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Natural Disasters and Macroeconomic Performance

by

Holger Strulik
and Timo Trimborn
Abstract. Recent empirical research has shown that output and GDP per capita in the aftermath of natural disasters are not necessarily lower than before the event. In many cases, both are not significantly affected and, surprisingly, sometimes they are found to respond positively to natural disasters. Here, we propose a novel economic theory that explains these observations. Specifically, we show that GDP is driven above its pre-shock level when natural disasters destroy predominantly durable consumption goods (cars, furniture, etc.). Disasters destroying mainly productive capital, in contrast, are predicted to reduce GDP. Insignificant responses of GDP can be expected when disasters destroy both, durable goods and productive capital. We extend the model by a residential housing sector and show that disasters may also have an insignificant impact on GDP when they destroy residential houses and durable goods. We show that disasters, irrespective of whether their impact on GDP is positive, negative, or insignificant, entail considerable losses of aggregate welfare.

Keywords: natural disasters; economic recovery; durable goods; residential housing; economic growth.

JEL: E20, O40, Q54, R31.
1. Introduction

In this paper, we propose an economic theory to analyze the macroeconomic effects of natural disasters. We mainly focus on hydro-meteorological disasters (e.g. floods, storms, droughts) and geophysical disasters (e.g. earthquakes, tsunamis), and the physical and monetary damage caused by these disasters. As documented by Cavallo and Noy (2011), these types of disasters are fairly common events across the globe, and occur with increasing frequency. For example, in the Asia-Pacific region, the most afflicted region, the incidence of natural disasters increased from 11 events per country in the 1970s to 28 events in the 2000s. In Western Europe, events per country and decade increased from 5 to 15 over the same time period.

To date, there exists a large and increasing empirical literature investigating the economic impact of natural disasters (e.g. Raddatz, 2007; Noy, 2009; Loyza et al., 2012; Fomby et al., 2013; Cavallo et al., 2013; see Cavallo and Noy, 2011 for a survey). One perhaps surprising conclusion suggested by the literature is that disasters do not necessarily decrease output and GDP per capita in the aftermath of the event. Loayza et al. (2012), for example, find no significant impact on a country’s GDP and industrial output across all disasters. In contrast, when floods are investigated separately, they are found to stimulate GDP while droughts are found to harm GDP in developing countries (but not everywhere). Similarly, Fomby et al. (2013) find a positive effect of floods and a negative effect of droughts but no effect of earthquakes and storms on post-disaster GDP in developing countries. In developed countries, all types of disasters appear to exert no significant impact on GDP. Using counterfactual analysis, Cavallo et al. (2013) find that disasters exert no significant influence on short- and long-run output when they control for potential post-disaster outbreak of social conflict.

These empirical observations seem puzzling when analyzed within the context of conventional neoclassical growth theory. Once we acknowledge that disasters destroy (potentially severely) the productive potential of an economy, we would expect that they harm subsequent economic performance. It is true that the neoclassical growth model predicts that the growth rate after an exogenous loss of capital stock (or other productive factors) is positive. This phenomenon is known as catch-up growth from below towards the steady-state. GDP per capita, however, is predicted to fall short of its pre-disaster level according to conventional growth theory.¹

¹The disaster literature, confusingly for growth economists, refers sometimes to the differential between pre- and post-shock levels of GDP as GDP growth (e.g. Loayza et al., 2012). This differential is predicted by the standard neoclassical growth model to be unambiguously negative. Moreover there exists also a smaller literature investigating...
In this paper, we show how a simple extension of the neoclassical growth model can be used to reconcile theory with empirical evidence and how the model can be used to motivate the diverse post-shock macroeconomic outcomes found in the disaster literature. The key ingredients are the introduction of variable labor supply and the distinction between productive capital stock and durable goods (like cars, furniture, or household appliances). In line with conventional theory, the model predicts that an exogenous loss of capital stock reduces post-disaster GDP. An exogenous loss of durable goods, in contrast, drives GDP above its pre-shock level. The reason is that individuals, suffering from the implied negative wealth shock, supply more labor. On the other hand, households want to (quickly) rebuild the stock of durable goods. The higher level of durable goods investment during the reconstruction phase also lifts labor demand. Higher employment in conjunction with an undestroyed capital stock implies higher output per capita during the reconstruction phase.

Considering simultaneous shocks on both state variables, the model predicts a negative impact on GDP for disasters destroying predominantly productive capital and a positive impact on GDP for disasters destroying predominantly durable goods. No significant effect on GDP is predicted when the effects on firm capital is somewhat smaller compared to that on durable goods. The theory is not only helpful to explain the frequently insignificant impact of disasters on GDP, it can also rationalize those cases for which the literature finds significant GDP effects. Intuitively, we may expect that droughts exert a negative impact on GDP because they leave durable goods mostly intact and destroy predominantly firm capital. This is in particularly the case in largely agrarian societies (where capital stock consists of seeds, livestock etc.). Conversely, we could imagine that floods stimulate GDP because they damage predominantly durable goods like cars, furniture or household appliances. Furthermore, some authors find positive employment effects of disasters supporting our proposed mechanism through which output might increase in the aftermath of a disaster (see Leiter et al., 2009, Ewing et al., 2009).

In order to present the mechanics behind the “durable goods–channel” in the cleanest way we first discuss in Section 3 the case of a small open economy with perfect capital mobility. This shuts down the “capital-channel” because the capital stock is pinned down to its steady-state value. In this framework, we generally prove that disasters damaging durable goods lead to higher GDP in the short and long-run. The reason for the positive effect on GDP is that disasters reduce the association between disaster risk and long-run growth (e.g. Skidmore and Toya, 2002; and Crespo Cuaresma et al., 2008). Here, we focus on the short- to medium-run impact of disasters.
the stock of durables, which motivates households to raise durable goods investment in order to quickly reconstruct the stock of durable goods. This triggers higher labor demand and raises output and GDP.

We then turn to the large economy case and show that the main mechanics of the wealth effect are preserved while another amplifying effect occurs through intertemporal substitution. Households want to reconstruct their destructed stock of durable goods quickly and increase their durable goods purchases in the aftermath of the disaster. Since international capital markets are no longer available to smooth consumption, resources for this purpose are freed by reducing investments in productive capital and by reducing consumption of nondurable goods. In order to mitigate the drop in consumption, households are motivated to raise their labor supply even further, beyond what has already been triggered by the wealth effect.

Because disasters may not only destroy physical capital and durable goods but also houses, we extend the benchmark model by residential housing and investigate if our results also hold in the extended model. We show that the main mechanisms yielding a negative, insignificant or positive impact of the disaster on output are still at work in the extended model. Furthermore, the model offers an additional channel through which a disaster might leave GDP unaffected: When a disaster leaves physical capital intact but destroys durable goods and residential housing, there exist a positive impact on GDP through higher labor supply, and a negative impact on GDP through a lower service flow from housing contributing to GDP. When both effects cancel out, GDP is predicted to remain unaffected in the aftermath of a disaster.

Our analysis of the individual effects of disasters on firm capital, durable goods, and residential housing shows that GDP can be a very misleading indicator of the economic damage caused by natural disasters. This is most obvious when we compare a disaster that destroys “only” productive capital with another one destroying additionally durable goods (and residential housing). The GDP damage is larger for the first one while the welfare loss is larger for the second one. At the end of the paper, we perform a welfare analysis for a numerically specified version of the model and find large welfare losses from natural disasters that leave GDP more or less unaffected.

There exist a small theoretical literature assessing the general economic impact of disasters (West and Lenze, 1994; Rose and Liao, 2005; Henriet et al., 2012). These studies focus mainly on a sectoral decomposition of the impact of disasters and do not try to explain the (potentially non-negative) impact of disasters on aggregate GDP. Crespo Cuaresma et al. (2008) speculate
that disasters may exert a positive impact on GDP through creative destruction. The proposed mechanism works through opportunities to update the partially destroyed capital stock. However, scrutinizing the mechanism in a theoretical model, Hallegatte and Dumas (2009) found that it cannot account for a positive response of GDP on disasters.

Another strand of related literature discusses how mitigation policies affect the economic consequences of disasters (see e.g. Kellenberg and Mobarak, 2008, Hallegatte, 2013, and Posch and Trimborn, 2013). Ex ante mitigation in response to disaster risk exposure is certainly an important aspect for a proper quantitative assessment of the impact of disasters on GDP. In the context of our model, however, it is important to emphasize that taking ex ante mitigation into account would not change any of our qualitative results. Mitigation would reduce the amount of capital and durable goods that get destroyed by a disaster but it would have no effect on the relation between the type of destroyed assets and GDP growth in the aftermath of the disaster, which is the focus of our study.

The paper is organized as follows. The next section introduces the general model. Section 3 presents the case of a small open economy and Section 4 presents the closed economy case. In Section 5, we specify the model numerically and investigate post-disaster adjustment dynamics quantitatively. We provide estimates of the incurred welfare loss under varying assumptions about the physical impact of disasters. Throughout the paper we focus on economic or “material” effects and ignore the fact that natural disasters kill people. The welfare estimates should thus be understood as lower bounds of the actual damage caused by natural disasters. Section 6 presents an extension of the benchmark model by a residential housing sector. In the main text, we assume that households own their houses. In the Appendix, we show robustness of results against an alternative setup in which households rent housing services from firms.

2. The Model

2.1. Households. The economy is populated by a continuum (0, 1) of households who take prices as given, supply \( \ell \) units of labor, and experience utility from consuming nondurable goods \( c \) and durable goods \( d \) as well as from enjoying leisure \( (1 - \ell) \). In order to keep the utility function general, we abstain from introducing exogenous technological growth. As well-known from the business cycle literature, introducing trend growth would entail severe restrictions on the functional form of the utility function in order to guarantee that leisure is stationary. Our variables (besides
leisure) could be interpreted as being measured in terms of deviation from trend growth. In order to derive theoretical results, we have to assume that utility is additively separable between nondurable consumption, durable consumption, and leisure. In the quantitative part of the paper, we also investigate non-separable utility and demonstrate robustness of the main results.

Households maximize lifetime utility

$$V = \int_0^\infty (u(c) + v(d) + q(1-\ell)) \cdot e^{-\rho t} dt,$$

where \(u, v,\) and \(q\) denote strictly concave sub-utility functions satisfying the Inada conditions and \(\rho\) is the time preference rate. To simplify the notation, we introduce \(\sigma_c, \sigma_d,\) and \(\sigma_\ell\) as the elasticity of subutility with respect to nondurable goods consumption, durable goods consumption, and leisure, respectively:

$$
\begin{align*}
\sigma_c(c) & := -\frac{c u''(c)}{u'(c)} \\
\sigma_d(d) & := -\frac{d v''(d)}{v'(d)} \\
\sigma_\ell(1-\ell) & := -\frac{(1-\ell) q''(1-\ell)}{q'(1-\ell)}.
\end{align*}
$$

Household income is spent on nondurable consumption goods \(c\) and on durable goods investment \(x\). Households earn a wage \(w\) per unit of labor supplied and hold assets \(a\), on which they earn a return \(r\), which altogether implies that they face the budget constraint

$$\dot{a} = w\ell + ra - c - px,$$

in which \(p\) denotes the price of durable goods purchases. We assume that a no-Ponzi-game condition for assets holds.

Durable goods depreciate at rate \(\delta_d\); hence, the stock of durables evolves according to

$$\dot{d} = x - \delta_d d,$$

with \(x \geq 0\). Households choose \(c, \ell,\) and \(x\) to maximize (1) subject to (3) and (4), and the initial conditions \(a(0) = a_0\) and \(d(0) = d_0\). The first order conditions are

$$
\begin{align*}
u'(c) & = \lambda \\
\mu & = p\lambda \\
\lambda w & = q'(1-\ell)
\end{align*}
$$

Iacoviello (2005) argues that separability between nondurable and durable goods consumption is supported by empirical evidence; see also Bernanke (1984).
where \( \lambda \) denotes the shadow price of one unit of financial assets and \( \mu \) denotes the shadow price of one unit of durable goods. From the first order conditions we derive the Euler equation for consumption and a relation equating the wage rate with the marginal rate of substitution between consumption of nondurables and leisure:

\[
\dot{\lambda} = \lambda \rho - \lambda r \tag{8}
\]
\[
\dot{\mu} = \mu \rho - v'(d) + \mu \delta_d \tag{9}
\]
\[
\lim_{t \to \infty} e^{-\rho t} \lambda a = \lim_{t \to \infty} e^{-\rho t} \mu d, \tag{10}
\]

2.2. Durable goods producing firms. There exists a continuum \((0, 1)\) of firms producing durable investment goods. These firms convert units of final goods into units of durable goods. Following the DSGE literature, we assume that firms face convex adjustment costs depending on the amount of durables they produce per unit of time. These costs can be understood as, for example, arising in terms of additional planning costs when sequential tasks have to be performed in a tight time frame, or when there is inefficient labor input due to fatigue during overtime hours. Since such costs arise when investment and thus the workload is especially high, they explain why marginal costs are increasing in investment per unit of time.

The empirical literature on capital and investment adjustment costs has identified costs arising at the plant level if firms adjust the capital stock or investment (see e.g. Cooper and Haltiwanger, 2006, for a recent study). Similar costs are likely to emerge for firms in the durable goods producing industry. In order to keep the analysis general, we assume that the total costs for producing \( x \) units of durables sum up to \( x + \psi(x) \) with a strictly convex function \( \psi \) satisfying \( \psi(0) = 0 \) and \( \psi'(0) = 0 \). The literature on capital adjustment costs usually assumes that adjustment costs additionally depend on the installed stock. For reasons of tractability we assume that costs depend only on investment. Our quantitative results are robust against alternative specifications with reasonable parametrization of adjustment costs.

Total firm revenue equals \( px \). Each household engages one construction firm per unit of time and may change the contracting party at any point of time. Free entry into the construction sector
implies that firms sell $x$ at unit costs:

$$p = 1 + \frac{\psi(x)}{x}.$$  \hfill (13)

Differentiating (13) with respect to time and using (5) – (9), we obtain the law of motion for durable goods investment as

$$\frac{\dot{x}}{x} = \left(\psi'(x) - \frac{\psi(x)}{x}\right)^{-1} \left[p(r + \delta_d) - \frac{\psi'(d)}{w'(c)}\right].$$ \hfill (14)

2.3. Final goods producing firms. The economy is populated by a continuum (0, 1) of firms producing final goods. Final goods are used as nondurable consumption goods, for durable goods investment, and for investment in firm capital. Each firm employs capital $k$ and labor $\ell$ to produce final output $y = Af(k, \ell)$, in which $A$ denotes total factor productivity and $f(k, \ell)$ is a neoclassical production function with positive and diminishing marginal returns. Firm capital depreciates at rate $\delta_k$. Firms have to pay adjustment costs to install capital, but we allow for the corner case of zero adjustment costs. We assume that adjustment costs $\phi(i/k)$ are convex with respect to the investment-capital ratio and we normalize adjustment costs and marginal adjustment costs to zero at the steady state. This means that adjustment costs $\phi(i/k)$ satisfy

$$\phi(\delta_k) = 0, \quad \phi'(\delta_k) = 0,$$ \hfill (15)

and, if $\phi \neq 0$,

$$\phi'(\cdot) > 0, \quad 2\phi'(\cdot) + \frac{i}{k\phi''(\cdot)} > 0.$$ \hfill (16)

Firms choose investment, $i$, and employment, $\ell$ to maximize

$$\int_0^{\infty} \left[Af(k, \ell) - w\ell - i - i\phi\left(\frac{i}{k}\right)\right] e^{-\int_s^t r(s) ds} dt$$ \hfill (17)

subject to

$$\dot{k} = i - \delta_k k.$$ \hfill (18)

First order conditions are

$$w = \frac{\partial Af(k, \ell)}{\partial \ell}$$ \hfill (19)
\[ q = 1 + \phi \left( \frac{i}{k} \right) + \left( \frac{i}{k} \right) \phi' \left( \frac{i}{k} \right) \]  

(20)

\[ \dot{q} = (r + \delta_k)q - \frac{\partial A f(k, \ell)}{\partial k} - \left( \frac{i}{k} \right)^2 \phi' \left( \frac{i}{k} \right), \]  

(21)

with multiplier \( q \). If \( \phi \equiv 0 \) these equation collapse to

\[ w = \frac{\partial A f(k, \ell)}{\partial \ell}, \]  

(22)

\[ r = \frac{\partial A f(k, \ell)}{\partial k} - \delta_k. \]  

(23)

3. The Small Open Economy

In a small open economy, firm capital adjusts via international capital movements and is independent from the savings decision of domestic households. We thus focus the disaster analysis of this section on the effects originating from destruction of durable goods. For this purpose, we assume that the economy rests at a steady-state before it is hit by a natural disaster. Furthermore, in order to focus on the dynamics of durables in the aftermath of a disaster, we assume that capital adjustment cost are zero, i.e. \( \phi \equiv 0 \).

Because of perfect capital mobility and zero adjustment costs firms can borrow at the world interest rate \( \bar{r} \), \( r = \bar{r} \). This means that the world interest rate pins down the domestic capital labor ratio (with equation (23)) and further the domestic wage rate to \( w = \bar{w} \) (with equation (22)). In order to simplify the formal analysis, we assume that \( \bar{r} = \rho \) such that households prefer a constant time profile of consumption and labor supply, \( \dot{c} = \dot{\ell} = 0 \) (from equations (11) and (12)).

The demand for nondurable consumption goods and durable goods purchases, as well as household labor supply is determined by the intertemporal budget constraint. This means that any shock or new information affecting the intertemporal budget constraint also affects the optimal level of \( c \) and \( \ell \). The implied dynamics of durable goods investment is then given by equations (4) and (14).

To analyze the households’ intertemporal budget constraint we have to specify domestic households’ asset composition. We assume that households own the entire domestic capital stock such that

\[ a = k + b, \]  

(24)
with $b$ denoting the net international investment position, i.e. the net asset position vis a vis the rest of the world. When domestic firms adjust their capital stock, domestic households, ceteris paribus, do not change their level of assets $a$ but restructure their portfolio such that they supply the demanded stock of capital and adjust $b$ residually.

In order to elaborate how a disaster destroying durable goods affects GDP, we begin by showing that the destruction of durables entails a negative wealth effect. Households respond to the wealth shock by consuming fewer nondurables and durables and by supplying more labor. Higher labor supply then lifts GDP above the pre-shock level.

Integrating equation (3) provides the households’ intertemporal budget constraint,

$$\int_0^\infty c e^{-\bar{r}t} dt = \ell \int_0^\infty \tilde{w} e^{-\bar{r}t} dt - \int_0^\infty p(x) x e^{-\bar{r}t} dt + a_0. \quad (25)$$

Using the fact that $c$ and $\ell$ are constant, the budget constraint simplifies to

$$\frac{c}{\bar{r}} = \ell \frac{\tilde{w}}{\bar{r}} - \int_0^\infty p(x) x e^{-\bar{r}t} dt + a_0. \quad (26)$$

Finally, substituting $w = u'(1 - \ell)/u'(c)$, we obtain

$$\frac{(u')^{-1}}{\tilde{r}} \left( \frac{d(1-\ell)}{w} \right) - \frac{\ell \tilde{w}}{\bar{r}} = a_0 - \int_0^\infty p(x) x e^{-\bar{r}t} dt. \quad (27)$$

Notice that the left-hand side of (27) depends negatively on $\ell$ since $d (u'(\cdot))^{-1} / d(\cdot) < 0$. This means that households supply more labor when they experience a negative wealth effect such that the right-hand side of (27) decreases.

We next show that an economy initially situated at a steady-state experiences indeed a negative wealth effect when it is exposed to a disaster that destroys part of $d$. For that purpose we define the net present value of durable goods investments as

$$X := \int_0^\infty p(x) x e^{-\bar{r}t} dt = \int_0^\infty x \left( 1 + \psi(x) \right) e^{-\bar{r}t} dt. \quad (28)$$

Intuitively, a disaster that destroys parts of $d$, raises $X$, because rebuilding the stock of $d$ requires higher temporary investments $x$, which raises the net present value of future investments. In order to verify this claim we focus on the dynamics of $d$ and $x$ summarized by

$$\dot{d} = x - \delta_d d \quad (29)$$
\[
\frac{\dot{x}}{x} = \left(\psi'(x) - \frac{\psi(x)}{x}\right)^{-1} \left[\left(1 + \frac{\psi(x)}{x}\right)(r + \delta_d) - \frac{v'(d)}{u'(c)}\right]
\]  
(30)

and \(d(0) = d_0\). The steady-state of the subsystem (29) and (30) is given by \(x = \delta_d d\) and 
\[(1 + \psi(x)/x)(\bar{r} + \delta_d) = v'(d)/u'(c).\]

Notice that the steady-state of the subsystem depends on \(c\) and that \(c\) is determined in conjunction with \(x\) by the intertemporal budget constraint (27) and the labor supply equation (12). This means that the steady-state itself depends on the evolution of the dynamic system towards the steady-state. In other words, the steady-state of the dynamic subsystem (29) and (30) depends on the initial situation \((c_0, x_0)\).\(^3\)

In order to demonstrate that adjustment dynamics towards the steady-state are unique, we construct a phase diagram, taking \(c\) as given and keeping in mind that \(c\) depends on \(X\) and therefore on the adjustment path of \(x\). The panel on the left-hand side of Figure 1 shows the phase diagram with adjustment dynamics towards the steady-state. The phase diagram is constructed by first noting that the \(\dot{d} = 0\) isocline is a ray with slope \(\delta_d\) starting at the origin (from \(x^* = \delta_d d^*\)). Above the line, \(\dot{d}\) is positive; and below the line, \(\dot{d}\) is negative. The \(\dot{x} = 0\) isocline is downward sloping and intersects the \(d\) axis at 
\[d = (v')^{-1}(u'(c)(\bar{r} + \delta_d)).\]

On the right-hand side of the line, \(\dot{x}\) is positive; and on the left hand side, \(\dot{x}\) is negative. The slopes of the two isoclines have opposite signs, implying that the isoclines intersect exactly once. In conclusion, the steady-state is saddlepath-stable. A higher value of \(c\) (lower \(X\)) shifts the \(\dot{x} = 0\) isocline upwards; hence, the steady-state value of \(d\) increases. The stable saddlepath towards the steady-state is downward sloping. In Appendix A we formally derive that a unique and saddle-point stable steady-state exists for subsystem (29) and (30).

In comparison with conventional growth theory, the usual argument for uniqueness of adjustment dynamics is slightly modified. To see this, consider an economy starting somewhere below the stable saddlepath. Following the arrows of motion, the economy would always remain below the saddlepath such that aggregate \(X\) would also be lower. Then, from the intertemporal budget constraint (27), consumption \(c\) must be higher. This in turn means that the \(\dot{x} = 0\) isocline, and thus the steady-state, shifts upwards. An economy starting below the stable saddlepath would thus never arrive at the steady-state because \(x\) is below the stable saddlepath everywhere during the transition and, secondly, because this very phenomenon shifts the steady-state even

\(^3\)A similar type of phase diagram for the analysis of dynamic subsystems has been popularized in growth economics by Galor and Weil (2000).
further upwards. Analogously an economy starting above the saddlepath would never reach the steady-state. This reasoning can be exploited to arrive at the following conclusion.

**Lemma 1.** If an economy rests at a steady-state and durable goods \(d\) gets destroyed, aggregate expenditure on durable goods \(X\) increases compared to the pre-shock steady-state level.

**Proof.** The lemma is proved by contradiction. The counterfactual phase diagram is shown in the panel on the right-hand side of Figure 1. Assume that the destruction of durables \(d\) reduces aggregate durables expenditure \(X\). In this case, equations (27) and (12) show that \(c\) would increase; thus the \(\dot{x} = 0\) isocline would shift upwards. This would increase the steady-state value of \(d\). However, along the adjustment path, \(x\) is strictly larger than its former steady-state because the stable saddlepath is downward sloping, \(x(t) > x_{old}^*\), implying increasing aggregate expenditure \(X\). In other words, reducing \(X\) in response to lower \(d\) would lead to a contradiction. If \(X\) would remain constant, this would lead to a contradiction in an analogous way. In conclusion, after a destruction of \(d\), \(X\) rises compared to its original steady-state level. \(\square\)

In short, the actual adjustment dynamics triggered by a natural disaster destroying \(d\) are shown in Figure 2. Notice that along the adjustment path \(x\) is higher compared to the old steady-state

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The local uniqueness of the saddlepath can also be proven by analyzing the full dynamic system of \(c\), \(x\), \(d\), and \(a\). It can be shown that the Jacobian matrix exhibits two positive, one negative, and one zero Eigenvalue. This indicates that the stable saddlepath is unique and that the steady-state to which the economy converges depends on the initial conditions \((a(0) = a_0\) and \(d(0) = d_0\)). We thank Franz X. Hof for this proof. The proof is available upon request.
for an initial period $[0, T]$. During the time interval $(T, \infty) \ x$ is smaller than $x_{old}^{*}$. Yet due to discounting of future expenditures and higher adjustment costs in the initial periods, $X$ increases in net terms. This leads to the following result.

**Figure 2: Phase Diagram: Adjustment Dynamics After a Disaster**

![Phase Diagram](image)

**Proposition 1.** If an economy rests at a steady state and durable goods ($d$) get destroyed, only parts of the stock of $d$ are rebuilt. The resulting new steady-state level of $d$ is lower compared to pre-shock level.

**Proof.** Inspecting the adjustment dynamics derived in Figure 2 confirms that the after-shock steady-state level of $d$ lies below the pre-shock steady-state level. □

The result implies that without trend growth, an economy never completely rebuilds the stock of durables destroyed by a disaster. If there is trend growth, $d$ would grow at the steady-state and the post-disaster growth rate of $d$ would be higher than the pre-shock rate. However, the level of $d$ would be lower compared to an economy not experiencing the disaster and growing along the balanced growth path.

The reason for the permanently lower level of durable goods is the negative wealth effect. Households response to the diminished wealth by reducing their consumption of nondurable goods, consumption of durable goods, and leisure. This leads to our next result.

**Proposition 2.** Aggregate welfare falls below its pre-shock level if durable goods gets destroyed in an economy resting at a steady-state.
Proof. According to Proposition 1, the level of durable goods after the disaster falls permanently below the pre-shock level. From Lemma 1, we conclude that consumption of nondurable goods is also permanently lower. Finally, we conclude from equation (27) that labor supply increases. Since instantaneous utility from all three components is strictly below pre-shock level, the lifetime utility of households, i.e. aggregate welfare, is below the pre-shock level as well.

Finally, we show that, although aggregate welfare is affected negatively, output and output growth increase in response to the disaster, and we derive conditions under which GDP also responds positively.

**Proposition 3.** If durable goods gets destroyed in an economy resting at a steady state, then aggregate output per capita increases permanently above the pre-shock level. Hence, output growth in the aftermath of the disaster is positive.

Proof. If $d$ gets destroyed, consumption of nondurable goods decreases (see equation (12)) and labor supply increases (see equation (27)). Higher labor supply triggers capital inflows from the rest of the world until the capital labor ratio reaches its pre-shock level and the domestic interest rate equals the world interest rate. Higher labor supply in conjunction with higher capital stock implies higher aggregate output per capita.

Finally, note that in an open economy model GDP differs from GNP by factor income received from abroad. This means that

$$ GNP = GDP + rb $$

holds.

To assess the response of GNP in the aftermath of a disaster recall that GDP increases. On the other hand $b$ deteriorates for two reasons. First, the capital stock of firms increases as a response to higher labor supply. Households restructure their portfolio to serve the higher domestic capital demand by reducing $b$. Second, expenditure on durable goods is higher in the initial years after the disaster compared to expenditure at the steady state. This means that the savings rate is negative in the aftermath of a disaster, i.e. $a$, and thus $b$, declines in the years following a disaster.

While GDP increases in the aftermath of a disaster and $b$ declines, the response of GNP is undetermined and depends on households’ preferences. We illustrate this fact by focussing on two extreme cases. The first case assumes inelastic labor supply. This assumption closes down
the labor supply channel of disasters on GDP. However, households still raise their investment in durable goods in the aftermath of the disaster and, hence, \( b \) declines. The impact of a disaster on GNP would thus be still negative.

The second extreme case assumes infinitely elastic labor supply. In this case nondurable consumption would not decline as a response to the disaster because households raise their labor supply so much that the negative wealth effect is fully compensated by higher labor income (see Strulik and Trimborn, 2016, for details). This implies that the stock of destroyed durables is completely rebuild in the long run. Evaluating equation (26) shows that the increase in net present value terms for labor income is equal to the decline in net present value terms of durable investment, i.e. \( \Delta \ell \cdot \bar{w}/\bar{r} = \Delta X \). We derive the impact on households’ assets at the new steady state by evaluating equation (26) again, now at the new steady state. There, \( \bar{c} \) and \( X \) are equal to their old steady state values, whereas labor income is higher. Assets \( a \), at the new steady state, are determined residually, i.e. they have to be lower exactly by excess labor income in order to balance the equation. This means that at the new steady state households’ assets are reduced by the net present value of reconstruction costs.

Comparing the old and the new steady state, we compute the change in the net foreign asset position as

\[
\Delta b = \Delta a - \Delta k = -\Delta X - \Delta k = -\Delta \ell \cdot \bar{w}/\bar{r} - \Delta k
\]

Using this result, we can compute the response of GNP at the new steady state as

\[
\Delta GNP^* = \Delta \ell w + \Delta kr + \bar{r} \Delta b = \Delta \ell w + \Delta kr - \Delta \ell \cdot \bar{w} - \bar{r} \Delta k = 0.
\]

To assess the response of GNP on impact recall that the decline in \( b \) fades in over time. Hence, GNP is higher on impact, compared to the new steady state, then converges towards the old steady state as time proceeds.

4. The Closed Economy Case

4.1. Setup of the Model. The closed economy case can be best understood as a standard neoclassical growth model augmented by a durable goods producing sector and variable labor
supply. In a closed economy, households save only in terms of domestic capital such that \( a = k \). The feature of diminishing marginal returns to capital provides a unique steady-state.

The evolution of the economy is described by the household’s first order condition, the durable good producing firms’ first order conditions, and the final good producing firm’s first order condition. In addition, equilibrium on the goods market requires that output is used up for nondurable consumption, physical capital investment, durable goods investment, depreciation, and adjustment costs for capital and durable goods investment. For convenience, we summarize below the equations describing the closed economy:

\[
\dot{k} = i - \delta_k k \tag{34}
\]

\[
\dot{d} = x - \delta_d d \tag{35}
\]

\[
\dot{c} = \frac{r - \rho}{\sigma_c} \tag{36}
\]

\[
\dot{x} = \left( \psi'(x) - \frac{\psi(x)}{x} \right)^{-1} \left[ p(r + \delta_d) - \frac{\psi'(d)}{w'(c)} \right] \tag{37}
\]

\[
\dot{q} = (r + \delta_k)q - \frac{\partial Af(k, \ell)}{\partial k} - \left( \frac{i}{k} \right)^2 \phi' \left( \frac{i}{k} \right) \tag{38}
\]

\[
q = 1 + \phi \left( \frac{i}{k} \right) + \left( \frac{i}{k} \right) \phi' \left( \frac{i}{k} \right) \tag{39}
\]

\[
Af(k, \ell) = \delta_k k + x + \psi(x) + c + i + i\phi \left( \frac{i}{k} \right) \tag{40}
\]

\[
\frac{\partial Af(k, \ell)}{\partial \ell} = \frac{q'(1 - \ell)}{w'(c)} \tag{41}
\]

4.2. Effect of Disasters on GDP: Intuition. To illustrate our argument, we distinguish between two polar types of disasters. The first type destroys only durable goods while the second type destroys only productive capital. In reality, of course, disasters usually destroy both \( d \) and \( k \). Real disasters can be conceptualized as a mix of the two polar cases. In the quantitative section, we first discuss results for the two polar types of disasters and then investigate the impact of “mixed disasters”.

\[\text{In the Appendix, we show that the dynamic system (34) to (41) has a unique steady-state. We also checked by numerically evaluating the eigenvalues that adjustment dynamics are unique for the relevant range of parameter values.}\]
We begin the analysis again by inspecting the intertemporal budget constraint. Integrating (3) and inserting \( a_0 = k_0 \), we obtain

\[
\int_0^\infty ce^{-\int_0^s r(u)du}ds = k_0 + \int_0^\infty \omega e^{-\int_0^s r(u)du}ds - \int_0^\infty p(x)e^{-\int_0^s r(u)du}ds.
\]

The intertemporal budget constraint differs from the open economy case (equation 27) mainly because the wage rate \( w \) and the interest rate \( r \) are now varying over time. A damaged stock of productive capital entails temporarily lower wages and higher interest rates and these changing factor prices impinge on household wealth. As demonstrated below, these "factor price effects" are quantitatively of second order compared to the wealth effect originating from the loss of productive capital or durables.

We start by analyzing a \( d \)-shock. As for the small open economy, destroyed durables entail reconstruction costs such that the present value of aggregate durable good investment \( \int_0^\infty p(x)e^{-\int_0^u r(u)du}du \) increases. This, in turn, implies lower household wealth. Households respond to the reduced wealth by lowering consumption of nondurable goods and by supplying more labor. In the quantitative section below we show that on top of the wealth effect there exists an intertemporal substitution effect that amplifies the response of employment. Since international capital markets are no longer available to smooth consumption, households are motivated to raise labor supply even further in the initial years after the disaster in order to mitigate the drop in consumption. At the same time, high reconstruction costs also raise labor demand. Productive capital, by assumption, was not affected by the disaster, which means that higher employment lifts output above its pre-shock level.

The impact of a disaster destroying productive capital can be investigated analogously. The \( k \)-shock reduces \( k_0 \) on the right-hand side of equation (42) and thus entails a negative wealth effect as well. Households respond by consuming less nondurable goods, by reducing durable goods investment, and by supplying more labor. The reason for this joint response lies in the fact that nondurables, durables, and leisure are normal goods such that the demand for all three components shifts in the same direction after a wealth shock. In contrast to the \( d \)-shock, higher labor supply is not sufficient to lift output above the steady-state level. For reasonable parameter values, the negative impact of the lower level of capital stock after the shock is the dominant force on output. This means that we observe conventional "neoclassical" adjustment dynamics: the economy starts at a lower level of output and converges towards the steady state from below.
5. Quantitative Analysis

5.1. Numerical Specification of the Model. In order to parameterize the model, we need to specify the form of the utility function and the production technology. We assume that households face an isoelastic utility function

\[
\int_0^\infty \left[ \frac{c^{1-\sigma_c} - 1}{1 - \sigma_c} + \frac{\beta d^{1-\sigma_d} - 1}{1 - \sigma_d} + \eta \frac{(1 - \ell)^{1-\sigma_\ell} - 1}{1 - \sigma_\ell} \right] \cdot e^{-\rho t} dt
\]

in which \( \beta \) and \( \eta \) denote the weights of durable goods and leisure, respectively. In Section 5.3, we relax the assumption of separability between nondurable and durable goods. Firms are assumed to produce according to the Cobb-Douglas production technology \( y = A k^\alpha \ell^{1-\alpha} \), in which \( \alpha \) denotes the elasticity of output with respect to capital. Adjustment costs for durable goods investment and capital investment are quadratic, as it is often assumed in the DSGE literature (see e.g. Christiano et al., 2011), i.e. \( \psi(x) = \gamma x^2 \) and \( \phi(i/k) = \kappa (i/k - \delta k)^2 \). We normalize the cost parameters \( \gamma \) and \( \kappa \) to unity.

For the benchmark case we take the values of \( \alpha, r^*, \ell^* \), and the Frisch elasticity of labor supply from our calibration of the neoclassical growth model for the U.S. economy (Strulik and Trimborn, 2012). For depreciation of physical capital we take the average depreciation rates measured for the U.S. between 1948 and 2008 (Davis and Heathcote, 2005). Parameter \( \eta \) is set in order to match steady-state labor supply \( \ell^* \), and \( \rho \) is set in order to fit the steady-state interest rate to \( r^* \). For given \( \ell^* = 0.25 \) and a given Frisch elasticity of unity, we obtain \( \sigma_\ell = 3 \). The inverse of the intertemporal elasticity of substitution for consumption of durables and nondurables, respectively, is set to 2, based on Ogaki and Reinhart (1998).

We set \( \beta \) in order to match the expenditure ratio of nondurable to durable goods, according to the US Consumption Expenditure Survey of 2014 (Bureau of Labor Statistics, 2014). For durable goods we have only estimates of depreciation rates when they are treated as investment goods (see House and Shapiro, 2008). These estimates range from 0.3 for communication equipment, software, computers and peripheral equipment, to 0.165 for cars, and to 0.06 for boats. We take a weighted average of the depreciation rate for all items, which provides the estimate \( \delta_d = 0.18 \).

The numerical specification of the benchmark model is summarized in Table 1. A sensitivity analysis with respect to the most decisive parameters is provided below.
Table 1. Parameter Values: Benchmark Model

<table>
<thead>
<tr>
<th>A</th>
<th>α</th>
<th>δ_k</th>
<th>δ_d</th>
<th>r*</th>
<th>(ρ)</th>
<th>γ</th>
<th>κ</th>
<th>ℓ*</th>
<th>(η)</th>
<th>Frisch</th>
<th>(σ_d)</th>
<th>σ_c</th>
<th>σ_d</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.38</td>
<td>0.058</td>
<td>0.18</td>
<td>0.038</td>
<td>0.038</td>
<td>1</td>
<td>1</td>
<td>0.25</td>
<td>10.7</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4.39</td>
</tr>
</tbody>
</table>

Notation in parenthesis indicates implied values.

5.2. **Numerical Experiment.** Quantitative results are calculated by employing the relaxation algorithm (Trimborn et al., 2008). The solution method calculates adjustment dynamics of non-linear models and provides the exact solution, up to a user-specified error. It is thus a suitable tool to investigate natural disasters, i.e. big shocks that drive the economy far away from the steady-state. We employ a recently developed numerical method to ensure that non-negativity constraints on capital investment and durable goods investment hold during the adjustment process (Trimborn, 2013).

Because we already showed analytically that disasters destroying durable goods exert an unambiguously positive effect on output in the small open economy case, we focus on the closed economy case and investigate three different types of disasters: a $d$-disaster destroying only durable goods, a $k$-disaster destroying only productive capital, and a "mixed disaster" destroying parts of both stocks. We assume that the economy rests at the steady state and that the disaster comes unanticipated. We normalize the size of disasters such that the value of assets destroyed is equal in all three cases. Because the capital stock at the steady state is about twice as large as the value of durables this implies that a shock destroys either 10% of the capital stock or 20% of the stock of durables, or – in case of a mixed disaster – 3% and 15% of capital and durables, respectively.

Figure 3 shows the impulse responses following a destruction of $d$ by 20% of the steady-state level. As a reaction to the disaster, households increase durable goods purchases above the steady-state level in order to rebuild the damaged stock of durables. The resulting negative wealth effect causes households to reduce nondurable consumption and to increase labor supply on impact. Higher labor supply in the aftermath of the disaster lifts output above steady-state level. Later, when the economy recovers and the damaged stock of durables is rebuilt, economic aggregates return to their pre-shock steady-state levels.

There exists, on top of the wealth effect, an intertemporal substitution effect, which amplifies the positive response of labor supply and output. Due to their damaged durables, households experience a high marginal utility from durable goods consumption, which induces them to rebuild their stock of durables quickly and to incur high costs of durable goods investments. Since
households cannot borrow on international capital markets, resources are scarce in the aftermath of a disaster. In order to free resources, households reduce capital investment and consumption. Then, in order to mitigate the drop in consumption, households further increase their labor supply. As a consequence, output in the initial periods after the disaster rises even further, beyond the increase triggered by the wealth effect.

As a side effect, lower investment in productive capital reduces the capital stock. Only after about three years – when about two thirds of the durable goods have been reconstructed – investment in capital is higher than depreciation and the capital stock returns to its steady-state level from below.

In our numerical simulations it turns out that as long as the Frisch elasticity of labor supply is positive, the initial response of labor supply and hence of output is positive. Our numerical analysis also reveals that the size of the intertemporal elasticity of substitution in durable and non-durable consumption has a large impact on the quantitative response of output. As a rule, the positive response of output is stronger when the intertemporal elasticity of substitution is low (high $\sigma_c$ and $\sigma_d$).

Although the disaster has a positive effect on GDP, the effect on welfare is clearly negative. Households experience lower utility from durable goods as a direct result of the disaster. Furthermore, they experience lower levels of nondurable consumption and leisure (as a result of the...
intertemporal substitution effect and the wealth effect). Hence, all three components of instantaneous utility, i.e. $u(\cdot)$, $v(\cdot)$, and $q(\cdot)$, are affected negatively. In order to make welfare losses comparable we measure them in consumption equivalents of nondurable goods. In the benchmark case the accumulated welfare loss amounts to 4.1% compared to the pre-shock steady-state welfare level (cf. the upper left entries in Table 2). This number means that a household living in the economy struck by the disaster suffers the same welfare loss as a household permanently losing 4.1% of nondurable consumption.

We next turn to a shock that destroys physical capital $k$. In Figure 4, we show the impulse responses caused by a reduction of $k$ by 10%. Consumption and labor supply respond in the same way as for the $d$-shock. On impact, households consume less nondurable goods and supply more labor because of the negative wealth effect. Similar to the $d$-shock, there is also an intertemporal substitution effect at work. Since high capital investments are needed in order to rebuild the capital stock, resources are scarce during initial periods. Hence, households react by reducing durable goods investments, nondurable consumption and increasing labor supply. The intertemporal substitution effect works on top of the wealth effect and thus amplifies labor supply in the aftermath of the disaster.

**Figure 4: Natural Disasters: Destruction of Productive Capital**

![Graph of impulse responses showing capital, durables, capital investments, consumption, labor supply, and GDP.](image)

Impulse responses to the destruction of productive capital by 10%.

Although labor supply is higher compared to the pre-shock level in the initial periods, output decreases after the $k$-shock. The effect of lower productive capital on output dominates. As already pointed out, this is not a general result but holds for reasonable parameterizations of the
neoclassical growth model. Only when the Frisch elasticity of labor supply is infinite and the share of capital in production is implausibly low, output would respond positively to a $k$-shock (see Strulik and Trimborn, 2015).

For welfare, the intuition developed in conjunction with the standard neoclassical growth model holds for the $k$-disaster as well. A lower capital stock unambiguously causes welfare to decline. This can be seen by inspecting the time path of durables ($d$), nondurable consumption ($c$), and labor supply ($\ell$). As a response to the negative wealth effect, households enjoy lower levels of nondurable consumption and leisure. In addition, temporarily lower durable goods investment reduces the stock of durables. Since all three components of utility are affected negatively, overall utility is lower. We calculate a welfare loss of 3.5% measured in consumption equivalents.

Finally, we investigate a shock that destroys both capital and durable goods simultaneously. This opens up the possibility that output remains unaffected in the aftermath of a disaster, because the positive effect through higher labor supply and the negative effect through lower productive capital balance each other out. For our benchmark calibration, a disaster destroying 15% of durables and 3% of productive capital has a negligible effect on output, as shown in Figure 5.

**Figure 5: Natural Disasters: Destruction of Productive Capital and Durable Goods**

In our example, the effects of destruction of productive capital and durable goods roughly balance each other out. Output per capita and GDP per capita remain largely unaffected. Nevertheless, an “GDP-neutral” disaster has substantial effects on other economic aggregates and on welfare because all three components of welfare are affected negatively. Households enjoy less
durable goods (as a direct result of the disaster) as well as lower consumption of nondurable goods and lower leisure (as a result of the intertemporal substitution effect and the wealth effect). As a result, the benchmark “mixed” disaster causes aggregate welfare to decline by 3.9%.

5.3. **Non-Separable Utility.** The assumption of a separable utility function tremendously simplifies the model analysis. Naturally, like us most of the related literature assumes that utility experienced from the consumption of durable and nondurable goods is separable. The study by Ogaki and Reinhart (1998), however, suggests that utility between nondurable and durable consumption is non-separable. In order to demonstrate robustness of our main results with respect to non separability, we adopt the utility function suggested by Ogaki and Reinhart (1998) and extend it with utility experienced from leisure. This means that households maximize

\[
\int_0^\infty \left( \left[ \frac{(c^{1-\epsilon} + \beta d^{1-\epsilon})^{1-\sigma}}{1-\sigma} - 1 + \eta (1 - \ell)^{1-\sigma\ell} - 1 \right] e^{-\rho t} \right)
\]

subject to the budget constraints (3) and (4). We adopt the parameter estimates from Ogaki and Reinhart for \(\epsilon\) and \(\sigma\), i.e. \(\epsilon = 1.167\) and \(\sigma = 2.2\), and we adjust \(\beta\) such that the ratio of nondurable to durable goods expenditure matches the data from the US Consumption Expenditure Survey of 2014. All other parameter values are taken from our benchmark calibration.

<table>
<thead>
<tr>
<th>Table 2. Robustness: Separable vs. Non-Separable Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>d-disaster</strong></td>
</tr>
<tr>
<td>separable utility</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>non-separable utility</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

\(\Delta y\) denotes the average percentage deviation of output compared to the pre-shock level during the first three years, \(\Delta W\) denotes the percentage deviation of welfare compared to the pre-shock steady-state level, measured in nondurable consumption equivalents. A d-disaster is a 20% reduction of durable goods compared to the steady-state level, a k-disaster is a 10% reduction of productive capital, and a mixed disaster is a 15% reduction of durables and a 3% reduction of productive capital.

Table 4 summarizes the results for the three different types of disasters. The first row reiterates the results for the case of separable utility and the second row shows the results for the case of non-separable utility. Apparently, the conclusions from the benchmark model are robust against the assumption of non-separable utility.
6. RESIDENTIAL HOUSING

Apart from physical capital and durable goods, disasters can also destroy residential housing. We cannot aggregate durable goods (like cars, furniture, etc.) and residential houses, because houses depreciate at a much lower rate compared to durable goods, and because both are treated very differently in the National Accounts (see below). To test the robustness of our results and to allow for disasters destroying residential housing in addition to other assets we extend the baseline model by a residential housing sector. Everything else from the benchmark model remains the same.

6.1. Model setup. In the extended model, households derive utility from nondurable consumption, durable consumption, residential housing services $h$, and leisure, and maximize

$$V = \int_0^{\infty} (u(c) + v(d) + v_h(h) + q(1 - \ell)) \cdot e^{-\rho t} dt,$$

subject to

$$\dot{h} = i_h - \delta_h h,$$

the augmented budget constraint

$$\dot{a} = w + ra - c - p_x x - p_h h,$$

the accumulation equation for durable goods (4), the initial conditions $a(0) = a_0$, $d(0) = d_0$, $h(0) = h_0$, and a no-Ponzi-game condition. The price of residential investment is denoted by $p_h$.

Furthermore, we have now a third type of firms producing residential investment. These firms convert units of final output into residential investment $i_h$ and face adjustment costs $\psi_h(i_h)$, as the firms producing durable goods. Each household is assumed to engage one construction firm per unit of time and may change the contracting party at any point of time. Free entry into construction implies that firms set the price for residential investment at unit costs:

$$p_h = 1 + \frac{\psi_h(i_h)}{i_h}.$$

We summarize the equations characterizing the evolution of the economy:

$$\dot{k} = i - \delta_k k$$
\[
\dot{d} = x - \delta_d d \\
\dot{h} = i_h - \delta_h h \\
\dot{c} = \frac{r - \rho}{\sigma_c} \\
\dot{x} = \left( \psi'(x) - \frac{\psi(x)}{x} \right)^{-1} \left[ p(r + \delta_d) - \frac{v'(d)}{u'(c)} \right] \\
\dot{i_h} = \left( \phi'(x) - \frac{\phi(i_h)}{i_h} \right)^{-1} \left[ p_i(r + \delta_h) - \frac{v'_h(d)}{u'(c)} \right] \\
\dot{q} = (r + \delta_k)q - \frac{\partial A f(k, \ell)}{\partial k} - \left( \frac{i}{k} \right)^2 \phi' \left( \frac{i}{k} \right) \\
A f(k, \ell) = \delta k + x + \psi(x) + i_h + \phi(i_h) + c + i + i \phi \left( \frac{i}{k} \right) \\
\frac{\partial A f(k, \ell)}{\partial \ell} = \frac{q'(1 - \ell)}{u'(c)}.
\]

6.2. Output and GDP. In most macroeconomic models, aggregate output equals GDP. Here, in a model with residential houses, we have to distinguish between these aggregates. The reason is that the System of National Accounts (SNA) requires to account for the service flow from already installed houses as part of consumption, and thus, as a part of GDP (see EC, IMF, OECD, UN & World Bank, 2009, pp. 466-467). In particular, SNA states that houses leased to other households are supposed to be treated equivalently to owner-occupied houses. A newly constructed house raises output and GDP according to its construction costs. In the following years, the house contributes to GDP – but not to output – according to the rent paid by its tenant (if it is leased) or according to an imputed rent (if it is owner occupied). Houses are the only durable good from which a service flow enters GDP calculations. For example, service flows from cars used for consumption purposes are not included as part of GDP.

In terms of our model, newly constructed houses are immediately accounted for in output and GDP in terms of their construction costs. Because houses are owner-occupied in our benchmark model, we introduce an imputed rent in order to account for the contribution of housing services to GDP. In Appendix B, we set up an alternative model in which houses are leased to households by real estate firms. We show that the alternative setup is equivalent to the benchmark model,
and that the equilibrium rental price \( p_h \) for renting one unit of \( h \) for one unit of time is equal to

\[
p_h = \frac{v'_h(d)}{u'(c)}. \tag{59}\]

Intuitively, the price ratio between durable consumption goods and residential houses, \( p_h/1 \), is equal to the marginal rate of substitution between durable consumption goods and nondurable consumption goods, \( v'_h(d)/u'(c) \). In the following, we use (59) as the imputed price for GDP accounting. Hence, GDP equals

\[
GDP_N = y + p_d d. \tag{60}\]

Finally, note that equation (60) denotes GDP in terms of the numeraire, i.e. in terms of consumption goods. In order to compare our results with the empirical literature we also calculate GDP in terms of a weighted price index of non-durables and durables, \( GDP = GDP_N/P \). The price index \( P \) is calculated as a weighted average from the Paasche and Laspeyres price indices using the geometric mean (Fisher index).

6.3. Calibration. We assume that households face isoelastic utility according to

\[
\int_0^\infty \left[ c^{1-\sigma_c} - 1 \frac{1}{1 - \sigma_c} + \beta d^{1-\sigma_d} - 1 \frac{1}{1 - \sigma_d} + \beta_h h^{1-\sigma_h} - 1 \frac{1}{1 - \sigma_h} + \eta (1 - \ell)^{1-\sigma_\ell} - 1 \frac{1}{1 - \sigma_\ell} \right] e^{-\rho t} dt. \tag{61}\]

where \( \beta_h \) denotes the weight of housing in utility. We further assume that adjustment costs are quadratic, i.e. \( \phi(i_h) = \gamma_h i_h^2 \).

For depreciation of residential houses we take the average depreciation rates measured for the U.S. between 1948 and 2008 (Davis and Heathcote, 2005; Eerola and Määttänen, 2013), i.e. \( \delta_h = 0.015 \). We set \( \beta_h \) in order to match the share of households’ housing assets on total assets of about 0.5 in the year 2008 (see Iacoviello, 2010 and 2011), and we reestimate \( \beta \) such that the spending share of durable goods investment and nondurable consumption match the data from the Consumption Expenditure Survey. This gives \( \beta_h = 6.0 \) and \( \beta = 4.34 \). Finally, we set the adjustment cost parameter \( \gamma_h \) equal to one. All other parameters are taken from the benchmark calibration.

6.4. Results. In the extended model, a disaster may not only destroy physical capital and durable goods, but also residential housing. However, it is unlikely that a disaster destroys residential housing and leaves durable goods intact. This would mean that the structure houses gets damaged
while the interior (e.g. furniture) remains intact. We therefore focus on three types of disasters: As before, we consider a $d$-disaster where only durable goods get destroyed and a $k$-disaster where only physical capital gets destroyed. Additionally, we consider a mixed disaster where residential housing and durable goods get destroyed. The mixed disaster offers another possibility how a disaster could be GDP-neutral. When residential houses get destroyed, the service flow from houses adding to GDP is lower in the aftermath of the disaster thus leading to the prediction of lower GDP through an accounting effect. However, as shown above, the destruction of both durables and residential houses exerts a positive impact on output, and thereby on GDP, through higher labor supply. A mixed disaster will thus be GDP-neutral when the output effect counterbalances the accounting effect. We summarize the impact of a mixed disaster destroying 15\% of durable goods and 2.5\% of residential houses in Figure 6. The amount of destroyed durables and houses is chosen such that the amount of destroyed assets is the same as in the previous scenarios, and such that the resulting response of GDP is insignificant.

Figure 6: Natural Disaster: Mixed disaster in the Extended Model

Impulse Responses to the destruction of durable goods by 15\% and residential housing of 2.5\%. The panel shows the response of capital ($k$), durable goods ($d$), residential housing ($h$), durable goods investment ($x$), residential investment ($i_h$), nondurable consumption ($c$), labor supply ($\ell$), imputed rents for houses ($p_h$), output ($y$), and GDP.
The results for output, GDP, and welfare for all three types of disasters are summarized in Table 3.

**Table 3. Impact of disaster on output, GDP, and Welfare**

<table>
<thead>
<tr>
<th>d-disaster</th>
<th>k-disaster</th>
<th>mixed-disaster</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y = +0.35%$</td>
<td>$\Delta y = -2.0%$</td>
<td>$\Delta y = +0.58%$</td>
</tr>
<tr>
<td>$\Delta GDP = +0.12%$</td>
<td>$\Delta GDP = -1.8%$</td>
<td>$\Delta GDP = 0.00%$</td>
</tr>
<tr>
<td>$\Delta W = -4.2%$</td>
<td>$\Delta W = -3.2%$</td>
<td>$\Delta W = -3.7%$</td>
</tr>
</tbody>
</table>

$\Delta y$ denotes the average percentage deviation of output compared to the pre-shock level during the first three years, $\Delta GDP$ denotes the average percentage deviation of GDP compared to the pre-shock level during the first three years, $\Delta W$ denotes the percentage deviation of welfare compared to the pre-shock steady-state level, measured in nondurable consumption equivalents. A $d$-disaster is a 20% reduction of durable goods compared to the steady-state level, a $k$-disaster is a 10% reduction of productive capital, and a mixed disaster is a 15% reduction of durables and a 2.5% reduction of residential housing.

7. Conclusion

In this paper, we developed a theory that offers an explanation for the puzzling empirical finding that output and GDP in the aftermath of natural disasters are not necessarily lower than before the event, and in some cases even higher than before. We have shown that disasters destroying predominantly durable goods drive output and GDP above the pre-disaster steady-state level and that disasters destroying mainly productive capital are predicted to reduce output and GDP. Insignificant responses of output can be expected when disasters destroy both, durable goods and productive capital. In an extension of the model we show that a neutral response of GDP can also be obtained if durable goods and residential structures are destroyed, while physical capital is left intact. The theory explains why the welfare losses entailed by GDP-neutral disasters are substantial and of about the same order of magnitude as those entailed by one-shock-only disasters. As a rule of thumb we estimate that a disaster destroying 20 percent of total assets entails a welfare loss of about 3 to 4 percent irrespective of its highly variable and disaster-type specific impact on output and GDP. This result turned out to be very robust against the type of the disaster and the assumed preferences of citizens.

We acknowledge that many countries afflicted by natural disasters receive foreign aid. Foreign aid can be conceptualized in our model as a transfer to households coming from abroad. This transfer would mitigate the negative wealth effect caused by the disaster and hence reduce the positive response of labor supply in the aftermath of a disaster. We therefore expect that foreign aid...
aid mitigates, but not reverses, the positive channel through which disasters affect output and GDP.

Our study suggests that output and GDP are inferior and misleading indicators of the damage caused by natural disasters. A better proxy would be the lost stock of durable goods, housing and/or productive capital or the discounted aggregate investment expenditure needed to reconstruct the lost stocks.

Our results are insignificantly influenced by the assumed absolute size of the disaster. This fact allowed us to focus the quantitative analysis exemplarily on disasters leading to a loss of 10 percent of total assets. For larger or smaller disasters, the estimated welfare loss and – in case of one shock disasters – the estimated GDP responses vary in proportion with the size of the disaster. Likewise, we can always find a shock composition implying zero disaster impact on GDP, irrespective of disaster size. This quantitative outcome is a natural consequence of iso-elastic utility and an iso-elastic, constant-returns-to-scale production function, the usual ingredients of quantitative macroeconomics.

For very large shocks, however, it seems reasonable to abandon the constant elasticity assumption. In particular, labor supply is likely to be bounded from above. The work day is limited by 24 hours and for most occupations, physiological limits are reached far earlier. Humans cannot sustain a physical activity level (PAL) of more than 2.4 times the basal metabolic rate for an extended period of time (Westerterp, 2001). For example, activities like ‘loading sacks on a truck’ and ‘carrying wood’ are associated with PAL values of 6.6 (FAO, 2001), implying that a worker’s energy needs would be 6.6 times his basal metabolic rate if he were occupied with these activities for 24 hours. Such a “heavy construction worker” could only manage to exert effort for $2.4 \cdot 24/6.6 = 8.7$ hours per day. Less energy consuming activities are, of course, sustainable for longer hours. In any case, upper limits to daily labor supply would help to explain why large disasters are more frequently found to exert a negative impact on GDP than small disasters (see Loayza et al., 2012). Extending our model by physiologically-constrained labor supply, for example based on Dalgaard and Strulik (2011), could be a promising task for future research on the macroeconomic implications of natural disasters.
Appendix A

We analyze the phase diagram for subsystem (29) and (30), and show that it has a unique and saddle-point stable steady-state. To do this, we show that the \( \dot{x} = 0 \) isocline and \( \dot{d} = 0 \) isocline intersect exactly once in the positive quadrant. Saddle-point dynamics can then be inferred from the phase diagram.

The \( \dot{d} = 0 \) isocline is given by \( x = \delta_d d \). Hence, it is linear with positive slope \( \delta_d > 0 \). The \( \dot{x} = 0 \) isocline is given by

\[
\left(1 + \frac{\psi(x)}{x}\right) (\bar{r} + \delta_d) = \frac{v'(d)}{u'(c)}.
\] (62)

Both isoclines intersect in the positive quadrant. To see this, notice that \( \lim_{x \to 0} \frac{\psi(x)}{x} = 0 \) and \( \lim_{x \to \infty} \frac{\psi(x)}{x} = \infty \) because \( \psi \) is strictly convex and \( \psi(0) = 0 \). Hence, for \( d \to 0 \), the right-hand side of equation (62) converges towards infinity implying that \( x \to \infty \), and for \( d \to \infty \) the right-hand side of equation (62) converges to 0 implying that the \( \dot{x} = 0 \) isocline intersects the \( d \) axis at a finite point. An illustration of both isoclines is shown in Figure 1.

In order to prove that the intersect of the isoclines is unique, we begin by showing that the slope of the \( \dot{x} = 0 \) isocline is negative for \( x > 0 \). By implicit differentiation of (62), we obtain

\[
\frac{dx}{dd} = -\frac{\frac{v''(d)}{u'(c)}}{\left(1 + \frac{\psi(x)}{x}\right) (r + \delta_d)} < 0 \quad \forall x > 0.
\] (63)

The numerator is negative, because \( v \) is strictly concave, and the denominator is positive, because the strict convexity of \( \psi \) together with \( \psi(0) = 0 \) implies that \( \psi'(x) > \psi(x)/x \) for \( x > 0 \). Together, this implies that the sign of the derivative is negative. Hence, the slopes of the isoclines have opposite signs and the intersection point is unique.

The isoclines divide the phase diagram into four areas. Saddle-point stability follows from the dynamics within these areas: Above the \( \dot{d} = 0 \) isocline, \( \dot{d} \) is positive and below \( \dot{d} \) negative; on the right-hand side of the \( \dot{x} = 0 \) isocline, \( \dot{x} \) is positive, and on the left-hand side, \( \dot{x} \) is negative.

Appendix B

Here, we present an alternative setup in which households rent housing services from firms. We show that this setup is equivalent to the model in Section 2. The alternative setup allows us to derive a rental price for durables, \( p_d \).
Households solve

\[
\max_{c,x,h,\ell} \int_0^\infty (u(c) + v(d) + v_h(h) + q(1 - \ell)) \cdot e^{-\rho t} dt,
\]

subject to

\[
\dot{\alpha} = w\ell + r\alpha - c - p_x x - p_h h
\]

\[
\dot{d} = x - \delta d,
\]

where \( p_h \) denotes the price for hiring one unit of residential housing \( h \) for one unit of time. The first order conditions are

\[
u'(c) = \lambda \tag{67}
\]

\[
\lambda p_x = \mu \tag{68}
\]

\[
v_h'(h) = \lambda p_h \tag{69}
\]

\[
\lambda w = q'(1 - \ell) \tag{70}
\]

\[
\lambda r = \lambda \rho - \dot{\lambda} \tag{71}
\]

\[
\dot{\mu} = \mu \rho - v'(d) + \mu \delta d. \tag{72}
\]

We derive an equation for \( p_h \) by combining equation (67) and (69):

\[
p_h = \frac{v_h'(d)}{u'(c)}. \tag{73}
\]

Furthermore, substituting equation (67) into (70) yields equation (12), and differentiating equation (67) with respect to time and combining with equation (71) yields the Keynes-Ramsey rule (11). Introducing firms supplying durable investment goods at price \( p_x = 1 + \psi(x)/x \) yields the differential equation for \( x \), equation (14).

There exists a continuum \((0, 1)\) of firms buying installed units of houses \( i_h \) from construction firms at price \( p_{i_h} \) and renting houses \( h \) to households. These firms can best be seen as real estate firms whose only role is to buy installed houses and rent them to households. Hence, these firms maximize

\[
\pi = \int_0^\infty (p_h h - p_{i_h} i_h) e^{-rt} dt \tag{74}
\]
s.t. \( \dot{h} = i_h - \delta_h h \) \hspace{1cm} (75)

The first order conditions are

\[
\begin{align*}
\rho_{i_h} &= \nu \hspace{1cm} (76) \\
\rho_h - \nu \delta_h &= \nu r - \dot{\nu} \hspace{1cm} (77)
\end{align*}
\]

with \( \nu \) denoting the shadow price of one installed unit of \( h \) in terms of marginal revenues.

There exists a continuum \( (0, 1) \) of construction firms converting final goods into housing. These firms face adjustment costs \( \phi(i_h) \) for installing \( i_h \) units of housing. Each real estate firm is assumed to engage one construction firm per unit of time but it can change the contracting party at any point of time. Free entry into the construction sector implies that firms sell \( i_h \) at unit costs:

\[
p_{i_h} = 1 + \frac{\phi(i_h)}{i_h}. \hspace{1cm} (78)
\]

Substituting equation (76) into (78), differentiation with respect to time, and substituting (77) and (73) yields

\[
\frac{\dot{i}_h}{i_h} = \left( \phi'(i_h) - \frac{\phi(i_h)}{i_h} \right)^{-1} \left[ 1 + \frac{\phi(i_h)}{i_h} \left( r + \delta_h \right) - \frac{\nu_r(h)}{u'(c)} \right], \hspace{1cm} (79)
\]

which is equal to equation (54). Hence, the alternative setup for households renting durable goods \( h \) from real estate firms is equivalent to the benchmark setup.

**Appendix C**

We show that the steady-state of the large economy described by system (34) - (41) is unique, if it exists. We begin by analysing the steady state relationship of investment, capital stock, interest rate and Tobin’s \( q \). At the steady state \( i = \delta_k k \) holds (from equation (34)), and thus capital adjustment costs and marginal capital adjustment costs are both zero, i.e. \( \phi(\delta_k) = \phi'(\delta_k) = 0 \). This means that \( q = 1 \) (from (39)), and \( r = \partial A f(k, \ell) / \partial k \). Finally, at the steady state \( r = \rho \) holds (from (36)), which pins down the capital-labor ratio.

Second, we exploit the analysis of subsystem (29) and (30) from Section 3. From the analysis above we know that a higher steady-state value of \( c \) leads to a higher steady-state of \( x \) and \( d \). We can express this insight as a steady-state relationship \( x = s(c) \) with \( s'(\cdot) > 0 \). This means that by
substituting for $x$ and rearranging equation (40) we obtain

$$k \left( \frac{Af(k, \ell)}{k} - \delta_k \right) = c + s(c) + \psi(s(c)).$$  \hspace{1cm} (80)

The equation states a positive relationship between $k$ and $c$ at the steady-state.

Next, we show that equation (41) constitutes a negative steady-state relation between $k$ and $c$, taking the capital-labor ratio (and thus the marginal product of labor) as given. We do this by evaluating the derivative at the steady-state for a constant capital-labor ratio:

$$\left. \frac{dk}{dc} \right|_{(k, \ell)^*} = \left. \frac{dk}{dl} \right|_{(k, \ell)^*} \cdot \left. \frac{dl}{dc} \right|_{(k, \ell)^*} = 1 \cdot \frac{-u''(c)}{u'(c)} q''(1 - \ell) < 0. \hspace{1cm} (81)$$

The overall derivative is negative because $u''(\cdot) < 0$, $q''(\cdot) < 0$, and $u'(\cdot) > 0$. Equations (80) and (41) both imply a steady-state relationship between $k$ and $c$. Finally, noticing that the slopes of the steady-state equations (41) and (80) are of opposite sign, we conclude uniqueness of the steady-state.
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