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**Article — Manuscript Version (Preprint)**

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Theoretical and Practical Research in Economic Fields

*Suggested Citation:* Vasilev, Aleksandar (2016) : Aggregation with sequential non-convex public- and private-sector labor supply decisions, Theoretical and Practical Research in Economic Fields, ISSN 2068-7710, ASERS Publishing, Craiova, Vol. VII, Iss. 2(14), Winter, <https://doi.org/10.14505/tpref>

This Version is available at:

<https://hdl.handle.net/10419/146518>

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# Aggregation with sequential non-convex public- and private-sector labor supply decisions

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September 1, 2016

## Abstract

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner. Still, the micro-founded representation obtained from explicit aggregation over homogeneous individuals again features different disutility of labor across the two sectors. Thus, there is little merit in the timing of the sectoral non-convex labor choice.

**Key words:** indivisible labor, public employment, sequential lottery, aggregation

**JEL Classification:** J22, J45

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# 1 Introduction

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner.

## 2 Model Setup

The theoretical setup follows to a great extent Vasilev (2015a), except for the timing of the sectoral labor supply decisions. The economy is static, there is no physical capital, and agents face a sequential non-convex decision in a two-sector economy.<sup>1</sup> Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by  $i$  and distributed uniformly on the  $[0, 1]$  interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index  $i$  to save on notation.

### 2.1 Households

Each household maximizes the following utility function

$$Max_{\{c, h^p, h^g\}} \{ \ln[c^\eta + S^\eta]^{\frac{1}{\eta}} + \alpha \ln(1 - h^p - h^g) \} \quad (2.1)$$

where  $c, S, h^p, h^g$  denote private consumption, consumption of the public good, hours worked in the private sector, and hours worked in the government sector. The parameter  $\alpha > 1$  measures the relative weight of leisure in the utility function. Total consumption is a Constant Elasticity of Substitution (CES) aggregation of private consumption and consumption of government services, where  $\eta > 0$  measures the degree of substitutability between private and public consumption.<sup>2</sup>

Each household is endowed with 1 unit of time that can be allocated to work in the private

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<sup>1</sup>Adding physical capital accumulation decision, and a dynamic structure to the model is then straightforward. Also, the absence of those elements in the current analysis does not affect in any major way the derivation of the optimality conditions characterizing the aggregate labor supply decisions.

<sup>2</sup>The separability of consumption and leisure is not a crucial assumption for the results that follow. A more general, non-separable, utility representation, does not generate new results, while significantly complicates the algebraic derivations, and thus interferes with model tractability.

sector, work in the government sector, or leisure

$$h^p + h^g + l = 1 \quad (2.2)$$

Labor supply in each sector is assumed to be discrete  $h^p \in \{0, \bar{h}^p\}, h^g \in \{0, \bar{h}^g\}$ . In contrast to Vasilev (2015a), within the period, each household decides first to look for a job in the private sector, and if unsuccessful, will search for work in the public sector. The wage rate per hour worked in the private and public sectors is  $w^p$  and  $w^g$ , respectively.

In addition to labor income, households hold shares in the private firm and receive an equal profit share  $\pi$ , with  $\int_0^1 \pi di = \Pi$ .<sup>3</sup> Income is subject to a (equal) lump-sum tax  $t$ , where  $\int_0^1 t di = T$ , with  $T$  denoting aggregate tax revenue. Therefore, each household's budget constraint is

$$c^j \leq w^j h^j + \pi - t, j = p, g \quad (2.3)$$

Households act competitively by taking the wage rates  $\{w^p, w^g\}$ , aggregate outcomes  $\{C, S, H^p, H^g\}$  and lump-sum taxes  $\{T\}$  as given. Each household chooses  $\{c^j, h^p, h^g\}$  to maximize (2.1) s.t. (2.2)-(2.3).

### 3 Firms

There is a representative firm in the private sector producing a homogeneous final consumption good, which uses labor as an only input. The production function is given by

$$Y = F(H^p), F' > 0, F'' < 0, F'(\bar{H}^p) = 0, \quad (3.1)$$

where the last assumption is imposed to proxy a capacity constraint. The firm acts competitively by taking the hourly wage rate  $\{w^p\}$ , aggregate outcomes  $\{C, S, H^g\}$  and policy variable  $\{T\}$  as given. Accordingly,  $\{H^p\}$  is chosen to maximize static aggregate profit:<sup>4</sup>

$$\max_{H^p \geq 0} F(H^p) - w^p H^p. \quad (3.2)$$

Given the assumption imposed on the production function, in equilibrium, the firm will realize positive economic profit.

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<sup>3</sup>This technical assumption guarantees a positive consumption to either of the two types, even if they choose not to work in their sector.

<sup>4</sup>This representation can be viewed as being isomorphic to a problem in which capital has already been optimized over.

## 4 Government

The government hires employees to provide public services and levies lump-sum taxes on households to finance the government wage bill. The technology of the public good provision uses labor  $H^g$  as an input, which is remunerated at a non-competitive wage rate  $w^g = \gamma w^p$ . Parameter  $\gamma \geq 1$  will measure the fixed gross mark-up of government sector wage rate over the private sector one.<sup>5</sup> The production function of public services is as follows:

$$S = S(H^g), S' > 0, S'' < 0, S'(\bar{H}^g) = 0, \quad (4.1)$$

where the last assumption guarantees that not everyone will work in the production of the public good.<sup>6</sup> In addition, the public good is a pure non-market output, thus it will not appear in the government budget constraint.

The government runs a balanced budget: The public sector wage bill is financed by levying a lump-sum tax  $T$  on all households

$$w^g H^g = T. \quad (4.2)$$

In terms of fiscal instruments available at the government's disposal, the government takes total public sector hours,  $H^g$ , as given, and sets the public sector wage rate,  $w^g$ , as a fixed gross mark-up above the competitive wage rate. In a sense, the government faces a supply curve for labor in the public sector and determines the demand for government employees. Lump-sum taxes will be then residually chosen to guarantee that the budget is balanced.

## 5 Decentralized Competitive Equilibrium

Given the choice of  $T$ , a DCE is defined by allocations  $\{c, h^p, h^g, S\}$ , wage rates  $\{w^p, w^g\}$ , and firm's profit  $\pi$  s.t (i) all households maximize utility; (ii) the private firm maximizes profit; (iii) the government budget constrain is balanced; (iv) all markets clear.

## 6 Characterizing the DCE

Given the restrictions imposed on the production functions in the private and public sector goods, it follows that not everyone will be employed in the private sector in the first stage.

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<sup>5</sup>Such a mark-up is a stylized fact for the major EU economies, e.g. Vasilev (2015b).

<sup>6</sup>The level of government services increases households' utility, hence marginal utility matters.

Therefore, everyone doing the same - working or not working in the first stage - is not an equilibrium.

*Proof:* Case (1): For any positive and finite wage in the private sector, *i.e.*  $0 < w^p < \infty$ , both sectors will want to hire a bit of labor. Hence,  $h^p = 0$  cannot be an equilibrium because firm will have a positive labor demand for any finite wage, and households will have zero consumption,  $c = 0$ , which is ruled out as an optimal choice from the monotonicity of the logarithmic utility.<sup>7</sup>

Case (2):  $h^p = \bar{h}^p$  only if  $w^p = 0$ , which follows from the assumptions on both production technologies. At such wage rates both the firm and government will want to hire everyone, but no household will want to supply any labor. Thus having everyone working is not optimal either. *QED*

Denote the proportion of households employed in the private sector by  $q$ . The rest of the households,  $1 - q$ , will go to the public sector to search for a job. Again, everyone working or not working in the second stage is not an equilibrium outcome. Thus, in the second stage, there will be a proportion  $\lambda$  of those who remained unemployed after stage one, or  $(1 - q)\lambda$  of all households, that will be employed in the public sector. The proportion  $\lambda$  will be chosen optimally to equate the utility of those who will be employed in the public sector, and those who will not.<sup>8</sup> To achieve this, and to guarantee that this sequential choice is time-consistent, we have to solve the game backwards. In this way households who decide not to work in the private sector cannot end up getting less utility. Alternatively, the government knows  $q$  from the first stage, when it chooses  $\lambda(1 - q)$  in the second stage that will equate the utilities of the all the three groups of individuals. In game-theoretic language,  $q$  is a best (Nash) response to  $\lambda$  and  $\lambda$  is a best (Nash) response to  $q$ .

Hence, if there is a DCE, it must be that in equilibrium not everyone will get the same private consumption. Still, everyone consumes the same level of public good, as it is assumed to be non-excludable and non-rivalrous. The households that work will have higher utility of private consumption, while those who do not work will enjoy more utility from

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<sup>7</sup>Note that when nobody works in the private sector,  $\pi = 0$  as well.

<sup>8</sup>This is a sort of rationing scheme, or a lottery. We assume that the government cannot hire all the people who do not work in the private sector.

leisure. Lastly, every household belonging to the same type will enjoy the same level of total utility.

Therefore, we will consider an equilibrium in which  $q$  of the people are employed in the private sector, and  $\lambda$  of the people who go to public sector are employed in the second stage,  $0 < q + (1 - q)\lambda < 1$ . Thus,  $H^p = q\bar{h}^p$ , and  $H^g = \lambda(1 - q)\bar{h}^g$ .

From the firm's optimization problem, we obtain the expression for the competitive hourly wage

$$F'(q\bar{h}^p) = w^p \quad (6.1)$$

Hence, there will be positive economic profits amounting to

$$\pi = \Pi = F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p > 0, \quad (6.2)$$

which follow from the assumption that the production function features decreasing returns to scale. Next, equilibrium government output is

$$S = S(\lambda(1 - q)\bar{h}^g), \quad (6.3)$$

and lump-sum tax revenue equals

$$T = w^g\lambda(1 - q)\bar{h}^g = \gamma w^p\lambda(1 - q)\bar{h}^g = \gamma F'(q\bar{h}^p)\lambda(1 - q)\bar{h}^g. \quad (6.4)$$

Now we will show the existence of a unique pair  $(q, \lambda) \in (0, 1) \times (0, 1)$  by analyzing a system of two non-linear equations. Those equations use the equality of utility of those who work and those who do not in the same sector. Households in the private sector are indifferent between working or not working:

$$\begin{aligned} \ln[(F'(q\bar{h}^p)\bar{h}^p + F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p - T)^\eta + (S(\lambda(1 - q)\bar{h}^g))^\eta]^\frac{1}{\eta} + \alpha \ln(1 - \bar{h}^p) \\ = \ln[(F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p - T)^\eta + (S(\lambda(1 - q)\bar{h}^g))^\eta]^\frac{1}{\eta} + \alpha \ln(1) \end{aligned} \quad (6.5)$$

Similarly, in the second stage, unemployed households are indifferent between working or not in the public sector

$$\begin{aligned} \ln[(w^g\bar{h}^g + F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p - T)^\eta + (S(\lambda(1 - q)\bar{h}^g))^\eta]^\frac{1}{\eta} + \alpha \ln(1 - \bar{h}^g) \\ = \ln[(F(q\bar{h}^p) - F'(q\bar{h}^p)q\bar{h}^p - T)^\eta + (S(\lambda(1 - q)\bar{h}^g))^\eta]^\frac{1}{\eta} + \alpha \ln(1) \end{aligned} \quad (6.6)$$

Substitute out the public sector wage rate with its equivalent expression from the government budget constraint

$$w^g(q) = \gamma w^p(q) = \gamma F'(q\bar{h}^p). \quad (6.7)$$

Then do the same for the lump-sum taxes to obtain

$$T(q, \lambda) = w^g \lambda (1 - q) \bar{h}^g = \gamma F'(q\bar{h}^p) \lambda (1 - q) \bar{h}^g. \quad (6.8)$$

Next, proving existence and uniqueness of optimal  $(q, \lambda) \in (0, 1) \times (0, 1)$  follows trivially from the Brower's Fixed Point and the assumptions on the functional forms of utility and production functions.<sup>9</sup>

Note that there are a lot of equilibria (in terms of the "names" of the people working), all of them with the same fraction of population  $q$  working in the private sector, and  $\lambda(1 - q)$  working in the public sector. Let  $c_w^j, c_n, j = p, g$  denote the private consumption of individuals that work in each sector, and those who do not. Note that those who do not work have not been selected to work in the private sector during the first stage, and then have remained unemployed after the second stage.

Because of the presence of the public good and the non-convexities, the First Welfare Theorem does not hold, so this equilibrium is not PO. Therefore, there exists an alternative allocation that a SP could choose that can make everyone better off. More specifically, the Social Planner (SP) can improve upon the initial equilibrium allocation by giving each household a consumption level independent of the fact whether they worked or not,  $c_w^j = c_n, j = p, g$ . For example, giving everyone  $c = qc_w^p + (1 - q)\lambda c_w^g + (1 - q)(1 - \lambda)c_n$  is Pareto improving, as the new consumption allocation is feasible, and gives a higher utility in expected terms<sup>10</sup>. Hence  $c_w^p = c_w^g = c_n$ , or perfect insurance is achieved. In conclusion, we can do better than the original equilibrium allocation if we allow the Social Planner to randomize allocations, or offer employment lotteries.

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<sup>9</sup>This theorem states that if a function is continuous and monotone in its argument, and crosses the origin only once, then a unique fixed point exists on the domain over which the argument of the function is defined.

<sup>10</sup>This follows from the concavity of the logarithmic utility function, the Constant Elasticity of Substitution (CES) aggregation of total consumption, and the restrictions imposed on the production function.



After equalizing private consumption across states, aggregate utility function becomes

$$U = \ln[c^\eta + s^\eta]^{\frac{1}{\eta}} + q\alpha \ln(1 - \bar{h}^p) + (1 - q)\lambda\alpha \ln(1 - \bar{h}^g)$$

Notice that  $H^p = q\bar{h}^p$ , then  $q = H^p/\bar{h}^p$ . Similarly,  $H^g = \lambda(1 - q)\bar{h}^g$ , then  $\lambda(1 - q) = H^g/\bar{h}^g$ . Substitute those expressions into the utility function to obtain

$$U = \ln[c^\eta + s^\eta]^{\frac{1}{\eta}} + \alpha \ln(1 - \bar{h}^p) \frac{H^p}{\bar{h}^p} + \alpha \ln(1 - \bar{h}^g) \frac{H^g}{\bar{h}^g}$$

Let

$$-\alpha \frac{\ln(1 - \bar{h}^p)}{\bar{h}^p} = A > 0, -\alpha \frac{\ln(1 - \bar{h}^g)}{\bar{h}^g} = B > 0$$

Then

$$U = \ln[c^\eta + s^\eta]^{\frac{1}{\eta}} - AH^p - BH^g$$

As it turns out, the assumption of sequentiality does not change the form of the aggregate utility function obtained in Vasilev (2015a).

## 7 Conclusions

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev (2015a), the sectoral labor choice is made in a sequential manner. Still, the micro-founded representation obtained from explicit aggregation over homogeneous individuals again features different disutility of labor across the two sectors. Thus, there is little merit in the timing of the sectoral non-convex labor choice.

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