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Debt Maturity and the Dynamics of Leverage *

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Abstract

This paper shows that long debt maturities eliminate equityholders' incentives to reduce leverage when the firm performs poorly. By contrast, short debt maturities commit equityholders to such leverage reductions. However, shorter debt maturities also lead to higher transactions costs when maturing bonds must be refinanced. We show that this tradeoff between higher expected transactions costs against the commitment to reduce leverage when the firm is doing poorly motivates an optimal maturity structure of corporate debt. Since firms with high costs of financial distress benefit most from committing to leverage reductions, they have a stronger motive to issue short-term debt.

Keywords: debt maturity, optimal capital structure choice
JEL: G3, G32

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1 Introduction

Significant progress has been made towards understanding firms' dynamic financing decisions. Major contributions to this literature model a firm's assets or cash flows as a stochastic process and assume that debt generates some benefit, such as a tax advantage, but also generates dead weight costs associated with excessively high leverage, such as bankruptcy costs.[1] While these models have been successful in explaining firms' optimal target leverage ratios and their decisions to dynamically increase debt levels in response to increases in their asset values or cash flows, they have been much less successful in explaining leverage reductions. In fact, these models generally imply that equityholders never find it optimal to reduce dividends or issue equity to reduce debt. As shown by Admati, DeMarzo, Hellwig, and Pfleiderer (2015), equityholders not only lack any incentive to actively repurchase outstanding debt but frequently have incentives to increase debt even if this reduces total firm value. Thus, in these models debt reductions only occur following bankruptcy.[2]

This implication is in contrast to empirical evidence that debt reductions frequently occur outside of bankruptcy and without negotiated debt forgiveness.[3] In this paper we develop a dynamic capital structure model where leverage reductions occur not only after bankruptcy or after renegotiations with debtholders. We show that such voluntary leverage reductions are closely related to the firm's debt maturities. Thus, we identify and analyze a largely unexplored aspect of debt maturity, namely its effect on future capital structure dynamics. We specifically address the following questions. How is debt maturity related to equityholders'

[1] See, for example, Fischer, Heinkel, and Zechner (1989), Leland (1994a), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), Dangl and Zechner (2004), Strebulaev (2007).

[2] Some models consider debt renegotiations and derive partial debt forgiveness outside of bankruptcy (See, e.g. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Christensen, Flor, Lando, and Miltersen (2014)). Mao and Tserlukevich (2015) present a model where non-coordinated debt holders may accept repurchase offers below the market price when firms pay with existing safe assets or cash. Lehar (2015) considers multilateral bargaining and explicitly regards renegotiation breakdowns and subsequent inefficient liquidation. In contrast to these papers we focus on situations where coordination problems among bondholders prevent renegotiation solutions.

[3] Leary and Roberts (2005) report that a fraction of 28% of capital structure adjustments in their 1984 to 2001 dataset comprises active debt repurchases. Surveying 392 CFOs, Graham and Harvey (2001) report that 81% of firms in their sample use at least flexible target leverage ratios. If highly levered, firms tend to issue equity to maintain their target ratios. Hovakimian, Opler, and Titman (2001) find strong evidence that firms use (time varying) target leverage ratios. They find the deviation from this target as the dominant economic factor in determining whether a firm retires debt.

dynamic leverage adjustments? How do firms optimally refinance expiring debt? What is the optimal debt maturity structure given its implications for dynamic capital structure adjustments and which firms are most likely to issue short-term debt? We address these questions in a framework that does not rely on information asymmetries or agency conflicts. In the model firms' equityholders are allowed to optimize the mix of debt and equity used to refinance maturing debt, but covenants do not allow them to increase the total face value of debt. If firms wish to increase the face value of debt they must first eliminate these covenants by repurchasing the existing debt before they can make discrete capital structure adjustments. They are allowed to do so at any point in time.

We find that firms' equityholders may not wish to roll over maturing debt by issuing a new bond with the same face value. Instead, it may be optimal for them to issue a bond with lower face value, i.e. to at least partly use equity to repay the maturing debt. This happens for firms with sufficiently short debt maturities following a deterioration in the firm's profitability. In this situation issuing new debt to refinance maturing old debt is costly for equityholders since the price of the new bonds reflects the increased default probability and the resulting increase in expected costs of financial distress. These costs would effectively be borne by the equityholders and they may therefore optimally reduce the face value of the new bond issue.

If, by contrast, debt maturity is sufficiently long, then replacing maturing debt with equity always leads to a significant wealth transfer to the remaining bonds outstanding, since debt with a longer maturity is subject to more credit risk. This creates a more severe debt overhang problem and makes the use of equity to refinance maturing debt suboptimal. In this case the firm's equityholders always prefer to roll over debt at the maximum rate, i.e. to issue a new bond with a face value that corresponds to the face value of the maturing bonds. This result is in accordance with empirical evidence provided by Hovakimian, Opler, and Titman (2001), who find that long debt maturities seem to be major impediments to debt reductions.

We also find that shorter debt maturities lead to more pronounced debt reductions since the short maturities require the firm to refund a larger fraction of its debt during any given period of time. This implies that a firm which refinances part of the retired debt with equity will lower its debt level more quickly in response to a drop in its profitability.

Firms will never find it optimal to repurchase debt over and above the contracted retire-

ment rate which is in accordance with Admati, DeMarzo, Hellwig, and Pfleiderer (2015). It is the ex-post decision how to repay of expiring debt that is influenced by debt maturity and which determines the firm's future capital structure. Hence, we identify short-term debt as an ex-ante commitment to engage in debt reductions should the firm eventually run into trouble.

Equityholders incentives to refund maturing debt with equity are non-monotonic in the firm's profitability and thus in firm value. For values around the initial cash flow level it is optimal to roll over maturing debt by issuing new bonds with the same face value. If the firms profitability drops sufficiently, then the equityholders reduce the rollover rate, as explained above. However, if the firm's cash flows continue to deteriorate and the firm is pushed towards the default boundary, then equityholders find it optimal again to choose the maximum rollover rate. Since the firm is close to bankruptcy a reduction in leverage largely benefits the remaining bondholders, even if the maturity of the remaining debt is short. Thus, the resulting debt overhang problem implies that equityholders are no longer willing to contribute capital to reduce debt.

One way to understand this result intuitively is to recall that the equityholders effectively own a put option which gives them the right to sell the firms assets to its bondholders at an exercise price equal to the face value of the bonds. Reducing this exercise price by retiring some or all of the maturing debt leads to a particularly significant reduction in the value of this option when it is at or in the money, i.e. if the value of the firms assets is already close to or less than the face value of debt. In this case the equityholders are willing to roll over maturing debt, even if the new bonds can only be issued at a low price.

Hovakimian, Opler, and Titman (2001) present strong empirical support for this non-monotonicity in voluntary debt reductions. Interestingly, existing literature such as Welch (2004) has interpreted the fact that highly levered firms issue debt as evidence *against* the trade off theory of capital structure choice, since it moves the leverage ratio away from the optimal target ratio. Our analysis demonstrates that this behavior is in full accordance with a dynamic tradeoff paradigm once multiple debt issues and optimal financing of maturing debt are considered.

In our setting, debt maturity significantly influences the expected probability of bankruptcy. This is so since short debt maturities lead to more rapid debt reductions when the firm's prof-

itability starts to decrease. Investors take this into account when they price the debt initially. This implies that firms' debt capacity generally *increases* as they choose shorter debt maturities. This result is also in contrast to existing literature which unanimously predicts that short-term debt leads to early and inefficient default. The intuition for this is that equityholders incur the rollover cost. I.e. when a new bond issue with the same face value cannot fully refinance the maturing bond, equityholders must cover the remaining funding gap. For shorter maturities, the firm must roll over a higher fraction of its debt and therefore equityholders face larger funding gaps. As a result they default sooner (see, e.g. Leland (1994b), Leland and Toft (1996), He and Xiong (2012a) or He and Milbradt (2014)). This aspect of short-term debt tends to reduce firms' debt capacities. In contrast to these papers we consider a new aspect of short-term debt, namely its effect on future leverage reductions. As we show, this implies that shortening debt maturity generally increases the firm's debt capacity and reduces the risk of bankruptcy ex ante.

Our analysis therefore generates a novel theory of debt maturity where, for reasonable parameter values, total firm value is maximized at an interior debt maturity. Since firms never engage in debt reductions for long debt maturities but still incur some transactions costs when debt is rolled over, total firm value is locally maximized for infinite-maturity debt. This saves transactions costs and prevents inefficient early default.[4] When shortening debt maturity sufficiently, however, firms start to engage in debt reductions when their profitability decreases, thereby reducing the probability of financial distress. In this maturity range, shorter maturity comes with higher debt capacity and total firm value starts to rise, until the transactions costs associated with refinancing maturing debt outweigh the benefits due to faster debt reductions along unfavorable cash flow paths. Thus, total firm value exhibits another local maximum at an interior value of debt maturity.[5] The exact location of this maximum depends on the parameters of the firm's cash flow process, such as its growth rate and its volatility, as well as on the transactions costs associated with rolling over debt and the magnitude of bankruptcy costs. For empirically reasonable model parameterizations we find that firm value is indeed

[4]Furthermore it is shown by Leland (1994b), Leland and Toft (1996), and Leland (1998) that the tax advantage of debt is maximized when issuing infinite-maturity debt. Hence, when finite-maturity debt does not imply more efficient downwards restructuring, it is dominated by debt with infinite maturity.

[5]Alternative rationales for short-term debt are based on agency costs originating from the 'asset substitution' problem, first introduced by Jensen and Meckling (1976).

maximized for interior debt maturities. Infinite-maturity debt maximizes firm value globally only if the costs of financial distress and/or the tax advantage of debt are very low and/or transactions costs for rolling over debt are high. In this case the benefit from increasing debt capacity and reducing the bankruptcy probability by committing to future leverage reductions via short debt is too low compared to the additional transactions costs associated with short-term debt.

Optimal debt maturity was first analyzed in tradeoff models by Leland (1994b), Leland and Toft (1996), and Leland (1998). Titman and Tsyplakov (2007) extend the analysis by endogenizing investment decisions. These papers have derived important modelling strategies allowing the analysis of debt maturity in a tractable continuous-time framework. They have also generated significant insights on the interplay between leverage and debt maturity. However, they cannot explain interior optimal debt maturities. In these models it would be optimal to issue perpetual debt.

Our model uses a similar modelling approach with one important difference. We allow firms to choose the mix of debt and equity to repay maturing debt, whereas firms in the above models *must* roll over maturing debt with new debt issues, keeping the face value of total debt constant. In contrast to these papers, we concentrate on debt maturity and its role in mitigating conflicts of interest between debtholders and equityholders on capital structure dynamics. [6]

Our paper relates to existing work on debt maturity which explores informational asymmetries. In this literature short debt maturities signal positive inside information, as demonstrated by the seminal work by Diamond (1991, 1993) and Flannery (1986, 1994). Other authors, such as Calomiris and Kahn (1991) and Diamond and Rajan (2001) have emphasized the disciplinary role of short term debt. Debt maturity has also been linked to the debt overhang or underinvestment problem. While the original work by Myers (1977) concludes

[6]Childs, Mauer, and Ott (2005) and Ju, Parrino, Poteshman, and Weisbach (2005) also explore debt maturity. However, in these models firms can only change their debt levels after the entire existing debt has matured. Also, at each point in time firms can only have one bond outstanding with a given maturity. In our model firms are allowed to change the debt level at any point in time. As a result, we are able to isolate the pure commitment effect of debt maturity on equityholders' willingness to adjust debt levels downwards after a decrease in profitability. Furthermore, firms in our model have many bonds with different maturities outstanding, as is frequently the case in practice. At any point in time firms retire only a fraction of outstanding bonds. Therefore, when some bonds mature and are refinanced with new debt or via equity, this influences the value of the existing bonds outstanding.

that short-term debt mitigates these problems, Diamond and He (2014) show that maturing short-term debt can lead to more severe debt overhang than non-maturing long-term debt. Finally, Hackbarth and Mauer (2012) and Dockner, Maeland, and Miltersen (2016) analyze the effect of debt seniority on the underinvestment problem.

There is also an interesting related literature on the interaction between debt maturity, rollover risk, and capital structure. Examples are He and Xiong (2012a,b), He and Milbradt (2014), Cheng and Milbradt (2012) and Chen, Cui, He, and Milbradt (2015). In a similar vein, He and Xiong (2012a) and Acharya, Gale, and Yorulmazer (2011) analyze debt maturity when short-term debt can lead to early and inefficient asset liquidation.

Recently, optimal debt maturity adjustments over time have been analyzed. See, for example, Brunnermeier and Oehmke (2013), and He and Milbradt (2016). Finally, our paper is also related to an emerging literature that analyzes rollover risk and the volatility of credit spreads and the optimal dispersion of debt maturities (see Choi, Hackbarth, and Zechner (2015) and Chaderina (2016)). While credit spreads at future roll-over dates are also stochastic in our model and therefore affect optimal maturity choices, we do not explicitly model the dispersion of debt maturities.[7]

None of the contributions discussed above addresses the main aspect of our paper, namely the effect of debt maturity on equityholders' future incentives to delever. While the existing literature assumes that the face value of debt is kept konstant at rollover dates, we allow firms to optimally choose the refinancing mix. This implies that a firm's leverage capacity increases if it chooses a capital structure which forces it to regularly roll over a non-trivial portion of its debt. This can only be achieved if the average debt maturity is not too long.

The remainder of the paper is structured as follows. Section 2 introduces the main building blocks of the model. The valuation of debt and equity claims and the optimal refinancing of expiring debt are derived in Section 3. Section 4 discusses dynamic capital structure strategies and their optimization. Section 5 describes how the model is calibrated to mirror a representative firm under the US tax code and Section 6 provides numerical examples and comparative statics results. Section 7 concludes.

[7] In fact, the maturity dispersion in our model is characterized by a constant proportion of bonds expiring in each instant of time.

2 The Model

Consider a firm that has debt outstanding with face value B_t and a fixed coupon rate i. Coupon payments are tax deductible so that there is a tax advantage of debt. See Table 1 for the notation used throughout this paper. Following the modeling of finite maturity debt in Leland (1994b), Leland (1998), and Ericsson (2000), we assume that debt has no single explicit maturity date but that a constant fraction m of the outstanding debt matures at any instant of time. Ignoring default and debt repurchase, the average maturity of a debt contract is then $1/m$ years.

The firm must repay maturing debt at par, and thus must maintain a flow of principal repayment mB_t. The retired portion of debt may be replaced by a new debt issue. However, we consider bond indentures ensuring that a new bond issue may not increase the total initial face value of debt, so that the rate δ_t at which the firm may issue new debt must satisfy $0 \leq \delta_t \leq m$. The new debt issue is associated with proportional transactions costs k_i, has the same priority as existing debt, and is amortized at the same constant rate m. This ensures that the entire debt of the firm is homogeneous and no distinction between early issues and later issues must be made. Although this modelling approach is a simplification it allows us to analyze the implications of debt maturity in the realistic setting in which firms have more than one debt issue outstanding and where the refinancing decision influences the value of the remaining bonds.

As discussed above, covenants prohibit the firm from issuing debt that would increase the total face value. The total amount of debt outstanding can therefore only be increased by repurchasing all outstanding debt contracts and subsequently issuing new bonds with higher face value. Again, proportional transactions costs k_r are associated with the new bond issue. The coupon rate of the new issue is set to ensure that it can be sold at par.

In contrast to existing models with finite average maturity, the firm is not required to roll over the entire amount of maturing debt. For certain leverage ratios, the firm may find it optimal to replace only part of the retired debt with new debt or it might entirely refrain from issuing new debt contracts. If the firm does not fully replace retired debt then the face value of debt outstanding is reduced at a rate $m - \delta_t$ which in turn may help the firm to avoid financial

Table 1: Notation

a firm's instantaneous free cash flow after corporate tax	c_t
expected rate of change of c_t	μ
risk adjusted drift of the cash flow process	$\hat{\mu}$
riskless rate of interest	r
instantaneous variance of the cash flow process	$c_t^2\sigma^2$
face value of debt	B_t
debt retirement rate	m
average debt maturity	$T = 1/m$
debt roll over rate	δ
value of equity	E
value of debt	D
total value of the firm	V
instantaneous coupon rate	i
firm's inverse leverage ratio	y_t
personal tax rate on ordinary income	τ_p
corporate tax rate	τ_c
proportional bankruptcy costs	g
proportional transactions costs for rolling over debt	k_i
proportional transactions costs for issuing debt after recapitalization	k_r
proportional call premium	λ

distress.

Debt covenants restrict the face value of debt issued in any given period to be less or equal to the face value of the retired debt. Therefore, after a phase of debt reduction the firm cannot return to the original debt level unless it eliminates the bond indenture by calling all outstanding debt.

If the firm's equityholders stop coupon payments and thereby trigger bankruptcy, all control rights over the firm's productive assets are handed over to debtholders who will then optimally relever the firm. As in Fischer, Heinkel, and Zechner (1989), bankruptcy costs are assumed to be a fraction g of the outstanding face value of the firm's debt.

We assume that the firm's instantaneous free cash flow after corporate tax, c_t, follows a

geometric Brownian motion given by

$$\begin{aligned} \frac{dc_t}{c_t} &= \mu dt + \sigma dW_t, \\ c_0 &= c(0), \end{aligned} \tag{1}$$

where the expected instantaneous drift and the instantaneous variance of the cash flow process are defined by $c_t\mu$ and $c_t^2\sigma^2$ respectively, and dW_t is the increment to a standard Wiener process.

Although there is considerable cross-country variation in the way corporate and personal income are taxed, many tax systems exhibit similar key features. First, the deductibility of interest expenses from taxable corporate income is frequently more generous than that of dividend payments. Second, the effective personal tax on equity income is frequently lower than that on debt income. The latter feature may be due to an outright favorable treatment of dividend income or due to the fact that a larger portion of equity income is generally realized in the form of capital gains, which are often treated more favorably than ordinary income. In addition capital gains income comes with a tax-timing option for the investor which is also contributes to the second feature of tax systems mentioned above.

We capture these two features by defining the firm's operating cash flow c_t as after corporate tax and by allowing any coupon payments to be deductible at the constant statutory corporate tax rate τ_c.[8] At the personal level, τ_p is interpreted broadly as the tax disadvantage of interest income over equity income. Therefore the appropriate discount rate to be applied to expected after-corporate-tax income from equity investment under the risk-neutral probability measure is given by $r(1-\tau_p)$, see Section 3. For a discussion of the calibration of the tax parameters and how they relate to the current US tax code we refer to Section 5.

At any point in time, equityholders can decide to adjust the amount of debt by a discrete amount to a new face value B_t^*. Alternatively, equityholders may maintain the current debt level and only decide on the rate $\delta_t \in [0,m]$ at which new debt is issued to roll-over (a fraction of) maturing debt. If $\delta_t = m$, then the firm issues new bonds with a face value exactly equal to the face value of the bonds retired at time t.[9] The dynamics of the face value of debt are

[8] Hence, we assume instant tax refunds for coupon payments and do not explicitly model any loss carry-back and loss carry-forward due to limited corporate taxable income.

[9] Depending on the market value of debt, the proceeds may be considerably less than what is required by

therefore given by

$$\begin{aligned} \frac{dB_t}{B_t} &= \begin{cases} \dfrac{B_t^*}{B_t} - 1 & : \text{ debt is increased from } B_t \text{ to } B_t^* \text{ at time } t, \\ -(m-\delta_t)\,dt & : \text{ firm replaces maturing debt at a rate } \delta_t \in [0,m] \text{ at time } t \end{cases} \\ B_0 &= B(0). \end{aligned} \tag{2}$$

We define y_t as the inverse leverage ratio with respect to the unlevered firm value

$$y_t = \frac{1}{B_t}\frac{c_t}{r(1-\tau_p)-\hat{\mu}}, \tag{3}$$

where τ_p is the personal income tax rate and $\hat{\mu}$ is the risk neutral drift rate of the free cash flow c_t.[10] Then the risk neutral dynamics of y_t are

$$\begin{aligned} \frac{dy_t}{y_t} &= \begin{cases} \dfrac{B_t}{B_t^*} - 1 & : \text{ debt is increased from } B_t \text{ to } B_t^* \text{ at time } t, \\ (\hat{\mu} + (m-\delta_t))\,dt + \sigma\,dW_t & : \text{ maturing debt is replaced at a rate } \delta_t \text{ at time } t \end{cases} \\ y_0 &= y(c_0, B_0) = \frac{1}{B_0}\frac{c_0}{r(1-\tau_p)-\hat{\mu}}. \end{aligned} \tag{4}$$

(See Appendix A.1 for the derivation of Equation (4).) A discrete adjustment of the debt level following a debt repurchase leads to an immediate jump in the inverse leverage ratio. When the face value of debt is maintained at a constant level (i.e., $\delta_t = m$), then the inverse leverage ratio follows a geometric Brownian motion with the same drift rate and volatility as the cash flow process c_t. When only part of the maturing debt is rolled over ($\delta_t < m$), then the drift rate of the inverse leverage ratio is $\hat{\mu} + (m - \delta_t) > \hat{\mu}$, i.e., due to the shrinking debt level, the firm's leverage ratio tends to fall, and thus, the inverse leverage ratio tends to rise.

Existing dynamic capital structure models such as Fischer, Heinkel, and Zechner (1989) utilize the fact that equity and debt values are homogeneous of degree one in the face value

repayment obligations even when $m = \delta_t$. In this case the remaining amount is financed by retained earnings or raising new equity. Alternatively, it may as well be the case that debt trades above par, then the net proceeds are paid out as a dividend to equityholders.

[10] For a discussion of the effect of personal taxes on debt dynamics, see Hennessy and Whited (2005).

of debt, B. Thus, all firm-relevant decisions can be made contingent on the leverage ratio y, and hence B serves as a scaling factor only. It is shown below, that this homogeneity can be preserved even in the case of finite-maturity debt and gradual debt reductions. Therefore, all claims contingent on the cash flow c_t can be re-interpreted as claims contingent on the two state variables, debt level B_t and inverse leverage ratio y_t. This facilitates to obtain closed form solutions for the optimal roll-over schedule δ_t and for the value of debt and equity of the firm.

3 Claim Valuation and Optimal Funding of Debt Repayment

In this section we derive the valuation equations for the firm's debt and equity as well as propositions on the optimal refinancing mix for maturing debt. Consider a firm which has debt outstanding with face value B_t. Contingent on the choice of δ_t, the firm's debt level changes at a rate $-(m-\delta_t)$ and, consequently, the drift rate of the inverse leverage ratio y_t is $\hat{\mu}+(m-\delta_t)$. The required instantaneous principal repayment is $mB_t dt$, the after-tax coupon payment is $i(1-\tau_p)B_t dt$, and debtholders pay $\delta_t D$ for the new debt issues. Therefore the value of debt, D, must satisfy the partial differential equation

$$\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 D}{\partial y^2} + (\hat{\mu}+(m-\delta_t))y\frac{\partial D}{\partial y} + \frac{\partial D}{\partial t} + B_t(i(1-\tau_p)+m) - \delta_t D = r(1-\tau_p)D. \tag{5}$$

Using the homogeneity with respect to the face value B_t, we can write $D = B_t\tilde{D}(y)$. The fact that the debt level changes at a rate of $-(m-\delta_t)$ then leads to $\partial D/\partial t = -(m-\delta_t)B_t\tilde{D}(y)$. Then the value of debt per unit of face value, $\tilde{D}(y)$, is not explicitly time dependent and satisfies the following differential equation

$$\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{D}}{\partial y^2} + (\hat{\mu}+(m-\delta_t))y\frac{\partial \tilde{D}}{\partial y} + (i(1-\tau_p)+m) = (r(1-\tau_p)+m)\tilde{D}. \tag{6}$$

We next turn to the valuation of equity. Equityholders must provide a cash flow of mB_t to service expiring debt contracts. Furthermore, debt requires coupon payments of iB_t which are

tax deductible. The tax-adjusted outflow to debtholders is therefore $(i(1-\tau_c)+m)B_t$. At the same time equityholders issue new debt at a rate δ_t to (partly) replace maturing debt. They receive the proceeds, i.e., the market value of the newly issued debt contracts, $\delta_t D(y,B)$, and have to bear proportional transactions costs k_i. The inflow from rolling over debt is therefore $\delta_t(1-k_i)D(y,B)$. Finally, equityholders receive the cash flow of the assets of the firm, $c=(r(1-\tau_p)-\hat{\mu})yB_t$.

Again using the homogeneity with respect to the face value of debt we write $E=B_t\tilde{E}(y)$, where $\tilde{E}$ is the equity value per unit of face value of debt. While the individual debt contract amortizes at a constant rate m, the firm's total debt level changes at a rate $m-\delta_t$ depending on the firm's current rollover decision δ_t. Consequently, the partial derivative of equity with respect to time is $\partial E/\partial t=-(m-\delta_t)B_t\tilde{E}(y)$. The value of equity therefore satisfies the following differential equation

$$\begin{aligned}\frac{1}{2}\sigma^2y^2\frac{\partial^2\tilde{E}}{\partial y^2}+(\hat{\mu}+(m-\delta_t))y\frac{\partial\tilde{E}}{\partial y}-(i(1-\tau_c)+m)& \\ +(1-k_i)\delta_t\tilde{D}(y)+(r(1-\tau_p)-\hat{\mu})y &= (r(1-\tau_p)+(m-\delta_t))\tilde{E}.\end{aligned} \tag{7}$$

We are now able to derive the equilibrium roll-over rate for maturing debt, δ. Suppose that a firm announces a roll-over rate δ' and the market prices the bonds accordingly. As long as the partial derivative of equity value with respect to the roll-over rate is positive at δ', the equityholders have an incentive to re-enter the market and issue more debt. Rational investors anticipate this and price the new bonds, conjecturing a roll-over rate from which equityholders have no incentive to deviate, given the price of the bonds.

Since it follows from the two Hamilton-Jacoby-Bellman equations (6) and (7) that there is no explicit time dependence, the optimal debt roll-over rate depends only on the current leverage of the firm, i.e., $\delta_t=\delta(y)$. The optimal roll-over schedule $\delta^*(y)$ is therefore determined as a rational expectations equilibrium (i.e., a Markovian Nash-equilibrium) of the game between equityholders (setting the roll over rate δ) and the market (valuing equity and debt).[11] To derive the equilibrium, the following corollary will be useful.

[11] For a game theoretic analysis of a trading environment in which buyers or sellers cannot commit to a single trade, see DeMarzo and Bizer (1993). For a comprehensive discussion of differential games, see Dockner, Jørgensen, Van Long, and Sorger (2000)

Corollary 1. *The partial derivative of equity with respect to the debt roll-over rate* δ *is given by*

$$\frac{\partial \tilde{E}}{\partial \delta} = \frac{K_1 - (r(1-\tau_p)+m)K_2}{(r(1-\tau_p)+(m-\delta))^2},$$

where K_1 *and* K_2 *are given by*

$$\begin{aligned} K_1 &= \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{E}}{\partial y^2} + (\hat{\mu}+m) y \frac{\partial \tilde{E}}{\partial y} \\ &\quad -(i(1-\tau_c)+m) + (r(1-\tau_p)-\hat{\mu})y, \\ K_2 &= y\frac{\partial \tilde{E}}{\partial y} - (1-k_i)\tilde{D}(y). \end{aligned}$$

The partial derivative of debt with respect to the debt roll over rate δ *is given by*

$$\frac{\partial \tilde{D}}{\partial \delta} = -\frac{y}{r(1-\tau_p)+m}\,\frac{\partial \tilde{D}}{\partial y}.$$

(See Appendix A.2 for the proof of Corollary 1.)

Corollary 1 implies that the sign of the partial derivative of equity with respect to the roll-over rate depends on the value of debt per unit of face value, $\tilde{D}(y)$. For sufficiently large values of debt it is positive whereas it is negative for sufficiently low values. The partial derivative is zero for a critical value $\tilde{D}^I$. These results imply the following proposition.

Proposition 1. *Equityholders are indifferent to changes in the debt roll over rate* $\delta(y)$ *if and only if the value of debt per unit of face value satisfies*

$$\begin{aligned} \tilde{D}(y) &= \frac{1}{1-k_i}\left(y\frac{\partial \tilde{E}}{\partial y}(y) - \tilde{E}(y)\right) \\ &=: \tilde{D}^I(y). \end{aligned}$$

If and only if $\tilde{D}(y) > \tilde{D}^I(y)$*, the firm optimally rolls over debt at* $\delta^* = m$*. If and only if* $\tilde{D}(y) < \tilde{D}^I(y)$ *the firm optimally finances debt repayments entirely with equity, i.e.,* $\delta^* = 0$*.*

(See Appendix A.3 for the proof of Proposition 1.)

Proposition 1 is very intuitive. Suppose the firm issues one additional unit of debt dB then it will receive the proceeds of this issue (net of transactions costs). In addition to the

proceeds there will be a change in equity value because the issue influences both B and y. Equityholders find it optimal to go ahead with this debt issue only if the sum of these effects is positive, i.e.,

$$\begin{aligned} 0 &< (1-k_i)\tilde{D}(y)dB + dE \\ &= (1-k_i)\tilde{D}(y)dB + \frac{\partial E}{\partial B}dB + \frac{\partial E}{\partial y}\frac{dy}{dB}dB \\ &= \left((1-k_i)\tilde{D}(y) + \tilde{E}(y) - y\frac{\partial \tilde{E}}{\partial y}(y)\right)dB, \end{aligned} \tag{8}$$

which is equivalent to the statement in Proposition 1.

On first inspection one may conclude that the optimal solution for δ is characterized by a 'bang-bang' solution, i.e., either full re-issuance of no re-issuance. This first intuition is, however, not correct since the value of debt per unit of face value, $\tilde{D}(y)$ reflects the roll-over rate δ^*. In many situations it will not be optimal to fully roll over maturing debt, since this would imply a $\tilde{D}(y)$ less than $\tilde{D}^I$. At the same time it will not be optimal to set the roll-over rate to zero, since this would imply a debt value larger than $\tilde{D}^I$, thus implying a positive partial derivative of equity value with respect to the roll-over rate. In these cases there exists an interior equilibrium which implies that $\tilde{D} = \tilde{D}^I$.

This situation represents a differential game between equityholders, who determine the roll over rate δ^* and the market, which determines the value of debt and equity. For a given value of $\tilde{D}$, the best response of equityholders is characterized by Proposition 1. The best response of the capital market to a given roll-over rate δ is to price debt at the value given by Equation (6). Therefore, the response curve is a straight line with slope $\partial\tilde{D}/\partial\delta = -y\,\partial\tilde{D}/\partial y \frac{1}{r(1-\tau_p)+m}$. Figure 1 illustrates the typical shape of the response functions $\delta(\tilde{D})$ and $\tilde{D}(\delta)$ in the case of an interior equilibrium.

The interior equilibrium with $0 < \delta^* < m$ is characterized by the following equilibrium conditions on $\tilde{E}$, $\tilde{D}$, and δ^*.

Proposition 2. *In an interior equilibrium for* δ *the value of equity, debt, and the roll over*

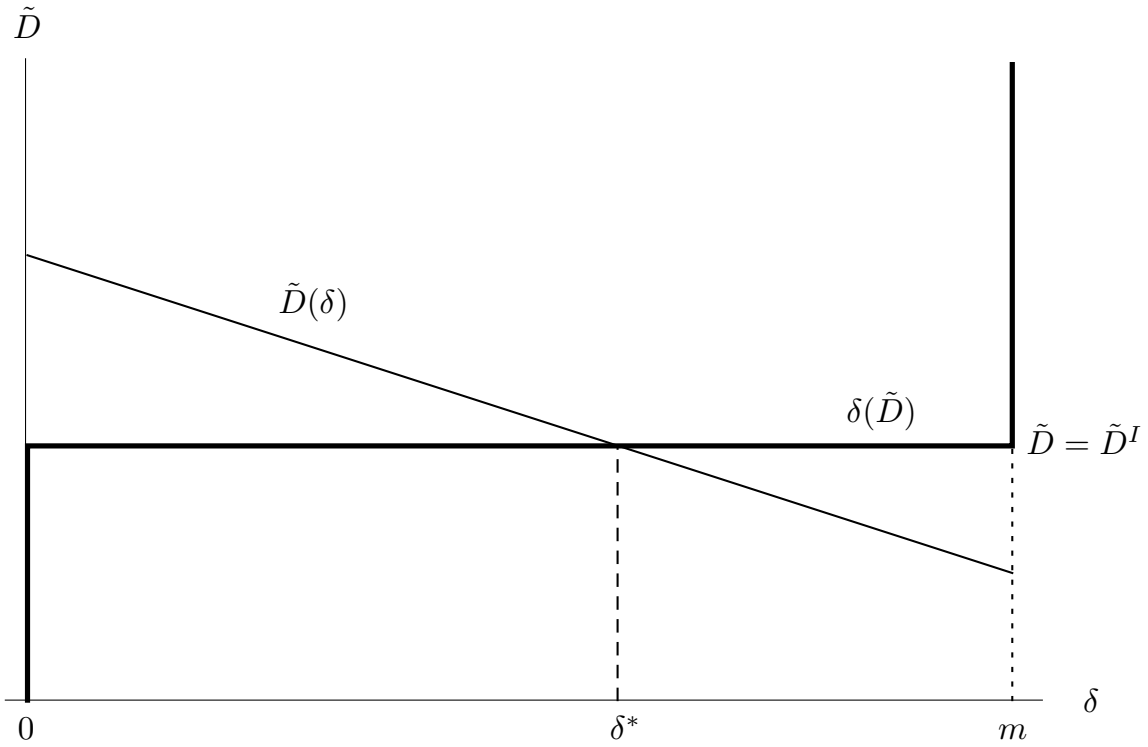


Figure 1: The shape of the response functions $\delta(\tilde{D})$ and $\tilde{D}(\delta)$ in the case of an interior equilibrium. The equilibrium debt roll over rate is δ^*

rate must satisfy

$$\begin{aligned}
\tilde{E} &= \frac{K_1}{(r(1-\tau_p)+m)}, \\
\tilde{D}(y) &= \tilde{D}^I(y), \\
0 < \delta^* &= \frac{1}{y\dfrac{\partial \tilde{D}}{\partial y}} \left[\frac{1}{2}\sigma^2 y^2 \frac{\partial^2 \tilde{D}}{\partial y^2} + (\hat{\mu}+m) y \frac{\partial \tilde{D}}{\partial y} \right. \\
&\qquad \left. +(i(1-\tau_p)+m) - (r(1-\tau_p)+m)\tilde{D} \right] < m.
\end{aligned} \tag{9}$$

Furthermore, the existence of an interior equilibrium requires

$$\begin{aligned}
\frac{\partial^2 \tilde{E}}{\partial y^2} &> 0, \\
\frac{\partial \tilde{D}}{\partial y} &> 0.
\end{aligned}$$

(See Appendix A.4 for the proof of Proposition 2.)

The following Proposition gives the analytic solutions for debt and equity for all possible roll-over rates. For $\delta = m$ and for $\delta = 0$, analytic solutions are straightforward. However, a closed-form solution can also be obtained for the case of an interior equilibrium since the valuation equations for equity and debt in Proposition 2 do not explicitly depend on the equilibrium roll-over rate, δ^*.

Proposition 3. *In a region where the firm fully rolls over its debt, i.e., $\delta = m$, the value of equity and debt are given by*

$$\begin{aligned}
\tilde{E}(y) &= E_1 y^{\beta_{m1}} + E_2 y^{\beta_{m2}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)} \\
&\quad + m(1-k_i)\left[\frac{1}{r(1-\tau_p)}\frac{i(1-\tau_p)+m}{(r(1-\tau_p)+m)}\right. \\
&\qquad + \frac{D_1 y^{\gamma_1}}{r(1-\tau_p) - \hat{\mu}\gamma_1 - \frac{1}{2}\sigma^2\gamma_1(\gamma_1-1)} \\
&\qquad \left. + \frac{D_2 y^{\gamma_2}}{r(1-\tau_p) - \hat{\mu}\gamma_2 - \frac{1}{2}\sigma^2\gamma_2(\gamma_2-1)}\right] + y, \\
\tilde{D}(y) &= D_1 y^{\gamma_1} + D_2 y^{\gamma_2} + \frac{i(1-\tau_p)+m}{r(1-\tau_p)+m}
\end{aligned}$$

In a region where the firm rolls over its debt at an interior optimum δ^*, the value of equity and debt are given by

$$\begin{aligned}
\tilde{E}(y) &= E_1 y^{\beta_{01}} + E_2 y^{\beta_{02}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)+m} + y, \\
\tilde{D}(y) &= \tilde{D}^I(y).
\end{aligned}$$

In a region where the firm funds repayment of retiring debt entirely with equity, i.e., where $\delta = 0$, the value of equity and debt are given by

$$\begin{aligned}
\tilde{E}(y) &= E_1 y^{\beta_{01}} + E_2 y^{\beta_{02}} - \frac{i(1-\tau_c)+m}{r(1-\tau_p)+m} + y, \\
\tilde{D}(y) &= D_1 y^{\beta_{01}} + D_2 y^{\beta_{02}} + \frac{i(1-\tau_p)+m}{r(1-\tau_p)+m}.
\end{aligned}$$

The exponents β and γ are the characteristic roots of the homogeneous differential equations

given by

$$\begin{aligned}
\beta_{m1,m2} &= \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2})^2 + \frac{2(r(1-\tau_p))}{\sigma^2}}, \\
\beta_{01,02} &= \frac{1}{2} - \frac{\hat{\mu}+m}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}+m}{\sigma^2})^2 + \frac{2(r(1-\tau_p)+m)}{\sigma^2}}, \\
\gamma_{1,2} &= \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2})^2 + \frac{2(r(1-\tau_p)+m)}{\sigma^2}}.
\end{aligned}$$

The constants $E_{1,2}$ and $D_{1,2}$ have to be determined separately for each of the regions by proper boundary conditions (see below).

(See Appendix A.5 for the proof of Proposition 3.)

In equilibrium, the financing strategy of the firm and the corresponding valuation given by Proposition 3 are in accordance with the optimality conditions stated by Proposition 1.

4 Capital Structure Strategy

Using the building blocks developed in previous chapters, we now discuss the features of the optimal capital structure strategy. We consider time-invariant barrier strategies on inverse leverage ratios, i.e., discrete capital structure adjustments are made when the inverse leverage ratio hits endogenously optimized barriers. In addition firms choose the refinancing policy (full roll-over, no roll-over, interior financing mix, as derived in Chapter 3) inside characteristic intervals with endogenously-optimized interval bounds. Within this class of strategies, we derive the firm's optimal capital structure strategy as a rational expectations equilibrium. Debtholders price the firm's debt under the anticipated capital structure strategy and equityholders find the strategy optimal under anticipated market valuations of debt.[12]

As discussed above, our firm model is homogeneous in the debt level of debt, B, and we rely on this fact already when deriving claim valuations in Section 3. To establish homogeneity, we also ensure that the boundary conditions applied in the case of a discrete debt restructuring and in the case of bankruptcy are linear-homogeneous in the debt level, as we will see in the following section.

[12] We do, however, not rule out the existence of equilibrium strategies that are arbitrarily general, e.g., explicitly time dependent strategies, strategies that depend on the specific path of y_t and B_t, etc.

4.1 Discrete Restructuring

We consider debt indentures that allow firms to replace expiring debt with new debt with identical or lower face value and identical maturity structure. Firms are, however, also allowed to discretely reorganize their capital structure through a buyback of all outstanding debt (possibly at a call premium) to implement an optimal target leverage ratio by issuing new debt. Firms will make use of this option in good states of nature, when cash flows have increased and additional tax shields can be utilized. In bad states of nature, firms will potentially use equity to finance the repayment of expiring debt. However, they never find it optimal to actively repurchase non-expiring debt. This fact is derived in Proposition 4 below.[13] Thus, in the absence of discrete leverage reductions, equityholders will default on their obligations to debtholders if cash flows deteriorate sufficiently fast.

We can now establish the boundary conditions for the cases of discrete restructuring, i.e. when equityholders decide to default and when they decide to increase leverage after eliminating existing bond indentures via a debt repurchase. We hereby assume that after each discrete restructuring (installing an inverse leverage ratio $\acute{y}$) new debt is sold at par so that the coupon rate i is determined endogenously by

$$\text{choose } i \text{ such that} \qquad D(\acute{y},B) = B. \tag{10}$$

In the case of default, which occurs at the trigger level $\underline{y}$, the boundary condition is determined by the fact that equity becomes worthless. At the opposite restructuring boundary $\bar{y}$, equityholders repurchase the entire debt, thereby paying a call premium of λ times the face value. In return they receive an all-equity firm which they immediately relever to achieve the inverse leverage ratio $\acute{y}$.[14] This leads to the conditions

$$E(\underline{y},B) = 0, \tag{11}$$

$$E(\bar{y},B) = \left[V(\acute{y},B\frac{\bar{y}}{\acute{y}}) - k_r B\frac{\bar{y}}{\acute{y}}\right] - (1+\lambda)B. \tag{12}$$

[13] This result is in accordance with Admati, DeMarzo, Hellwig, and Pfleiderer (2015), who also find that equityholders resist leverage reductions.

[14] As introduced in Section 2 we allow the transactions costs k_i for rolling over debt and transactions costs k_r for placing a discrete portion of debt in the case of a recapitalization of the firm to differ.

Condition (10) is already used in (12).

Next we consider the boundary conditions for debt. Following bankruptcy, debtholders take control over the unlevered productive assets of the firm. They incur bankruptcy costs and transactions costs to relever the firm. When debt is called by equityholders, debtholders receive the face value plus a proportional call premium λ. This implies

$$D(\underline{y},B) = \max\left\{\left[V(\acute{y},B\underline{y}\tfrac{1}{\acute{y}}) - k_r B\underline{y}\tfrac{1}{\acute{y}}\right] - gB,\ 0\right\}, \tag{13}$$

$$D(\overline{y},B) = (1+\lambda)B. \tag{14}$$

The inverse leverage ratio y is a diffusion that can move freely inside the interval $[\underline{y},\overline{y}]$. Thus, to ensure consistent expectation formation under the equivalent martingale measure, both equity and debt must be continuous and smooth in the entire interval $[\underline{y},\overline{y}]$, independent of the segmentation into sub-regions induced by the choice of $\delta(y)$.

Before discussing the optimization of the restructuring thresholds $\underline{y}$, $\acute{y}$, and $\overline{y}$ in Section 4.3 we will characterize the firms' refinancing strategy regarding expiring debt.

4.2 Debt Refinancing Decisions

The key insight to understand the optimal refinancing mix for expiring debt can be obtained by considering existing models with exponential debt, like Leland (1994b), as a starting point. These models require equityholders to fully roll over all expiring debt, i.e. to issue a new bond with a face value exactly equal to the face value of the expiring debt. Figure 2 shows the partial derivative of the equity value with respect to the roll-over rate, $\frac{\partial \tilde{E}}{\partial \delta}$, for two different levels of average debt maturity for a firm that fully rolls over the face value of expiring debt.[15] When $\frac{\partial \tilde{E}}{\partial \delta} > 0$, equityholders' incentives are consistent with the model assumption of full debt roll-over. In regions where $\frac{\partial \tilde{E}}{\partial \delta} < 0$, however, equityholders would prefer a roll-over rate strictly less than m, thereby reducing the level of outstanding debt.

Close to the upward recapitalization threshold $\overline{y}$, equityholders are reluctant to roll over debt at the maximum possible rate. This is so, because new debt issues close to the reorganization trigger are likely to be recalled only a short time thereafter, thereby causing transaction

[15] For this illustration we use the base-case parameterization which is delineated in Section 5.

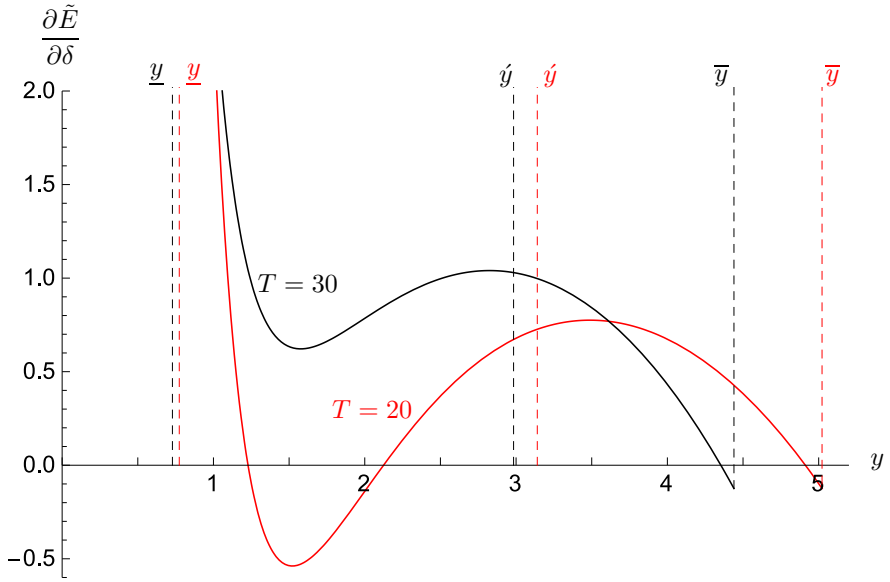


Figure 2: The partial derivative $\frac{\partial \tilde{E}}{\partial \delta}(y)$ for a firm that always rolls over all expiring debt. With long debt maturity (e.g., $T = 30$ a), the assumed full roll-over is mainly consistent with equityholders' incentive, i.e., $\frac{\partial \tilde{E}}{\partial \delta}(y)$ is positive for almost the entire range of (inverse) leverage ratios. Only close to the upward restructuring threshold $\bar{y}$ equityholders have an incentive to use equity to finance debt repayment. With shorter debt maturity ($T = 20$ a), a considerable inconsistency arises in rather bad states of the firm when inverse leverage is substantially below the initial level $\acute{y}$. In the region with $\frac{\partial \tilde{E}}{\partial \delta}(y) < 0$, equityholders' would strictly prefer using also equity to effectively reduce the amount of outstanding debt.

costs that are high compared to the tax shield these issues generate over their short expected lifetime. We therefore augment the capital structure strategy by an interval $[\tilde{y}_3, \bar{y}]$ with $\delta < m$, with the time-invariant trigger $\tilde{y}_3$ chosen by equityholders. Since for reasonable call premiums, λ, we have $\partial \tilde{D}/\partial y < 0$ for y close to debt repurchasing at $\bar{y}$, in which case it is optimal to entirely stop reissuing new debt, i.e., $\delta = 0$, see Proposition 2.

When debt maturity is sufficiently short and the firm's cash flows deteriorate, they have an incentive to reduce debt roll-over below the maximum rate, $\delta < m$. Therefore, we augment the strategy space of equityholders by another time-invariant interval $[\tilde{y}_1, \tilde{y}_2]$, within which the firm engages in debt reduction.

This strategy space allows equityholders to choose $\tilde{y}_1 = \tilde{y}_2$ and / or $\tilde{y}_3 = \bar{y}$, in which case the model resembles a Leland (1994b)-type model where equityholders always hold the face value of debt constant by fully rolling over all expiring debt.

4.3 Optimality Conditions

Implementing the optimal capital structure strategy as a time-invariant barrier strategy, initial firm owners choose a starting capital structure $\acute{y}$ and the average maturity m to maximize total firm value, fully anticipating the resulting capital structure dynamics. Equityholders endogenously optimize the default trigger $\underline{y}$ and the the optimal level where they repurchase the firm's entire debt, $\bar{y}$. Immediately following the debt repurchase, equityholders own an unlevered firm. Due to homogeneity, they will find it optimal to reestablish the initial capital structure $\acute{y}$, m. Finally, equityholders optimize the intervals $[\tilde{y}_1, \tilde{y}_2]$ and $[\tilde{y}_3, \bar{y}]$, where debt reduction occurs. Debtholders price bonds in rational anticipation of equityholders' strategy choices.

Thus, we regard capital structure strategies that are characterized by

$$
\begin{aligned}
&(\underline{y} \leq \tilde{y}_1 \leq \tilde{y}_2 \leq \tilde{y}_3 \leq \bar{y};\, \acute{y}, m), \\
\text{with:} \quad \underline{y} &\leq \acute{y} \leq \bar{y}, \\
\delta(y) &= \delta^*(y) \quad \text{for } y \in [\tilde{y}_1, \tilde{y}_2], \\
\delta(y) &= 0 \qquad \text{for } y \in [\tilde{y}_3, \bar{y}], \\
\delta(y) &= m \qquad \text{elswhere.}
\end{aligned}
\tag{15}
$$

The time-invariant thresholds $\underline{y}$, $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{y}_3$, and $\bar{y}$ are set to maximize equity value. The initial capital structure $\acute{y}$ and m is set to maximize total firm value.

The first order conditions of optimality at the upper and the lower reorganization threshold follow from a 'smooth pasting' condition (see Dixit (1993) for a discussion of these optimality conditions)

$$
\frac{\partial E}{\partial y}(y,B)|_{y=\underline{y}} = 0, \tag{16}
$$

$$
\frac{\partial E}{\partial y}(y,B)|_{y=\bar{y}} = \frac{1}{\acute{y}}\left[E(\acute{y},B) + B(1-k_r)\right]. \tag{17}
$$

The thresholds $\tilde{y}_1$, $\tilde{y}_2$, and $\tilde{y}_3$ are set to satisfy so called super-contact conditions, see Dumas (1991).

Recognizing that the optimal values of $\underline{y}$, $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{y}_3$, $\bar{y}$, and $\delta(y)$ are functions of $\acute{y}$ and m,

the initial firm value V can be written as $V(y,B;\acute{y},m)$. Taking into account transaction costs the maximand is

$$\max_{\acute{y},m}(V(y,B;\acute{y},m)-k_rB|_{y=\acute{y}}), \tag{18}$$

with the first order conditions

$$\begin{aligned}\frac{\partial V}{\partial m}(\acute{y},B;\acute{y},m) &= 0, &(19)\\ \frac{\partial V}{\partial y}(y,B;\acute{y},m)|_{y=\acute{y}}+\frac{\partial V}{\partial \acute{y}}(\acute{y},B;\acute{y},m)-\frac{1}{\acute{y}}(V(\acute{y},B;\acute{y},m)-k_rB) &= 0. &(20)\end{aligned}$$

4.4 Debt Amortization Beyond Contracted Retirement

Under our model assumptions, equityholders have the choice over the financing mix used to pay for expiring debt, but the retirement rate, m, is fixed once the initial capital structure decision is made. In this subsection we explore whether firms have – under certain conditions – an incentive to amortize outstanding debt at a rate exceeding m. To answer this question we calculate the partial derivative of the firm's equity with respect to the retirement rate m. We distinguish two cases. First, the case where the optimal roll over rate δ is a corner solution with $\delta = m$ and second, the case where the optimal roll over rate is below the allowed maximum, $\delta < m$.

Proposition 4. *The effect of a local deviation from the given retirement rate on equity is*

$$\frac{d\tilde{E}}{dm}=\begin{cases}\dfrac{\partial\tilde{E}}{\partial m}+\dfrac{\partial\tilde{E}}{\partial\delta} = (1-k_i)\tilde{D}(y)-1 & : \ \textit{if}\ \delta(y)=m\\ \dfrac{\partial\tilde{E}}{\partial m} = \dfrac{y\dfrac{\partial\tilde{E}}{\partial y}-\tilde{E}-1}{r(1-\tau_p)+(m-\delta)} & : \ \textit{if}\ \delta(y)<m\end{cases} \tag{21}$$

The firm has no incentive to locally accelerate repayment over and above the contracted rate m if and only if $\frac{d\tilde{E}}{dm}<0$.

See Appendix A.7 for a proof of Proposition 4.

In the case where $\delta(y)=m$, the firm has an incentive to increase the retirement rate and

immediately re-issue retired debt if the after-transactions cost proceeds from re-issuing one unit debt, $(1-k_i)\tilde{D}$, exceed the cost of repaying one unit of debt, i.e. 1. Since $\delta(y) = m$, the firm does not delever in this case, but would simply expropriate debtholders by repaying debt at par and re-issuing above par in good cash-flow states. We do not allow for such strategies. Furthermore, although high values of debt may create an incentive to locally deviate from the contracted retirement schedule, such a behavior cannot be part of an equilibrium strategy. Rational debtholders will not price debt above par in such a situation and, thus, transaction costs make the increased debt retirement rate non-optimal since $\tilde{D} = 1$ implies $\frac{d\tilde{E}}{dm} < 0$.

In the alternative case in which $\delta(y) < m$, increasing the retirement rate above m effectively leads to accelerated leverage reductions. In the discussion of Proposition 1 we derive $\frac{dE}{dB} = \tilde{E} - y\frac{\partial \tilde{E}}{\partial y}$. The firm has an incentive to retire one extra unit of debt if the effect on equity, $-\frac{dE}{dB}$ minus associated cost of repayment, 1, yields a positive net effect. This is exactly what Proposition 4 states.

Proposition 5. *If the firm has no incentive to accelerate debt repayment at a given inverse leverage ratio y_0, i.e. $\delta(y_0) \leq m$ and equity is convex for all $y < y_0$, then the firm will never seek accelerated repayment for all $y < y_0$. Thus, for $y < y_0$ we have $\partial\tilde{E}/\partial m < 0$.*

See Appendix A.8 for a proof of Proposition 5.

For all parameter constellations explored in Section 6 we find that firms choose the initially optimal $\acute{y}$ such that there is no incentive to immediately repay debt at a rate exceeding m. Furthermore, we find that equity is convex for all $y < \acute{y}$, hence we conclude that firms never find it optimal to accelerate repaying debt at a rate which exceeds m. In particular, equityholders will sometimes use equity to repay debt at the contracted repayment rate m but never at a rate that exceeds m.

5 Calibration

For our numerical analysis the base-case parameters are calibrated to the US tax code and listed in Table 2. Taxation of investors' personal income is captured by a single tax rate τ_p. This tax rate is applied to debt income and reflects the fact that income from interest bearing

investments is taxed more heavily than equity income from equity capital. Specifically, interest income is treated as ordinary income in the US and we calibrate this to the maximum tax rate on wage income. This is currently 39.6% for high income earners plus a 3.8% Medicare surtax on investment income. We therefore set the base-case personal tax rate on interest income of 43.4%. By contrast, high income earners only pay 20% tax on dividend income plus 3.8% for Medicare, i.e. 23.8%. To capture the disadvantage of interest income over income from equity investment we therefore set $\tau_p = 19.6\%$.

On the corporate level we assume that income is taxed at a constant statutory rate τ_c, which we calibrate to empirical effective marginal tax rates. For this purpose we use two sources of information. First, we use marginal tax rates from COMPUSTAT MTR database, which employs the nonparametric estimation method introduced by Blouin, Core, and Guay (2010) that explicitly takes care of mean reversion of corporate income. We merge the MTR database with COMPUSTAT firm characteristics to calculate total-asset-weighted average marginal tax rates after interest expense over the available horizon from 1994 to 2012, which yields 30.6%. Average marginal tax rates peak in 1993 (33.0%) and are lowest in 2010 (22.0%).

Second, as a robustness check we analyze John Graham's file of simulated tax rates.[16] The average marginal tax rate after interest expense over the last 20 years, i.e., from 1994 to 2013, is estimated to be 25.9%. Again, average simulated marginal tax rates in the sample period are lowest in 2010 (18.7%) and highest in 1995 (30.7%). The total-asset-weighted average marginal tax rate before interest expenses over the stated period is 33.1%. In the numerical analysis below also provide comparative statics with respect to our tax parameters to illustrate the effect of deviations from the base case parameters.

Recent empirical estimates of corporate bankruptcy costs have considerably changed the academic community's view of their magnitude. Early papers estimate bankruptcy costs investigating sets of defaulted firms and estimated these costs to be only a few percent of

[16] We thank John Graham for providing us with his comprehensive set of simulated marginal tax rates covering the range from 1980 to 2013 and for his advice on calibrating our model to the US tax code. Please see Graham (1996a) and Graham (1996b) for details on the applied simulation procedure. Graham and Mills (2008) use federal government tax return data and show that simulated marginal tax rates provided in the file are close approximations.

Table 2: Base case parameters

parameter		
riskless rate of interest r	5	%
personal tax rate τ_p	19.6	%
corporate tax rate τ_c	30.6	%
volatility of the cash flow process σ	13	%
risk adjusted drift $\hat{\mu}$	0	%
bankruptcy cost g	34.39	%
transactions costs for rolling over debt k_i	0.5	%
transactions costs after recapitalization k_r	1	%
call premium λ	0	%

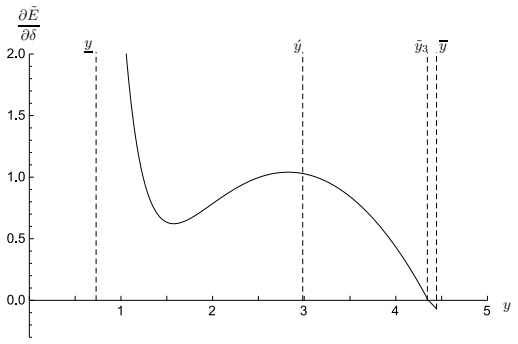
the firm's asset value.[17] More recently, researchers accounted for the fact that a subset of defaulted firms is likely to produce a biased bankruptcy cost estimate for the entire population of firms. They argue that low-destress cost firms are overrepresented in this sample and, thus, existing estimates of bankruptcy costs might be significantly downward biased. Reindl, Stoughton, and Zechner (2015) infer implied distress costs from market prices of equity and prices of put options employing a dynamic capital structure model. They show that estimated bankruptcy costs vary considerably across industries from below 10% to well over 60% with typical values in the range between 20% to 30%. In our calibration we refer to Glover (2014), who estimates parameters of a structural trade-off model of the firm with time-varying macroeconomic conditions by employing simulated methods of moments. He estimates the mean firm's costs of default with 45% and the median firm's cost with 37% of asset value. Our model specifies bankruptcy costs as a fraction g of the face value of debt. Thus, aiming for a base-case parameterization that resembles median bankruptcy costs, we select g sucht that a firm with optimally chosen debt maturity experiences bankruptcy costs of 37% of its asset value. This leads to a base-case parameter of $g = 34.39\%$. Below we provide comparative statics to estimate the effect of varying bankruptcy costs (e.g., across industries).

[17] See, for example, the following papers for studies on default costs, estimated averages are in parenthesis: Warner (1977) (5.3%), Ang, Chua, and McConnell (1982) (mean 7.5%, median 1.7%), Weiss (1990) (3.1%), Altman (1984) (6.0%), Andrade and Kaplan (1998) (10% to max. 23%).

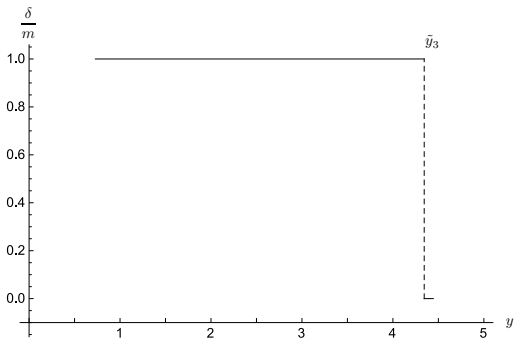
6 Debt Maturity, Capital Structure Dynamics and Firm Value

We start by analyzing the effect of average debt maturity on the firm's optimal refinancing decision. If not otherwise mentioned, base case parameters listed in Table 2 are used. First, we explore firms which have issued debt with long maturities. Panels (a) and (b) of Figure 3 illustrate the partial derivative $\frac{\partial \tilde{E}}{\partial \delta}(y)$ and the optimal roll over rate normalized by the retirement rate δ/m over the inverse leverage ratio y (which in our model is proportional to the firm's cash flow level), both for $m = 0.0\dot{3}$ ($T = 30$ years). As can be seen, the partial derivative of equity with respect to δ is always positive, except for the small region near the upper restructuring threshold $\bar{y}$, where it is negative. Consequently, equityholders will not engage in voluntary debt reductions when the firm's cash flow decreases but instead the firm always fully rolls over all debt by setting $\delta = m$. Only immediately before calling the bonds to subsequently issue more debt, i.e. in the region $y \in [\tilde{y}_3, \bar{y}]$ does it become optimal for equityholders to use equity to repay maturing debt. The intuition for this latter result is straightforward. In this leverage region, it is optimal to use retained earnings to finance principal repayments since it would be inefficient to incur transactions costs for a new bond issue, knowing that the bond will be called in the near future with high probability.

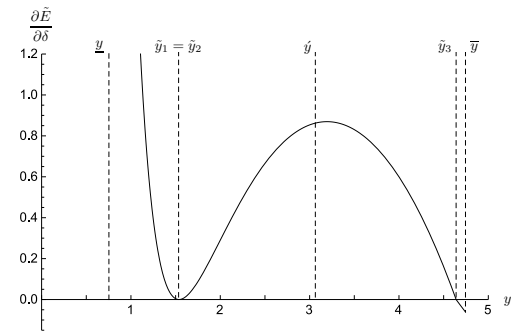
Next, we analyze shorter debt maturities. We observe that shorter debt maturity weakens the incentive to roll over debt, $\frac{\partial \tilde{E}}{\partial \delta}$, especially in states in which the firm is not very profitable (low but not too low levels of the firm's cash flow). Eventually, for sufficiently low debt maturities, $\frac{\partial \tilde{E}}{\partial \delta}$ reaches zero. Panels (c) and (d) of Figure 3 show the partial derivative $\frac{\partial \tilde{E}}{\partial \delta}(y)$ and the optimal roll over rate δ/m for this critical debt maturity of $T = 23.86$. years ($m = 0.04192$). This is the lowest average maturity for which there is no voluntary debt reduction, given the base case parameterization. We see that the partial derivative of equity with respect to the roll-over rate, $\frac{\partial \tilde{E}}{\partial \delta}(y)$, touches zero between $\underline{y}$ and $\acute{y}$. That is, there is one point between $\underline{y}$ and $\acute{y}$ at which equityholders are indifferent between rolling over all debt and refraining from issuing debt to replace retired debt. Thus, shortening the debt maturity from $T = 30$ to $T = 23.86$ considerably weakens equityholders' incentives to always fully roll-over maturing debt. We illustrate below that if the average maturity is less than the critical value of $T = 23.86$, there exists a region where equityholders choose an interior roll-over rate, δ^*.



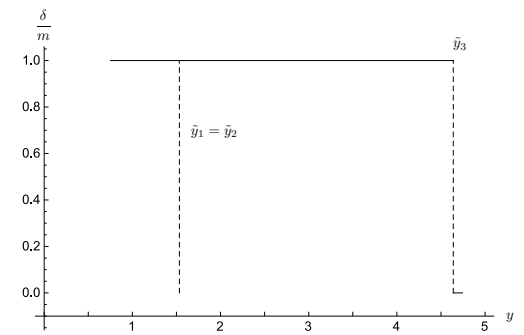
(a) $\frac{\partial \tilde{E}}{\partial \delta}(y)$ for $m = 0.0333$ ($T = 30$)



(b) $\frac{\delta(y)}{m}$ for $m = 0.0333$ ($T = 30$)



(c) $\frac{\partial \tilde{E}}{\partial \delta}(y)$ for $m = 0.04192$ ($T = 23.86$)



(d) $\frac{\delta(y)}{m}$ for $m = 0.04192$ ($T = 23.86$)

Figure 3: (a) and (b): The partial derivative of equity value $\tilde{E}$ with respect to the roll over δ, together with the implied optimal roll-over rate $\frac{\delta}{m}$ for base-case parameters and debt maturity of $T = 30$ years, which is so long that equityholders will not engage in debt reductions in bad states of the firm.
(c): Debt with a critical maturity of $T^{\text{crit}} = 23.86$ years lowers $\frac{\partial \tilde{E}}{\partial \delta}$ in bad states of the firm such that it touches the critical value of zero at $\tilde{y}_1 = \tilde{y}_2$, at wich the firm is indifferent to the particular choice of δ.
(d): Optimal choice of the roll-over rate δ/m for debt with critical maturity of $T^{\text{crit}} = 23.86$. At $\tilde{y}_1 = \tilde{y}_2$, the firm is indifferent with respect to the choice of $\delta \in [0, m]$.

The example plotted in Figure 4 considers an even shorter debt maturity. Now m is set to 0.2350 which corresponds to an average debt maturity of $T = 4.255$.[18] We find that there exists a region $[\tilde{y}_1, \tilde{y}_2]$ between the bankruptcy threshold $\underline{y}$ and the initial inverse leverage ratio $\acute{y}$ where equityholders find it optimal to reduce δ below m to voluntarily reduce the debt level. Figure 5 shows the partial derivative of equity with respect to δ. For $y \in [\tilde{y}_1, \tilde{y}_2]$, this derivative vanishes, thus, the choice how to fund debt repayments results in an interior

[18] As can be seen from below, these values correspond to the optimal maturity choice of a base case firm.

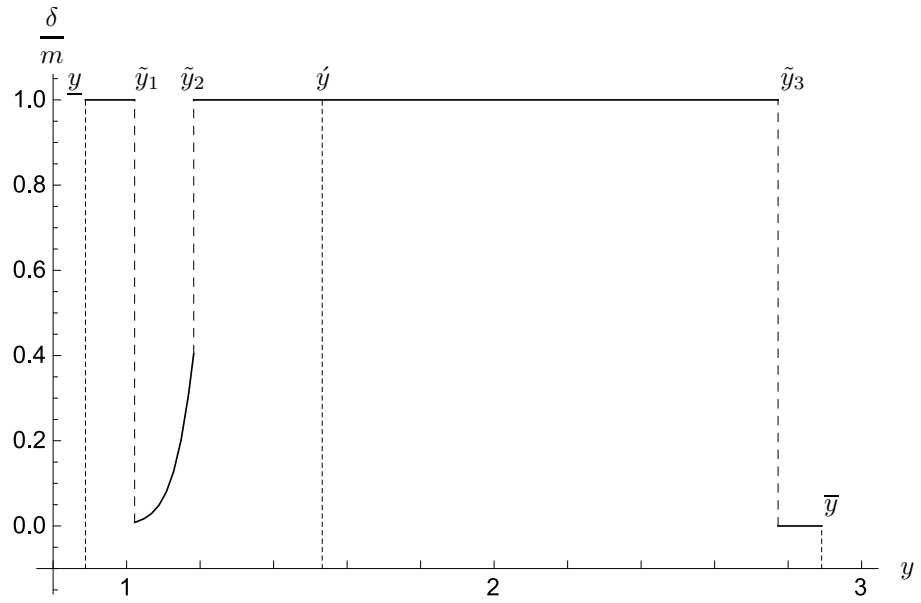


Figure 4: Endogenously determined roll over rate δ/m for base-case parameters and optimally-chosen debt maturity $m = 0.2350$ ($T = 4.255$). Optimally chosen, debt maturity is sufficiently short to create an incentive for debt reductions in the range $[\tilde{y}_1, \tilde{y}_2]$. There, only part of expiring debt is rolled over.

equilibrium. Intuitively, equityholders find the prices of new bonds too low, since they reflect high leverage and future costs of financial distress. It is in their own interest to partly use equity to refund maturing debt, despite the fact that it implicitly also benefits the remaining bondholders. For all parameter values we have used, voluntary debt reduction was associated with an interior choice of δ. I.e., we could not find a case where equityholders stopped issuing debt completely and funded debt repayment exclusively with equity. However, we do not have an analytic proof that this is a general result.

Interestingly, the firm's willingness to use equity to repay debt is non-monotonic in the inverse leverage ratio, y. When the firm approaches bankruptcy, i.e. for $y < \tilde{y}_1$ equityholders terminate their effort to reduce debt. In this region they once again fully roll over debt and exploit existing debtholders by re-issuing the expired debt. Thus, when pushed very close to bankruptcy, equityholders are no longer willing to make additional voluntary equity investments in the firm. On the contrary, they would rather issue new debt at the maximum rate allowed by debt covenants, even if this can only be done at unfavorable prices, i.e. at high

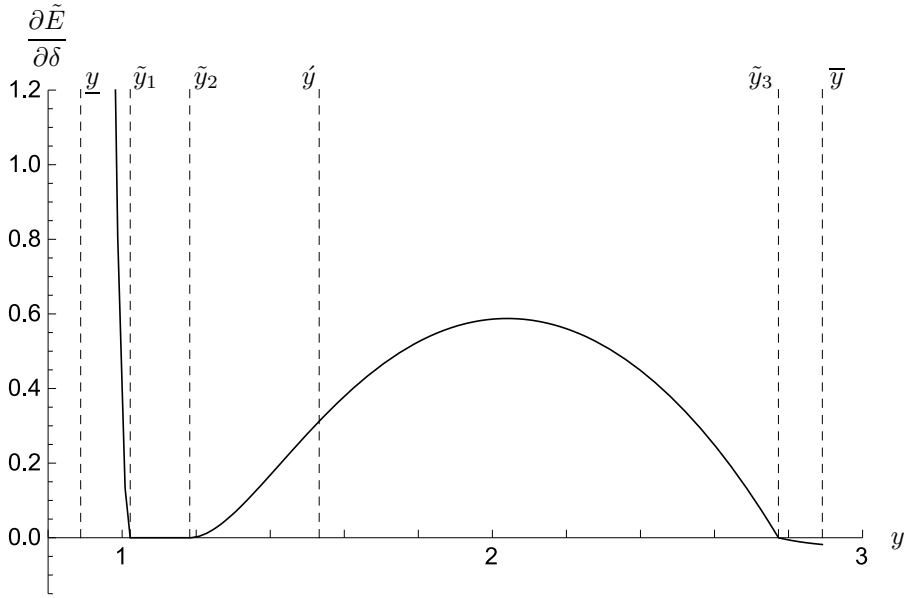


Figure 5: The partial derivative $\frac{\partial \tilde{E}}{\partial \delta}(y)$ for base-case parameters and optimally-chosen debt maturity $m = 0.2350$ ($T = 4.255$). In $[\tilde{y}_1, \tilde{y}_2]$ the derivative vanishes, i.e., the firm is indifferent towards the choice of δ. This is the requirement for an interior optimum.

credit spreads.

To summarize, there are four main insights that the above numerical analysis provides. First, for sufficiently long maturities, equityholders never use retained earnings or equity issues to repay maturing debt except immediately before a discrete leverage increase. This result changes if the average debt maturity is shortened. In this case there exists a range of leverage ratios strictly above the initial optimum for which equityholders find it optimal to partly use retained earnings of equity to repay maturing debt. This is in accordance with the empirical findings of Hovakimian, Opler, and Titman (2001) who report that long-term debt is an impediment to movements toward the target leverage ratio.

Second, at the initial leverage ratio $\acute{y}$ the firm always holds its debt level constant and fully rolls over maturing debt, $\delta = m$. This follows directly from the optimality of the initial leverage ratio. Since the initial issue of debt is associated with proportional transactions costs k_r, equityholders would not incur these costs if they would immediately find it optimal to reduce debt by repaying debt with equity.

Third, near the restructuring threshold $\bar{y}$ the firm entirely refrains from issuing debt. This

is so because approaching $\bar{y}$ is associated with the repurchase of all debt in order to reestablish the optimal initial capital structure. Therefore, near this threshold, equityholders do not find it optimal to incur costs k_i for rolling over contracts which will (with high probability) be repurchased after a short period. With $k_i \to 0$ this region of $\delta = 0$ vanishes.

Fourth, near the bankruptcy threshold $\underline{y}$ the firm fully rolls over all expiring debt, i.e. $\delta = m$. Thus, even with short-term debt outstanding, equityholders resume issuing debt if the leverage ratio becomes sufficiently high. In this case the equityholders are no longer willing to invest in debt reductions to keep their equity option alive. This latter result can be derived analytically.

Proposition 6. *If loss-given-default is strictly less than 100%, it is optimal to roll over debt at the maximum rate $\delta = m$ in a neighborhood above the bankruptcy threshold $\underline{y}$.*

(See Appendix A.6 for the proof of Proposition 6.)

Bankruptcy costs, corporate taxes and critical debt maturity: We find that bankruptcy costs as well as the magnitude of the tax shield of debt financing represent the main determinants for the critical average maturity that triggers voluntary debt reductions. The lower the bankruptcy costs the shorter the maturity required to provide incentives for voluntary debt reductions. Figure 6 plots the critical average maturity over bankruptcy costs for two different levels of corporate tax, τ_c. The lower line represents our base case where the corporate tax rate is calibrated to the average marginal tax rate provided by COMPUSTAT MTR database (τ_c = 30.6%, see Section 5).

The upper line shows critical debt maturities over bankruptcy costs when using the lower average effective corporate tax rate implied by the marginal-tax rate data provided by John Graham (τ_c = 25.9%,). It is evident, that in the case of higher tax shields it requires shorter debt maturities to induce sufficient incentive for equityholders to engage in active debt reduction when the firm's cash flows deteriorate. This result is quite intuitive, since actively replacing retired debt with equity reduces the firms tax shields and, hence, providing larger tax shields reduces the incentive to substitute debt with equity.

With base-case parameterization, i.e., $\tau_c = 30.6\%$, $g = 34.39\%$, debt maturity below the critical maturity of 23.86 years induces debt reductions in bad times. With lower tax shields, using $\tau_c = 25.9\%$ the critical debt maturity at $g = 34.39\%$ is 33.4 years. Bankruptcy costs

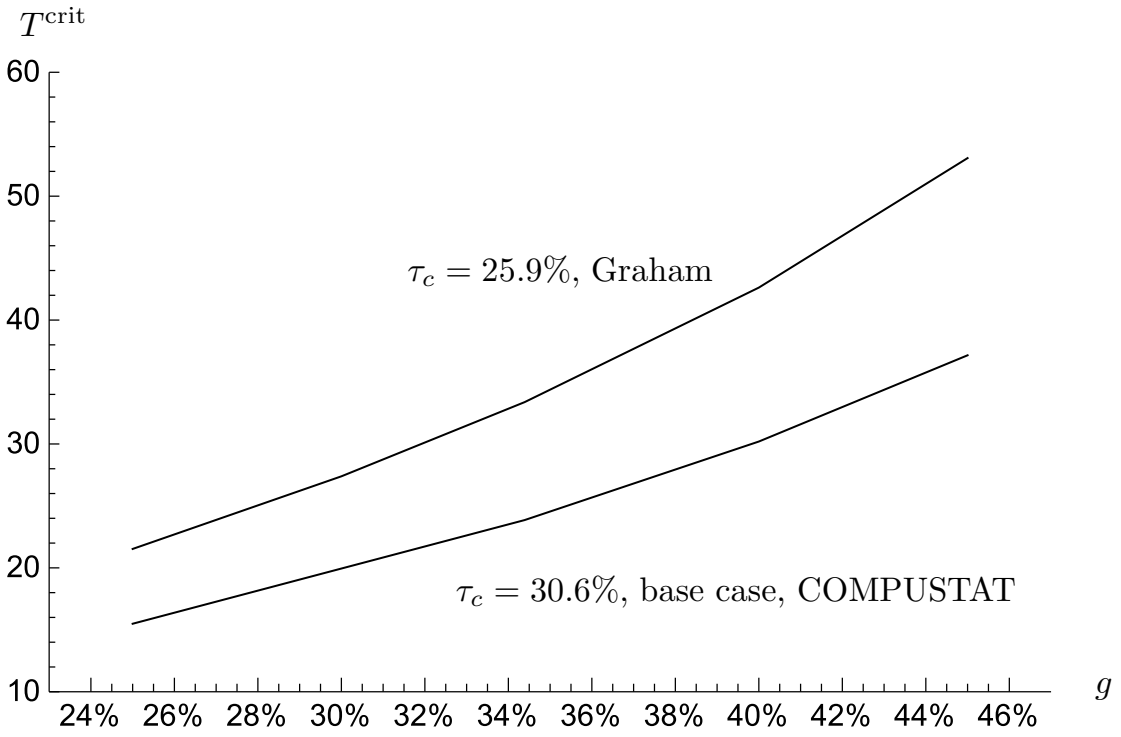


Figure 6: Critical average debt maturity below which the commitment to debt reductions in bad times is credible as a function of bankruptcy costs. Critical maturities are plotted for the base case parameterization $\tau_c = 30.6\%$, which is the average marginal tax rate estimated from COMPUSTAT MTR database, which employs the approach of Blouin, Core, and Guay (2010). Additionally, critical maturity for the average tax rate from John Graham's database, i.e., $\tau_c = 25.9\%$, is also plotted. See Section 5 for more details.

as low as $g = 25\%$ require average maturities of 15.49 years and 21.53 years when using corporate tax rates of 30.6% and 25.8% respectively. Bankruptcy costs as high as $g = 45\%$ induce debt reduction for average maturities below 37.16 years and 53.07 years respectively. Thus, with lower bankruptcy costs, it needs shorter-term debt to induce debt reductions.

Debt maturity and firm value: We next consider the effect of debt maturity on firm value and illustrate the potential benefit of a short debt maturity with base-case parameters.Results for different parameterizations are reported below. Figure 7 displays the initial tax advantage, i.e., the extent to which the value of the optimally levered firm exceeds the value of the unlevered productive assets as a function of the retirement rate of debt, m.

The figure also displays the relative value of a reference firm (dotted line) which is as-

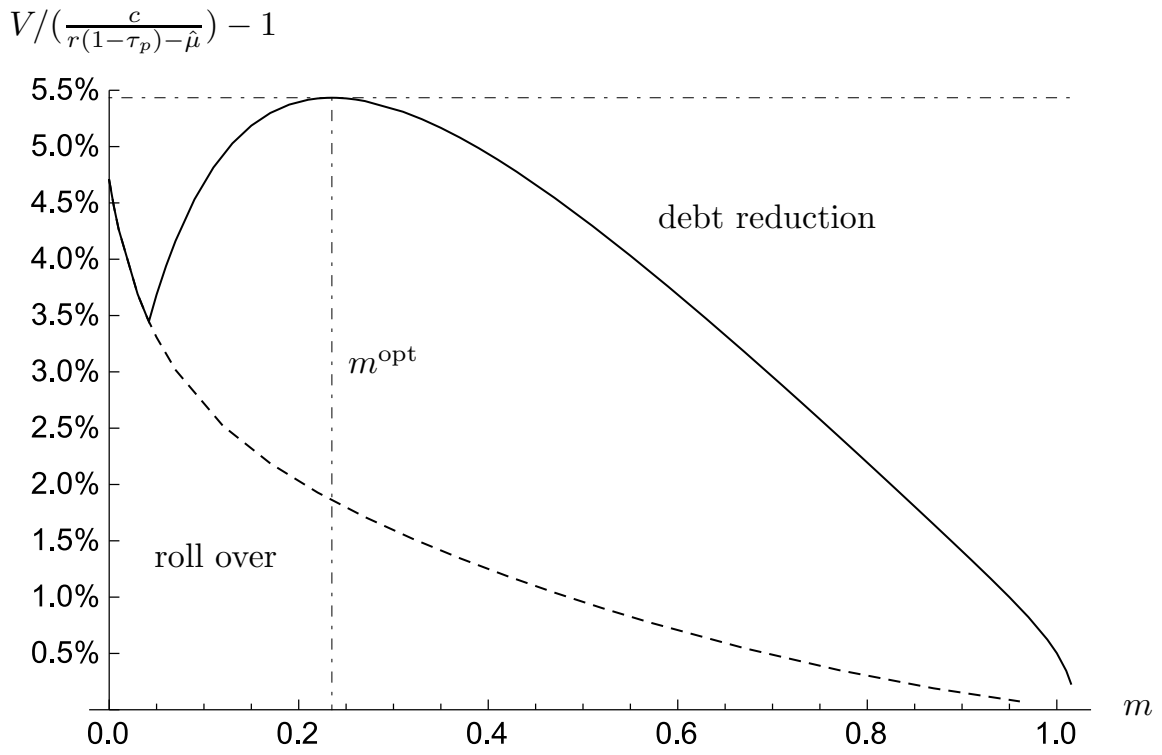


Figure 7: The tax advantage of debt at the optimal initial leverage for the base-case firm plotted against the retirement rate *m*. The dotted line shows the corresponding tax advantage for a firm that has to keep the debt level constant and therefore rolls over all expiring debt. The relation between the maturity structure of debt and firm value is non-monotonous. Firm value is maximized at an optimal endogenous average maturity of ≈ 4.26 years

sumed to always fully roll-over maturing debt with new debt issues.[19] For the reference firm, total firm value is maximized by choosing the longest possible maturity for its debt, as reported in Leland (1994b) and Leland and Toft (1996). By contrast, if the firm can engage in debt reductions, the relationship between total firm value and the maturity structure of debt is not monotonic.[20] This is so because debt with sufficiently short maturity induces more efficient capital structure adjustments by equityholders when the firm's cash flows decrease, thereby lowering probability of default and, hence, expected bankruptcy costs.

As illustrated in Figure 7, the beneficial effect of shorter debt maturity on future capital

[19]This is modelled as in Leland (1994b). In addition, we also allow the firm to increase its debt by repurchasing all debt outstanding and to issue a higher amount of debt.

[20]Empirical evidence for this nonmonotonicity is provided by Guedes and Opler (1996) who report that investment grade firms seem to be indifferent between issuing debt at the long end of the maturity spectrum and issuing debt at the short end of the spectrum.

structure dynamics outweighs the disadvantage due to higher transactions costs from rolling over maturing debt. In the base case, overall firm value is maximized at a debt maturity of ≈ 4.26 years.

Debt capacity: The commitment effect of debt maturity also has a significant effect on the optimal initial leverage ratio. In contrast to existing results in the finance literature we find that shorter debt maturities lead to higher debt capacities.

This effect is illustrated by Figure 8, which plots the initial optimal leverage as a function of m for the base-case firm. Unlike firms which must roll over all maturing debt, firms that choose the roll-over rate optimally actually *increase* their debt capacity as they shorten their debt maturities. The optimal initial leverage increases from approximately 39% percent for perpetual bonds and reaches its maximum with 80% at an average debt maturity of approximately 1.5 years. At the firm-maximizing debt maturity of 4.26 years, the firm's debt capacity is approx. 65%. For very short maturities, debt capacity decreases, due to transactions costs incurred when rolling over debt.

6.1 Comparative Statics

In this section we explore the effect of various model parameters on firm value, optimal debt maturity and dynamic capital structure policy. **Bankruptcy costs:** We first focus on the role of bankruptcy costs. The key role of bankruptcy costs for the commitment to debt reductions was already discussed above. Figure 9 plots the tax advantage of debt, i.e., the extend to which the initial firm value exceeds the unlevered firm value, for different levels of bankruptcy costs. Several effects can be seen: (i) lower bankruptcy costs require a shorter debt maturity in order to induce voluntary debt reductions, (ii) lower bankruptcy costs reduce the maximum attainable tax advantage of debt, (iii) lowering bankruptcy costs moves the optimal finite maturity towards shorter maturities, (iv) for very low bankruptcy costs it becomes relatively more advantageous to issue console bonds.

The most surprising effect is that higher bankruptcy costs imply higher firm values. Higher bankruptcy costs make it easier for equityholders to credibly commit to debt reductions. The resulting decrease in the expected probability of bankruptcy more than offsets the effect of the increased costs given a default.

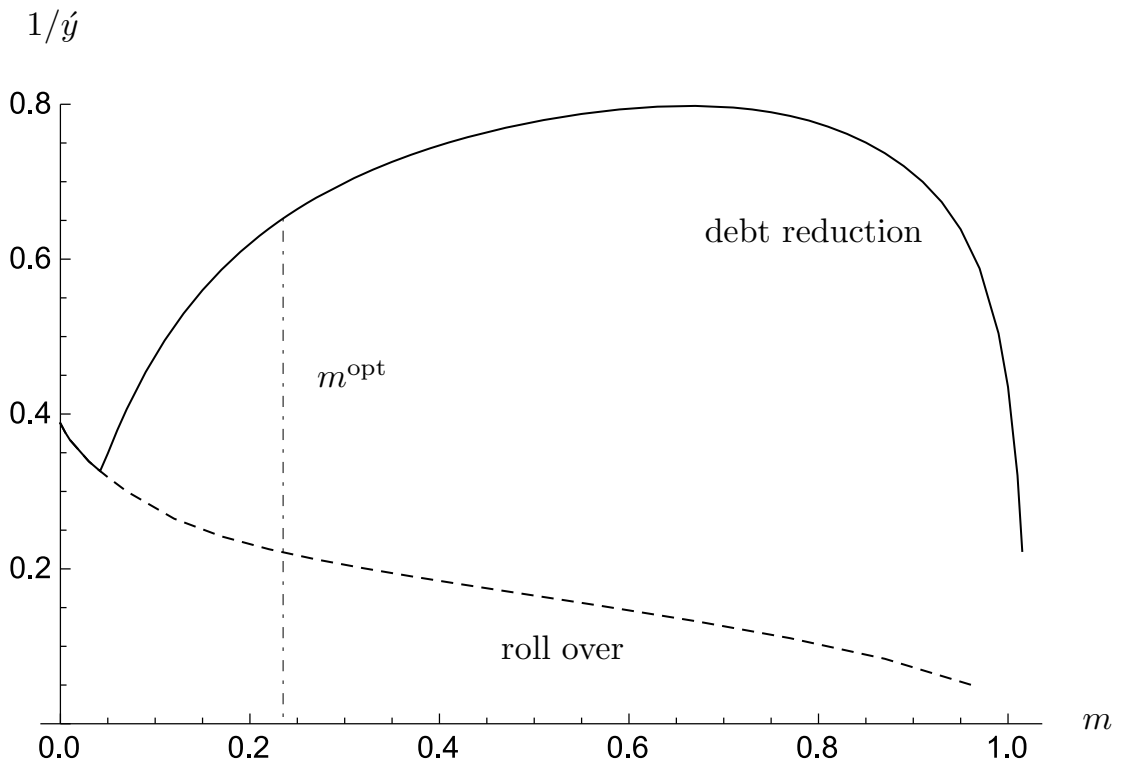


Figure 8: Optimal initial leverage ratios $1/\acute{y}$ plotted over the retirement rate m. Without allowing for downward restructuring, debt capacity decreases when moving from long to short-term debt. For firms that explicitly consider debt reduction, debt capacity increases once maturity is sufficiently short in order to commit to debt reductions to avoid financial distress. Only at the very short end, transaction costs lead to a deterioration in debt capacity.

Transactions costs: The costs associated with rolling over debt are another key determinant of firm value when finite maturity debt is issued. Figure 10 illustrates the effect on firm value for different values of k_i. When moving to lower values of k_i we observe that (i) firms with shorter-term debt gain relatively more and (ii) the local maximum of total firm value for finite debt maturity moves towards shorter maturities.

Cash flow characteristics: Figure 11 shows how changes in cash flow characteristics affect total firm value. Panel 11a plots the initial tax advantage as a function of the retirement rate for several values of cash flow volatility σ. Moving to higher volatilities (i) results in lower firm value, (ii) requires shorter debt maturity to induce debt reductions, (iii) moves the local maximum of total firm value towards shorter maturity debt. High cash flow volatilities reduce the firm's debt capacity but increase the option value for equityholders and thus make

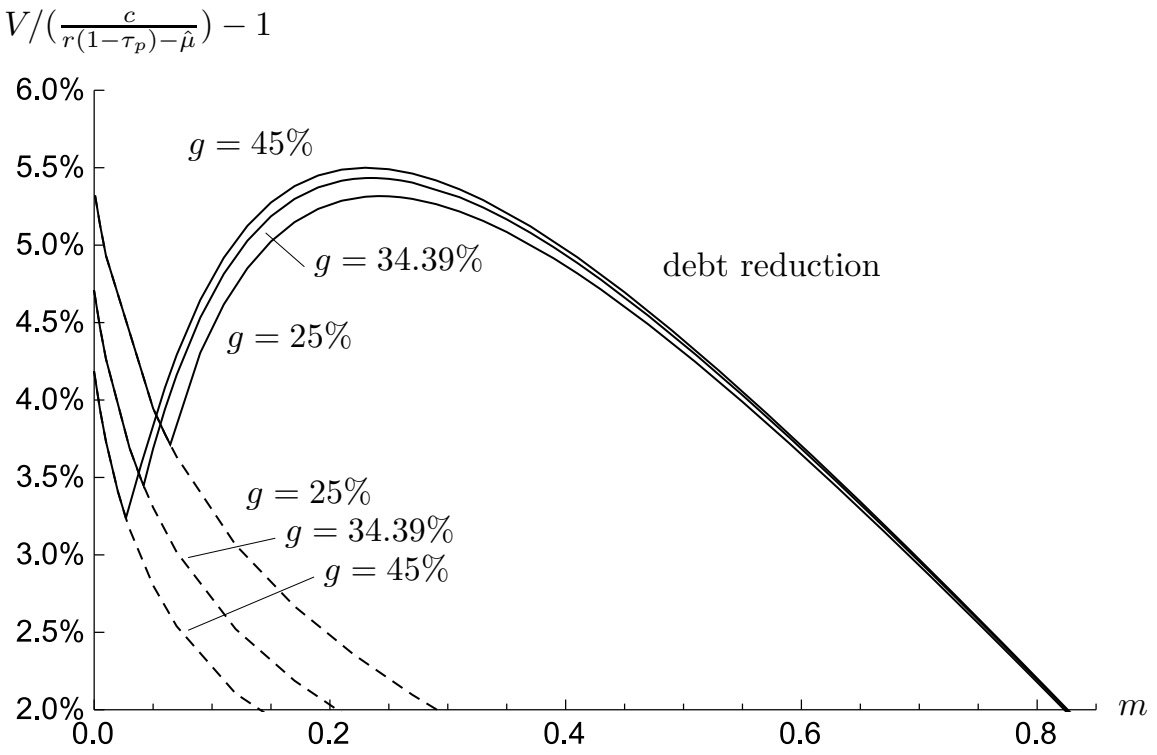


Figure 9: Initial tax advantage plotted over the retirement rate *m* for different levels of bankruptcy costs. Lower bankruptcy costs require a shorter debt maturity in order to induce voluntary debt reductions, lower bankruptcy costs reduce the maximum attainable tax advantage of debt, lowering bankruptcy costs moves the optimal finite maturity towards shorter maturities.

them more reluctant to default on their debt obligations. This makes the commitment effect of short-term debt relatively less advantageous and requires shorter debt maturities to induce voluntary debt reductions.

Panel 11b displays the tax advantage of debt as a function of the retirement rate for several values of the risk-adjusted cash flow growth rate $\hat{\mu}$. Moving to higher growth (i) increases firm value and (ii) shifts optimal maturity towards long-term debt. This is so since the commitment to decrease leverage in response to decreasing cash flows is less valuable for firms with high expected cash flow growth rates.

Bankruptcy costs and debt capacity: Next, Figure 12 plots the firm's optimal initial leverage ratio, which we refer to as the firm's debt capacity, for different debt maturities and for different levels of bankruptcy costs. Consistent with the findings reported above, higher

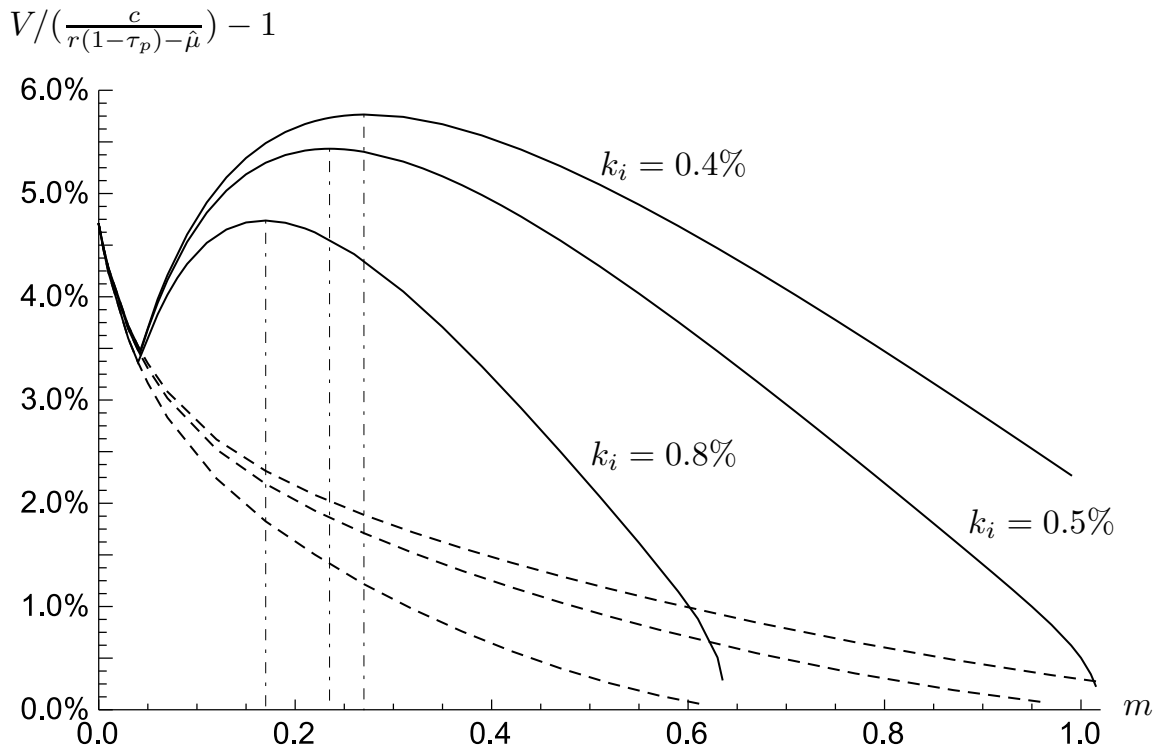
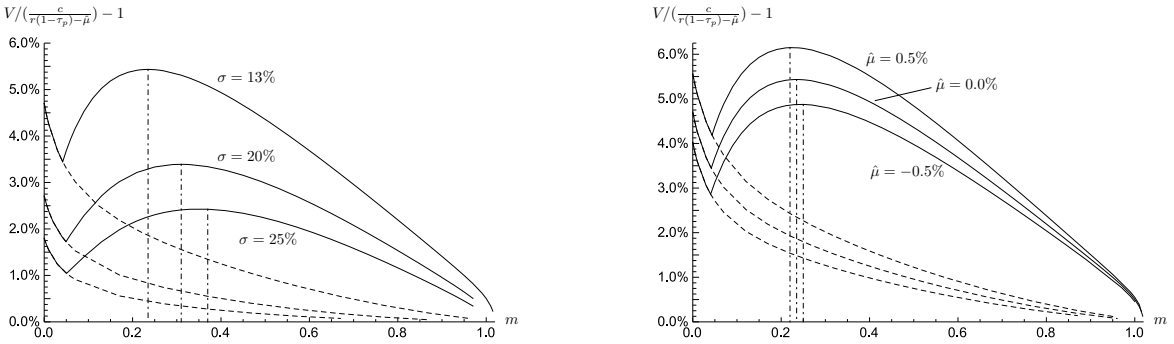


Figure 10: Optimal initial tax advantage for different costs k_i associated with rolling over debt plotted over the retirement rate m. With lower k_i, the optimal maturity moves towards shorter-term debt.



(a) tax advantage for different values of σ

(b) tax advantage for different values of $\hat{\mu}$

Figure 11: Optimal initial tax advantage plotted over the retirement rate m for different values of σ and $\hat{\mu}$.

bankruptcy costs are associated with a higher debt capacity since equityholders can commit to debt reductions when cash flows decrease. This results in a reduced bankruptcy probability

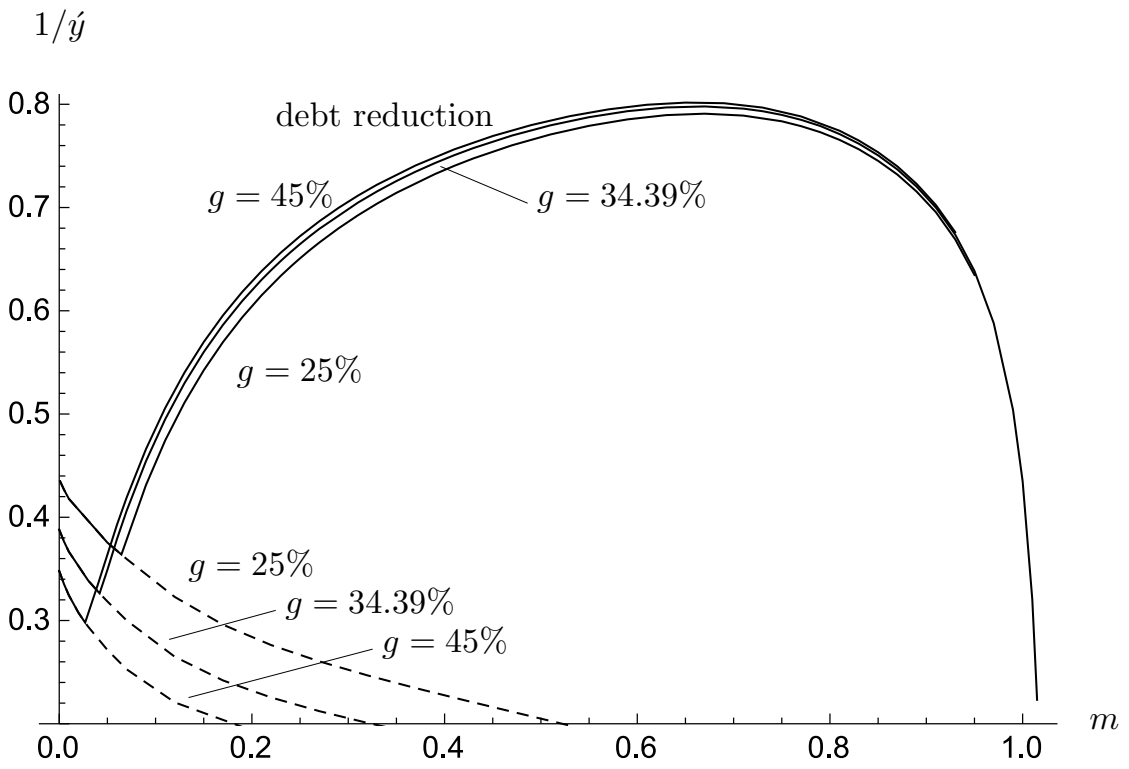


Figure 12: Optimal initial leverage ratios $1/\acute{y}$ plotted over the retirement rate m. High bankruptcy costs lead to high debt capacity if using short-term debt.

which more then offsets the higher bankruptcy costs conditional on default.

Firm value and corporate tax rates: Finally, Figure 13 shows the tax advantage of debt plotted against the retirement rate m for the base-case firm with $\tau_c = 30.6\%$. As a comparison we plot the tax advantage when corporate taxes are estimated from John Graham's marginal tax rate data, $\tau_c = 25.9\%$. Higher tax shields caused by higher corporate tax rates lead to (i) lower optimal debt maturity and (ii) a higher tax advantage at the optimal debt maturity. While it is intuitive that a higher corporate tax rate leads to higher tax advantage if debt is used optimally, the result that higher tax rates reduce optimal maturity is less obvious. As a direct consequence, higher tax rates make debt reduction less desirable, because reducing debt diminishes the associated tax shield. A secondary effect is that debt capacity increases with shorter debt, and higher debt capacity ex ante allows the firm to use debt more aggressively, which increases the debt tax shield. From Figure 13 we see that the latter effect dominates the direct effect and, over all, higher corporate tax rates reduce optimal debt maturity, from

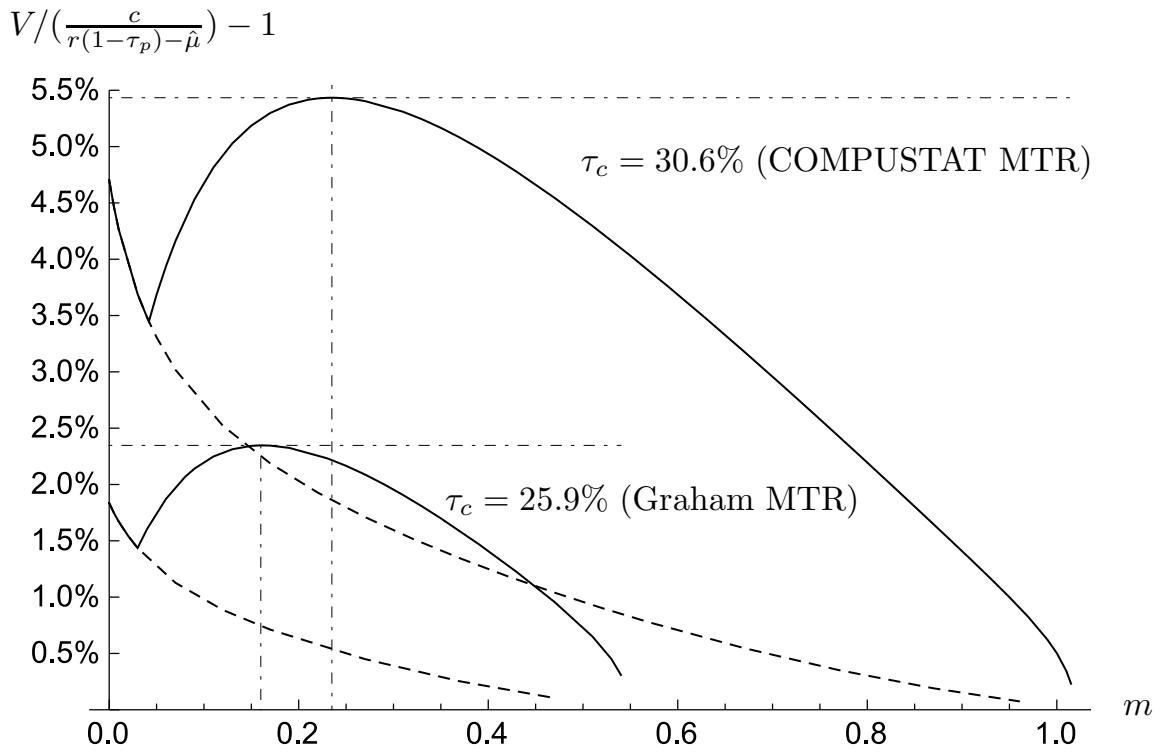


Figure 13: Tax advantage of debt for different levels auf corporate tax, τ_c, plotted over the retirement rate *m*. High corporate tax rates lead to higher tax advantages and lower optimal maturity.

an optimal average maturity of 6.25 years for $\tau_c = 25.9\%$ to 4.26 years in the base case with an effective corporate tax rate of $\tau_c = 30.6\%$.

7 Conclusions

This paper explores the effects of debt maturity on subsequent dynamic capital structure adjustments. We find that long debt maturities eliminate equityholders' incentives to engage in future voluntary debt reductions. By contrast, short debt maturities serve as a commitment to lower leverage in times when the firm's profitability decreases. This value-enhancing effect of short debt maturities must be traded off against increased transactions costs associated with the higher frequency of rolling over maturing bonds. The resulting tradeoff generates a new theory of optimal debt maturity.

We find that the equityholders' incentives to engage in debt reductions is non-monotonic

in the firm's leverage. For moderate drops in the firm's profitability, equityholders find it in their own best interest to repay maturing debt at least partly with equity, thereby mitigating the leverage increasing effect of the reduction in cash flows. However, if the firm's profitability continues to drop until it is pushed close to bankruptcy, then equityholders resume issuing new debt and gamble for resurrection.

Ex ante, the debt capacity of the firm increases if it uses debt with sufficiently short maturity. We find that high costs of bankruptcy induce a stronger incentive to use short-term debt since this reduces the expected probability of bankruptcy for given debt level. Higher tax shields caused by a higher corporate tax rate also makes shorter-term debt more advantageous, since increased debt capacity associated with short-term debt allows for a better utilization of debt tax shields. Comparative statics results reveal that increased cash flow risk reduces optimal debt maturity, whereas the growth rate of the cash flow process and the transactions costs of rolling over debt have the opposite effect.

All our main results are in accordance with existing empirical studies which confirm that firms tend to readjust their capital structure if they are highly levered and that firms with long-term debt are more reluctant to reduce leverage when their profitability drops compared to firms with a high portion of short-term debt. Other empirical predictions of our theory, such as the effects of growth and firm risk on firms' leverage adjustments in financial distress remain to be tested.

A Appendix

A.1 Derivation of Equation 4

The inverse leverage ratio with respect to the unlevered firm value, y_t, depends on two state variables, the cash flow of the firm's productive assets, c_t, and the current face value of debt, B_t. Thus one can write $y_t = y(c_t, B_t)$. If the debt level is adjusted by repurchasing all existing debt with face value B_t and issuing new debt with face value B_t^*, the leverage ratio

immediately jumps to the new value, i.e., in this case we have

$$dy_t = (\frac{1}{B_t^*} - \frac{1}{B_t})\frac{c_t}{r(1-\tau_p)-\hat{\mu}}, \tag{22}$$

and therefore

$$\frac{dy_t}{y_t} = \frac{B_t}{B_t^*} - 1. \tag{23}$$

In the absence of a discrete adjustment, the inverse leverage ratio, y_t, follows a diffusion and its dynamics can be determined using a Taylor-series expansion and Itô's Lemma

$$dy_t = \frac{\partial y}{\partial c}dc + \frac{\partial y}{\partial B}dB + \frac{1}{2}\left(\frac{\partial^2 y}{\partial c^2}(dc)^2 + \frac{\partial^2 y}{\partial B^2}(dB)^2\right) + \frac{\partial^2 y}{\partial c\,\partial B}dc\,dB. \tag{24}$$

Neglecting all terms that are $o(dt)$ gives

$$\begin{aligned} dy_t &= \frac{1}{B_t}\frac{1}{r(1-\tau_p)-\hat{\mu}}c_t\,(\mu dt + \sigma dW_t) \\ &\quad -\frac{1}{B_t^2}\frac{c_t}{r(1-\tau_p)-\hat{\mu}}(-(m-\delta)B_t\,dt) \\ &= y_t\,((\hat{\mu}+(m-\delta))\,dt + \sigma dW_t) \end{aligned} \tag{25}$$

for $0 \leq \delta \leq m$.

A.2 Proof of Corollary 1

From Equation (7) it follows that $\tilde{E}$ can be written as

$$\tilde{E} = \frac{K_1 - \delta K_2}{r(1-\tau_p)+(m-\delta)}, \tag{26}$$

hence, the partial derivative of $\tilde{E}$ with respect to δ is given by the expression in Corollary 1.

The partial derivative of $\tilde{D}$ with respect to δ can be directly determined from Equation (6) to be equal to $-y\frac{\partial \tilde{D}}{\partial y}\frac{1}{r(1-\tau_p)+m}$.

A.3 Proof of Proposition 1

Consider the expression for $\frac{\partial \tilde{E}}{\partial \delta}$ from Corollary 1. Since $\delta \leq m$ it follows that the denominator in this expression is always strictly positive, the sign of the partial derivative equals the sign of $K_1 - (r(1-\tau_p)+m)K_2$. From (7) it follows that

$$K_1 = (r(1-\tau_p)+m-\delta)\tilde{E} + \delta K_2, \tag{27}$$

so we have

$$K_1 - (r(1-\tau_p)+m)K_2 = (r(1-\tau_p)+m-\delta)(\tilde{E}-K_2). \tag{28}$$

Since $(r(1-\tau_p)+m-\delta) > 0$ the sign of the partial derivative $\frac{\partial \tilde{E}}{\partial \delta}$ equals the sign of

$$\tilde{E} - K_2 = \tilde{E} - (y\frac{\partial \tilde{E}}{\partial y} - (1-k_i)\tilde{D}). \tag{29}$$

Consequently, the firm is indifferent with respect to δ if and only if $\tilde{D}$ satisfies

$$\tilde{D}(y) = \frac{1}{1-k_i}\left(y\frac{\partial \tilde{E}}{\partial y}(y) - \tilde{E}(y)\right). \tag{30}$$

$\frac{\partial \tilde{E}}{\partial \delta} > 0$ if and only if the value of $\tilde{D}$ exceeds the value of the right-hand-side expression and it is optimal to choose $\delta = m$. $\frac{\partial \tilde{E}}{\partial \delta} < 0$ if and only if the value of $\tilde{D}$ is lower than the value of the right-hand-side expression and it is optimal to choose $\delta = 0$.

A.4 Proof of Proposition 2

In a region of internal optimum for the roll-over rate $0 < \delta^* < m$ we require $\tilde{D} = 1/(1-k_i)[y\frac{\partial \tilde{E}}{\partial y} - \tilde{E}]$. From Proposition 1 we know that under this condition we have $\frac{\partial \tilde{E}}{\partial \delta} = 0$, thus, the value of equity determined by valuation equation (7) is independent of the particular choice of δ. For simplicity, we substitute $\delta = 0$ into (7) to receive the expression for $\tilde{E}$ stated in Proposition 2.

The equilibrium roll-over rate δ^* is then determined by solving Equation (6) for δ. Since

equityholders are indifferent with respect to the choice of δ the particular choice δ^* does not change the valuation of equity.

The local response function of the value of debt to a re-issuance rate δ has the slope $\partial\tilde{D}/\partial\delta = -y\,\partial\tilde{D}/\partial y \frac{1}{r(1-\tau_p)+m}$, (see Equation (6)). From Figure 1 we can conclude that the equilibrium is stable only if $\tilde{D}(\delta)$ is downward sloping, i.e., $\partial\tilde{D}/\partial\delta < 0$ which requires $\partial\tilde{D}/\partial y > 0$. The latter condition simply requires that the value of debt per unit of face value increases as the inverse leverage ratio increases, i.e. leverage decreases.

In the case of an internal equilibrium we have $\tilde{D}(y) = \tilde{D}^I(y)$, hence $\frac{(1-k_i)}{y}(\partial\tilde{D}/\partial y) = (\partial^2\tilde{E}/\partial y^2) > 0$. which concludes the proof.

A.5 Proof of Proposition 3

In regions where $\delta = m$ or $\delta = 0$, the value function for $\tilde{D}$ and $\tilde{E}$ are the general solutions of the second-order ordinary differential equations (6) and (7) which can be proved by substituting the solution into the equation. In a region of an interior equilibrium $0 < \delta^* < m$ we know from Proposition 2 that $\tilde{D} = \tilde{D}^I$. The value of equity must be the solution of the Hamilton-Jacobi-Bellman equation (7) with δ^* from Proposition 2 substituted for δ. However, since we know that in an internal equilibrium the value of equity is invariant with respect to the choice of δ we solve (7) for $\delta = 0$ and argue that this solution must hold for every $0 \leq \delta \leq m$, and in particular for $\delta = \delta^*$. Substitution of this solution together with the equilibrium conditions of Proposition 2 into (7) constitutes an alternative proof.

A.6 Proof of Proposition 6

Suppose loss-given-default is less than 100%. This is the case, if $g < 1$ and optimal bankruptcy occurs at a level $\underline{y}$ such that the value of the remaining assets exceeds bankruptcy costs. Then it follows from boundary condition (13) that $\tilde{D}(\underline{y}) > 0$. However, for the value of equity and its partial derivative with respect to the inverse leverage ratio it follows from boundary condition (11) and optimality condition (16) that

$$\lim_{y\to\underline{y}} \tilde{E}(y) = \lim_{y\to\underline{y}} \frac{\partial\tilde{E}(y)}{\partial y} = 0.$$

Therefore, in a neighborhood of $\underline{y}$ it is true that

$$\tilde{D}(y) > \frac{1}{1-k_i}\left(y\frac{\partial \tilde{E}}{\partial y}(y) - \tilde{E}(y)\right).$$

According to Proposition 1 this implies that $\delta = m$ is the optimal strategy.

A.7 Proof of Proposition 4

This proposition is derived from Equation (7) which determines the value of equity per unit of debt. After dividing (7) by $r(1-\tau_p)+(m-\delta_t)$ and calculating partial derivatives $\partial\tilde{E}/\partial m$ and $\partial\tilde{E}/\partial\delta$, the proposition immediately follows.

A.8 Proof of Proposition 5

Take some good state of the firm, y_0, where it fully rolls over expiring debt. From Proposition 4 we know that in equilibrium firms will not want to accelerate repayment there. Combining Propositions 1 and 4 we get

$$\begin{aligned} 1 > (1-k_i)\tilde{D}(y_0) &\geq y_0\frac{\partial \tilde{E}}{\partial y}(y_0) - \tilde{E}(y_0) \\ \Rightarrow \quad \tilde{E}(y_0) - y_0\frac{\partial \tilde{E}}{\partial y}(y_0) &> -1. \end{aligned} \tag{31}$$

The tangent on $\tilde{E}$ at y_0 can be written as

$$t_0(y) = d_0 + y\frac{\partial \tilde{E}}{\partial y}|_{y=y_0}.$$

We conclude $d_0 > -1$ from the fact that $\tilde{E}(y_0)$ lies on t_0 and from (31).

Now take a state $y_1 < y_0$. Convex equity implies that $\tilde{E}(y_1)$ lies above the tangent t_0 and, furthermore, the tangent t_1 on $\tilde{E}$ at y_1 is flatter than t_0, thus, it follows

$$t_1(y) = d_1 + y\frac{\partial \tilde{E}}{\partial y}|_{y=y_1} \quad \Rightarrow \quad d_1 \geq d_0 > -1$$

By construction $\tilde{E}(y_1)$ lies on t_1, consequently,

$$\tilde{E}(y_1) - y_1 \frac{\partial \tilde{E}}{\partial y}|_{y=y_1} = d_1 > -1, \quad \Rightarrow \quad \frac{\partial \tilde{E}}{\partial m}(y_1) < 0, \tag{32}$$

which proves the proposition.

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