

Sprecher, Arno

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Sales force deployment by genetic concepts

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Manuskripte
aus den
Instituten für Betriebswirtschaftslehre
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No. 514

Sales Force Deployment by Genetic Concepts¹

Arno Sprecher



Abstract

Sales force management decisions belong to the major issues considered in marketing research. Among others, the alignment of sales territories has been one of the main field of work for years. Recent results have reported a strong impact of the alignment of sales territories on profit, and, thus, have directed the focus from the balancing approach to profit maximization.

One of the latest models proposed is the so-called sales force deployment problem. Employing a sales response function the sales force deployment problem simultaneously considers several interacting subproblems: (1) Sales force sizing, (2) sales force location, (3) sales territory alignment, and (4) sales effort allocation are the subjects of investigation.

We provide a heuristic solution approach that builds on genetic concepts. The approach is evaluated on a set of benchmark instances with sizes of practical relevance. The approach produces solutions of competitive quality at far less CPU-time than required by the state-of-the-art procedure.

Moreover, simple modification of the concepts allow to deal with alternated problem settings as well. First, the per-period fixed cost of setting up a sales center can be considered as a discrete function of the amount of selling time made available. Second, the balancing approach can be portrayed.

Additionally, the concepts can support man-machine interactions in an online decision support system required to adjust sales territory alignments, e.g., when new products are launched, markets shift or mergers change the portfolio of the company. As such, the operations can be employed by a decision maker in a step-by-step approach to manipulate given territory alignments.

Keywords: Sales Force Management, Decision Support, Mathematical Programming, Genetic Algorithm, Experimental Evaluation.

1 Introduction

Sales territory alignment can be considered as the problem of grouping smaller geographic units, the so-called sales coverage units (SCUs), into larger geographic clusters, i.e., sales territories. (see, e.g., [11]). The SCUs, as a rule relate to a given decomposition of the overall sales territory,

as through zip code areas, districts, or census tracts. The alignment to sales territories (STs) follows pre-specified managerially criteria like balancing the workload, balancing the potential, or maximizing the profit (see, e.g., [11]).

Over years the balancing approach that seeks to build sales territories that are balanced with respect to one or more criteria has attracted much attention and was mainly employed by practitioners. The main focus is on the input requirements workload and the output opportunities potential to provide the foundation of fair payment of the salesmen (see [3]).

Recent studies have revealed that the results of the balancing approach leave much room for the improvement of profit contribution. Skiera and Albers (see [10]) developed COSTA (Contribution Optimizing Sales Territory Alignment). The model employs a sales response function to maximize the total profit contribution. The approach simultaneously considers two subproblems: First, the SCUs are assigned exclusively to given sales territory centers, i.e., salespersons, and, second, the available selling times of the salespersons are allocated to the SCUs that built the sales territories. The objective is to maximize the total profit contribution, which is built by sum of the differences of individual (territorial) profit contributions and individual travel costs. The individual profit contributions and travel cost are expressed by functions of the calling time and travel time, respectively. The calling time and the travel time are measured in fractions of the allocated selling time. The approach solves the assignment of SCUs by a backward deletion algorithm and the allocation of selling time by an analytic formula. Typical decision problems of large companies (pharmaceutical industry, 2000 SCUs, 100 salespersons) can be solved on a personal computer (Pentium, 133 Mhz) within 100 minutes.

Recent advances have lead to an extension of the model. Drexler and Haase ([4]) propose the sales force deployment problem (SDP), which simultaneously considers several interacting subproblems: (1) Sales force sizing, (2) sales force location, (3) sales territory alignment, and (4) sales effort allocation. That is, instead of fixed set of sales territory centers from which the salespersons operate, the authors assume that there is a set of candidate locations, of which a subset can be selected in order to perform the selling activities. Setting up a sales territory center provides the selling time at some fixed cost. The fixed cost may vary from candidate to candidate location. The constraints of the mathematical programming model are linear and the objective function is non-linear. Drexler and Haase provide a generation scheme that produces contiguous sales territories. The heuristic search strategy assumes concavity of the

objective function with respect to the number of sales centers (salespersons). Consequently the line specifying the number of sales centers can be searched via bisection. Given a certain number of sales centers, the set of actual sales centers is determined via tournament selection. The SCUs are assigned via some sort of steepest ascend. Moreover, an improvement routine is employed. It tests if moving a SCU to another sales territory center improves the objective.

In order to derive upper bounds, and thus benchmarks, Drexler and Haase approximate the objective function by a piecewise linear upper envelope. The linearization allows to solve the problem by a standard LP-solver, and provides upper bounds for the optimal solution. The computational results reveal, that problems with up to 500 SCUs and 50 candidate locations can be solved within some 10 minutes on a personal computer (Pentium, 133 Mhz). The solution gap decreases with an increasing number of SCUs and is about two percent for 500 SCUs.

We provide a heuristic solution approach that builds on genetic concepts. The approach is evaluated on a set of benchmark instances with sizes of practical relevance. The procedure produces solutions of competitive quality at far less CPU-time than required by the state-of-the-art procedure.

Moreover, simple modification of the method allow to deal with altered problem settings as well. First, the per-period fixed cost of setting up a sales center can be considered as a discrete function of the selling time made available. Second, the balancing approach can be portrayed. Additionally, the concepts can support man-machine interactions in an online decision support system required to adjust sales territory alignments, e.g., when new products are launched, markets shift or mergers change the portfolio of the company. As such, the operations can be employed by a decision maker in a step-by-step approach to manipulate given territory alignments.

The paper is organized as follows: In Section 2 we give a brief summary of the model. In Section 3 we present the genetic concepts developed to maximize the profit. In Section 4 we analyze variants of the problem and propose modifications of the concepts. In Section 5 we evaluate the different components of the algorithm, and compare the approach with the state-of-the-art procedure. Conclusions are drawn in Section 6.

2 The Sales Force Deployment Problem

We consider an overall sales territory that is decomposed in disjoint SCUs. The SCUs are referred to by an index j from the set J of all SCUs. In certain SCUs, more specific, in subset I of the set of SCUs J , sales territory centers (STCs) can be located. For sake of simplicity a STC is referred to by the index of the related SCU. A STC located in SCU i , $i \in I$, causes per-period fixed cost of f_i monetary units and provides T_i units of selling time per period. A sales force (salesperson) located in STC i is in charge of the SCUs that are aligned to it (him). This collection of SCUs is considered as a sales territory, it is denoted by J_i . Practical considerations require: (1) a STC serves the SCU it is located in, (2) a sales territory is connected, (3) each SCU is assigned to exactly one salesperson, i.e., the sales territories are disjoint, that is, it is, $\bigcup_{i \in I} J_i = \{1, \dots, \bar{J}\}$, $J_i \cap J_{i'} = \emptyset$, for $i \neq i'$.

The selling time consists of the travel time and the calling time. The expected sales $S_{i,j}$ of STC i in SCU j is then a function of the calling time $z_{i,j}$

$$S_{i,j} = g_j(z_{i,j})^{b_j}, \quad i \in I, j \in J$$

Following Skiera and Albers (see [10]) the calling time $z_{i,j}$ can be expressed as a portion $p_{i,j}$ of the selling time $t_{i,j}$, i.e., $z_{i,j} = p_{i,j}t_{i,j}$. That is, the portion depends on the location of the STC and the SCU. Employing $c_{i,j} = g_j(p_{i,j})^{b_j}$ we obtain

$$\begin{aligned} S_{i,j} &= g_j(p_{i,j}t_{i,j})^{b_j}, \quad i \in I, j \in J \\ &= c_{i,j}(t_{i,j})^{b_j}. \end{aligned}$$

Using the parameters summarized in Table 1 we can present the sales force deployment problem (SDP) as developed by Drexler and Haase ([4]). The model employs binary and continuous decision variables. It is displayed in Table 2. The binary variables $x_{i,j}$, $i \in I$, $j \in J$, indicate if a salesman located in SCU i is in charge of SCU j ($x_{i,j} = 1$) or not ($x_{i,j} = 0$). The continuous variables $t_{i,j}$, $i \in I$, $j \in J$, represent the selling time the salesman located in STC i allocates to SCU j . The model has linear constraints and a non-linear objective function. The objective (1) is defined by the maximization of the profit contribution. The profit contribution is built by the difference of the expected per-period sales and the per-period fixed cost. The constraint (2) guarantees that each SCU is assigned to exactly one salesman. The constraint (3) allows a salesman only to allocate selling time to a SCU if the SCU is assigned to him. The constraint

J	: set of sales coverage units (SCUs)
\bar{J}	: number of sales coverage units (SCUs), i.e., $\bar{J} = J $
j	: index of a SCU
\mathcal{N}_j	: set of SCUs that are neighbors of SCU j
b_j	: calling time elasticity of SCU j
I	: set of SCUs, that are candidates for a STC, $I \subseteq J$
\bar{I}	: number of SCUs, that are candidates for a STC, i.e., $\bar{I} = I $
i	: index of a candidate for a STC
f_i	: per period fixed cost of STC i
T_i	: total selling time of STC i

Table 1: Problem Parameters

$$\text{Maximize } Z_{TSC}(x, t) = \sum_{i \in I} \sum_{j \in J} c_{i,j}(t_{i,j})^{b_j} - \sum_{i \in I} f_i x_{i,i} \quad (1)$$

s.t.

$$\sum_{i \in I} x_{ij} = 1 \quad j \in J \quad (2)$$

$$t_{i,j} \leq T_i x_{i,j} \quad i \in I, j \in J \quad (3)$$

$$\sum_{j \in J} t_{i,j} \leq T_i x_{i,i} \quad i \in I \quad (4)$$

$$\sum_{j \in \bigcup_{i \in V} (\mathcal{N}_i - V)} x_{i,j} - \sum_{k \in V} x_{i,k} \geq 1 - |V| \quad i \in I, V \subseteq J - \mathcal{N}_i - \{i\} \quad (5)$$

$$x_{i,j} \in \{0, 1\} \quad i \in I, j \in J \quad (6)$$

$$t_{i,j} \geq 0 \quad i \in I, j \in J \quad (7)$$

**Table 2: The Sales Force Deployment Problem for Maximizing the
Total Sales Contribution – SDP-TSC**

(4) ensures that the selling time of a salesman is not exceeded. The constraint (5) ensures that the sales territory assigned to the salesmen are connected.

3 The Genetic Algorithm

Basically a genetic algorithm aims at finding high quality solutions by simulating the principles of evolution of biological organism. The foundations have been developed by Holland (see [7]). The stochastic concepts operate on the individuals of a population, i.e., a set of feasible solutions. In order to define the operations of a genetic algorithm, we have to represent an individual, i.e., a feasible solution of the problem, by a string. Given an initial population three operators are employed to form a new population from the current population: The crossover, the mutation, and the selection. The crossover combines chromosomes of different individuals to form new individuals. The mutation makes random changes to single individuals. The selection decides which individuals from the current population enter the new population. Commonly the selection is governed by the fitness of the individual as given by the objective function value.

Subsection 3.1 summarizes some basic calculations required for optimally allocating the selling time effort to the SCUs of a specified sales territory. Subsection 3.2 outlines the representation of a feasible solution. Subsection 3.3 addresses the generation of the initial population. Subsection 3.4 describes and illustrates the crossover. Subsection 3.5 presents two mutation operators. Finally, Subsection 3.6 specifies the selection. The notation employed in this section are listed in Table 3.

3.1 Basic Calculations

For a given set J_i of SCUs assigned to a STC i , $i \in I$, we denote by $(t_{i,j})_{j \in J_i}$ a selling time allocation with $\sum_{j \in J_i} t_{i,j} \leq T_i$. Note, by definition it is $t_{i,j} = 0$ for $j \in J - J_i$. The sales contribution resulting from the selling time allocation is denoted by $S_i(J_i, (t_{i,j})_{j \in J_i})$. It is

$$S_i(J_i, (t_{i,j})_{j \in J_i}) = \sum_{j \in J_i} c_{i,j} (t_{i,j})^{b_j}$$

The related optimal sales contribution is denoted by $S_i^*(J_i, T_i)$. It is realized if optimal selling times $t_{i,j}^*(J_i, T_i)$ are allocated. If $b_j = b$ for all $j \in J_i$ holds, then, with $a = 1/(1 - b)$, the optimal selling time allocation is obtained (see [5] and [2]) by

$$t_{i,j}^*(J_i, T_i) = \frac{(c_{i,j})^a}{\sum_{h \in J_i} (c_{i,h})^a} T_i, \quad j \in J_i$$

\mathcal{N}_j^-	: lower indexed neighbors of SCU j , i.e., $\mathcal{N}_j^- = \{l \in \mathcal{N}_j; l < j\}$
\mathcal{N}_j^+	: higher indexed neighbors of SCU j , i.e., $\mathcal{N}_j^+ = \{l \in \mathcal{N}_j; l > j\}$
J^{ass} (J^{unass})	: set of SCUs currently (not) assigned to a sales territory center
I^{sel} (I^{unsel})	: candidate locations currently (not) selected as sales territory center, $I^{sel} \subseteq I$ ($I^{unsel} \subseteq I$)
J^{el}	: set of eligible SCUs, i.e., SCUs that can be assigned to a sales territory center without violation of the connectivity of the sales territory
$I^{perm}(j)$: set of permissible sales territory centers, i.e., sales territory centers that can be assigned to SCU j without violation of the connectivity of the ST
J_i	: set of SCUs assigned to sales territory center i , i.e., the sales territory of sales territory center i
$t_{i,j}(J_i, T_i)$: selling time allocation of sales territory center i to sales territory J_i
$S_i(J_i, (t_{i,j})_{j \in J_i})$: sales of sales territory center i in sales territory J_i induced by selling times $(t_{ij})_{j \in J_i}$
$t_i^*(J_i, T_i)$: optimal allocation of T_i units of selling time of territory center i within sales territory J_i
$S_i^*(J_i, T_i)$: optimal sales of sales territory center i in sales territory J_i if selling time T_i is allocated
F (M)	: individual of population selected as father (mother)
S (D)	: son (daughter) of father and mother
X	: individual of population
x_j	: sales territory center to which SCU j is assigned within individual X
X_j	: sales territory to which SCU j belongs within individual X

Table 3: Symbols

with related optimal sales contribution

$$S_i^*(J_i, T_i) = \sum_{j \in J_i} c_{i,j} \cdot \left(\frac{(c_{i,j})^a}{\sum_{h \in J_i} (c_{i,h})^a} T_i \right)^b$$

The general case with calling time elasticities depending on the SCU is covered by bisection search as outlined by Skiera and Albers ([9]).

3.2 Representation of a Solution

The representation of a solution is displayed in Table 4. An individual X is described by (a) the STC x_j that performs the selling activities in SCU j , $j = 1, \dots, J$, (b) the ST X_j to which SCU j belongs, $j = 1, \dots, J$, (c) the optimal selling time allocation $t_{x_j,j}^*(X_j, T_{x_j})$ of STC x_j to SCU j . Certainly, for the proper definition of a feasible solution it suffices to store for each SCU

SCU	1	2	...	\bar{J}
STC	x_1	x_2	...	$x_{\bar{J}}$
aligned SCUs	X_1	X_2	...	$X_{\bar{J}}$
allocated selling time	$t_{x_1,1}^*(X_1, T_{x_1})$	$t_{x_2,2}^*(X_2, T_{x_2})$...	$t_{x_{\bar{J}},\bar{J}}^*(X_{\bar{J}}, T_{x_{\bar{J}}})$

Table 4: Representation of the Sales Force Alignment and Time Allocation of an Individual X

the STC to which it is assigned. The remaining quantities can be computed. However, the extended representation will simplify the description of the algorithmic elements substantially. Note, feasibility of the solution requires additionally, (1) that a selected STC performs the selling activities in the SCU j it is located in, i.e., it is $x_j = j$, for all the sales territory centers, and (2) that the STs X_j are connected. Whereas the former condition is easily verified the latter requires some additional preparations. Since the preparation is of fundamental importance it is summarized as a remark:

Remark 1 (Relabeling)

Assuming connectivity of the overall sales territory we can relabel the SCUs such that the subgraphs with nodes $G_j = \bigcup_{k=1}^j \{k\}$, $j = 1, \dots, \bar{J}$, are connected. That is all the SCUs but the first one have a lower indexed neighbor. The lower (higher) indexed neighbors of SCU j are denoted by \mathcal{N}_j^- (\mathcal{N}_j^+).

3.3 The Initial Population

In order to generate the initial population we employ priority rules to guide the selection of sales territory centers.

NoN	number of neighbors	$\Pi_i = \mathcal{N}_i $	max
FC	fixed cost per period	$\Pi_i = f_i$	min
FCpN	fixed cost per period and neighbor	$\Pi_i = f_i/ \mathcal{N}_i $	min
S-FC	sales contribution of STC reduced by fixed cost	$\Pi_i = c_{i,i} \cdot (T_i)^{b_i} - f_i$	max
SCaN-FC	sales contribution of neighbors reduced by fixed cost	$\Pi_i = S_i^*(\mathcal{N}_i \cup \{i\}, T_i) - f_i$	max

Table 5: Priority Rules

The priority rules are listed in Table 5. The first column shows the abbreviation of the rule. The second column contains the description, the third column displays the formulae to calculate the priority value, and the fourth column shows the preference of selection. We denote the minimum (maximum) priority value by Π^{min} (Π^{max}) and the current set of (un)selected STCs by I^{sel} (I^{unsel}) and obtain probabilities

$$P_i(I^{sel}) := \frac{(\Pi_i - \Pi^{min} + 1)}{\sum_{k \in I^{unsel}} (\Pi_k - \Pi^{min} + 1)}, i \in I^{unsel}$$

if larger values are preferred and

$$\bar{P}_i(I^{sel}) := \frac{(\Pi^{max} - \Pi_i + 1)}{\sum_{k \in I^{unsel}} (\Pi^{max} - \Pi_k + 1)}, i \in I^{unsel}$$

if smaller values are preferred. Using the probabilities an individual of the initial population is generated by the steps of Table 6.

-
- Step 1:** Determine number n of STC by a Laplace-distributed random number out of $[1, \bar{I}]$; determine set I^{sel} of STCs randomly with respect to priority rule Π ; assign SCUs related to STCs, i.e., $J_i := \{i\}, i \in I^{sel}; J^{ass} := I^{sel}; J^{unass} := \{1, \dots, \bar{J}\} - J^{ass}$;
- Step 2:** If ($J^{unass} = \emptyset$) then STOP;
- Step 3:** Select randomly an eligible SCU j out of $J^{el} := \{j \in J^{unass}; \mathcal{N}_j \cap J^{ass} \neq \emptyset\}$; assign SCU j randomly to an STC i out of $I^{perm}(j) := \{i \in I^{sel}; \mathcal{N}_j \cap J_i \neq \emptyset\}; J_i := J_i \cup \{j\}; J^{ass} := J^{ass} \cup \{j\}; J^{unass} := J^{unass} - \{j\}$; goto Step 2;
-

Table 6: Determination of Initial Population

In Step 1 the number of STCs is randomly selected out of an pre-specified interval $[1, \bar{I}]$. Subsequently, employing a priority rule Π the set I^{sel} of STCs is determined, and the sales territories

are initialized. Then, until all the SCUs have been assigned, in Step 3, randomly selected eligible SCUs j are added to randomly selected permissible STCs. Note, the construction process, i.e., the assignment of eligible SCUs to permissible STs, guarantees that the sales territories are connected.

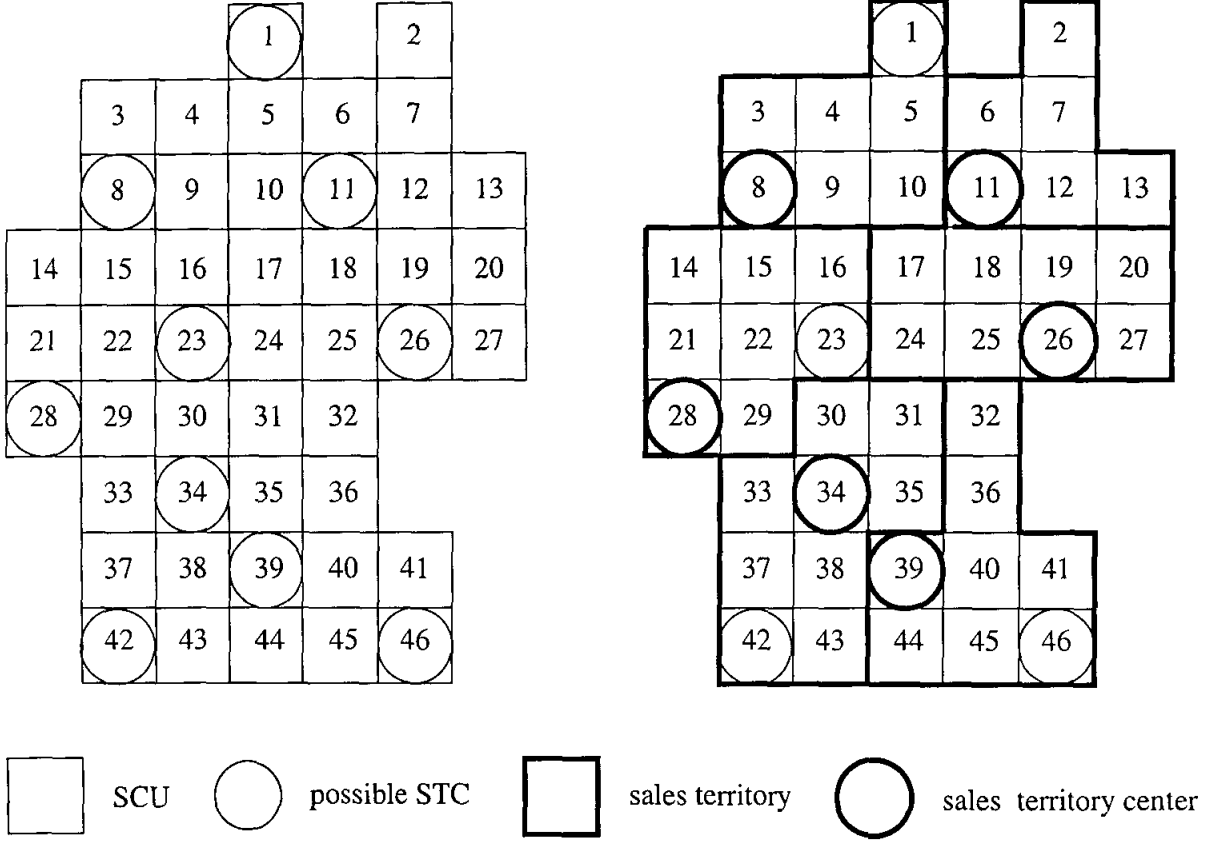


Figure 1: Overall Sales Territory Decomposed in SCUs and Partitioned in STs

Figure 1 shows an overall sales territory decomposed in SCUs and partitioned in sales territories as a possible outcome of the generation stochastic process. We obtain the set of sales centers $I^{sel} = \{8, 11, 26, 28, 34, 39\}$, and the sales territories $J_8 = \{1, 3, 4, 5, 8, 9, 10\}$, $J_{11} = \{2, 6, 7, 11, 12, 13\}$, $J_{26} = \{17, 18, 19, 20, 24, 25, 26, 27\}$, $J_{28} = \{14, 15, 16, 21, 22, 23, 28, 29\}$, $J_{34} = \{30, 31, 33, 34, 35, 37, 38, 42, 43\}$, $J_{39} = \{32, 36, 39, 40, 41, 44, 45, 46\}$.

3.4 Crossover

The crossover operation is performed in two steps. First, the population is randomly partitioned into pairs (parents). Second, for each pair a position in the string is randomly selected, and the prefixes of the strings are exchanged. If only one position is selected a so-called one-point crossover is determined. The crossover produces two new individuals, i.e., children, the son and the daughter. However, the constraints of the problem, connectivity and disjointness of the sales territories, require some additional considerations to guarantee that the children meet the constraints. For a given subset A of Z we denote the complement of A with respect to Z by A^c . Moreover, we abbreviate the neighbors n of SCU j with $n < j$ to \mathcal{N}_j^- . Randomly selecting an element from a set A is represented by the expression $\text{rand}(A)$.

We will use the individuals of Figure 2 to illustrate the operation. We denote the father by F , the mother by M , the son by S , and the daughter by D . As displayed in Table 4 we will use the indexed upper-case (lower-case) letter to refer to the sales territory (sales territory center) to which the SCU belongs (is assigned) within the given individual. That is, a SCU j is assigned to STC f_j (m_j , s_j , d_j) within ST F_j (M_j , S_j , D_j) of individual F (M , S , D). The son S and the daughter D are determined as follows: First, randomly select cp from $\{1, \dots, \bar{J}\}$. Second, define the son by

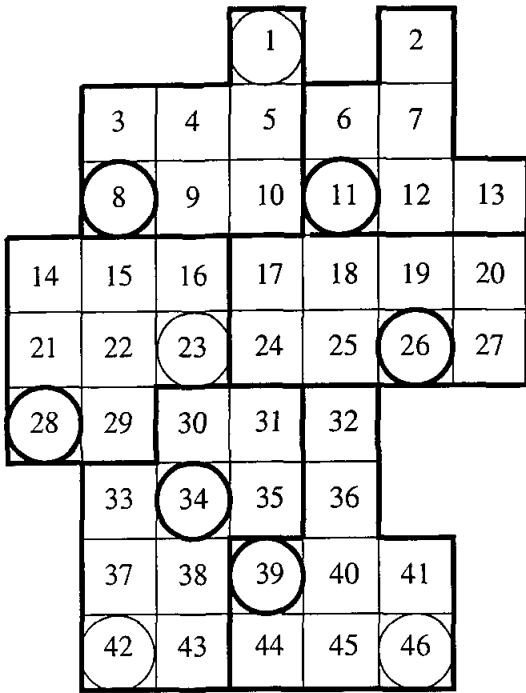
$$s_j = \left\{ \begin{array}{ll} f_j & , \text{ if } (F_j \subset (\bigcup_{k=1}^{cp} F_k)) \\ m_j & , \text{ else if } (D_j \subset (\bigcup_{k=1}^{cp} F_k)^c) \\ x_j & , \text{ with } x_j = (\text{rand}(\bigcup_{k \in \mathcal{N}_j^-} \{s_k\})) \text{ otherwise} \end{array} \right\} \quad j = 1, \dots, \bar{J}$$

and the daughter by

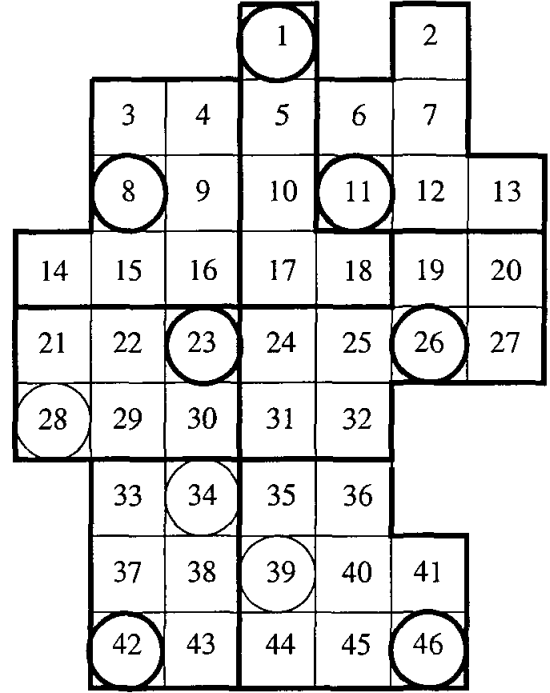
$$d_j = \left\{ \begin{array}{ll} m_j & , \text{ if } (M_j \subset (\bigcup_{k=1}^{cp} M_k)) \\ f_j & , \text{ else if } (F_j \subset (\bigcup_{k=1}^{cp} M_k)^c) \\ y_j & , \text{ with } y_j = (\text{rand}(\bigcup_{k \in \mathcal{N}_j^-} \{d_k\})) \text{ otherwise} \end{array} \right\} \quad j = 1, \dots, \bar{J}$$

The sales territories of the son S and the daughter D are then determined through

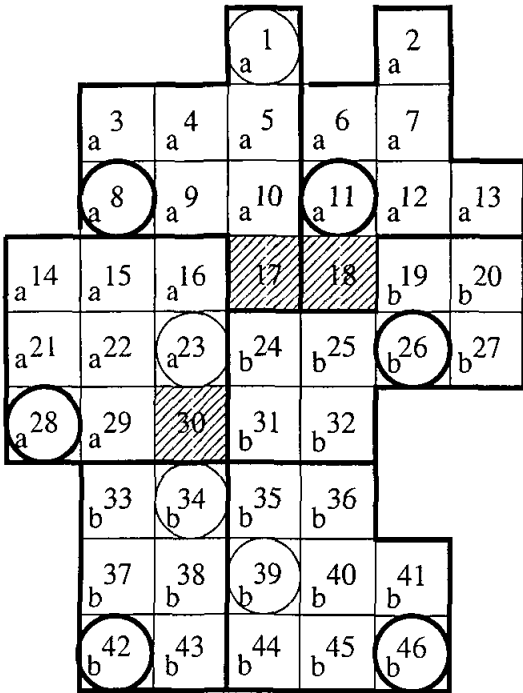
$$S_i := \{l; s_l = i, l = 1, \dots, \bar{J}\}, \quad i = 1, \dots, \bar{I}$$



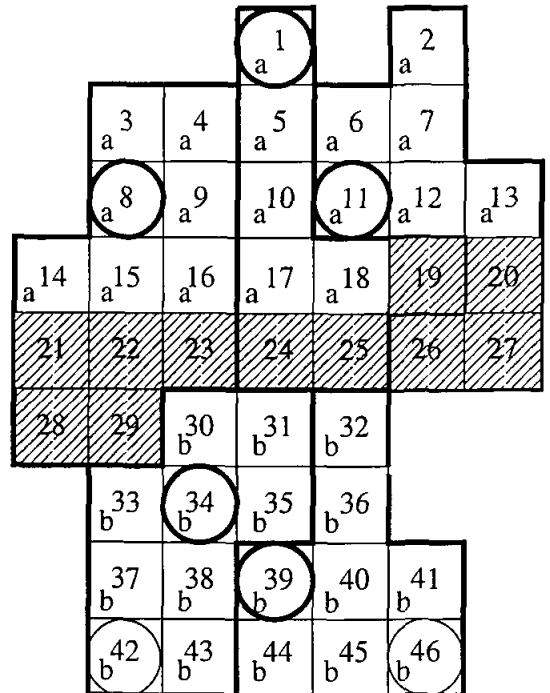
Father



Mother



Son



Daughter

Figure 2: One-point Crossover - $cp = 16$

$$D_i := \{l; d_l = i, l = 1, \dots, \bar{J}\}, \quad i = 1, \dots, \bar{I}$$

For the instance, assuming $cp = 16$, we obtain the son and the daughter as displayed in the lower part of Figure 2. The SCUs that are assigned via the first (second) case are marked by a (b), and the SCUs that are assigned via the third case are shaded. The son is determined by first, taking $S_8 = F_8$, $S_{11} = F_{11}$, $S_{28} = F_{28}$ from the father, second, randomly assigning the shaded SCUs 17 and 18, to S_8 and S_{11} , respectively, third, taking $S_{26} = M_{26}$ from the mother, fourth, randomly assigning the shaded SCU 30 to S_{28} , and finally taking $S_{42} = M_{42}$ and $S_{46} = M_{46}$ from the mother. The daughter is determined by first, taking $D_1 = M_1$, $D_8 = M_8$, $D_{11} = M_{11}$ from the mother, second, randomly assigning the shaded SCUs, i.e., SCU 19 to D_1 , SCU 20 to D_{11} , SCUs 21, 22, 23 to D_8 , SCUs 24, 25 to D_1 and SCUs 26, 27 to D_{11} , SCUs 28, 29 to D_8 and finally, third, taking $D_{34} = F_{34}$, and $D_{39} = F_{39}$ from the father.

Note, first, assuming that the STs of the father as well as the STs of mother are disjoint, the STs of the son (the daughter) are disjoint, by construction. Second, assuming that the STs of the father as well as the STs of the mother are connected, the STs of the son (the daughter) are connected.

Given the sales territory centers and the sales territories we can now calculate the fixed cost and the optimal sales and selling times as summarized in Subsection 3.1. Note, the sales of the STs have to be recalculated only if a SCU is assigned randomly, via the third case, otherwise the sales can be taken from the related parent.

3.5 Mutation

The mutation operation randomly changes single elements of the population. It is applied to the elements of the new generation, i.e., the children. The mutation probability defines the probability for a mutation. It is usually set to 0.05 or 0.10. The crossover described in the previous subsection bears two systematic biases produced by the random assignment. First, the SCUs that are only assigned via a lower indexed neighbor. Second, the individuals of the child population tend to larger sales territories and a lower number of sales territory centers. Consequently, the mutation operators we have developed will take the biases into account. The first of mutation is obtained through relocation of SCUs and the second through decomposition of sales territories. The mutations are illustrated by Figure 3. We assume that by probabilistic decision the individual X has to be mutated.

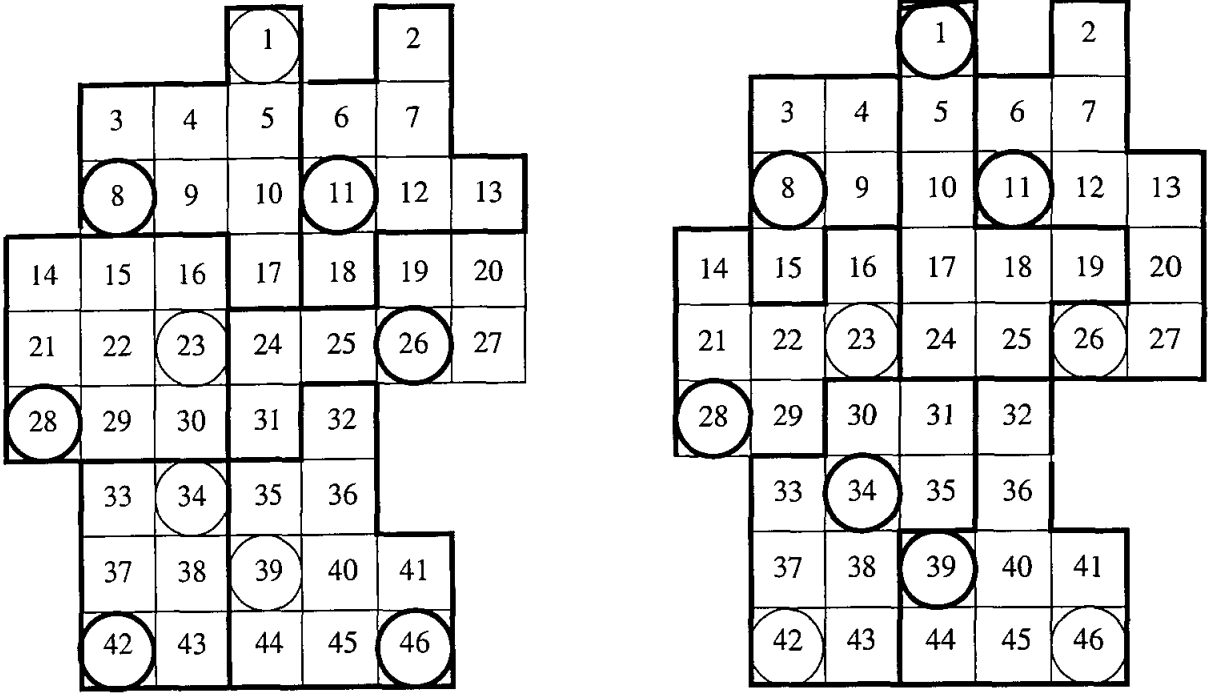


Figure 3: Mutation by Relocation and Splitting

The mutation by relocation randomly selects a SCU j , if the SCU can be removed from the sales territory X_j without violation of connectivity, i.e., $X_j - \{j\}$ is connected and, if there is at least one higher indexed neighbor, i.e., $\mathcal{N}_j^+ \neq \emptyset$ then a higher indexed neighbor n is randomly selected and SCU j is assigned to the sales territory including SCU n . An example is given in the left part of the figure. The alignment results from the mutation of the son. It is $j = 32$, $n = 36$ which leads to $X'_{26} = \{19, 20, 24, 25, 26, 27, 31, 32\} - \{32\} = X_{26} - \{j\}$ and $X'_{46} = \{35, 36, 39, 40, 41, 44, 45, 46\} \cup \{32\} = X_{46} \cup \{j\}$.

The mutation by splitting randomly selects a sales territory X_i that contains at least one unselected candidate \bar{i} for a sales territory center. Subsequently, the sales territory X_i is randomly decomposed in two disjoint and connected sales territories X'_i and X''_i with $X_i = X'_i \cup X''_i$. An example is given in the right part of the figure. The alignment results from the mutation of the daughter. It is $i = 8$ and $\bar{i} = 28$. We obtain $X_8 = \{3, 4, 8, 9, 15\}$ and $X_{28} = \{14, 16, 21, 22, 23, 28, 29\}$.

3.6 Selection

Several strategies have been developed to select the new population from the parent population and child population. The individuals of the parent population and the child population are collected in a temporary population \mathcal{TP} . The temporary population consists of $2 \cdot SPOP$ individuals. An individual of the temporary population \mathcal{TP} enters the new population with a probability proportional to its fitness. As a rule the fitness is defined by the value of the objective function. However, several strategies have been developed (see [8]): We denote $\Phi(X)$ the objective function value of individual X and by Φ^{max} the maximum objective function value from the individuals of the entire population.

The ranking method selects the best $SPOP$ individuals from the temporary population as a member of the new population.

The proportional selection is a randomized rule. We define probabilities $P^{death}(X)$ for an individual X to die. It is

$$P^{death}(X) := \frac{(\Phi^{max} - \Phi(X) + \epsilon)^\lambda}{\sum_{E \in \mathcal{TP}} (\Phi^{max} - \Phi(E) + \epsilon)^\lambda} \quad (8)$$

The constant ϵ causes (a) that the worst element can survive, and (b) that even if all individuals are of identical quality the random selection can be performed. The constant λ levels ($\lambda \simeq 0$) or increases ($\lambda \gg 1$) the differences of the quality of the individuals. Consequently the ranking method can be approximated by employing ($\lambda \gg 1$).

Now, as far as the temporary population consists of more than $SPOP$ individuals, an element X is randomly selected from the temporary population, the element is determined to die, i.e., removed from the temporary population, with a probability $P^{death}(X)$. The selection is repeated until $SPOP$ individuals are left. The finally remaining individuals form the new population.

Now we are able to describe the overall algorithm. The symbols employed are listed in Table 7, and the algorithm is displayed in Table 8. Note, $\text{Connected}(X_j - \{j\})$ is a boolean function that returns 1 iff $X_j - \{j\}$ is connected. If any of the operations improves the currently best known solution then improved solution is stored in X^* . In **Step 1**, the initial population of size $ISPOP$ is generated in accordance with the selected priority rule. Subsequently, each individual X , $X \in \mathcal{IP}$, is considered and for all its sales territories X_1, \dots, X_j the optimal location of the center is determined, i.e.,

$$Z_{TSC}^*(X_j) = \max\{S_i^*(X_j, T_i) - f_i; i \in X_j \cap I\} \quad (9)$$

$ISPOP$: initial size of population
$SPOP$: size of population
G	: number of generations
\mathcal{IP}	: initial population
\mathcal{TP}	: temporary population
\mathcal{PP}	: parent population
\mathcal{CP}	: child population
$\mathcal{F}(\mathcal{M})$: population of fathers (mothers)
$Z_{TSC}^*(X_j)$: maximal sales contribution of the sales territory centers out of X_j , i.e., $Z_{TSC}^*(X_j) = \max\{S_i^*(X_j, T_i) - f_i; i \in X_j \cap I\}$
$X(X^*)$: current (best) individual
$I(X)$: set of sales territory centers of individual X
x_j	: sales territory center that serves SCU j
X_j	: sales territory to which SCU j belongs
P_S	: probability to split a sales territory of an individual
P_R	: probability to relocate a SCU of an individual

Table 7: Symbols Employed in the Algorithm

The individual is adapted accordingly and a temporary population is formed by the adapted individuals of the initial population.

In **Step 2**, the selection of the individuals of the succeeding generation is performed in accordance with probability (8). The surviving individuals built the parent population of the new generation. In **Step 3** the parent population is randomly divided into fathers \mathcal{F} and mothers \mathcal{M} . In **Step 4** a father and a mother is randomly selected for crossover. The reproduction forms the son and the daughter. The reproduction is repeated as far as each father and each mother has been selected once for reproduction. The new individuals enter the child population. In **Step 5** and **Step 6**, if the random number drawn decides so, the children are mutated by splitting a sales territory, or relocating a SCU. In **Step 7**, the selection of the sales territory centers is reoptimized via formula (9). In **Step 8** it is checked if the required number of generations has been studied, if so the procedure proceeds with the improvement, otherwise the Step 2 to Step 8 are repeated. The best sales territory alignment found is stored in individual X^* . In **Step 9** the best individual found so far is studied for improvement. The

Step 1: (Generate Initial Population)

Select priority rule Π , and randomly generate the initial population \mathcal{IP} of *ISPOP* individuals in accordance with priority rule Π ; optimize for each individual X , $X \in \mathcal{IP}$ and each sales territory the selection of the sales territory center; $g := 1$; $\mathcal{TP} := \mathcal{IP}$;

Step 2: (Selection)

Build temporary population \mathcal{TP} by selecting *SPOP* individuals from population \mathcal{TP} in accordance with probability (8);

Step 3: (Divide Population)

Randomly divide temporary population \mathcal{TP} in populations of identical sizes of fathers \mathcal{F} and of mothers \mathcal{M} ; $\mathcal{CP} := \emptyset$;

Step 4: (Crossover)

Randomly select $F \in \mathcal{F}$ and $M \in \mathcal{M}$; apply crossover to father F and mother M to build daughter D and son S and set $\mathcal{CP} := \mathcal{CP} \cup \{S, D\}$; set $\mathcal{F} := \mathcal{F} - \{F\}$, $\mathcal{M} := \mathcal{M} - \{M\}$; if $(\mathcal{F} \cup \mathcal{M} \neq \emptyset)$ then repeat Step 4;

Step 5: (Mutation by Splitting Sales Territories)

For each individual X out of \mathcal{CP} draw random number, if $(rand[0, 1] < P_S)$ then mutate individual by splitting a randomly selected ST;

Step 6: (Mutation by Relocation of SCUs)

For each individual X out of \mathcal{CP} draw random number, if $(rand[0, 1] < P_R)$ then mutate individual by relocating a randomly selected SCU;

Step 7: (Reoptimize Children)

For each individual X out of \mathcal{CP} reoptimize the selection of sales territory centers;

Step 8: (Termination)

If $(g < G)$ then $\{ \mathcal{TP} := \mathcal{TP} \cup \mathcal{CP}; g := g + 1; \text{goto Step 2} \}$;

Step 9: (Improvement Best Individual)

for $j = 1$ to \bar{J} do $\{$ if $((j \neq x_j^*)$ and $\text{Connected}(X_j^* - \{j\}))$ then $\{$ for $(b \in I(X^*) - \{x_j^*\})$ with $(X_b^* \cap \mathcal{N}_j \neq \emptyset)$ do if $(S_b^*(X_b^* \cup \{j\}, T_b) + S_{x_j^*}^*(X_j^* - \{j\}, T_{x_j^*}) \geq S_b^*(X_b^*, T_b) + S_{x_j^*}^*(X_j^*, T_{x_j^*}))$ then $\{ X_j^* := X_j^* - \{j\}; X_b^* := X_b^* \cup \{j\}; \text{goto Step 9}; \}$ }

Table 8: Maximizing the Total Profit Contribution

step is repeated until the relocation of a single SCU cannot improve the total sales contribution anymore.

4 Variants of the Alignment Problem

In this section we give an idea about the versatility of the concepts. The concepts presented can be employed to support man-machine interactions in a decision support system to manipulate sales territory alignments. Adjustments allow that (a) different sales territory alignments, e.g., originating from different products, or different companies, can be merged; (b) preselected sales territory centers, e.g., to guarantee a minimum level of exposure can be taken into account; (c) different levels of available selling time can be considered; and (d) different objectives can be studied. We limit our detailed considerations to the selling time variation in Subsection 4.1 and the balancing of the potential in Subsection 4.2.

4.1 Selling Time Variation

If the selling time of a sales territory center is not considered as the capacity of a single salesman but as the capacity of a sales crew then it is additionally necessary to decide on the size of the sales crew. We divide the fixed cost f_i of a sales territory center i , $i \in I$, in a capacity independent part f_i^{CI} and a capacity dependent part $f_{i,s}^{CD}$ for allowing $T_{i,s}$, $s = 1, \dots, C_i$ units of selling time. We assume $T_{i,s} < T_{i,s+1}$, $s = 1, \dots, C_i - 1$. The objective becomes

$$\text{Maximize } Z_{TSC}^{CD}(x, y, t) = \sum_{i \in I} \sum_{j \in J} c_{i,j} (t_{i,j})^{b_j} - \sum_{i \in I} f_i^{CI} x_{i,i} + \sum_{s=1}^{C_i} f_{i,s}^{CD} y_{i,s} \quad (10)$$

where $y_{i,s}$ is a binary decision variable that equals 1 exactly if $T_{i,s}$ units of selling time are made available for STC i . To guarantee that exactly one level is chosen we add

$$\sum_{s=1}^{C_i} y_{i,s} = x_{i,i}, \quad i \in I$$

The constraints (3) and (4) are replaced by

$$t_{i,j} \leq T_{i,C_i} x_{i,j}, \quad i \in I, j \in J \quad (11)$$

and

$$\sum_{j \in J} t_{i,j} \leq \sum_{s=1}^{C_i} T_{i,s} y_{i,s}, \quad i \in I \quad (12)$$

Note, the line search employed by Drexler and Haase [4] does not allow solve the variable selling time problem directly. In order to deal with the altered problem setting it is necessary to

consider several SDPs. For finding a reasonable solution, apart from dominances, $\Pi_{i=1}^{\bar{I}} C_i$ SDPs have to be analyzed.

However, only minor adaptations of the algorithm of Section 3 are necessary to take the capacity variation into account: First, when constructing the initial population, the capacity is randomly selected. Second, when applying the crossover, the capacity is determined in accordance with the origin of the sales territory. If the father (mother) determines the sale territory then the capacity is defined by the capacity of the father (mother). Third, when applying the mutation by splitting, the capacity of the newly introduced sales territory center is randomly determined. Fourth, a new mutation operator that randomly adapts the capacity can be defined.

4.2 Balancing the Sales Potential

For years, the balancing approach (see [11]) has been the most considered strategy for building sales territories. The objective of the balancing approach is to form sales territories that are well balanced with respect to several sales territory attributes, as, e.g., workload and sales potential. Further relevant design attributes are given in [11]. The model developed considers connectivity of the sales territories, unambiguous and unshared alignment of SCUs to STCs. Furthermore, pre-specified lower and upper bounds on given design attributes have to be taken into account. One of the design attributes is selected for optimization.

Employing workload and potential as design attributes aims at forming a foundation of a fair payment of the salesmen. If the sales territories are well balanced with respect to both criteria then the payment can be independently of the sales territory (sales man) linked to the input requirements and output opportunities. Alternatively, Albers (see [1]) suggests rewards for achieving sales territory specific quotas, and concludes no further need of balanced territories. However, considering motivational aspects induced by the question for the contribution to the success of the company, it might be de-motivating for the top performer as well as for the under performer to compare the results of the efforts allocated.

The SDP can consider balanced workloads simply by the definition of identical selling times, i.e., $T_i = T$, for $i \in I$. However, the sales potential can differ substantially. We model the balancing of the potential. For sake of simplicity a conceptual model is provided. Given the set I^{sel} , $I^{sel} \subset I$, of sales territory centers selected we obtain the model of Table 9.

The objective (13) minimizes the standard deviation of the sales contribution of the STs, more

$$\text{Minimize } Z_{BP}((J_i)_{i \in I^{sel}}) = \left(\frac{1}{|I^{sel}|} \sum_{i \in I^{sel}} \left(\frac{S_i^*(J_i, T_i)}{\bar{S}} - 1 \right)^2 \right)^{\frac{1}{2}} \quad (13)$$

s. t.

$$\bar{S} = \frac{\sum_{i \in I^{sel}} S_i^*(T_i, J_i)}{|I^*|} \quad (14)$$

$$J = \bigcup_{i \in I^{sel}} J_i \quad J_i \subset J, i \in I^{sel} \quad (15)$$

$$\emptyset = J_i \cap J_{i'} \quad i, i' \in I^{sel}, i \neq i' \quad (16)$$

$$J_i \subseteq J \quad i \in I^{sel} \text{ is connected} \quad (17)$$

Table 9: The Sales Force Deployment Problem for Balancing the Potential – SDP-BP

precisely, the standard deviation of the percentage of the realization of of average sales is minimized. The objective function Z_{BP} equals zero if all the sales territories have identical sales contributions. The constraints (15), (16) and (17) ensure unambiguous assignment of the SCUs, disjointness and connectivity of the STs. Finally, (14) defines the average sales. Note, we assume that the sales territory centers allocate the selling time so that the maximum sales contribution is realized.

The potential can be balanced by an adaptation of the genetic algorithm presented in Section 3. First, the generation of the initial population is modified. Instead of determining new sales territory centers from individual to individual, the sales territory centers are selected once and remain unchanged for the entire population. Consequently (only) the sales territories may vary from individual to individual. However, we further assume that the center optimally allocates the selling time. Second, the crossover is adapted. Since the crossover defined earlier tends to reduce the number of sales territory centers – even when the individuals selected for reproduction have identical sales territory centers – we now have to preserve the sales territory centers. We redefine the son S obtained from mother M and father F for a given crossover point cp as follows

$$s_j = \left\{ \begin{array}{ll} f_j & , \text{ if } (F_j \subset (\bigcup_{k=1}^{cp} F_k)) \\ m_j & , \text{ else if } (D_j \subset (\bigcup_{k=1}^{cp} F_k)^c) \\ f_j & , \text{ else if } (f_j = j) \\ x_j & , \text{ with } x_j = (\text{rand}(\bigcup_{k \in \mathcal{N}_j^-} \{s_k\})) \text{ otherwise} \end{array} \right\} \quad j = 1, \dots, \bar{J}$$

The daughter is defined analogously. However, particularly the third case induces extremely small sales territories, which can consist only of the SCU the STC is located in. Third, the mutation by multiple relocation is therefore developed to reduce the effect. It is performed by the 3 steps of Table 10. In Step 1, a ST is randomly selected. The probability of selection is determined by the deviation of the sales from average sales. In Step 2, an adjacent ST is selected. If the sales of the formerly selected ST is less (larger) than average sales only STs with larger (less) sales are considered. The probability to select a ST from the reduced set is defined by the absolute deviation of the sales. Finally, in Step 3, new sales territories are built by randomly assigning the SCUs to the STCs i' and \bar{i} .

Fourth, the mutation by (single) realignment is employed as defined for profit maximization.

Fifth, the selection now employs probabilities to die

$$P^{death}(X) = \frac{(Z_{BP}(X) - Z_{BP}^{min} + \epsilon)^\lambda}{\sum_{E \in POP} (Z_{BP}(E) - Z_{BP}^{min} + \epsilon)^\lambda}.$$

Note, the reoptimization of selection of sales territory centers as described for the profit maximization is not employed.

5 Computational Results

The algorithm described in the previous section has been implemented in GNU C, Version 2.7.2.1, and run on personal computer (Pentium 166 MHz, 64 MB).

The problem instances have been introduced by Drexler and Haase (see [4]). The instance sets consist of SDPs with a smaller number of SCUs, i.e., 50 and 100, as well as of instance sets with a larger number of SCUs, i.e., 250 and 500 SCUs. The smaller sets contain subsets with 10, 25, 50 candidates for a sales territory center, and the larger sets allow up to 50 candidates for sales territory center. Ten instances in each class give a total of 80 instances.

Step 1: Randomly select a sales territory $J_{i'}$, $i' \in I^{sel}$ in accordance with probabilities p_i ,

$$p_i = ((\frac{S_i^*(J_i, T_i)}{\bar{S}} - 1)^2) / (\sum_{k \in I^{sel}} (\frac{S_k^*(J_k, T_k)}{\bar{S}} - 1)^2), \quad i \in I^{sel}.$$

Step 2: If $(S_{i'}^*(J_{i'}, T_{i'}) < \bar{S})$ then

$$I^{red} := \{i \in I^{sel}; S_i^*(J_i, T_i) > S_{i'}^*(J_{i'}, T_{i'}), \mathcal{N}(J_i) \cap J_{i'} \neq \emptyset\}$$

else

$$I^{red} := \{i \in I^{sel}; S_i^*(J_i, T_i) < S_{i'}^*(J_{i'}, T_{i'}), \mathcal{N}(J_i) \cap J_{i'} \neq \emptyset\};$$

randomly determine a sales territory $J_{\bar{i}}$ with $\bar{i} \in I^{red}$ with respect to probabilities p_i ,

$$p_i = |(S_i^*(J_i, T_i) - S_{i'}^*(J_{i'}, T_{i'}))| / \sum_{k \in I^{red}} |(S_k^*(J_k, T_k) - S_{i'}^*(J_{i'}, T_{i'}))|, i \in I^{red}.$$

Step 3: The sales territories are newly determined by successively assigning eligible SCUs out of $J_{i'} \cup J_{\bar{i}}$ randomly to a permissable ST of $J_{i'}$ and $J_{\bar{i}}$ with initial definition $J_{i'}' := \{i'\}$ and $J_{\bar{i}} := \{\bar{i}\}$.

Table 10: Mutation by Multiple Relocation

In the first experiment we have studied the different priority rules guiding the selection of sales territory centers. We employed stochastified maximum number of neighbors maxNoN, minimum fixed cost minFC, minimum fixed cost per neighbor minFCpN, maximum sales within center reduced by fixed cost maxS-FC, and maximum sales among neighbors reduced by fixed cost maxSaN-FC. For each instance and priority rule we have generated 100, 500, 1000 and 5000 solutions. The results are displayed in Table 11. The table lists the average percentage deviation Δ of the total sales contribution determined from an upper bound of the total sales contribution. The bounds have been provided by Haase it improve the LP-relaxation considered in [4]. We observe: The average CPU-times [sec.] for generating a solution by the different rules varies with the rule by less than five percent. The overall average CPU-times are displayed in the last column. The stochastified minimum fixed cost rule performs best for the instances with up to 250 SCUs. The results of minFC, minFCpN, and maxSaN-FC are nearly identical for 500 SCUs to be aligned. However, the deviation from the upper bound decreases only slightly even if the number of individuals is greatly increased. The results, of the overall algorithm are stable

\bar{J}	\bar{I}	ISPOP	Priority Rule					CPU
			max	min	min	max	max	
			NoN	FC	FCpN	S-FC	SaN-FC	
			Δ	Δ	Δ	Δ	Δ	
50	10	100	6.99	5.94	7.32	6.47	7.18	0.08
		500	5.35	4.56	5.59	5.45	5.25	0.40
		1000	4.85	3.85	5.20	4.90	4.75	0.70
		5000	4.08	3.29	3.71	4.12	4.26	3.50
	25	100	13.45	9.70	11.14	11.53	11.55	0.10
		500	10.85	7.96	10.17	9.15	9.50	0.45
		1000	10.02	7.56	8.69	8.72	8.54	0.87
		5000	8.38	6.11	7.90	7.39	6.97	4.37
	50	100	16.32	12.79	17.32	13.79	15.21	0.14
		500	15.80	9.91	14.97	12.24	12.61	0.56
		1000	14.72	9.71	14.10	10.63	11.97	1.09
		5000	12.40	8.98	11.42	9.51	10.92	5.10
100	10	100	3.38	3.16	3.22	3.32	3.45	0.16
		500	2.79	2.73	2.75	2.92	2.60	0.72
		1000	2.65	2.54	2.53	2.73	2.27	1.47
		5000	2.11	2.06	2.22	2.39	1.97	7.36
	25	100	8.28	6.68	7.85	8.53	7.58	0.21
		500	7.25	5.36	6.91	6.54	6.93	0.94
		1000	6.67	5.12	6.33	5.87	6.56	1.86
		5000	5.69	4.26	5.57	5.45	5.14	9.26
	50	100	11.94	8.78	11.34	10.12	10.81	0.30
		500	10.45	7.99	9.94	8.44	9.20	1.32
		1000	9.99	7.56	9.35	8.40	8.98	2.63
		5000	9.28	6.49	8.43	7.68	8.26	12.82
250	50	100	7.79	5.87	7.74	7.87	7.64	0.84
		500	7.13	5.39	6.87	6.37	7.06	3.89
		1000	6.98	5.14	6.73	6.07	6.43	7.50
		5000	6.42	4.61	6.08	5.64	6.04	37.57
500	50	100	5.48	5.39	5.49	5.96	5.20	1.80
		500	5.08	4.89	4.74	5.42	4.75	8.33
		1000	4.75	4.85	4.65	5.05	4.71	16.51
		5000	4.53	4.33	4.35	4.73	4.31	83.64

Table 11: Quality and CPU-Time for Different Priority Rules and Sizes of Initial Population

with respect to the priority rule employed to generate the initial population. We continue our considerations employing maxSaN-FC.

In the second experiment we have studied different numbers of generations G and different sizes of populations sizes $SPOP$. The initial population consists of 500 individuals. We compare

\bar{J}	\bar{I}	Genetic Algorithm								Drexl / Haase	
		G	$SPOP$	G	$SPOP$	G	$SPOP$	G	$SPOP$	–	–
		Δ	CPU	Δ	CPU	Δ	CPU	Δ	CPU	Δ	CPU
50	10	500	10	200	20	500	20	500	100	3.39	0
	25	1.75	1.01	1.58	0.87	1.58	1.61	1.54	8.47	3.65	0
	50	2.60	1.25	2.25	1.06	2.23	1.96	2.23	10.08	3.30	1
100	10	500	10	200	20	500	20	500	100	0.68	0
	25	2.11	1.54	2.26	1.40	2.18	2.55	1.86	12.80	1.69	1.90
	50	0.40	1.70	0.41	1.56	0.41	2.58	0.41	1.84	3.04	6.4
250	50	1.27	2.13	1.43	1.97	1.30	3.25	1.35	2.33	1.09	47.30
	50	1.55	2.81	1.55	2.57	1.41	4.34	1.45	3.16		
	50	500	10	200	100	500	50	500	100		
500	50	1.12	7.13	0.94	16.67	0.88	17.52	0.80	32.32		
	50	500	10	100	100	1000	50	1000	100		
	50	0.77	15.72	0.69	22.38	0.64	53.94	0.60	100.19		

Table 12: Quality and CPU-Time versus Number of Generations and Sizes of Population

the results with the state-of-the-art approach developed by Drexl and Haase, DH for short. Table 12 shows the results. Each block in the center of the table presents in its first row the number of generations G and the size of the population $SPOP$. The remaining rows of the block show the average percentage deviation of the best alignment found from the upper bound and the average CPU-time for the different numbers of candidate locations. We see: First, the genetic algorithm produces far better results at far less CPU-time than the construction procedure employed to obtain the initial population. Second, for the instances with $\bar{J} = 50$ and $\bar{I} = 10, 25, 50$ the genetic algorithm produces at slightly increased CPU-time far better results than DH. Third, for the instances with $\bar{J} = 100$ and $\bar{I} = 10, 25, 50$ the approach produces better results than DH in nearly identical CPU-times. Fourth, for the instances with $\bar{J} = 250$ and $\bar{I} = 50$ as well as for the instances with $\bar{J} = 500$ and $\bar{I} = 50$, the new solution strategy provides at least competitive results within fractions of the time required by DH. Fifth, the new approach is with respect to solution time far less sensitive towards an increase of the number of candidates for sales territory center than DH.

In the third experiment we have studied the effect of the mutation operators. The results are

\bar{J}	\bar{I}	Genetic Algorithm								Drexl / Haase	
		P_S	P_R	P_S	P_R	P_S	P_R	P_S	P_R		
		0.0	0.0	0.1	0.0	0.0	0.1	0.1	0.1		
		Δ	CPU	Δ	CPU	Δ	CPU	Δ	CPU	Δ	CPU
50	10	2.16	0.80	1.87	0.84	1.99	0.89	1.71	0.91	3.39	0
	25	3.63	0.99	2.98	1.04	2.95	1.10	2.47	1.11	3.65	0
	50	3.40	1.20	2.65	1.30	2.75	1.33	2.16	1.42	3.30	1
100	10	0.56	1.46	0.42	1.55	0.45	1.55	0.42	1.60	0.68	0
	25	1.99	1.85	1.64	1.90	1.70	1.95	1.24	2.01	1.69	1.90
	50	2.63	2.39	2.07	2.50	2.30	2.54	1.50	2.64	3.04	6.4
250	50	1.83	6.73	1.54	6.82	1.65	6.89	1.19	6.98	1.09	47.30
500	50	1.27	16.05	0.55	14.09	1.25	16.01	0.76	16.38	0.79	184.00

Table 13: Combinations of Mutation by Splitting and Mutation by Relocation at $ISPOP = 500$, $G = 200$ and $SPOP = 20$

listed in Table 13. We have employed $ISPOP = 500$, $G = 200$ and $SPOP = 20$. The results for different combinations of the mutation probabilities of splitting P_S and of relocation P_R can be found two columns per combination. The left column shows the average deviations and the right column the average CPU-times. The rows belong to the different instance sets. The mutation operators substantially improve the quality of the solution at the cost of a marginal increase of CPU-time through the mutation operators. In a further experiment we have increased the probabilities of mutation without being able to proof a significant improvement.

In the fourth experiment we have studied the variant to balance the potential. The algorithm has been executed as follows. First, the sales force deployment problem for maximization of the total sales contribution (SDP-TSC) has been considered. The best solution found has been employed to define the sales force deployment problem for balancing the potential (SDP-BP). From the solution of SDP-TSC we have extracted the sales territory centers, and defined them as input for the problem SDP-BP. Subsequently the balancing approach seeks to balance the sales territories by evaluating different sales territory alignments. The results are shown in Table 14. The first two columns specify the instances and the third column shows the average number of sales territory centers obtained when considering SDP-TSC. The table contrasts the results of the consideration of SDP-TSC in the left block with the results of the consideration

\bar{J}	\bar{I}	\overline{BL}	SDP-TSC				SDP-BP				
			$\bar{\Delta}$	Z_{BP}	$\bar{\lambda}^{min}$	$\bar{\lambda}^{max}$	$\bar{\Delta}$	Z_{BP}	$\bar{\lambda}^{min}$	$\bar{\lambda}^{max}$	\overline{CPU}
50	10	5.42	1.62	0.11	0.83	1.15	6.13	0.01	0.97	1.02	0.95
	25	6.70	2.58	0.11	0.83	1.17	6.40	0.02	0.97	1.03	1.10
	50	7.35	2.12	0.10	0.84	1.15	6.37	0.02	0.97	1.03	1.23
100	10	9.03	0.44	0.14	0.79	1.22	8.07	0.02	0.97	1.03	1.80
	25	12.07	1.27	0.11	0.82	1.21	6.72	0.02	0.96	1.04	2.05
	50	14.18	1.44	0.09	0.84	1.16	5.43	0.03	0.95	1.05	2.26
250	50	27.45	1.07	0.09	0.81	1.20	4.08	0.05	0.89	1.11	6.29
500	50	40.88	0.77	0.14	0.70	1.27	3.63	0.11	0.74	1.27	16.43

Table 14: Balancing the Potential – $ISPOP = 500$, $G = 200$ and $SPOP = 20$

of SDP-BP in the right block. The left block shows from the left to the right: the average deviation of the solution of SDP-TSC from the upper bound on sales contribution, the average of the balancings value Z_{BP} of the best solutions of SDP-TSC, the average of the minimum realization of mean sales of the solution of SDP-TSC, the average of the maximum realization of mean sales of the solution of SDP-TSC. The right block shows from the left to the right: the average deviation of the solution of SDP-BP from the upper bound on sales contribution, the average of the balancing values Z_{BP} of the best solutions of SDP-BP, the average of the minimum realization of mean sales of the solution of SDP-BP, the average of the maximum realization of mean sales of the solution of SDP-BP. Finally, the last column displays the average CPU-time necessary to determine the solution of SDP-BP. We notice that, although the sales territory optimally allocate the available selling times, the modifications proposed for balancing the potential substantially reduce the standard deviation substantially. However, the more balanced territories are only realized at the cost of a lower total sales contribution.

6 Conclusions

We have studied the sales force deployment problem that simultaneously considers several interacting subproblems. Employing a sales response function sales force sizing, sales force placement, sales territory alignment, and sales effort allocation are the subjects of investiga-

tion. Genetic concepts have been proposed for the maximization of the total profit contribution. The concepts are capable to deal with practical requirements, such as connectivity of the sales territories, and adaptable to consider problem variations, such as capacity variation and objective variation. Moreover, the concepts can be employed to support man-machine interactions in a DSS. Sales territory alignments originating from different product or companies can be merged.

The computational analysis shows that the algorithm produces competitive results at far less CPU-time than required by the state-of-the-art procedure. Moreover, the CPU-time is less sensitive to the variation of the size of the problem, i.e., number of SCUs and number of potential sales territory centers, than the state-of-the-art approach.

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