

Schetter, Ulrich

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## Comparative Advantages with Product Complexity and Product Quality

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# Comparative Advantages with Product Complexity and Product Quality\*

## **Preliminary**

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## **Abstract**

We analyze the interplay between product-intrinsic complexity and endogenously chosen product quality in international trade. Our work reveals a novel mechanism that can explain a rich set of empirical observations: (1) how specialization within products on quality can equalize comparative advantages across products, (2) why poor countries do not export a broad range of products nonetheless, and (3) why the share of products for which this is the case tends to be decreasing over time. Our theory motivates the use of a censored regression model to estimate the link between a country's GDP per capita and the quality of its exports. Following this empirical strategy, we find a much stronger relationship than when using OLS, in line with our theory.

Keywords: Comparative Advantage, Product Complexity, Product Quality

JEL: F11, F14

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# 1 Introduction

Countries compete over a heterogeneous set of products. While this heterogeneity is multidimensional, it is certainly true that products differ largely in their complexity – ranging, at the hs4 classification level, from cocoa beans and cotton shirts, through hydraulic turbines and inorganic acids, to nuclear reactors and various kinds of high-tech machines. A standard Ricardian argument suggests that countries should specialize according to their comparative advantages, i.e. we would expect that industrialized countries specialize on complex products, whereas developing countries specialize on simple products.<sup>1</sup> Yet often both rich and poor countries successfully export the same products, and the share of products for which this is the case tends to increase over time.<sup>2</sup>

Why do we not observe a stronger specialization of countries in products? Empirical evidence suggests that this might happen because countries specialize within products in quality.<sup>3</sup> There is undoubtedly ample room for industrialized countries to compete by producing high quality.<sup>4</sup> As an example, while you can buy an analog watch for less than a Euro on the Internet, many Swiss watches are sold at a price of several thousand Euros and Vacheron Constantin even sells its ‘Tour de l’Ile’ at the price of more than one million Euros.<sup>5</sup> Yet it is not clear what such specialization within products implies for comparative advantages across products, and the underlying mechanisms have not been studied in the literature so far. We will analyze the interplay between product-intrinsic complexity and endogenously chosen quality in a general equilibrium model of international trade. Our work reveals a novel mechanism that can explain a rich set of empirical observations, in particular:

- how specialization within products in quality can equalize comparative advan-

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<sup>1</sup>More generally, comparative advantages, whether they are arising from production technologies or factor endowments, should presumably give rise to a (block-) diagonal pattern of specialization in international trade. Costinot (2009a) shows that in a Ricardian model specialization occurs if countries can be ranked according to some characteristic (e.g. institutions), if products can be ranked according to some other characteristic (e.g. complexity), and if factor productivity is log-supermodular in both characteristics.

<sup>2</sup>Cf. Schott (2004) and Pham (2008), for example. China is an important driver of this development (Pham, 2008).

<sup>3</sup>Cf. Schott (2004), Hummels and Klenow (2005), Pham (2008), Khandelwal (2010), and Hallak and Schott (2011), for example.

<sup>4</sup>For the purpose of our discussion here and below, a product’s quality summarizes all product attributes that increase a consumer’s willingness to pay for that product.

<sup>5</sup><http://www.manager-magazin.de/magazin/artikel/a-357485.html>, retrieved on 25 October 2013.

tages across products;

- why, nonetheless, poor countries cannot successfully compete for a broad range of products;
- why the share of products for which this is the case tends to diminish over time.

We further show that this mechanism motivates the use of a censored regression model to estimate the link between a country's GDP per capita and the quality of its exports.

We start from the simple Ricardian rationale outlined above. In our model countries differ in the skill level of their labor, while products differ in complexity. High-skill countries are better at producing all products. Yet they have a comparative disadvantage for simple products, because the skill intensity increases with the complexity of a product. This changes, however, if we introduce an endogenous choice of product quality into our model. Then high-skill countries can successfully compete for simple products by producing high quality, and across-product specialization is replaced by within-product specialization as suggested above.

Does this rationale imply that there are no comparative advantages across products? Our answer is no. The reason for this is the existence of minimum-quality requirements. These minimum-quality requirements arise from different sources. In many cases, they are product-intrinsic. Referring to the watch example, even the cheapest version of a watch requires a balance wheel (pendulum), a spring, and a suspension of reasonable quality, and these parts need to be assembled in a reasonably accurate manner for the watch to serve its intended purpose. Similarly, banknotes and computer software certainly have to meet minimum requirements in terms of safety, air beds and glass in terms of resistance, photo lenses and clinical diagnostics in terms of precision, and autopilots and refrigerated trucks in terms of reliability. Yet (stricter) minimum-quality requirements are also often introduced by law. Many products sold within the European Economic Area, for example, have to bear the *CE* mark indicating that they conform to European product requirements.<sup>6,7</sup>

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<sup>6</sup>Such legal product requirements are a significant barrier to trade and, among others, feature prominently in the negotiations on a free trade agreement between the EU and the US (cf. e.g. <http://online.wsj.com/news/articles/SB10001424127887324162304578301662368145012>, retrieved on 27 January 2014).

<sup>7</sup>Minimum-quality requirements may also implicitly arise from the competitive fringe. As an example, firms in the semiconductor equipment industry compete to provide high-tech machines enabling the production of ever smaller and more powerful computer chips. The market is strongly concentrated in the hands of the technology leader: Currently, ASLM dominates this market, with a market share of around two thirds, whereas the market was dominated by Canon and Nikon in 1990 (cf.

The crucial observation is that these minimum-quality requirements are product specific, and in particular, that satisfying them is more demanding for complex products than for simple ones. Producing a functional air bed is certainly less of a challenge than producing an autopilot that can safely navigate you through Moscow’s traffic snarl.<sup>8</sup> Minimum-quality requirements thus impose critical restrictions on the specialization of countries on quality. They prevent low-skill countries from successfully competing for complex products. Products like nuclear reactors and high-tech machines are just too difficult to produce, even in a minimum-quality version.

Hence the interplay between product-intrinsic complexity and endogenously chosen quality gives rise to an upper-triangular structure of comparative advantages. While high-skill countries can always compete for simple products by producing high quality, low-skill countries cannot always compete by producing low quality, due to the minimum-quality requirements.

These implications are in line with what we observe in the data. Industrialized countries are successfully exporting the complex products, but also most of the simpler products.<sup>9</sup> At the other extreme, countries like Algeria, Somalia, and Turkmenistan, for example, are successfully exporting only a few – presumably simple – products.<sup>10</sup> More to the point, an upper-triangular structure of international specialization is observed by Hausmann and Hidalgo (2011) and Tacchella et al. (2012).<sup>11</sup>

Moreover, the proposed rationale provides an intuitive explanation why the share of products that are co-exported by poor and rich countries tends to increase over time, as observed by Schott (2004). If comparative advantages stem from minimum-quality requirements, they naturally subside as countries develop.<sup>12</sup>

Our work reflects the empirical observation that richer countries export higher quality.

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[http://corporate.zeiss.com/content/dam/Corporate/pressandmedia/downloads/innovation\\_ger\\_20.pdf](http://corporate.zeiss.com/content/dam/Corporate/pressandmedia/downloads/innovation_ger_20.pdf), retrieved on 27 January 2014).

<sup>8</sup>Moscow is the city with the worst traffic congestions worldwide according to the TomTom traffic index (cf. <http://www.tomtom.com/news/category.php?ID=4&NID=1487&Year=2013&Language=3&TT=16a0bfb2-baba37bd-00000000-00000000-0000001b-4j1f8h7dvlbtibhj84n7mccg0>, retrieved on 28 January 2014).

<sup>9</sup>In 2010, Germany, the USA, Belgium, and the Netherlands, for example, had a *revealed comparative advantage* of at least 0.05 for around 95% of the products at the hs4 classification level, with revealed comparative advantage referring to the measure originally proposed by Balassa (1965).

<sup>10</sup>In 2010, these countries had revealed comparative advantage of at least 0.05 for less than 10% of the products at the hs4 classification level.

<sup>11</sup>We briefly discuss and summarize this evidence in appendix A.

<sup>12</sup>Note that when making this observation, Schott (2004) classifies countries as rich and poor based on a comparison with the cross-section of countries.

Yet our work also has important implications for this strand in the literature. If low-skill countries cannot successfully compete for complex products because they are bounded by a minimum-quality constraint, then this information could – and should – be exploited in an empirical analysis of the link between a country’s GDP per capita and the quality of its exports. We show that our theoretical set-up rationalizes the use of a censored regression model. Taking this model to the data, we observe a much stronger link between a country’s GDP per capita and the quality of its exports than when using OLS, as to be expected according to our theory.

### *Relation to the literature*

Our work complements a growing literature that studies various aspects related to quality upgrading in international trade. Flam and Helpman (1987), Stokey (1991), Murphy and Shleifer (1997), and Matsuyama (2000), for example, consider non-homothetic preferences for quality in models of north-south trade to study product cycles and the welfare effects from trade, among others. More recently, Baldwin and Harrigan (2011), Kugler and Verhoogen (2012), Johnson (2012), Hallak and Sivadasan (2013), and Benedetti Fasil and Borota (2013), for example, integrate quality into trade models with firm-level heterogeneity to derive richer predictions on the exporting behavior of firms and countries. Yet none of these strands in the literature addresses the implications of quality differentiation for the comparative advantages of countries over a heterogeneous set of products, which is the main focus here.

A key element of our model is the existence of a two-dimensional commodity space. In particular, we consider horizontally differentiated *products* that differ vertically in *quality*.<sup>13</sup> Only a few models consider a horizontally and vertically differentiated commodity space in international trade. To the best of our knowledge, Jaimovich and Merella (2014) and Alcalá (2012) are the only papers analyzing the interplay between horizontal specialization across products and vertical specialization within products. Both these models exhibit a positive relationship between a country’s comparative advantage for a product and its export quality. By contrast, we stress that quality differentiation might actually attenuate comparative advantages of countries across

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<sup>13</sup>Another strand in the literature considers horizontally differentiated varieties within products (or industries). Helpman and Krugman (1985), Bernard et al. (2007), Okubo (2009), and Fan et al. (2011), for example, consider multi-sector versions of the Krugman (1979, 1980) and the Melitz (2003) model, respectively. Chor (2010) and Costinot et al. (2012), for example, present multi-sector versions of the Ricardian trade model developed by Eaton and Kortum (2002).

products.

To study the implications of within-product specialization on quality for comparative advantages of countries across products, we consider a stylized economy. In our model countries differ in one reduced-form parameter only, which captures their economic strength. To be precise, we will assume that countries differ in the skill level of their labor, but the origins of this economic strength are not essential for any of our results. Following Kremer (1993), we assume that production is based on an O-ring process that uses labor as the only input. Production requires successful accomplishment of all tasks. The skill level of labor determines the probability of successful accomplishment of any given task. Products differ in the number of simultaneous tasks – their *complexity*.<sup>14</sup> At constant quality, high-skill countries then have comparative advantage for complex products, as originally shown by Kremer (1993).<sup>15</sup>

We combine this production technology with a new way of modeling the endogenous choice of quality in an O-ring process. In particular, we suggest that producing higher output quality requires higher quality of every individual task involved in production, which, in turn, renders the successful accomplishment of every task more demanding.<sup>16</sup> This modeling choice generalizes Kremer’s rationale in a natural way. It nicely reflects his guiding example of the failure of the space shuttle *Challenger*, as well as the concept of *Total Quality Management*, which is well established in management science.<sup>17</sup> Moreover, it provides a simple rationale for quality-biased efficiency and cross-product

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<sup>14</sup>Costinot (2009b) considers the same source of product heterogeneity in a model of international trade. However, as opposed to our model, workers have to spend a fixed amount of their endowment with efficient labor on learning each task they are working on. Moreover, a worker fails if and only if he shirks, implying that the probability of a worker failing is independent of the number of tasks he is working on. Firms then face a simple trade-off: Increasing the division of labor reduces learning costs but increases the probability of at least one worker shirking and hence failure of production. This trade-off implies that countries with higher human capital and better institutions have a comparative advantage for the more complex products.

<sup>15</sup>Formally, the productivity of labor is log-supermodular in product complexity and the skill level of labor.

<sup>16</sup>Antràs and Chor (2013) also consider quality upgrading in a production process with a continuum of tasks. In their model, these tasks are sequential and they study vertical integration of firms. As in our model, all tasks are essential. However, in the model presented by Antràs and Chor (2013), higher quality in one task can partly compensate lower quality in other tasks.

<sup>17</sup>The idea can also be illustrated by the prototype of the *Devel Sixteen* presented at the *Dubai International Motorshow 2013*. According to *Defining Extreme Vehicles Car Industry L.L.C.*, the firm presenting the prototype, this prototype is equipped with an engine of 5000hp. At present, however, it is not possible to drive the *Devel Sixteen* because there are neither tires, nor gear drives, nor clutches available on the market that could cope with such a powerful engine (cf. <http://www.spiegel.de/auto/aktuell/devel-sixteen-premiere-eines-brachial-autos-aus-dubai-mit-5000-ps-a-932513.html>, retrieved on 12 November 2013).



differences in the scope for quality differentiation, two assumptions that are common in the literature and that are supported by the data. Skills are more valuable in the production of high quality for the very same reason that they are more valuable in the production of complex products.<sup>18</sup> And quality upgrading is more difficult for complex products because it requires higher quality of every task involved in production.<sup>19</sup>

In the context of our model, this extension of Kremer's O-ring theory implies that high-skill countries specialize on producing high quality, in line with what we observe from the data. This within-product specialization replaces across-product specialization. The basic intuition is that comparative advantages refer to the difficulty of production. With an endogenous choice of quality, this difficulty is no longer exogenously determined by the product complexity, but it becomes endogenous as well.

As argued previously, we further suggest that specialization within products is subject to product-specific minimum-quality requirements. This precludes low-skill countries from being competitive for complex products and gives rise to an upper-triangular structure of specialization of countries on products, in line with what we observe from the data.

Our insights also have important implications for two related strands in the literature. First, Hidalgo and Hausmann (2009) and Tacchella et al. (2012) propose new measures for the economic strength of countries and the complexity of products, based on a binary country-product matrix that indicates for every country the products for which it has a revealed comparative advantage. Broadly speaking, these measures classify a country as strong if it has revealed comparative advantage for many, complex products – a product being considered complex if few, strong countries have a revealed comparative advantage for it. Empirical evidence suggests that these measures can uncover important information on the economic strength of countries. We provide a general-equilibrium rationale for the proposed algorithms. We show that the interplay between product complexity and product quality introduces a systematic link between the economic strength of a country – as captured by a single reduced-form parameter

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<sup>18</sup>Quality-biased efficiency is assumed by Alcalá (2012) and Jaimovich and Merella (2014), for example, as discussed above. It is also in line with reduced-form specifications that directly link the quality of inputs to the quality of outputs, as in Verhoogen (2008), for example.

<sup>19</sup>Khandelwal (2010), Jaimovich and Merella (2012), and Kugler and Verhoogen (2012), for example, assume product-specific scope for quality upgrading. They model this scope by introducing parameters either in the production function (Jaimovich and Merella, 2012, and Kugler and Verhoogen, 2012) or in the consumers' indirect utility function (Khandelwal, 2010). Khandelwal (2010) presents empirical evidence in support of such cross-product differences in the scope for quality differentiation.



– and the range of products it can successfully compete for on the world market. This link can be exploited by the proposed algorithms.

Second, as already indicated, our work motivates the use of a censored regression model to estimate the link between a country’s GDP per capita and the quality of its exports. The use of a censored regression model is new to this strand in the empirical literature, and it has important implications on the results, indicating a much stronger link than the one observed when using OLS. We briefly discuss the related literature at the onset of our empirical section below.

### *Organisation of the paper*

The remainder of this paper is organized as follows. Section 2 presents our model. In section 3 we derive the equilibrium in our economy. We discuss the equilibrium pattern of comparative advantages and specialization in international trade in section 4. In section 5 we derive the censored regression model to estimate the link between a country’s GDP per capita and the quality of its exports, and take this model to the data. Section 6 concludes.

## **2 Model**

The world is composed of  $N_c$  countries. We consider the case of a footloose economy, where firms are free to locate production in whichever countries they deem best and to supply the world market from there. There are no tariffs, transportation costs, or other barriers to trade. Hence, in our economy, there is a single world market and a single price for every good.

### **2.1 Households**

The world is populated by a continuum of households  $h \in [0, 1]$  who derive utility from consumption of a continuum of products  $i \in [0, N]$ . Consumption of each product  $i$  is split across a set of varieties that differ in their quality  $q \in \mathcal{Q}_i$ . Utility depends on the quantity and the quality consumed, where quality can be interpreted as a reduced

form capturing any product attributes valued by the household:<sup>20</sup>

$$U^h \left( \{c_{i,q}^h\}_{(i,q) \in [0,N] \times \mathcal{Q}_i} \right) = C^h \quad (1)$$

$$C^h := \left( \int_0^N \left( \int_{q \in \mathcal{Q}_i} q c_{i,q}^h dq \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where  $c_{i,q}^h$  denotes the amount of the variety with quality  $q$  of product  $i$  consumed by household  $h$ .<sup>21</sup> There is a one-to-one mapping from qualities to varieties, and we therefore subsequently refer to qualities directly, unless this might cause confusion.

Qualities of the same product are perfect substitutes. The elasticity of substitution between products is given by  $\sigma$ .<sup>22</sup> We assume  $\sigma < 1 + \lambda$ , where  $\lambda$  is a parameter determining how difficult it is for firms to increase quality. This assumption guarantees that all products will be consumed in equilibrium.<sup>23</sup>

Households live in one of  $N_c$  countries. They differ in the efficiency of their labor,  $r$ . Across the world, these efficiencies are distributed according to  $F_r(r)$  on the interval  $[\underline{r}, \bar{r}]$ , with  $0 < \underline{r} \leq \bar{r} < 1$ , i.e.  $F_r(r)$  is the total mass of households with efficiency less than or equal to  $r$ . We use  $\mathcal{R}$  to denote the support of the associated probability distribution function, i.e.  $\mathcal{R}$  is the set of efficiency levels of labor available, and assume  $\underline{r}, \bar{r} \in \mathcal{R}$ , without loss of generality. We are most interested in analyzing how countries with different levels of economic strength compete over a heterogeneous set of products. For the main part, we will therefore assume that all households living in country  $k$  have the same efficiency level  $r^k$  and that  $r^k \neq r^l, \forall k, l \in \{1, 2, \dots, N_c\}$  with  $k \neq l$ . In principle, however, households can also be heterogeneous within countries, the efficiency levels of households can overlap across countries, and the distribution of efficiencies can be continuous or discrete. To simplify notation, we will henceforth

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<sup>20</sup>Baldwin and Harrigan (2011) vividly describe these preferences as *box-size-quality* preferences: Consumers are indifferent between a box of size 1 with quality 2 and a box of size 2 with quality 1.

<sup>21</sup> $\mathcal{Q}_i$  may be a discrete set. Thus, the integral sign should be interpreted as representing a Lebesgue integral, simply representing a sum in such case.

<sup>22</sup>As long as all products are consumed in equilibrium, the exact functional form of the outer utility does not matter for the main insights. CES utility is assumed as a convenient way of closing the model.

<sup>23</sup> $\sigma > 1$  implies that households love variety.  $\sigma < 1 + \lambda$  ensures that households' love-for-variety is sufficiently large. In the words of Bernard et al. (2003, p. 1276), it ensures that: '*goods are sufficiently heterogeneous in consumption relative to their heterogeneity in production so that buyers do not concentrate their purchases on a few low-price goods.*' This assumption is not needed in a variant of our model where all products involve the same number of tasks but differ in their minimum-quality requirements (cf. appendix F).

identify countries by the efficiency of their labor,  $r$ , and drop the country index.<sup>24</sup> To be concrete, we will speak of  $r$  as the *skill level* of a country (of its labor), but the origins of the country-specific efficiency  $r$  do not matter, and it may reflect institutions, production technologies, and human capital, for example. Labor is perfectly mobile across products, but immobile across countries.

Each household inelastically supplies  $L$  units of labor and maximizes utility subject to its budget constraint, which for the representative household is given by:

$$\int_0^N \int_1^{\bar{q}_i} p_{i,q} c_{i,q} dq di \leq L \int_{\underline{r}}^{\bar{r}} w_r dF_r(r) + \bar{I}, \quad (3)$$

where  $p_{i,q}$  is the price of quality  $q$  of product  $i$ ,  $w_r$  is the equilibrium wage rate earned by labor with skill level  $r$ , and  $L \int_{\underline{r}}^{\bar{r}} w_r dF_r(r)$  is the average labor income.<sup>25</sup> To solve the representative household's decision problem, we note that perfect substitutability between different qualities of the same product implies that all qualities of product  $i$  will be sold at the same *effective price*  $\rho_i := \frac{p_{i,q}}{q}$  in equilibrium. This, in turn, implies that the representative household is indifferent between consuming any combination of these qualities and, hence, its demand is defined at the product level only. Let  $\tilde{c}_i := \int_1^{\bar{q}_i} q c_{i,q} dq$  denote total *effective consumption* of product  $i$  by the representative household. Following standard steps, we then get:

$$\tilde{c}_i = CP^\sigma [\rho_i]^{-\sigma} \quad (4)$$

$$PC = L \int_{\underline{r}}^{\bar{r}} w_r dF_r(r) + \bar{I} \quad (5)$$

$$\text{with } P := \left( \int_0^N [\rho_i]^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

## 2.2 Firms

There is a continuum of monopolistically competitive firms. Firm  $i \in [0, N]$  has a global patent for all qualities of product  $i$ . Total demand for product  $i$  is given by equation (4). Production is organized in production sites. Each production site can

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<sup>24</sup>Analogously, we will use the set of efficiency levels of labor available,  $\mathcal{R}$ , to represent the set of countries.

<sup>25</sup>In equilibrium, labor with skill level  $r$  will earn the same wage rate,  $w_r$ , irrespective of its country of residence, and hence a household's labor income will depend only on the skill level of its labor. This is trivially the case if we assume that the skill levels of labor do not overlap across countries. Note, however, that this is the case more generally, as show in an extended version of this paper.

produce any amount of one specific quality of product  $i$ . We have a footloose economy, i.e. the firm is free to open up production sites at no costs anywhere in the world and to supply the world market from there. There are no transportation costs, tariffs, or other barriers to trade.<sup>26</sup>

### 2.2.1 Production technology

Production is based on an O-ring technology in the spirit of Kremer (1993), but with an endogenous choice of output quality. In particular, if firm  $i \in [0, N]$  opens up a production site in country  $r \in \mathcal{R}$  and hires a mass  $L_i(r)$  of labor to produce quality  $q$  of its product, then expected output,  $E[x_{i,q}]$ , is given by:

$$E[x_{i,q}] = [r]^{iq^\lambda} L_i(r), \quad q \geq 1. \quad (7)$$

This technology has the following interpretation: Producing product  $i$  requires successful accomplishment of a continuum of measure  $i$  of simultaneous tasks. If the firm hires a worker with skill level  $r$  to work on a set of tasks with measure  $\Delta$ , then the worker will successfully accomplish these tasks with probability  $[r]^{\Delta q^\lambda}$ .<sup>27</sup> This probability is the same, irrespective of the tasks the worker is working on, i.e. there are no gains from specialization of labor on a specific set of tasks. Each product has a standard version with quality  $q = 1$ . It can be refined by producing higher quality, but the inverse is not true: The product has to be at least of standard quality.<sup>28</sup> Higher quality, i.e.  $q > 1$ , renders the successful accomplishment of each individual task more demanding and, hence, lowers the probability of success.<sup>29</sup>  $\lambda > 0$  is a parameter determining how difficult it is to raise quality. The production technology implies that more *complex* products, i.e. products with a higher index  $i$ , are more difficult to produce, and that quality improvements are more demanding for these products. It also implies that skills are *complexity-biased* and *quality-biased* in the sense that they are of higher value in production of more complex and/or higher-quality products.

Labor is organized in a continuum of teams. The probability of successful operation of each team is  $[r]^{iq^\lambda}$ . Its expected output is given by this probability, times the mass of labor employed in the team. With a continuum of teams, we can apply the law of large

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<sup>26</sup>As alternatives, we could assume perfect competition or Bertrand competition for each product. The results would essentially be the same.

<sup>27</sup>Note that, by assumption of  $0 < \underline{r} \leq \bar{r} < 1$ , we have  $r \in (0, 1)$ .

<sup>28</sup>Cf. the discussion in section 1 for a motivation.

<sup>29</sup>Our guiding rationale is that higher output quality requires higher quality of every single task involved in production. Cf. section 1 for a discussion.

numbers and ignore the expectation operator in the production function henceforth. The production technology exhibits constant returns to scale with respect to  $L_i(r)$ . We note that the complexity of the product increases the difficulty of the production process and thus lowers the probability of successful operation of any given team, but not more labor is needed to accomplish the increased number of tasks: Given the same success probability,  $[r]^{iq^\lambda}$ , the same mass of labor employed in production,  $L_i(r)$ , yields the same output, irrespective of the complexity of the product, and hence the number of tasks involved in the production process.<sup>30</sup>

We further illustrate the production technology by means of the following simple example.<sup>31</sup>

### Example 1

*Paula wants to run a Swiss-watch business. Each watch is composed of three main parts: (1) A balance wheel (pendulum), (2) a spring, and (3) a suspension. A watch only works properly if all three components are well-functioning, which can only be observed upon assembly of the watch. Otherwise, the watch is worthless. Let  $\lambda = 1$ , for simplicity, and let  $q_w$  denote the quality of the watch produced. Then a worker with skill level  $r$  working on component  $j \in \{1, 2, 3\}$  successfully produces this component with probability  $(r)^{q_w}$ . Suppose one workday is needed for every attempt to produce a watch. Then the production function for watches is given by:*

$$E[x_{w,q_w}] = [(r)^3]^{q_w} L_w(r) ,$$

where  $x_{w,q_w}$  is the number of watches with quality  $q_w$  produced, and  $r$  and  $L_w(r)$  are the skill level and the mass in workdays of labor employed, respectively.

*Paula can produce two types of watches: A standard watch with  $q_w = 1$  and a high precision watch with  $q_w = 2$ . There are three types of workers: John with skill level  $r^J = \exp(-\frac{1}{2}) \approx 0.61$ , Thomas with  $r^T = \exp(-\frac{1}{3}) \approx 0.72$ , and Amy with  $r^A = \exp(-\frac{1}{6}) \approx 0.85$ . Now suppose that Paula hires John for two days to produce standard watches. Then her (expected) output is given by:  $[(r^J)^3]^1 * 2 \approx 0.45$ . Suppose, by contrast, she hires Amy for three days to*

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<sup>30</sup>The specification of the production technology in equation (7) implies that all workers working in a team have the same skill level of labor, i.e. we rule out the possibility that the firm hires different skill levels of labor to work on different tasks involved in production. With the assumptions made on the cross-country distribution of skills, this is trivially not possible. Note, however, that a standard result following Kremer (1993) implies that it is never optimal for a firm to form heterogeneous teams, i.e. ruling out this possibility does not impair the applicability of our subsequent analyses to alternative cross-country distributions of skills. We briefly discuss the more general set-up in an extended version of this paper.

<sup>31</sup>Throughout the remainder of this and in the following section, we will repeatedly refer back to this example to illustrate our results. The example may be skipped, if desired.

produce high-precision watches. Then her (expected) output is given by:  $[(r^A)^3]^2 * 3 \approx 1.1$ .

### 2.2.2 Firm's decision problem

Firm  $i \in [0, N]$  chooses a set of countries where to open up production sites,  $\mathcal{R}_i \subseteq \mathcal{R}$ . This choice is driven by the skill level of labor living in a country. For each  $r \in \mathcal{R}_i$ , the firm chooses a quality of product  $i$  to produce,  $q_i(r)$ , its price level,  $p_{i,q_i(r)}$ , total output,  $x_{i,q_i(r)}$ , and the mass of labor employed,  $L_i(r)$ .<sup>32</sup> It maximizes its profits, taking as given the production technology, (7), the input prices, i.e. the wage rate in each country,  $\{w_r\}_{r \in \mathcal{R}}$ , and the demand for product  $i$ , (4). This demand is specified in total quality-adjusted consumption of all qualities with the lowest quality-adjusted price,  $\rho_i$ . Hence firm  $i$ 's choice of output prices reduces to the choice of  $\rho_i$ , and it can freely allocate its quality-adjusted output to qualities.

In summary, we get the following profit maximization problem of firm  $i$ :

$$\begin{aligned} & \max_{\substack{\mathcal{R}_i, \rho_i, \{q_i(r)\}_{r \in \mathcal{R}_i} \\ \{x_{i,q_i(r)}\}_{r \in \mathcal{R}_i}, \{L_i(r)\}_{r \in \mathcal{R}_i}}} \int_{r \in \mathcal{R}_i} [\rho_i q_i(r) x_{i,q_i(r)} - L_i(r) w_r] dr & (8) \\ \text{s.t. } & x_{i,q_i(r)} = [r]^{iq_i(r)\lambda} L_i(r), \quad \forall r \in \mathcal{R}_i \\ & \int_{r \in \mathcal{R}_i} q_i(r) x_{i,q_i(r)} dr = CP^\sigma [\rho_i]^{-\sigma} \\ & q_i(r) \geq 1 \quad \forall r \in \mathcal{R}_i \\ & \mathcal{R}_i \subseteq \mathcal{R}. \end{aligned}$$

We now analyze firm  $i$ 's decision problem in detail. Let us refer to quality-adjusted output as *effective output* and let  $\chi_i := \int_{r \in \mathcal{R}_i} q_i(r) x_{i,q_i(r)} dr$  denote the total effective output of product  $i$ . Then, in essence, firm  $i$ 's decision problem boils down to the following two basic decisions:

- (i) Choose locations for production sites and the qualities they produce to minimize the costs per unit of effective output;
- (ii) Given these costs per unit of effective output, choose a (quality-adjusted) price to maximize profits.

If several production sites share the minimal costs per unit of effective output, then the allocation of total effective output,  $\chi_i$ , to these production sites is a matter of

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<sup>32</sup>In principle, the firm could produce several qualities in country  $r \in \mathcal{R}_i$ . However, as we show in Lemma 1, this will never be the case in equilibrium. To simplify notation, we thus ignore this possibility here.

indifference.

We proceed by considering the firm's cost minimization problem first.

*(i) Cost minimization problem*

When opening up a production site, firm  $i \in [0, N]$  chooses a location,  $r$ , and an output quality,  $q$ , for product  $i$  to solve the following cost minimization problem:

$$\begin{aligned} \min_{q,r} \quad & \frac{w_r}{q[r]^{iq^\lambda}} \\ \text{s.t.} \quad & q \geq 1 \\ & r \in \mathcal{R}, \end{aligned}$$

where  $\frac{w_r}{q[r]^{iq^\lambda}}$  are the costs per unit of effective output. It will be instructive to solve this problem in two steps. First, we take the location of a production site, i.e. the skill level of labor employed, as given and derive the optimal choice of quality. Then we discuss the choice of skill levels.

Suppose firm  $i$  operates a production site in a country with skill level  $r \in \mathcal{R}$ . With  $r$  – and hence the wage rate – fixed, minimizing the costs per unit of effective output is equivalent to maximizing the effective output per worker, which is given by  $q[r]^{iq^\lambda}$ . Then the choice of  $q$  involves a simple trade-off: When raising  $q$ , the firm weighs the gain from a more valuable product against the loss of a lower output due to the increased difficulty of production. Formally, the choice of  $q$  solves the following first order condition:

$$[r]^{iq^\lambda} = -\lambda q^\lambda i \log(r) [r]^{iq^\lambda}, \quad (9)$$

and it turns out that there is a unique cost-minimizing choice of quality,  $q_i(r)$ :

**Lemma 1**

$\forall (i, r) \in [0, N] \times \mathcal{R}$ , let firm  $i$  run a production site in country  $r$ . Then it produces quality:

$$q_i(r) = \max \left\{ 1, \left[ -\frac{1}{\lambda i \log(r)} \right]^{\frac{1}{\lambda}} \right\}.$$

A proof of Lemma 1 is given in appendix B.1. For  $-\frac{1}{\lambda i \log(r)} > 1$ , quality increases with the skill level of labor used in production,  $r$ , reflecting a quality-bias of skills.<sup>33</sup>

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<sup>33</sup>Formally, the productivity of labor in terms of effective output is log-supermodular in the skill level of labor and output quality.



Furthermore, optimal product quality decreases with the complexity of the product,  $i$ , and  $\lambda$ , the factor characterizing how difficult it is to increase quality. When operating in a low-skill country, the firm would ideally simplify production by producing a variety with a quality that is lower than the minimum quality. As this is not feasible, it can do no better than producing the minimum quality instead.

**Example 1 (continued)**

*Amy optimally produces high precision watches:  $-\frac{1}{3\log(r^A)} = 2$ . Thomas optimally produces standard watches:  $-\frac{1}{3\log(r^T)} = 1$ . John, however, is bounded by the minimum-quality constraint. For him, producing standard watches is very demanding and he would preferably produce quality  $-\frac{1}{3\log(r^J)} = \frac{2}{3}$ . As this is not feasible, however, his best alternative is to also produce standard watches instead.*

The optimal choice of product quality pins down the costs per unit of effective output up to the choice of the location of the production site, i.e. the skill level of labor employed. Depending on this choice of  $r$ , the firm can produce *preferred quality* or not, with preferred quality given by  $\left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}}$ . Let us introduce the following notation for threshold complexity levels as a function of skills,  $\tilde{i}(r)$ , and threshold skill levels as a function of complexity,  $\tilde{r}(i)$ :

$$\begin{aligned}\tilde{i}(r) &:= -\frac{1}{\lambda \log(r)} \\ \tilde{r}(i) &:= e^{-\frac{1}{\lambda i}}.\end{aligned}$$

$\tilde{i}(r)$  denotes the maximal complexity of the products that can be produced at preferred quality in country  $r$ , i.e. the complexity level for which the minimum-quality constraint is just binding when labor with skill level  $r$  is used in production. Conversely,  $\tilde{r}(i)$ , which is the inverse of  $\tilde{i}(r)$ , denotes the minimal skill level needed in production of product  $i$  to produce at preferred quality, i.e. the skill level for which the minimum-quality constraint is just binding when producing product  $i$ .

**Example 1 (continued)**

*The threshold complexity levels for Amy, Thomas, and John, are  $-\frac{1}{\log(r^A)} = 6$ ,  $-\frac{1}{\log(r^T)} = 3$ , and  $-\frac{1}{\log(r^J)} = 2$ , respectively. The threshold skill level for watches is  $\exp\left(-\frac{1}{3}\right) \approx 0.72$ , Thomas' skill level.*

Let  $\frac{L_i(r)w_r}{\chi_i} \Big|_q$  denote firm  $i$ 's costs per unit of effective output if it produces quality  $q$

in country  $r$ . Then, with the optimal choice of quality,  $q_i(r)$ , we have:

$$\frac{L_i(r)w_r}{\chi_i} \Big|_{q_i(r)} := \frac{w_r}{q_i(r) [r]^{iq_i(r)\lambda}} = \begin{cases} w_r [-e\lambda i \log(r)]^{\frac{1}{\lambda}} & \text{if } r \geq \tilde{r}(i) \\ w_r [r]^{-i} & \text{otherwise} \end{cases}. \quad (10)$$

Firm  $i$  opens up production sites in an arbitrary combination of the countries where its costs per unit of effective output, (10), are minimal. This choice depends on the shape of the wage scheme,  $\{w_r\}_{r \in \mathcal{R}}$ , and we thus analyze it in connection with our discussion of the equilibrium wage scheme in section 3.1 below. For now, we take the choice of countries  $\mathcal{R}_i \subseteq \mathcal{R}$  as given and briefly discuss firm  $i$ 's profit maximization problem first.

(ii) *Profit maximization problem for  $\mathcal{R}_i$  given*

All qualities of product  $i$  are sold at the same quality-adjusted price. we can follow standard steps to get:

$$\rho_i = \begin{cases} \frac{\sigma}{\sigma-1} w_r [-e\lambda i \log(r)]^{\frac{1}{\lambda}} & \text{if } r \geq \tilde{r}(i) \\ \frac{\sigma}{\sigma-1} w_r r^{-i} & \text{otherwise} \end{cases}. \quad (11)$$

The quality-adjusted price is equal to the well-known constant mark-up over the marginal costs of producing effective output.

### 3 Equilibrium

In this section we analyze the equilibrium, starting with its definition.

#### Definition 1 (Equilibrium)

An equilibrium is:

- (i) for each firm  $i \in [0, N]$ , a set of countries where the firm operates a production site,  $\{\mathcal{R}_i\}_{i \in [0, N]}$
- (ii) for each production site of each firm, a quality,  $\{q_i(r)\}_{(i,r) \in [0, N] \times \mathcal{R}_i}$ , an output level,  $\{x_{i,q_i(r)}\}_{(i,r) \in [0, N] \times \mathcal{R}_i}$ , and a mass of labor employed,  $\{L_i(r)\}_{(i,r) \in [0, N] \times \mathcal{R}_i}$  a set of consumption levels of the representative household for each quality of each product,  $\{c_{i,q_i(r)}\}_{(i,r) \in [0, N] \times \mathcal{R}_i}$
- (iii) a set of good prices,  $\{p_{i,q_i(r)}\}_{(i,r) \in [0, N] \times \mathcal{R}_i}$
- (iv) a set of wage rates,  $\{w_r\}_{r \in \mathcal{R}}$

such that:

- (i)  $\frac{p_{i,q_i(r)}}{q_i(r)} = \rho_i, \forall (i, r) \in [0, N] \times \mathcal{R}_i$  and some  $\rho_i \geq 0$
- (ii)  $\mathcal{R}_i, \{q_i(r)\}_{r \in \mathcal{R}_i}, \{x_{i,q_i(r)}\}_{r \in \mathcal{R}_i}, \{L_i(r)\}_{r \in \mathcal{R}_i}$ , and  $\rho_i$  solve firm  $i$ 's profit maximization problem, (8),  $\forall i \in [0, N]$
- (iii)  $\{c_{i,q_i(r)}\}_{(i,r) \in [0,N] \times \mathcal{R}_i}$  maximize the representative household's utility, (1), subject to its budget constraint, (3)
- (iv) good markets clear for each quality of each product
- (v) labor markets clear in all countries

From our discussion of the firm we know that equilibrium outcomes depend on whether or not firm  $i$  locates production in a country with skill level  $r \geq \tilde{r}(i)$ . We start by analyzing the labor market and identify a condition for *sufficient skills* in the economy which guarantees that  $r \geq \tilde{r}(i), \forall (i, r) \in [0, N] \times \mathcal{R}_i$  in equilibrium. A sequence of preliminary results, along with this condition, eventually allow to characterize equilibrium wages as outlined in Proposition 1. We then derive the remaining equilibrium outcomes for the case of sufficient skills and summarize these findings in Proposition 2.

### 3.1 Equilibrium wage

In our economy there is a separate labor market in each country. Labor is immobile across countries, but firms are not. They can freely locate production wherever they deem best. We thus start our analysis of the labor market by reconsidering the optimal choice of a location for a production site by firm  $i \in [0, N]$ , i.e. its demand for skills. Firm  $i$  chooses  $\mathcal{R}_i$  to minimize its costs of producing one unit of effective output as specified in equation (10). This optimal choice depends on the shape of the wage scheme  $\{w_r\}_{r \in \mathcal{R}}$ . It is instructive to consider the choice between two countries with skill levels  $r^h, r^l \in \mathcal{R}$  first, where  $r^h > r^l \geq \tilde{r}(i)$ . Then, from equation (10), it follows that firm  $i$  would be indifferent between producing in either of the two countries if wages satisfied:

$$w_{r^h} = \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}} w_{r^l} .$$

This holds true  $\forall r^l, r^h \in [\tilde{r}(i), \bar{r}]$ . More generally, take  $w_{\underline{r}} = 1$  to be the numéraire and let wages satisfy:

$$w_r = \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R} . \quad (12)$$

Then firm  $i$  is indifferent between producing in any country with a skill level at least as high as its threshold level  $\tilde{r}(i)$ . The wage premium earned by workers in the high-skill country when compared to workers in the low-skill country just compensates for their higher productivity, provided that the firm is not bounded by the minimum-quality constraint  $q \geq 1$  in both countries.

Next, consider the trade-off between two countries  $r^h \geq \tilde{r}(i)$  and  $r^l < \tilde{r}(i)$  for the case of wages given by equation (12) above. Then we have:

$$\begin{aligned} \frac{L_i(r^l)w_{r^l}}{\chi_i} \Big|_{q_i(r^l)} &= w_{r^l} [r^l]^{-i} \\ &> w_{r^l} [-e\lambda i \log(r^l)]^{\frac{1}{\lambda}} = w_{r^h} [-e\lambda i \log(r^h)]^{\frac{1}{\lambda}} = \frac{L_i(r^h)w_{r^h}}{\chi_i} \Big|_{q_i(r^h)} . \end{aligned}$$

The inequality follows from the fact that producing quality  $q_i(r^l) = \left[ -\frac{1}{\lambda i \log(r^l)} \right]^{\frac{1}{\lambda}} < 1$  would be uniquely cost minimizing in country  $r^l$  if it was feasible. Hence firm  $i$  strictly prefers producing in country  $r^h$  to producing in country  $r^l$ . Intuitively, in country  $r^l$  it cannot produce preferred quality,  $\left[ -\frac{1}{\lambda i \log(r^l)} \right]^{\frac{1}{\lambda}} < 1$ , but produces quality  $q = 1$  instead. This quality constraint implies an additional advantage of high-skill over low-skill countries which is not compensated for by their wage premium.

We summarize these insights on firm  $i$ 's demand for skills in Lemma 2 below. In addition to what was discussed previously, this lemma states that firm  $i$  also prefers producing in country  $r^h$  to producing in country  $r^l$  for the case of  $\tilde{r}(i) > r^h > r^l$ . The intuition is that the minimum quality  $q = 1$  is closer to the preferred quality in the high-skill country than to the preferred quality in the low-skill country. A proof of this statement is given in appendix B.2.

## Lemma 2

Let wages be given by equation (12). Then:

- (i)  $\forall (i, r^l, r^h) \in [0, N] \times \mathcal{R} \times \mathcal{R}$  such that  $r^h > r^l \geq \tilde{r}(i)$ , firm  $i$  is indifferent between producing in country  $r^l$  and in country  $r^h$ ;
- (ii)  $\forall (i, r^l, r^h) \in [0, N] \times \mathcal{R} \times \mathcal{R}$  such that  $r^h > r^l$  and  $r^l < \tilde{r}(i)$ , firm  $i$  strictly prefers producing in country  $r^h$  to producing in country  $r^l$ .

**Example 1 (continued)**

Let John's wage be the numéraire, i.e. we have  $w^J = 1\$$ . Further, let wages for Thomas and Amy satisfy equation (12) above, i.e. we have  $w^T = \frac{\log(r^J)}{\log(r^T)} = 1.5\$$  and  $w^A = \frac{\log(r^J)}{\log(r^A)} = 3\$$ . Now suppose Paula hires John. John needs  $\frac{2}{(r^J)^3} \approx 8.96$  workdays on average to produce two watches of standard quality. This costs Paula  $\sim 8.96\$$ . Thomas, by contrast, needs  $\frac{2}{(r^T)^3} \approx 5.44$  workdays on average to produce two watches of standard quality. This costs Paula  $\sim 8.15\$$ . Finally, Amy needs  $\frac{1}{[(r^A)^3]^2} \approx 2.72$  workdays to produce one high-precision watch which is as good as two watches of standard quality. This costs Paula  $\sim 8.15\$$  as well. Hence Paula is indeed indifferent between hiring Amy and Thomas, and strictly prefers hiring either one of them to hiring John.

Lemma 2 immediately implies that equilibrium wages must satisfy the following condition:<sup>34</sup>

**Corollary 1**

*In equilibrium we must have:*

$$\frac{w_{r^h}}{w_{r^l}} \geq \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}} \quad \forall r^l, r^h \in \mathcal{R} : r^h \geq r^l . \quad (13)$$

We now analyze under which conditions the equilibrium wages are characterized by equation (12). From Lemma 2 we know that such an equilibrium is associated with indeterminacy in terms of where firms locate their production sites. Every firm  $i \in [0, N]$  is indifferent between producing in any country with skill level  $r \geq \tilde{r}(i)$ . It follows that the mass of labor employed by firm  $i$  is also undetermined. To analyze the equilibrium on the labor market, we therefore introduce the following concept of *effective labor*,  $\tilde{L}(r)$ :

$$\tilde{L}(r) := L(r) \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} . \quad (14)$$

This concept of effective labor normalizes labor of skill level  $r \in \mathcal{R}$  in terms of labor with the lowest skill level,  $\underline{r}$ , for the case of both skill levels being able to operate at preferred quality. It follows that firm  $i$ 's demand for effective labor is uniquely

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<sup>34</sup>To show the result stated in Corollary 1, consider two countries with skill levels  $r^l, r^h \in \mathcal{R}$  with  $r^h > r^l$  and suppose, by contradiction, that their respective wages  $(\hat{w}_{r^l}, \hat{w}_{r^h})$  satisfy  $\frac{\hat{w}_{r^h}}{\hat{w}_{r^l}} < \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}}$ . Then we can conclude from Lemma 2 that all firms would strictly prefer producing in country  $r^h$  to producing in country  $r^l$ . Hence there would be excess supply of labor in country  $r^l$ , a contradiction to  $(\hat{w}_{r^l}, \hat{w}_{r^h})$  being the equilibrium wages in countries  $r^l$  and  $r^h$ , respectively.

determined, irrespective of the exact skill level  $r \geq \tilde{r}(i)$  it uses in production.<sup>35</sup>

With this notation, we can identify conditions such that the wage scheme (12) is an equilibrium. In particular, because firm  $i$  is willing to produce in any country with skill level  $r \geq \tilde{r}(i)$ , the following two conditions are sufficient for labor market clearing in all countries  $r \in \mathcal{R}$ : First, every firm  $i \in [0, N]$  must be able to satisfy its total demand for effective labor,  $\tilde{L}_i$ , in countries with skill level  $r \geq \tilde{r}(i)$ , i.e. there must be no excess demand for skills. Second, the overall labor market must clear, i.e. total supply of effective labor must equal total demand.

Let us turn to the former condition first. Consider some firm  $\hat{i} \in [0, N]$ .  $\tilde{r}(i)$  is increasing in  $i$ . Hence a necessary condition for all firms  $i \in [\hat{i}, N]$  being able to satisfy their demand for effective labor in a country with skill level  $r \geq \tilde{r}(i)$  is that the total supply of effective labor in countries with skill level  $r \geq \tilde{r}(\hat{i})$  is no less than total demand for effective labor by firms  $i \in [\hat{i}, N]$ . Now suppose that this condition is satisfied  $\forall \hat{i} \in [0, N]$ . Then firms can locate their production sites such that  $r \geq \tilde{r}(i) \forall (i, r) \in [0, N] \times \mathcal{R}_i$ , i.e. such that the minimum-quality constraint is never binding, and we say that we have *sufficient skills* in the economy.<sup>36</sup>

### Definition 2 (Sufficient Skills)

We say that there are sufficient skills in the economy if the following condition is satisfied:

$$L \int_{e^{-\frac{1}{\lambda}}}^{\bar{r}} \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} dF_r(r) \geq \int_{\hat{i}}^N \tilde{L}_i di, \quad \forall \hat{i} \in [0, N]. \quad (\text{SSC})$$

Condition (SSC) rules out that there is excess demand for skills.<sup>37</sup> Hence, if, in addition,

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<sup>35</sup>Suppose firm  $i \in [0, N]$  wants to produce  $\chi_i$  units of effective output in country  $r \geq \tilde{r}(i)$ . Then it needs:

$$\begin{aligned} \tilde{L}_i(r) &= L_i(r) \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \\ &= [-e\lambda i \log(\underline{r})]^{\frac{1}{\lambda}} \chi_i \end{aligned}$$

units of effective labor, which is indeed independent of the exact skill level  $r \geq \tilde{r}(i)$ .

<sup>36</sup>Note, however, that of course more skills would always be desirable, as high-skill workers are more productive.

<sup>37</sup>In principle, labor in high-skill countries may be fully employed in production of products with low complexity, and thus not be available for use in production of the most complex products, even though we have sufficient skills in the economy. To avoid this caveat, we assume that in case of indifference, labor will always opt to work in production of the most complex product. We motivate this ‘tie-breaking-rule’ by the following thought experiment: Consider an economy with two firms  $i$  and  $j$ ,  $i > j$ , and two countries  $r^l$  and  $r^h$ ,  $r^h > r^l$ . Let  $r^h \geq \tilde{r}(i) > \tilde{r}(j)$  and  $\tilde{r}(i) > r^l \geq \tilde{r}(j)$  and suppose labor in country  $r^l$  is selling at wage 1. Then firm  $j$ ’s maximal willingness to pay for labor

the overall market for effective labor clears, i.e. if condition (SSC) holds with equality for  $\hat{i} = 0$ , labor markets are in equilibrium.

We summarize our insights on equilibrium wages in the following proposition, in which we use  $\tilde{L}_i(\{\hat{w}_r\}_{r \in \mathcal{R}})$  to denote the effective labor input used by firm  $i$  when confronted with the wage scheme  $\{\hat{w}_r\}_{r \in \mathcal{R}}$ :

**Proposition 1**

Let  $\{\hat{w}_r\}_{r \in \mathcal{R}}$  be a wage scheme satisfying  $\hat{w}_r = \left[ \frac{\log(r)}{\log(\hat{r})} \right]^{\frac{1}{\lambda}} \forall r \in \mathcal{R}$ .

(i)  $\{\hat{w}_r\}_{r \in \mathcal{R}}$  is the unique equilibrium wage scheme if and only if  $\left\{ \tilde{L}_i(\{\hat{w}_r\}_{r \in \mathcal{R}}) \right\}_{i \in [0, N]}$  satisfies condition (SSC).

(ii) Otherwise, the equilibrium wage scheme,  $\{w_r^*\}_{r \in \mathcal{R}}$  satisfies:

$$w_r^* \begin{cases} = \left[ \frac{\log(r)}{\log(\hat{r})} \right]^{\frac{1}{\lambda}} & \text{if } r \leq \hat{r} \\ > \left[ \frac{\log(r)}{\log(\hat{r})} \right]^{\frac{1}{\lambda}} & \text{otherwise} \end{cases},$$

for some  $\hat{r} \in \mathcal{R}$  such that  $\hat{r} < \max \{ \mathcal{R} \}$ , and where

$$\frac{w_{r^h}}{w_{r^l}} \geq \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}} \forall r^l, r^h \in \mathcal{R} : r^h \geq r^l.$$

A proof of Proposition 1(i) is given in appendix B.3. Proposition 1(ii) follows immediately from Proposition 1(i) and Corollary 1. Intuitively, if condition (SSC) is not satisfied given  $\{\hat{w}_r\}_{r \in \mathcal{R}}$ , there are fewer skills available in the economy than demanded given a wage scheme  $\{\hat{w}_r\}_{r \in \mathcal{R}}$ . This excess demand for skills implies that workers in high-skill countries must earn an extra wage premium, leading to a wage scheme as characterized in Proposition 1(ii).

Without sufficient skills, we will always observe some block-diagonal pattern of specialization of countries on products.<sup>38</sup> We are most interested in analyzing how countries specialize on quality for a heterogeneous set of products. Hence, in the remainder of this paper, we will make the following assumption:

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in country  $r^h$  is  $w_{r^h} = \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}}$ . By contrast, being able to produce in country  $r^h$  has some extra value for firm  $i$ , as it enables production with preferred quality, which is not the case for country  $r^l$ . Hence, if necessary, firm  $i$  will be willing to offer a marginally higher wage to labor in country  $r^h$  to break these workers' indifference between joining firm  $i$  and  $j$ .

<sup>38</sup>In an equilibrium according to Proposition 1(ii), all firms  $i \in [0, \tilde{i}(\hat{r})]$  will only produce in countries with skill level  $r \leq \hat{r}$ . On the other hand, if some firm  $\hat{i} > \tilde{i}(\hat{r})$  chooses to produce in some country  $r^h > \hat{r}$ , then all firms  $i \in (\hat{i}, N]$  will only produce in countries with skill level  $r > \hat{r}$ . This follows



### Assumption 1

$$\frac{\int_{e^{-\frac{1}{\lambda i}}}^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)}{\int_r^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)} \geq 1 - \left(\frac{i}{N}\right)^{\frac{1+\lambda-\sigma}{\lambda}}, \quad \forall i \in [0, N]$$

Assumption 1 restricts the set of feasible distributions  $F_r(r)$ . As we will show, it guarantees that we have sufficient skills in the economy, i.e. that we have an equilibrium according to Proposition 1(i) above. While Assumption 1 may seem technical, it is important to bear in mind that, in economic terms, it simply states that there are enough high-skill countries such that these countries do not only produce complex products, but also some of the simple products. This is exactly what we observe from the data.

### 3.2 Other equilibrium values

From above we know that with sufficient skills equilibrium wages are given by:

$$w_r^* = \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R},$$

where the superscript  $*$  denotes equilibrium values. The derivations of the other equilibrium values are straightforward and are thus relegated to appendix B.4. There, we also use  $\tilde{L}_i^*$  in condition SSC to show that indeed Assumption 1 implies sufficient skills.

We summarize our insights in the following proposition:

#### Proposition 2

*Let Assumption 1 be satisfied. Then in any equilibrium it holds that:*

- (i)  $w_r^* = \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R}$
- (ii)  $\mathcal{R}_i^* \subseteq \{r \in \mathcal{R} : r \geq \tilde{r}(i)\} \quad \forall i \in [0, N]$
- (iii)  $q_i^*(r) = \left[ -\frac{1}{\lambda i \log(r)} \right]^{\frac{1}{\lambda}} \quad \forall (i, r) \in [0, N] \times \mathcal{R}_i^*$

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from the fact that with an optimal choice of output quality we have:

$$\frac{\partial}{\partial i} \left[ \frac{\left. \frac{X_i}{L_i(r)} \right|_{q_i(r^h)}}{\left. \frac{X_i}{L_i(\tilde{r})} \right|_{q_i(\tilde{r})}} \right] > 0, \quad \forall i > \tilde{i}(\tilde{r}),$$

where  $\left. \frac{X_i}{L_i(r)} \right|_q$  denotes firm  $i$ 's effective output per unit of labor input if it produces quality  $q$  in country  $r$ .

$$\begin{aligned}
(iv) \quad \rho_i^* &= \frac{\sigma}{\sigma-1} [-e\lambda i \log(\underline{r})]^{\frac{1}{\lambda}} \quad \forall i \in [0, N] \\
(v) \quad \chi_i^* &= \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} \frac{1+\lambda-\sigma}{\lambda} N^{-\frac{1+\lambda-\sigma}{\lambda}} [i]^{-\frac{\sigma}{\lambda}} \quad \forall i \in [0, N] \\
(vi) \quad \tilde{L}_i^* &= \tilde{L} \frac{1+\lambda-\sigma}{\lambda} N^{-\frac{1+\lambda-\sigma}{\lambda}} [i]^{\frac{1-\sigma}{\lambda}} \quad \forall i \in [0, N] \\
(vii) \quad P^* &= \frac{\sigma}{\sigma-1} [-e\lambda \log(\underline{r})]^{\frac{1}{\lambda}} \left[ \frac{\lambda}{1-\sigma+\lambda} \right]^{\frac{1}{1-\sigma}} N^{\frac{1-\sigma+\lambda}{(1-\sigma)\lambda}} \\
(viii) \quad C^* &= \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} \left[ \frac{\lambda}{1+\lambda-\sigma} \right]^{\frac{1}{\sigma-1}} N^{\frac{1+\lambda-\sigma}{(\sigma-1)\lambda}}.
\end{aligned}$$

The equilibrium is unique up to the allocation of total effective output of product  $i$ ,  $\chi_i^*$ , to production sites,  $\mathcal{R}_i^*$ , and hence the choice of qualities and actual output, price, and labor input levels for these qualities.

## 4 Comparative advantages with sufficient skills

Our discussions so far have focused on the firm. Let us now consider the implications of sufficient skills for specialization in international trade. From above we know that, in an equilibrium with sufficient skills, the minimum-quality constraint is never binding. Hence the following corollary follows immediately from Lemma 1:

### Corollary 2

*With sufficient skills in the economy, (high-) low-skill countries specialize on producing (high) low quality.*

This specialization of countries evens out comparative advantages across products that exist in the absence of a quality choice. Suppose, for example, that all products have minimum quality,  $q_i = 1 \quad \forall i \in [0, N]$ . Let  $\left. \frac{\chi_i}{L_i(r)} \right|_q$  denote firm  $i$ 's effective output per unit of labor input if it produces quality  $q$  in country  $r \in \mathcal{R}$ . With  $q = 1$ , we have:

$$\left. \frac{\chi_i}{L_i(r)} \right|_1 = [r]^i. \quad (15)$$

Now consider two countries with skill levels  $r^h, r^l \in \mathcal{R}$ , where  $r^h > r^l$ . Equation (15) implies that the more complex the product, the more productive is the high-skill country relative to the low-skill country:

$$\frac{\left. \frac{\chi_i}{L_i(r^h)} \right|_1}{\left. \frac{\chi_i}{L_i(r^l)} \right|_1} = \left[ \frac{r^h}{r^l} \right]^i,$$

i.e. high-skill countries have a comparative advantage for complex products.<sup>39</sup> This is a standard result already shown by Kremer (1993). We introduce a second dimension of product differentiation: the endogenous choice of product quality. With an interior solution for product quality, we have:

$$\frac{\chi_i}{L_i(r)} \Big|_{q_i(r)} = \left[ -\frac{1}{\lambda i \log(r)} \right]^{\frac{1}{\lambda}} [r]^{-\frac{1}{\lambda \log(r)}} = [-e\lambda i \log(r)]^{-\frac{1}{\lambda}} , \quad (16)$$

and for the ratio of productivities of the two countries  $r^h$  and  $r^l$  it follows:

$$\frac{\frac{\chi_i}{L_i(r^h)} \Big|_{q_i(r^h)}}{\frac{\chi_i}{L_i(r^l)} \Big|_{q_i(r^l)}} = \left[ \frac{\log(r^l)}{\log(r^h)} \right]^{\frac{1}{\lambda}} . \quad (17)$$

Indeed, this ratio is independent of the complexity of the product, i.e. with an interior solution for quality, there are no comparative advantages of countries for products. Intuitively, high-skill countries differentiate by producing higher quality. A simple revealed preference argument then suggests that they gain in absolute advantage. The crucial observation is that because quality upgrading is more demanding for complex products, this increase in absolute advantage will be smaller for the complex products. These are exactly those products for which high-skill countries have a comparative advantage at constant quality and, hence, quality-differentiation attenuates comparative advantages across products.<sup>40</sup>

In essence, by introducing an endogenous choice of product quality, across-product specialization is replaced by within-product specialization in the spirit of Schott (2004). Within-product specialization is truncated by the minimum-quality constraint,  $q \geq 1$ . In equilibrium, this implies that high-skill countries can successfully compete for even

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<sup>39</sup>Formally, labor productivity is log-supermodular in the skill level of labor and the complexity of the product.

<sup>40</sup>Formally, within the context of our model, quality differentiation breaks the strict log-supermodularity of the productivity of labor in terms of *effective* output. Let  $h(r, i, q) := qr^{iq^\lambda}$  denote this productivity. At constant quality,  $\bar{q}$ , we have:

$$\frac{\partial^2 \ln(h(r, i, \bar{q}))}{\partial r \partial i} = \frac{\bar{q}^\lambda}{r} > 0 .$$

With quality differentiation, however, we have to take into account that the optimal choice of quality varies with  $r$  and  $i$ , i.e. what matters for comparative advantages is the total cross derivative which, by the envelope theorem, simplifies to:

$$\frac{d^2 \ln(h(r, i, q_i(r)))}{dr di} = \frac{\partial^2 \ln(h(r, i, q_i(r)))}{\partial r \partial i} + \frac{\partial^2 \ln(h(r, i, q_i(r)))}{\partial r \partial q} \frac{dq_i(r)}{di} .$$

It is easy to verify that this cross-derivative is indeed equal to 0.

the simplest products by specializing on high quality, but not vice versa. Low-skill countries cannot successfully compete for complex products because these products are just too difficult, even in their minimum-quality version. Formally, the following corollary follows immediately from Proposition 2:

**Corollary 3**

*In an equilibrium with sufficient skills, each country  $r \in \mathcal{R}$  is competitive for all products  $i \in [0, \tilde{i}(r)]$ .*

Hence an equilibrium with sufficient skills is associated with an upper-triangular structure of competitiveness of countries for products. A country with skill level  $r^h \in \mathcal{R}$  is competitive for all products a country with a lower skill level  $r^l < r^h$  is competitive for, plus some additional – more complex – products. The exact mapping of products to countries is undetermined in equilibrium. We therefore consider a simple numerical example next, to illustrate how the described pattern of comparative advantages translates into an upper-triangular structure of specialization in international trade, in line with what we observe from the data.<sup>41,42</sup>

**4.1 Numerical example**

We consider the equilibrium in a world with  $N_c$  countries,  $N_p$  products, and sufficient skills. For simplicity, we assume that countries are equally sized, each having one unit of labor, i.e. we have  $L = N_c$ . We present a numerical example where we randomly allocate products to countries, subject to the constraint that all firms  $i = 1, 2, \dots, N_p$  produce only in countries that are competitive for their product.

To calibrate our model, we first set  $N_c = 149$  and  $N_p = 1239$  to match the number of countries and products, respectively, considered in Figures 5 to 7. We then require that

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<sup>41</sup>Cf. appendix A.

<sup>42</sup>To be precise, we need to slightly strengthen Assumption 1 for an upper-triangular structure of specialization to occur in equilibrium:

**Assumption 1(a)**

$$\frac{\int_{e^{-\frac{1}{\lambda i}}}^{\tilde{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)}{\int_{\underline{r}}^{\tilde{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)} \geq 1 - \left(\frac{i}{N}\right)^{\frac{1+\lambda-\sigma}{\lambda}}, \quad \forall i \in [0, N],$$

*with the inequality being strict  $\forall i > 0$ .*

If the condition stated in Assumption 1 was to hold with equality for some  $\hat{i} \in (0, N]$ , then while countries with skill level  $r \geq \tilde{r}(\hat{i})$  would still be competitive for products  $i < \hat{i}$ , their labor force would be fully employed in production of products  $i \geq \hat{i}$ . Hence high-skill countries would never produce products with low complexity in equilibrium.

the distribution of equilibrium wages,  $w_r^*$ , matches the distribution of GDP per capita in purchasing power parities (PPP) in 2010 for the selection of countries considered in Figures 5 to 7, as observed from World Bank (2013).<sup>43</sup> Out of the selection of 149 countries included in Figure 5, we observe data on GDP per capita in PPP for 140 countries only, i.e. this calibration step reduces  $N_c$  to 140. It determines the distribution of skill levels,  $F_r(r)$ , up to the choice of  $\underline{r}$ , which we set equal to  $\underline{r} = 0.01$ . We assume that the complexities of products are uniformly distributed on the set  $\left\{ \frac{N}{N_p} \left[ i - \frac{1}{2} \right] : i = 1, 2, \dots, N_p \right\}$ , and choose  $N = 4$ . As to the remaining parameter values, we assume  $\lambda = 1$  for simplicity, and  $\sigma = 1.5$ , the midpoint of its feasible range.

We divide production of product  $i$  in small production steps. Each such step consists of a fixed amount of effective labor and is randomly allocated to countries as outlined in appendix C. The simulation results in one specific realization of equilibrium outcomes. For this equilibrium, we can observe the effective output of product  $i$  produced in country  $r$  for every country-product pair  $(r, i) \in \mathcal{R} \times \{1, 2, \dots, N_p\}$ . We follow Hausmann and Hidalgo (2011) and Tacchella et al. (2012) in visualizing the implied pattern of international specialization graphically. In particular, we use the equilibrium allocation of production to derive a binary matrix  $M$  that indicates for every country the products for which it has a revealed comparative advantage.<sup>44</sup> We then order countries from the lowest to the highest skill level, and products from the simplest to the most complex. We plot the accordingly rearranged matrix  $M$  in Figure 1. Comparing Figure 1 to the real-world counterparts shown in appendix A reveals that, indeed, our model gives rise to the same basic pattern of international specialization as the one we observe from the data.

## 4.2 Discussion

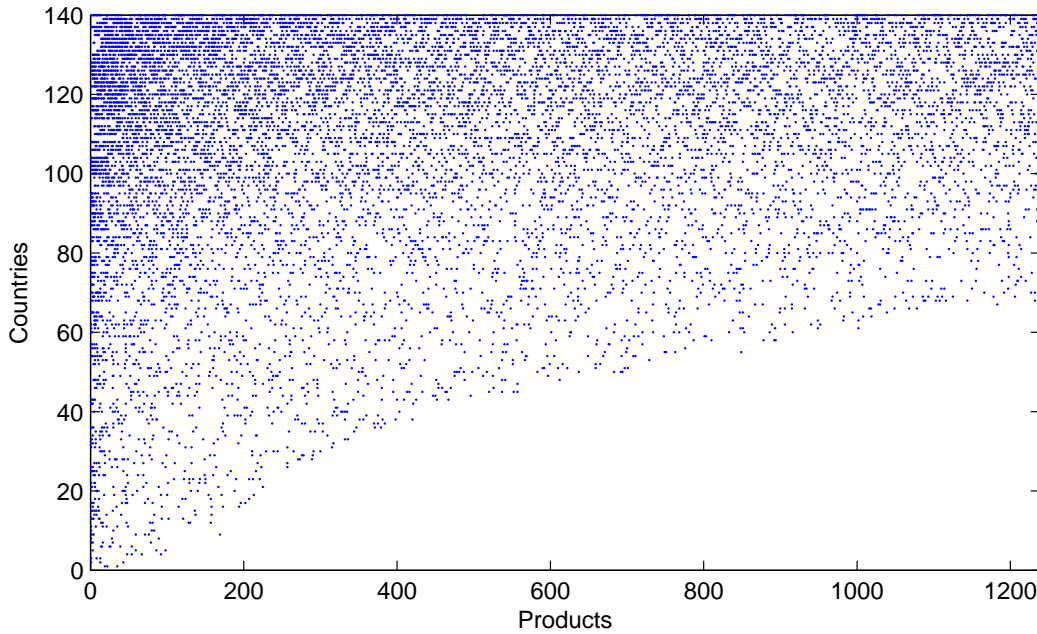
We present a parsimonious model to analyze the interplay between product complexity and product quality in international trade. In this model, there is one dimension

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<sup>43</sup>Note that, in our model, GDP per capita is equal to  $\frac{\sigma}{\sigma-1} w_r$  and, hence, is proportional to equilibrium wages.

<sup>44</sup>This can be achieved by first multiplying the effective outputs with quality-adjusted prices to get equilibrium revenues. Second, by computing the revealed comparative advantages and connecting products to all countries exporting them with revealed comparative advantage of at least 1, as originally proposed by Hidalgo and Hausmann (2009). As opposed to the real-world application of Hidalgo and Hausmann (2009), we base our computations on total production and not on exports only. Using total production should give a more comprehensive picture of a country's productive capacities (cf. also Hausmann and Hidalgo, 2011).

Figure 1: Revealed comparative advantages in an equilibrium with sufficient skills



*Source:* Own illustration, based on the numerical example as described in the main text and in appendix C. Countries and products are ranked according to the skill level of their workforce,  $r$ , and their complexity,  $i$ , respectively. For every country-product pair, a dot indicates that the country has *revealed comparative advantage* of at least 1 for that product.

of country heterogeneity – the skill level of a country’s labor,  $r$  – and one dimension of product heterogeneity – a product’s complexity,  $i$ . Countries specialize on quality. We show that this specialization eliminates comparative advantages across products. Our model thus introduces a new theoretical mechanism showing how countries successfully compete for a heterogeneous set of products. We further suggest that the specialization on quality is subject to product-specific minimum-quality requirements. These requirements impose no restrictions on high-skill countries, as these countries deliberately produce higher quality. Low-skill countries, however, are bounded by the minimum-quality requirements for complex products. For them, even the simplest versions of these products are very difficult to produce, and they cannot successfully compete for these products on the world market. Hence we introduce an alternative mechanism underlying comparative advantages, which is rooted in product complexity. In an equilibrium with sufficient skills, this mechanism gives rise to an upper-triangular structure of specialization in international trade, in line with what we observe from the data.

The basic mechanism we consider can also explain why the share of products that are co-exported by rich and poor countries tends to increase over time. If minimum-quality requirements are an important source of comparative advantages, then these comparative advantages naturally subside as the world economy develops. In fact, according to our model, comparative advantages subside as the low-skill countries develop, irrespective of the development of high-skill countries.

To simplify the exposition, we assumed that all workers living in country  $k \in \{1, 2, \dots, N_c\}$  have the same skill level  $r^k$ . However, allowing for heterogeneity of labor within countries would not affect our main insights as long as countries differ as to the highest skill level available. Also, for concreteness, we attributed the productivity of labor living in a country as captured in  $r^k$  to the skill level of this labor. Yet the origins of the differences in  $r^k$  do not matter for any of the implications of our model, and we can think of  $r^k$  as a reduced form for institutions, production technologies, and/or human capital, for example.

Our work has important implications for related fields of the literature. Hidalgo and Hausmann (2009) and Tacchella et al. (2012) propose new measures for the economic strength of countries and for the complexity of products, based on trade data. Precisely, these measures are based on a binary country-product matrix, indicating for each country the products it has a revealed comparative advantage for on the world market. Broadly speaking, they classify a country as strong if it has a revealed comparative advantage for many, complex products – a product being considered as complex if few, strong countries have a revealed comparative advantage for it. Our model can provide an economic rationale for the proposed algorithms. It introduces a systematic link between the economic strength of a country and the range of products it can successfully compete for on the world market. It follows that the binary country-product matrix indeed entails important information on the economic strength of countries and the complexity of products. As opposed to the rationale proposed by Hidalgo and Hausmann (2009) and formalized by Hausmann and Hidalgo (2011), our rationale is not based on a large set of non-tradeable capabilities and product-specific capability requirements, but on the interplay between product complexity and product quality. Country heterogeneity is summarized in a single reduced-form parameter,  $r$ , which – as mentioned above – can reflect various sources of economic strength discussed in the literature. Hence our model suggests that these new measures may well be informative on a more general scale about a country’s economic strength, without relying on a



heterogeneous set of capabilities. We substantiate this conjecture by means of a simple Monte Carlo experiment in appendix D, where we apply the proposed algorithms to binary country-product matrices that we derive from our numerical example of section 4.1. We then compare the rankings based on the proposed algorithms with the fundamental rankings of countries and products according to our model. This simple experiment suggests that the proposed algorithms can indeed well recover the fundamental rankings of countries and products, at least in a world as described by our model.

If low-skill countries cannot successfully compete for complex products because they are bounded by a minimum-quality constraint, this will also have important implications for empirical analyses to estimate the link between a country's skill level and the quality of its exports. We discuss these implications next.

## 5 Empirical analysis

The rationale developed in our theoretical model is centered on the observation that richer countries export higher quality. As already mentioned in the introduction, this observation is well established in the empirical literature. Schott (2004) estimates the elasticities of unit values of exports to the US with respect to the exporter's GDP per capita and its factor endowments. For a large and increasing share of product categories, this elasticity is positive.<sup>45</sup> Hummels and Klenow (2005) estimate the elasticities of the extensive and the intensive margin of a country's exports with respect to its GDP per worker and total employment. They observe that the extensive margin is more important for richer as opposed to larger economies, in line with the interpretation that a higher skill level allows a country to diversify into a broader set of products. They further decompose the intensive margin into a price and a quantity component, also concluding that richer countries tend to export higher quality goods. Khandelwal (2010) estimates product quality using information on a country's market share, controlling for its price level. Regressing this measure on log GDP per capita, he also finds a positive relationship.<sup>46</sup>

The studies by Schott (2004) and Khandelwal (2010) share in common that they esti-

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<sup>45</sup>Pham (2008) substantiates this result, considering imports to Brazil, India, and Japan.

<sup>46</sup>Similarly, Hallak and Schott (2011) use information on a country's trade balance, controlling for its export prices, to infer on the quality of its exports. They find that their measure of a country's export quality is significantly positively correlated with its GDP per capita.

mate the link from a country’s GDP per capita to its export quality from a regression using the data on observed export qualities. The rationale we develop suggests that this data might be censored from below. In particular, if countries with low skill levels are bounded by the minimum-quality constraint for complex products and can therefore not compete for them, then the fact that a country does not export a product could – and should – be exploited in an empirical analysis. As we will show next, this motivates the use of a censored regression model.

## 5.1 A censored regression model for a country’s export quality

In an equilibrium with sufficient skills, a country with skill level  $r \in \mathcal{R}$  is competitive only for products that it can produce with preferred quality,  $\left[-\frac{1}{\lambda i \log(r)}\right]^{\frac{1}{\lambda}} \geq 1$ , i.e.  $\forall i \in [0, \tilde{i}(r)]$ . Now suppose that each country exports all products that it is competitive for.<sup>47</sup> Then our theoretical set-up naturally leads to a censored regression model to estimate the link between a country’s skill level  $r$  and the quality of its exports. In particular, taking logs of the preferred quality, we get:

$$\log(q_{i,t}^k) = \mathbf{d}_{i,t} \boldsymbol{\alpha} + \beta \log[-\log(r_t^k)] + u_{i,t}^k, \quad u_{i,t}^k | \mathbf{d}_{i,t}, r_t^k \sim N(0, \sigma^2) \quad (18a)$$

$$\log(\tilde{q}_{i,t}^k) = \begin{cases} \log(q_{i,t}^k) & \text{if } q_{i,t}^k \geq 1 \\ \text{NaN} & \text{otherwise} \end{cases}, \quad (18b)$$

where  $q_{i,t}^k$  is the *latent* preferred quality of product  $i$  if produced in country  $k$  in period  $t$ ,  $\tilde{q}_{i,t}^k$  denotes the *observed* quality,  $r_t^k$  denotes the skill level of country  $k$  in period  $t$ ,  $\mathbf{d}_{i,t}$  is a  $1 \times T \cdot N_p$  vector of product-time dummies capturing (time-varying) product characteristics,  $\boldsymbol{\alpha}$  is a  $T \cdot N_p \times 1$  vector of coefficients on these dummies,  $u_{i,t}^k$  is an error term, and where we have assumed that the distribution of  $\log(q_{i,t}^k)$  given  $\mathbf{d}_{i,t}$  and  $r_t^k$  is homoskedastic normal.

It is not possible to take the censored regression model (18) directly to the data, as both  $\tilde{q}_{i,t}^k$  and  $r_t^k$  are unobservable. However, we can estimate the link between a country’s GDP per capita,  $GDP_{cap,t}^k$ , and the quality of its exports, and use export prices,  $p_{i,t}^k$ , as a proxy for quality. As we argue in detail in appendix E.1, this leads to the following censored regression model:

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<sup>47</sup>In 2010, countries such as Austria, Australia, Belgium, Canada, China, France, Germany, Japan, Switzerland, and the US, for example, had some exports in more than 97% of the products at the hs4 classification level.

## Hypothesis 1

$$\log(p_{i,t}^k) = \mathbf{d}_{i,t}\boldsymbol{\alpha} + \beta_i \log(GDP_{cap,t}^k) + u_{i,t}^k, \quad u_{i,t}^k | \mathbf{d}_{i,t}, GDP_{cap,t}^k \sim N(0, \sigma_i^2) \quad (19a)$$

$$\log(\tilde{p}_{i,t}^k) = \begin{cases} \log(p_{i,t}^k) & \text{if } p_{i,t}^k \geq \min_{k \in \{1,2,\dots,N_c\}} p_{i,t}^k \\ NaN & \text{otherwise} \end{cases} \quad (19b)$$

The latent variable model, equation (19a), is, in essence, regression model (2) in Schott (2004) and regression model (17) in Khandelwal (2010). Both papers estimate their models using OLS. The reasoning developed here suggests that using OLS on the subsample with  $p_{i,t}^k \geq \min_{k \in \{1,2,\dots,N_c\}} p_{i,t}^k$  is inconsistent, and that we should rather use maximum likelihood instead.<sup>48,49</sup> Precisely, we may expect OLS to underestimate the true link between a country's GDP per capita and the quality of its exports.<sup>50</sup>

## 5.2 Data description

Export data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013), which reports bilateral data on export values and export quantities for more than 200 countries at the hs6 classification level for the period from 1995 to 2011. From this dataset, we exclude countries with less than 1m inhabitants in 2008. We then sum up a country's exports over all destinations and summarize data at the hs4 classification level in our base-case scenario.<sup>51</sup>

Unit values are computed as the ratio of export values over export quantities. The resulting data is trimmed by excluding observations with extreme unit values. Let  $uv_{i,t}^k$  denote the unit value of exports of product  $i$  by country  $k$  in period  $t$ . Then

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<sup>48</sup>Cf. Wooldridge (2002, p. 524).

<sup>49</sup>Consistency of the maximum-likelihood estimator relies on the distributional assumptions. In particular, heteroskedasticity and nonnormality would result in inconsistent estimators. As a more robust alternative, we could apply the censored least absolute deviations estimator proposed by Powell (1984), which is based on the assumption  $Med(u_{i,t}^k | \mathbf{d}_{i,t}, GDP_{cap,t}^k) = 0$ . We could also estimate a richer model. For example, country  $k$  may not export product  $i$  in period  $t$  even though it is competitive for that product, i.e. even though  $q_{i,t}^k \geq 1$ . We could introduce this possibility into our regression model by modeling the probability of not exporting a product conditional on being competitive.

<sup>50</sup>Schott (2004) considers what he calls LMH products only, i.e. products that are co-exported by low- and high-income countries to the US. Following the line of reasoning presented here, this should mitigate the OLS estimation bias.

<sup>51</sup>Our main findings are robust to classifying products at the hs6 level. Cf. appendix E.2.1.

observations are dropped whenever:

$$\begin{aligned}
& uv_{i,t}^k \geq 10 \times \text{median}_k(uv_{i,t}^k) \wedge uv_{i,t}^k \geq 5 \times \text{median}_t(uv_{i,t}^k) \\
& \qquad \qquad \qquad \vee \\
& uv_{i,t}^k \leq \frac{1}{10} \times \text{median}_k(uv_{i,t}^k) \wedge uv_{i,t}^k \leq \frac{1}{5} \times \text{median}_t(uv_{i,t}^k) ,
\end{aligned}$$

i.e. observations are classified as outliers whenever they deviate strongly from the median observation across countries in the same year and from the median observation over time for the same country.<sup>52</sup>

Data on GDP per capita is taken from World Bank (2013). Following Hummels and Klenow (2005), we use data in purchasing power parities to avoid mechanical relationships between export prices and GDP stemming from market exchange rates.<sup>53</sup>

### 5.3 Estimation results

We start by estimating equation (19a) by OLS, using the subsample of data for which we observe an exporter's unit value. We run the estimation separately for each of the 1241 hs4 product categories included in our data. Standard errors are clustered by exporting country. The estimation results are summarized in Figure 2 and Table 1.

These results are remarkably close to the results of Schott (2004). A share of 57.5 % of the coefficients on  $\log(GDP_{cap})$  is positive and significant at the 5% level. Moreover, the average of the  $\beta$ 's indicates that a 10% increase in GDP per capita is associated with a 1.4% increase in unit values.

We next estimate the censored regression model (19) by maximum likelihood, using the full sample available. Again, we run the estimation separately for each of the 1241 hs4 product categories included in our data, and cluster standard errors by exporting country. The estimation results are summarized in Figure 3 and Table 2.

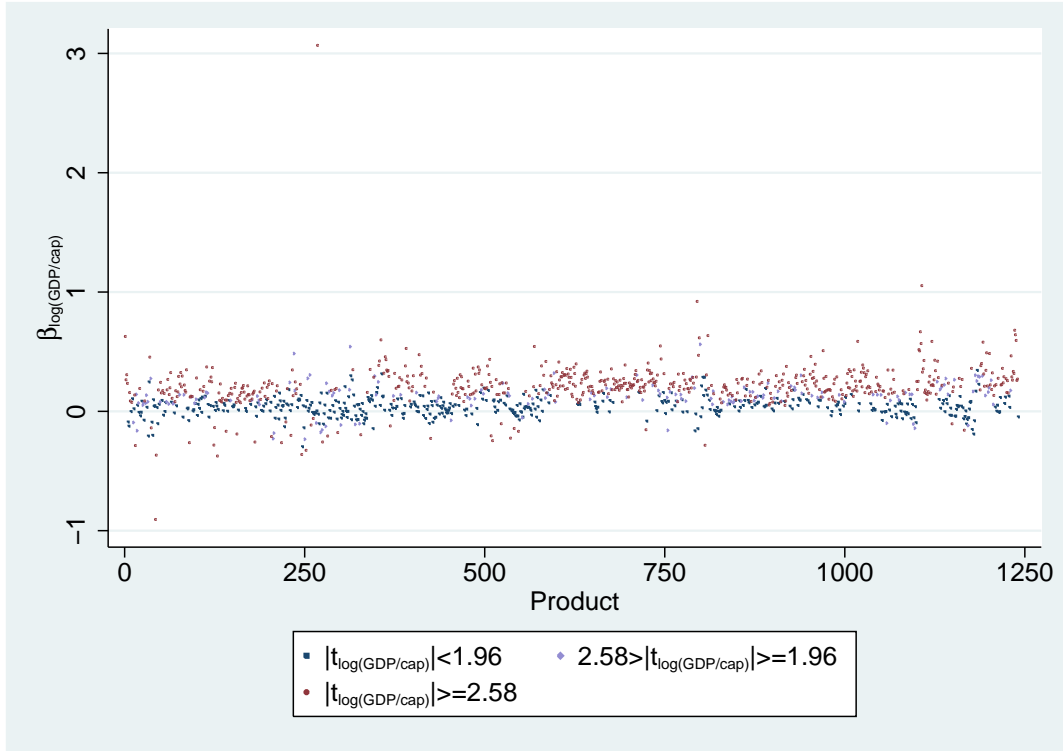
We next estimate the censored regression model (19) by maximum likelihood, using the full sample available. Again, we run the estimation separately for each of the 1241 hs4 product categories included in our data, and cluster standard errors by exporting country. The estimation results are summarized in Figure 3 and Table 2.

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<sup>52</sup>Our main findings are robust to using different selection criteria for outliers. Cf. appendix E.2.2.

<sup>53</sup>Our main findings are robust to using data at market exchange rates. Cf. appendix E.2.3.

Figure 2: OLS estimates of  $\beta_{\log(GDP_{cap})}$  – base case



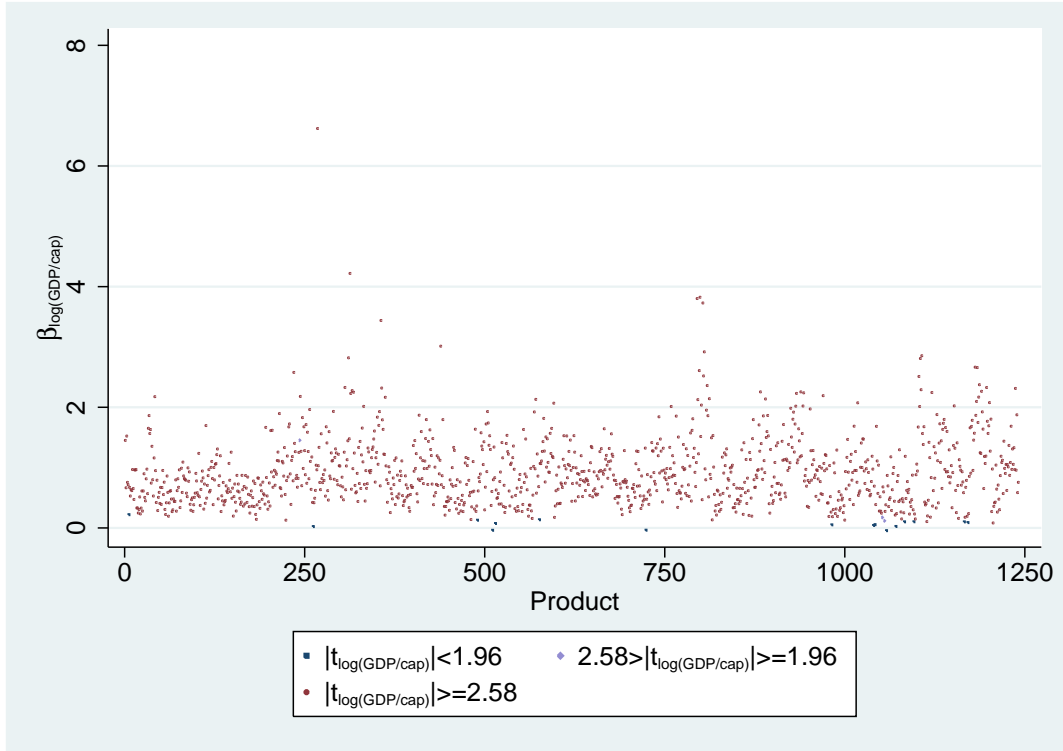
*Notes:* This figure plots the OLS estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a), using the subsamples with observed unit values. Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

Table 1: OLS estimates of  $\beta_{\log(GDP_{cap})}$  – base case

Mean $\beta_{\log(GDP_{cap})}$	0.136
Share $\beta_{\log(GDP_{cap})}$ significantly positive at 5% level	57.5%
Share $\beta_{\log(GDP_{cap})}$ significantly negative at 5% level	4.4 %
Product $\times$ year FEs	YES
Mean R-squared	0.643
Mean # observations	1460

*Notes:* This table reports summarizing statistics of the OLS estimates of equation (19a), using the subsamples with observed unit values. Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

Figure 3: ML estimates of  $\beta_{\log(GDP_{cap})}$  – base case



*Notes:* This figure plots the ML estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a). Standard errors are clustered by exporting countries. The trade data is taken from Centre d’Etudes Prospectives et d’Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

Table 2: ML estimates of  $\beta_{\log(GDP_{cap})}$  – base case

Mean $\beta_{\log(GDP_{cap})}$	0.915
Share $\beta_{\log(GDP_{cap})}$ significantly positive at 5% level	98.7%
Share $\beta_{\log(GDP_{cap})}$ significantly positive at 1% level	98.5 %
Share $\beta_{\log(GDP_{cap})}$ significantly negative at 5% level	0 %
Product $\times$ year FEs	YES
Mean # observations	2258

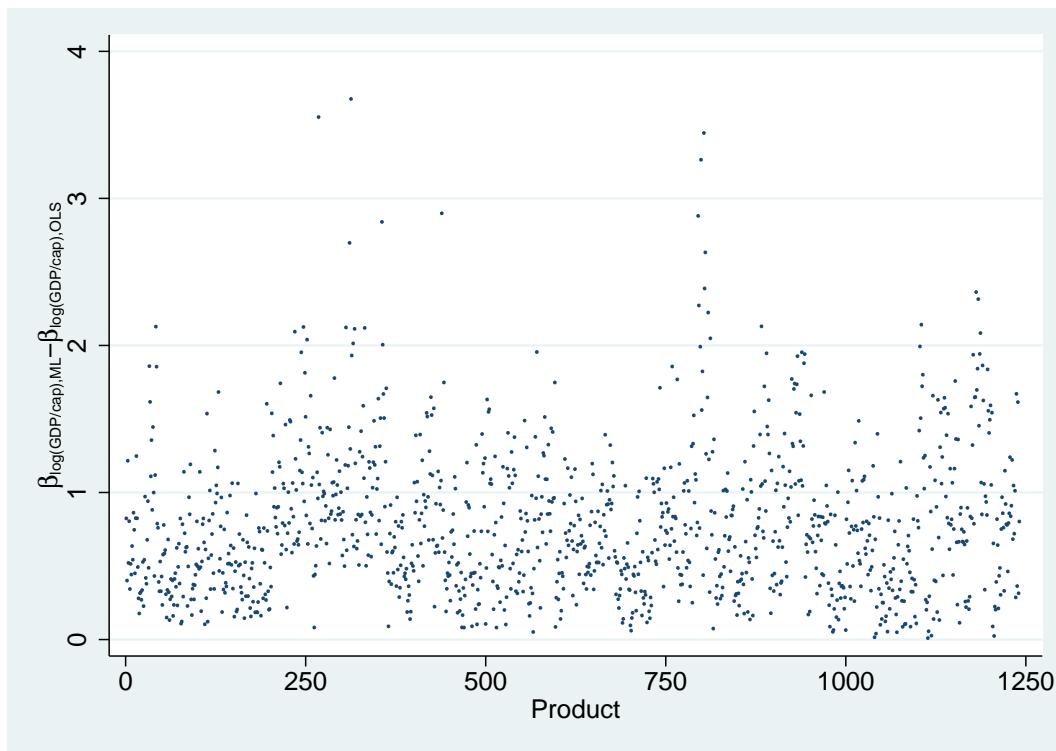
*Notes:* This table reports summarizing statistics of the ML estimates of regression model (19). Standard errors are clustered by exporting countries. The trade data is taken from Centre d’Etudes Prospectives et d’Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

These results reveal a much stronger link from a country’s GDP per capita to the unit values of its exports. The coefficient on  $\log(GDP_{cap})$  is significantly positive even at the

1% level in more than 98% of the 1241 regressions. We obtain a negative coefficient in only 3 out of 1241 regressions, none of which is significant at any conventional level.<sup>54</sup> Furthermore, the estimated effect is substantially larger: The maximum likelihood estimates suggest that on average across all product categories, a 10% higher GDP per capita is associated with a 9.1% higher unit value of the same product.

Our findings are in line with a downward bias of the OLS estimator. For each of the 1241 hs4 product categories considered, we plot in Figure 4 the difference between the maximum likelihood and the OLS estimate of the coefficient on  $\log(GDP_{cap})$ . This difference is positive in all cases.<sup>55</sup>

Figure 4: Comparison of estimated betas:  $\beta_{\log(GDP_{cap}),ML} - \beta_{\log(GDP_{cap}),OLS}$



To summarize, our theoretical set-up motivates the use of a censored regression model to estimate the link between a country's GDP per capita and the quality of its exports.

<sup>54</sup>The three hs4 product categories with negative coefficients are: 8504 – Transformers; 6309 – Worn Textiles; 4403 – Wood.

<sup>55</sup>Of course, according to our regression model, the OLS estimate using the subsample of data with observed export prices is a linear approximation to  $E[\log(p_{i,t}^k) \mid \mathbf{d}_{i,t}, GDP_{cap,t}^k, p_{i,t}^k \geq \rho_{i,t}]$  and hence, technically, a direct comparison of the maximum likelihood and the OLS estimates is not meaningful. Yet, if we ignored the censoring structure in our data, we would typically interpret the OLS estimates as indicating the link between a country's GDP per capita and the unit values of its exports.

In line with our theory, this censored regression model takes into account the preferred quality of countries that are not exporting a given product. Taking this model to the data, we find a strong and significant relationship. The estimated link is much stronger than when using OLS on the subsample of countries exporting the product under scrutiny. This observation is in line with a downward bias of OLS, as expected based on our theory.

## 6 Conclusion

This paper introduces a mechanism underlying comparative advantages that is new to the literature. Our mechanism is centered on the interplay between product complexity and product quality. It is well known that industrialized countries are able to produce more complex products. Based on this observation, a classical Ricardian argument would suggest that the countries with highest level of human capital have a comparative disadvantage for the simplest products. Yet the situation changes if firms can choose product quality. In that case, industrialized countries compete in simple products through product quality. As we have shown, this results in a triangular structure of comparative advantages. The intuitive argument is that products such as nuclear reactors and high-tech machines are simply too difficult to produce and distribute for the least productive countries. This pattern of comparative advantages was found empirically and has come to the front of policy discussions. To the best of our knowledge, this paper is the first to provide a theoretical underpinning.

The link from a country's GDP per capita to the quality of its exports was examined in many empirical studies. In our empirical section, we show how our rationale naturally leads to a censored regression model to estimate this link. Taking this model to the data, we find the said link to be much stronger than by using OLS.

In future work, it may be interesting to apply our rationale to development economics: Embedding our model in a dynamic framework may provide us with new insights on the drivers of the economic development of countries. It may also be interesting to analyze the implications of our work in the context of a richer model of international trade, allowing for outsourcing and trade in intermediate goods, for example.



# Appendix

## A Revealed comparative advantages of countries

As argued in the main text, empirical evidence presented by Hausmann and Hidalgo (2011) and Tacchella et al. (2012) indicates an upper-triangular structure of specialization of countries on products. In this part of the appendix, we briefly discuss and summarize this evidence.

Hausmann and Hidalgo (2011) and Tacchella et al. (2012) summarize *revealed comparative advantages* of countries for products in a binary country-product matrix that indicates for each country the products for which it has a revealed comparative advantage of at least 1. Their new idea is to rank countries from the weakest to the strongest economically, and products from the least to the most complex, according to the new measures developed by Hidalgo and Hausmann (2009) and Tacchella et al. (2012).

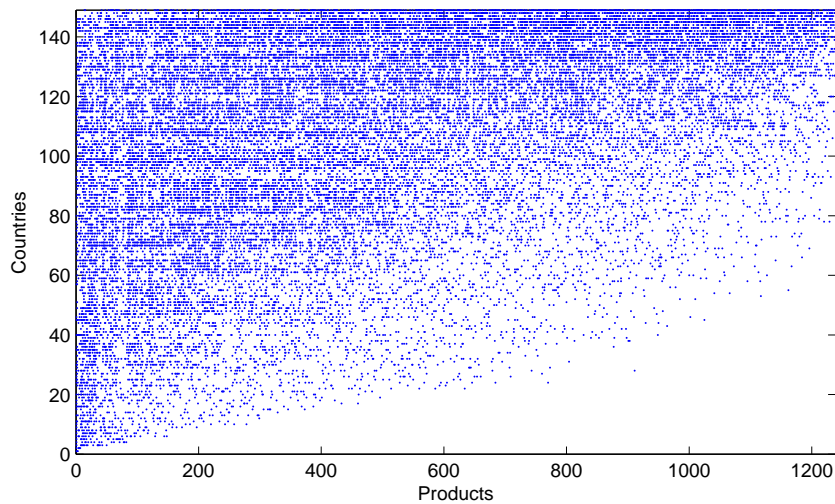
We follow their approach and visualize the ordered country-product matrix in Figures 5 to 7. In Figure 5 countries and products are ranked according to the measures proposed by Tacchella et al. (2012); in Figure 6 they are ranked according to the measures of economic complexity proposed by Hidalgo and Hausmann (2009); and in Figure 7 they are ranked according to their diversification and their ubiquity, respectively.<sup>56,57</sup>

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<sup>56</sup>Hidalgo and Hausmann (2009) define a country's *diversification* as the number of products it has revealed comparative advantage for. A product's *ubiquity* is defined as the number of countries exporting this product with revealed comparative advantage.

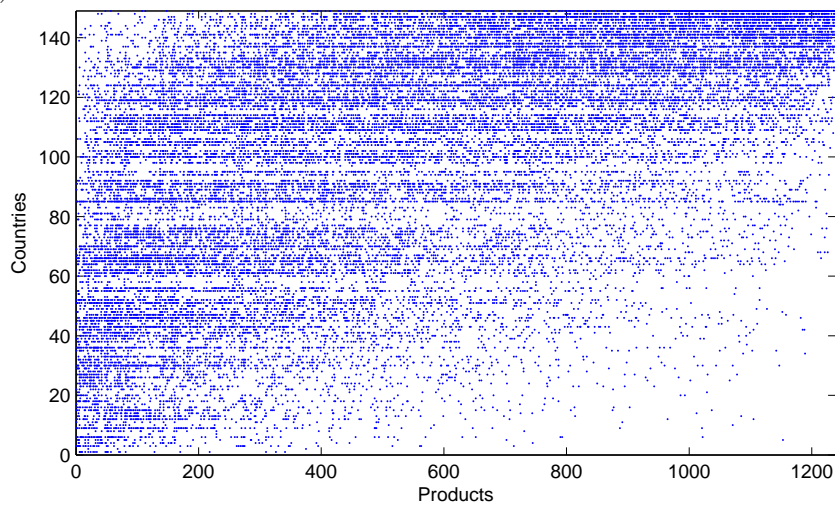
<sup>57</sup>We note, however, that the measures as proposed by Hidalgo and Hausmann (2009) and Tacchella et al. (2012) tend to accentuate the observed upper-triangular structure. In essence, these algorithms classify products as simple when they are strongly exported by weak countries.

Figure 5: Revealed comparative advantages – ranking according to Tacchella et al. (2012)



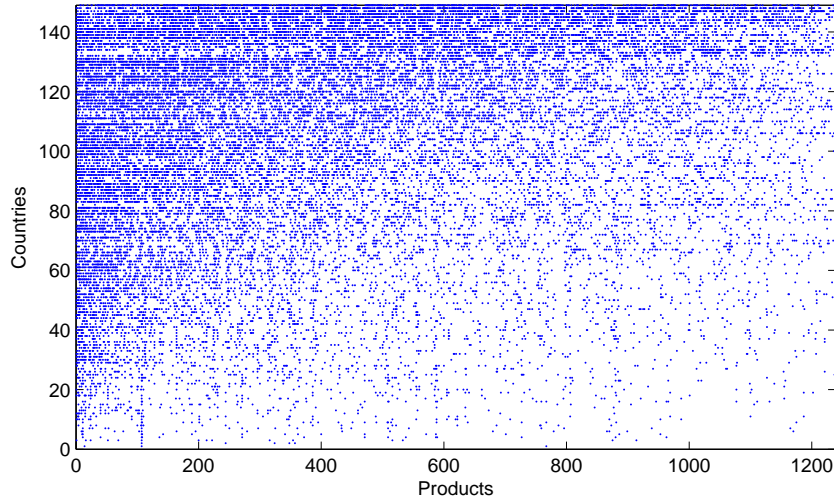
*Source:* Own illustration, based on Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). Products are grouped according to the 'hs4' classification codes. Countries and products are ranked according to the measures of *fitness of countries* and *complexity of products*, respectively, proposed by Tacchella et al. (2012). For every country-product pair, a dot indicates that the country has *revealed comparative advantage* of at least 1 for that product. The data refers to 2010 and was downloaded in August 2013.

Figure 6: Revealed comparative advantages – ranking according to Hidalgo and Hausmann (2009)



*Source:* Own illustration, based on Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). Products are grouped according to the 'hs4' classification codes. Countries and products are ranked according to the measures proposed by Hidalgo and Hausmann (2009). For every country-product pair, a dot indicates that the country has *revealed comparative advantage* of at least 1 for that product. The data refers to 2010 and was downloaded in August 2013.

Figure 7: Revealed comparative advantages – ranking according to diversification and ubiquity



*Source:* Own illustration, based on Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). Products are grouped according to the 'hs4' classification codes. Countries and products are ranked according to their diversification and their ubiquity, respectively. For every country-product pair, a dot indicates that the country has *revealed comparative advantage* of at least 1 for that product. The data refers to 2010 and was downloaded in August 2013.

## B Proofs

### B.1 Proof of Lemma 1

Solving the first order condition stated in the main text, equation (9), yields:

$$q_i(\hat{r}) = \left[ -\frac{1}{\lambda i \log(\hat{r})} \right]^{\frac{1}{\lambda}},$$

the expression stated in Lemma 1. Furthermore,

$$\frac{\partial \frac{X_i}{L_i(\hat{r})}}{\partial q} = [\hat{r}]^{iq^\lambda} [1 + \lambda i q^\lambda \log(\hat{r})] \begin{cases} < 0 & \text{if } q > q_i(\hat{r}) \\ > 0 & \text{if } q < q_i(\hat{r}) \end{cases},$$

and hence, the effective output is strictly decreasing as we move away from  $q_i(\hat{r})$  in either direction. We conclude that  $q_i(\hat{r})$  uniquely maximizes the effective output per worker.

Taking into account the minimum-quality constraint  $q \geq 1$ , yields the result as stated in Lemma 1.

□

## B.2 Proof of Lemma 2

We prove Lemma 2(ii) for the case of  $r^h < \tilde{r}(i)$ . The remainder of Lemma 2 follows immediately from the discussions in the main text.

To prove the result, we show that  $\forall i \in (\tilde{i}(r^h), N]$ , firm  $i$ 's costs per unit of effective output are strictly larger when producing in country  $r^l$  than when producing in country  $r^h$ . For the case of  $\tilde{r}(i) > r^h > r^l$  we have:

$$\frac{L_i(r^j)w_{r^j}}{\chi^i} \Big|_{q_i(r^j)} = [r^j]^{-i} w_{r^j}, \quad j \in \{l, h\},$$

and  $\frac{L_i(r^l)w_{r^l}}{\chi^i} \Big|_{q_i(r^l)} > \frac{L_i(r^h)w_{r^h}}{\chi^i} \Big|_{q_i(r^h)}$  follows from the fact that:

$$\frac{d[r^{-i}w_r]}{dr} = [-\log(r)]^{\frac{1}{\lambda}} r^{-i-1} [-\log(r)]^{-\frac{1+\lambda}{\lambda}} \left[ -\frac{1}{\lambda} + i \log(r) \right] < 0.$$

□

## B.3 Proof of Proposition 1 (i)

We proof necessity (i) and sufficiency (ii) of condition (SSC) separately.

(i) Suppose that for some  $\hat{i} > \tilde{i}(\min\{\mathcal{R}\})$  condition (SSC) is not satisfied.<sup>58</sup> Then it must hold:

$$L \int_{e^{-\frac{1}{\hat{i}\lambda}}}^{\bar{r}} \left[ \frac{\log(r)}{\log(r)} \right]^{\frac{1}{\lambda}} dF_r(r) < \int_{\hat{i}}^N \tilde{L}_i(\{\hat{w}_r\}_{r \in \mathcal{R}}) di. \quad (\text{B.1})$$

Condition (B.1) implies that total supply of effective labor in countries with skill level  $r \geq e^{-\frac{1}{\hat{i}\lambda}}$  is less than total demand of effective labor by firms  $i \in [\hat{i}, N]$ . However, from Lemma 2 we know that firms  $i \in [\hat{i}, N]$  will employ labor with skill level  $r \geq \tilde{r}(\hat{i}) = e^{-\frac{1}{\hat{i}\lambda}}$  only. Hence total demand for labor with skill level  $r \geq e^{-\frac{1}{\hat{i}\lambda}}$  exceeds total supply thereof, a contradiction to  $\{\hat{w}_r\}_{r \in \mathcal{R}}$  being the equilibrium wage scheme.

The contradiction establishes necessity of condition (SSC).

(ii) If for some wage scheme  $\{\hat{w}_r\}_{r \in \mathcal{R}}$  condition (SSC) is satisfied  $\forall \hat{i} \in [0, N]$ , then  $\{\hat{w}_r\}_{r \in \mathcal{R}}$  is an equilibrium, as discussed in the main text. We prove uniqueness by contradiction.

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<sup>58</sup>Note that for  $i \leq \tilde{i}(\min\{\mathcal{R}\})$  condition (SSC) is always satisfied for any feasible allocation of total effective labor.

Suppose the equilibrium is not unique. Then there exists an alternative equilibrium wage scheme  $\{\check{w}_r\}_{r \in \mathcal{R}} \neq \mu \{\hat{w}_r\}_{r \in \mathcal{R}}, \forall \mu > 0$ . By Corollary 1,  $\{\check{w}_r\}_{r \in \mathcal{R}}$  satisfies:

$$\check{w}_r \begin{cases} = \left[ \frac{\log(\hat{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \check{w}_{\hat{r}} & \text{if } r \geq \hat{r} \\ < \left[ \frac{\log(\hat{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \check{w}_{\hat{r}} & \text{otherwise} \end{cases},$$

for some  $\hat{r} \in (\min \{\mathcal{R}\}, \max \{\mathcal{R}\}]$ . Then by Lemma 2 all firms  $i \in [0, \tilde{i}(\hat{r}))$  strictly prefer producing in a country with skill level  $r < \hat{r}$  to producing in a country with skill level  $r \geq \hat{r}$ .<sup>59</sup> Now it is clear that firms  $i \in [\tilde{i}(\hat{r}), N]$  decrease their demand for effective labor vis-à-vis the equilibrium with  $\{\hat{w}_r\}_{r \in \mathcal{R}}$ . But then, there must be excess supply of effective labor with skill level  $r \geq \hat{r}$ , a contradiction to  $\{\check{w}_r\}_{r \in \mathcal{R}}$  being an equilibrium. □

## B.4 Proof of Proposition 2

We proceed in two steps. We first derive the equilibrium values, assuming that we have sufficient skills. We then use the expression for the equilibrium demand for effective labor by firm  $i \in [0, N]$ ,  $\tilde{L}_i^*$ , in condition (SSC) to observe when we have sufficient skills in equilibrium.

As discussed in the main text, with  $w_r^* = \left[ \frac{\log(r)}{\log(\underline{r})} \right]^{\frac{1}{\lambda}} \forall r \in \mathcal{R}$ , the allocation of total effective output of product  $i$  to countries  $r \in \mathcal{R}$  with  $r \geq \tilde{r}(i)$  is a matter of indifference. To simplify the exposition, we will assume here that firm  $i$  produces its entire effective output in one country  $r_i \geq \tilde{r}(i)$ .

Using  $\{w_r^*\}_{r \in \mathcal{R}}$  along with the fact that  $r_i \geq \tilde{r}(i)$  in equation (11) yields:

$$\begin{aligned} \rho_i^* &= \frac{\sigma}{\sigma - 1} w_{r_i} [-e\lambda i \log(r_i)]^{\frac{1}{\lambda}} \\ &= \frac{\sigma}{\sigma - 1} [-e\lambda i \log(\underline{r})]^{\frac{1}{\lambda}}. \end{aligned} \tag{B.2}$$

The quality-adjusted price of firm  $i$  is given by its marginal costs of producing effective output times a constant mark-up of  $\frac{\sigma}{\sigma - 1}$ . The marginal costs, and hence  $\rho_i^*$ , are in-

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<sup>59</sup>To be precise, our reasoning is based on  $F_r(r)$  being continuous in the neighborhood of  $\hat{r}$ . However, the reasoning can easily be adapted to discrete distributions. With  $F_r(r)$  being discrete, all firms  $i \in [0, \tilde{i}(r^-)]$ ,  $r^- := \max \{r \in \mathcal{R} : r < \hat{r}\}$ , strictly prefer producing in a country with skill level  $r < \hat{r}$  to producing in a country with skill level  $r \geq \hat{r}$ .

creasing in the complexity of the product,  $i$ , reflecting the fact that the more complex a product, the more difficult it is to produce.<sup>60</sup>

Substituting equation (B.2) in equation (6) and solving the integral yields the equilibrium price index  $P^*$ :

$$P^* = \frac{\sigma}{\sigma - 1} [-e\lambda \log(\underline{r})]^{\frac{1}{\lambda}} \left[ \frac{\lambda}{1 - \sigma + \lambda} \right]^{\frac{1}{1-\sigma}} N^{\frac{1-\sigma+\lambda}{(1-\sigma)\lambda}} . \quad (\text{B.3})$$

Note that here we used the assumption  $\sigma < 1 + \lambda$ .

To derive  $C^*$ , we first have to analyze labor-market clearing for the overall market for effective labor. Using equation (B.2) in the demand for product  $i$ , we obtain:

$$\chi_i = CP^\sigma \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} [-e\lambda i \log(\underline{r})]^{\frac{-\sigma}{\lambda}} . \quad (\text{B.4})$$

Combining this result with equation (16) and solving for  $L_i(r_i)$  yields:

$$L_i(r_i) = CP^\sigma [e\lambda i]^{\frac{1-\sigma}{\lambda}} \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} [-\log(r_i)]^{\frac{1}{\lambda}} [-\log(\underline{r})]^{\frac{-\sigma}{\lambda}} ,$$

which implies:

$$\tilde{L}_i = CP^\sigma [-e\lambda i \log(\underline{r})]^{\frac{1-\sigma}{\lambda}} \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} . \quad (\text{B.5})$$

Now condition (SSC) guarantees that there is no excess demand for skills in our economy. In addition, labor-market clearing requires that total demand for effective labor equals total supply:

$$\begin{aligned} \tilde{L} &\stackrel{!}{=} \int_0^N \tilde{L}_i di \\ &= CP^\sigma [-e\lambda \log(\underline{r})]^{\frac{1-\sigma}{\lambda}} \left[ \frac{\sigma}{\sigma - 1} \right]^{-\sigma} \frac{\lambda}{1 + \lambda - \sigma} N^{\frac{1+\lambda-\sigma}{\lambda}} , \end{aligned}$$

where the second equality follows from using firm  $i$ 's demand for effective labor, and where  $\tilde{L}$  denotes the aggregate supply of effective labor in the economy as defined by:

$$\tilde{L} := L \int_{\underline{r}}^{\bar{r}} \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} dF_r(r) . \quad (\text{B.6})$$

Solving for  $C$  and using equation (B.3) yields:

$$C^* = \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} \left[ \frac{\lambda}{1 + \lambda - \sigma} \right]^{\frac{1}{\sigma-1}} N^{\frac{1+\lambda-\sigma}{(\sigma-1)\lambda}} . \quad (\text{B.7})$$

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<sup>60</sup>In a variant of our model where products differ only in the minimum quality, but not in the number of tasks involved in production, quality-adjusted prices are the same for all products in an equilibrium with sufficient skills. Cf. appendix F.

The consumption aggregator  $C$  is increasing in the number of products  $N$  and in the total effective labor available in the economy,  $\tilde{L}$ .  $\tilde{L}$  is increasing in both hours worked per household,  $L$ , and the skills available in the economy, as summarized in their distribution  $F_r(r)$ .

Using equations (B.3) and (B.7) in equation (B.4), we obtain:

$$\chi_i^* = \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} \frac{1 + \lambda - \sigma}{\lambda} N^{-\frac{1+\lambda-\sigma}{\lambda}} [i]^{-\frac{\sigma}{\lambda}} .$$

$\chi_i^*$  is decreasing in  $i$ , implying that the increased difficulty associated with producing more complex products is reflected in lower effective output in equilibrium, as we would expect.

Finally, using equations (B.3) and (B.7) in equation (B.5) yields:

$$\tilde{L}_i^* = \tilde{L} \frac{1 + \lambda - \sigma}{\lambda} N^{-\frac{1+\lambda-\sigma}{\lambda}} [i]^{\frac{1-\sigma}{\lambda}} . \quad (\text{B.8})$$

Effective labor used in production is also decreasing in  $i$ . Hence the lower effective output of more complex products is not only a consequence of the higher difficulty of production, but also of less labor input used in production.

Our equilibrium analysis outlined so far was conditional on  $\left\{ \tilde{L}_i (\{w_r^*\}_{r \in \mathcal{R}}) \right\}_{i \in [0, N]}$  satisfying condition (SSC). We can now use the equilibrium demand for effective labor by firm  $i$ , equation (B.8), to further analyze when this is the case:

$$L \int_{e^{-\frac{1}{i\lambda}}}^{\bar{r}} \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} dF_r(r) \geq \int_{\hat{i}}^N \tilde{L} \frac{1 + \lambda - \sigma}{\lambda} N^{-\frac{1+\lambda-\sigma}{\lambda}} [i]^{\frac{1-\sigma}{\lambda}} di, \quad \forall \hat{i} \in [0, N] .$$

Solving the integral on the right-hand side, using the definition of  $\tilde{L}$  given in equation (B.6), and rearranging terms, we get a condition for sufficient skills in the economy based on parameter values alone:

$$\frac{\int_{e^{-\frac{1}{\lambda i}}}^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)}{\int_{\underline{r}}^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)} \geq 1 - \left( \frac{i}{N} \right)^{\frac{1+\lambda-\sigma}{\lambda}}, \quad \forall i \in [0, N] .$$

This is exactly the condition stated in Assumption 1. □

## C Details on the numerical example

In this part of the appendix, we outline the details of the random allocation of products to countries underlying our numerical example of section 4.1.

We divide production of product  $i$  into small production steps. Each such step consists of a fixed amount of effective labor and is randomly allocated to countries. We require that this random allocation satisfies the following two conditions:

1. Each firm  $i \in \{1, 2, \dots, N_p\}$  produces in countries with skill level  $r \geq \tilde{r} \left( \frac{N}{N_p} \left[ i - \frac{1}{2} \right] \right)$  only.<sup>61</sup>
2.  $\forall r^l, r^h \in \mathcal{R}$ , with  $r^l < r^h$ , the relative odds of allocating a production step to country  $r^l$  or  $r^h$  are the same  $\forall i \leq \tilde{i}(r^l) \frac{N_p}{N} + \frac{1}{2}$ .<sup>62</sup>

The first condition is satisfied by recursive allocation of production, starting from the most complex product. The second condition is satisfied by randomly allocating the production of product  $\hat{i} \in \{1, 2, \dots, N_p\}$  based on the effective labor available in expectation for the production of products  $i = 1, 2, \dots, \hat{i}$ . In particular, let  $\tilde{L}_{\hat{i}, \hat{r}}$  denote effective labor available in expectation in country  $\hat{r} \in \mathcal{R}$  for use in production of products  $i = 1, 2, \dots, \hat{i}$ . Then a production step of product  $\hat{i} \in \{1, 2, \dots, N_p\}$  is allocated to country  $\hat{r}$  with probability:

$$pr_{\hat{i}, \hat{r}} = \begin{cases} \frac{\tilde{L}_{\hat{i}, \hat{r}}}{\sum_{r \in \mathcal{R}: r \geq \tilde{r} \left( \frac{N}{N_p} \left[ \hat{i} - \frac{1}{2} \right] \right)} \tilde{L}_{\hat{i}, r} & \text{if } \hat{r} \geq \tilde{r} \left( \frac{N}{N_p} \left[ \hat{i} - \frac{1}{2} \right] \right) \\ 0 & \text{otherwise} \end{cases} . \quad (\text{C.1})$$

Now for  $\hat{i} = N_p$  we have:<sup>63</sup>

$$\tilde{L}_{N_p, \hat{r}} = \frac{\log(r)}{\log(\hat{r})} . \quad (\text{C.2})$$

Effective labor available in expectation in country  $\hat{r}$  for use in production of products  $i = 1, 2, \dots, N_p$  is simply the total effective labor available in country  $\hat{r}$ . Using  $\tilde{L}_{N_p, \hat{r}}$  in equation (C.1) yields  $pr_{N_p, \hat{r}}$ . Combining this information with total effective labor used in equilibrium in production of product  $N_p$ ,  $\tilde{L}_{N_p}^*$ , allows to derive  $\tilde{L}_{N_p-1, \hat{r}}$  via the

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<sup>61</sup>In the model as presented in section 2, the complexity of firm  $i$ 's product is  $i$ . In the discrete example considered here, firm  $i$ 's product has complexity  $\frac{N}{N_p} \left[ i - \frac{1}{2} \right]$  and, hence, the minimum skill level needed to produce product  $i$  with preferred quality is given by  $\tilde{r} \left( \frac{N}{N_p} \left[ i - \frac{1}{2} \right] \right)$ .

<sup>62</sup>The maximum complexity that country  $r^l$  can produce with preferred quality is given by  $\tilde{i}(r^l)$ , i.e. country  $r^l$  can produce all products  $i \leq \tilde{i}(r^l) \frac{N_p}{N} + \frac{1}{2}$  with preferred quality.

<sup>63</sup>Remember that  $\lambda = L = 1$ .



following recursive formula:<sup>64</sup>

$$\tilde{L}_{N_p-1,\hat{r}} = \begin{cases} \tilde{L}_{N_p,\hat{r}} - pr_{N_p,\hat{r}}\tilde{L}_{N_p}^* & \text{if } \hat{r} \geq \tilde{r}\left(\frac{N}{N_p}\left[N_p - \frac{1}{2}\right]\right) \\ \tilde{L}_{N_p,\hat{r}} & \text{otherwise} \end{cases}.$$

In words, effective labor available in expectation in country  $\hat{r}$  for use in production of products  $i = 1, 2, \dots, N_p - 1$  is equal to total effective labor available in country  $\hat{r}$  minus effective labor used in expectation for production of product  $N_p$ . In general, we can derive  $\tilde{L}_{\hat{i},\hat{r}}, \hat{i} \in \{1, 2, \dots, N_p - 1\}$  recursively as follows:

$$\tilde{L}_{\hat{i},\hat{r}} = \begin{cases} \tilde{L}_{\hat{i}+1,\hat{r}} - pr_{\hat{i}+1,\hat{r}}\tilde{L}_{\hat{i}+1}^* & \text{if } \hat{r} \geq \tilde{r}\left(\frac{N}{N_p}\left[\hat{i} + \frac{1}{2}\right]\right) \\ \tilde{L}_{\hat{i}+1,\hat{r}} & \text{otherwise} \end{cases}. \quad (\text{C.3})$$

Combining equations (C.1) and (C.3) with the initial values (C.2) yields probabilities for the random allocation of products to countries. These probabilities are based on expected levels of labor available. In the numerical implementation, we use these probabilities, but subject to the constraint that total allocation of production to country  $\hat{r}$  may not exceed its total effective labor available.<sup>65</sup>

## D Measuring economic complexity in a world as described by our model

In this part of the appendix, we present a simple experiment to illustrate that in a world as described by our model, the algorithms proposed by Hidalgo and Hausmann (2009)

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<sup>64</sup>To compute  $\tilde{L}_i^*$ , we use the formulas for the model with a continuum of countries and products as outlined in the main text. In the discrete counterpart considered here, we have divided the continuum of complexities from 0 to  $N$  in  $N_p$  equally sized intervals, and attributed to product  $i$  a complexity level which corresponds to the midpoint of the  $i^{\text{th}}$  interval.  $\tilde{L}_i$  is strictly convex in the complexity of the product. It follows that we underestimate total demand for effective labor when using the formulas from the continuous model. We correct for this error by proportionally scaling the demand for effective labor of each firm  $i \in \{1, 2, \dots, N_p\}$ .

<sup>65</sup>Note that the resulting probabilities do indeed satisfy the second condition as specified above. To see this, consider two countries  $r^l, r^h \in \mathcal{R}$ , with  $r^l < r^h$  and product  $\hat{i} \leq \tilde{i}(r^l) \frac{N_p}{N} + \frac{1}{2} < \tilde{i}(r^h) \frac{N_p}{N} + \frac{1}{2}$ . Then we have:

$$\frac{pr_{\hat{i},r^l}^*}{pr_{\hat{i},r^h}^*} = \frac{\tilde{L}_{\hat{i},r^l}}{\tilde{L}_{\hat{i},r^h}} = \frac{\tilde{L}_{\hat{i},r^l} \left[ 1 - \frac{\tilde{L}_{\hat{i}}^*}{\sum_{r \in \mathcal{R}: r \geq \tilde{r}\left(\frac{N}{N_p}\left[\hat{i}-\frac{1}{2}\right]\right)} \tilde{L}_{\hat{i},r} \right]}{\tilde{L}_{\hat{i},r^h} \left[ 1 - \frac{\tilde{L}_{\hat{i}}^*}{\sum_{r \in \mathcal{R}: r \geq \tilde{r}\left(\frac{N}{N_p}\left[\hat{i}-\frac{1}{2}\right]\right)} \tilde{L}_{\hat{i},r} \right]} = \frac{\tilde{L}_{\hat{i}-1,r^l}}{\tilde{L}_{\hat{i}-1,r^h}} = \frac{pr_{\hat{i}-1,r^l}^*}{pr_{\hat{i}-1,r^h}^*}.$$

and Tacchella et al. (2012) can indeed reveal important information on the economic strength of countries and on the complexity of products. In particular, we apply the proposed algorithms to binary country-product matrices that are generated according to our simple numerical example of section 4.1. We then compare the derived rankings of countries and products to the fundamental rankings underlying our model. Table 3 shows the mean and the standard deviations for the according rank correlations as observed from a Monte Carlo simulation with 1000 random draws of the equilibrium in our economy. These rank correlations are generally high, suggesting that the proposed algorithms perform well indeed in uncovering the economic strength of countries and the complexity of products from the bipartite country-product network, at least in a world as described by our model.

Table 3: Rank correlations between measures derived from proposed algorithms and fundamental values<sup>a</sup>

	Model	
	Countries	Products
Hidalgo and Hausmann (2009)	0.7138 / 0.0456 (mean / std)	0.3431 / 0.0344 (mean / std)
Tacchella et al. (2012)	0.9907 / 0.0014 (mean / std)	0.7639 / 0.0104 (mean / std)

<sup>a</sup> *Source*: Own calculations. The data was retrieved from a Monte Carlo simulation with 1000 iterations.

## E Details on the empirical analysis

### E.1 Derivation of Hypothesis 1

In our model GDP per capita is proportional to the wage rate:

$$GDP_{cap,t}^k = \frac{\sigma}{\sigma - 1} w_{r^k,t} ,$$

which implies:

$$-\frac{1}{\lambda} \log [-\log (r_t^k)] = \log (GDP_{cap,t}^k) - \log \left( \frac{\sigma}{\sigma - 1} \right) - \frac{1}{\lambda} \log [-\log(\underline{r})] ,$$

and hence:

$$\log (q_{i,t}^k) = -\log \left( \frac{\sigma}{\sigma - 1} \right) - \frac{1}{\lambda} [\log(\lambda) + \log(i) + \log(-\log(\underline{r}))] + \log (GDP_{cap,t}^k) .$$

Now in equilibrium all qualities of product  $i$  are sold at the same quality-adjusted price,  $\rho_{i,t}$ . Hence, if product  $i \in [0, N]$  is produced in country  $k$  at time  $t$ , its price  $p_{i,t}^k$  is proportional to its quality,  $p_{i,t}^k = q_{i,t}^k \rho_{i,t}$ , i.e. the elasticity with respect to a country's GDP per capita is the same for product quality and for product price, and we can use the latter instead. Taking logs and using the equilibrium values of  $q_i$  and  $\rho_i$ , we obtain:

$$\log(p_{i,t}^k) = \frac{1}{\lambda} + \log(GDP_{cap,t}^k) \quad , \quad (\text{E.1})$$

an expression we can use to estimate the relationship between a country's GDP per capita and its export quality. In particular, taking into account that, according to our model, we can observe  $\log(p_{i,t}^k)$  only if country  $k$  is competitive for product  $i$ , i.e. only if  $p_{i,t}^k \geq \rho_{i,t}$ , and assuming normally distributed errors, we get the following censored regression model:

$$\log(p_{i,t}^k) = c + \beta \log(GDP_{cap,t}^k) + u_{i,t}^k, \quad u_{i,t}^k | GDP_{cap,t}^k \sim N(0, \sigma^2) \quad (\text{E.2a})$$

$$\log(\tilde{p}_{i,t}^k) = \begin{cases} \log(p_{i,t}^k) & \text{if } p_{i,t}^k \geq \rho_{i,t} \\ \text{NaN} & \text{otherwise} \end{cases} \quad , \quad (\text{E.2b})$$

where  $\tilde{p}_{i,t}^k$  denotes the observed price level.

To take this model to the data, we reintroduce product-time dummies capturing (time-varying) product characteristics,  $\mathbf{d}_{i,t}$ . We further allow the effect of  $GDP_{cap,t}^k$  on output prices and the variance of the error term to differ across products, i.e. we have  $\beta_i$  and  $\sigma_i$  in equation (E.2a) above. Finally, we cannot observe the quality-adjusted price of a product,  $\rho_{i,t}$ . However, as shown by Carson and Sun (2007), we can use the minimum price level that we observe for product  $i$  in period  $t$ ,  $\min_{k \in \{1, 2, \dots, N_c\}} p_{i,t}^k$ , instead. This does not affect consistency and asymptotic efficiency of the maximum-likelihood estimator.

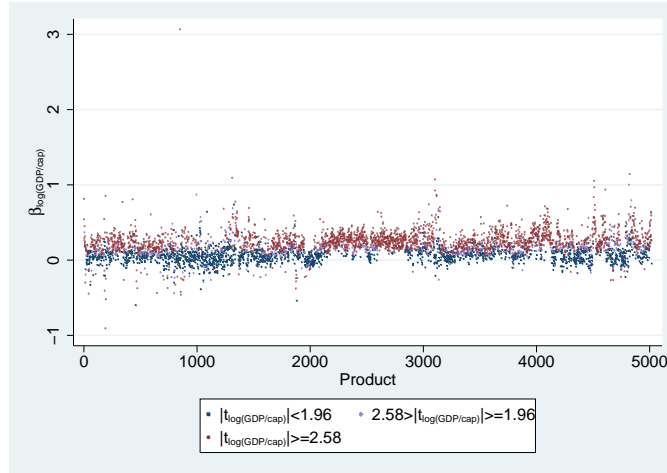
In summary, the previous derivations give rise to the censored regression model outlined in Hypothesis 1.

## E.2 Further estimation results

In this part of the appendix, we present some robustness tests for the empirical observations outlined in section 5.3.

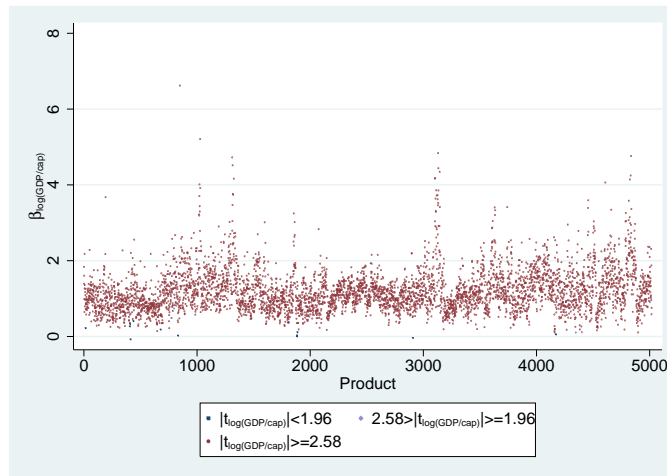
## E.2.1 Product classification at the hs6 level

Figure 8: OLS estimates of  $\beta_{\log(GDP_{cap})}$  – hs6 product classification



*Notes:* This figure plots the OLS estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a), using the subsamples with observed unit values. Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

Figure 9: ML estimates of  $\beta_{\log(GDP_{cap})}$  – hs6 product classification



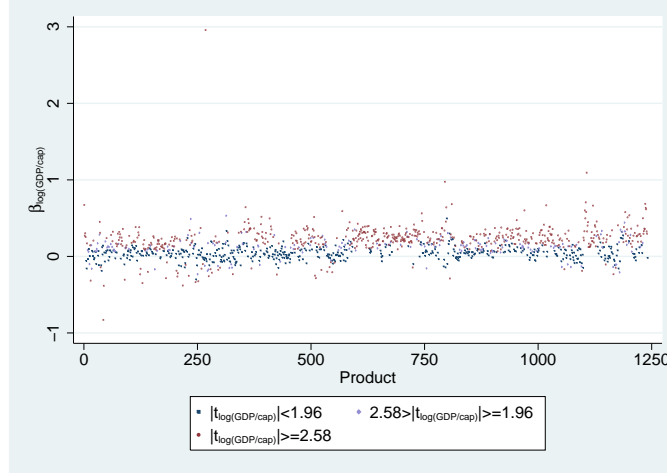
*Notes:* This figure plots the ML estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a). Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

## E.2.2 Alternative selection criterion for outliers

In this section, we classify observations as outliers whenever:

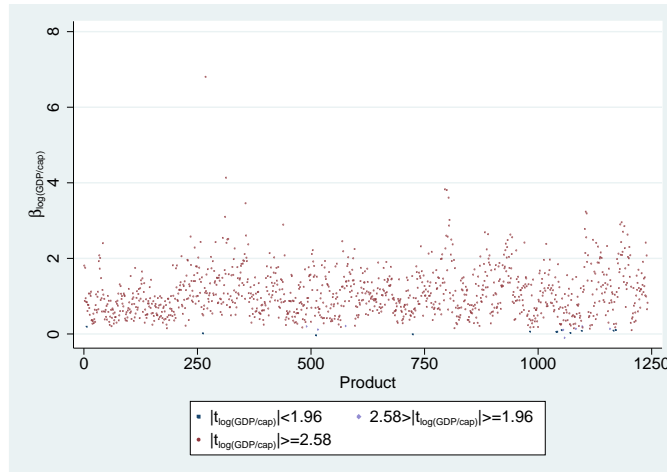
$$\begin{aligned}
 & uv_{i,t}^k \geq 100 \times \text{median}_k(uv_{i,t}^k) \wedge uv_{i,t}^k \geq 50 \times \text{median}_t(uv_{i,t}^k) \\
 & \vee \\
 & uv_{i,t}^k \leq \frac{1}{100} \times \text{median}_k(uv_{i,t}^k) \wedge uv_{i,t}^k \leq \frac{1}{50} \times \text{median}_t(uv_{i,t}^k) .
 \end{aligned}$$

Figure 10: OLS estimates of  $\beta_{\log(GDP_{cap})}$  – alternative selection criterion for outliers



*Notes:* This figure plots the OLS estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a), using the subsamples with observed unit values. Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

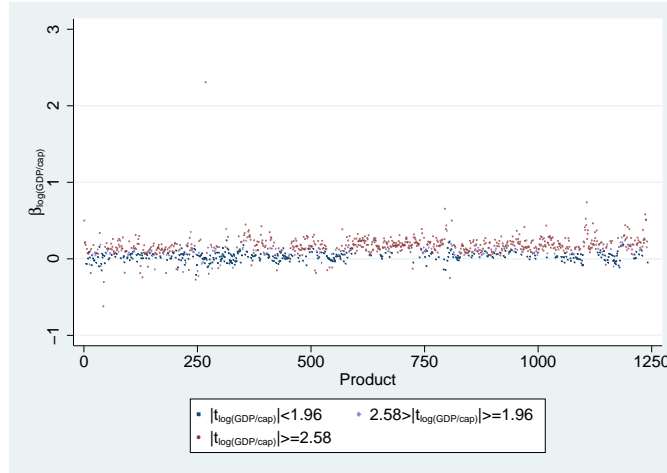
Figure 11: ML estimates of  $\beta_{\log(GDP_{cap})}$  – alternative selection criterion for outliers



*Notes:* This figure plots the ML estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a). Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is in purchasing power parities and is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

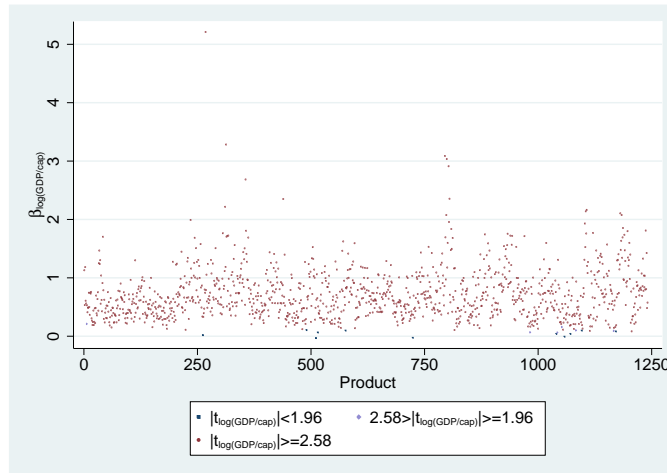
### E.2.3 GDP at market exchange rates

Figure 12: OLS estimates of  $\beta_{\log(GDP_{cap})} - \text{GDP}$  at market exchange rates



*Notes:* This figure plots the OLS estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a), using the subsamples with observed unit values. Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

Figure 13: ML estimates of  $\beta_{\log(GDP_{cap})} - \text{GDP}$  at market exchange rates



*Notes:* This figure plots the ML estimates of  $\beta_{\log(GDP_{cap})}$  in equation (19a). Standard errors are clustered by exporting countries. The trade data is taken from Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) (2013). The data on GDP per capita is taken from World Bank (2013). The data ranges from 1995 to 2011 and was downloaded in August 2013.

## F A variant with product-specific minimum-quality requirements

In the model presented in the main text, we introduce two dimensions of product heterogeneity: product-intrinsic complexity and endogenously chosen product quality. Complexity is defined as the number of tasks involved in production. Production is subject to a minimum-quality constraint, where the minimum quality is the same for each product.

In this part of the appendix, we present a variant of our model, where we assume that every product involves a continuum of measure 1 of tasks, irrespective of its complexity. We newly define the complexity of a product as the product-specific minimum quality. In particular, we assume that product  $i$  has minimum quality,  $q_{i,\min} = [i]^{\frac{1}{\lambda}}$ ,  $\forall i \in [0, N]$ . It turns out that this model exhibits the same qualitative characteristics. The main difference is that now, in an equilibrium with sufficient skills, the quality-adjusted price and output level is the same for each product. This follows naturally from the fact that production is equally difficult for any two products of same quality.

With the described changes in assumptions, the production function (7) changes to:

$$E[x_{i,q}] = [r]^{q\lambda} L_i(r), \quad q \geq [i]^{\frac{1}{\lambda}} .$$

This is, up to the minimum-quality constraint, the same production function as the one faced by firm  $i = 1$  in the version of the model presented in the main text. The discussions of section 2.2 apply to all firms  $i \in [0, N]$  and hence also to firm  $i = 1$ . It follows that Lemma 1 also applies in the present case, with the only difference that:

$$q_i(r) = \max \left\{ [i]^{\frac{1}{\lambda}}, \left[ -\frac{1}{\lambda \log(r)} \right]^{\frac{1}{\lambda}} \right\} ,$$

which does not affect the threshold complexity and skill levels,  $\tilde{i}(r)$  and  $\tilde{r}(i)$ . Also, as long as both countries produce at preferred quality, the relative productivity of two countries with different skill levels of labor is still given by equation (17). This implies that in an equilibrium with sufficient skills, the equilibrium wage is still given by Proposition 1(i). Now, however, this equilibrium is characterized by symmetry across firms in terms of quality-adjusted prices and output levels, as well as in their demand for effective labor. It follows that Assumption 1 simplifies to:

## Assumption 2

$$\frac{\int_{e^{-\frac{1}{\lambda i}}}^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)}{\int_{\underline{r}}^{\bar{r}} [-\log(r)]^{-\frac{1}{\lambda}} dF_r(r)} \geq 1 - \frac{i}{N}, \quad \forall i \in [0, N],$$

and we can show that the equilibrium satisfies:

### Proposition 3

Let Assumption 2 be satisfied. Then in any equilibrium it holds:

- (i)  $w_r^* = \left[ \frac{\log(\underline{r})}{\log(r)} \right]^{\frac{1}{\lambda}} \quad \forall r \in \mathcal{R}$
- (ii)  $\mathcal{R}_i^* \subseteq \{r \in \mathcal{R} : r \geq \tilde{r}(i)\} \quad \forall i \in [0, N]$
- (iii)  $q_i^*(r) = \left[ -\frac{1}{\lambda \log(r)} \right] \quad \forall (i, r) \in [0, N] \times \mathcal{R}_i^*$
- (iv)  $\rho_i^* = \frac{\sigma}{\sigma-1} [-e\lambda \log(\underline{r})]^{\frac{1}{\lambda}} \quad \forall i \in [0, N]$
- (v)  $\chi_i^* = \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} N^{-1} \quad \forall i \in [0, N]$
- (vi)  $\tilde{L}_i^* = \tilde{L} N^{-1} \quad \forall i \in [0, N]$
- (vii)  $P^* = \frac{\sigma}{\sigma-1} [-e\lambda \log(\underline{r})]^{\frac{1}{\lambda}} N^{\frac{1}{1-\sigma}}$
- (viii)  $C^* = \tilde{L} [-e\lambda \log(\underline{r})]^{-\frac{1}{\lambda}} N^{\frac{1}{\sigma-1}}$



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