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Fuzzy multicriteria flight gate assignment

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Manuskripte
aus den
Instituten für Betriebswirtschaftslehre
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No. 605

Fuzzy multicriteria flight gate assignment¹

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Working Paper, May 2006

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Abstract

This paper addresses the multiple criteria flight gate assignment problem under uncertainty, which is naturally modeled by fuzzy numbers. The problem examined is a special kind of multicriteria multi-mode resource-constrained project scheduling problem with generalized precedence constraints or time windows. Fuzziness is introduced for basic problem parameters, that is, arrival and departure times, in order to cover possible earliness and tardiness of flights. We solve the problem directly by a special multicriteria metaheuristic, known as fuzzy Pareto simulated annealing, in order to get a representative approximation of the Pareto front.

Keywords: flight gate assignment, multicriteria optimization, project scheduling, Pareto simulated annealing, fuzziness

1 Introduction

The main purpose of flight gate scheduling is to assign different aircrafts to distinct stands or gates. From a practical point of view flight gate assignment has to take into account two important factors: First, multiple optimization criteria. As it was generally shown in [13], a solution which is optimal with respect to one single objective might be arbitrarily bad with respect to other criteria and thus will be unacceptable for the decision maker. Second, uncertainty of problem parameters which in the flight gate assignment arena is mainly due to delayed arrival of flights.

In [5] mathematical models and state-of-the-art solution techniques developed for solving a general flight gate scheduling problem have been presented. Two models which reflect the peculiarities adopted in airports of the United States and of the European Union have been specified. The first model is based on the quadratic assignment problem (studied in detail in [7]). The second model which we consider in this paper has originally been proposed in [4]. The purpose of the model is to assign three possible aircraft activities (arrival; optional intermediate parking, the length of which depends on the ground time; departure) to the available airport flight gates and to schedule start and completion times of the activities at the positions. The problem is a multicriteria multi-mode resource-constrained project scheduling problem with generalized precedence constraints or time windows (for a survey see, e.g., [2]). While in previous work the problem has been simplified to a single objective counterpart, in [8] it is solved directly by means of a multicriteria metaheuristic.

The main drawback, however, of the model as it has been dealt with so far is its deterministic nature, that is, the assumed crispness of basic parameters such as flight arrival and departure times. The first attempt to eliminate this drawback is the clique partitioning approach of [6], the single objective of which covers an additional component. This extra component addresses the buffer time, originally proposed in [1], and it is intended to avoid the assignment of two flights with low buffers time to the same gate. Apparently, this policy tends to produce schedules which are more stable with respect to small tardiness and earliness of flights. However, the price the authors of [6] do have to pay in order to get a clique partitioning reformulation, is to take the starting and finishing time of parking activities as constants instead of decision variables. This restriction reduces the original project scheduling model proposed in [4], to a particular assignment problem. In this research we propose a novel way to escape the disadvantage of crispness of arrival and departure times by defining uncertainty using the concept

of fuzzy numbers. Doing so, we are able to keep the original characteristics and features of the model from [4] which are very attractive from a practical point of view.

The paper is organized as follows. In section 2 we shortly describe the deterministic model. Section 3 is devoted to the concepts of fuzziness, fuzzy numbers and fuzzy arithmetic. In section 4 we show how to transfer the deterministic model into a fuzzy counterpart. Additionally we describe a fuzzy Pareto simulated annealing algorithm which is capable to approximate the Pareto front. A summary is given in section 5.

2 Model with crisp arrival and departure times

To make our paper self-contained we give a brief description of the deterministic model. For more details, see [4] and [8].

Let n be the number of aircraft (flights) and m be the number of terminal stand positions (gates). Each aircraft activity (arrival, departure or parking) i can be described by its start time S_i and by its completion time C_i . The start time for an arrival activity t_i^a and the completion time of a departure activity t_i^d are fixed and given a priori according to some time-table. All other start and completion times are decision variables of the model. Thus, we have $3n$ aircraft activities, and $4n$ decision variables related to time. Additionally, we have n implicit variables related to gate assignment. In total the number of variables is equal to $5n$.

Let V denote the set of all activities as a unification of the sets of arrival, parking and departure activities, respectively, that is, $V = V^a \cup V^p \cup V^d$. Each activity i has a minimum required processing time p_i^{min} . Activity i can be assigned to different flight gates (that is, can be processed in different modes) M_i from the associated mode set \mathcal{M}_i which is a subset of the set of all possible modes \mathcal{M} . To cope with the situation where the constraints do not allow to assign all aircraft to real gates, a fictitious gate 0 with unlimited capacity is introduced. Every set \mathcal{M}_i contains this dummy gate, and an assignment to the dummy gate will be penalized in the objective.

If two interrelated (linked) activities are assigned to different flight gates, then they require a towing procedure in order to be moved from one position to another. Two activities are linked if they are subsequently served by the same aircraft (e.g., arrival/parking or parking/departure). Towing activity i assigned to gate M_i to activity j assigned to gate M_j takes some fixed processing time $d_{iM_i j M_j}^{tow}$. Let \mathcal{E}^{tow} represent the set of linked activities. It follows that the completion and start times of two linked activities i and j should satisfy the equality $C_i + d_{iM_i j M_j}^{tow} = S_j$ to provide continuous processing. In fact, this means that among all $4n$ decision variables only $2n$ variables are independent. The values $d_{iM_i j M_j}^{tow}$ are usually defined by a function $d_{iM_i j M_j}^{tow} := f^{tow}(i, M_i, j, M_j)$.

Gates are disjunctive resources that can only process one aircraft at a time (the only exception is, of course, dummy gate 0). Between the processing of two activities i and j at the same gate a fixed setup time $d_{iM_i j M_j}^{setup} \in \mathbb{N}_0$ must pass. The setup time can reflect the time required to push the first aircraft back from the gate and for moving the second aircraft to the gate as well as the time required for setting up gate equipment. Observe also that, if i and j are two activities served by the same aircraft, i.e., $(i, j) \in \mathcal{E}^{tow}$, then $d_{iM_i j M_j}^{setup} = 0$. In analogy to the time needed for towing, the setup time is usually calculated by some function $d_{iM_i j M_j}^{setup} := f^{setup}(i, M_i, j, M_j)$.

Additionally to these basic disjunctive constraints we have to consider so-called ‘shadow’ restrictions (i, M_i, j, M_j) between gates M_i and M_j which can be interpreted as follows: if mode $M_i \in \mathcal{M}_i$ is assigned to activity i , then activity j must not be processed simultaneously in mode $M_j \in \mathcal{M}_j$. The set of shadow restrictions is denoted as \mathcal{E}^{shadow} .

Now, the model can be summarized as follows: Find a schedule (S, C, M) which assures constraints (1) to (5) and optimizes goals (6a) to (6c).

Minimal processing time

$$S_i + p_i^{min} \leq C_i \quad \forall i \in V \quad (1)$$

Continuous processing

$$C_i + d_{iM_i j M_j}^{tow} = S_j \quad \forall (i, j) \in \mathcal{E}^{tow} \quad (2)$$

Disjunctive activities and setup times

For all activities $i, j \in V$ such that either $M_i = M_j \neq \emptyset$ or $\exists (i, M_i, j, M_j) \in \mathcal{E}^{shadow}$ one of the following two conditions must be fulfilled:

$$C_i + d_{iM_i j M_j}^{setup} \leq S_j \quad (3a)$$

$$C_j + d_{jM_j i M_i}^{setup} \leq S_i \quad (3b)$$

Start and completion time

$$S_i = t_i^a \quad \forall i \in V^a \quad (4a)$$

$$C_i = t_i^d \quad \forall i \in V^d \quad (4b)$$

$$S_i, C_i \in \mathbb{N}_0 \quad \forall i \in V \quad (4c)$$

Mode selection

$$M_i \in \mathcal{M}_i \quad \forall i \in V \quad (5)$$

Objectives

The objective function is a vector containing the goals: (i) maximization of the total assignment preference score, (ii) minimization of the number of required towing operations, and (iii) minimization of the deviation from a given reference gate schedule. Hence, the objective function $z(M) := (z_1, z_2, z_3)$ is a vector with three partial objectives where

$$z_1 := \sum_{i \in V} w_i u_{iM_i}, \quad (6a)$$

$$z_2 := |(i, j) \in \mathcal{E}^{tow} : M_i \neq M_j|, \text{ and} \quad (6b)$$

$$z_3 := \sum_{i \in V : M_i \neq M_i^r} w_i. \quad (6c)$$

Typically $u_{i\mu} \in [0, 1]$ is a preference value associated with every activity-mode combination, $w_i \in [0, 1]$ is a priority weight associated with every activity and M_i^r denotes the reference gate of activity i , respectively. Apparently, the choice of appropriate preference weights and priorities may have a substantial impact on the optimal gate schedule. Note that any maximization

(minimization) objective can be converted into a minimization (maximization) objective by simply changing its sign.

Generally speaking project scheduling is concerned with single-item or small batch production where scarce resources have to be allocated to dependent activities over time. Accordingly, applications can be found in diverse industries such as construction engineering, software development, etc. Due to the variety of potential applications there do exist two main algorithmic streams of research. One of them does not take into account any specifics of potential applications, that is, it is concerned with the development of general purpose algorithms (for a general project scheduling model type under consideration). On the contrary researchers develop algorithms for specific 'project scheduling' application domains (as we do here). Of course, both streams are interrelated and mutually stimulate each other. A review of algorithms belonging to the first and to the second variety can be found in [8].

3 Uncertainty modelling, fuzzy numbers and operators

Among the most natural and popular ways of defining uncertainty is one which uses the concept of fuzzy numbers. In many respects, fuzzy numbers depict real-life more realistically than single-valued (exact) numbers. The definition of fuzzy numbers is based on the notion of fuzzy sets, a classical description of which will be give below (see, e.g., [15], too).

A fuzzy set is denoted by an ordered set of pairs, the first companion of which denotes an element of the ground set X and the second companion denotes the degree of membership

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}.$$

In order to introduce the concept of fuzzy numbers we also need the widespread definition of a level cut. The crisp set of elements that belongs to the fuzzy set \tilde{A} at least to the degree α is called the α -level cut of \tilde{A} , that is,

$$\tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}.$$

The set \tilde{A}'_α is called the strong α -level cut of \tilde{A} if

$$\tilde{A}'_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha\}.$$

Usually the decision maker has no comprehensive information about membership function. In [12] it was proposed to express expert estimates concerning parameter uncertainty on some prominent membership levels $\alpha \in (0, 1]$:

- $\alpha = 1$: $\mu(x) = 1$ means that x certainly belongs to X (absolute confidence level);
- $\alpha = \lambda$: $\mu(x) > \lambda$ means that x with $\mu(x) \geq \lambda$ has a good chance of belonging to X , that is, an expert is willing to consider x as desirable (high confidence level);
- $\alpha = \epsilon$: $\mu(x) < \epsilon$ means that x with $\mu(x) < \epsilon$ has a very small chance of belonging to X , that is, an expert may neglect it (low confidence level).

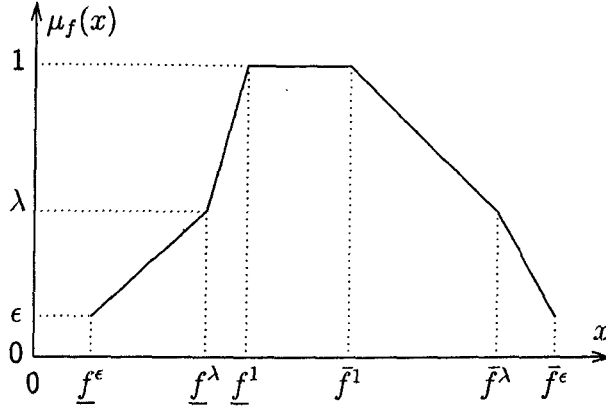


Figure 1: Six point representation of flat fuzzy number \tilde{F}

A plane (flat) fuzzy number is a special kind of fuzzy set, namely a normalized convex fuzzy set whose membership function is at least segmentally continuous, which has the membership value $\mu(x) = 1$ at precisely one element, and the ground set X of which represents the set of all rational numbers $X = \mathbb{R}$, respectively.

We will follow [10], where the six-point convention of a fuzzy number \tilde{F} represented by six real numbers is given as follows:

$$\tilde{F} := (\underline{f}^\epsilon, \underline{f}^\lambda, \underline{f}^1, \bar{f}^1, \bar{f}^\lambda, \bar{f}^\epsilon).$$

In other words a fuzzy number \tilde{F} is given by three nested intervals determined by three α -levels: 1-level cut $[\underline{f}^1, \bar{f}^1]$, λ -level cut $[\underline{f}^\lambda, \bar{f}^\lambda]$ and ϵ -level cut $[\underline{f}^\epsilon, \bar{f}^\epsilon]$. Observe that $[\underline{f}^1, \bar{f}^1] \subseteq [\underline{f}^\epsilon, \bar{f}^\epsilon] \subseteq [\underline{f}^\lambda, \bar{f}^\lambda]$. Notice also that in case $f = \underline{f}^1 = \bar{f}^1 = \underline{f}^\epsilon = \bar{f}^\epsilon = \underline{f}^\lambda = \bar{f}^\lambda$ we have a crisp number f .

We adopt the piece-wise linear form of representation of fuzzy numbers (see Figure 1). A well-known extension of Zadeh's principle [14] can be formulated as follows: the α -cut on a general arithmetic operator with two fuzzy numbers is equivalent to the arithmetic operator on the respective α -cuts. Therefore, the following binary operators are defined for two fuzzy numbers $\tilde{A} = (\underline{a}^\epsilon, \underline{a}^\lambda, \underline{a}^1, \bar{a}^1, \bar{a}^\lambda, \bar{a}^\epsilon)$ and $\tilde{B} = (\underline{b}^\epsilon, \underline{b}^\lambda, \underline{b}^1, \bar{b}^1, \bar{b}^\lambda, \bar{b}^\epsilon)$ in six-point representation:

$$\tilde{A} \oplus \tilde{B} = (\underline{a}^\epsilon + \underline{b}^\epsilon, \underline{a}^\lambda + \underline{b}^\lambda, \underline{a}^1 + \underline{b}^1, \bar{a}^1 + \bar{b}^1, \bar{a}^\lambda + \bar{b}^\lambda, \bar{a}^\epsilon + \bar{b}^\epsilon)$$

$$\tilde{A} \ominus \tilde{B} = (\underline{a}^\epsilon - \underline{b}^\epsilon, \underline{a}^\lambda - \underline{b}^\lambda, \underline{a}^1 - \underline{b}^1, \bar{a}^1 - \bar{b}^1, \bar{a}^\lambda - \bar{b}^\lambda, \bar{a}^\epsilon - \bar{b}^\epsilon)$$

$$\tilde{A} \times \tilde{B} = (\underline{a}^\epsilon \times \underline{b}^\epsilon, \underline{a}^\lambda \times \underline{b}^\lambda, \underline{a}^1 \times \underline{b}^1, \bar{a}^1 \times \bar{b}^1, \bar{a}^\lambda \times \bar{b}^\lambda, \bar{a}^\epsilon \times \bar{b}^\epsilon)$$

$$\tilde{A} / \tilde{B} = (\underline{a}^\epsilon / \underline{b}^\epsilon, \underline{a}^\lambda / \underline{b}^\lambda, \underline{a}^1 / \underline{b}^1, \bar{a}^1 / \bar{b}^1, \bar{a}^\lambda / \bar{b}^\lambda, \bar{a}^\epsilon / \bar{b}^\epsilon)$$

$$\widetilde{\max}\{\tilde{A}, \tilde{B}\} = (\max\{\underline{a}^\epsilon, \underline{b}^\epsilon\}, \max\{\underline{a}^\lambda, \underline{b}^\lambda\}, \max\{\underline{a}^1, \underline{b}^1\}, \max\{\bar{a}^1, \bar{b}^1\}, \max\{\bar{a}^\lambda, \bar{b}^\lambda\}, \max\{\bar{a}^\epsilon, \bar{b}^\epsilon\})$$

$$\widetilde{\min}\{\tilde{A}, \tilde{B}\} = (\min\{\underline{a}^\epsilon, \underline{b}^\epsilon\}, \min\{\underline{a}^\lambda, \underline{b}^\lambda\}, \min\{\underline{a}^1, \underline{b}^1\}, \min\{\bar{a}^1, \bar{b}^1\}, \min\{\bar{a}^\lambda, \bar{b}^\lambda\}, \min\{\bar{a}^\epsilon, \bar{b}^\epsilon\})$$

In order to compare two fuzzy numbers one can use two different rules – the weak comparison rule (WCR) and the strong comparison rule (SCR) – a detailed description of which is given in [10].

According to the WCR the degree C to which a fuzzy number \tilde{A} is greater than or equal to \tilde{B} is defined as

$$C(\tilde{A} \geq \tilde{B}) = J(\tilde{A}) - J(\tilde{B}), \quad (7)$$

where $J(\tilde{A})$ and $J(\tilde{B})$ are defuzzification functions calculated in six-point representation as follows:

$$J(\tilde{A}) = \frac{1}{4(1-\varepsilon)} \{(\lambda - \varepsilon)(\underline{a}^\lambda + \underline{a}^\varepsilon + \bar{a}^\lambda + \bar{a}^\varepsilon) + (1 - \lambda)(\underline{a}^\lambda + \underline{a}^\varepsilon + \underline{a}^1 + \bar{a}^1)\} \quad (8)$$

$$J(\tilde{B}) = \frac{1}{4(1-\varepsilon)} \{(\lambda - \varepsilon)(\underline{b}^\lambda + \underline{b}^\varepsilon + \bar{b}^\lambda + \bar{b}^\varepsilon) + (1 - \lambda)(\underline{b}^\lambda + \underline{b}^\varepsilon + \underline{b}^1 + \bar{b}^1)\} \quad (9)$$

According to the SCR we have

$$\tilde{A} \gg \tilde{B} \Leftrightarrow \max\{\tilde{A}, \tilde{B}\} = \tilde{A}. \quad (10)$$

Application of WCR (7) is most suitable when comparing fuzzy optimization results (that is, objective values). However, direct application of WCR during a scheduling algorithm might lead to unfeasibility (see, e.g., [10]). In order to avoid this this happen we will use the SCR (10). It is clear that if there are two fuzzy numbers \tilde{A} and \tilde{B} such that $\tilde{A} \gg \tilde{B}$ according to SCR (10), then after defuzzification using rules (8) – (9) we get the same type of inequality $A \geq B$ for crisp values A and B .

4 Fuzzy flight gate scheduling model

4.1 Fuzzy arrival and departure times

Fuzzification can be illustrated easily by means of an example. Assume that for a particular flight 100 observations concerning past arrival time delays are available. The duration of the delays (in minutes) and the observed frequency are given in Table 1; the data are illustrated graphically in Figure 2. Notice that longer delay does not imply lower probability with which it might occur.

Now we construct the so-called cumulative distribution. Doing so we consider an event “delay” not as independent (as in probability theory), but as a consequence of preceding events. In other words, a delay with duration 10 minutes never happens until we get a sequence of preceding delays from 1 minute up to 9 minutes considering delays as discrete events with discretization factor 1. In our particular example, e.g., the delay “5 minutes” is covered in all delays which are greater than or equal to 5 minutes, that is, it happens in 70 out of 100 delays. The cumulative distribution for our example is given in Table 2 and it is depicted in Figure 3.

Note that contrary to the explicit distribution, for the cumulative distribution longer delay ultimately means a value which is lower (or equal) than the probability value with which it might occur.

From the cumulative distribution we can easily derive a normalized possibility distribution for arrival delays assuming piece-wise linear approximation. The meaning of possibility in our case is relative “easiness” or “simplicity” with which a certain delay might occur. Table 3 and Figure 4 provide the possibility distribution for the example.

Since a decision maker has three levels of confidence, he or she can be assured of correct data only for those points where confidence levels cross the possibility distribution. All other data

Delay duration (min)	0-5	5-10	10-15	15-20	20-25	25-30
Number of delays	30	15	25	10	10	10

Table 1: Distribution of delays

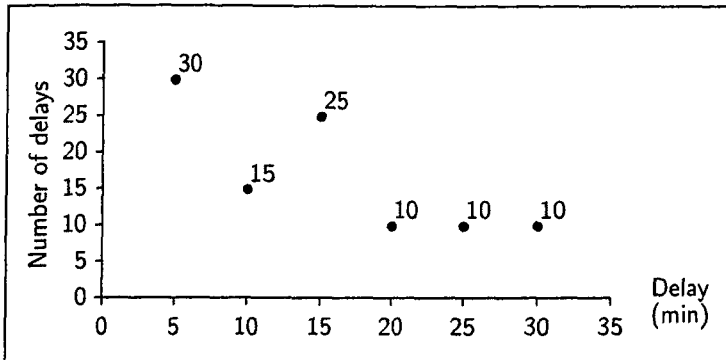


Figure 2: Graphical illustration of delay distribution

Cumulative delay duration (min)	> 5	> 10	> 15	> 20	> 25	> 30
Cumulative number of delays	70	55	30	20	10	0

Table 2: Distribution of cumulative delays

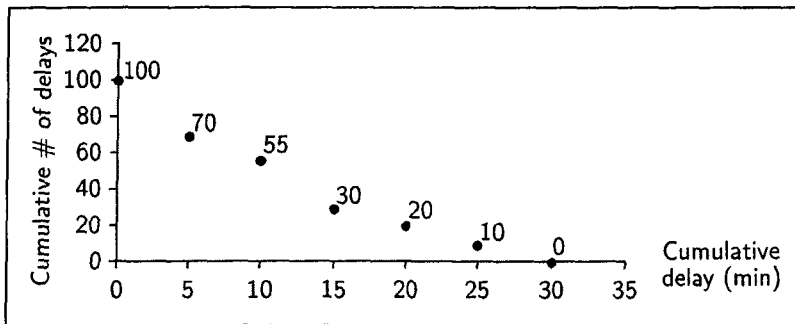


Figure 3: Graphical illustration of cumulative delays

Delay duration (min)	0	5	10	15	20	25	30
Amount of cumulative delays	1	0.7	0.55	0.3	0.2	0.1	0

Table 3: Possibility distribution of delays

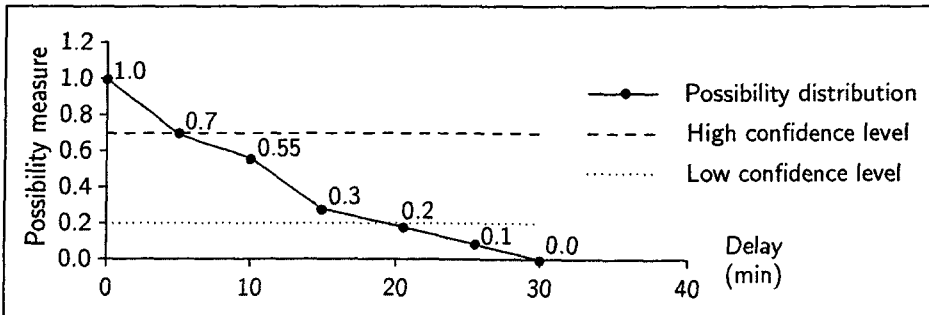


Figure 4: Graphical illustration of possibility distribution of delays

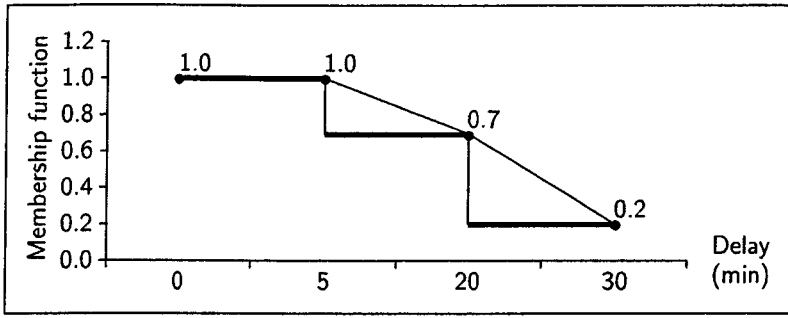


Figure 5: Nested interval approximation

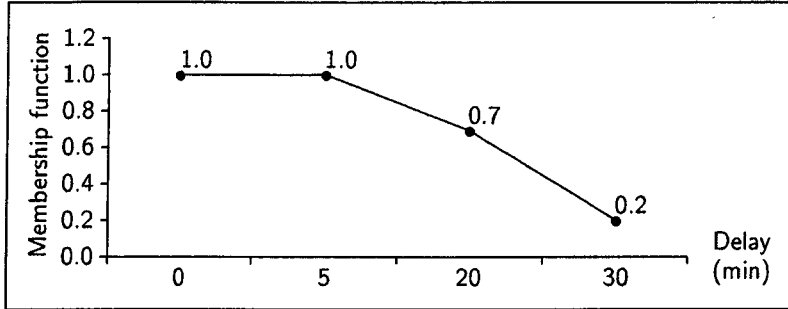


Figure 6: Fuzzy numbers representing possible arrival delays

should be approximated with some guarantee of “robustness”. For example, suppose that for another observed arrival delay the explicit distribution changes, then we might get another shape of the possibility distribution. Therefore, we need to construct a “universal” possibility distribution with a certain degree of freedom related to possible changes of observed data.

How to achieve this is shown in Figures 5 and 6. We simply assume that all points from nested intervals related to lower confidence interval are approximated with adjacent higher confidence level. Then we join right-hand-side endpoints using piece-wise linear approximation. This way of approximation gives a firm and robust way of presenting possibility distribution as fuzzy numbers. It is robust to changes of possibility distribution both inside and outside nested intervals. Moreover, it guarantees the correctness of the well-known probability/possibility consistency principle which states that possibility can be treated as upper bound of probability, and any possibility distribution defines a unique set of admissible probability distributions (see, e.g., [15]).

In a similar way we can construct a left-hand-side of the fuzzy number using explicit probability distribution of arrival earliness. Combining left-hand and right-hand shapes, we get the complete shape of fuzzy numbers describing possible earliness and tardiness of arrival times for a particular flight. It is based on information we can easily collect from observed data, but it has a more universal and robust form since it uses the concepts of possibility and fuzziness.

4.2 Fuzzy model

Now, the model can be summarized using fuzzy start and completion times as follows: Find a schedule $(\tilde{S}, \tilde{C}, M)$ which assures constraints (11) to (15) and optimizes goals (6a) to (6c).

Minimal processing time

$$\tilde{S}_i + p_i^{min} \ll = \tilde{C}_i \quad \forall i \in V \quad (11)$$

Continuous processing

$$\tilde{C}_i + d_{iM_i j M_j}^{tow} = \tilde{S}_j \quad \forall (i, j) \in \mathcal{E}^{tow} \quad (12)$$

Disjunctive activities and setup times

For all activities $i, j \in V$ such that either $M_i = M_j \neq \emptyset$ or $\exists (i, M_i, j, M_j) \in \mathcal{E}^{shadow}$ one of the following two conditions must be fulfilled:

$$\tilde{C}_i + d_{iM_i j M_j}^{setup} \ll = \tilde{S}_j \quad (13a)$$

$$\tilde{C}_j + d_{jM_j i M_i}^{setup} \ll = \tilde{S}_i \quad (13b)$$

Start and completion time

$$\tilde{S}_i = \tilde{t}_i^a \quad \forall i \in V^a \quad (14a)$$

$$\tilde{C}_i = \tilde{t}_i^d \quad \forall i \in V^d \quad (14b)$$

$$\tilde{t}_i^a = (\underline{t}_i^{ae}, \underline{t}_i^{a\lambda}, \underline{t}_i^{a1}, \bar{t}_i^{a1}, \bar{t}_i^{a\lambda}, \bar{t}_i^{ae}) \quad \forall i \in V^a \quad (14c)$$

$$\tilde{t}_i^d = (\underline{t}_i^{de}, \underline{t}_i^{d\lambda}, \underline{t}_i^{d1}, \bar{t}_i^{d1}, \bar{t}_i^{d\lambda}, \bar{t}_i^{de}) \quad \forall i \in V^d \quad (14d)$$

$$\underline{t}_i^{ae}, \underline{t}_i^{a\lambda}, \underline{t}_i^{a1}, \bar{t}_i^{a1}, \bar{t}_i^{a\lambda}, \bar{t}_i^{ae}, \underline{t}_i^{de}, \underline{t}_i^{d\lambda}, \underline{t}_i^{d1}, \bar{t}_i^{d1}, \bar{t}_i^{d\lambda}, \bar{t}_i^{de} \in \mathbb{N}_0 \quad \forall i \in V \quad (14e)$$

Mode selection

$$M_i \in \mathcal{M}_i \quad \forall i \in V \quad (15)$$

Objectives

Since the objective functions depend only on the particular gate assignment (while being independent of the particular realization of start and completion times which guarantee solution feasibility) they are identical to (6a) – (6c) as in the deterministic model.

The problem parameters p_i^{min} , $d_{iM_i j M_j}^{tow}$, $d_{iM_i j M_j}^{setup}$ are crisp and fixed, but for calculation purposes they can be treated as fuzzy numbers $\tilde{F} = (\underline{f}^\epsilon, \underline{f}^\lambda, f, \bar{f}^\lambda, \bar{f}^\epsilon)$ with $\underline{f}^\epsilon = \underline{f}^\lambda = f = \bar{f}^\lambda = \bar{f}^\epsilon$, too.

4.3 Mode shaving and disjunctive activities under fuzziness

In order to solve the flight gate scheduling problem with fuzzy start and completion times we extend the Pareto simulated annealing algorithm described in [8]. This algorithm contains several essential ingredients: mode shaving, disjunctive activity detection method, and a time domain reduction procedure for linked and non-linked activities, respectively. A detailed description of these methods is given in [8].

Pareto simulated annealing (PSA) proposed in [3] is a multiple criteria metaheuristic that uses some general building blocks developed for single objective simulated annealing (SA) as well as several specific concepts which take into account the multiple criteria nature of the problem

(see [11]):

Standard concepts of SA

- *Neighborhood concept*
- *Probabilistic acceptance of a new neighborhood solution*
- *Parameter (temperature) dependent acceptance probability*
- *Cooling schedule*
- *Termination criterion*

Specific concepts of PSA

- *Use of aggregation function-based probabilities for acceptance of new neighborhood solutions*
- *The concept of generating solutions or agents*
- *Management of the population of generating solutions or repulsion*
- *Normalization of objectives*
- *Updating the set of potentially Pareto optimal solutions.*

The characteristics of PSA may vary dependent on particular characteristics of the problem under consideration and, of course, substantially influence the effectiveness of PSA. Initial values of the following parameters have to be defined in advance: the number of generating solutions, the initial annealing temperature, the repulsion coefficient, the cooling schedule and the termination criterion.

The algorithm presented here mainly requires to adapt mode shaving and disjunctive activity detection in order to cope with the new comparison rule for fuzzy numbers (SCR). This adaptation is described in the sequel. The modifications of time domain reduction for linked and non-linked activities are left to the reader. One only has to replace the arithmetic operators over real numbers with their fuzzy counterparts. Afterwards the general PSA algorithm can be applied to solve the fuzzy flight gate scheduling problem. In what follows we will use the

notation of [8].

Mode shaving. Let i and i' be two activities served by different aircraft, and assume the temporal constraint i before i' (denoted as $i \rightarrow i'$) to be valid. If no such constraint exists for a particular gate assignment $\mu \in \Delta_{M_i}$, then μ can be removed from Δ_{M_i} . Formally, we have

$$i \rightarrow i', \mu \in \Delta_{M_i} \wedge EC_i + \min_{\nu \in \Delta_{M_{i'}}} d_{i\mu i'\nu}^{setup} > LS_{i'} \Rightarrow \Delta_{M_i} = \Delta_{M_i} \setminus \{\mu\}.$$

In other words, before each new activity assignment we have to scan all activities with their mode domains and update the domains by excluding those modes which are not feasible at the moment. Such a procedure is known as 'mode shaving' (see [4]). We include the apron into mode domains in order to assure that the domain set always contains at least one element, given the fact that the apron has unlimited capacity and assignment is never forbidden.

When we try to compare two fuzzy numbers using SCR, the main problem is that inequality $\tilde{A} \gg \tilde{B}$ does not imply immediately that $\tilde{B} \ll \tilde{A}$ as in crisp algebra. Fuzzy numbers \tilde{A} and \tilde{B} can be incompatible, that is, for one realization of uncertainty we have $A > B$ and opposite for another. So, when using SCR, we have to take this fact into account and guarantee that the proper meaning of each method under any possible realization of uncertainty is preserved. Thus, the mode shaving procedure can be rewritten in the fuzzy case as follows:

$$i \rightarrow i', \mu \in \Delta_{M_i} \wedge \tilde{L}S_{i'} \gg \tilde{E}C_i + \min_{\nu \in \Delta_{M_{i'}}} d_{i\mu i'\nu}^{setup} \Rightarrow \mu \text{ is feasible, otherwise } \Delta_{M_i} = \Delta_{M_i} \setminus \{\mu\}.$$

Doing so we guarantee that mode μ is admissible if a precedence relation $i \rightarrow i'$ which has already been fixed is not corrupted for any possible realization of fuzzy parameters $\tilde{L}S_{i'}$ and $\tilde{E}C_i$.

Disjunctive activities. Gates are disjunctive resources. They can process one activity at a time. The only exception is dummy gate 0 that represents the apron. It has infinite capacity and can process an unlimited number of flights simultaneously. The set of disjunctive activities, that is, activities that may potentially overlap and for which constraints (13a) to (13b) must be explicitly defined, can be described as follows:

$$\begin{aligned} \mathcal{D} := \{ & (i, i') \in V \times V \mid LC_i + \max_{\mu \in M_i, \nu \in M_{i'}} d_{i\mu i'\nu}^{setup} > ES_{i'} \wedge \\ & LC_{i'} + \max_{\mu \in M_i, \nu \in M_{i'}} d_{i'\nu i\mu}^{setup} > ES_i \wedge \\ & [M_i \cap M_{i'} \neq \emptyset \vee \exists (i, \mu, i', \nu) \in \mathcal{E}^{shadow} : \mu \in M_i, \nu \in M_{i'}]\} \end{aligned}$$

In fact, any disjunctive conflict depends on the gate assignment, so we delay its resolution until the corresponding activities are scheduled. In other words, if a disjunctive conflict appears, that is, if (i) the same mode is assigned to activities i and i' or if (ii) shadow restrictions between i assigned to gate μ and i' assigned to gate ν arise, and, additionally, i may precede i' as well as i' may precede i , then we have to resolve this conflict by ordering the activities, i.e., defining whether activity i or i' should be processed first.

The set of disjunctive activities can be rewritten in the fuzzy case as follows:

$$\mathcal{D} = \{(i, i') \in V \times V \mid [M_i \cap M_{i'} \neq \emptyset \vee \exists (i, \mu, i', \nu) \in \mathcal{E}^{shadow} : \mu \in M_i, \nu \in M_{i'}]\} \setminus \bar{\mathcal{D}},$$

where

$$\begin{aligned} \bar{D} := \{ & (i, i') \in V \times V \mid \tilde{E}S_{i'} \gg \tilde{L}C_i + \max_{\mu \in \mathcal{M}_i, \nu \in \mathcal{M}_{i'}} d_{i\mu i'\nu}^{setup} \wedge \\ & \tilde{E}S_i \gg \tilde{L}C_{i'} + \max_{\mu \in \mathcal{M}_i, \nu \in \mathcal{M}_{i'}} d_{i'\nu i\mu}^{setup} \wedge \\ & [\mathcal{M}_i \cap \mathcal{M}_{i'} \neq \emptyset \vee \exists (i, \mu, i', \nu) \in \mathcal{E}^{shadow} : \mu \in \mathcal{M}_i, \nu \in \mathcal{M}_{i'}]\}. \end{aligned}$$

As an output of PSA we have a set of potentially Pareto optimal schedules, in which all flights have been assigned to their stand positions (modes) and all possible resource conflicts have been resolved. The start and completion time domains are generally reduced. Possible defuzzification of start and completion times for all activities can be performed according to the rules (8) – (9).

5 Summary

This paper addresses the multiple criteria flight gate assignment problem under uncertainty, which is naturally modeled by fuzzy numbers in six-point piece-wise linear form. Fuzziness is introduced for basic problem parameters – arrival and departure times – in order to model possible earliness and tardiness of flights. We solve the problem directly by a special multicriteria metaheuristic, known as fuzzy Pareto simulated annealing, in order to get a representative approximation of the Pareto front. We describe a modification of the main parts of an algorithm originally developed for the deterministic variant of the problem in order to preserve its correctness under fuzziness.

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