

Kociński, Marek

Article

Transaction costs and market impact in investment management

e-Finanse: Financial Internet Quarterly

Provided in Cooperation with:

University of Information Technology and Management, Rzeszów

Suggested Citation: Kociński, Marek (2014) : Transaction costs and market impact in investment management, e-Finanse: Financial Internet Quarterly, ISSN 1734-039X, University of Information Technology and Management, Rzeszów, Vol. 10, Iss. 4, pp. 28-35, https://doi.org/10.14636/1734-039X_10_4_003

This Version is available at:

<https://hdl.handle.net/10419/147111>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

TRANSACTION COSTS AND MARKET IMPACT IN INVESTMENT MANAGEMENT

MAREK KOCIŃSKI¹

Abstract

The aim of this article is to analyse the major sources of transaction costs in financial markets, in particular to find the amounts of such costs on the Warsaw Stock Exchange (WSE). Sources of transaction costs are considered: commissions, bid-ask spread and market impact. The commissions are only briefly described since they are explicitly stated and easily measured. More attention is paid to the bid-ask spread which is one of the main causes of trading costs. It is shown that the investor who wants to outperform the Polish market should usually expect a much higher bid-ask spread than it follows from the officially used calculations. Then it is demonstrated how historical spreads can be used in predicting their future values. This seems to be important from the practical point of view, since forecasting trading costs is a compelling task for financial managers. Next, market impact and market impact costs are considered. The practical method of measuring these is applied and discussed.

JEL classification: C5, G11, G24

Keywords: transaction costs, bid-ask spread, market impact

Received: 21.06.2014

Accepted: 31.01.2015

¹ Marek Kociński, Faculty of Applied Informatics and Mathematics, Warsaw University of Life Sciences-SGGW, e-mail: marek_kocinski@sggw.pl.

INTRODUCTION

Measuring and understanding financial costs is important for practitioners and regulators, and has attracted much attention in the academic literature. Transaction costs are widely recognized as a substantial determinant of investment performance since the net gains are affected by such costs. Moreover, they are closely related to market liquidity which is a very important factor in the trading world, since it represents the ease with which financial instruments can be traded. Transaction costs are an integral component of market microstructure. They cannot be eliminated but they can and should be managed (Amihud & Mendelson, 2013, p. 421). To define the cost of a transaction it is good to consider separately the cases of buying and selling. In the case of buying assets, the transaction cost is the difference between the value of money spent for purchase of the assets and the market value of the assets just before the transaction. In the case of selling, to obtain the considered cost one must subtract from the market value of the traded assets just before the order the amount of money obtained in exchange for selling the assets. Though a trading cost may be a small fraction of the value of the single transaction, over the long-term horizon such expenses can significantly lower the return attained by application of the investment strategy, especially when a large number of purchases or sales is required. Therefore the transaction costs should be taken into account in the assessment of the efficiency of dynamic portfolio strategies and there exists an extensive literature on this subject which analyses various market models. To such recent works belongs, for example, the article of Kim and Viens (2012) who evaluate the optimal trading frequency when transaction costs are incurred, the article of Garleanu and Pedersen (2013) who provide closed-form expressions for the optimal portfolio policy in the case with quadratic transaction costs, and many other papers. Trade execution costs substantially affect strategies which hedge risky positions in derivatives.

In fact, the presence of transaction costs invalidates the famous and celebrated Black and Scholes (1973) model. Even for any small trading costs, for a European call option the minimal initial wealth needed to hedge an option in a Black-Scholes market with proportional transaction costs, is just the price of one share of the underlying stock.

This means that the minimal hedging strategy is the trivial, buy-and-hold strategy - buying a share, and holding it until the maturity time of the option (Levental & Skorohod, 1997). The obtained theoretical price is then too high to be used as the actual price of a claim. The problem of appropriate pricing derivatives under transaction costs was considered in numerous papers. In this context, discrete time models proved to be highly useful. Kociński (2010) proved that, for large class of options, the option's price in a simple binomial structure of stock price evolution, is the same as in the case of a quite general discrete model. Apart from influencing the practice of asset management, the transaction costs are important in the theory of finance – they explain the phenomena which seem to make it possible to outperform systematically the market, which contradicts the market efficiency: autocorrelation in stock returns, abnormally high returns of stocks with low price-earnings or middle price-book value ratios (Czekaj, Woś & Żarnowski, 2001, p. 126-130). It may be said that without taking into account transaction costs it is impossible to understand properly the workings of a financial system. Moreover, in order to optimize a trading strategy it is also necessary to measure the impact of the transaction on the asset's price. The security's price evolves mostly according to market forces that occur randomly and independently of the considered transaction but there is also an additional factor influencing the price and it is called market impact. This notion denotes a change in the asset price induced by the transaction. It is intuitive that a buy order should drive the price up, and a sell order should drive it down. This is not only easily demonstrated empirically but also postulated by standard economic theory: an increase in demand should increase prices and an increase in supply should decrease prices. The analysis of a multi-period investing strategy should take such price changes into account, especially for large transactions. Moreover, it is easily seen that market impact generates trading cost since the average costs of buying a share is larger and the average cost of selling is smaller than the price just before transaction. This cost depends on how the transaction is carried out, that is, how assets are transferred in a specified period of time. The quicker the large order is executed, the larger the market impact and related to it costs occur. Splitting the order and executing it incrementally over a longer period reduces market impact but incurs the risk of loss of the opportunity of taking advantage of the profitable pre-transaction price.

The taxonomy of trade execution costs is quite complex and questionable. For example, Fabozzi, Focardi and Kolm (2010, p. 420) categorize the bid-ask spread as an explicit cost and Huang (2013, p. 234) classify it as implicit. In general, typically transaction costs are categorized in two dimensions: fixed costs versus variable costs, and explicit costs versus implicit costs (Fabozzi et al., 2010, p. 446). In practice of investment management three major sources of transaction costs are taken into account: commissions (and similar payments), bid-ask spreads and market impact (Elton, Gruber, Brown & Goetzmann, 2010, p. 39; Sharpe, Alexander & Bailey, 1999, p. 69-74). They are analysed in detail in the following chapters.

COMMISSIONS

A commission is the amount of money paid to the brokerage firm for its services, including investment advice and execution of the investors' orders on the trading floor of the exchange. Normally such costs are negotiable. Additionally the brokerage firm may charge for transfers of cash to the bank and for holding the account. Fees charged by an institution that holds the securities in safekeeping for an investor are called custodial fees and costs of transfer of the ownership over a stock are referred to as transfer fees. The buyers and sellers do not trade directly, in Poland for each transaction there are simultaneously three intermediaries: the brokerage house which operates the trading account, the stock exchange which organizes and regulates the market and the institution responsible for the management and supervision of the depository, clearing and settlement system to enable the trading of financial instruments (Krajowy Depozyt Papierów Wartościowych). In practice all payments for those services are charged by the brokerage which shares them with the remaining two institutions. In a given range of trading values the commission is proportional to the value of the trade, with a usually added condition that it can not be lower than some minimal cost. The commissions also depend on whether an order is placed personally, by telephone or through the Internet. Commissions vary widely from brokerage to brokerage. The web portal Bankier.pl published the rank of brokerage firms operating on the Polish stock market (Retrieved from: <http://www.bankier.pl/wiadomosc/Ranking-rachunkow-maklerskich-1-kwartal-2014-3063196.html>). The ranking list was established on January 2014 and 21 brokerage houses

were taken into account. The commissions for the transaction valued at 2500 PLN varied from 0,19% to 0,39%.

BID-ASK SPREAD

The bid-ask spread is the distance between the quoted sell and buy order. There are several factors that influence the difference between the bid and ask prices. The most evident factor is liquidity which is measured by the volume or amount of stocks traded daily. For large, actively traded stocks the spread usually is less than 1% of the price per share and is inversely related to the amount of trading activity in a stock. That is, stocks with less trading volume tend to have a greater spread. The explanation of this phenomenon is straightforward. When a stock has a low trading volume, the market participants consider it illiquid because it is not easily converted into cash. As a result they will require compensation for providing the investor with liquidity, accounting for the larger spread. Another important factor that affects the bid-ask spread is volatility. Volatility usually increases during a period of rapid market decline or growth. Then, the bid-ask spreads are much wider because investors want to take advantage of the change. When assets are increasing in value, investors are willing to pay larger costs for buying them and when prices of the assets are decreasing, their holders agree to bear larger expenses for getting rid of them. At times, volatility is low, risk and uncertainty is also low and the bid-ask spread is narrow. The price of the stock is also a factor influencing the spread. If the price is lower then the bid-ask spread expressed as the percentage of the mid-price is considerably larger. It is also important to note that the bid-ask spread is a proxy for liquidity of the stock (Barucci, 2003, p. 302).

A natural question arises: what is the value of the spread the investor should take into account when planning a transaction on the Warsaw Stock Exchange (WSE)? Since the spread is not a constant, the approximate answer could be the average WSE spread in a given period.

The methodology used by the Warsaw Stock Exchange in historical bid-ask spread calculation is based on the algorithm endorsed by the Federation of European Stock Exchanges and the historical average spread is obtained by the following formulas (personal communications, April 9 and May 23, 2014):

$$S = \frac{\sum_{i=1}^N O_i * S_i}{O_i}, S_i = 20000 \left| \frac{k(b) - \left(\frac{k+s}{2}\right)}{\frac{k+s}{2}} \right|$$

where S = the average spread weighted turnover
 O_i = the value of the i -th transaction
 $k(b)$ = the trading price of the i -th transaction,
 k = the best buy offer preceding the transaction
 s = the best sell offer preceding the transaction

The result of the considered formula is expressed in basis points. It is easily seen that the spreads of bid-ask prices of stocks with greater values of share trading are better represented in the average value than the spreads of stocks with less valued transactions. This is because the weights used in calculation of the average are proportional to the values of transactions. However, since stocks with less valued transactions tend to have greater spreads, there is ground for supposing that the real spread the investor has to face is often greater than the average shown by the official statistics. To check this, compare, for the period 2009-2013, the official WSE average spreads to the arithmetic means of average spreads of stocks, without distinguishing between stocks in terms of any financial indicator. Surprisingly, the number of easily available sources concerning the WSE average spread seems to be very small. In fact, the only one found, which showed the indicator explicitly called “average spread on WSE” for all companies, is a document entitled “Universal Trading Platform (UPT):

WSE’s State-of-The-Art Trading System” on the website of WSE (http://www.gpw.pl/trading_system_utp) where suitable data on spread are available just for the years 2009-2012. However, there are also WSE Monthly Bulletins available on the website of the Warsaw Stock Exchange (Retrieved from: <http://www.gpw.pl/2151>) and the December issues of these bulletins in years 2009-2013 provide indicators described as January-December average spreads which are viewed as official average spreads on WSE in the appropriate years. The results in this area for years 2009-2012 retrieved from pages http://www.gpw.pl/trading_system_utp and <http://www.gpw.pl/2151> are sufficiently similar from a practical point of view. The average spreads of individual stocks needed for mean calculations are taken from annual WSE Statistics Bulletins (Retrieved from: http://www.gpw.pl/statystyki_roczne_en). The results are presented in Chart 1.

From Chart 1 one can see that spread is likely to be a major constraint for outperforming the market. In fact, large differences between the results of the two methods of the average spread calculation is a strong argument against the possibility of outperforming the market. In order to achieve better results than the market portfolio (or its proxy which can be the WIG index), it seems that the investor should pay attention to stocks with market price substantially different from their real value. Then the shares should be sold when overestimated and bought when underestimated. However, market mispricing of the stock is related to smaller interest of market participants. This means lower transaction values

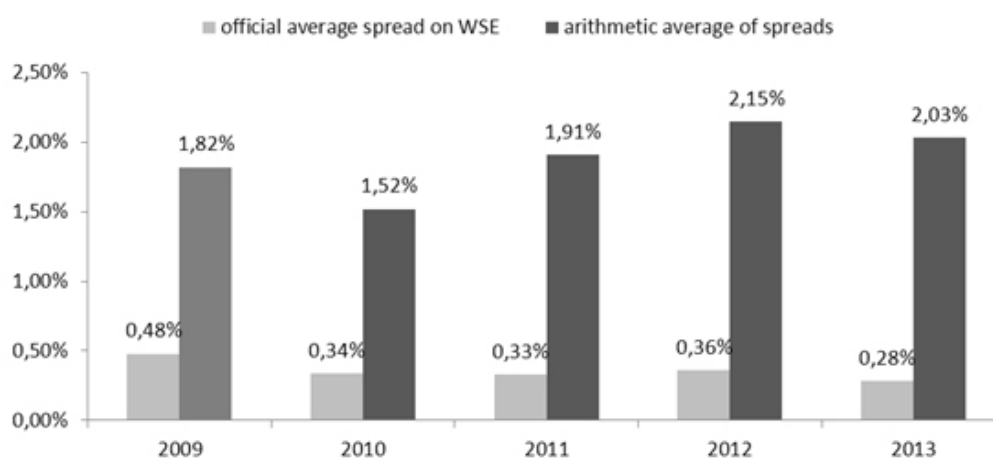


Chart 1. Two views of average spreads on the WSE

Source: Retrieved from: <http://www.gpw.pl/2151>,
http://www.gpw.pl/statystyki_roczne_en and own calculations

and, as Chart 1 shows, a much larger bid-ask spread. From a practical point of view it is important whether the historical spreads can be used in predicting the future. In order to check that, a random sample of 50 monthly average spreads, from WSE Statistic Bulletins for the period 2009-2013, was taken into consideration²).

To each element in the sample, which is the spread of the drawn stock (the value of the explanatory variable) corresponds the value of a dependent variable which is the spread of the same stock in the succeeding month. The following results were obtained from linear regression:

$$S_{t+1} = 0,34\% + 0,925S_t, \quad \rho = 0,865, \quad p = 5,87 * 10^{-16}, \quad R^2 = 74,76\%$$

where S_{t+1} = the average monthly spread in month $t+1$
 S_t = the average monthly spread in month t
 ρ = the correlation coefficient
 p = the p-value of the test of significance of explanatory variable (t-test)
 R^2 = the coefficient of determination

It is apparent from the calculations that the most recent average monthly spread for a given stock contains a lot of information about the value of the considered parameter in the next month. The p-value is very small and this provides a strong evidence that the current spread has a significant impact upon the next spread value. The value of R^2 indicates that about 75% of variability of current spread around its mean is explained by the considered model. The strong relationship between bid-ask spreads in two consecutive months can be also seen in Chart 2.

THE POLICY OF SHOWING INCOME

Market impact refers to the effect of the order (to buy or to sell) on the price of the traded security. This effect tends to move the price against the order maker, that is upward when buying and downward when selling. Thus, it easily seen that such impact is a source of trading costs. Especially for large investors, the market impact is very important and can strongly

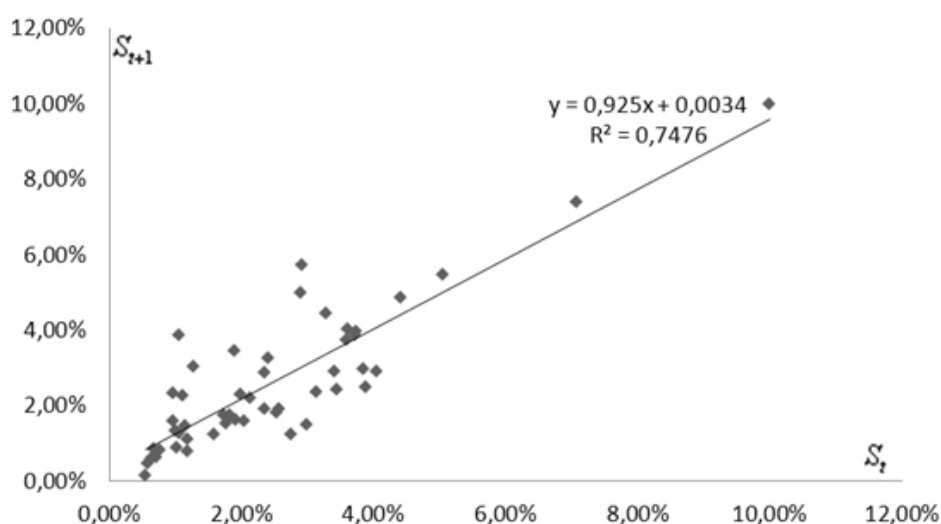


Chart 2. Linear Interdependence of spreads in two consecutive months

Source: Retrieved from: http://www.gpw.pl/statystyki_roczne_en and own calculations

² (Retrieved from: http://www.gpw.pl/statystyki_miesieczne_en)

influence the effect of an investment decision. For small transactions such an effect is negligible. There are two kinds of market impact. Temporary impact refers to temporary price movement away from equilibrium caused by the transaction and it may be assumed in practice that such impact affects only this transaction. Permanent impact means change in the equilibrium price due to trading and it affects every future transaction.

The total market impact is a combination of these two effects. The temporary component may be considered as a cost of an order execution in a short time, the cost of additional liquidity and the

permanent component reflects the persistent price change which is a result of an adjustment of the market to the information content of the trade. Intuitively, a sell transaction is a signal to the market that the security may be overvalued whereas the buy transaction signals that the security may be undervalued. The distinction between two types of impact is important in research referring to simulating the trading strategies in a given model of asset price dynamics. Exhibit 1 shows the idealized market impact of a market order to buy. The horizontal and vertical axes display the time and stock price, respectively.

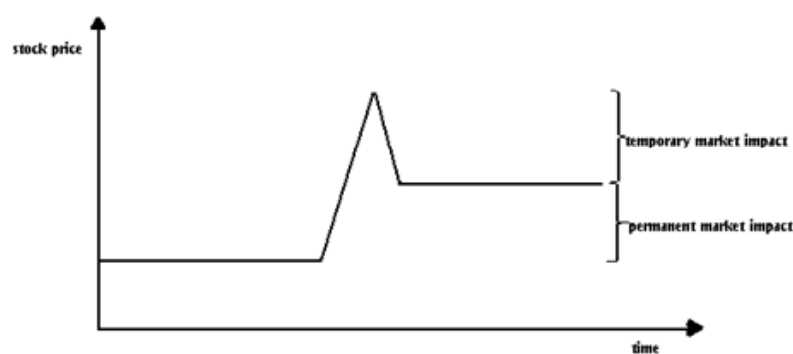


Exhibit 1: Idealized market impact of buy order

Source: Own elaboration

influence the effect of an investment decision. For small transactions such an effect is negligible. There are two kinds of market impact. Temporary impact refers to temporary price movement away from equilibrium caused by the transaction and it may be assumed in practice that such impact affects only this transaction. Permanent impact means change in the equilibrium price due to trading and it affects every future transaction.

The total market impact is a combination of these two effects. The temporary component may be considered as a cost of an order execution in a short time, the cost of additional liquidity and the The permanent impact has a feature which seems to be rarely taken into account in literature on transaction costs. That is, the move of the price during the transaction is on one hand unfavorable because it increases average cost of trading but on the other hand this move is also favorable for portfolio market value. Namely, the rise of the price induced by the buy order means that bought assets in the investor's portfolio are now more precious than before the transactions and it positively affects the value of the portfolio.

Market impact is widely discussed, but relatively rarely measured. However, in the literature various functional forms of market impact are considered. The simplest assumption is that impact is both linear in the traded volume and permanent in time, but more sophisticated, non-linear functions are also considered. One of the well-known impact models which can be used to predict changes in price due to trading activity and market impact costs is the model worked out by Almgren, Thum, Hauptmann and Li (2005). They managed to describe market impact and its costs in terms of a small number of input variables and their results can be directly applied to a trade schedule of the investor's strategy. Almgren et al. (2005) use the data set of almost 700,000 US stock trade orders, in which a trade's direction (buyer or seller initiated) is known, executed by Citigroup equity trading desks for the 19-month period from December 2001 to June 2003. They define two variables I and J . I is the permanent change of the stock price induced by the order, is the difference between the realized value of transaction and such value calculated at the price before

the impact of the trade begins, per unit of the stock. Thus, if J is positive it denotes additional expenses for buying shares and negative values of J means decrease in the payment the investor obtains for selling shares. Both I and J are expressed as a fraction of the pre-trade price. The average values of I and J are given as follows:

$$I = \gamma\sigma \frac{X}{V} \left(\frac{\Theta}{V} \right)^{\frac{1}{4}} \quad \text{and} \quad J = \frac{I}{2} + \text{sgn}(X)\eta\sigma \left| \frac{X}{VT} \right|^{\frac{3}{5}}$$

where X = the number of shares in the order
 V = the average daily volume in shares
 Θ = the total number of shares outstanding
 T = T is a trade duration (days)
 σ = the daily volatility
 γ = the estimated coefficient
 η = the estimated coefficient

The values of γ and η , called the universal coefficients of market impact, were determined by linear regression by Almgren et al. (2005) and the results are as follows: $\gamma = 0.314 \pm 0.041$ and $\eta = 0.142 \pm 0.0062$. In practice, these coefficients may be continually updated to reflect the most recent data. Consider now a specific numerical example for the shares of the company NETIA SA quoted on the WSE. Assume that the investor wants to buy shares on May 27, 2014 and one day before the transaction would like to estimate the expected permanent change of the stock price induced I by his or her order I and the average transaction cost J . The values of the variables necessary for calculations are given in Table 1. Daily volatility σ is estimated from historical returns (Sharpe et al., 1999, p. 626-627) as follows:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{t=1}^n \left(R_t - \frac{1}{n} \sum_{t=1}^n R_t \right)^2}, \quad R_t = \ln \left(\frac{S_t}{S_{t-1}} \right)$$

where S_t = The close price of the stock at time t , $t = 0, \dots, n$
 n = The size of the sample of logarithmic returns R_t

The number n is arbitrary (but must not be too small or too large). In this article $n = 30$, which means that in the sample there are 30 historical logarithmic returns. In more detail, the oldest return in the sample is the close on April 10, 2014 and the latest is the close on May 26, 2014. Average daily volume V is calculated as the average of daily trading volumes in the considered 30 days².

Table 1. Data of the stock NETIA SA needed to calculate I and J , on May 26, 2014

Variable	Value
Shares outstanding Θ	347910774
Average daily volume V	622779.30
Daily volatility σ	1,38%

Source: Retrieved from <http://gpwinfostrefa.pl> and own elaboration

The investor submits the buy order to set the number of shares to buy X and time T during which the transaction should be carried out. Table 2 provides the obtained results of applying formulas for I and J . Assuming that the trading day lasts 8 hours, T equal to 0,002 represents approximately 1 minute and T equal to 0,125 is 1 hour. A major advantage of the model of Almgren et al. (2005) is that it takes into account the trade duration, which seems to be rare among impact models. However, the question arises as to whether the bid-ask spread is inserted into the considered model.

Table 3. Price impact and average impact cost for various orders and order durations

X	T	I	J
10000	0,002	0,03%	0,70%
10000	0,125	0,03%	0,07%
10000	1	0,03%	0,03%
200000	0,002	0,68%	4,46%
200000	0,125	0,68%	0,68%
200000	1	0,68%	0,44%
500000	0,002	1,69%	8,00%
500000	0,125	1,69%	1,44%
500000	1	1,69%	1,02%
1000000	0,002	3,38%	12,53%
1000000	0,125	3,38%	2,60%
1000000	1	3,38%	1,95%

Source: own elaboration based on Table 1

² There are no guarantees that the parameters of a stock will be the same in the future, but 30 trading days seems to be enough data to have a reasonable degree of certainty that they will be similar, at least for the next few days.

A major advantage of the model of Almgren et al. (2005) is that it takes into account the trade duration, which seems to be rare among impact models. However, the question arises as to whether the bid-ask spread is inserted into the considered model. Almgren et al. (2005) define the market price S_0 which they, in the same time, call the pre-trade price, as the bid-ask mid-point, and compute it from the latest quote just preceding the start of the transaction. The price S_0 is a reference point in calculations of the market impact and its costs and this suggests that the bid-ask spread is taken into account in the cost formula. On the other hand, the spread is immediate transaction cost, the price for supplying immediacy Fabozzi, Focardi and Kolm (2006, p. 54). This implies that in the model of Almgren et al. (2005) is 0 and, since it is in the denominator, the cost cannot be computed. The doubt is increased by the fact, that in the economic literature concerning transaction costs the bid-ask spread and the market impact are usually considered as separate components of total cost. The practical solution of this problem could be the assumption that market order execution takes place over some very short but non-zero time (which is obvious) and in this framework J contains both bid-ask spread and market impact costs. There is, however, some mathematical doubt.

By high-frequency algorithmic trading which uses sophisticated technological tools and computer algorithms one can rapidly buy or sell securities in even fractions of a second. Thus, T can be very small and when $\frac{X}{V}$ is also very small, the fraction $\frac{X}{VT}$ approaches indeterminate form which makes it computationally unstable.

SUMMARY

Transaction costs are a very important factor affecting investment performance in general, and in particular on the Polish stock market. Two of three major sources of trading costs such as commissions and bid-ask spreads are relatively easy to assess for practical purposes. However, the actual amount of those costs suggests that dynamic strategies requiring frequent portfolio rebalancing, in practice underperform reasonably balanced at the start, buy-and-hold strategies. This conclusion is supported by the phenomenon of market impact which is another source of transaction costs, especially for large orders. Market impact, although considered in many papers, still remains a challenge for researchers and financial managers. One of the well-known mathematical models capturing market impact is cited and applied in this paper. The practical question which arises from this model is stated.

REFERENCES

- Almgren, R., Thum, C., Hauptmann, E., Li. H. (2005). *Direct Estimation of Equity Market Impact*. Risk, 18(7), 58–62.
- Amihud Y., Mendelson H. (2013). *Transaction Costs and Asset Management*. In M. Pinedo, I. Walter, (Ed.), *Global Asset Management – Strategies, Risk, Processes, and Technologies*, (p. 414-434). London: Palgrave Macmillan.
- Barucci, E. (2003). *Financial Markets Theory – Equilibrium, Efficiency and Information*. London: Springer-Verlag.
- Black, F., Scholes, M. (1973). *The Pricing of Options and Corporate Liabilities*. Journal of Political Economy. 81, 637–659.
- Czekaj, J., Woś M., Żarnowski, J. (2001). *Efektywność giełdowego rynku akcji w Polsce. Z perspektywy dziesięciolecia*. Warszawa: Wydawnictwo Naukowe PWN.
- Elton, E. J., Gruber, M. J., Brown, S. J., Goetzmann, W. N. (2010). *Modern Portfolio Theory and Investment Analysis*. Hoboken: John Wiley & Sons.
- Fabozzi, F. J., Focardi S. M., Kolm P. N. (2006). *Financial Modeling of the Equity Market: From CAPM to Cointegration*. Hoboken: John Wiley & Sons.
- Fabozzi, F. J., Focardi S. M., Kolm P. N. (2010). *Quantitative Equity Investing*. Hoboken: John Wiley & Sons.
- Garleanu, N., Pedersen L. (2013). *Dynamic Trading with Predictable Returns and Transaction Costs*. The Journal of Finance 68, 2309-2340.
- Huang, Y.C., (2013). Determinants of Trading Costs. In H. K. Baker, H. Kiyamaz, (Ed.), *Market Microstructure in Emerging and Developed Markets*, (p. 233-252). Hoboken: John Wiley & Sons.
- Kim, H.-Y., Viens, F. G. (2012). *Portfolio Optimization in Discrete Time with Proportional Transaction Costs under Stochastic Volatility*. *Annals of Finance*, 8, 405–425.
- Kociński, M. A., (2010). *Hedging of the European Option in Discrete Time under Transaction Costs Depending on Time*. *Applicationes Mathematicae*, 37, 201-214.
- Levental, S., Skorohod, A. V. (1997) *On the Possibility of Hedging Options in the Presence of Transaction Costs*. *Annals of Applied Probability*, 7, 410–443.