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Preprint

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September 19, 2016

Abstract

Recent developments in identifying economic and social phenomena within the theory and laws of physics do not provide a sufficient basis of information for economists. In this article, an interdisciplinary analysis in microeconomics, is proposed to overcome this limit by establishing a new economic theory in the way of better analyzing economic problems from the methodological perspectives of physics. This new theoretical approach is tested on a model using individual time use values that identifies how life satisfaction may be defined through economic utility so as to better investigate the income-happiness paradox as pointed out by Easterlin. The theoretical analysis point out that an increase in happiness would depend on the ability to replace subsidiaries with conspicuous consumption so long as they have common characteristics that satisfy the same needs. This consumption pattern implies that consumers would draw more satisfaction from less usage of conspicuous goods than from higher usage of subsidiaries, under given budget and time constraints.

Keywords:Time use, Relativistic Satisfaction, Satisfaction Waves, Speed of Satisfaction

1 Introduction

Econophysics is a field of research that aims to empirically discover and interpret in terms of models regularities, through physics formulas, in the temporal, spatial and social settings of economic and social phenomena. This approach mainly focuses on the observation of real systems in model-building and development without emphasizing or explaining the socio-economic theoretical background of a system. These attempts are generally designed to evaluate specific research questions regarding finance or certain economic structures through comparisons between physics and economics. Ten principles of economics proved to be analogous with the theory and the laws of physics (Unimed, 2014): the law of conservation of energy, the laws and theory of thermodynamics, opportunity theory, the theory of statistical physics, Newton's law, Ohm's law, equilibrium theory, the theory of atomic bonding, composite bonding theory and the Stefan

Boltzmann law. There seem to be two principles underpinning those abovementioned research aspects (Mantegna and Kertész, 2011). First, as financial instruments have become increasingly complicated and mathematically demanding, physicists have become more eligible and trained to model phenomena and to work in a process of continuous feedback with data. Second, the flood of data that results from the extraordinary and growing speed and ubiquity of computers provides attractive, spectacular results with new job opportunities for physicists.

On the other hand, econophysics providing an opportunity to reflect on the problem, inquiring "do we need to revise our thinking regarding the nature of interaction between economic variables and, furthermore, other sciences?". As a matter of fact, it is already known that the methodological ideal in physics (and in psychology) has been extremely influential for the formation of mainstream economic methodology since from 19th century by Jevons, Walras, notably Edgeworth and later by Pareto, Fisher and Samuelson (Mirowski, 1991). These works are useful for understanding how to use methodology and tools taken from physics. For instance, a special theory of relativity has recently been used to explain new economic theories like interstellar trade (Krugman, 2010) or space time finance (Haug, 2004). But maybe one of the most important applications in this case, is the use of elasticity analysis borrowed from physics shows improvements in consumer theory (Marshall, 1890; Frisch, 1959; Deaton and Muellbauer, 1980). The same can be said for psychology. Decreasing marginal utility hypothesis is also borrowed from psychology. This hypothesis states that the logarithmic form of the relationship between the magnitude of sensation and the discrimination ratio (expressed as percentage changes in intensity and original intensity), the so-called logarithmic psychophysical law, has been explored for more than a century by studies that err towards the psychological and economic perspectives (Arago, 1858; Fechner, 1860; Edgeworth, 1881; Blanchard, 1918; Thurstone, 1931; Masin et al., 2009). Furthermore, more recent studies using psychological approaches based on experimental case studies performed by behavioral economists and later by neuro-economists represent an interesting alternative in identifying the determinants of irrational (or boundedly rational) decisions (Kahneman, 2012; Schmid, 2010; De Martino et al., 2006; Tom et al., 2007).

Based on these factors, this paper proposes to tackle the identification problem between satisfaction and utility, by pointing out the nature of the relationship between the psycho-economic variables from a physics point of view. Therefor, the translation of subjective well-being into utility is questionable and has far-reaching implications for economic analysis (Lévy-Garboua and Montmarquette, 2007). This problem today may be seen within discussions surrounding the Easterlin Paradox which reveal that raising the incomes of all does not increase the happiness of all and demonstrates that utility is a function of relative income, since the latter is not affected by uniform economic growth (Easterlin, 1973; Easterin, 1974; Stiglitz et al., 2009). In fact, as first posited by behavioral economists, people may express their judgments of satisfaction towards their own experience which in turn may render the satisfaction different from utility.

Thus, inconsistencies in utility and satisfaction can, for instance, be explained by postulating that there indeed exist two different kinds of utility that depend on the effects of the experience (whether expected or not) on actual state before and after the choice is made (Kahneman and Thaler 2006; Kahneman et al., 1997).[1]However, the main problematic areas about theoretical identification of satisfaction as a functions of decision-making in economic theory exist which are yet to be worked on.

In this respect, the aim of this study is to develop a new micro theory to better analyze the nature of the relationship between life satisfaction and utility through the methodology proposed by physics in order to answer such questions as:

1) How should the relationship between life satisfaction and utility based on individual time use and consumption values be defined?

2) How to modelize life satisfaction drawn from consumption activities related to the relative position of the household within a reference population?

3) Which consumption pattern occurs within the Easterlin Paradox?

We named our methodology Physiconomics. Section 2 presents the new theory with definitions and later proves the satisfaction waves in order to answer the first question. Section 3 answers the second question by defining "the Relativistic Theory of Life Satisfaction" with "Relativistic Satisfaction Theory". Section 4 reports the results by way of clarifying the consumption phenomenon behind the Paradox. Section 5 concludes the paper.

2 Theory and definition of satisfaction waves

The curiosity is to understand the relationship between life satisfaction and utility based on individual time use and consumption values. This section is dedicated to giving a canonical definition of satisfaction and consumption in terms of time spend and beckerian utility function in order to prove the existence of satisfaction waves.

2.1 *Satisfaction, Utility and Consumption*

Definition of satisfaction, as assumed in this paper,can be understood more easily through the question "How long would a normal person accept to stay in a perfectly isolated room without any activity except consuming high quality food?" At first, note that "normal" in this question refers to an individual whose satisfaction is equal to the average satisfaction level of the preference group which he/she represents. It is clear that having only a good meal means nothing in exclusion of other activities. Our normal person probably refuses

[1]A brief explanation of decision-utility and experienced utility. Decision-utility refers to a preference index describing how choices are made while experienced utility refers to the measure of pleasure and pain before making a decision, as suggested by Bentham in 1789. In this respect, the satisfaction and utility would to be the same if and only if there is no sort of dissatisfaction stemming from the difference between these two utilities.

this proposition or accepts it only for short-time period because of the fact that not being hungry is only useful as long as it allows the realization of other activities without suffering. Be that as it may, normal person would decide to satisfy as fast as possible his/her need. For this special case, satisfaction from a meal would depend on quality and/or quantity of food and on time spent in eating activity in order to reach a desired utility level. As a matter of fact, this phenomenon has already been studied by various time allocation models about how to reduce travel or transportation time in consumer decision (see Johnson, 1966; Oort , 1969; Small, 1982; Jara-Diaz et al., 2013)[2].

DEFINITION 1. Satisfaction from an economic good consumption activity i is given by consumption good $x_i \in \mathbf{X}$, multiplied by utility over individual time use for an activity i:

$$\sum_i s_i = x_i \sum_i \frac{u_i}{t_i} \tag{1}$$

Let t_i, be individual time use for an activity i, $t_i \in \mathbf{T}$, with $\mathbf{T}$ which is the set of time spent activities as the available individual total time. (1) is the total satisfaction where u is the utility of activity *i=1,2,...,n*. Satisfaction (s) of an individual is derived from a unitary utility per time use (t_i) felt from the consumption of good x_i. The identity given in (1) can also be interpreted as the satisfaction determined by the time it requires to reach a utility level by the consumption good x_i. Therefore, $\sum u_i/t_i$ in (1) implies that an increase in utility yields higher satisfaction which is negatively conditional on time spent in activity i. The main argumentation underpinning this relationship is that the demand for the consumption good x_i depends on the share of the market input cost in the total cost of producing this commodity, and on the elasticity of substitution between goods and time: households substitute the time intensive goods with goods intensive ones since an increase in the shadow price of time immediately raises the relative price of time-intensive commodities(Gronau, 1986)[3]. Thus, an increase in x_i necessitates having lower t_i ; hence higher s_i. Furthermore, some additional properties are assumed in (1): (i) Additive separability of satisfaction holds for $f(x_i, u_i, t_i)$, an additive satisfaction function if and only if it is a satisfaction function and functions $f_1, \ldots, f_n$ exist so that $f(x_1,u_1,t_1;,\ldots;x_n,u_n,t_n)= f(x_1,u_1,t_1)+\ldots+f_n(x_n,u_n,t_n)$. (ii) Following Lancaster (1966), it is supposed that utility, hence satisfaction, depend on the characteristics of an economic good[4].

The figures for consumption are supposed to be combined with market goods to transform them into final commodities (Becker, 1965). This is the Beckerian

[2]Furthermore, another fact which confirms this satisfaction phenomenon is that higher opportunity costs of not-working may promote having good intensive leisure instead of time intensive leisure consumption.

[3]The effect of characteristics of economic good on decision-making will be clarified at Section 4.

[4]When there exists more than two commodities, the outcome depends on the cross-elasticities of the substitution between the commodities other than i ; see Atkinson, A. B. and N. H. Stern (1979).

type of utility where final goods, themselves produced through household activities, are directly represented in the utility function.

$$MaxU = U\left(c_1, ..., c_n; t_1, ..., t_n\right) \tag{2}$$

In (2), the satisfactory action supposes to maximize the utility function depending on the optimal time allocation and consumption. Let consumption activity vector $\mathbf{C} = \mathbf{X}.\mathbf{P}$ is the dot product of quantity $\mathbf{X}$ and price $\mathbf{P}$ vector. The utility function, $\mathbf{U}(\mathbf{C}, \mathbf{T})$ defined over for $c_i \in \mathbf{C}$ and for $t_i \in \mathbf{T}$ is given in from of Cobb–Douglas (Gardes, 2014; Jara-Diaz et. al, 2013; Gunes and Aktuna-Gunes, 2015).

$$U_i = a \prod_i t_i^{\alpha} \prod_i c_i^{\beta} \tag{3}$$

α, β are the exponents of consumption activity and amount of consumption goods for commodity i respectively.

Definition 2. The consumption amount for an activity i is determined by the consumption quantity x_i times the effect on the time required to reach a utility, of the change in time use.

$$c_i = x_i \frac{\partial(u_i/t_i)}{\partial t_i} \tag{4}$$

Let $v_i = u_i/t_i$, (4) reveals that an individual continues to consume x_i as long as the marginal utility during change in time spent is positive. In fact, $\partial v_i/\partial t_i$ in (4) measures the intensity of satisfaction depending on the change in speed of satisfaction of a need derived from consumption[5]. Consumption stops, $c_i = 0$, whenever $\partial u_i = 0 \rightarrow \partial v_i = 0$ or having $\partial t_i \rightarrow \infty$; briefly, $\lim_{\partial v_i \rightarrow 0} c_i = 0$ or/and $\lim_{\partial t_i \rightarrow \infty} c_i = 0$.

2.2 *Satisfaction Waves*

Theorem 1. Satisfaction wave is the displacement of households' satisfactory state (*i.e.* satisfaction) defined in a satisfaction string in which the duration of the satisfactory state for a given period is measured by the relationship between utility and time use, itself dependent on the density and tension of each satisfaction string.

$$\frac{\partial^2 u_i}{\partial {t_i}^2} = \frac{1}{x_i}\left(\left(\frac{\partial v_i}{\partial s_i}\right)\frac{\partial^2 s_i}{\partial {u_i}^2}\right) \tag{5}$$

Proof1. Traditional theory assumes that satisfaction in (1) is identical to utility (3). This identification can simply be denoted by a simple case of a unique consumption good i, *cetaris paribus*, as

[5]In fact, $\frac{\partial v_i}{\partial t_i} = \frac{\partial v_i}{\partial s_i} c_i$ since $c_i = \frac{\partial s_i}{\partial t_i}$. This will be clarified at satisfaction waves.

$$(s_i = x_i v_i) \equiv (t_i c_i = u_i) \tag{6}$$

Thus, the proof of (5) can be given in seven steps:

- 1. The change in utility would depend on the change on consumption realized during time use as $\partial t_i v_i = \partial u_i$. Thus, $\partial v_i / \partial t_i$ in (4) is equal to

$$a = \partial^2 u_i / \partial t_i^2 \tag{7}$$

- 2. By using $s_i = x_i v_i$ in (6) with (4) we get $s_i = \frac{c_i}{a_i} \frac{u_i}{t_i}$ where $\frac{\partial s_i}{\partial u_i} = \frac{c_i}{a_i t_i}$ and further $c_i = \frac{\partial s_i}{\partial u_i} a_i t_i$

- 3. Thus, $s_i = x_i v_i$ in form of $s_i = c_i \frac{1}{a_i} v_i$ can be rewritten as $s_i = a_i t_i \frac{\partial S_i}{\cancel{\partial a_i}} \frac{\cancel{\partial a_i}}{\cancel{\partial t_i}} \frac{\cancel{\partial t_i}}{\partial v_i}$ where it can be simplified as $s_i = a_i t_i \frac{\partial s_i}{\partial v_i}$ or further as $\frac{s_i}{t_i} \frac{\partial v_i}{\partial s_i} = a_i = \frac{c_i}{x_i}$

- 4. Later by using $\partial v_i = a_i \partial t_i$ and assuming that $\partial s_i = c_i \partial t_i$ from (6) together, $\frac{s_i}{t_i} \frac{\partial v_i}{\partial s_i} = \frac{c_i}{x_i}$, the third step can be denoted as $\frac{1}{t_i} \frac{s_i}{c_i} a_i = \frac{c_i}{x_i}$

- 5. Again by using $\partial s_i = c_i \partial t_i$ and (7) step four yields $\frac{s_i}{t_i} c_i \frac{\partial^2 u_i}{\partial s_i^2} = \frac{c_i}{x_i}$

- 6. The right hand side in step five is also equals to $\frac{s_i}{t_i} \frac{1}{x_i}$, thus by eliminating s_i / t_i from both sides we have $c_i = \frac{\partial^2 s_i}{\partial u_i^2} \frac{1}{x_i}$

- 7. Since $x_i = \frac{c_i}{a_i} = \frac{\partial s_i / \partial t_i}{\partial v_i / \partial t_i} = \frac{\partial s_i}{\partial v_i}$; hence

$$c_i = \frac{\partial^2 s_i}{\partial u_i^2} \frac{\partial v_i}{\partial s_i} \tag{8}$$

Finally, by using (7) and (8) in (4) together yields

$$\frac{\partial^2 s_i}{\partial u_i^2} \left(\frac{\partial v_i}{\partial s_i} \right) = x_i \frac{\partial^2 u_i}{\partial {t_i}^2} \tag{9}$$

Which finally identifies (5) as

$$\frac{\partial^2 u_i}{\partial {t_i}^2} = \frac{1}{x_i} \left(\frac{\partial v_i}{\partial s_i} \right) \frac{\partial^2 s_i}{\partial {u_i}^2} \tag{10}$$

$\partial v_i / \partial s_i$ in (10) determines the degree of tension for $\partial^2 s_i / \partial {u_i}^2$ which also determines the intensity of needs depending on the effect on the satisfaction, of the change in time required to reach a utility level by the consumption of x_i. The intensity of need in (10) defined for each satisfaction string has an inverse relationship with the consumption quantity as $(\partial v_i / \partial s_i) / x_i$. In other words, the intensity of needs would depend on the effect of satisfaction on the speed of satisfaction per consumption good. The satisfactory power of an extra consumption good approaches zero $\partial v_i = 0$ when as the marginal utility of consumption (also) approaches zero.

3 Relativistic Theory of Life Satisfaction

The identification of satisfaction with utility in (6) has important implications. An increase in consumption with time spent of i necessarily yields higher speed of satisfaction derived from consumption goods if and only if the time requires to reach a utility level decreases[6]. Therefore, increase in satisfaction will be limited due to decreasing marginal utilities.

COROLLARY 1. The higher the level of satisfaction is, the more difficult any further acceleration becomes.

PROOF 2. The main argumentation underpinning this corollary is that an increase in income forces one to consume a higher quality of market goods requiring less time use (Gelber and Mitchell, 2012) . The idea is that the new needs indexed on the household level of well-being change with income (Gardes and Merigan, 2008) . However, for a given satisfaction level, better quality of goods increases the satisfaction speed as the intensity of utility per time use.

To show this, it is enough to multiply the both sides with $t_i u_i$ in (6), which gives

$$(u_i)(t_i)x_i\frac{u_i}{t_i} \equiv t_i c_i(t_i)(u_i) \tag{11}$$

By rearranging the equation (11), we can get

$$c_i u_i \equiv x_i v_i^2 \tag{12}$$

The evidence given in (12) implies that consumption multiplied by utility is equivalent to the speed of satisfaction squared times the consumption good quantity. In this respect, one overarching question is why greater quality of goods demand is required? Furthermore, it can be argued that the satisfaction derived from an activity that uses a higher-quality of good is the same as the satisfaction derived from a lower-quality good using activity, since the time period over which one feels satisfaction from i would be the same.

3.1 *Relativistic Satisfaction Theory*

The satisfaction supposed to be influenced by the given consumption level also depends on its relative magnitude in the social group. This motivation justifies the assumptions on consumption decisions which are motivated by "relative" consumption (Duesenberry, 1948 and 1949). The strength of any individual's desire to increase his consumption expenditure is a function of the ratio of his expenditure to some weighted average of the expenditures of others with whom he comes into contact. This motivation would have two dimensions:

i) First, these households would have a desire to be close to the consumption structure of the upper bound (rich households) within

[6]This is $x_i\frac{\overline{u_i}}{\partial t_i\downarrow} \equiv \partial(t_i c_i)\uparrow$; therefore ∂x_i is unknown at first side. Quantity and/or quality change in x_i is clarified at Relativistic Satisfaction Theory

their reference group. This is the inner relationship between the individual and his/her reference group.

ii) Second, motivation would be, respecting Duesenberry's idea of the socio-psychological difficulties due to reducing the given expenditure scheme of households, a desire to be away from the consumption structure of the individuals who are included in the upper bound of the lower reference group. This is the external relationship between the individual and other reference groups.

In other words, the poorest individuals in the poor population would want to be close to the consumption structure of the relatively richer poor in their reference group but not to that of the poor ones in the lower groups. The high level income distribution of inequalities, especially in developing economies, would promote this motivation. Therefore, such a phenomenon justifies the existence of high level luxury goods-buying in these countries.

THEOREM 2. Relativistic satisfaction theory explains that satisfaction increases when v_i in (12) approaches that of the reference group and which, in turn, necessitates the consumption of a higher quality of market goods in order to stay at the same level of life satisfaction.

$$c_i u_i \equiv x_i \left(v_i^{'}\right)^2 = \frac{x_{i_{o,c}} \left(v_i^{'}\right)^2}{\sqrt{1 - \left(v_{i_{c,o}}\right)^2 \Big/ \left(v_i^{'}\right)^2}} \tag{13}$$

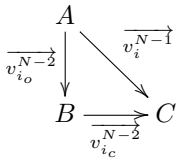
PROOF 3. Theorem 2 proposes that the consumption amounts are corrected with relativistic satisfaction speed of the persons (relatively riches) in upper bounds.

It can be assumed, at first, that the time period over which one feels satisfaction derived from a higher-quality good using activity and the satisfaction derived from a lower-quality good using in an identical activity would be the same when we compare the quality of goods used in the activity within the reference group for each individual. In other words, let reference groups be determined through the populations given for each income inequality quartile (N). Thus, we can find the average utility for each quartile to be $\overline{u}_{N-1}$. The number for the upper reference group is N-1 since there won't be any upper bound for the richest persons within the country. Thus, given the time use for the N-1 group is t_{N-1}, it can be assumed at first sight that $v_1 = v_2 = \ldots = v_{N-1}$ for the optimum quantities of x_i.

However, relative consumption hypotheses (i) and (ii) reveal that the peer consumption effect exists among the group members. In this case, speed of satisfaction for each group would probably be biased; where

$$v_1 \leq v_2 \leq \ldots \leq v_{N-1} \begin{cases} \bar{u}_1 \leq \bar{u}_2 \leq \ldots \leq \bar{u}_{N-1} \\ with \\ t_1 \geq t_2 \geq \ldots \geq t_{N-1} \end{cases} \tag{14}$$

(14) shows that the difference in satisfaction speeds of reference groups would be caused either by different average utilities between groups for any chosen i or by inconsistencies in times use among quartile groups. In fact, the time required to reach a desired higher utility level, by consuming a higher quality good x_i, for lower references groups is bigger than that of upper groups. In other words, time spent per consumption good i is different for references groups. To show this, it can be supposed that each group has the desire to reach a similar consumption good pattern to the closest upper reference group to them. This is possible only by way of some additional time required to derive extra satisfaction in order to catch the utility values at the upper bound. In the vectorial space this instance can be shown as



(15)

Where, o is an index used for denoting consumption for subsistence, whilst c represents the conspicuous consumption of the lower bound, $(x_{i_o}, x_{i_c}) \in \mathbf{X}$. $v_i^{N-1}$is the upper bound satisfaction speed for i. The lower bound (N-2) first spends time satisfying basic needs and later in satisfying less basic needs through conspicuous consumption so as to attain the upper bound consumption pattern. A Pythagorean equation taken from (15) enables us to demonstrate this relationship as

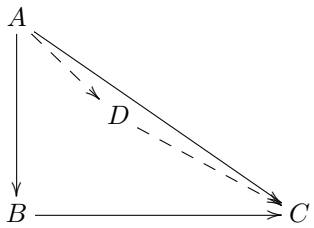
$$\left(v_i^{N-1}\right)^2 = \left(v_{i_c}^{N-2}\right)^2 + \left(v_{i_o}^{N-2}\right)^2 \tag{16}$$

Where we get for conspicuous and subsistence consumption that,

$$v_{i_{c,o}}^{N-2} = v_i^{N-1}\sqrt{1 - \frac{\left(v_{i_{o,c}}^{N-2}\right)^2}{\left(v_i^{N-1}\right)^2}} \tag{17}$$

Hypothesis 1. The poor in the lower bound intend to attain a consumption pattern closer to that of the upper bounds (See. Arrow and Dasgupta, 2009) .

Since it was first pointed out by Veblen (1899) about conspicuous consumption that the existence of a negative relationship between dispersion of social group income and conspicuous consumption driven by a reduction in spending on conspicuous goods by high income group members(Charles et al.2009; Chai and Kaus, 2012). In fact, the hypothesis explains that desire to replace the subsistence expenditures with more conspicuous and luxury ones is relatively higher in poor populations. Such tendency implies that



(18)

Following (15), this behavior can be interpreted by the magnitude order of $||AB|| + ||BC|| > ||AD|| + ||DC|| > ||AC||$ in (19). Thus, the hypothesis explains the decision whereby people living in the lower bound have the desire to have$||AD||+||DC|| \geq ||AC||$ where they are ready to replace subsidiaries with conspicuous ones as much as possible in order to reach the upper bounds' utility level as fast as possible[7]. The reason for this being that they can increase their satisfaction speed in order to reach the satisfaction level of the upper bound by lowering the time spent in subsidiaries. Thus, in terms of utility functions in (3), $||AD|| + ||DC|| = ||AC||$, $\lim_{v_{i_o} \to 0} c_{i_o} = 0$ implies,

$$t_{i_c}^{N-2} x_{i_c}^{N-2} = t_i^{N-1} x_i^{N-1} \quad with \quad t_{i_c}^{N-2} x_{i_c}^{N-2} \geq t_{i_o}^{N-2} x_{i_o}^{N-2} \tag{19}$$

The equality in (19) can also be specified as

$$\frac{x_i^{N-1}}{x_{i_c}^{N-2}} = \frac{t_{i_c}^{N-2}}{t_i^{N-1}} \tag{20}$$

Where the time use ratios is also determined by

$$\frac{t_{i_c}^{N-2}}{t_i^{N-1}} = \frac{u_{i_c}^{N-2} / v_{i_c}^{N-2}}{u_i^{N-1} / v_i^{N-1}} = \frac{v_i^{N-1}}{v_{i_c}^{N-2}} \tag{21}$$

Where lower bound has desire to have same utility as that of upper bound $u_{i_c}^{N-2} = u_i^{N-1}$. Thus, by using (17) with (21)

$$\frac{t_{i_c}^{N-2}}{t_i^{N-1}} = \frac{1}{\sqrt{1 - \frac{\left(v_{i_o}^{N-2}\right)^2}{\left(v_i^{N-1}\right)^2}}} \tag{22}$$

Further, (22) allows to interpret (20) as

$$\frac{x_i^{N-1}}{x_{i_c}^{N-2}} = \frac{t_{i_c}^{N-2}}{t_i^{N-1}} = \frac{1}{\sqrt{1 - \frac{\left(v_{i_o}^{N-2}\right)^2}{\left(v_i^{N-1}\right)^2}}} \tag{23}$$

or

[7]This is true especially for the subsidiary and conspicuous consumption goods used for satisfying the same need. This case will be explained at Section 4

$$x_i^{N-1} = \frac{x_{i_c}^{N-2}}{\sqrt{1 - \frac{\left(v_{i_o}^{N-2}\right)^2}{\left(v_i^{N-1}\right)^2}}} \tag{24}$$

Finally, to show this relative satisfaction as different from the definition 1 given above, let relative satisfaction $h_i = c_i u_i$ with $h \in \mathbf{H}$ in (12) which can be denoted in the vectorial space as

$$H = \int_0^u C \, du \tag{25}$$

By using $c_i = dx_i v_i / dt_i$ from (6) in (25), we have

$$H = \int_0^u \frac{dx_i v_i}{dt_i} \, du \tag{26}$$

Later by using (24) in (26),

$$\int_0^{xv} v_i \; d(x_i v_i) = \int_0^{v} v_i \; d\left[\frac{x_{i_c} v_i}{\sqrt{1 - (v_{i_o})^2 \Big/ \left(v_i'\right)^2}} \right] \tag{27}$$

Note that, in (27), $v_i^{'} = v_i^{N-1}$, $v_{i_o} = v_{i_o}^{N-2}$and $v_i = v_i^{N-2}$for the simplicity. By integrate by parts

$$\int x \, dy = xy - \int y \, dx \tag{28}$$

to yield,

$$H = \frac{x_{i_c} v_i}{\sqrt{1-(v_{i_o})^2 \Big/ \left(v_i'\right)^2}} - x_{i_o} \int_0^u \frac{v_i \; dv_i}{\sqrt{1-\left(v_{i_o}\right)^2 \Big/ \left(v_i'\right)^2}}$$

$$\Leftrightarrow \frac{x_{i_c} v_i}{\sqrt{1-\left(v_{i_o}\right)^2 \Big/ \left(v_i'\right)^2}} - \left[x_{i_c} \left(v_i^{'}\right)^2 \frac{x_{i_c} v_i}{\sqrt{1-\left(v_{i_o}\right)^2 \Big/ \left(v_i'\right)^2}} \right]_0^{v_i}$$

$$\Leftrightarrow \frac{x_{i_c} \left(v_i^{'}\right)^2}{\sqrt{1-\left(v_{i_o}\right)^2 \Big/ \left(v_i'\right)^2}} - x_{i_c} \left(v_i^{'}\right)^2$$

$$H = x_i\left(v_i^{'}\right)^2 - x_{i_c}\left(v_i^{'}\right)^2 \tag{29}$$

This result (19) shows that the relative satisfaction H is equal to increase in consumption good as a consequence of its relative consumption amount multiplied with speed of satisfaction of upper bound. (29) can be rearranged to show

$$x_i\left(v_i^{'}\right)^2 = H + x_{i_c}\left(v_i^{'}\right)^2 \tag{30}$$

If the relative satisfaction decreased so that $H = 0$, the satisfaction will be stationary, but will still possess satisfaction owing to conspicuous consumption, $x_{i_c}\left(v_i^{'}\right)^2$. In other words, the individual have satisfaction s_c when stationary relative to its frame and will have consumption good x_{i_c}. This is shown as

$$S = s_c + H \tag{31}$$

Where

$$s_c = x_{i_c}\left(v_i^{'}\right)^2 \tag{32}$$

This, then, completes the derivation of $H = x_i\left(v_i^{'}\right)^2$ for the stationary satisfaction. For individuals having relative satisfaction, total satisfaction is given by

$$H = x_i\left(v_i^{'}\right)^2 = \frac{x_{i_c}\left(v_i^{'}\right)^2}{\sqrt{1 - \left(v_{i_o}\right)^2 \Big/ \left(v_i^{\prime}\right)^2}} \tag{33}$$

Furthermore, respecting to the condition $t_{i_c}^{N-2} x_{i_c}^{N-2} \geq t_{i_o}^{N-2} x_{i_o}^{N-2}$ given in (19), the resolution from (16) to (32) also implies that the rest satisfaction for subsistence with : $S = s_o + H$ and $s_o = x_{i_o}\left(v_i^{'}\right)^2$ as

$$H = x_i\left(v_i^{'}\right)^2 = \frac{x_{i_o}\left(v_i^{'}\right)^2}{\sqrt{1 - \left(v_{i_c}\right)^2 \Big/ \left(v_i^{\prime}\right)^2}} \tag{34}$$

Where subsidiary consumption amount is corrected with relative satisfaction speed $\left(v_i^{'}\right)^2 \Big/ \sqrt{1 - \left(v_{i_c}\right)^2 \Big/ \left(v_i^{\prime}\right)^2}$.

4 Theoretical Results

The main hypothesis behind (32) and (33) is that an increase in unitary utility values per time use as the satisfaction speed requires a greater quality of good demand in order to stay at the same level of satisfaction. This equation implies that "the higher the satisfaction is, the more difficult any further acceleration becomes". However, it becomes hard to increase satisfaction by additional goods due to a decrease in marginal utilities and budget with time constraints.

In fact, a careful reader may note for $H = x_i(v_i^{'})^2$that any given state of satisfaction, consumption goods and speed of satisfaction are inversely related. This equation reveals that for an identical state of satisfaction, the higher the speed of satisfaction becomes, the less consumption goods are required and vice versa. Such a consumption pattern implies that consumers would prefer to satisfy their needs via conspicuous goods rather than via subsidiaries. This would especially be true of the countries where there is a large income distribution difference. As mentioned in hypothesis 1, poor in lower bound groups may prefer to replace subsidiaries with conspicuous consumption in order to increase the speed of satisfaction so as to attain that of the rich in upper bound. One difference is that the frequency of using conspicuous goods would necessarily be less than with subsidiary ones due to budget and time constaints.

This result emphasizes for under developed countries that relatively poor populations would have a lexicographic preference for subsidiaries and conspicuous consumption goods. To show this, let C_s and C_c are respectively the subsidiary and conspicuous consumption amounts of i under the assumption that both commodities are eligible to use for the satisfaction of the same need within $i{:}c, s \in \mathbf{I}$. Furthermore, Y, p_s and p_c represents income, unit price of C_s and C_c . Suppose that there is three p_c such as when $p_c^+ \geq p_c$, p_c^- when $p_c^- \leq p_c$ and $p_c^{\pm} = p_c$

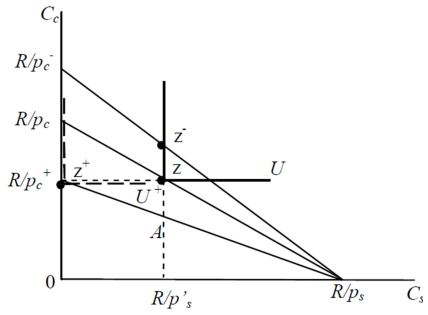


Figure 1: Lexicographic preference for subsidiaries and conspicuous consumption goods

The implication of the equation is shown in Figure 1 where the consumer might have the equilibrium z, z^- and z^+corresponding to U and U^+. At first, the consumer does not want to change the consumption amount C_s but is willing to increase C_c when $p_c^- \leq p_c$ at z^-whilst our consumer is ready to decrease C_s

in order to consume same amunt of $C_c$when $p_c^+ \geq p_c$ with $z^+ \in](R/p^c), A[$ since it is supposed that $\forall C_s \in]0, R/p_s^{'}[$ we have $z^+ = minU^+(C_s, C_c)$.

However, the satisfaction derived from C_s at z^+ is minimum but not zero (*i.e.* absence of a corner solution). The traditional analysis in Figure 1 does not have sufficient basis to explain this condition. In fact, a consumption decision would also depend on the characteristics of C_s and C_c by which it can be assumed that there exists a feasible region of characteristics combinations determined by the set K and by prices p_s, p_c, p_c^+, p_c^- can be indicated in the characteristics space. For the case of two characteristics: K_1 as the socio-economic characteristic of a good. K_1 is supposed to be determined by each reference groups. That is to say,K_1 for C_s and C_c of poor in lower bounds is small (*vice versa* for relatively rich person). K_2 is simply the rest of the characteristics as the complement of K_1 in K.

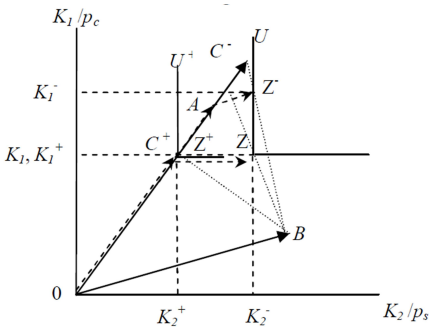


Figure 2: Characteristics combinations determined for subsidiaries and conspicuous consumption goods

According to the Lancaster(1966) model given in Figure 2, the combinations that are ultimately chosen depend on the utility function and the indifference curves U, U^+ of the individual. C^-, C^+ and B correspond to the maximum amount of characteristics that the consumer can buy with the income budget for C_c when p_c^- , C_c when p_c^+ and C_s when p_s respectively. Let BC^+ and BC^- are the two efficiency frontiers respectively for p_c^+ and p_c^-. The optimum is placed in Z^- when $p_c^- \leq p_c$ where the consumer buys the amount of C_c and C_s correspond to the total characteristic vector $\overrightarrow{OA}$ and the $\overrightarrow{AZ^-}$. As can be seen in Figure 2, $Z^- > Z$ owing to $K_1^- > K_1$. When $p_c^+ \geq p_c$ a consumer buys characteristics of (K_2^+, K_1^+) at C^+ which strictly dominate any other accessible ones could be obtained by the combination of C_s and C_c. Hence BC^+ becomes the new efficiency frontier where a two-fold analysis is required:

i) When income decreases, the optimum points move to the north-west as parallel to initial efficiency frontiers. Thus, analyses in Figure 1 and 2 clearly point out that the condition that keeps consumers' happiness at a certain level when their income decreases or when there is an increase in prices.

However, one may argue that the amount of chance in satisfaction by means of substitution between C_s and C_c has technical limits.

ii) Thus, an increase in income and/or decrease in prices would yield higher utilities only when C_s and C_c could be used to satisfy the same needs. In other words, C_s and C_c are technically limited to satisfy a given need. These limitation would come, for example, from time or budget constairns or from lack of information on usage of new technology or from social norms(Alpman, 2013)...etc.

To explain this later condition (ii), the equilibrium will be established only at z_1, z_2, z_1^- or z_1^+after the price of C_c decrease or income increase as seen in Figure 3:

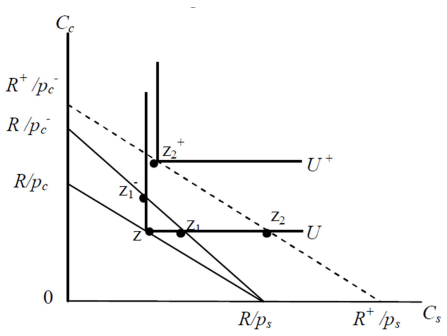


Figure 3: Preference under limitations for subsidiaries and conspicuous consumption goods

The line $[R/p_c, R/p_s]$ in Figure 3 gives the intial budet. The decrease in prices C_c yileds to move the initial budget to $[R/p_c^-, R/p_s]$. When income increases, let the new budget line is defined at $[R^+/p_c, R^+/p_s]$. Following the analysis given at Figure 1, if the substitution between C_c and C_s is technically bounded, individual will necessarily consume at z_1 instead of z_1^- when $p_c^- \leq p_c$. On the other hand, individual consumes at z_2 when $R^+ > R$ since z_2^+ is inaccessible due to technical limit where $U < U^+$. In terms of characteristics, an equilibrium at z_2 implies less K_1 than that can be obtained at z_2^+.

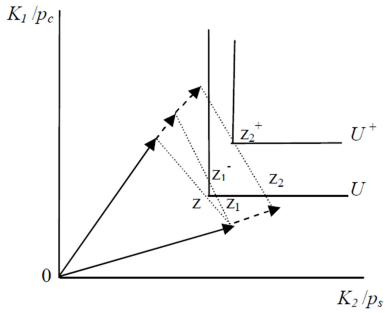


Figure 4: Characteristics combinations under limitations for subsidiaries and conspicuous consumption goods

Where, $U^{+}(z_2^{+}) > U(z_2)$ in Figure 4.

5 Conclusion

It is hard to differentiate proper economic methodology from that of other sciences. In this paper, an interdisciplinary analysis is proposed to solve one major problem in economics by synthesizing the methodology of physics with economics by way of analyzing well-established psychological-economic concepts of decision-making. Speed of satisfaction, satisfaction wave equation, relativistic satisfaction equations are all introduced so as to better analyze the relationship between satisfaction and utility through emphasizing the role of consumption and time use values. The analysis presented here has very interesting implications and opens up new perspectives on the happiness research. We find out that the condition of an increase in consumer happiness owing to higher income levels would depend on the ability to substitute subsidiaries with conspicuous ones, so long as they have common characteristics to satisfy given specific needs. This phenomenon is particularly applicable to developing countries, in describing how consumers may wish to buy more luxury goods but may prefer to use them only when it is necessarily required due to budget and time use constraints. In terms of frequency of use, this consumption pattern implies that consumers would draw more satisfaction from less usage of conspicuous goods than from higher usage of subsidiaries. The idea underpinning this behavior is attain the satisfaction level of upper income groups by means of reducing the time required to reach upper groups' utility level. These results are consistent with Engel's law stating that as income rises, the proportion of income spent on basic needs falls. However, they stand in contrast to Adam Smiths' idea that *basic needs* are the real happiness of human life (in the *Theory of Moral Sentiments*) especially in underdeveloped countries.

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