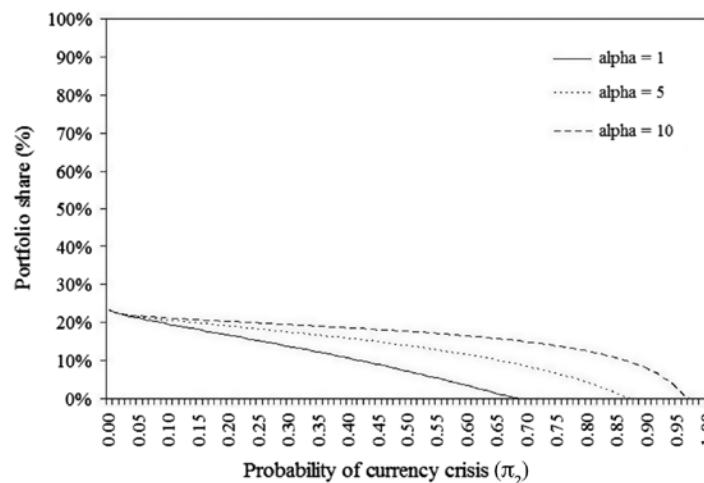


Figure 7. Optimal demand for equities.



4. Results

The results allow us to conjecture about the relevance of ambiguity as an explanatory factor for portfolio dollarization in Argentina. Asset holdings in Calibration 1, without ambiguity ($\pi_2=0$), consist entirely of term deposits in local currency. However, under ambiguity, this result changes dramatically. With a subjective probability $\pi_2=10\%$ that returns behave consistently with the empirical distribution observed in 1981/1983, the share of term deposits denominated in US dollars lies between 38 and 50%, depending on the CAAA coefficient considered. These shares increase sharply, achieving values between 45 and 66% when $\pi_2=15\%$. In addition, Calibration 2 shows that ambiguity aversion reduces the demand for assets denominated in US dollars and increases the demand for investment real estate and equities. These results are consistent with Proposition 1. In fact, in Calibrations 1 and 2, $E_{\mu_2} u > E_{\mu_1} u$ for $\pi_2 > .05$ and $\pi_2 > .02$, respectively (see Figures 8 and 9). Hence, a rise in ambiguity aversion implies that $\left\langle -E_{\mu_2} u'_v / E_{\mu_1} u'_v \right\rangle_{\phi^*} > -E_{\mu_2} u'_v / E_{\mu_1} u'_v$ consistently with the increase in the distortion ratio $\xi_1^*(v^{**}) / \xi_2^*(v^{**}) > \xi_1(v^*) / \xi_2(v^*)$. Conditioned to the assumed empirical distributions μ_1 and μ_2 , the new value for the expected marginal utilities ratio will be met for an optimal asset allocation vector v^{**} with lesser participation of assets denominated in US dollars.

Figure 8. Calibration 1: expected utility values.

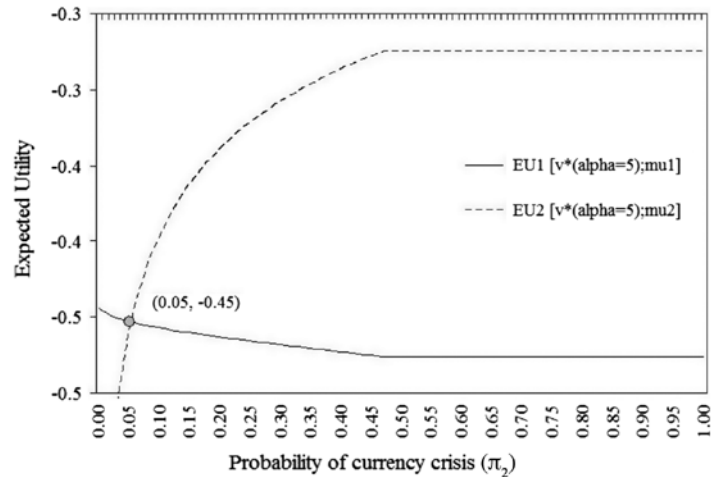
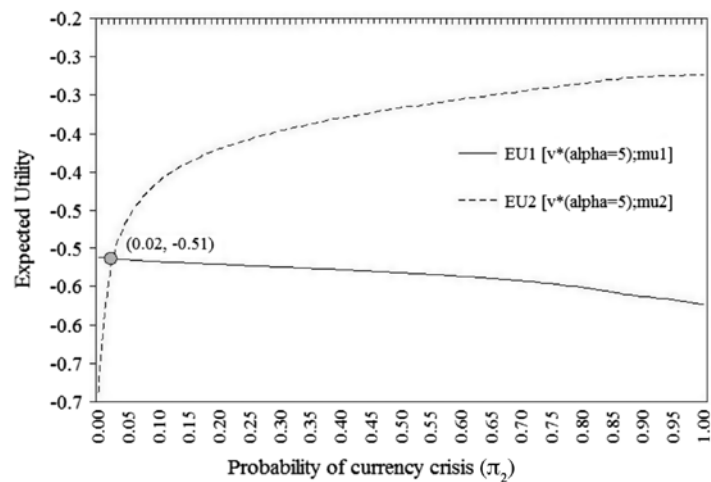


Figure 9. Calibration 2: expected utility values.



5. Conclusions

Ambiguity can be a relevant factor explaining portfolio dollarization in Argentina. In addition, ambiguity aversion explains part of the demand for investment real estate as an element to preserve the real value of wealth. A particular feature of this asset in Argentina is that its probability distribution on real returns remains relatively invariant between the various multivariate distributions μ_i that are considered feasible. This makes investment real estate an especially appealing store of value for agents showing ambiguity aversion.

Funding

The authors received no direct funding for this research.

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Citation information

Cite this article as: Ambiguity, ambiguity aversion and stores of value: The case of Argentina, E.A. Corso, *Cogent Economics & Finance* (2014), 2: 947001.

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References

- Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. *The Quarterly Journal of Economics*, 75, 643–669. <http://dx.doi.org/10.2307/1884324>
- Epstein, L., & Schneider, M. (2010). Ambiguity and asset markets. *Annual Review of Financial Economics*, 2, 315–346. <http://dx.doi.org/10.1146/annurev-financial-120209-133940>
- Gilboa, I., & Schmeidler, D. (1989). Maxmin expected utility with a non-unique prior. *Journal of Mathematical Economics*, 18, 141–153. [http://dx.doi.org/10.1016/0304-4068\(89\)90018-9](http://dx.doi.org/10.1016/0304-4068(89)90018-9)
- Guidolin, M., & Rinaldi, F. (2013). Ambiguity in asset pricing and portfolio choice: A review of the literature. *Theory and Decision*, 74, 183–217. <http://dx.doi.org/10.1007/s11238-012-9343-2>
- Hansen, L., & Sargent, T. (2001). Robust control and model uncertainty. *American Economic Review*,

91, 60–66.
<http://dx.doi.org/10.1257/aer.91.2.60>
 Klibanoff, P., Marinacci, M., & Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73, 1849–1892.
<http://dx.doi.org/10.1111/ecta.2005.73.issue-6>

Maccheroni, F., Marinacci, M., & Rustichini, A. (2006). Ambiguity aversion, robustness, and variational representation of preferences. *Econometrica*, 74, 1447–1498.
<http://dx.doi.org/10.1111/ecta.2006.74.issue-6>
 Strzalecki, T. (2011). Axiomatic foundations of multiplier preferences. *Econometrica*, 79, 47–73.

Appendix 1

Proof of Theorem 1

Let v^* be the vector of optimal asset allocation consistent with a preferences structure (u, ϕ) , a set of priors $\Pi = \{\pi(\mu_1), \pi(\mu_2)\}$ and a set of feasible probability distributions $M = \{\mu_1, \mu_2\}$, such that

$$E_{\mu_1} u(v^*) \leq E_{\mu_2} u(v^*)$$

So that

$$E_{\mu_1} u \leq \sum_{i=1}^2 \pi(\mu_i) \cdot E_{\mu_i} u \text{ and } E_{\mu_2} u \geq \sum_{i=1}^2 \pi(\mu_i) \cdot E_{\mu_i} u$$

Given that ϕ is a strictly concave increasing function,

$$\phi(E_{\mu_1} u) \leq \phi(E_{\mu_2} u) \text{ and } \phi'(E_{\mu_1} u) \geq \phi'(E_{\mu_2} u)$$

Hence, we have:

$$\phi'(E_{\mu_1} u) \geq \pi_1 \cdot \phi'(E_{\mu_1} u) + \pi_2 \cdot \phi'(E_{\mu_2} u) \tag{A.1}$$

So that

$$\xi_1 = \frac{\phi'(E_{\mu_1} u)}{\sum_{i=1}^2 \pi_i \cdot \phi'(E_{\mu_i} u)} \geq 1 \tag{A.2}$$

In addition,

$$\phi'(E_{\mu_2} u) \leq \pi_1 \cdot \phi'(E_{\mu_1} u) + \pi_2 \cdot \phi'(E_{\mu_2} u) \tag{A.3}$$

So that

$$\xi_2 = \frac{\phi'(E_{\mu_2} u)}{\sum_{i=1}^2 \pi_i \cdot \phi'(E_{\mu_i} u)} \leq 1 \tag{A.4}$$

Appendix 2

Proof of Theorem 2

Let ϕ and ϕ^* be strictly concave increasing functions, with ϕ^* more concave than ϕ . From expression (A.1) in proof of Theorem 1, we have:

$$\frac{\phi^*(E_{\mu_1} u)}{\sum_{i=1}^2 \pi_i \cdot \phi^*(E_{\mu_i} u)} \geq \frac{\phi'(E_{\mu_1} u)}{\sum_{i=1}^2 \pi_i \cdot \phi'(E_{\mu_i} u)} \tag{B.1}$$

So that $\xi_1^* \geq \xi_1$.

The reverse can be proved from (A.3), so that $\xi_2^* \leq \xi_2$.



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