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Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 583

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Suggested Citation: Nikulin, Yury V. (2004) : Robustness in combinatorial optimization and scheduling theory: An annotated bibliography, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 583, Universität Kiel, Institut für Betriebswirtschaftslehre, Kiel

This Version is available at:
https://hdl.handle.net/10419/147643

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Robustness in Combinatorial Optimization and Scheduling Theory: An Annotated Bibliography

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October, 2004

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Abstract

This short annotated bibliography focuses on what has been published during the last ten years in the area of combinatorial optimization and scheduling theory concerning robustness and other similar techniques that deal with worst case optimization under uncertainty and non-accuracy of problem data.

Keywords: *robustness, tolerance, worst-case scenario, input data uncertainty, non-accuracy, flexibility of solution, maximal regret.*

1 Introduction

One of the most interesting branches of combinatorial optimization that have emerged over the past 20 -30 years is robust optimization. Since the early 1970s there has been an increasing interest in the use of worst-case optimization models. The theory of robustness is a relatively new and quickly developing area of combinatorial optimization. It deals with uncertainty of problem parameters. The presence of such parameters in optimization models is caused by inaccuracy of initial data, non-adequacy of models to real processes, errors of numerical methods, errors of rounding off and other factors. So it appears to be important to identify classes of models in which small changes of input data lead to small changes of the result under worst possible scenario of distribution of problem parameters. The models with such properties are called robust counterpart. It is obvious that any optimization problem arising in practice can hardly be correctly formulated and solved without use of results of the theory of robustness and post-optimal analysis. During the last ten years many authors concentrate on robust optimization and related approaches in which one optimizes against the worst instances that might arise by using min-max objective.

This survey gives an idea of the variety of modern techniques and may serve as a short introduction into the theory of robustness. The main goal of this annotated bibliography is to collect all existing papers together in order to present a complete description of new avenues that have not been explored earlier.

In article annotations the original phrases and abstract fragments are used in order to be as close as possible to authors ideas and descriptions. Of course, the author remains responsible for reformulation errors or omissions that might exist. Unfortunately, I was not able to take into account papers which were written in any language different from English.

2 Terms and concepts

We consider a general optimization problem in the following standard form:

$$minimize \quad f(c, x)$$

subject to

$$g_i(a, x) \leq 0,$$
$$x \in X, \ i = 1, ..., n,$$

where
- x is the vector of variables
- X is the set of feasible solutions
- f and g_i are convex function
- c and a are problem parameters (uncertain).

Uncertainty may be related to coefficients of objective function, coefficients of restrictions or both of them. It seems very naturally to give a definition of a robust solution as follows: an optimal solution is robust if it remains optimal under any realization (scenario) of the data (problem parameters). But this definition can hardly be regarded as desirable, because it is too restrictive. Most unlikely such a solution exists. Another definition may be considered more appropriate: our subject is to find a robust solution which minimizes maximum regret (minimizes worst case scenario). The major part of the papers presented in this bibliography deals with problems like that. Authors of most papers attempt to answer to the following closely related questions: How can one represent uncertainty? What is a robust solution? How to calculate a robust solution? How to interpret worst case realization under uncertainty? and others. Different answers to these questions lead to different approaches and directions. Bibliographical analysis provides us with a list of contributors who proposed several main avenues in the theory of robustness:
- Averbakh [1] – [2] (minmax regret for optimization over a uniform matroid)
- Ben-Tal and Nemirovski [4] – [7] (ellipsoidal uncertainty)
- Bertsimas and Sim [8] – [9] (robust optimization with control of conservatism of a solution)
- Kouvelis and Yu [16] (minmax regret)
- Mulvey, Vanderbei and Zenios [18] (worst possible scenario and penalty functions)
- Yaman and Karasan [19] (absolute and relative robustness for spanning tree problem).

3 References

Part 1. Robustness in Combinatorial Optimization

[1] **Averbakh I.** (2001). On the complexity of a class of combinatorial optimization problems with uncertainty. *Mathematical Programming 90*, 263 – 272.

Under the minmax regret approach adopted in this paper it is shown that polynomial solvability is preserved for a specific (optimization over a uniform matroid) discrete optimization problem. Unfortunately, the direct generalization of this technique to other discrete problems is not clear.

[2] **Averbakh I.** (2004). Minmax regret linear resource allocation problems. *Operations Research Letters 32*, 174 – 180.

In this paper the minmax regret version of the problem of selecting p objects of minimum total weights out of set of m objects is considered. There are two types of uncertainty of problem parameters considered in the paper. In the case of scenario-represented uncertainty (even if there are only two possible scenarios) it is shown that the robust version of the problem of finding the minimum weight base of a uniform matroid of rank p on a ground set of cardinality m is NP-hard. The second case of uncertainty, so-called interval uncertainty, is traditionally more complicated to analyze. All weights can take on any values from some interval. It means that the set of possible realizations has a form of rectangle box in the space of problem parameters. However, in that case a new polynomial algorithm with the order of complexity $O((min(p, m-p))2m)$ is proposed. The author claims that it is the first known example of a minmax regret combinatorial optimization problem that is polynomially solvable in the case of interval representation of uncertainty while being NP-hard in the case of scenario represented uncertainty. This is a very interesting and surprising result and it may give an additional stimulant to researchers for finding polynomial algorithms for the large variety of robust combinatorial optimization problems where complexity in the practically important case of interval representation of uncertainty is still open.

[3] **Aron I. and P. van Hentenryck.** (2004). On the complexity of the robust spanning tree problem with interval data. *Operations Research Letters 32*, 36 – 40.

This paper is focused on the robust spanning tree problem where the edge costs are given by intervals under the robust deviation framework (minimization of the maximum deviation of total cost from the costs of the minimum spanning tree for all possible realizations of the edge cost within the given intervals). This papers proves a conjecture, which originally stated in [16]. It claims that the there is no polynomial algorithm for such type of robustness. The authors prove that the robust spanning tree problem at least as hard as the central tree problem (it consists in finding a tree in graph G such that the rank of its cospanning tree is minimal over all the cospanning trees of G), the NP-completeness of which is well-known. Furthermore, it is shown that the robust spanning tree problem remains hard on complete graphs, even though the central tree can be found in polynomial time on such graphs.

[4] **Ben-Tal A. and Nemirovski A.** (1998). Robust convex optimization. *Mathematics of Operations Research 23*, 769 – 805.

Ben-Tal and Nemirovski address the over conservatism of robust solutions by allowing the uncertainty sets for the data to be ellipsoids, and propose efficient algorithms to solve convex optimization problems under data uncertainty.

[5] **Ben-Tal A. and Nemirovski A.** (1999). Robust solutions to uncertain programs. *Operations Research Letters 25*, 1 – 13.

[6] **Ben-Tal A. and Nemirovski A.** (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming 88*, 411 – 424.

[7] **Ben-Tal A., El Ghaoui L. and Nemirovski A.** (2000). Robust semidefinite programming. In Saigal R., Vandenberghe L. and Wolkowicz H. editors, Handbook of Semidefinite programming and applications, Kluwer Academic Publishers, Waterloo.

A number of important formulations as well as applications are introduced in [5] – [7] and some other papers of these authors. A detailed analysis of the robust optimization framework in linear programming is provided.

[8] **Bertsimas D. and Sim M.** (2004). The price of robustness. *Operations Research 52*, 35 – 53.

A robust approach to solving linear optimization problems with uncertain data was proposed in the early 1970s and has recently been extensively studied and extended. The authors propose another approach which is different from previous ones in order to control the level of conservatism in the solution. This approach has the advantage that it leads to a linear optimization model and it can be directly applied to discrete optimization problems (it was done later in [9]).

[9] **Bertsimas D. and Sim M.** (2002). Robust discrete optimization. *Working Paper. Operations Research Center, MIT.*

A robust version (counterpart) of integer programming problems is proposed for the case when both the cost coefficients and the data constraints are subject to uncertainty. When only the cost coefficients are subject to uncertainty and the problem is $0-1$ discrete optimization problem on n variables the procedure of solving the robust counterpart by solving $n+1$ instances of the original problem is described. As the consequence a very interesting fact stated: if the original problem is polynomially solvable, than the robust counterpart problem also remains polynomially solvable. It means that robust versions of such well-known problems as matching, spanning tree, shortest path, matroid intersection etc. are polynomially solvable. Some results concerning the α-approximation of $0-1$ discrete optimization problems are given.

[10] **Bertsimas D. and Sim M.** (2003). Robust discrete optimization and network flows. *Mathematical Programming 98*, 49 – 71.

Additionally to the results of [9], the authors propose an algorithm for robust network flows that solves the robust counterpart by solving a polynomial number of nominal minimum cost

flow problems in a modified network.

[11] **Bertsimas D. and Sim M.** (2004). Robust conic optimization. *Working Paper. Operations Research Center, MIT.*

In earlier proposals the robust counterpart of a conic optimization problem exhibits an increase in complexity, i.e. robust linear programming problems [6] become second order cone problems, robust second order cone problems [4] become semidefinite programming problems and robust semidefinite programming problems become [7] NP-hard. In this paper a relaxed robust counterpart for general cone optimization problems that preserves the computational tractability of the nominal problem is proposed. Namely, under this concept the robust cone optimization retains the original structure, i.e. robust linear programming problems remain LPs, robust second order cone programming problems remain SCCPs and robust semidefinite programming problems remain SDP. Moreover, when that data entries are independently distributed, the size of the proposed robust problem especially under l_2 norm is practically the same as the original problem.

[12] **Bertsimas D. and Sim M.** (2004). Robust discrete optimization under ellipsoidal uncertainty sets. *Working Paper. Operations Research Center, MIT.*

It is probably the first attempt to investigate robust discrete optimization under ellipsoidal uncertainty sets. It is shown that the robust counterpart of a discrete optimization problem with correlated objective function data is NP-hard even though the original problem is polynomially solvable. For uncorrelated and identically distributed data, it is proved that the robust counterpart retains the complexity of the original problem. A generalization of the robust discrete optimization approach proposed earlier is given which presents the tradeoff between robustness and optimality.

[13] **Bertsimas D., Pachamanova D. and Sim M.** (2004). Robust linear optimization under general norms. *Operations Research Letters* (to appear).

The explicit characterization of the robust counterpart of a linear programming problem with uncertainty set is described by an arbitrary norm. This approach encompasses several approaches from the literature and provides guarantees for constraint violation under probabilistic models that allow arbitrary dependencies in the distribution of the uncertain coefficients.

[14] **El Ghaoui L. and Lebret H.** (1997). Robust solution to least square problems to uncertain data matrices. *SIAM Journal Matrix Analysis Appl. 18*, 1035 – 1064.

[15] **El Ghaoui L., Oustry F. and Lebret H.** (1998). Robust solutions to uncertain semidefinite programs. *SIAM Journal Optimization 9*, 33 – 52.

El Ghaoui et al. derived in [14] – [15] results similar to [4] – [7]. In particular, they deal with robust reformulation of optimization model by adapting robust control techniques under the assumption that the coefficient matrix data may vary inside ellipsoidal uncertainty set. The robust counterpart of some important problems are either exactly or approximately tractable problems that are efficiently solvable with interior point methods. However, the difficulty of the robust problems increases.

[16] **Kouvelis P. and Yu G.** (1997). Robust discrete optimization and its application. Kluwer Academic Publishers, Norwell, M.A.

A comprehensive treatment of the state of the art (up to 1997) in robust discrete optimization and extensive references are presented in this work. However, there still are more open problems than solved ones. Most of the known results correspond to scenario-represented models of uncertainty, i.e. where there exists a finite number of possible scenarios each of which is given explicitly by listing the corresponding values of parameters. It is shown that most classical polynomially solvable combinatorial optimization problems loose this nice property and become NP-hard in a robust version with scenario-represented uncertainty.

[17] **Kozina G. and Perepelitsa V.** (1994). Interval spanning tree problem: solvability and computational complexity. *Interval Computing 1*, 42 – 50.

Kozina and Perepelitsa study the minimum spanning tree problem with interval edge costs. They define a relation order on the set of feasible solutions and generate the Pareto set.

[18] **Mulvey J., Vanderbei R. and Zenios S.** (1995). Robust optimization of large-scale systems. *Operations Research 43*, 264 – 281.

Mulvey et al. present an approach that integrates goal programming formulations with scenario-based description of the problem data. They use penalty functions to develop robust models to hedge against the worst possible scenario.

[19] **Mulvey J., Vanderbei R.** (1995). Robust optimization of large-scale systems: an emerging new technology. *Scientific Report A204992, Princeton University.*

A generalized interior point algorithm for convex objective functions has been developed and applied to various large scale problems.

[20] **Yaman H., Karasan O. and Pinar M.** (2001). The robust spanning tree problem with interval data. *Operations Research Letters 29*, 31 – 40.

A robust version of the minimum spanning tree problem under interval type of uncertainty (each edge cost can take any value in its interval, independent of the other edge) is considered. The authors propose to compute a robust spanning tree. Two types of robustness are studied. A spanning tree whose absolute worst case scenario (i.e. scenario in which the cost of this spanning tree is the maximum) is minimum is called an absolute robust spanning tree. Under the second concept, a robust spanning tree whose total cost minimizes the maximum deviation from the optimal spanning tree over all realizations of the edge costs is called a relative robust spanning tree. In this paper it is proved that the absolute robust spanning tree problem can be solved in polynomial time. Additionally, a mixed integer programming reformulation of the relative robust spanning tree problem is presented.

Part 2. Robustness in Scheduling Theory

Uncertainty in scheduling may have many reasons:
- equipment (machine) breakdown
- early or late arrival time
- uncertainty of processing time
- changes of release data
- nature and human factors and others.

No one can guarantee the reliability of input data in the contemporary non-static permanently changing world. That is one reason why almost all modern scheduling techniques try to find a solution (being probably non-optimal but as close as possible to an optimal one), which has the property of flexibility to changes of input data. In this section we give a brief description of the literature concerning both theoretical and practical aspects of scheduling in the robustness framework.

[21] **Davenport A. and Beck J.** (2000). A survey of techniques for scheduling with uncertainty. Manuscript in preparation, Toronto.
(available via web: http://www.eil.utoronto.ca/profiles/chris/chris.papers.html)

This paper surveys some robust scheduling techniques that have been appeared during the last decade since the well-known survey by McKay et al. about the state of the art in job-shop scheduling was written. Many new approaches are discussed: redundancy-based techniques (a reservation of extra time and resources for unexpected events), probabilistic techniques (they do not construct a robust schedule, but we have the possibility to measure the probability of uncertainty and, moreover, to construct an optimal schedule so to maximize), different on-line and off-line approaches as well as rescheduling techniques.

[22] **Jensen M.** (2001). Robust and flexible scheduling with evolutionary computation. *Ph.D. thesis, University of Aarhus, Department of Computer Science*, Denmark.

This thesis presents two fundamentally different approaches for scheduling job shops facing machine breakdowns. The first method is called neighborhood based robustness and is based on an idea of minimizing the cost of a neighborhood of schedules. The scheduling algorithm attempts to find a small set of schedules with an acceptable level of performance. The other method for stochastic scheduling uses the idea of co-evolution to create schedules with a guaranteed worst-case performance for a known set of scenarios. The method is demonstrated to improve worst-case performance of the schedules when compared to ordinary scheduling; it substantially reduces running time when compared to a more standard approach explicitly considering all scenarios.

[23] **Kouvelis P., Daniels R. and Vairaktarakis G.** (2000). Robust scheduling of a two-machine flow shop with uncertain processing times. *IIE Transactions 36*, 667 – 682.

This paper is one of the first attempts to introduce the concept of robustness for scheduling problems. The authors suggest a robust schedule when processing times are uncertain, but they compute this robust schedule based on maximum absolute deviation between the robust solution and all the possible scenarios, but this requires knowledge of all possible scenarios.

Moreover, the optimal solution of each scenario is supposed to be known a priori.

[24] **Leus R. and Herroelen W.** (2004). Robust and reactive project scheduling: a review and classification of procedures. *International Journal of Production Research 42*, 1599 – 1620.

Predictive-reactive scheduling refers to the process where a baseline schedule is developed prior to the start of the project and updated if necessary during project execution. The objective of this paper is to review possible procedures for the generation of proactive (robust) schedules, which are as good as possible protected against schedule disruptions, and for the deployment of reactive scheduling procedures that may be used to revise or re-optimize the baseline schedule when unexpected events occur. Finally, the authors propose a survey of the basics of critical chain scheduling and indicate in which environments it is useful.

[25] **Policella N., Smith S., Cesta A. and Oddi A.** (2004). Generating robust schedules through temporal flexibility. *Proceedings 14th International Conference on Automated Planning and Scheduling, Whistler CA.* (to appear)

This paper considers the problem of generating partial order schedules that retain temporal flexibility and thus provide some degree of robustness. Two different orthogonal procedures for construction a POS were proposed. The first, which the authors call the resource envelope based approach, uses computed bounds on cumulative resource usage (i.e., a resource envelope) to identify potential resource conflicts, and progressively winnows the total set of temporally feasible solutions into a smaller set of resource feasible solutions by resolving detected conflicts. The second, referred to as the earliest start time approach, instead uses conflict analysis of a specific (i.e., earliest start time) solution to generate an initial fixed-time schedule, and then expands this solution to a set of resource feasible solutions in a post-processing step.

[26] **Sevaux M. and Sörensen K.** (2004). A genetic algorithm for robust scheduling in a just in-time environment. *Technical Report LAMIH, SR-2003-1, University of Valencieness, France.*

It is shown how a robust genetic algorithm can be applied to single machine scheduling problem when release data are subject to small variations. This method leads to a robust solution, meaning that the value of the objective function remains high when small variations in some release data occur.

[27] **Sevaux M. and Zann Le Quere.** (2003). Solving a robust maintenance scheduling problem at the French railways company. *Technical Report LAMIH, SR-2003-3, University of Valencieness, France.*

This paper gives an extension of the results derived by Sevaux and Sörensen for a different scheduling problem with a different objective. The authors focus on solving a robust scheduling problem at the French railway company, which can be modelled as a special case of a resource-constrained scheduling problem with additional constraints. The objective is to construct a robust schedule, i.e. a sequence of the tasks on each resource for which the makespan value can be predicted when the duration of the task is increased.

[28] **Sörensen K.** (2001). Tabu search for robust solutions. In *Proceedings of the 4th Metaheuristics International Conference, Porto, Portugal,* 707 – 712.

The robust tabu search firstly introduced in this paper is a new and original technique based on ideas taken from theory of robust optimization for continuous mathematical functions. This approach can be combined with the ideas of [29] – [30] on robust genetic algorithm.

[29] **Tsutsui S. and Ghosh A.** (1997). Genetic algorithms with a robust solution searching scheme. *IEEE Transactions on Evolutionary Computation 1,* 201 – 208.

[30] **Tsutsui S. and Jain J.C.** (1998). Properties of robust solution searching in multi-dimensional space with genetic algorithms. In *Proceedings of the 2nd International Conference on Knowledge-Based Electronic Systems.*

Additionally we give a short list of references [31] – [39] on robustness in economics, namely in portfolio optimization, supply chain management and monetary policy.

[31] **Ben-Tal A., Margalit T. and Nemirovski A.** (2000). Robust modelling of multi-stage portfolio problems. In K. Roos, T. Terlaky, and S. Zhang, editors, High Performance Optimization, Kluwer Academic Publisher, Dordrecht, 303 - 328.

[32] **Ben-Tal A., Golany B., Nemirovski A. and Vial J.-P.** (2003). Supplier-retailer flexible commitments contracts: A robust optimization approach. *Technical report, Dept. of Management Studies, University of Geneva.*

[33] **Bertsimas D. and Thiele A.** (2003). A robust optimization approach to supply chain management. *Technical report, MIT.*

[34] **El Ghaoui L., Oks M. and Oustry F.** (2003). Worst-case value-at-risk and robust portfolio optimization: A conic programming approach. *Operations Research 51,* 543 - 556.

[35] **Giannoni M.** (2002). Does model uncertainty justify caution? robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics 6,* 111 - 144.

[36] **Goldfarb D. and Iyengar G.** (2003). Robust portfolio selection problems. *Mathematics of Operations Research 28,* 1 - 38.

[37] **Levin A. and Williams J.** (2003). Robust monetary policy with competing reference models. *Working Paper, Federal Reserve Bank of San Francisco.*

[38] **Lutgens F. and Sturm J.** (2002). Robust option modelling. *Technical report, University of Maastricht.*

[39] **Popescu I.** (2003). Robust mean-covariance solutions for stochastic optimization. *Technical report, INSEAD, Technology Management Area.*

Finally, we should mention text book [40] (written and available in German only), which contains a comprehensive description of a large variety of robustness concepts.

[40] **Scholl A.** Robust scheduling and Optimization: Basics, Concepts and Methodology, Physics, Heidelberg, 2001 (in German: Robuste Planung und Optimierung: Grundlagen, Konzepte und Methoden, Experimentelle Untersuchungen. Physica-Verlag, Heidelberg).

Acknowledgements: I wish to express my thanks to Prof. Andreas Drexl for his help in editing text of the paper. I also gratefully acknowledge the helpful suggestions of Prof. Marek Libura concerning an extension of the reference list.