Cordes, Christian; Su, Tong-Yaa; Strimling, Pontus

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Going Through a Crisis: Firm Development and Firm Size Distributions

Christian Cordes
Tong-Yaa Su
Pontus Strimling
Going Through a Crisis:  
Firm Development and Firm Size Distributions  

Christian Cordes\textsuperscript{a}, Tong-Yaa Su\textsuperscript{a}, Pontus Strimling

\textsuperscript{a} University of Bremen, Faculty of Business Studies and Economics, Hochschulring 4, 28359 Bremen, Germany (Corresponding author)

\textsuperscript{b} Mälardalen University, Department of Mathematics and Physics, Högskoleplan 1, Rosenhill, Västeås, Sweden

(c.cordes@uni-bremen.de, tong-yaa.su@uni-bremen.de, Pontus.strimling@mdh.se).

Abstract

This paper relates firm-level processes and size distributions of firms at the industry level. An analytically tractable model explores how firm growth, exit, and spinoff activity in combination with systematically appearing growth crises in organizational development translate into specific steady-state firm size distributions (FSDs). Based on anthropological, social-psychological, and economic evidence on the effects of increasing group size on performance, the model features a critical organizational size that triggers these growth crises. Together, these processes generate empirically observable size distributions of firms including right-skewed and Pareto distributions as well as self-reinforcing spinoff processes that affect an industry’s FSD.

\textit{Keywords:} Firm Size Distributions, Firm Growth, Critical Firm Size, Industry Evolution

\textit{JEL classification:} L11, D21, C61
1. Introduction

The distribution of firms by size is an interesting empirical phenomenon to be explained by economists and an important aspect of industrial structures and dynamics (see, e.g., Hart and Prais, 1956; Simon and Bonini, 1958; Mansfield, 1962; Ijiri and Simon, 1974; Sutton, 1997; de Wit, 2005). In fact, it is one of the most fundamental stylized facts in industrial economics (for a comprehensive survey of the relevant literature see Coad, 2009). In addition, there is a long-standing interest of scholars in the theory of the firm in the determinants and consequences of various firm-level developments (e.g., Penrose, 1959; Ijiri and Simon, 1967; Albach et al., 1984; Audretsch and Mahmood, 1994; Langlois and Robertson, 1995; Langlois, 1998; Foss, 2000; Witt, 2007; Witt and Schwesinger, 2013). As the evolution of business organizations moves through typical phases, one observation is that they experience common problems and challenges arising systematically in the course of their development (see Churchill and Lewis, 1983; Greiner, 1998; Cordes et al., 2010).

It is to these strands of literature that our contribution adds new insights: based on certain assumptions concerning firm-level developments, especially as to the existence of a critical organizational size that systematically induces a growth crisis, the proposed analytical model produces several empirically well-established shapes of firm size distributions (FSDs). It captures firm dynamics involving certain patterns of growth and exit of incumbent firms as well as entry of new firms via spinoffs. The cumulated results of these firm dynamics give rise to specific steady-state FSDs including both right-skewed and Pareto distributions as well as self-reinforcing spinoff dynamics that affect an industry’s FSD. Our theoretical concept of industry FSD accounts for a broader range of dynamic processes at the firm level as compared to existing avenues and it does not need specific stability devices to impose a steady state.

Early work in the field of FSDs has been done by Gibrat (1931) on the size of French firms in terms of employees and Hart and Prais (1956) with data on UK firms whose empirical evidence led them to the conclusion that firm sizes follow a right-skewed, approximately lognormal distribution (also Cabral and Mata, 2003). This kind of distribution is, however, only one possible candidate among several skewed distributions. Other researchers opted for Pareto or exponential distributions for they are – compared to the lognormal distribution – better suited to describe the upper tail of the firm-size distribution (see, e.g., Steindl, 1965; Ijiri and Simon, 1974; Stanley et al., 1995). A crucial problem with this type of distribution is, however, that the empirically observed FSDs are characterized by many more middle-sized firms and fewer very large firms.
(e.g., Vining, 1976). The mixed results of these inquiries hint at another problem: regularities in FSDs observed at the aggregate level of one industry may not hold in others or with sectoral disaggregation (e.g., Dosi et al., 1995; Bottazzi et al., 2011). Our model, therefore, suggests several factors that influence FSD and that may differ across industries leading to different patterns of size distributions.

Moreover, in order to substantiate the assumption of a critical organizational size in firm development, we draw on insights from anthropology, social psychology, and economics. A good deal of research in these disciplines has been done on the effects of increasing group size (e.g., Levine and Moreland, 1998; Ostrom, 2000; Mukhopadhaya, 2003; Marlowe, 2005; Forsyth, 2006). While people find it easy and natural to function prosocially in small groups, problems, such as free-riding, bickering, coordination failures, and misbehavior, arise when group size increases: in the case of firm organizations, these then undergo a growth crisis. Based on this evidence, we argue that firms systematically reach a critical organizational size in the course of their development. By doing so, we root industry-level observations in behavioral reality at the firm level. As shown, the existence of a critical firm size can have far-reaching repercussions on the shape of an industry’s FSD.

The article is organized as follows. Section 2 introduces an analytically tractable model of organizational development and industry evolution that produces FSDs found in empirical work. Next, based on this formal analysis, Section 3 offers a discussion that shows how different patterns of firm growth, exit, and spinoff activity in combination with a critical organizational size translate into specific steady-state FSDs. Section 4 concludes the paper.

2. A model of organizational development and industry evolution

In this section, we devise a formal model that provides original insights into industry evolution including firm growth processes, exit, spinoff generation, and aspects of organizational development, such as growth crises. Moreover, it is capable of explaining important aspects of the stylized facts that emerge from empirical work in the field of firm size distributions.

Our indicator of firm size is employment. Let firm size \( i \) \((i = 1,2,\ldots,n)\) be the number of employees within a firm, given that there are \( n \) different firm sizes in an industry. \( t \) \((t = 1,2,\ldots,k)\) captures progressing time, while \( k \) is the time step in which the equilibrium
frequencies of firm sizes is reached. Accordingly, \( e_{i,t} \) gives the number of firms with a certain number of employees \( i \) at time \( t \). \( g_i \in [0; 1] \) captures our definition of firm growth: it represents the fraction of firms of size \( i \) that grows into the next firm size category, \( i + 1 \). The fraction of firms that exits at each size \( i \) is given by \( d_i \in [0; 1] \).\(^1\) Firms do not decline. We first derive a recursion that determines the number of firms of size \( i \) in the next time step, \( e_{i,t+1} \), for \( i > 1 \), i.e., for the time being excluding firms of the smallest size \( i = 1 \):

\[
(1) \quad e_{i,t+1} = e_{i,t}(1 - d_i)(1 - g_i) + e_{i-1,t}(1 - d_{i-1})g_{i-1}.
\]

We can now calculate the equilibrium frequency of each firm size after \( k \) iterations. \( \hat{e}_i \) denotes the equilibrium frequency of firms of size \( i \) in an industry. At equilibrium, the number of firms of a certain size \( i \) does not change, so that \( e_{i,t+1} - e_{i,t} = 0 \). Setting \( \hat{e}_i = e_{i,t+1} = e_{i,t} \), we obtain for \( i > 1 \):

\[
(2) \quad \hat{e}_i = \frac{e_{i-1,t}(1 - d_{i-1})g_{i-1}}{d_i + g_i - d_i g_i}.
\]

In a similar manner, with \( i > 2 \), we can solve for \( \hat{e}_{i-1} \) to get

\[
(3) \quad \hat{e}_{i-1} = \frac{e_{i-2,t}(1 - d_{i-2})g_{i-2}}{d_{i-1} + g_{i-1} - d_{i-1} g_{i-1}}.
\]

Given Equations (2) and (3) and \( i > 2 \), \( \hat{e}_i \) can also be expressed by

\[
(4) \quad \hat{e}_i = \frac{e_{i-2,t}(1 - d_{i-2})g_{i-2}(1 - d_{i-1})g_{i-1}}{(d_{i-1} + g_{i-1} - d_{i-1} g_{i-1})(d_i + g_i - d_i g_i)}.
\]

which can be simplified so that \( \hat{e}_i \) depends on \( \hat{e}_1 \), the equilibrium frequency of firms of the smallest size \( i = 1 \):

\[
(5) \quad \hat{e}_i = \hat{e}_1 \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1} + g_{j+1} - d_{j+1} g_{j+1}}.
\]

\(^1\) We assume exiting to take place prior to firm growth processes.
PROOF: Equation (5) is true for \( i > 2 \).

By complete induction, we show that Equation (5) obtains the correct results for all given \( i \). First, we start with the smallest firm size \( i = 3 \). In that case, Equation (5) yields

\[ (5a) \quad \hat{e}_3 = \hat{e}_1 \frac{(1-d_3)g_1(1-d_2)g_2}{(d_2+g_2-d_2g_2)(d_3+g_3-d_3g_3)}, \]

which is the same result as the one following from Equation (4). Equation (5), therefore, is true for \( i = 3 \). Since the equilibrium case is analyzed here, \( \hat{e}_1 = e_{1,t} \) also holds.

Second, we investigate the case including the largest firm size plus one employee, i.e., \( i = n + 1 \). In that case, Equation (5) gives

\[ (5b) \quad \hat{e}_{n+1} = \hat{e}_1 \prod_{j=1}^{n} \frac{(1-d_j)g_j}{d_{j+1}+g_{j+1}-d_{j+1}g_{j+1}} = \hat{e}_1 \prod_{j=1}^{n} \frac{(1-d_j)g_j}{d_{j+1}+g_{j+1}-d_{j+1}g_{j+1}}. \]

Supposing Equation (5) is true, we get

\[ (5c) \quad \hat{e}_n = \hat{e}_1 \prod_{j=1}^{n-1} \frac{(1-d_j)g_j}{d_{j+1}+g_{j+1}-d_{j+1}g_{j+1}} \]

by inserting \( i = n \). Assuming, in turn, Equation (5c) is true for \( n \), we can show that it is also true for \( n + 1 \):

\[ (5d) \quad \hat{e}_{n+1} = \hat{e}_1 \prod_{j=1}^{n} \frac{(1-d_j)g_j}{d_{j+1}+g_{j+1}-d_{j+1}g_{j+1}} \cdot \frac{(1-d_{n+1})g_{n+1}}{d_{(n+1)+1}+g_{(n+1)+1}-d_{(n+1)+1}g_{(n+1)+1}} \]

which gives us the same result as in (5b). Thus, Equation (5) is true for \( i = n + 1 \).

To conclude, Equation (5) yields the correct results for the smallest firm size, \( i = 3 \), and the largest size plus one employee, \( i = n + 1 \). This implies that Equation (5) is also true for all firm sizes in between, \( i = 4, 5, \ldots, n \). Hence, with the help of this complete induction, we have demonstrated that Equation (5) holds for \( i > 2 \).
Next, the occurrence of potential spinoffs is introduced to the model. Let $b_i \in [0; 1]$ be the share of firms of size $i$ that generates a potential spinoff, i.e., an employee considering leaving the firm to start her own business in the same industry. We assume firms of the smallest size $i = 1$ to not generate potential spinoffs. Accordingly, the total number of potential parent firms at equilibrium, measured by $\hat{R}$, is given by

$$\hat{R} = \sum_{i=2}^{n} \hat{e}_i \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{a_{j+1} + g_{j+1} - d_{j+1}g_{j+1}}. \quad (6)$$

Then, the total number of potential spinoffs at equilibrium, denoted by $\hat{B}$, is expressed by

$$\hat{B} = \sum_{i=2}^{n} b_i \hat{e}_i \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{a_{j+1} + g_{j+1} - d_{j+1}g_{j+1}}. \quad (7)$$

Accordingly, $B_t$ gives the number of potential spinoffs and $R_t$ the number of parent firms for these potential spinoffs in the industry at time $t$. In addition, we account for other market entrant types, such as startups or diversifying firms originating from other industries. $s_1$ measures the level of continuous entry activity of this kind. All firms enter at minimum size. We can now define a recursion that determines the number of firms of the smallest size, $i = 1$, in the next time step, $t + 1$:

$$e_{1,t+1} = e_{1,t}(1-d_1)(1-g_1) + s_1 + \delta \frac{B_t}{R_t}. \quad (8)$$

On the rightmost side of this expression, the total number of actually realized spinoffs at time $t$ is given by the term $\delta \frac{B_t}{R_t}$. Within this expression, the ratio $\frac{B_t}{R_t}$ measures the number of potential spinoffs per firm in an industry at time $t$. Then, parameter $\delta$ scales spinoff activity by determining how the spinoff potential per firm translates into real market entry. In this context, $\delta$ may vary across regions, cultural environments, or stages of an industry’s life cycle.

Setting $\hat{e}_1 = e_{1,t+1} = e_{1,t}$, $\hat{B} = B_t$, and $\hat{R} = R_t$, we obtain for the equilibrium frequency of firms of size $i = 1$:  

6
Solving for \( \hat{e}_1 \), we have

\[
\hat{e}_1 = \hat{e}_1 (1 - d_1) (1 - g_1) + s_1 + \delta \frac{\sum_{i=2}^{n_b} b_i \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1}^* + g_{j+1} - d_{j+1}g_{j+1}}}{\hat{e}_1 \sum_{i=2}^{n} \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1}^* + g_{j+1} - d_{j+1}g_{j+1}}).
\]

Given \( \hat{e}_1 \), we can determine the equilibrium frequency of firms of any size \( i \), \( \hat{e}_i \), independent of \( \hat{e}_1 \). Considering Equations (5) and (10), we thus get

\[
\hat{e}_i = \left( \delta \frac{\sum_{i=2}^{n_b} b_i \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1}^* + g_{j+1} - d_{j+1}g_{j+1}} + s_1}{\sum_{i=2}^{n} \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1}^* + g_{j+1} - d_{j+1}g_{j+1}}} \right) \frac{1}{d_1 + g_1 - d_1g_1} \prod_{j=1}^{i-1} \frac{(1-d_j)g_j}{d_{j+1}^* + g_{j+1} - d_{j+1}g_{j+1}},
\]

which gives us the equilibrium number of firms of size \( i \) in an industry. Finally, for an equilibrium to exist, the condition \( \sum_{i=1}^{n_B} \hat{e}_i d_i = s_1 + \frac{\delta}{R} \) must hold, i.e., the inflow of firms has to equal the outflow of firms in equilibrium.

### 3. Firm-level dynamics and industry-specific firm size distributions

In this Section, we derive some interesting insights from our formal model relating firm-level dynamics and steady-state FSDs at the industry level. It gives insight into what kind of firm dynamics may be underlying specific FSDs. Parameters \( g_i, d_i, \) and \( b_i \), capturing firm growth, exit, and spinoff generation respectively, proof to be crucial for matching empirically observed FSDs with the model’s predictions. We discuss several scenarios to analyze the interplay of these parameters and FSDs.
3.1. Growth crises in firm development and right-skewed FSDs

Group and firm size affects many aspects of group life and organizational performance. As a group or firm grows larger, many problems appear: members of larger groups tend to be less satisfied with their membership, are absent more often, contribute less often to group activities, are less likely to cooperate with one another, and more likely to behave opportunistically (e.g., Markham et al., 1982; Albanese and van Fleet, 1985; Kerr, 1989; Levine and Moreland, 1990; Ostrom, 2000; Levine and Moreland, 1998; Forsyth, 2006). A decline in a group’s members willingness to cooperate with increasing group size is a common phenomenon in social psychology and experimental economics (e.g., Olson, 1994; Güth and van Damme, 1998; Fehr and Gächter, 2000; Spoor and Kelly, 2004). Hence, coordination problems and motivation losses are more frequent in larger, more anonymous groups. Moreover, employees who are willing to contribute to the benefit of the organization and who are motivated by a cooperative culture, rather suddenly change behavior when the firm reaches a critical group size (Schelling, 1972; Grofman, 1974; Gladwell, 2000; Card et al., 2008; Cordes et al., 2014). Given this evidence, we assume that in many industries firm organizations systematically undergo a growth crisis in the course of their development (also Churchill and Lewis, 1983; Greiner, 1998; Cordes et al., 2010) and we expect this observation at the firm level to have repercussions on industry-level FSDs.

On the other hand, bigger organizations may reap the productive potential that larger groups offer beyond the critical group size: for example, a corporation may recover from a growth crisis when organizational restructuring enables further firm growth by keeping its single organizational units below the critical size (e.g., Witt, 2000, 2007). Other driving-forces of subsequently higher growth rates of larger firms comprise economies of scale (see Jovanovic and MacDonald, 1994), the reaching of a critical technological minimum size (see Pratten, 1971; Audretsch and Mahmood, 1994), success-breeds-success dynamics (e.g., Klepper, 1996), or the absence of financial constraints for bigger corporations (e.g., Cabral and Mata, 2003). These larger organizations would then feature corporate cultures relying more on formal leadership, hierarchy, and monitoring.

Taking these observations together, we suggest the fraction of firms growing into the next size category, \( g_i \), to first decline with growing firm size, then to approach a critical organizational size at which \( g_i \) reaches a minimum, and to subsequently increase again due to positive size effects. If we assume such a crisis-induced growth pattern to systematically appear in firm development, we can account for this regularity by defining a specific relationship between firm growth, \( g_i \), and
firm sizes $i$, i.e., we define for every observed size class the share of firms that grows into the next category ($g_i := g(i)$). The simplest shape of such a correlation is a U-shaped relation between firm growth and firm size, as shown in Figure 1. It includes a firm growth crisis around a critical size of 40-50 employees, a number that can be entertained by anthropological evidence (see below). Moreover, we assume an asymmetric firm-level relationship between growth and size that accounts for the observation that larger corporations have a higher probability to grow into the next size category relative to small ventures, i.e., once they survive and grow beyond the critical size, organizations may experience success-breeds-success dynamics and reap size-related advantages that spur growth. Figure 1 also features different levels of growth rates ($g_i^a > g_i^b > g_i^c$). Moreover, there is a constant stream of small new firms entering the industry at the minimum firm size, as measured by parameter $s_1$.

The corresponding FSDs at equilibrium for a certain range of firm sizes produced by the model are given in Figure 2 ($\hat{e}_i := \hat{e}(i)$). These distributions become skewed to the right, implying the mass of firm size observations concentrated on the left of the mean and fewer large enterprises in the relatively longer upper tail. Therefore, the assumed asymmetric pattern of organizational growth including a crisis in development translates into right-skewed distributions of firm sizes that may approach an approximately lognormal distribution where we observe many middle-sized firms and more small than large enterprises. This result is in accordance with empirical observations in several industries (e.g., Gibrat, 1931; Hart and Prais, 1956; Hashemi, 2000; Cabral and Mata, 2003; Growiec et al., 2008).

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1 See, for a similar idea, Simon (1955). For the sake of simplicity, we abstract from negative growth rates, i.e., shrinking firm sizes. Below, we will, however, account for certain patterns of firm exit.
Figures 1 and 2. Asymmetrically shaped relationship between firm growth and size (1) and the corresponding right-skewed FSDs (2) 

$$ (d_l = 0.001, b_i = 0, s_1 = 3, \delta = 10).$$

Growth dynamics at the firm level generate these specific FSD patterns. The reason for the observation of a relatively high number of middle-sized firms around the critical size lies in the relationship between $g_i$, the fraction of firms that grows into the next firm size class $i + 1$, and $g_{i+1}$, the fraction of firms leaving this class to grow into $i + 2$. Before an organization reaches the crisis-induced minimum of growth rates, $g_i > g_{i+1}$ holds, i.e., the number of firms entering size class $i + 1$ is greater than the number of firms leaving it. This effect causes firms to amass at sizes where this condition is met. In our model, this phenomenon is salient around the assumed critical firm size. If, on the other hand, $g_i < g_{i+1}$, more organizations leave size category $i + 1$ than entering it originating from class $i$. Consequently, the number of firms in class $i + 1$ decreases in this case. Therefore, until reaching the minimum of firm growth rates, the continuously decreasing fraction of firms that grows into the next category causes firm numbers to “pile-up” around these size classes. If we expect growth crises to systematically appear in firm development, they can manifest themselves in FSDs as described above.

Moreover, all steady-state FSDs shown in Figure 2 comprise the same total number of firms active in the industry. This final number of organizations is determined by the steady-state condition, $\sum_{i=1}^{\infty} \hat{e}_i d_i = s_1$ (for $b_i = 0$), where firm entry equals firm exit and the population of
firms stops growing. Given the disproportionate amassing of firms around the critical size for different levels of growth rates, this implies that the curves describing the right-skewed FSDs have an intersection at some point on the upper tail of the distribution, beyond which we see different levels of large firm observations (as illustrated in Figure 2). FSDs, therefore, differ as to the distribution of firm size observations over the range of size classes.

Another effect is underlying a – at first view – counterintuitive result: *ceteris paribus*, the lower the level of firm growth rates is \( g_l^a > g_l^b > g_l^c \), see Figure 1, the disproportionately higher is the number of observed firms around the critical firm size (see corresponding FSDs in Figure 2). This effect is due to different ratios of firm growth rates between neighboring size categories, \( \frac{g_l}{g_{l+1}} \), that vary between \( g_l^a, g_l^b \), and \( g_l^c \): \( \frac{g_l}{g_{l+1}} \) is higher, the lower the level of growth rates. For \( g_l > g_{l+1} \), i.e., for organizations that have not yet reached the minimum of growth rates, it holds that the higher \( \frac{g_l}{g_{l+1}} \), the disproportionately higher is the number of firms that can be observed in class \( i + 1 \).\(^1\) Accordingly, a relatively lower value of \( \frac{g_l}{g_{l+1}} \) implies that relatively more firms grow into the next size class, \( i + 2 \), preventing firms from amassing at certain size classes to the same extent and rendering the effect of a critical organizational size less pronounced. On the other hand, we then see more large firms active in the industry beyond the point of intersection of the FSDs on the upper tail. The concrete features of crisis-induced growth dynamics at the firm level, therefore, determine the steady-state distribution of organizations over the whole range of size observations.

In order to isolate the “piling-up” phenomenon of organizations around a critical organizational size, we assumed firm exit rates to be constant over the whole range of firm sizes \( d_i = 0.001 \). However, with a, for example, stylized inverted U-shaped relationship between \( d_i \) and organizational size with a maximum of firm exit at the critical firm size (for empirical evidence for such an exit pattern see, e.g., Boone et al., 2004), the disproportionate increase in firm numbers around this size may vanish and the resulting FSDs may be altered in significant ways. Exit dynamics of this kind are approximated in Figure 3 by different levels of inverted U-shaped exit rates with their maximums around the critical firm size and different levels of variance in exit rates. The underlying growth dynamics are taken from those described in Figure

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\(^1\) The opposite holds true beyond the minimum of growth rates: then, the ratio \( \frac{g_l}{g_{l+1}} \) is lower, the lower the level of growth rates. In this case, we see a disproportionate decrease in firm numbers. The strength of the “amassing effect” corresponds with the strength of the later decline in size observations.
1 above \(g_l^b\) and \(g_l^f\), i.e., the scenario combines a growth crisis and higher exit rates around the critical organizational size.

**Figures 3 and 4.** An inverted U-shaped relationship between firm size and exit (3) and the corresponding right-skewed FSDs (4)

\[g_i = g_l^b\text{ or } g_i = g_l^f, \quad b_i = 0, \quad s_1 = 3, \quad \delta = 10.\]

Figure 4 shows the FSDs generated by these differently shaped exit patterns in combination with growth dynamics including a critical organizational size. While one exit-growth pattern \((d_l^b\) and \(g_l^f\)) still allows for a “piling-up” phenomenon with a higher number of firm size observations around the organizational size irrespective of lower growth rates within this range (as compared to \(d_l^a\) and \(g_l^b\); \(g_l^b > g_l^f\)), the other pattern reduces firm numbers in the industry to an extent that prevents the amassing of firm observations around the critical size \((d_l^c\) and \(g_l^c\)). Moreover, differing with exit-growth patterns, we see different distributions of size observations over the whole range of classes. Depending on the patterns’ means and variances, we find more or less middle-sized or large firm organizations. For example, the probability to observe relatively large firms varies greatly across these exit-growth patterns.

Starting with Gibrat’s classic work (1931) on aggregate FSD, much empirical evidence suggests that right-skewed distributions, including the lognormal, are useful approximations describing firm sizes within an industry (also Simon and Bonini, 1958; Cabral and Mata, 2003;
Our model shows that for reasonable assumptions on firm-level dynamics, especially as to the existence of a crisis in organizational development, we yield right-skewed FSDs as found in this literature. Particularly in complex, uncertain business environments, this observation should be salient: firms then have to rely on a cooperative corporate culture that depends on the discretionary contributions of highly autonomous members to maintain flexibility of response, coordination, and competitive advantage (see Barney, 1986; Gittell, 2000; Cooter and Eisenberg, 2001; Rob and Zemsky, 2002). Since employees’ motivation to contribute services of this kind depends on organizational size, firms in industries characterized by such an environment should face such a pronounced critical size where corporate culture changes and developmental crises emerge. Our arguments and observations, therefore, lead to the following proposition:

**Proposition 1**: If a critical size inducing an organizational growth crisis exists in an industry, a right-skewed distribution describes the steady-state of the final firm size distribution.

Anthropology, social psychology, and economics provide some concrete numbers and observations for the existence of a critical group size that has deep roots in humans’ evolutionary past (e.g., Aronson et al., 2002; Robson and Kaplan, 2003; Dunbar, 2008; Ostrom, 2009). This evidence substantiates the argument developed above, informs the concrete definition of the shape of the functional relationship between firm size and growth in Figure 1, and finally enables empirical testing of the model’s theoretical predictions. For instance, Marlowe (2005) reviews the group sizes among hunter-gatherers whose way of life most closely resembles those of our Pleistocene ancestors. Based on a sample size of 294 cases, local residential groups (bands) averaged 48 (median 30) people. These local groups are nested within ethno-linguistic groups (tribes), whose sizes average 1749 (sample size 396). The author found no indication of local group sizes depending on resources. Instead, the upper limit on their size is determined by the frequency of bickering, reflecting an increase in free riding and opportunism. Hence, these findings suggest that hunter-gatherer bands tend to equilibrate at sizes around 50 individuals at the band-level and around 1750 individuals at the tribal-level. Both numbers constitute potential thresholds at which organizations face developmental crises and may, therefore, manifest themselves in an industry’s (possibly multimodal) FSD.

1 There is a human disposition to identify with larger, symbolically marked groups and their norms and institutions. Such groups still depend, however, upon the moral dispositions that help stabilize cooperation in local band-scaled groups as their constituents (see Richerson and Boyd, 2005).
Similarly, Dunbar (1993, 2008) showed that human social groupings exhibit unique distinct size and structure. Thereby, he draws on insights from different disciplines that indicate the existence of cognitive constraints on our ability to maintain social, personalized relationships at a given level of emotional intensity (also Sawaguchi and Kudo, 1990; Zhou et al., 2005). To a great extent, the evolution of primate brains was driven by the need to coordinate and manage increasingly large social groups. The finding that average species social group size correlates with relative neocortex size gives several expected critical sizes of human groups, some of which correspond with the observations of Marlowe (2005) above. We might expect that other human groups based on informal leadership and management by intensive face-to-face contacts and communications will tend to equilibrate at similar sizes.

In line with these observations, studies from economics on village scale commons management suggest that small, band-based systems can work well and be maintained by informal agreements, but that larger systems require formal rules as well as monitoring and sanctions to avoid crisis development (e.g., Ostrom, 2000, 2009). Band-sized groups may represent the limits of cooperation organized by more freewheeling means in organizations and would thus define the approximate range of a lower critical firm size. Most firms that significantly exceed this first critical size will begin to fashion more formal leadership, rule bound management (including monitoring), and explicit norms and institutions as well as subdivisions to proceed through their growth crisis when they get above a size of 50 or so, if the analogy with bands and tribes is correct.¹

Hence, organizational developments common to all firms might well be stronger determinants of growth than production-related factors, which differ for companies producing different goods (also Stanley et al., 1996). Given the fundamentals of humans’ social psychology operating through the evolution of organizations’ cultures, firms should – in the course of their development – systematically face growth constraints and corresponding crises. Moreover, we expect these growth patterns to lead to specific FSDs.

¹ This observation is also given some anecdotal weight by the existence of tailored packages of management services offered by consultancies that aim at firms reaching a critical size at 50 employees. Similarly, in management circles, there has long been a verbal take that firms reach a critical size at approximately this number of employees.
3.2. Selection effects, exit rates, and Pareto FSDs

There are significant differences across industries in the distribution of firm sizes. Especially for data sets including large numbers of smaller firms, researchers have opted for exponential, Pareto, or power law distributions due to their better fit for the upper tail of the FSD (e.g., Steindl, 1965; Stanley et al., 1995). In many industries, small numbers of large firms coexist alongside monotonically increasing numbers of progressively smaller firms (e.g., Axtell, 2001; Dinlersoz and MacDonald, 2009). A lognormal distribution, for example, cannot reproduce such a pattern.

The intensity of competition is expected to significantly influence the final steady-state distribution of firm sizes. For instance, an industry that is characterized by a mature, stable business environment allows for investments in expensive capital goods for mass production leading to intensive (price-based) competition among firms. In such a stable setting, larger corporations do relatively better at exploiting existing possibilities, such as refinement, production, and execution, than smaller firms. Instead of relying on a cooperative corporate culture, organizations can then employ more formal mechanisms of coordination and control, such as rules, routines, and hierarchical modes of communication, to cope with increasing market selection (e.g., Thompson, 1967, p. 71; Crémer, 1993). In the course of an industry’s life cycle, the business environment can be expected to change in this direction: the emergence of a dominant product design (see Utterback and Suárez, 1993), an emerging high critical technological firm size (e.g., Audretsch and Mahmood, 1994; Jovanovic and MacDonald, 1994), or a shift toward process technologies (see Klepper, 1997) lead to fiercer competition among firms and favors larger corporations. In this context, issues of a critical organizational size are expected to be less relevant, for firm cultures would emphasize efficiency and monitoring instead of high, size-sensitive, levels of cooperation.

Our model shows how the intensity of competition translates into approximate Pareto distributions in industries. For this purpose, it captures selection effects on industry-level FSDs by incorporating certain exit dynamics at the firm level: (a) increasing overall competition is reflected by rising levels of firm exit over the whole range of firm sizes and (b) a relative advantage of larger firms leads to a relatively lower likelihood for these ventures to exit the industry. As shown in Figure 5, the level of competition-induced exit rates, $d_i$ ($d_i := d(i)$), increases step-by-step ($d_i^d < d_i^e < d_i^f$). Within the single levels of exit rates, we assume $d_i$ to
decrease with increasing firm size, i.e., small firms exit more often than larger ones ($\frac{dd_i}{dt} < 0$; see also Sutton, 1997). Again, growth rates over size classes, $g_i$, include a growth crisis in organizational development, as depicted by Figure 1.

\textbf{Figures 5 and 6.} The functional relationship between firm exit and firm sizes (5) and approximate Pareto distribution as a result of increasing competition (6)

\[(g_i = g_i^e, b_i = 0, s_1 = 3, \delta = 10).\]

Figure 6 displays the development of steady-state FSDs in an industry given a stepwise increase of competition intensity ($\hat{e}_i := \hat{e}(i); \sum_{i=1}^{\infty} \hat{e}_i d_i = s_1$ for $b_i = 0$). The assumed firm-level exit dynamics make the size distribution shift toward an approximate Pareto distribution. The fiercer competition becomes, as measured by increasing levels of $d_i$, the more the shapes of the FSDs resemble a Pareto distribution. The total number of firms active in the industry falls as well as the relative likelihood to observe larger firms. Klepper (1996; 1997), for example, empirically shows that in many infant industries the initial number of firms is high and then experiences a sharp decline in the course of industry development. The shifting curves in Figure 6 potentially represent a detail of such a shakeout process taking place in a maturing industry within a certain range of firm sizes. These observations are captured by our second proposition that pays tribute to the fact that industries may differ in selection intensity:
Proposition 2: *Ceteris paribus*, high exit rates in a fierce business environment lead to firm size distributions that approximate a Pareto distribution.

Consequently, given an increasing intensity of competition, the proposed model predicts a development toward approximate Pareto FSDs, which have been observed in many empirical studies (see Axtell, 2001; de Wit, 2005). We expect final FSD of this kind for industries characterized by a stable business environment and accordingly fierce competition, where a critical corporate size based on social dispositions of human agents presumably does not play the same role as in industries with a more volatile, dynamic business environment.

3.3. Crisis-induced spawning of spinoffs in an industry and a “level effect”

If organizations systematically face a growth crisis at a certain firm size, this potentially leads to the spawning of spinoffs. Some agents will leave the organization due to changes in corporate culture and the growth-induced crisis in firm development. With increasing firm size, a cooperative corporate culture is becoming more and more difficult to sustain and the final drop in the level of cooperation is motivating entrepreneurially minded agents to leave the organization to found a spinoff. For instance, Cordes et al. (2014) present a model of cultural evolution that shows that organizations reaching a critical size experience a collapse of a cooperative culture that triggers the exodus of personnel founding own firms. Garvin (1983) argues that employees start their own firms after becoming frustrated with their employers. Decreasing cooperativeness in the course of firm development as well as a growth crisis are sources of frustration. Hence, a parent firm’s evolving corporate culture and systematically appearing growth crises are both considered to be triggering mechanisms of spinoffs.

In our FSD model, we account for the systematic generation of spinoffs around a critical organizational size by assuming a certain stylized functional relationship between \( b_l \) (\( b_l := b(i) \)), denoting the share of firms of size \( l \) that host an employee considering leaving the organization to found a spinoff, and \( g_l \) (\( g_l := g(i) \)) capturing firm growth that includes a pronounced growth crisis in organizational development (as illustrated by Figure 1 above). Figure 7 displays such a relationship between \( b_l \) and \( g_l \), where crisis-induced low firm growth rates result in a higher share of firms of a certain size that generate a potential spinoff founder. As shown in Equation (8) in Section 2, the number of actually realized spinoffs, \( \delta \frac{B_{l}}{R_{l}} \), depends on the number of potential spinoffs per firm and \( \delta \), a parameter that scales spinoff activity in a market by determining the
number of actually realized spinoffs. δ may vary across industries, institutional set-ups, national cultures, or stages of an industry’s life cycle.

Accordingly, Figure 8 shows the right-skewed steady-state FSDs generated by the model given the additional entry of minimum-sized firms via spinoffs \((\hat{e}_i := \hat{e}(i); \sum_{i=1}^{\infty} \hat{e}_i d_i = s_1 + \delta \frac{\delta}{R}\) in the steady state). It can be seen that the total number of firms active in the industry is increasing. Moreover, if we assume exit rates, denoted by \(d_i\), to be constant over the whole range of observed firm sizes, the additional entry of small firms via spinoffs leads to a more than proportionate increase in the number of firms active in the market.\(^1\) This effect is due to second and later generation spinoffs that come out of these new firms in the course of their development. The industry, therefore, experiences a “level effect” as to entrepreneurial activity: the crisis-induced exodus of personnel at the firm level initiates a self-reinforcing industry-level process of firm creation that nonlinearly shifts regional or industrial development to a higher level of economic activity (also Garvin, 1983; Kenney and von Burg, 1999).

\[\text{Figures 7 and 8. The stylized relationship between } b_i \text{ and } g_i (7) \text{ and FSDs including spinoff generation (8)}\]

\[
(d_i = 0.001, b_i = 0.65e^{-4g_i}, s_1 = 3, \delta = 10).
\]

\(^1\) This holds true although FSDs may have intersections on the upper tail of the distribution.
As a result, we suggest the following proposition:

Proposition 3: Ceteris paribus, the generation of spinoffs via growth crises in organizational development reshapes right-skewed firm size distributions by a “level effect” that more than proportionately increases the number of firms active in an industry.

Successful regions or industries can be assumed to be driven by such a nonlinear dynamic in entrepreneurial activity, i.e., high rates of new firm formation are vitally important to their success (e.g., Christensen, 1993; Buenstorf and Klepper, 2009). The genealogy of, for instance, Silicon Valley start-up firms shows the importance of such a “fissioning” process based on spinoff activity (see Moore and Davis, 2004; Klepper and Sleeper, 2005; Klepper, 2010). A similar role of spinoffs as the driving-force of industry evolution is shown by Klepper (2002) for the automobile industry and by Kenney and von Burg (1999) for the LAN industry. The FSDs generated by our model based on the assumption of growth crisis-induced spinoff activity reflect this implication of self-reinforcing firm entry on industry evolution.

4. Conclusions
We have proposed an analytically tractable model of industry evolution that explored how firm-level processes, such as growth, exit, and spinoff activity, in combination with a critical organizational size translate into specific steady-state firm size distributions (FSDs) at the industry-level. The model generates a set of predictions that are in line with the empirical evidence on FSDs including right-skewed and Pareto distributions as well as a level-effect due to self-reinforcing spinoff dynamics that modifies an industry’s FSD. Moreover, we accounted for an anthropological, behavioral constant: the existence of a critical firm size systematically triggering growth crises in organizational development that arguably can manifests itself in FSD data via an “amassing phenomenon” of firm size observations around this critical size. By doing so, we delivered new theoretical explanations for some interesting features of empirically observable patterns in FSDs in industries.

Finally, a caveat is in order: obviously, a steady-state approach to FSDs can only explain size distributions in industries that are in, or moving toward, a steady-state. In practice, many industries’ FSDs may be in an evolving state. Hence, future research should focus on transitional states before the steady-state sets in, a procedure that can be based on an extended version of the
proposed model. This avenue will facilitate the dynamic analysis of some aspects of an industry’s life cycle.

References


