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**Article**

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# A real options approach to the baseball game betting

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Applications of real options can be found in real assets, but not in sports. This article proposes the real options model and applies it to baseball games, allowing the option to switch between the two teams during the game. We believe this is the first attempt to apply the switching options to the baseball game scheme. To do so, we first design some rules in our assumptions and define variables that are relevant to the baseball options. We combine features of the wager volatility and options to determine the optimal time of team switching, and we find that with low switching cost, the expected investment costs fall and so does risk. The application of switching options to baseball game also presents a feasible method to other sport games.

## I. Introduction

Baseball has been one of the most popular sports in the world; many countries regard baseball as the national sport. Because of the popularity, not only the number of professional teams has increased, but also the baseball fans. To mitigate the illegal game-cheating problem, which often involves gang threats and violence, regulation and legalizing baseball game gambling such as using public welfare lottery has been proposed. Since baseball games have large number of fans, and the trading of derivatives has gained an important footing in the financial markets, the proposed combination of baseball games and switching options is indeed a new endeavour to our best knowledge. Legalizing the trading of such a new product may increase the trading volume of options, reduce illegal gambling and enhance government tax revenues. To shed light on such a proposal, we propose and design the use of switching option to add to the flexibility of baseball game gambling. To be sure, application of the switching option is also feasible for other sport games.

A bettor owns an option and has the opportunity to switch from one team to another team as soon as future wager volatility favours such switch. This opportunity is comparable to a call option, since in effect the bettor pays a price (the option premium) for the right to pay a fixed exercise price (the investment cost) and obtain the switching option. With some costs, a bettor owns the switching option to exploit wager volatility movements but can switch back to the holding strategy when losses occur. The result is that, just like the holder of a financial option, the bettor can in principle benefit from unlimited profit opportunities, while cap their potential loss.

Therefore, uncertainty of wager volatility is not only a source of concern to the bettors; it also creates the potential for abnormal payoffs. In the ideal situation, winning takes place wherever the bettors have more information or take advantage of wager volatility movements. The more uncertain the wager volatility and the

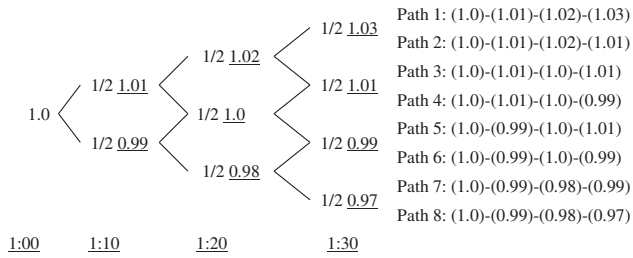
lower the switching cost, the more valuable is the bettor's ability to respond to new information swiftly. We combine features of the wager volatility and the bettor's ability to switch between teams in this article. We start out with the premise that bettors can choose team or combination of teams to bet, and have the options to switch between teams by purchasing an option after paying a switching cost. The relative cost or return for the holding versus the switching strategy is determined by the team-specific wager volatility and is thus variable over time.

The rest of this article is organized as follows: Section II presents the literature review about baseball option and models of sport researches. In Section III, we discuss the concept of baseball options where we present the application of switching options to the valuation of baseball game betting. Some rules in the assumptions and variables to estimate the variance in the baseball options are proposed. Section IV gives numerical examples to compare and analyse the results of baseball options. We use Stochastic Dynamic Programming (SDP) procedure to discuss risk implications and flexibility. Conclusions are presented in Section V.

## II. Literature Review

Early researchers sought evidence of inefficiencies in the form of systematic biases in bookmakers' prices, such as the home-away team or favourite-longshot biases. More recently, forecasting models have been used to establish whether historical information available in previous match results can be extrapolated to formulate profitable betting strategies. *Gandar et al. (2001)* are sceptical over the existence of systematic biases favouring bets on home teams in baseball and basketball.

*Moroney (1956)* and *Reep et al. (1971)* use the Poisson and negative binomial distributions to model at an aggregate level the



**Fig. 1. Numerical example – paths of wager in the first four innings**

distributions of the numbers of goals scored per game, but this approach precludes the generation of specific forecasts for individual matches based on information about the respective strengths of the two teams. Hill (1974) demonstrates that individual match results do have a predictable element, and are not determined solely by chance. Maher (1982) develops a model in which the home and away team scores follow independent Poisson distributions, with means reflecting the attacking and defensive capabilities of the two teams. A full set of attacking parameters and a set of defensive parameters for each team are estimated *ex post*, but the model does not predict scores or results *ex ante*. A tendency to underestimate the proportion of draws is attributed to interdependence between the home and away scores, and corrected using the bivariate Poisson distribution to model scores. Dixon and Coles (1997) develop a forecasting model capable of generating *ex ante* probabilities for scores and match outcomes. The home and away team scores follow independent Poisson distributions, but for low-scoring matches an ad hoc adjustment allows for interdependence. Using a similar framework, Rue and Salvesen (2000) assume that the time-varying attacking and defensive parameters of all teams vary randomly over time.

In the real world of sport, once the bettor bets on a team, he/she seldom can switch bet between teams, which greatly limits the flexibility of betting. In fact, the best team in baseball has the lowest probability to win an average game compare with other sports such as football and basketball. Switching options thus add decision dynamics to the betting. We explain in more details about the switching between teams in Fig. 1, Tables 4 and 5. With the introduction of switching options in the baseball game, it serves three purposes. First, the ability to switch teams offers the bettor capability to ‘hedge’ during the game with little cost; second, the introduction of the switching options to the baseball game betting enhances the liquidity of the wager; and finally, introducing options to the baseball game betting offers the bettors more time to observe and to make decisions, hence delay his/her ‘investment decisions’ until more information becomes available. Most of the game models use the Poisson distribution, bivariate Poisson distribution and negative binomial distributions model (e.g. Dixon and Coles, 1997; Rue and Salvesen, 2000; Crowder *et al.*, 2002). This article thus uses the bivariate Poisson dynamic stochastic model outlined above to identify potentially profitable betting strategies.

### III. An Application of Switching Options to Baseball Game Betting

Although applications of option pricing models to financial assets are abundant, applications to real assets are less popular. To be sure, there are applications of option model to subject matters such as R&D project evaluation and selection (e.g. Newton and Pearson, 1994; Robert, 1998); nevertheless, application to sports is absent.

We build our scheme based upon the concept of consumer switching costs (Grossman, 1981; Klemperer, 1995), and the notion of the value of an option to exchange one asset for another (Margrabe, 1978).

#### The money line and switching options

In the traditional baseball games, sportsbooks use a money line for betting baseball. The money line determines the amount of money laid and money won. The money line, however, does not provide the flexibility of allowing the bettor to switch bet between teams during the game. The bettor cannot change his/her original bet during the game when condition changes. Therefore, this article proposes a switching options approach to baseball games, allowing bettor the option to switch bets between the two teams during the game. Hence it is a combination of traditional money line with the real option model.

The proposed baseball option relies on the concept of switching option – a variation of real options. To do so, we first design the rules for the model; then we demonstrate how to measure primary variables and calculate the value of these variables related to the baseball options. Finally, we use these variables to price switching baseball options.

#### The rules of baseball options

This article presents a framework for baseball options in a manner similar to stock options. We assume that there is a home team competing against the guest team in a single game in which there are nine innings. To facilitate the discussions, the following assumptions are made without losing generality:

- (1) There are no more than nine innings in a single game; tie is permitted.
- (2) The quality of teams is very competitive through a fair scheme of public selection and a free market for trading bettors.
- (3) Game continues until the end of the ninth inning.
- (4) The duration of the game is assigned a value of 1.5 hours, hence for each inning is 10 minutes.
- (5) Standard deviation is estimated based upon the team’s past performance.
- (6) The bettor incurs a switching cost for changing bet between teams.
- (7) Switching is no longer permitted when the difference in scores exceeds ‘y’ points in a single game.
- (8) Betting is no longer allowed after the *n*th inning (e.g. fourth inning).

#### The value of the switching options

Switching options allow the bettors to switch bet from one team to another during the game at a fixed cost. This option is more versatile than the Black–Scholes options pricing model (1973), and is more adapted to the real-world situation.

In the real options framework, the bettor holds the real option of switching between the two teams. It is therefore interesting to determine the value of this switching option, i.e. what additional value the bettor receives from having the option to switch bet between teams. We define the value of switching options as follows:

$$v_{AB}(p_0) = \max(v_A(p_0), v_B(p_0)), \quad (1)$$

where  $p_0 > 1$  is the initial present value of wagering on team A. In Equation 1,  $v_{AB}(p_0)$  is the maximum value to the bettor from

having the option of switching bets between teams A and B and  $\max(v_A(p_0), v_B(p_0))$  is the maximum value from betting only on one of the two teams. We assume optimal policies exist within each of the game determined by the boundary conditions at the switching points at which bettor switches between teams (Dixit, 1989).

Assume that a bettor has already betted on A. Due to the seesaw game, the bettor is now considering the following options: continue the bet with team A; or buy a flexible switching option that allows him/her to switch from team A to B for a cost of  $\$C_{AB}$  (or from B to A for  $\$C_{BA}$  if the initial bet is on B) We then use the Real Options Analysis (ROA) method to value the flexibility feature of the option (Copeland and Antikarov, 2001). Since the value of switching option is the marginal value of such flexibility, the first question we need to answer is: given the choice of only either team A or team B, which one should be wagered on? A simple decision can be made based on the Net Present Value (NPV) of the expected cash flows each team provides. Following the binomial model, we can compute the value of a real option as (Cox et al., 1979)

$$ROV = V_0 B(n \geq a|T, p) - K(1+r)^{-T} B(n \geq a|T, p'), \quad (2)$$

where  $p$  is the up-movement probability such that  $p = (1+r-d)/(u-d)$ ,  $p' = u/(1+r)p$ .

ROV denotes the current real option value;  $V_0$  is the value of expected cash flow;  $B(n \geq a|T, p)$  is the cumulative probability of having an in-the-money option (i.e.  $n \geq a$ ), where the probabilities are the certainty-equivalent probabilities determined by the risk-free hedge portfolio; and  $K(1+r)^{-T}$  is the discounted initial cash out-flow. Assuming uniform through the game, the up-movement factor for each inning is defined as  $u = e^{\sigma\sqrt{\Delta t}}$ , and the down-movement factor is defined as  $d = e^{-\sigma\sqrt{\Delta t}}$ , where  $\sigma$  quantifies the uncertainty of expected cash flows,  $r$  denotes the risk-free interest rate and  $T = 1.5$  hours so that  $t$  is 10 minutes (nine innings in a game). Giving the risk-adjusted discount rate per period ( $k$ ), the risk-free rate of return and the objective probability for up- (down-) movement on every step, we can discount the expected cash flows along the event tree for team A and obtain a corresponding tree with the present values. The present value for team A at state  $y$  can thus be expressed in the following:

$$PV_{Ay} = \frac{p \times PV_{Aw} + (1-p)PV_{Az}}{(1+k)} \quad (3)$$

Equation 3 indicates that the present value of betting on team A at each possible stage of nature (inning) is equal to the expected value the team offers in the future discounted at a risk-adjusted discount rate. Next, let us assume that the bettor betted on team A at the previous state, he/she can stay with team A or switch to team B and pay the switching cost  $C_{AB}$ . The optimal decision rules at stage  $y$  are:

$$\begin{aligned} S_{Ay} &= \text{MAX}(PV_{Ay}, PV_{By} - C_{AB}), \text{ if} \\ PV_{Ay} &> PV_{By} - C_{AB}, \text{ stay with A} \\ PV_{Ay} &< PV_{By} - C_{AB}, \text{ switch to B} \end{aligned} \quad (4)$$

After we have identified an optimal switching for the ending period state in both decision trees, we work backward to analyse the states before the ending period state, and find the present value for the two decision trees. We then check to see whether it is optimal to stay with their current team or to switch to a different team. Repeat the same process and work backward for each state all the way to the beginning period state. The final result provides an optimal contingent plan for executing the available options. We now can price

the options-to-switch between the two teams. The marginal value of real options A (B) with the flexibility to switch to team B (A) is thus the difference between the real options value with switching option ( $ROV_{A,B}$ ) and the NPV of betting on one team without the switching feature ( $NPV_{A,B}$ ), that is

$$V_{A,B} = (ROV_{A,B}) - (NPV_{A,B}) \quad (5)$$

### The estimation of variance in the baseball options

Since our idea of baseball options evolves from the concept of real options, it is necessary to find the underlying asset standard deviation-equivalent in the baseball option. Therefore, one of the important concepts of this article is to objectively determine a feasible standard deviation for computing the baseball options price. We propose to estimate the expected variance of the underlying asset value,  $\sigma^2$ , based upon three important factors that determine the outcome of a baseball game. Since the underlying asset of real options is often not traded, a proxy for volatility has to be found. (Copeland) This proxy can be a Monte Carlo simulation of the value of the project, another traded asset, or a synthetic portfolio of volatility that affect the outcome of a baseball game. We choose a synthetic portfolio of volatility that affects the outcome of a baseball game and these factors are the past performance of baseball professionals. To create a synthetic portfolio of volatility that affect the outcome of a baseball game, we use the following three factors, which are generally considered the most influential factors by baseball professionals (see Yang and Swartz, 2004 for a very similar argument). These three factors are: (1) Earned Run Average (ERA), (2) line-up players' batting average (AVG) and (3) fielding percentage (FLP). They are defined as:

$$\begin{aligned} \overline{\text{ERA}} &= \sum_{i=1}^n (\text{innings pitched by pitcher } i/9^* \\ &\times \text{pitcher } i\text{'s ERA}), \text{ where} \end{aligned}$$

$$\text{ERA} = (\text{earned runs} * 9)/(\text{innings pitched})$$

$$\text{AVG} = \frac{\text{Hit}}{\text{At Bats}} \text{ where}$$

$$\text{At Bats} = (\text{Plate appearances} - \text{walks} - \text{sacrifice hits})$$

$$\text{FLP} = \frac{(\text{Total Chances} - \text{Errors})}{\text{Total Chances}}$$

Without the loss of generality, we assume that these three factors share equal weights; i.e. one-third each, in determining the variability of the payoffs. Moreover, since there are nine innings and two types of pitchers – starting and relief, we use two weights for the ERA,  $X/9$  and  $(9-X)/9$ , where  $X$  is the number of innings pitched by the starting pitcher. Therefore, proxy of the variability of the payoffs can be written as:

$$\begin{aligned} \sigma^2 &= \frac{1}{3} \sigma_{(\text{AVG of line-up players} - \text{AVG of all players in league})}^2 \\ &+ \frac{1}{3} \sigma_{(\text{FLP of line-up players} - \text{FLP of all players in league})}^2 \\ &+ \frac{1}{3} \left[ \frac{X}{9} \sigma_{(\text{ERA of starting pitcher} - \text{ERA of all players in league})}^2 \right. \\ &\left. + \frac{9-X}{9} \sigma_{(\text{ERA of relief pitcher} - \text{ERA of all players in league})}^2 \right] \end{aligned} \quad (6)$$

where  $X$  is the number of innings pitched by the starting pitcher.

**Table 1. Team A line-up players' ERA, AVG, FLD**

Line-up (player)	ERA	AVG (batting average)	FLD (fielding percentage)
3B		0.256	0.960
RF		0.161	0.955
SS		0.421	0.957
DH		0.500	1
1B		0.444	0.974
CF		0.462	1
LF		0.400	0.929
2B		0.476	0.969
C		0.310	1
SP	5.290		
RP	0		
CP	7.360		
Average	5.170	0.381	0.972

This estimated  $\sigma^2$  then can be used to calculate the node values of the decision tree, the present value event tree and finally the real options value of betting on each baseball teams in the game. Incorporating the ROA, bettors are allowed to switch between teams with a switching cost. This flexibility offers bettors the best decision-making tool during each innings, and the value of switching right can also be calculated using ROA. Such real option is known as the switching option.

#### IV. Switching Options of the Baseball Game: Numerical Examples

In the following subsections, we use an example of a single game to analyse the baseball options. Home team A and guest team B are assumed very competitive teams although the historical statistics slightly favour team A.<sup>1</sup>

##### An example of baseball game

To simplify the discussions, we assume two teams are competitive; hence it is difficult to know which team will win the game. Following the standard binomial tree option valuation procedure, we first compute the option value for each node in the last period state, discounting back to the present value for the state before the last one, and continue this process until the option value for the beginning period state is obtained.

To employ the baseball options, however, we must first estimate the value of wager volatility following Equation 6. To estimate this value, the AVG, FLP and ERA of home team A and guest team B are shown in Tables 1 and 2, while Table 3 shows the statistics for all league players' averages. The switching cost from team A to team B is assumed to be \$25. Based upon our hypothetical example, the estimated wager variability for team A using Equation 6 is thus  $\sigma_A^2 = 4.369$  (see Table 3). For team B, we obtain very similar up- and down-movement factors, i.e.  $u \approx 1.01$ . Hence the probability for up- and down-movement is also close to 0.5.

For example, assuming annual risk-free rate of 3%, each inning is about 10 minutes, we have  $\Delta t = 10/(365*24*60) = 1.90259E-05$ , hence  $r = 0.03*10/(365*24*60) = 5.70776E-07$ , both are very

**Table 2. Team B line-up players' ERA, AVG, FLD**

Line-up (player)	ERA	AVG (batting average)	FLD (fielding percentage)
CF		0.304	1
SS		0.333	1
2B		0.258	0.942
3B		0.176	0.867
DH		0.308	1
1B		0.297	0.969
LF		0.242	0.941
C		0.200	1
RF		0.250	1
DH		0.222	0.917
C		0	1
SP	7.260		
RP	0		
CP	0		
Average	4.300	0.236	0.967

**Table 3. All players' average statistics**

League AVG	League FLP	League ERA	Team A's $\sigma^2$	Team B's $\sigma^2$
0.243	0.805	5.234	4.369	3.224

small numbers. From Table 3, since  $\sigma^2 = 4.369$  for team A, hence the up- and down-movement factors are calculated as  $u \approx 1.01, d = 1/u = 0.99$ , respectively. The probabilities of up-movement (and down-movement) are thus  $p \approx .5$ . Since the risk-free interest rate and time are very small unit, therefore, for team B  $u \approx 1.01, p \approx .5$ , which is very similar to team A's statistics. Based upon the above explanations, the calculated up- and down-movement factors are close to zero irrespective of  $\sigma$  and interest rate. This is because the short time horizon renders the impact of  $\sigma$  and interest rate negligible.<sup>2</sup>

This result reflects that both teams are competitive in our hypothetical example. In the real-world games, teams are often competitive and the ending results are not easy to predict. The most recent World Cup soccer game is a good example, in which the 'favourites' (e.g. Brazil and Argentina) lost to the 'underdogs' (e.g. Germany and the Netherlands).

Risk is managed by means of purchasing bets for which there are two strategies: (1) Holding team A or B incurring a beginning investment of \$1000; (2) Switching between teams A and B incurring an additional cost of (non-recurrent) \$25. At the beginning of each new 10 minutes (innings), the casinos obtain an estimate of cash flow in the first four innings. So, for example, at one o'clock estimated cash flows are obtained for the next 30 minutes, i.e. second, third and fourth innings. We assume bettors are allowed to bet only during the first four innings because outcome becomes less uncertain as games progress. To facilitate our discussions and compare with the switching strategy in the numerical example, we further assume that the bettor makes a wager in the first four innings. The wager volatility determines whether investment in team A or B is cheaper. For

<sup>1</sup> Handicap-point spread is often used in cases where teams are not competitive, we skip this possibility to avoid unnecessary analytical complications.

<sup>2</sup> Interest rate in the option pricing model is the annual interest rate because time unit in such model is a fraction of a year. In the baseball game, the time duration in each inning is very short (typically 10 minutes or so), hence converting the annual interest rate to per inning equivalent rate results in a near-zero risk-free rate. The near-zero risk-free rate makes the discount rate  $(1 + r)$  equal to one. As a result, only the expected cost at the  $k$ th inning is taken into consideration.



**Table 4. Deriving cost-minimizing decisions by means of SDP when switching cost = \$25**

Wager paths	Existing strategy	\$ Costs of continuing existing strategy	\$ Costs of switching to alternative strategy
<i>Cost-minimizing decisions at 1:30</i>			
1.03	A	<u>1000</u>	1030 + 25 = 1055
1.01	A	<u>1000</u>	1010 + 25 = 1035
0.99	A	<u>1000</u>	990 + 25 = 1015
0.97	A	1000	970 + 25 = <u>995</u>
1.03	B	1030	1000 + 25 = <u>1025</u>
1.01	B	<u>1010</u>	1000 + 25 = 1025
0.99	B	<u>990</u>	1000 + 25 = 1025
0.97	B	<u>970</u>	1000 + 25 = 1025
<i>(Expected) Cost-minimizing decisions at 1:20</i>			
1.02	A	$1000 + 1/2(1000 + 1000) = \underline{2000}$	$1020 + 25 + 1/2(1025 + 1010) = 2062.5$
1.0	A	$1000 + 1/2(1000 + 1000) = \underline{2000}$	$1000 + 25 + 1/2(1010 + 990) = 2025$
0.98	A	$1000 + 1/2(1000 + 995) = 1997.5$	$980 + 25 + 1/2(990 + 970) = \underline{1985}$
1.02	B	$1020 + 1/2(1025 + 1010) = 2037.5$	$1000 + 25 + 1/2(1000 + 1000) = \underline{2025}$
		$1000 + 1/2(1010 + 990) = \underline{2000}$	
1.0	B	$980 + 1/2(990 + 970) = \underline{1960}$	$1000 + 25 + 1/2(1000 + 1000) = 2025$
0.98	B		$1000 + 25 + 1/2(1000 + 995) = 2022.5$
<i>(Expected) Cost-minimizing decisions at 1:10</i>			
1.01	A	$1000 + 1/2(2000 + 2000) = \underline{3000}$	$1010 + 25 + 1/2(2025 + 2000) = 3047.5$
0.99	A	$1000 + 1/2(2000 + 1985) = \underline{2992.5}$	$990 + 25 + 1/2(2000 + 1960) = 2995$
1.01	B	$1010 + 1/2(2025 + 2000) = \underline{3022.5}$	$1000 + 25 + 1/2(2000 + 2000) = 3025$
0.99	B	$990 + 1/2(2000 + 1960) = \underline{2970}$	$1000 + 25 + 1/2(2000 + 1985) = 3017.5$
<i>(Expected) Cost-minimizing decisions at 1:00</i>			
1.0	A	$1000 + 1/2(3000 + 2992.5) = \underline{3996.25}$	$1000 + 25 + 1/2(3022.5 + 2970) = 4021.25$
1.0	B	$1000 + 1/2(3022.5 + 2970) = \underline{3996.25}$	$1000 + 25 + 1/2(3000 + 2992.5) = 4021.25$

Note: The \$ costs of the decision that minimizes (expected) costs is underlined.

simplicity the initial wager is assumed to be \$1000 at one o'clock and the sequence follows a basic binomial process with an equal chance of up and down where the up-movement factor is 1.01, while the down-movement factor is 0.99. Hence, expectations in the present case are assumed static (the current rate is expected to prevail in the future). The possible wager developments over the bettor's planning horizon are reflected in the binomial tree shown in Fig. 1. Following this procedure, the wager variation of betting on team A over the first four innings can be shown in the following binomial tree as:

The statistics shown in Fig. 1 can be interpreted as the wagers during different stages of the game given the volatility estimate. For example, the wager increases to 1.03 at 1:30 if one wishes to bet at this stage of the game. This is because the chance for team A to win the game at this point of time is enhanced, given the path of the development. On the other hand, the wager decreases to 0.97 in the event of moving along the down-path consecutively. The wagers shown in different nodes of the tree thus represent the cost of betting if one enters the game betting at different time of the game. These numbers are calculated based upon a hypothetical estimate of  $\sigma^2$ , which may vary in the real-world setting.

**SDP and risk implications**

In this subsection, the cost-minimizing decisions are determined first and then their risk implications are assessed. Cost-minimizing decisions are not directly obvious, since the optimal decisions at any time depend on decisions taken in the past due to the existence of switching costs and uncertainty in payoffs. SDP (Dixit and Pindyck, 1994) is the standard procedure for such decision-making. The first step is to calculate optimal decisions for the last period in a binomial

tree for all possible payoffs and for the two investment strategies (i.e. switching or status quo). Cost-minimizing decisions for all possible states are obtained by comparing the costs of continuing the existing strategy to the costs of switching to the other strategy. Then, having obtained the best decisions for 1:30, those for 1:20 can be determined and the procedures go on. This procedure is illustrated in Table 4 which also shows that the optimal decision in each period depends on the prevailing wager and the team the bettor invested before. Note that the SDP procedure used in these calculations is the same as the procedure used to determine the value of an American option (Cox et al., 1979).

The probability distribution of investment costs, and hence the risk implications of cost minimization can be obtained by tracking the eight possible wager paths (Fig. 1) and determining cost-minimizing decisions in each case (Table 4).

If the switching cost is \$25, the cost-minimizing decisions are shown in Table 4. The calculations in Table 4 show that a cost-minimizing bettor initially betting on team A should definitely stick to this strategy until 1:30 and should switch to team B only when the wager path has become 0.97. If the bettor's initial bet is on team B, it should definitely stick to this strategy until 1:30 and should switch to team A only when the wager path has become 1.03. Using the procedure of SDP, we can obtain the minimum expected costs of \$3996.25. The total investment is \$4000 (assuming \$1000 each inning for the ease of comparing with the switching strategy); but because the risk-free interest rate is taken to be near zero, the discount factor = 1, then the NPV = 4000. As a result, the value of switching option is \$4000-\$3996.25 = \$3.75.

If the switching cost is lowered to \$10, the cost-minimizing decisions are shown in Table 5. The calculations in Table 5 show that a

**Table 5. Deriving cost-minimizing decisions by means of SDP when switching cost = \$10**

Wager path	Existing strategy	\$ Costs of continuing existing strategy	\$ Costs of switching to alternative strategy
<i>Cost-minimizing decisions at 1:30</i>			
1.03	A	<u>1000</u>	1030 + 10 = 1040
1.01	A	<u>1000</u>	1010 + 10 = 1020
0.99	A	<u>1000</u>	990 + 10 = <u>1000</u>
0.97	A	1000	970 + 10 = <u>980</u>
1.03	B	1030	1000 + 10 = <u>1010</u>
1.01	B	<u>1010</u>	1000 + 10 = <u>1010</u>
0.99	B	<u>990</u>	1000 + 10 = 1010
0.97	B	<u>970</u>	1000 + 10 = 1010
<i>(Expected) Cost-minimizing decisions at 1:20</i>			
1.02	A	$1000 + 1/2(1000 + 1000) = \underline{2000}$	$1020 + 10 + 1/2(1010 + 1010) = 2040$
1.0	A	$1000 + 1/2(1000 + 1000) = \underline{2000}$	$1000 + 10 + 1/2(1010 + 990) = 2010$
0.98	A	$1000 + 1/2(1000 + 980) = 1990$	$980 + 10 + 1/2(990 + 970) = \underline{1970}$
1.02	B	$1020 + 1/2(1010 + 1010) = 2030$	$1000 + 10 + 1/2(1000 + 1000) = \underline{2010}$
1.0	B	$1000 + 1/2(1010 + 990) = \underline{2000}$	$1000 + 10 + 1/2(1000 + 1000) = 2010$
0.98	B	$980 + 1/2(990 + 970) = \underline{1960}$	$1000 + 10 + 1/2(1000 + 980) = 2000$
<i>(Expected) Cost-minimizing decisions at 1:10</i>			
1.01	A	$1000 + 1/2(2000 + 2000) = \underline{3000}$	$1010 + 10 + 1/2(2010 + 2000) = 3025$
0.99	A	$1000 + 1/2(2000 + 1970) = 2985$	$990 + 10 + 1/2(2000 + 1960) = \underline{2980}$
1.01	B	$1010 + 1/2(2010 + 2000) = 3015$	$1000 + 10 + 1/2(2000 + 2000) = \underline{3010}$
0.99	B	$990 + 1/2(2000 + 1960) = \underline{2970}$	$1000 + 10 + 1/2(2000 + 1970) = 2995$
<i>(Expected) Cost-minimizing decisions at 1:00</i>			
1.0	A	$1000 + 1/2(3000 + 2980) = \underline{3990}$	$1000 + 10 + 1/2(3010 + 2970) = 4000$
1.0	B	$1000 + 1/2(3010 + 2970) = \underline{3990}$	$1000 + 10 + 1/2(3000 + 2980) = 4000$

Note: The \$ costs of the decision that minimizes (expected) costs is underlined.

**Table 6. The value of flexibility: outcome of cost-minimizing decisions under different wager path scenarios**

Wager path	Switching cost = \$10		Switching cost = \$25	
	A (a)	B (b)	A (c)	B (d)
Path 1: (1.0)-(1.01)-(1.02)-(1.03)	4000	4010	4000	4035
Path 2: (1.0)-(1.01)-(1.02)-(1.01)	4000	4020	4000	4060
Path 3: (1.0)-(1.01)-(1.0)-(1.01)	4000	4020	4000	4020
Path 4: (1.0)-(1.01)-(1.0)-(0.99)	4000	4020	4000	4000
Path 5: (1.0)-(0.99)-(1.0)-(1.01)	4020	4000	4000	4000
Path 6: (1.0)-(0.99)-(1.0)-(0.99)	4000	3980	4000	3980
Path 7: (1.0)-(0.99)-(0.98)-(0.99)	3980	3960	4020	3960
Path 8: (1.0)-(0.99)-(0.98)-(0.97)	3950	3940	3975	3940
Average costs	3993.75	3993.75	3999.375	3999.375

cost-minimizing bettor initially betting on team A should definitely stick to this strategy until 1:30 and should switch to team B only when the wager path has become 0.97. If the initial bet is on team A, there is no difference either to switch or to stay status quo at 1:30 when the wager path is 0.99. Similarly, if the initial bet is on team B, there is no difference whether to switch or to hold onto the original team when the wager path has become 1.01. When the switching costs are lowered from \$25 to \$10; the value of switching option is changed from \$3.75 to \$10 (\$4000–\$3990). Therefore, we find that the lower the switching cost, the higher the value of switching option.

**The value of flexibility**

Based upon the statistics shown in Tables 4 and 5, Table 6 summarizes the value of flexibility. The statistics shown in columns (a) and

(b) in Table 6 can be derived directly from Table 5. Columns (c) and (d) can be derived directly from Table 4.

Teams with relatively low switching costs benefit from wager uncertainty: Table 6 shows that the lower the switching cost, the lower the investment cost. When switching cost = 10, the expected investment cost for a bettor that switches optimally is \$3993.75 regardless of its initial strategy, whereas a bettor that starts with team A and sticks to it pays a certain \$1000. Moreover, wager volatility should be assessed relative to the degree of uncertainty.

When the switching costs are lowered from, say, \$25–\$10, we find that investing in flexibility decreases expected costs and lowers risk: Comparing (c) and (d) with (a) and (b) shows that investing in lower switching costs yields two benefits: expected investment costs fall (from \$3999.375 to \$3993.75) and so does risk (the worst scenario costs now \$4020 instead of \$4060). Hence, the investment in switching should definitely be made if it costs \$5.625 or less (i.e.

\$3999.375–\$3993.75). If it costs more, nevertheless, it may still be worthwhile if enough value is attached to the risk reduction.

## V. Conclusions

Investing in flexibility serves an important purpose, especially if uncertainty is high. The uses of options are based on an assumption of aversion to downside risk. This implies that the objective of managing risk should be to exploit the wager volatility in order to decrease the bettor's cost, while ensuring that the downside risk remains sufficiently small. In this article, we introduce the concept of baseball options with a switching feature. More specifically, we propose the use of switching options to facilitate the pricing of baseball games. We believe this is the first attempt to apply the real options theory to baseball game scheme. To do so, we first design some rules in our assumptions and define variables that are related to the baseball options. We then use the ROA to demonstrate the decision rules for betting a baseball game. The concept of switching options provides the bettors with flexibility to switch between bets.

In Section IV, we suggest that a bettor who can switch between a home team and a guest team is better off than an otherwise identical bettor that has only access to one strategy. It is shown that by investing in lower switching costs, the bettors can attain his/her dual objective of minimizing expected costs and limiting the downside risk of wager uncertainty. Moreover, strategies that at first sight seem unattractive may actually be optimal by virtue of their higher flexibility. In general, an investment in flexibility also incurs costs. These costs should be weighed against the benefits; i.e. reduction in risk.

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