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A new class of valid inequalities for the round robin tournament problem

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Abstract

We introduce a new class of valid inequalities for the maximum round robin tournament (MRRT) problem. They strengthen the linear relaxation and improve the upper bound. We also answer negatively the question of Briskorn if the optimal objective value of the MRRT problem and that of its conventional linear relaxation always coincide.

Keywords: round robin tournaments; valid inequalities; linear relaxation

1 Problem formulation and previous results

We are given an even set T of n teams and a home-away-pattern set (HAP set) which is a binary matrix h of dimension $n \times (n - 1)$. Each component h_{ip} represents a requirement for team i to play at home (if $h_{ip} = 1$) or away (if $h_{ip} = 0$) in period p .

The problem is to find an assignment of pairs of teams to the periods with maximum number of assigned pairs such that no team is assigned to one and the same period twice, no pair is assigned more than once and if a pair (i, j) is assigned to a period p , then $h_{ip} \neq h_{jp}$. More precisely, the maximum round robin tournament problem is to find an optimal solution of the following integer problem:

$$\begin{aligned}
 z_h = \max \quad & \sum_{i \in T} \sum_{j \in T, j < i} \sum_{p \in P} x_{ijp} \\
 \text{s.t.} \quad & \sum_{p \in P} x_{ijp} \leq 1 & i, j \in T, j < i \\
 & \sum_{j \in T, j < i} x_{ijp} + \sum_{j \in T, j > i} x_{jip} \leq 1 & i \in T, p \in P \\
 & x_{ijp} \leq |h_{ip} - h_{jp}| & i, j \in T, j < i, p \in P \\
 & x_{ijp} \in \{0, 1\} & i, j \in T, j < i, p \in P.
 \end{aligned} \tag{1}$$

We call an integer feasible solution of (1) a *schedule*. We call an unordered pair (i, j) with $i, j \in T, i \neq j$ a *match*. A schedule is *maximal* if it can not be extended by setting one zero-valued variable to one without adjusting the values of other variables. Given a schedule x we say a match $(i, j), j < i$, is *missing* in the schedule, if $x_{ijp} = 0$ holds for each period p . If no match is missing in a schedule, then the schedule is *complete*. A

Team	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	0	0	0	1	1	1	1	0	1	1	1	0
2	0	0	0	0	1	1	1	1	0	0	1	1	1
3	0	0	0	0	1	1	1	1	1	0	0	1	1
4	0	0	0	0	1	1	1	1	1	1	0	0	1
5	0	0	0	0	1	1	1	1	1	1	1	0	0
6	0	0	0	0	0	0	0	0	1	1	1	1	0
7	1	1	1	1	0	0	0	0	0	1	1	1	0
8	1	1	1	1	0	0	0	0	0	0	1	1	1
9	1	1	1	1	0	0	0	0	1	0	0	1	1
10	1	1	1	1	0	0	0	0	1	1	0	0	1
11	1	1	1	1	0	0	0	0	1	1	1	0	0
12	0	0	1	1	0	1	1	1	0	0	0	0	0
13	1	1	0	1	1	0	1	1	0	0	0	0	1
14	1	1	1	0	1	1	0	0	0	0	0	0	0

Table 1: Infeasible HAP set of Kashiwabara

complete schedule consists of $\binom{n}{2}$ matches. A HAP set is called infeasible if there is no complete schedule satisfying this set.

To calculate an upper bound for z_h one can solve the linear relaxation of (1) which is obtained by relaxing the integrality constraints on variables. We denote this upper bound by \bar{z}_h .

Briskorn [1] shows that the necessary condition $\bar{z}_h = z_h$ for the feasibility of h dominates the necessary condition of Miyashiro, Iwasaki, and Matsui [4] which is:

$$\sum \min\{h(T', p), |T'| - h(T', p)\} \geq \binom{|T'|}{2} \quad (2)$$

holds for each $T' \subset T$. Here, $h(T', p) := \sum_{i \in T'} h_{ip}$.

Kashiwabara [3] proposes an unfeasible HAP set h for 14 teams that satisfies this necessary condition (see table 1). Briskorn shows the infeasibility of this HAP set by calculating \bar{z}_h . This value is 90 and can be obtained by means of any contemporary linear solver. He also states a question whether the optimal value of (1) always coincide with that of the linear relaxation of (1).

2 A class of valid inequalities

The following proposition helps us to answer the question negatively.

Proposition 1 *Each maximal schedule has zero or at least two missing matches.*

Proof. Suppose one match is missing in the schedule. Then some team i is not playing in some period p . There must be another team j that is not playing in p . Since the schedule is maximal the match (i, j) is scheduled in some other period. We have a missing match with i as well as a missing match with j , and these matches do not coincide, since (i, j) is scheduled. ■

Let us consider the instance in Table 1. A complete schedule should consist of $\binom{14}{2} = 91$ matches. The upper bound from the LP-relaxation is 90. Therefore, by the proposition, at least two matches are missing in each schedule for the instance, i.e the optimum value is at most $\binom{14}{2} - 2 = 89$. By means of an integer solver (we used ILOG CPLEX [2]) we can prove that this upper bound is tight, i.e. it is also the optimal objective value of the integer problem.

Can we tighten the LP-relaxation to obtain this better upper bound? The answer is *yes*. It can be done by expanding the set of constraints of the LP-relaxation of (1). We add the following inequalities which are valid for all maximal schedules.

$$\sum_{(i,j \neq k, j < i) \vee (p \neq q)} x_{ijp} - \left(\sum_{i > k} x_{ikq} + \sum_{i < k} x_{k iq} \right) \leq \binom{n}{2} - 2 \quad k \in T, q \in P \quad (3)$$

The validity follows from the proposition. Indeed, if the second term (in parentheses) is one, then the first term is at most $\binom{n}{2} - 2$ and the inequality is satisfied with equality. If for a maximal schedule the second term is zero and hence one match is missing, then there is a second missing match in the maximal schedule that makes the first term to be at most $\binom{n}{2} - 2$.

To show that (3) tighten the LP-relaxation of (1) we exploit the instance of Kashiwabara again. ILOG CPLEX gives the optimal value of 90 for the linear relaxation of (1).

The optimal value of the linear relaxation strengthened by (3) is 89 which is the optimal value of the MRRT problem.

The fact that there are $n \times (n-1)$ inequalities in the class makes the separation problem rather trivial. They can be easily used in practice to strengthen the linear relaxation of MRRT problem or for variations of the problem with arbitrary linear objective function with non-negative coefficients.

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