Mann, Katja; Davenport, Margaret

Conference Paper

Demography, Capital Flows and Asset Allocation over the Life-cycle

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Demographic Change and the Macroeconomy II, No. D24-V2

Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association

Suggested Citation: Mann, Katja; Davenport, Margaret (2016) : Demography, Capital Flows and Asset Allocation over the Life-cycle, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Demographic Change and the Macroeconomy II, No. D24-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:
http://hdl.handle.net/10419/145948

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

www.econstor.eu
Abstract

This paper studies the effect of population aging on portfolio choice, asset prices and international asset trades. In a multi-period OLG model, we analyze how an increase in longevity or a decrease in fertility in a country affects the demand for safe and risky assets. In a closed economy, given a fixed supply, the riskfree rate falls and the risk premium rises, because retirees prefer to hold a larger share of safe assets in their portfolio than working-age households. In a financially integrated world, countries with more extreme demographic trends export risky assets and import safe assets. We quantify these effects for the United States.

JEL Classification Numbers: G11, G15, J11

Keywords: Demographic change, portfolio choice, international asset trades, pension system sustainability
1 Introduction

Many societies are currently undergoing major changes to their demographic structure. Low birth rates and increasing longevity together shift the mass of the population distribution towards the elderly: The world population share of people aged 60 and older, currently at 12.3%, will rise to 21.5% by 2050. The old-age dependency ratio (the ratio of people older than 64 to the working-age population) is projected to rise from 12.6% in 2015 to 25.6% in 2050 (UN Population Division 2015). But while all world regions are aging, the trend and level of this transformation differs over time. Even among developed countries, projections range from an old-age dependency in 2050 of 36.9% in the United States to 70.9% in Japan.

Population aging will have drastic consequences for the savings behavior and portfolio choice of households. Agents with longer life expectancy will desire to save more during their working years, preparing for a longer period of dis-saving when old. In addition, the portfolio composition be affected by changing age structure of the population over time. As we document below, the choice between assets that are considered relatively safe and risky is age-dependent: agents increase their portfolio share of risky assets over the course of their working life, but shift towards safe assets when retired.\footnote{One common reasoning goes that agents facing a short remaining life period cannot smooth income shocks as well and therefore suffer greater losses of utility. See Jagannathan and Kocherlakota (1996) for an economic evaluation of this and other explanations.} A larger share of retirees in the economy therefore implies a stronger relative demand for safe assets. Declines in fertility will also shift the balance between savers (young cohorts) and dis-savers (old cohorts), generating additional effects on asset prices and returns. The extent of which aging influences asset holdings, portfolio allocation and prices will additionally depend on the pension system of the country and on the borrowing capacities offered by the domestic financial system. In this way, even countries with similar demographic trends may experience very different dynamics of asset demand.

As a consequence, there will be cross-country differences in autarky asset
prices. In a financially integrated world, we will observe cross-border asset flows in risky and safe assets, which may respond differently to demographic change. Aging societies will want to export risky assets while importing safe assets. Countries with a less drastic population trend should become exporters of safe assets and importers of risky assets.

The exact nature of demographic-induced international asset trade and its effect on asset prices will depend not only on the life-cycle savings and structure of the portfolio choice problem. Additionally, one must consider the aggregate implications of evolving relative size of cohorts and how this compares to demographic trends in the rest of the world. It is the interaction of these elements, along with the impact of state-supplied pension systems and asset supply that we will explore in this paper.

We consider the joint effect of increases in longevity and decreases in fertility or population aging in a structural model of overlapping generations of households and model their portfolio choice over the life-cycle. By varying the parameters that capture demographics, we show the effect of population aging on portfolio choice and asset prices first in a closed economy, then in a small open economy. In a closed economy, when supply of assets is fixed, all demographics-induced changes in the demand for assets are reflected in return rates changes. In an open economy, where the country takes the rest of the world’s return rates as given, asset trades result. We quantify these effects by calibrating the model to the United States. In a world where only the U.S. is aging as projected, whereas all other countries have a stable population distribution, the U.S. would experience sustained capital inflows as households save more for in expectation of increased longevity. In addition, the composition of these inflows into safe and risky assets is also affected. The share of safe assets in total wealth increases with demographic projections to 2050 in excess of 50%. This is mirrored by a decline in the share of risky assets in total wealth. These results are stronger for continued demographic trends to 2100. The results offer a novel demand-side explanation for the low interest rates that are currently observed in the U.S. and other developed countries,
while making a prediction about future bond and stock market returns.

This paper is a work in progress. In future versions of this paper, we will consider the important role for changing asset supply with demographic change, which we abstract from in our baseline approach. In addition, we highlight the transition for the United States as a small open economy experiencing demographic change against a stable rest of the world. Important next steps will be threefold. First, we will consider how our results will change if the rest of the world is also aging. Here, we pay special attention to the relative demographic trends among regions in the world. Second, we consider a small open economy in our baseline model; at a later stage, we would extend this model to a two large economy framework in order to determine important outcomes for demographic change impact on asset prices, comparing two large regions of the world: the US and Europe. Lastly, we will incorporate a more realistic analysis of state-supplied pension systems in order to assess how the this element changes our predictions for cross-border trades in risky and safe assets.

In what follows, we first provide an overview of the related literature. Next, we set up the model. We describe how we calibrate the benchmark version of the model to match the U.S. data. Then, we present simulation results for the effects of demographic change on key parameters of the benchmark model. We also outline how we are going to extend the benchmark model.

2 Review of Existing Research

Our research is at the intersection of three main strands of literature: Life-cycle theory, portfolio choice and the literature on demographics and international capital flows.

Life-cycle theory aims to explain the savings behavior of individual agents over the course of their lives, and across cohorts. In life-cycle models of saving, consumption smoothing motives induce agents to save during working life and
to dis-save during retirement (Modigliani and Brumberg, 1954, 1990; Ando and Modigliani, 1963). Economic growth may reduce the need for life-cycle saving. A hump-shaped age-earnings profile over the life-cycle, buffer stock savings of the young and retirement risk or bequest motives of the old can be incorporated to bring the baseline model closer to the data. (see e.g. Yaari, 1965; Kotlikoff, 1988; Gourinchas and Parker, 2002). The effect of changes in the dependency rate on life-cycle savings have been widely studied. Mason (1981, 1987) and Fry and Mason (1982) analyze the effect of increased population growth, which should have the opposite effect from a drop in fertility. Lee et al. (2000) and Bloom et al. (2003) study the role of a higher life expectancy. Population aging should increase aggregate savings in the short-run (working age households are saving for a longer retirement period) while decreasing it in the long-run (a larger relative size of cohorts that are dis-saving). A drop in fertility will decrease aggregate savings, given the larger relative size of the older cohort.

Over the course of their life, agents adjust the composition of their portfolio. The literature on portfolio choice over the life-cycle explores the optimal mix between safe assets (like cash or government bonds) and risky assets (like stocks) at various stages of life. Merton (1971) shows that when labor income substitutes for safe asset holdings, agents should invest a constant fraction of their wealth in risky assets. Cocco et al. (2005) characterize optimal portfolio choice when labor income is risky and positively correlated with the returns to the risky asset. In a society with a larger share of old agents, demand for safe assets is relatively higher, which should decrease the riskless rate and increase the risk premium. Papers studying the effect of demographic change on asset prices often focus on the U.S. “baby boomer” generation (e.g. Brooks (2000, 2006), Abel (2001, 2003), Poterba (2001), Ang and Maddaloni (2005)). While generally agreeing on the direction of change in asset prices, they differ in their estimates of the size of the effect.

A third strand of literature studies how international capital flows can result from cross-country differences in the aggregate demand and supply of savings. In a small open economy, higher longevity in one country will increase
the aggregate supply of savings by the working-age population. In a transition period, when the young are saving more, but the old do not have a higher retirement wealth to run down, the country will run a current account surplus. Once the new steady state with higher life expectancy is reached, the surplus will turn into a deficit. The net foreign asset position of that country will increase permanently. A decrease in fertility will likewise result in a temporary current account surplus, since the number of savers decreases while the number of dis-savers stays unchanged. This has been analyzed in structural models by Brooks (2003), Domeij and Floden (2006), Krueger and Ludwig (2007), Backus et al. (2014) and most recently by Barany et al. (2015). Börsch-Supan et al. (2006) study the effect of pension reform in an open economy. Empirical studies include Taylor and Williamson (1994), Higgins and Williamson (1997), Higgins (1998), Lane and Milesi-Ferretti (2002) and Chinn and Prasad (2003).

To our knowledge, we are the first to combine the insights of these three strands of literature into a comprehensive model that can explain the effect of population aging on international portfolio choice and allows to study the implications of different policies. There is one empirical paper, De Santis and Lührmann (2009), which finds that a higher old-age dependency induces net equity inflows and net outflows of debt instruments. This is in line with what we are expecting to find.

3 Stylized Facts

Demographic change is a universal phenomenon: Figure 1 shows data and projections on longevity and fertility for different world regions. While all regions are characterized by an increase in longevity and a decrease in fertility, they differ in the speed and timing of demographic change. Of all world regions, North America currently has the highest life expectancy, which will increase over the next decades. Other regions are projected to have similar trends, but at a lower level. These differences in life expectancy are projected to persist throughout the whole 21st century. Regarding fertility, most world
regions have already experienced a strong decline over the last 60 years. Only Africa still has a fertility rate of around four births per woman.

Figure 2: Demographics and international asset positions

Source: United Nations Population Division; External Wealth of Nations Dataset (Lane and Milesi-Ferretti, 2007).

Note: The observations are by region and year, covering 1970 to 2010. The correlations are robust to excluding outliers.
Regions with a higher share of old people tend to have a positive net debt position and a negative net equity position, as the plots in Figure 2 show. Given that debt can be considered a safe asset, and equity a risky asset, this supports our hypothesis that aging societies should go long in safe assets and short in risky assets. The plots do not represent a causal relationship, and in particular, the findings could be driven by other regional differences like in GDP. We will explore the data in more detail in the paper.

Figure 3: Risk-free rate

![Risk-free rate graph](image)

*Source: Federal Reserve Bank of St.Louis.*

Figure 3 shows the three-month U.S. treasury bill secondary market rate, which can be considered as a prototype safe asset. There has been a downward trend since the 1980’s. While there can be many explanations for this phenomenon, including supply-side approaches such as a shortness of other types of safe assets (Gourinchas and Jeanne, 2012), it is in line with what our model predicts about the relation between demographics and the demand for safe assets: the decrease in fertility and increase in longevity over the last decades should have increased the demand for assets like U.S. treasury bills, and, in absence of supply changes, have depressed the returns.

A look at household data shows how savings and the portfolio share of risky assets varies with household age. Figure 4 confirms the prediction of life-cycle
theory that agents save when in working age, and dis-save when retired. The share of risky assets also exhibits a hump-shaped pattern over the life-cycle (Figure 5): Over the course of their life, households gradually increase their risky portfolio share roughly up to retirement age, and decrease it afterwards. The hump-shaped form of risky asset shares is preserved when conditioning on participation, while participation itself is also hump-shaped over the lifecycle. We find the same pattern when considering European household data from the Eurosystem Household Finance and Consumption Survey. This finding is also supported by a large body of literature (e.g. Cocco et al., 2005, Haliassos and Michaelides, 2002, Chang et al., 2014).

Figure 4: Financial asset holdings

Note: Financial assets are the sum of safe and risky assets as defined below. Source: Survey of Consumer Finances 2010 (U.S. Federal Reserve Board)
4 A Simple Multi-Period OLG Model

We augment the quantitative multi-period overlapping generations model of Auerbach and Kotlikoff (1987) to an open economy case with aggregate risk and two assets.

Demographics

Agents\(^2\) have a known maximum life expectancy of \(N\) years. There is a probability of survival to the next period conditional on age \(n\) of \(\delta_{n,t}\). Given an initial size of this cohort, \(L_{0,t-n}\), the current size of each cohort at time \(t\)

\(^2\)We use the terms agents and households interchangeably.
is,

\[ L_{n,t} = \left( \prod_{l=0}^{n-1} \delta_{l,t-n+l} \right) L_{0,t-n}. \]  

(1)

Each successive cohort grows at a potentially time-varying rate \( \gamma_{t+1} \) so that the youngest cohort evolves,

\[ L_{0,t+1} = (1 + \gamma_{t+1}) L_{0,t}. \]  

(2)

With this formation of the population growth, \( \gamma \) coupled with survival probabilities, \( \delta \), we are able to model more richly two features of global trends in demographic change of interest to us: declining birth rates and increased longevity.

**Household Optimization Problem**

The economy is populated with agents who live for a maximum \( N \) periods. Households maximize expected lifetime utility of consumption at each age, \( n \).

\[ U_t = \sum_{n=0}^{N} \left( \prod_{l=0}^{n-1} \delta_{l,t+l} \right) \beta^n u(c_{n,t+n}). \]  

(3)

Where \( \delta_{n,t+n} \) represents the probability of survival for an agent of age \( n \) into the next period conditional upon being alive. The household is risk-averse with CRRA preferences: \( u(c_{n,t}) = \frac{c_{n,t}^{1-\vartheta}}{1-\vartheta} \) when \( \vartheta \neq 1 \) and \( u(c_{n,t}) = \ln(c_{n,t}) \) when \( \vartheta = 1 \), where \( \vartheta \) is the risk aversion parameter, which is at the same time the inverse of the intertemporal elasticity of substitution.

**Budget Constraint**

Households are subject to a flow budget constraint that takes into account their various forms of income and saving possibilities. Income and asset prices are denominated in units of the consumption good. Newborn households im-
mediately start working and stay in the workforce until age \(J\). Each period, they supply a fixed amount of labor, earn labor income \(Y_{n,t}\) and pay a proportional tax \(\tau_t\) to finance the pensions of the current old. After retirement at age \(J + 1\), labor income is zero and replaced with pension income, \(\tilde{Y}_t\). Cash on hand \(x_{n,t}\) of working and retired households evolves according to

\[
x_{n,t} = R_t^f b_{n-1,t-1} + R_{t-1} s_{n-1,t-1} + (1 - \tau_t) Y_{n,t} \quad \text{if} \quad n \leq J \tag{4}
\]

\[
x_{n,t} = R_t^f b_{n-1,t-1} + R_{t-1} s_{n-1,t-1} + \tilde{Y}_t \quad \text{if} \quad n \geq J + 1, \tag{5}
\]

Households allocate their resources respecting the budget constraint

\[
x_{n,t} = c_{n,t} + b_{n,t} + s_{n,t} \tag{6}
\]

where \(b_{n,t}\) are assets of households allocated to the non-risky asset: a period bond, paying a real return of \(R_t^f\) in each period. \(s_{n,t}\) are household assets allocated to the risky asset, a stock paying a stochastic dividend. The return on the risky asset is \(R_t\). Denote total asset holdings of the household by \(a_{n,t} = b_{n,t} + s_{n,t}\). The share of risky assets in the household portfolio is defined as \(\Omega_{n,t} = \frac{s_{n,t}}{a_{n,t}}\).

**Financial Assets**

Agents can choose to save in two assets, a riskless period bond that pays a time-varying, but certain return of \(R_t^f\) in each period and a risky stock with a stochastic return. The risky asset gross return follows the process

\[
R_t = R_t^f + r_t^p + \epsilon_t \tag{7}
\]

where \(\epsilon_t \sim \mathcal{N}(0, \sigma^2)\) is a period innovation to risky asset returns and \(r_t^p\) is a risk premium, which is time-varying but known to agents when they make their investment decision.
Aggregate asset demand and supply

The aggregate demand for safe and risky assets is denoted by \( B_t^d = \sum_{n=1}^{N} L_{n,t} b_{n,t} \) and \( S_t^d = \sum_{n=1}^{N} L_{n,t} s_{n,t} \), respectively.

For now, we assume that both risky and safe assets are in fixed supply \( \bar{B}^s \) and \( \bar{S}^s \), so that return rates are driven entirely by the demand for assets. While this is certainly not an accurate description of the long-run, the simplification allows us to isolate the demand channel. Under the realistic assumption that the supply of assets is not fully elastic, the benchmark model is informative about the direction of international asset trades following demographic changes. In the extended version of the model (to be developed), supply of risky and safe assets will be generated endogenously and we will be able to make a quantitative statement. For example, we may endow agents with Lucas trees that yield a risky and a safe tranche of dividends. The number of agents in the economy will determine the aggregate supply of both types of assets. Alternatively, we will consider modeling a production economy.

Labor Income

Pre-tax labor income for a household of age \( n \) is defined as

\[
Y_{n,t} = P_{n,t} D_{n,t}
\]

where

\[
P_{n,t} = G_n P_{n,t-1} U_{n,t}.
\]

This specification is frequently used in the life-cycle literature, going back to Zeldes (1989) and Carroll (1992). \( P_{n,t} \) is a permanent income component, the log of which evolves as a random walk with drift. \( G_n \) is an age-specific component of income. We can think about this either as a premium on work experience, or assume that the amount of labor supplied by the households or their productivity depends deterministically on their age. Cocco et al. (2005)
have estimated the permanent income process as a third-order polynomial function of $n$. In this way, the labor income of households can be modeled to exhibit the hump shape over the life cycle that is observed in the data.

The permanent income shock, $U_{n,t}$, is log-normally distributed, $\ln(U_{n,t}) \sim \mathcal{N}(0, \sigma^2_U)$. We allow $U_{n,t}$ to be correlated across households, so as to generate aggregate fluctuations in permanent income, but not serially correlated. $D_{n,t}$ is a transitory shock, which is independently and identically log-normally distributed, $\ln(D_{n,t}) \sim \mathcal{N}(0, \sigma^2_D)$, and is uncorrelated with $U_{n,t}$.

While income shocks in general induce agents to accumulate precautionary savings, the aggregate component becomes particularly important when being correlated with the risky asset return. A positive correlation would deter agents from holding stocks when they are young, because their endowment of lifetime income is high relative to their total wealth. As relative lifetime labor income falls, investment in stocks increases relative to bonds. A negative correlation would have the opposite effect, since stocks can be used as a hedge against labor income shocks. Several papers have studied how correlating stochastic labor income with stock returns can generate realistic portfolio allocations over the life-cycle (e.g. Heaton and Lucas, 1997, Davis and Willen, 2000, Gomes and Michaelides, 2005, Chang et al., 2014). Let $\rho^{dl}$ denote the correlation coefficient between $Y_{n,t}$ and $R_t$. We will explore various specifications of these parameters in our calibration.

\textbf{Pension Income}

We consider a PAYGO social security system in which taxes of working agents are directly transferred to retirees social security benefits. In each period the government budget balances, and pension income of a retiree (regardless of age) is

$$\bar{Y}_t = \frac{\sum_{j=0}^{J} \tau_j Y_{j,t} L_{j,t}}{\sum_{j=J+1}^{N} L_{n,t}}$$ (10)
Pension income is larger when the income generated by the working-age population is larger, the tax rate is higher, or the number of retirees is smaller.

Alternatively, we consider a specification where pension income is a constant fraction $\lambda$ of permanent labor income in the last working-year:

$$\tilde{Y}_{n,t} = \lambda P_{J,t-(n-J)}$$

(11)

We do not explicitly model how the retirement income is generated, but this set-up can be thought of as an abstract version of a defined-contribution funded pension scheme: One possible rationale is that the pension income is paid by the household’s former employer, who in earlier years discounted the wage accordingly. Note that in this formulation, contrary to PAYGO, pension income is household specific: A household that had a higher permanent labor income in the pre-retirement period will have a higher constant pension income.

Borrowing Constraints

Households are restricted in the extent to which they can short financial assets. We introduce a borrowing constraint for each asset,

$$b_{n,t} \geq b, \quad s_{n,t} \geq s.$$  

(12)

where $b$ and $s$ are lower bounds on the amount borrowed. The borrowing constraints can be motivated by some kind of moral hazard story. Models including young low-income agents have shown that short-sale constraints hinder these agents from borrowing against future income, affecting aggregate demand and returns on safe and risky assets (Constantinides et al., 2002). In our model, given that households immediately start earning after being born, the constraints will be relevant only for retired households in their last periods of life. We will show this in the calibration below.
Open Economy

The country is a small open economy, which differs from the rest of the
world only in terms of total population size and demographic characteristics.
Financial markets are perfectly integrated, so households of both regions are
free to buy bonds and stocks anywhere in the world. We denote foreign bond
and stock holdings by domestic agents with an asterisk, \(b_{n,t}^*\) and \(s_{n,t}^*\).

For simplicity, we abstract from modelling (real) exchange rates and assume
that the law of one price always holds.\(^3\) In our model world, there is only one
good, the price of which is normalized to 1 in both regions. Therefore, trade
in goods does not play a role.

International Asset Structure

Denote foreign bond and stock returns by \(R_t^f\) and \(R_t^*\), respectively. Foreign
risky asset returns are characterized by the same process as domestic risky
assets,

\[
R_t^* = R_t^f + r_t^p + \eta_t^*
\]

(13)
The risk premium \(r_t^p\) is identical to that in the small open economy and shocks
follows the same distribution, \(\epsilon_t^* \sim \mathcal{N}(0, \sigma_t^2)\). Therefore, \(\mathbb{E}(R_t) = \mathbb{E}(R_t^*)\).
However, shocks may not be perfectly correlated across regions.

In autarky, international differences in demographics may lead to \(R_t \neq R_t^*\)
and \(R_t^f \neq R_t^f^*\). Due to arbitrage opportunities, the same asset cannot be
priced differently in an integrated world. Domestic and foreign riskless assets
are perfect substitutes, so the riskless rate is equalized through asset trade. For
risky assets, the degree of substitutability depends on the correlation between
domestic and foreign stock returns, \(\rho_{df}\). In the extreme case of \(\rho_{df} = 1,\)

\(^3\)This assumption can be revoked. Exchange rate volatility deters households from
investing in foreign assets, whereas the correlation of exchange rate fluctuations and innovations
to domestic and foreign stock returns may have both positive and negative effects on
the international portfolio share. Michaelides (2003) shows these effects in a model similar
to ours, but which does not consider demographics.
domestic and foreign equity are identical assets and \( R_t = R_t^* \). Households will then be indifferent between holding domestic and foreign assets. When \( \rho^{df} < 1 \), domestic and foreign stocks become two different assets, and households will want to diversify their portfolio in order to minimize return fluctuations. In general, the ideal strategy would then be to purchase the same amount of both assets.

In the real world, despite well integrated financial markets, we observe a pronounced home bias in the portfolio choice of households. Agents choose to hold a much smaller portfolio share in foreign assets than predicted by standard theory, a discrepancy which has been termed international diversification puzzle (see Lewis, 1999, and Coeurdacier and Rey, 2013, for surveys of the literature). While there is an ongoing controversy about the reasons for the equity home bias, several modeling tools have been proposed in order to bring theoretical predictions more in line with empirical observations (Michaelides, 2003, Coeurdacier and Rey, 2013) These include behavioral biases, informational frictions, transaction or participation costs for foreign stock markets or non-hedgible exchange-rate risk.

In a future version of the paper, we will explore these tools to model realistic domestic and foreign asset holdings. For now, we simply assume that \( \rho^{df} = 1 \), which allows us to circumvent the equity home bias.

**International Portfolio Allocation**

In an integrated world, agents allocate their net worth to four different assets:

\[
a_{n,t} = b_{n,t} + b^*_{n,t} + s_{n,t} + s^*_{n,t}
\]

(14)

Asset markets now clear globally instead of locally. Domestic asset supply is still fixed at \( \bar{B}^s \) and \( \bar{S}^s \). Any excess demand will be directed abroad. Let \( \Psi_t = B^d_t - \bar{B}^s_t \) denote net outflows of risky assets (which is equal to the net external equity position, since all assets are held for one period), whereas \( \Phi_t = \)
denotes net outflows of safe assets. These are key parameters of the model.

**Equilibrium and Timing**

At the beginning of each period $t$, the youngest cohort $L_{0,t}$ is born with no initial wealth. Each surviving household receives the average accidental bequest, $\bar{a}_t$. All living households receive interest income on existing savings in safe and risky assets, returning $R_f^t$ and $R_t$, respectively. Households of working age receive their after-tax labor income, $(1 - \tau_t)Y_{n,t}$ and retired households receive pension benefits, $\tilde{Y}_t$. Together, these income streams constitute the household’s cash on hand, $x_{n,t}$, which they allocate optimally consumption and asset purchases. Since returns are allocated to end-of-period wealth, households are unaware of future returns when choosing portfolio allocation. After households have made their consumption and saving decisions, a share of each cohort, $(1 - \delta_{n,t})$, dies. Their remaining assets expire.

An equilibrium consists of paths for consumption $\{c_{n,t}\}$, bonds $\{b_{n,t}\}$ and stocks $\{s_{n,t}\}$ taking as given the paths for prices, interest rates, and wages. In a closed economy, the household chooses $c_{n,t}$, $b_{n,t}$ and $s_{n,t}$ as to maximize (3) subject to (4) through (12). The state variables are age and cash on hand, $x_{n,t}$. In an open economy, agents have two additional choice variables, foreign safe assets $b_{n,t}^*$, and foreign risky assets, $s_{n,t}^*$. Asset markets clear globally.

We solve for the household’s policy functions for consumption and the optimal risky share of net worth as a function of cash on hand, using the endogenous grid point method (for details, see Carroll, 2006; Hintermaier and Koeniger, 2010). Using an exogenous, time-invariant grid of net end of period wealth, $a_{n,t}$, we solve for the policy functions in each period, which producing endogenous, model consistent values for cash on hand in each period.

We begin by solving in the second to last period of life in $T - 1$ using the terminal condition, $c_{n,T} = x_{n,T}$ agents consume all of their remaining wealth in the final period. Taking the policy function for consumption in the second
to last period, we continue via backward induction.

5 Quantitative Exercise

Calibration

We calibrate the model to the U.S. economy. We interpret the demographic structure of the U.S. in the years 2010-2015 as the pre-aging steady state, and assume that the historically observed U.S. asset returns are the equilibrium rates prevailing in this economy. We characterize the consumption and savings decisions and in particular the portfolio choice in the pre-aging steady state. Then, we simulate a shift in U.S. demographics according to population projections for 2015 through 2100 in five-year increments, while assuming that the rest of the world has stable demographics. We characterize the effects on asset positions, returns and portfolio choice. We then derive the volumes of asset trades and compare the changes in cohort specific risky shares and asset.

Preferences

The coefficient of relative risk aversion is set to 9, which is within the range considered reasonable by Mehra and Prescott (1985). This is at the same time the inverse of the elasticity of intertemporal substitution. The discount factor is 0.96.

Demographics

Agents are born at the beginning of their working life, which we set to age 20, and die with probability 1 at the age of 100. The maximum lifetime is thus 80 years. The retirement age is set to 65. Total life time is uncertain. Survival probabilities for each age group are taken from the United Nations World Population Prospects (2015 Revision). For
the 2050-2055 projections, we consider the medium variant including migration. Figure (6) plots conditional survival probabilities for both time periods considered. Note that the UN provides survival probabilities only up to age 85. For the remaining 15 years that the agents in our model may survive, we use cubic spline interpolation. Forcing survival probabilities to go down to 0 by age 100 understates the population share of the eldest. We will get back to this point when comparing dependency ratios of model and data.

Figure 6: Survival probabilities


Data on population growth are also taken from the United Nations World Population Prospects. The annual birth rate is 12.6 births per 1,000 population in 2010-2015 and projected to be 12.1 for 2050-2055 and 11.9 for 2095-2100.

**Labor Income**

Labor income plays a key role for the allocation of assets over the life-cycle. Not only in a life-cycle pattern for the level of assets held by households,
but also for the allocation between safe and risky assets over the life cycle. Cocco et al. (2005) have estimated the shape of the labor income process over the life-cycle for three different educational groups using data from the Panel Study of Income Dynamics (PSID). Their definition of labor income includes unemployment compensation and other social assistance to correctly assess the risk connected to the labor income process. They regress household labor income on dummies for age and other household characteristics and, for three different educational groups, fit a third-order polynomial to the age dummies. We use their estimation for the medium educational group, high school graduates,

\[
\ln(G_n) = 2.7004 - 2.1700 + 0.1682 \times n - 0.0323 \times \frac{n^2}{10} + 0.0020 \times \frac{n^3}{100}.
\]  

(15)

The corresponding estimates for the variance of the permanent and transitory shocks are \(\sigma_U^2 = 0.0106\) and \(\sigma_D^2 = 0.0738\), respectively.

In our benchmark calibration, we set the correlation of labor income with stock returns, \(\rho_{dl}\), to zero. This is not an implausible assumption. Cocco et al. (2005) only get a small correlation coefficient in their estimates, the sign of which depends on the educational group considered. Davis and Willen (2000) also report very small (positive) values. Later, we will analyze how our results change when we allow \(\rho_{dl}\) to take non-zero values.

**Pension Income**

For now, we only consider the case where pension income is the constant fraction \(\lambda\) of the labor income generated in the last working-age period \(J\). We take the value estimated by Cocco et al. (2005), \(\lambda = 0.6821\). Since we do not model PAYG, the government does not levy any taxes, so \(\tau_t = 0\).
Asset Returns

In our benchmark version of the model, we set $\rho^{fd} = 1$, which facilitates the characterization of international asset positions because it implies $R_t = R_t^*$. Agents are hence indifferent between holding domestic and foreign equity.

As the real-world equivalent to safe assets we consider government bonds, whereas the equivalent to risky assets are firm shares traded on the U.S. stock market. The risk-free rate is 2%. For U.S. stock market returns, we use historical estimates by Fama and French (2002). For the period 1951-2000, they have estimated an average risk premium of 4.32%, with a standard deviation of 14.02%. Since we assume perfect international integration of asset markets, these rates are at the same time those of the rest of the world. The rest of the world is not experiencing demographic changes, so it continues to be in its steady state throughout the whole simulation period. This means that the U.S., being financially fully integrated into the world economy, will continue to face the same rates even when undergoing demographic transitions. What changes are the autarky asset returns, as we will show below.

Borrowing Constraints

We set $b = s = -\infty$, so agents can borrow an unlimited amount throughout their life. To prevent agents from accumulating large amounts of debt towards the end of their lives, we require them to have zero debts at the end of age 100. Later, we will experiment with different borrowing limits. The general insights will stay the same.

Simulation Results

We simulate life-cycle paths for consumption and portfolio choice for multiple households and report both life-cycle and cross-sectional means.

Before we discuss the simulation results for consumption, portfolio choice
and asset trades, we first present the policy functions generating these results. Figure 7 shows for households of age 99 how consumption and the risky share depend on cash on hand. The higher the cash on hand, the more the households choose to consume, and the more safe assets they prefer to hold relative to risky ones. To understand the rationale underlying the portfolio choice, it is important to bear in mind that retirement income is risk-free. For poorer agents, their income constitutes a larger share of cash-on-hand, thereby they hold a relatively large risk-free position and will hence want to invest their savings in risky assets. For richer agents, retirement income constitutes a smaller part of wealth, hence they prefer to hold relatively more safe assets. The policy function for the risky share is a function of age (not shown here): The closer the household comes to retirement, the more the function shifts in, since more and more of the remaining life time income is risk-free.
Pre-Demographic Change Steady State

Figure 8 shows how income, consumption and cash on hand evolve over the life-cycle. The consumption path is smoother than the income path: households use their financial assets to smooth consumption over their life time, as predicted by the permanent income hypothesis. Households start saving already in the first period of their lives, because their income is already considerable. Financial assets are accumulated until close to retirement age, and then slowly run down. Cash on hand jumps at the point of retirement because income jumps down to the fixed pension income. We force agents to die with zero debt at the age of 100, which induces them never to make debt even though not formally being constrained. Therefore, introducing a stricter borrowing limit will not make a difference to our results.

---

4This is in contrast to models that incorporate a young generation that has not yet entered the labor market. There, the permanent income hypothesis predicts that young agents will want to borrow against future income.
The share of risky assets over the life-cycle is U-shaped in our model. The large share of risky assets at the beginning of working age is explained by the fact the income of young agents is relatively low and increases fast. The present discounted value of future income rises with age. Towards the end of the lifetime, the risky share of 1 reflects that agents run down their assets. Cash on hand consists almost entirely of the risk-free retirement income. It is therefore in the agents’ interest to hold risky rather than safe assets.

The model results stand in contrast with the data: In Section 3 we showed that the risky share is hump-shaped over the life-cycle. This problem is widely recognized in the literature. There have been proposed several cures for this problem: Introduce a positive correlation between labor income and risky asset returns (as discussed above), in particular for agents at the beginning of their career, or stock market participation costs to drive down the share of risky assets for young households (Vissing-Jorgensen, 2002, Ameriks and Zeldes, 2004). Make retirement income uncertain, e.g. by introducing risk of high
medical expenditure in old age (DeNardi et al., 2015) to decrease the share of risky assets in the portfolio of the old.

**Demographic Change and Portfolio Choice**

Figure 10 shows the increases in total assets and declines in risky share of assets for the demographic projections to 2050 and 2100 relative to 2010. The further demographic change proceeds, the more assets households will want to accumulate: Their asset holdings over the life-cycle are increasing functions of the projection year. This is because the agents prepare for a longer retirement period in expectations. They will choose to hold more of both types of assets in order to optimally balance their portfolio, but the increase in safe assets is stronger. Consequently, the risky portfolio share of households in 2050 is smaller for most of the life-cycle, becoming significantly less during retirement years.
Demographic Change and External Assets

Keeping the supply of safe and risky assets fixed at initial steady state values taken to be 2010 values, we examine the external asset holdings for the two simulated demographic projections. We do this by considering aggregated asset holdings by cohort, which takes into account the relative size of cohorts over time.

Figure 11 depicts the aggregate differences by cohort between the initial supply of assets and the demand in 2050 and 2100, respectively. Thus, this figure shows the net external positions in both safe and risky assets for each cohort. Several features are worth noting. First, the net holdings in safe assets are increasing across all cohorts in both cases, the magnitude is increasing with longer term projections. For the risky assets, we see that younger cohorts are increasing their net holdings, which relative to 2010, older cohorts are exporting risky assets.
Examining the relationship for projective demographic change over time, we use projections in five year increments from 2010 to 2100. For each projection year, we calculate the dependency ratio defined as the total number of retired households over the total number of working age households. We compare it to the share of total wealth held in risky assets and safe assets. The results are depicted in Figure 12.

Consistent with our stylized facts shown in Section 3, there is a clear, positive relation between old-age dependency and the share of safe assets in total wealth and a negative relationship between the dependency ratio and risky share of total wealth: while aging populations demand more safe and more risky assets, the demand is relatively stronger for safe assets. Our results would be further strengthened should the risky share continuously decrease with age.

Note that the dependency ratios presented here slightly differ from those found in the data: For example, the ratio of over 64-year olds to those of age
20 to 64 was 0.21 in 2010 and is projected to be 0.41 in 2050 (United Nations World Population Prospects). The reason is that we are not able to accurately track the number of over 85-year olds in our simulation. Forcing the number of surviving households to be zero at age 100 leads to lower survival ratios for 86- to 99-year olds in our interpolation than in the data. Nevertheless, our model results come reasonably close to the data.

Figure 13 shows more clearly how the external positions of the U.S. evolve over time. Exports in both safe and risky assets will increase as demographic change proceeds. This could potentially generate large net foreign asset positions, a phenomenon we already observe in fast-aging countries like Japan and Germany. The effect will be less strong when taking into account that the rest of the world is aging as well. This is one of the routes along which we are going to develop the calibration further.
6 Extensions of the Benchmark Model

We are going to extend the benchmark model along several lines:

- **Endogenous supply of assets.** The dynamics of asset supply play an important role for asset returns. Demographic change is a long-run phenomenon, so we can expect asset supply to react to demand changes. The volume of international asset trades might become larger or smaller in consequence.

- **Aging in the rest of the world.** So far, we have made the simplifying assumption that the rest of the world is not aging. However, as shown in Section 3, the other world regions are aging as well. Japan and Southern Europe have even extremer demographic trends than the U.S. We are going to calibrate a two-region model where the second region consists of some of these countries. This is a necessary feature to get realistic quantitative results for international asset trades.

- **Nonzero correlation of domestic and foreign risky returns** ($\rho_{fd} \neq 0$). The effects of this important assumption have been explained in detail in Section 4.

- **Endogenous retirement income.** This applies both to the defined-contribution funded pension system of the benchmark model, and to a PAYGO system that is financed by taxes on labor income. We will be able to compare these two pension schemes and draw implications for their sustainability.

- **Nonzero correlation between labor income and risky returns** ($\rho_{dl} \neq 0$). As explained in the main text, this extension will allow us to generate more realistic patterns of the risky asset share over the lifecycle. We will also consider allowing for disastrous retirement income shocks as a way of modeling mortality risk.
- **Accidental bequests.** Assets of deceased agents are so far not re-distributed to living agents. We will revoke this assumption in future versions of the paper. The effect will be minor.

References


Fry, M. J. and A. Mason (1982). The variable rate-of-growth effect in the


