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Local public goods as perfect substitutes -- centralization vs. decentralization

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Local public goods as perfect substitutes – centralization vs. decentralization

**New insights on the role of spillovers, heterogeneity and strategic
delegation**

Work in Progress, this version: March 1, 2016

The main message of recent evaluations of decentralization efforts around the world is that these efforts were unable to generate the beneficial effects they were thought to induce. This finding constitutes a contrast to the rich body of literature on centralization and decentralization which was itself one driving factor of these efforts of decentralization. By arguing that (local) public goods can be viewed as perfect substitutes, this paper provides an explanation for these recent empirical findings and helps to reintegrate them into the theoretical literature on the subject. The main finding of this paper is that centralized and decentralized structures can induce identical provision levels of public goods. This ambivalence is generated by the interaction between electorates and representatives. Whereas both of these actors behave differently in the two scenarios, the overall outcomes are identical due to the leveling effects of strategic delegation. This finding is robust with respect to the assumption of a multistage government.

JEL Classification: C71, C72, H41, H77

1 Introduction

The question where the power of decision in a political system is best situated is an old one which has been the starting point of many political scientists' work. The two natural counterparts regarding this issue are centralized and decentralized decision structures. Whereas centralization allocates political power at a central – meaning higher – tier of the political system, decentralization puts a higher share of this power on rather regional – thus lower – tiers. Following this logic, the advantage of decentralized decision structures is that they can better cater to different preferences. In contrast to this, centralized systems offer the possibility to internalize interregional effects like spillovers which lead to inefficiencies in decentralized systems. Hence, as prominently stated by [Oates \(1972\)](#), decentralization leads to better outcomes than centralization in the absence of spillovers whilst centralization is the adequate system whenever there are no differences in preferences in the different regions. In the presence of heterogeneous preferences and spillovers one has to weigh the advantages and disadvantages of the two systems.

[Oates \(2005\)](#) provides a comprehensive overview of the literature in the field of fiscal federalism. Focusing on the emergence and development of this field and inspired by [Qian and Weingast \(1997\)](#), the author separates the literature in a “*first-generation theory of fiscal federalism*” and a “*second-generation theory*”. The first-generation theory was mainly shaped by the spirit of the works of Paul Samuelson, Kenneth Arrow and Robert Musgrave¹. While the first-generation literature on fiscal federalism was soon enriched by aspects like spillovers, limitations on feasible policies, issues of optimal tax schemes, mobile households and governmental budget constraints, the role of the government as an active corrective of failing markets (for public goods) remained unchanged. In the light of this perspective, the perceived overall motive of scholars working in this field was to compare and identify political systems which would promote welfare best given the obstacles mentioned above. Over time contributions from the field of public-choice and political economy were challenging the paradigms of this standard first-generation theory and paving the way for a second-generation theory of fiscal federalism were . Notably [Brennan and Buchanan \(1980\)](#) strengthened the link to the field of principal-agent theory by extending the “*view to the proposition that the public sector can itself be envisioned as a monolithic agent*” ([Oates, 2005](#), p. 355).

The second-generation theory of fiscal federalism is separated by [Oates \(2005\)](#) into two main strands: The first takes a fresh look on the behavior of actors in political systems and recognizes them as maximizers of their own objective functions which can (very likely) be different from overall welfare. Hence the focus lies on the “*modeling of political institutions with explicit attention to the incentives they embody*” ([Oates, 2005](#), p. 356). The second strand focuses on the issue of information or rather information asymmetries. Since optimal provision of public goods crucially depends on heterogeneous individual preferences which are private knowledge, an optimal political system rather seeks to obtain this information since it is the prerequisite of an optimal provision.

¹See [Oates \(2005, p. 350\)](#) for a collection of works of the mentioned authors which were especially influential.

This paper focuses on the aspect of strategic delegation in decentralized and centralized political systems which arises when providing local public goods. Thus this work belongs to the first strand of literature in the second-generation theory of fiscal federalism. Early contributions like [Brennan and Buchanan \(1980\)](#) introduced the idea that measures of decentralization could be used to impose constraints on central governments which limit their opportunities for political action and thereby work against their tendency to grow. In contrast to this, strategic delegation is not an intentionally used mechanism of voters to discipline politicians, but rather a byproduct of the political system itself which influences the incentives of voters. The underlying assumption of this approach is that politicians or all other representatives behave as personal utility maximizers² and are not interested in welfare directly³. Thus strategic delegation can be understood as the feedback of the interaction of multiple representatives with different goals on the voters' incentives where the catalyst of this feedback is the considered political system.

The simplest way to identify strategic delegation and its magnitude is to look at the desired representatives of voters. Due to heterogeneous preferences, every voter exhibits a different preference for public goods. Now, given a voter has a choice among different possible representatives, strategic delegation occurs if the voter votes for a representative who exhibits a different preference than the voter herself. Whether there exists an incentive for strategic delegation to representatives of higher or lower preference can be determined by comparing the preference types of voters and the respective representatives. The work of [Besley and Coate \(2003\)](#) and the extension by [Dur and Roelfsema \(2005\)](#) are prominent examples which identify incentives for strategic delegation: Voters will elect “*public good lovers*” ([Dur and Roelfsema, 2005](#), p. 395) whenever public good provisions are financed by a common budget and rather conservative (with respect to spendings for the public good) representatives if each region has to finance its provided amount individually. This finding which is in line with the one of [Besley and Coate \(2003\)](#) is quite intuitive. In the first case there are hidden, i.e. not internalized costs to the overall community which lead to an overprovision of the public good. In the second case underprovision is the logical consequence of the uninternalized welfare gains which occur in other regions due to spillovers. However, the driving force in these scenarios is the cost-scheme of public good provision. Apart from this link, there are no other channels of interaction between the different regions.

Looking back at the rich and long debate on federalism fueled by numerous political scientists, economists and scholars from many other fields, one can conclude that “*not all scholars are sure that decentralization is always beneficial*” ([Treisman, 2007](#)). Analogously, “*the rapid spread of decentralization worldwide, which at its peak led to elections of local governments in 90% of countries around the world, has subsided*” ([Malesky et al., 2014](#), p. 145). In their recent overview [Mansuri and Rao \(2013\)](#) evaluate numerous studies which tried to identify effects of decentralization. Their main findings were that hopes linked to decentralization

²This does by no means imply that politicians never promote welfare. The task of selecting appropriate politicians can be understood as finding politicians whose objective functions are somehow close to overall welfare. Furthermore, pursuing individual interests does not necessarily resemble selfishness as argued later.

³Politicians might be interested in promoting welfare in their voting district due to electoral pressure. Additionally to benevolence, this (more realistic) argument was the main basis for the assumption of welfare maximizing politicians in the first-generation theory of fiscal federalism ([Oates, 2005](#), p. 350).

efforts were “*not met by real-world improvements in outcomes*” (Malesky et al., 2014, p. 145) or that decentralization had different effects dependent on other characteristics. The essence of these findings is that much remains unclear related to the mechanics of different political systems and this debate is far from over.

This work builds on the efforts of Besley and Coate (2003) as well as Dur and Roelfsema (2005) and is targeted to provide new insights on several aspects which are of importance in the area of conflict between centralization and decentralization. This is possible since the model which is used in this paper – although it is seemingly of the same type as the models in the two mentioned papers – constitutes a different approach with respect to the substitutability of the individual provision levels of the public good. Taking the characterization of the individual provision levels as substitutes seriously implies that these provision levels can not only be substituted easily, but should also affect marginal utility from consuming the public good. More precisely, this work offers three main contributions to the existing literature: It identifies incentives for strategic delegation in both centralized and decentralized systems which fit nicely into the existing literature. However, the element causing these incentives is of a different kind than in previous works, for example the cost-scheme in Besley and Coate (2003) and Dur and Roelfsema (2005). Here strategic delegation occurs since voters can increase the provision level of other regions by committing to a representative who favors very low provision levels of the public good. Furthermore, the heterogeneity in the preferences for the public good between the different regions takes on an entirely new role in the emergence of incentives for strategic delegation. Strategic delegation worsens the conflict between the representatives leading to a harmful race to the bottom (in the provision levels of the public good). This race to the bottom becomes worse whenever spillovers are high⁴. Hence a region will defect and leave this damaging scenario whenever providing the public good alone is better than staying in the race to the bottom. Thus strategic delegation can only occur if the preferences of the two regions are rather similar. Lastly, this work challenges the view that the distinction between decentralized and centralized systems is as clear as it is often assumed and questions whether this distinction can be made by identifying the location where a decision has been made. Due to the interplay of strategic delegation and the interaction of representatives, the proficiency of different political systems to lead to different allocations can actually be lost.

The structure of this paper is the following: Section two discusses different aggregation technologies that can be used to determine the accessible quantity of the public good and explains the model in detail. Section three provides a benchmark by determining the welfare maximizing provision levels of the public good. Section four looks at the effects of a decentralized political system and section five at the ones which are induced by a centralized system. Section six briefly compares the findings of the two different scenarios in sections four and five with respect to public good provision and strategic delegation. Finally, section seven explores how the incentives for and the magnitude of strategic delegation are affected if one considers a political system with several stages. Section eight provides a short summary and concludes the work.

⁴The obtained mechanism is similar to the one in (Roelfsema, 2007).

2 Framework of the model

2.1 Aggregation technologies of public good provisions and utility

The totally accessible quantity of the public good is usually considered to be given by the sum of individual provision levels. This assumption originates from the work of [Bergstrom, Blume and Varian \(1986\)](#). Since the authors focus on the private provision of a public good within one community, the term individual can in the context of their work be understood as personally. In the following, the notion of this term will be rather based on the regional level since decisions about the provision levels and the actual provision happens on this level. From the formulation of public goods as a simple sum it was only a small step towards a weighted sum where the weights on individual provision levels resemble the degree of spillovers. [Hirshleifer \(1983\)](#) was among the first who expressed concerns that the formation of an aggregate stock of a public good might proceed differently as commonly thought and provided vivid examples like the dyke on the island of “*Anarchia*” [Hirshleifer \(1983, p. 371\)](#) – Hirshleifer’s example of a weakest-link aggregation function. However, like the vast majority in the literature, this paper is based on the ideas of [Hirshleifer \(1983\)](#) and builds on a summation technology: Let $G_j = \sum w_{j,k} \times g_k$ be the total accessible quantity of the public good in region j where g_k characterizes the individual provision level in region k and $w_{j,k}$ is a measure of spillovers between the regions k and j .

The difference of this work to the efforts of [Besley and Coate \(2003\)](#) and [Dur and Roelfsema \(2005\)](#) as well as many others is that individual public good provisions are perfect substitutes. This is not true for the mentioned works since they impose an “*assumption of separability of local public goods in the utility function*” ([Dur and Roelfsema, 2005, p. 399](#)) such that utility (in region j) is not a function of G_j , but rather of all individual provision levels what is somehow in contrast to the formulation of the aggregated public good as a sum. According to the approach of [Dur and Roelfsema \(2005\)](#), an arbitrary citizen in region j who has a preference for the public good according to the preference parameter λ_j draws utility $u_j = \lambda_j [b(g_j) + w \times b(g_{-j})] - c$ from the consumption of the public good. There are only two regions such that g_j is the provision in the citizen’s home region and g_{-j} the level of provision in the other region. $b(\cdot)$ is a concave function. Any costs or taxes and utility from a private good is covered by c . In – seemingly not so stark – contrast, I propose the alternative specification $u_j = \lambda_j b(G) - c = \lambda_j b(g_j + w \times g_{-j}) - c$. [Table 1](#) compares the two specifications.

Note that I do by no means claim that the alternative specification proposed in this work is more general than the framework of the works mentioned earlier. A brief look at the two utility functions would leave any claim like that baseless. The presented way of modeling the aggregation of individual public good provisions and utility rather focuses a different kind of public goods: In the [Besley and Coate \(2003\)](#) framework, these individual provision levels can be substituted. However, this become harder due to a falling marginal rate of substitution. This is not true in the alternative model, here individual provisions are perfect substitutes and the marginal rate of substitution is constant and does only depend on the degree of spillovers. This is

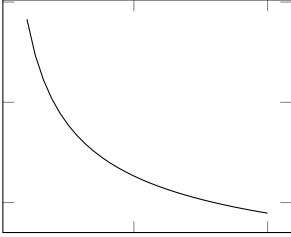
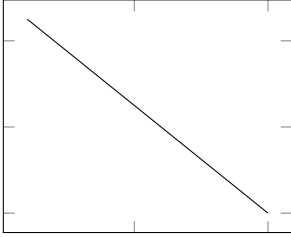
specification	Dur and Roelfsema (2005)	alternative
	$u_j = \lambda_j (b(g_j) + w \times b(g_{-j})) - c$	$u_j = \lambda_j b(g_j + w \times g_{-j}) - c$
$\frac{\partial u_j}{\partial G}$	n.a.	$\lambda_j b'(G)$
$\frac{\partial u_j}{\partial g_j}, \frac{\partial u_j}{\partial g_{-j}}$	$\lambda_j g'(g_j), \lambda_j w g'(g_{-j})$	$\lambda_j b'(G), \lambda_j w b'(G)$
$\frac{\partial^2 u_j}{\partial g_j \partial g_{-j}}$	0	$\lambda_j w b''(G) < 0$
$MRS = \frac{\partial u_j / \partial g_j}{\partial u_j / \partial g_{-j}}$	$\frac{\frac{1}{w} \frac{b'(g_j)}{b'(g_{-j})}}{\frac{1}{w}}$	$\frac{1}{w}$
isoquant	$g_{-j} = b^{-1} \left(\left[\frac{u_j - c}{\lambda_j} - b(g_j) \right] \frac{1}{w} \right)$	$g_{-j} = \left[b^{-1} \left(\frac{u_j - c}{\lambda_j} \right) - g_j \right] \frac{1}{w}$
		
	g_j	g_j

Table 1 Comparison of the framework of Dur and Roelfsema (2005) – a model in the tradition of Besley and Coate (2003) – to the framework proposed here.

achieved by dropping the mentioned assumption of separability of the public good provisions in the utility function and modeling the latter directly as a function only dependent on G (directly) – obviously G depends on g_j and g_{-j} . The falling marginal rate of substitution in the existing models could be explained by a falling degree of spillovers in the quantity provided in the other region. This in turn would be hard to align with the image of the total accessible quantity of the public good as the sum of individual provision levels. Considering environmental protection as a real world example – or more specifically, the reduction of carbon dioxide emissions – it is clear that spillovers are huge such that the location of the “provision” of this public good is irrelevant for consumers. Furthermore and more importantly, due to the perfect substitutability of the individual provision levels, the marginal rate of substitution should be constant irrelevant of the regions’ individual provision levels. Keeping the image of provision levels which are perfect substitutes, the introduction of falling marginal utility from g_j in g_{-j} makes sense when assuming $b(\cdot)$ to be concave.

Adopting this alternative specification of the model has severe consequences. In the following I assume $b(\cdot) = \ln(\cdot)$ as in Besley and Coate (2003). Hence in the framework of these scholars, both $g_j = 0$ and $g_{-j} = 0$ result in utility of minus infinity. Therefore the issue of free riding which is of importance in the complex of public goods provision and imperfectly internalized benefits and costs cannot arise in their framework. Regions always have to provide a positive amount of the public good themselves. Additionally, related to the argument of falling marginal utility above, the optimal quantity in a region does not depend on the amount which is provided in the other region. In the proposed formulation of the model, regions can decide not to provide the public themselves and can fully free ride on the efforts of the other region.

Again, this feature might resemble reality better for some kinds public goods – the example of environmental protection seems to be adequate in this context as well if one observes the great variety in these efforts among different nations.

2.2 The model

As hinted before, the model consists of the two regions $j = 1, 2$ which are of the same size with respect to the number of citizens. This amount is normalized to unity in both regions. Each citizen draws utility from consuming a private good (x) and a public good (g). Each citizen is initially endowed with the same amount of the private good which can be used to produce the public good. One unit of the private good can be converted to one unit of the public good. Like in other models, the private good is needed to model costs of the public good. The initial endowment of the private good is therefore assumed to be large enough to produce any desired amount of the public good such that binding budget constraints are no issue.

Citizens differ in their preference for the public good: This is captured by the preference parameter λ which is distributed according to some symmetric, single-peaked distribution. Importantly, the distribution of the preference parameter of citizens from region one is different from the one of citizens living in region two. Thus, there exists heterogeneity within a region as well as heterogeneity between the two regions. Due to this, the preference parameter is usually denoted as λ_j such that one can infer the home region of the respective citizen. The heterogeneity between the two regions can be measured with the help of the medians of the two distributions. The median of the distribution of λ_j is denoted as m_j . Furthermore and without loss of generality, $\mu \equiv \frac{m_1}{m_2} \geq 1$ holds. Hence on average region one exhibits a higher preference for the public good. To guarantee symmetry, the support of both preference parameters' distributions is $[0, 2m_j]$.

The utility of a citizen from region j who is of type λ_j consuming the amount x of the private good given region j provides g_j units of the public good and the other region provides g_{-j} units of the public good is given by

$$u_j = x + l \ln \left(g_j + \frac{g_{-j}}{1+d} \right) .$$

The parameter d which is weakly larger than zero constitutes a discount on the public good of the other region. This discount is rising in d , such that the parameter can be considered as a measure of the distance between the two regions. However, d is not a measure in kilometers, but rather proximity in terms of spillovers: Taking a reduction of carbon dioxide emissions as the regarded public good, d would be very low or even zero. Citizens do not care where these reductions are achieved. Regarding spendings on public education, like financing a university, d should take on some positive value: European Graduates can apply to German universities like graduates from German high-schools, however they still have to move to Germany and do more paperwork. Consumers do not discriminate between the two regions if $d = 0$ is true. For $d \rightarrow \infty$ there are no spillovers. A different interpretation of d is variation in the two individual provision

levels. Rather than having the exactly similar public good provided at two different locations, one could consider d to measure the similarity of g_{-j} to g_j . If d is positive, the public good provided in region $-j$ is not the same in character as the one provided in region j . Furthermore, note that the private good will be dropped in the analysis to come since it is a constant⁵.

3 Welfare maximizing provision levels

The welfare maximizing provision levels can be achieved with the help of a social planner who maximizes the sum of expected utility of both regions – what is equivalent to the sum of utility of both regions’ median citizens. Due to constant marginal costs the question of payment is irrelevant. The social planner faces the following optimization problem, a scheme of the solution is depicted in figure 1. See section A in the appendix for the calculus leading to this solution.

$$\max_{g_1, g_2} W = E[u_1] + E[u_2] = m_1 \ln \left(g_1 + \frac{g_2}{1+d} \right) + m_2 \ln \left(g_2 + \frac{g_1}{1+d} \right) - (g_1 + g_2) \quad \text{s.t.} \quad g_1, g_2 \geq 0$$

The solution is highly intuitive: Whenever region two’s median preference for the public good is sufficiently low and/or when spillovers are sufficiently high, the public good will only be provided in region one.

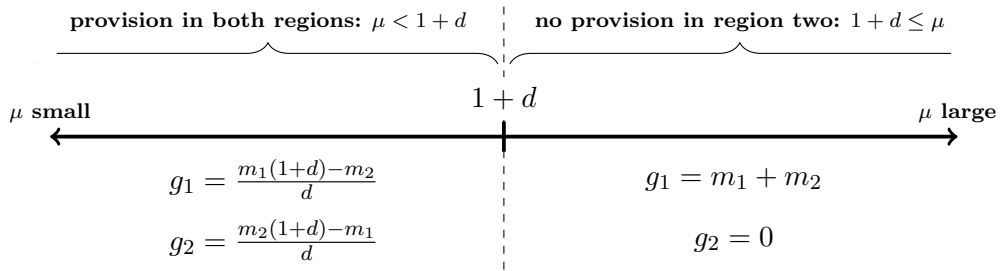


Figure 1 Welfare maximizing provision levels. Region two cannot be the sole provider of the public good due to $\mu > 1$.

To conclude this part, the total accessible quantity of the public good in region j conditional on the parameters μ and d is given by

$$G_j^{BM} = \begin{cases} m_j \frac{2+d}{1+d} & \text{for } \mu < 1 + d \\ m_1 + m_2 & \text{for } \mu \geq 1 + d \text{ and } j = 1 \\ \frac{m_1 + m_2}{1+d} & \text{for } \mu \geq 1 + d \text{ and } j = 2 \end{cases} \quad (\text{G-BM})$$

⁵The model presented above is also linked to the effort of [Buchholz, Haupt and Peters \(2005\)](#). They identify strategic delegation in a scenario of environmental policymaking. In the most extreme case they find that power is transferred to representatives who do not draw utility from the environment. This, however, results in disutility of minus infinity (due to standard Inada conditions they impose) and can therefore not be a solution of the model. The present approach adds to the mentioned work by explicitly analyzing the behavior of electorates and representatives in these settings.

4 Decentralized provision without representatives

In this section the decisions about the provision levels of the public good are made separately in each region. In contrast to the subsequent section, there are no representatives who decide about these provision levels. Both regions select their respective provision level directly by a popular vote. This setting might appear quite unrealistic especially if the regions are thought of as large economies. However, the results of this part are needed to determine the degree of strategic delegation when there are representatives who determine the provision levels of the public good. Furthermore and on the aspect of closeness to reality, Switzerland constitutes an example where national referendums on the governmental budget are even mandatory in some districts.

Whenever every citizen of a region is asked to specify her desired provision level, the median voter of region j – whose vote is decisive in this region – faces the following optimization problem:

$$\max_{g_j} u_j(m_j, g_j, g_{-j}) = m_j \ln \left(g_j + \frac{g_{-j}}{1+d} \right) - g_j \quad \text{s.t.} \quad g_j \geq 0$$

The optimal voting behavior of region j 's median citizen is described by the best-response function which originates from the above optimization problem:

$$g_j^* = BR_j(g_{-j}|m_j) = \begin{cases} m_j - \frac{g_{-j}}{1+d} & \text{for } g_{-j} < m_j(1+d) \\ 0 & \text{for } g_{-j} \geq m_j(1+d) \end{cases}$$

The possible Nash equilibria which are induced by these best response functions are depicted in figure 2. Since $\mu > 1$ is always true, region two will never solely provide the public good as it has been in the benchmark scenario.

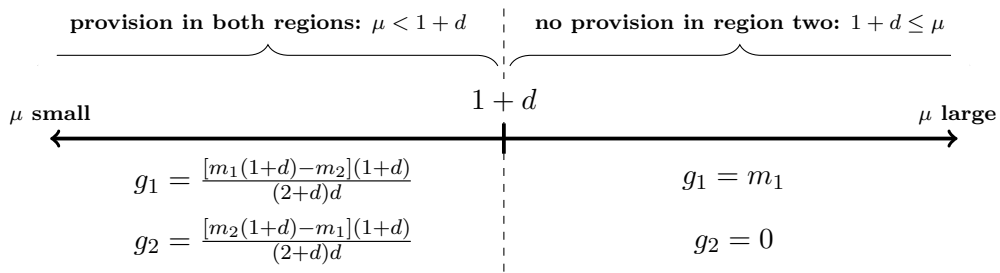


Figure 2 Possible Nash equilibria given both regions decide directly about the provision of the public good by popular vote.

However, the provision levels which arise from this scenario are strictly smaller than the ones stemming from the benchmark scenario. Obviously, this is due to the competitive character of the setting. The sum of median utility is no longer the function which is sought to maximize. Additionally and even worse, both regions' incentives are characterized by a classical free rider problem: Both regions' median citizens do not

induce the welfare maximizing provision levels since they do not benefit from gains which are realized in the other region and they themselves seek to benefit from the efforts undertaken in this other region. The accessible quantities of the public goods in both regions which result from the voting behavior of the median citizens in the two Nash equilibria are given by **G-DEC1**:

$$G_j^{DEC1} = \begin{cases} m_j & \text{for } \mu < 1 + d \\ m_1 & \text{for } \mu \geq 1 + d \text{ and } j = 1 \\ \frac{m_1}{1+d} & \text{for } \mu \geq 1 + d \text{ and } j = 2 \end{cases} \quad (\text{G-DEC1})$$

Since the current model does not rely on delegation, it cannot include any form of strategic delegation. However, the competitive character of the form of provision in this setting alone is sufficient to reduce the accessible quantities G_j . Comparing **G-BM** and **G-DEC1** under the assumption $\mu < 1 + d$ allows the finding $G_j^{DEC1} = G_j^{BM} \frac{1+d}{2+d}$. This relation is due to the fact that the representatives in the benchmark case react to an increase in spillovers (a fall in d) by raising the provision levels such that G_j^{BM} rises. This is not true in the present formulation on a decentralized provision since G_j^{DEC1} is independent of d . Thus the reduction in the accessible quantity G_j relative to the benchmark case becomes larger for rising spillovers (for $\mu < 1 + d$). Furthermore, one can infer that despite there is no direct influence of these parameters on G_j , they affect these values by determining the relevant Nash equilibrium. Whenever only region one provides the public good, the underprovision of the public good is independent from d and does only depend on the median preferences. Therefore, since the motive of any form of strategic delegation is to improve ones position in an upcoming competition, the aspect of strategic delegation which will be of central importance in the next section should be of a different form, whether both regions or only one region provides the public good.

5 Decentralized provision with representatives

Here in this part, the decisions about the provision level of the public good are made separately in each region by a representative. This representative is a citizen of the respective region and is elected by all citizens of this region. Note that the approach presented here is not a citizen-candidate-model. In the following, whenever a representative has to be nominated, we assume that every citizen is eligible. This simplification is made to ignore issues of endogenous candidates since the main interest lies on the effects a certain design of the political system exerts on the voters' incentives. In line with this, there are no costs or benefits of office apart from being allowed to determine the course of action. The model is solved by backward induction: First, the behavior of the representatives and the resulting provision levels of the public good conditional on the types of these representatives is determined. Second, the voting behavior of the citizens who anticipate the behavior of their future representatives is analyzed. A solution of the political system considered in this section is given by a pair of mutually majority preferred (mmp) representatives. This implies that no region has an incentive to elect a different representative given the other region's representative. Hence the concept of mmp representatives is equivalent to the Nash equilibrium in representatives. As before, I only

focus on pure equilibria in which the types of the representatives are clearly identified and not stochastic.

Section B in the appendix is of the same structure as this section and provides the calculus and proofs of all lemmas and theorems introduced in this section of decentralization with representatives.

5.1 Behavior of representatives

A representative maximizes her utility given her type and the provision level of the other region. This assumption should not be confused with selfishness but rather be interpreted in the spirit that politicians act according to and are bound by their agenda (their type) which is publicly known. The costs of providing the public good have to be raised among the citizens of the region in which the public good is provided. Hence a representative of region j who is of type λ_j whilst the provision level of the other region is given by g_{-j} faces the optimization problem

$$\max_{g_j} u_j = \lambda_j \ln \left(g_j + \frac{g_{-j}}{1+d} \right) - g_j \quad \text{such that} \quad g_j \geq 0 .$$

The optimal behavior of this representative is characterized by the best-response-function implied by the considered maximization problem:

$$g_j^* = BR_j(g_{-j}, \lambda_j) = \begin{cases} \lambda_j - \frac{g_{-j}}{1+d} & \text{for } g_{-j} < \lambda_j(1+d) \\ 0 & \text{for } g_{-j} \geq \lambda_j(1+d) \end{cases} .$$

So far, this was more or less equivalent to the approach in the decentralized scenario in which provision levels were determined by a popular vote – just exchange λ_j by m_j (and λ_{-j} by m_{-j}). However in the present scenario, dependent on the representatives' types and the parameter d , three possible (pure-strategy) Nash equilibria can emerge. Define L as the relative difference in the representatives' types, $L \equiv \frac{\lambda_1}{\lambda_2}$, then those three solutions are separated according to the scheme depicted in figure 3.

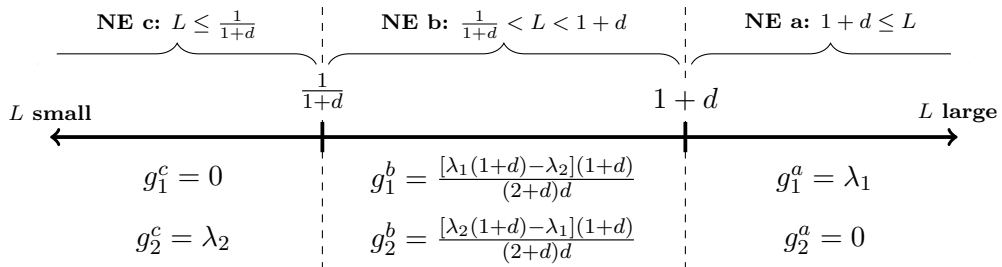


Figure 3 (Pure) Nash equilibria in the stage of the representatives dependent on the types of the representatives in the decentralized scenario. In contrast to the benchmark case (figure 1) and the decentralized scenario without representatives (figure 2) there are three solutions since $L < 1/(1+d)$ is actually possible.

The interpretation of these three Nash equilibria is straightforward. Whenever the difference in the preferences of the representatives is larger than the discount on the public good of the other region ($L > 1 + d$ respectively $L^{-1} > 1 + d$), the region with the low-preference representative will not provide the public good and fully free ride on the provided amount of the other region. Due to the low type of the representative in the idle region (and the relatively large amount which is provided by the other region), the marginal utility of providing the public good herself is negative already for the first unit of the public good. Both regions supply a positive amount of the public good whenever the types of the representatives are sufficiently similar. The actual provision levels of the public good in both regions can be found in the lower part of figure 3.

5.2 Behavior of the electorates

The last subsection carved out the behavior of the representatives given their types and the three Nash equilibria in the stage of the representatives (named a, b and c) which are of interest. It is left to determine on which representative the citizens of region j can agree given the type of the other region's representative and in anticipation of the outcome of the game played by the elected representatives. We approach this issue from the points of view of both regions' median citizens since we are interested in pairs of mmp representatives and the median voters will be decisive.

Given both representatives' types, region j can either be idle, provide the public good jointly with the other region or provide the public good alone. The utility of the median citizen of region j given the types of the representatives λ_j and λ_{-j} is given by

$$u_j(\lambda_j, \lambda_{-j}) = \begin{cases} m_j \ln \left(\frac{\lambda_{-j}}{1+d} \right) & , \lambda_j \leq \frac{\lambda_{-j}}{1+d} \quad (\text{region } j \text{ idle}) \\ m_j \ln(\lambda_j) - \frac{[\lambda_j(1+d) - \lambda_{-j}](1+d)}{(2+d)d} & , \frac{\lambda_{-j}}{1+d} < \lambda_j < (1+d)\lambda_{-j} \quad (\text{joint provision}) \\ m_j \ln(\lambda_j) - \lambda_j & , (1+d)\lambda_{-j} \leq \lambda_j \quad (\text{provision alone by region } j) \end{cases} \quad . \quad (\text{UD})$$

This figures are obtained by inserting the resulting provision levels shown in figure 3 into the median citizens' utility. Note that with this alternative naming *idle* stands for equilibrium c if we consider region one and equilibrium a when considering region two. The same is true for *alone*, whereas *joint* always refers to equilibrium b.

Naturally, the motivation of each citizen is to maximize her utility. They try to achieve this by electing a representative of a type which promotes their utility best. However, altering the type of the representative can affect median utility either while keeping the current equilibrium or by inducing a different equilibrium. The optimal representative within an equilibrium, meaning assuming the realization of a certain equilibrium for any pair of representatives, can be determined easily. Table 2 exhibits these representatives⁶ as well as

⁶Again, be aware that the types of representatives shown in table 2 are first-best types. These types λ_j^n are independent of λ_{-j} and might thus not be sufficient to imply the assumed equilibrium. For a complete overview of the first-best as well as

the resulting median utility levels.

scenario (s)	λ_j^{s*}	g_j^s	u_j^s
<i>idle</i>	any λ_j	0	$m_j \ln \left(\frac{m_{-j}}{1+d} \right)$
<i>joint</i>	$m_j \frac{(2+d)d}{(1+d)^2}$	$\frac{m_j(1+d)-m_{-j}}{1+d}$	$m_j \ln \left(m_j \frac{(2+d)d}{(1+d)^2} \right) - \frac{m_j(1+d)-m_{-j}}{1+d}$
<i>alone</i>	m_j	m_j	$m_j \ln (m_j) - m_j$

Table 2 Utility maximizing types of representatives (first-best, meaning within equilibrium n) for the median citizen of region j (first-best solutions), the resulting provision levels and the corresponding median utility – given that both regions set the first-best representatives.

Due to lemma 1, one can dismiss the potential outcome that region two is the sole provider of the public good. Hence there exists no pair of mmp representatives which induces this scenario. This seems quite intuitive. For a sufficiently large difference in the two representatives' types, it is theoretically possible that region two can become the sole provider (see figure 3). However, it is clear that this outcome can hardly be the result of any pair of mmp representatives: From the perspective of region two, being the sole provider can only be optimal for very small values of λ_1 – what is itself very unlikely since $m_1 > m_2$. However, if region one's representative is of a sufficiently low type, it is region one itself who faces an incentive to deviate.

Lemma 1 *There exists no pair of mmp representatives such that region two provides the public good alone. Or equivalent: There exists no pair of mmp representatives such that region one is idle. [Proof: B.3]*

As in the sections before, the parameter space can be split in the following way: For $\mu < 1 + d$, we are considering regions which are rather similar in (median) preferences and/or which lie rather far apart, while $\mu \geq 1 + d$ characterizes regions which are rather different in preferences and/or rather far apart. The two remaining potential outcomes of pairs on mmp representatives – both regions providing the public good jointly or region one providing the public good alone – can be roughly⁷ assigned to the two different areas of the parameter space.

Theorem 1 – mmp representatives for rather similar regions and/or small spillovers:

Given $\mu < \frac{(2+d)d}{1+d}$, the unique pair of (pure) mmp representatives is given by $\lambda_j^{\text{joint}^} = m_j \frac{(2+d)d}{(1+d)^2}$ (for $j = 1, 2$) such that both regions provide a positive amount of the public good. [Proof: B.4]*

Theorem 1 specifies the representatives $\lambda_1^{\text{joint}^*}$ and $\lambda_2^{\text{joint}^*}$ (see table 2) as the unique pair of mmp representatives for a slightly stricter restriction on the parameter space than introduced above. This finding is obtained by establishing these representatives as a pair of mmp representatives – see lemmas 2 and 3 – and showing the nonexistence of another pair of mmp representatives – refer to lemma 4. In the following, a

the second-best (corner solution) representatives, see IR in the appendix.

⁷Whilst the intuition in both theorems 1 and 2 is stemming from this split, the actually allowed parameter spaces in both theorems is slightly different.

sketch of this proof is presented.

Lemma 2 *Given $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ (for $i = 1, 2$) and $\mu < 1 + d$, the regions provide the public good jointly and no median citizen has an incentive to lower her representative's type such that the other region will become the sole supplier of the public good. [Proof: B.4.1]*

The intuition of Lemma 2 is clear: Regions which prefer a rather similar amount of the public good and/or are rather far apart must provide the public good themselves. Free-riding on the other region's effort is not an option since the utilizable quantity of the public good will be too small due to the rather large discount.

According to this argumentation, becoming the sole supplier is not as harmful as having to free-ride. This finding is captured by lemma 3: The restriction $\mu < 1 + d$ is too weak and leaves incentives to actually become the sole supplier. This is quite intuitive: The competition between the two regions (when electing their representatives) and the competition between these representatives itself becomes fiercer for smaller values of d – assuming joint provision which itself becomes more unlikely for small values of d . This effect is due to the increased usability of the other region's public good. Therefore both regions might profit from escaping this race to the bottom by electing a representative who makes them the sole provider of the public good. In order to eliminate these incentives, some parameter combinations must be excluded. Furthermore, note that the boundary established in lemma 3 is a conservative approximation. The actually allowed amount of parameter combinations which yield no incentive to deviate is larger (see figures 9 and 10 in the appendix). Yet the present boundary is used due to the lack of a closed form solution of the relevant threshold.

Lemma 3 *Given $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ (for $i = 1, 2$) and $\mu < \frac{(2+d)d}{1+d}$, the regions provide the public good jointly and no median citizen has an incentive to increase her representative's type such that her home region will become the sole supplier of the public good. [Proof: B.4.2]*

Lemma 4 *Given $\mu < \frac{(2+d)d}{1+d}$, there exists no pair of mmp representatives which implies that a region provides the public good alone. [Proof: B.4.3]*

Lemma 4 discards all other possible pairs of mmp representatives which are all cases in which only one region provides the public good. Note that there are two different ways how this can occur: First, whenever the idle region's type is sufficiently low, the active region provides a positive amount with the help of the first-best representative (m_j , see table 2). Second, if the type of the idle region's representative is too high, the optimal representative's type of the active region is given by a (second-best) corner solution: the representative who marginally still leads to sole provision ($\lambda_{-j}(1+d)$, see IR in the appendix). However, in both cases there are incentives for the regions to elect a different representative: In the first scenario it is the idle region who can be better off by inducing joint provision. In the second scenario it is the solely providing region who will switch to joint provision. This can be explained intuitively: Whenever the idle region does not prevent the active region (by the choice of their representative) to elect the first-best representative, the accessible quantity of the public good in the idle region is so low that a transition to joint provision is beneficial. In turn however, if the idle region manages to increase the providing region's quantity (by

forcing them to elect a second-best representative of above median type), it is the active region who faces an incentive to induce joint provision.

Theorem 2 covers the remaining parameter space $\mu > \frac{(2+d)d}{1+d}$ and is a little bit more complicated to establish than theorem 1 since it does not characterize a unique pair of mmp representatives, but rather a continuum of pairs of mmp representatives (dependent on μ and d) which all induce sole provision by region one. After the theorem itself, a sketch of the proof is presented.

Theorem 2 *Given $\mu > \frac{(2+d)d}{1+d}$, there exists a continuum of mmp representatives characterized by $\lambda_1 = m_1$ and $\lambda_2 \in [0, \varepsilon_2^* m_2]$ which leads to sole provision by region one for every pair of mmp representatives in the continuum. ε_2^* depends on the parameters μ and d such that there exists a continuum of mmp representatives of different size for all possible combinations of μ and d .*

Due to lemma 7 (see section B.2 in the appendix) one is able to know that whenever region one is the sole provider of the public good – what is the last potential outcome left to analyze – this has to be achieved with the first-best representative m_1 of this scenario. Building on this knowledge, theorem 2 is established by two lemmas: Lemma 5 proves that region two will never induce a different scenario if region one is the sole provider with the region’s first-best representative. This is straightforward since since the two alternatives of region two – joint provision or even becoming the sole provider itself – lead to the same median utility in the first case and to a decrease in median utility in the latter case.

Lemma 5 *Given $\lambda_1 = m_1$, $\lambda_2 \leq \frac{m_1}{1+d}$ and $\mu > \frac{(2+d)d}{1+d}$, region one is the sole provider of the public good and region two’s median citizen has no incentive to induce a different scenario. [Proof: B.5.1]*

Lemma 6 addresses the possibility that region one might want to leave the role of being the sole provider and maybe would profit from inducing joint provision⁸. Unfortunately, the initial condition $\lambda_2 \leq \frac{m_1}{1+d}$ which makes region one the sole provider given $\lambda_1 = m_1$ actually allows values of λ_2 which yield an incentive to switch to joint provision. Thus – conditional on the parameters μ and d – the representative of region two must be of an even lower type such that all incentives for region one to induce joint provision are dissolved.

Lemma 6 *Given $\mu > \frac{(2+d)d}{1+d}$ and $\lambda_1 = m_1$, there exists a critical representative of region two $\lambda_2 = \varepsilon_2^* m_2$ characterized by the parameter ε_2^* (which is conditional on μ and d) such that region one will be the sole provider of the public good and does not face an incentive to induce a different scenario given $\lambda_2 \leq \varepsilon_2^* m_2$. [Proof: B.5.2]*

The intuition behind lemma 6 is similar to the concept of entry deterrence in the field of industrial organization. Entry deterrence refers to a monopolist who charges prices lower than the monopoly prices⁹ to eliminate the incentives of potential competitors to enter the market. Similar to this phenomenon, the idle region – region two – has to lower the potential median utility region one would obtain by switching to joint provision such that this deviation does not lead to an increase in median utility. If this is done properly, a

⁸The option of making region two the sole provider is included in the present analysis since it leads to the same utility as inducing joint provision with the lower second-best representative, see section B.5.2 in the appendix.

⁹By this I mean the profit maximizing prices in a single period.

nonempty continuum of pairs of mmp representatives can emerge.

5.3 Equilibrium provision levels in decentralization with representatives

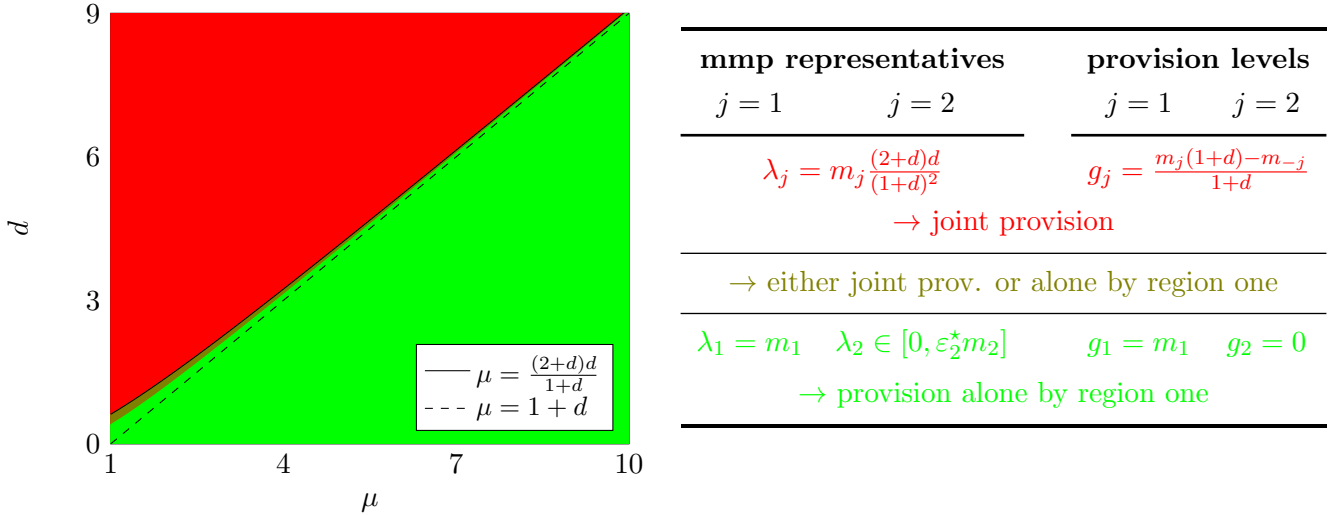


Figure 4 The structure of stable outcomes in the decentralized scenario with representatives.

Figure 4 displays the stable outcomes for decentralized provision with the help of representatives – meaning the provision levels which result from pairs of mmp representatives. The red area covers all parameter combinations allowed by $\mu < \frac{(2+d)d}{1+d}$. In this area of the parameter space there exists – due to theorem 1 – a unique pair of mmp representatives and thus also unique provision levels of the public good. The green area’s outcome is sole provision by region one. However, according to theorem 2, despite this outcome can be generated by a continuum of pairs of mmp representatives, this outcome is unique since region one’s representative is always given by the region’s median type. Obviously, this result is intuitively highly appealing and fits perfectly to previous results: Region one will provide the public good solely if its median preference is sufficiently larger than the one of region two and joint provision emerges if these two values are sufficiently close.

The small area between the red and green area characterizes a subset of the parameter space which allows both of the outcomes described above as stable outcomes. However, to get rid of this – not too troubling – ambiguity I assume that the realized outcome in the relevant parameter space is given by sole provision of region one according to theorem 2. First, this assumption comes handy since there exists no closed form solution for the boundary between the green and the orange area (see the proof of lemma 3 in section B.4.2 the appendix). Second and more important, region two obtains a strictly higher median utility in the orange region whenever region one is the sole provider compared to joint provision¹⁰. Note that the difference in

¹⁰Subtract the median utility in the joint provision case from the one of the idle case:

$$\Delta u_2 = m_2 \ln \left(\frac{m_1}{1+d} \right) - m_2 \ln \left(m_2 \frac{(2+d)d}{(1+d)^2} \right) + m_2 - \frac{m_1}{1+d} = m_2 \ln \left(\mu \frac{1+d}{(2+d)d} \exp \left[1 - \frac{\mu}{1+d} \right] \right) > 0$$

region one's median utility is not straightforward, such that this argument cannot identify sole provision by region one as the payoff dominant Nash equilibrium. However, I argue that the assumption of region one being the sole provider is fairly reasonable since region two – who experiences higher median utility when being idle – can always force region one to become the sole provider by electing a representative of sufficiently low preference for the public good. Thus the outcome of the decentralized scenario with representation under this assumption on the ambiguous part of the parameter space becomes even simpler in its structure. To allow a comparison between the two considered decentralized scenarios, the total accessible quantity in region j for decentralization with the help of representatives is given by equation G-DEC2:

$$G_j^{DEC2} = \begin{cases} m_j \frac{(2+d)d}{(1+d)^2} & \text{for } \mu < \frac{(2+d)d}{1+d} \\ m_1 & \text{for } \mu \geq \frac{(2+d)d}{1+d} \text{ and } j = 1 \\ \frac{m_1}{1+d} & \text{for } \mu \geq \frac{(2+d)d}{1+d} \text{ and } j = 2 \end{cases} \quad (\text{G-DEC2})$$

Comparing these figures to the ones from equation G-DEC1 allows the following findings which arise due to the formulation of local public goods as perfect substitutes: First, regional provision levels and the overall accessible quantity of the public good are only affected by strategic delegation if the regions provide the public good jointly. Indeed, strategic delegation – which can be understood as the try of electorates to nominate a representative who will have an advantage over the other representatives – does only occur in these cases. Whenever region one is the sole provider, there happens no competition between the representatives, such that there exists no incentive for strategic delegation. The effect of strategic delegation on the accessible quantity of the public good is negative such that $G_j^{DEC1} > G_j^{DEC2}$ is always true for joint provision. Since $G_j^{DEC1} < G_j^{BM}$ is always true, this constitutes a further decline in welfare.

6 Centralized provision with representatives

Now we analyze a centralized scenario – the natural counterpart to decentralized decision structures. Rather than having two regional governments, we now introduce a central government which decides about the provision levels of the public good. This central government is assumed to take on the form of a coalition consisting of one member stemming from region one and one from region two. Thus – similar to the decentralized case – we have representatives from both regions which are elected in these regions. However, in contrast to the previous setting, these representatives are now part of and constitute a superordinate political body and decide on the two provision levels jointly and in a more cooperative way: The representatives maximize the sum of their utility. The central government is not considered to be determined by a minimum winning coalition as in Besley and Coate (2003), since this would introduce an element of uncertainty which is not the focus of this work. Additionally, the effects of such a different formulation have been analyzed in other works including the mentioned. The possibility of a central government only consisting of one

In the orange are – which is bounded by $\mu \geq \frac{(2+d)d}{1+d}$ (on the left or above) and $\mu < 1+d$ (as an approximation on the right or below) – this difference is positive since the argument of the logarithm is always larger than one.

representative is discarded due to the following reasons: First, strategic delegation will not emerge without the interaction of representatives – independent from the form or intensity of this competition. Thus the chosen formulation is the appropriate given the purpose and focus of this work. Second, since the central decider’s type is obviously given by the median across both regions, further assumptions on this matter would be necessary.

Hence the different form of interaction of the two representatives is the unique distinction of this centralized scenario to previous decentralized settings. As before, the costs of provision are assigned to the region in which provision occurs. Despite this cost scheme is in contrast to early works on fiscal federalism which usually assumed uniform provision and cost sharing in centralized settings, it resembles the developments in more recent contributions like [Dur and Roelfsema \(2005\)](#) which introduce regional specific provision levels and more advanced cost schemes. Furthermore, sharing of costs is ignored here as it would introduce additional incentives for strategic delegation and render the obtained results incomparable to the previous results. In addition to this rather technical motive, one can argue that by allowing different provision levels in the regions and refraining from any form of cost sharing one has implemented the most desirable features of decentralization in centralized decision structures: The cooperative nature of the representatives’ stage allows an internalization of spillovers whereas the possibility of region specific provision levels can account for different preferences in the two regions. This formulation of the model is again solved by backward induction. All calculations which are omitted here can be found in section [C](#) in the appendix.

6.1 Behavior of representatives

The two members of the central government coalition of the types λ_1 and λ_2 seek to maximize the sum of their private utility. Again, this should be interpreted such that the coalition tries to find a middle course between the two agendas which have been determined as the majorities’ wills in both regions. The relevant optimization problem is the following:

$$\max_{g_1, g_2} \Sigma = u_1 + u_2 = \lambda_1 \ln \left(g_1 + \frac{g_2}{1+d} \right) + \lambda_2 \ln \left(g_2 + \frac{g_1}{1+d} \right) - (g_1 + g_2) \quad \text{such that} \quad g_1, g_2 \geq 0$$

Note that the optimization problem above is equivalent to the one from the benchmark scenario. The only difference is the type of the acting citizens. Whereas it were the median types in the benchmark scenario (as the objective function was the sum of expected utility), it are the representatives of arbitrary types λ_1 and λ_2 . Due to this change, region two can now actually become the sole provider whenever $\lambda_2 \geq \lambda_1(1+d)$ is true. The optimal provision levels are displayed in figure [5](#).

The separating conditions of those three Nash equilibria are the same as in all previous formulations of the model. Comparing the provision levels to the one of the decentralized scenario displayed in figure [3](#) reveals that the provision levels which emerge in the centralized setting are always strictly larger than the ones induced by decentralization while holding the types of the representatives constant. This resembles the fact

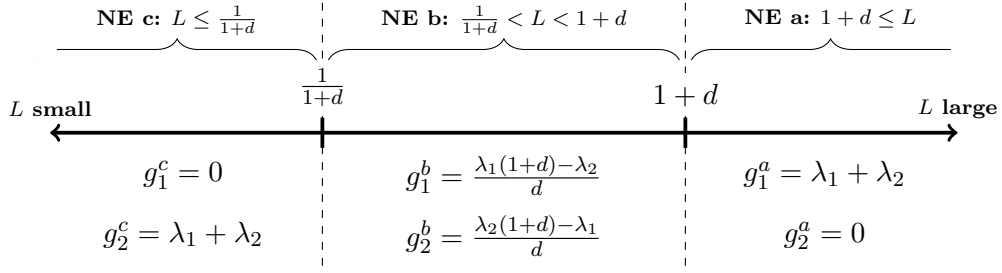


Figure 5 (Pure) Nash equilibria in the stage of the representatives dependent on the types of the representatives in the centralized scenario.

that the outcome of the representatives' stage under centralization does not suffer from free riding since – due to the maximization of the sum of utility – benefits in other regions are taken into account such that spillovers are internalized.

6.2 Behavior of the electorates

Similar to the procedure in section 5, the next step after determining the behavior of the central government is to analyze the incentives of the electorates in the two regions. As before, a solution of the model for centralization is given by pairs of mmp representatives and the resulting provision levels induced by these representatives. Dependent on L and d , region j can either provide the public good alone, jointly with the other region or be idle. The utility of the median citizen in region j given these three cases is

$$u_j(\lambda_j, \lambda_{-j}) = \begin{cases} m_j \ln \left(\frac{\lambda_j + \lambda_{-j}}{1+d} \right) & , \lambda_j \leq \frac{\lambda_{-j}}{1+d} \quad (\text{region } j \text{ idle}) \\ m_j \ln \left(\lambda_j \frac{2+d}{1+d} \right) - \frac{\lambda_j(1+d) - \lambda_{-j}}{d} & , \frac{\lambda_{-j}}{1+d} < \lambda_j < (1+d)\lambda_{-j} \quad (\text{joint provision}) \\ m_j \ln (\lambda_j + \lambda_{-j}) - (\lambda_j + \lambda_{-j}) & , (1+d)\lambda_{-j} \leq \lambda_j \quad (\text{provision alone by region } j) \end{cases} . \quad (\text{UC})$$

A first striking finding from UC is that an idle region can actually increase the amount of the public good which is provided by raising its representative's type. Since the idle region is not required to pay for the provision, it faces an incentive to push the provision of the public good as far as possible¹¹. Looking at the incentives of region j 's median citizen when this region is the sole provider of the public good ($\lambda \geq \lambda_{-j}(1+d)$ has to hold) helps to make progress on this issue. The solely providing region's median citizen should opt for a representative of type $\lambda_j = m_j - \lambda_{-j}$. Obviously, the type of this region must not be negative. However, this is not a problem: Since the idle region's representative λ_{-j} must fulfill $\lambda_{-j} \leq \frac{\lambda_j}{1+d}$ – such that region j stays the sole provider – and due to $\lambda_j = m_j - \lambda_{-j}$, we know that λ_{-j} cannot be larger than $\frac{m_j}{2+d}$. Hence region j will at least elect a representative of type $\lambda_j = m_j - \frac{m_j}{2+d} = m_j \frac{1+d}{2+d} > 0$. Even more important

¹¹Assuming that the outcome of the game between the (central) representatives is held fixed such that the providing region will keep on providing the public good solely, implies that the idle region should send the highest possible representative. This in turn renders the currently regarded outcome of the representatives' stage highly unlikely.

than ruling out negative representatives for the region providing the public good alone is that one can now clearly determine the provided amount in the scenarios where only one region provides the public good. Due to the interplay of the regions described above, region j provides the amount $g_j = m_j$ whenever it is the sole provider of the public good. Representatives which give raise to this provision level are $\lambda_j = m_j - \lambda_{-j}$ and $\lambda_{-j} \in \left[0, \frac{m_j}{2+d}\right]$. The ideal representative if the assumed outcome is joint provision does not depend on the other region's representative and is given by $\lambda_j = \frac{m_j d}{1+d}$. Table 3 exhibits these representatives and the resulting provision levels as well as implied median utility in the same way as table 2 for the decentralized scenario with representatives.

scenario (s)	λ_j^{s*}	g_j^s	u_j^s
<i>idle</i>	any $\lambda_j \leq \frac{m_{-j}}{2+d}$	0	$m_j \ln\left(\frac{m_{-j}}{1+d}\right)$
<i>joint</i>	$m_j \frac{d}{1+d}$	$\frac{m_j(1+d) - m_{-j}}{1+d}$	$m_j \ln\left(m_j \frac{(2+d)d}{(1+d)^2}\right) - \frac{m_j(1+d) - m_{-j}}{1+d}$
<i>alone</i>	$m_j - \lambda_{-j}$	m_j	$m_j \ln(m_j) - m_j$

Table 3 Utility maximizing types of representatives (first-best, meaning within equilibrium n) for the median citizen of region j (first-best solutions), the resulting provision levels and the corresponding median utility – given that both regions set the first-best representatives.

Having obtained this information, it immediately follows that the resulting structure of equilibria is identical to the one of the decentralized scenario. The values displayed in table 2 are identical to the ones found here and displayed in table 3. Hence refer to section 5.3 for the equilibrium values.

6.3 Uniform provision of the public good

To provide a contrast to the centralized scenario analyzed so far, I now consider a centralized setting in which the representatives have to decide on a uniform provision level g which will be implemented in both regions. Every region has to pay for the amount g . With a uniform provision level g across both regions, there obviously cannot be case in which the public good is not provided. The total accessible quantity of the public good in both regions is given by $G = g + \frac{g}{1+d} = g \frac{2+d}{1+d}$. Both representatives (of types λ_1 and λ_2) maximize the sum of their utility (see subsection C.2 in the appendix for the calculus):

$$\max_g \Sigma = u_1 + u_2 = [\lambda_1 + \lambda_2] \ln\left(g \frac{2+d}{1+d}\right) - 2g \quad \text{such that} \quad g \geq 0$$

This optimization problem gives rise to the optimal provision level $g^* = \frac{\lambda_1 + \lambda_2}{2}$ and thereby leads to $G = \frac{[\lambda_1 + \lambda_2](2+d)}{2(1+d)}$. Both regions' median citizens maximize their utility by selecting their region's representative:

$$\max_{\lambda_j} u_j = m_j \ln\left(\frac{[\lambda_j + \lambda_{-j}](2+d)}{2(1+d)}\right) - \frac{\lambda_j + \lambda_{-j}}{2} \quad \text{such that} \quad \lambda_j \geq 0 \quad \text{for} \quad j = 1, 2$$

The optimal type of region j 's representative is characterized by the best response function $\lambda_j^* = 2m_j - \lambda_{-j}$. Due to $m_1 \geq m_2$, the sum of both representatives' types is always clearly determined and given by $2m_1$. This resembles the change in the scenario: With the assumption of uniformity, there are no longer different provision levels of similar public goods which are perfect substitutes. Now, the two representatives – or rather their liking of the public good – are perfect substitutes. The obtained overall provision level is the following:

$$G_j^{CEN2} = m_1 \frac{2+d}{1+d} \quad \text{for} \quad j = 1, 2 \quad (\text{G-CEN2})$$

This finding yields several insights. First, related to the size of G , the provided quantities induce the benchmark provision, whenever the regions' median preferences are sufficiently similar or spillovers are sufficiently small respectively: For $\mu < 1 + d$, $G_j^{BM} = G_j^{CEN2}$ is true. This finding can be aligned to previous observations and argumentation. Given $\mu < 1 + d$, the welfare maximizing provision structure is joint provision. Whenever provisions are set by uncooperative representatives (decentralization as in section 5), this parameter space will also induce joint provision. However, the free-riding representatives create an incentive for strategic delegation which ultimately puts the welfare maximizing provision levels beyond reach. Now, considering uniform provision, there exists no conflict between the representatives and thus the incentive for strategic delegation does not arise. Looking at the parameter combinations which fulfill $\mu \geq 1 + d$ leads to a second aspect: The present setting leads to overprovision in both regions relative to the benchmark levels (equation G-BM). This is due to a combination of known mechanisms. Central provision is – due to the uniformity assumption – unable to cater to regional specific preferences. The overprovision in region two can be interpreted in this light. However, the overprovision in region one cannot be explained using this narrative. One would expect to observe underprovision since the uniform provision level would be some kind of mean between the regionally preferred provision levels. The element which leads to overprovision in region one is the cost scheme. In the current setting, every region can raise the uniform provision level by electing a representative of sufficiently high type. This puts – due to the higher median preference – region one in a position to dictate the uniform provision level. In interplay with the cost scheme which forces region two to finance half of the provision (which is dictated by region one), this leads to a provision level which is too high, compared to the benchmark case, even for region one.

Since these findings – centralization internalizes spillovers, is unable to handle heterogeneous preferences and cost sharing induces incentives for overspending – resemble well established knowledge, this part can be considered a test of robustness which proves that the insights on the complex of centralization and decentralization are not obtained at the expense of already existing knowledge.

7 Comparison between centralized and decentralized provision

Firstly, the equivalence in the achieved provision levels of the two political structures is striking. It perfectly fits to the experimental works used to motivate the present analysis. The driving factor of this finding is the emergence of strategic delegation: Comparing the mutually majority preferred representatives whenever the public goods are provided in both regions (see the values for λ_j^{joint*} in tables 2 and 3) – what are the outcomes which suffer from strategic delegation – one is able to see that the degree of strategic delegation is stronger in the centralized setting:

$$\text{decentralization} \quad m_j \frac{(2+d)d}{(1+d)^2} = m_j \frac{2+d}{1+d} \frac{d}{1+d} > m_j \frac{d}{1+d} \quad \text{centralization}$$

The mechanism which leads to this finding is the behavior of the electorates. The distortion towards below median representatives occurs since both regions' citizens try to elect a representative who is in a better position when it comes to determining the actual provision levels in interaction with the other region's representative. Voters face an incentive to elect an even more conservative representative (with respect to the preference for the public good) in the centralized setting due to the cooperative nature of the representatives' behavior: If both representatives maximize joint utility, a conservative representative will be better able to free-ride on the efforts of the other region. A different interpretation could be that the voters anticipate and account for the less fierce competition between the representatives in the centralized setting by electing more conservative representatives.

Another interesting insight from this work might be the one that there exists still – after a long lasting and rich debate – vagueness concerning the concepts of centralization and decentralization. The analyzed setting of centralized provision which included representatives and allowed regional different provision levels led to the same provision levels as the decentralized setting with representatives. This parity appears reasonable if one recalls the sole distinction between the two settings: In the centralized case, representatives maximize the sum of their utility whereas they were maximizers of private utility in the case of decentralized provision. Hence the behavior of the two representatives in the two scenarios somehow resembles the one in a Cournot duopoly. The decentralized setting corresponds to the case in which the firms behave competitively. The centralized setting which incorporates cooperative representatives is equivalent to the behavior of the firms if they form a cartel. Thus there has not been a change in the structure of the model itself, merely in the maximizing objectives of the agents. This, as argued above, is utilized (and by doing so, mitigated) by the citizens of the regions since they counter less fierce competition between representatives by electing more competitive representatives (with respect to the willingness to spend money on public good provision). Only a change of the structure towards uniform provision was able to induce different outcomes.

8 Strategic delegation in a multistage government

This section addresses an issue that may arise due to the findings in the previous parts: If two representatives are elected by certain electorates such that they exhibit below median preferences (relative to these electorates), what might happen if these electorates are themselves the outcomes of an election? A first thought can be that each stage of a representative democracy worsens the bias towards below median policymakers by creating a subset of the initial electorate which itself is biased. A well known phenomenon in economics which leads to a similar pattern is a chain of monopolies: The producer, (vertical) distributors and finally the seller, each of these firms adds a price premium such that the quantity of the good which is sold to consumers is lower than it would be if all these firms would act as a single monopolist. However, the question whether such a pattern arises in the present framework, cannot be answered straightforward.

The structure of the extended model in this section is the following: There exist T_j intermediary boards in region j . The term B_j^t refers to the t -th board in region j . For simplicity, assume that the first boards in both regions B_j^1 (for $j = 1, 2$) are equivalent to and given by the overall populations of both regions. The representatives who decide about the provision levels of the public good are elected by the last boards $B_j^{T_j}$ – obviously, board $B_j^{T_j}$ of region j elects this region’s representative λ_j . In turn, these two boards are formed by members of subordinate boards named $B_j^{T_j-1}$. Each board B_j^t (for $t = 1, \dots, T_j$ and $j = 1, 2$) is characterized by a size s_j^t and a median member m_j^t . The size can be understood as a proxy for the number of board members relative to the total population (unity). In line with the assumption above, $s_j^1 = 1$ and $m_j^1 = m_j$ hold for both regions. Note that, due to the formulation introduced above, the structure of the two multistage governments in the two regions can be different: This can be true regarding the number of intermediary boards and also for the sizes of these boards.

To be able to provide tractable results, I assume that the distribution of the preference parameter λ_j in region j is uniform across the support $[0, 2m_j]$ – as specified beforehand. Due to this assumption, the distribution of preferences in all other boards is also uniform and the support depends on the respective boards’ median member and size. The support in board B_j^t is given by the closed interval $[m_j^t - s_j^t m_j, m_j^t + s_j^t m_j]$. Obviously, greater board size results in greater variation of board members with respect to their types. Furthermore, there exist clearly defined lowest and highest types within a board – refer to these limiting members as $\underline{\lambda}_j^t = m_j^t - s_j^t m_j$ for the member who values the public good the least and $\bar{\lambda}_j^t = m_j^t + s_j^t m_j$ for the member who has the highest preference for the public good in board B_j^t . For the first board in region j – the region’s total population – these boundaries reduce to $\underline{\lambda}_j^1 = 0$ and $\bar{\lambda}_j^1 = 2m_j$ (due to $m_j^1 = m_j$ and $s_j^1 = 1$). Since boards can only send their own members to a superior board¹², $s_j^{T_j} < s_j^{T_j-1} < \dots < s_j^1 = 1$ has to hold for both regions¹³. Lastly, note that there exist constraints related to the type of a superior board’s median type. Due to the uniformity assumption, rather extreme members of a board are infeasible

¹²Dropping this requirement, such that boards can select delegates from the overall population, simplifies the analysis to a great extent and is included in the present analysis.

¹³Being strictly, this would only imply weak inequalities. However, without loss of generality, we assume that the boards become strictly smaller as board B_j^t would be identical to board B_j^{t-1} for $s_j^t = s_j^{t+1}$.

as median members of the next superior board because there exist too few members which are even more extremers such that they could be set as the middle member of the next board. The possible median types of board B_j^t depend on the size of board B_j^{t-1} and must lie in the interval $[\underline{\lambda}_j^{t-1} + s_j^t m_j, \bar{\lambda}_j^{t-1} - s_j^t m_j]$. Rephrase the boundaries of this interval as \underline{m}_j^t and \bar{m}_j^t . The type of the representative who is determined by the last board and who must be a member of this board is obviously not restricted by this constraint such that every member of board $B_j^{T_j}$ is a potential representative of region j . Figure 6 displays a scheme of the multistage structure of the government analyzed here and illustrates the constraint related to median members of superior boards.

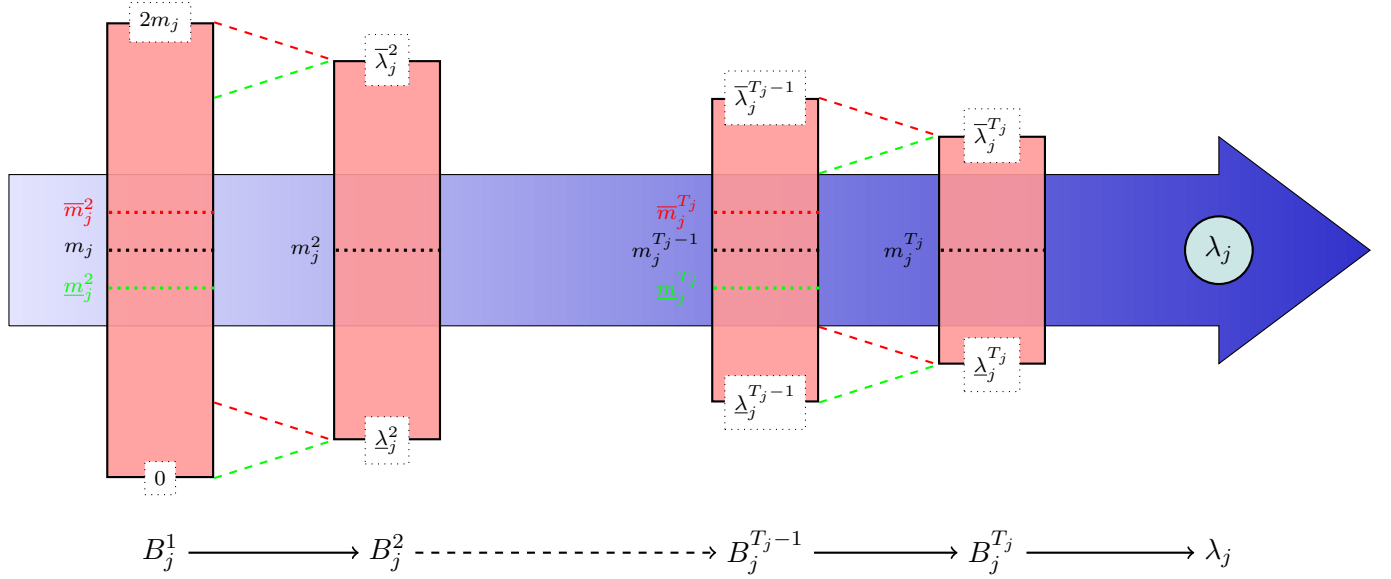


Figure 6 Structure of the multistage government in region j consisting of T_j different boards of sizes s_j^t ($t = 1, \dots, T_j$). The boundaries of the support of the types in each board which depend on the respective board's size and median member are denoted.

Due to the motivation of this part – to assess the effects of multistage governments on incentives for strategic delegation – the focus lies on outcomes in which both regions provide positive amounts of the private good as strategic delegation only occurs in these scenarios. The model is solved by backward induction (the formal part can be found in section D in the appendix). The result which seems surprising at first glance is that the issue of strategic delegation is unaffected by a multistage government. Theorem 3 below summarizes this finding. The intuition behind theorem 3 is presented in the following.

Theorem 3 *For joint provision and given uniformity of preferences, the magnitude of strategic delegation in multistage governments as defined in section 8 is neither affected by the number of boards in the two regions (T_j) nor the sizes of these boards.*

Altering the structure of the model such that there occur multiple delegation decisions does not affect the emergence and the magnitude of strategic delegation. The rationale behind this observation can be explained briefly: Every citizen in a region (characterized by her preference for the public good) has unique

preferences concerning the type of the region's representative which only depend on the considered scenario and not on the structure of the multistage government. As joint provision is assumed here, these ideal representatives for a citizen of type λ_j are $\lambda_j^*(\lambda_j) = \lambda_j \frac{(2+d)d}{(1+d)^2}$ under decentralization and $\lambda_j^*(\lambda_j) = \lambda_j \frac{d}{1+d}$ under centralization (see the optimal representatives for the *joint* scenario in tables 2 and 3). Obviously, the median members of boards are pivotal when it comes to determining the identity of the next boards' median members. Therefore, whenever a pivotal citizen delegates power such that the pivotal member of the next board exhibits different preferences, the delegating citizen has left her pivotal role. This implies that the former pivotal citizen is no longer able to determine the course of action and – even worse – is replaced by a decision maker who favors a different course of action for sure. A different interpretation of the result obtained here might be that the creation of a board (by a subordinate board) does not constitute a commitment device which is equivalent to the nomination of a representative. This is due to the fact that the change in competences occurs between boards and the representative: Whereas all boards except the last one are only capable of creating other superordinate boards with identical competences and even narrowing action spaces, the representative is allowed to set the actual provision level of the public good.

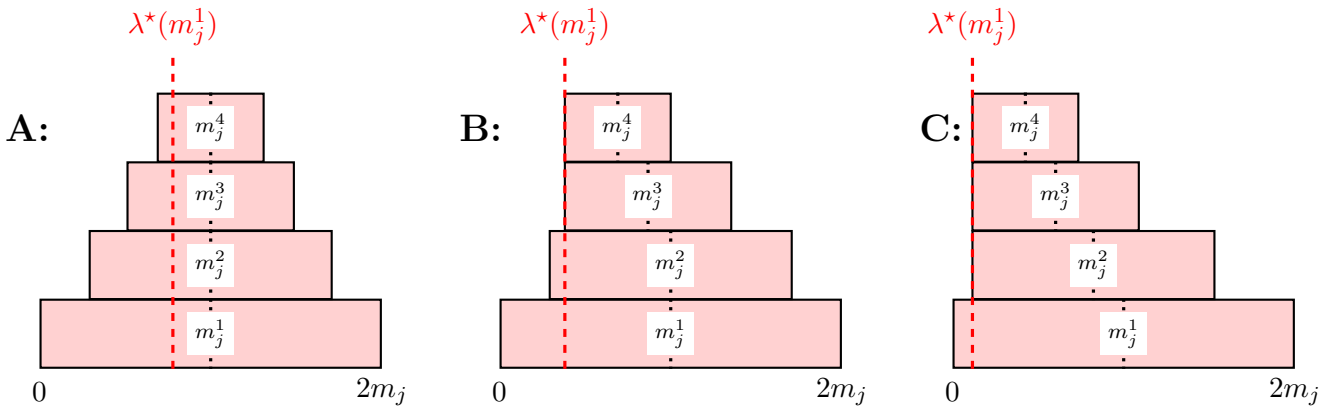


Figure 7 asd.

Figure 7 provides an overview of the three different outcomes which may arise in this multistage structure (for simplicity and without loss of generality, the multistage government shall consist of four boards). Outcome A on the left is the simplest: A region's median citizen creates a median preserving board structure by gradually dropping the most extreme board members in the process of delegation. This strategy is optimal for region j 's median citizen, if this median citizen's ideal representative is a viable option in the last board given that the median citizen reproduced herself as the median member in all superior boards. This fits to the argumentation above. Every citizen exhibits unique preferences, thus – if the unconstrained optimal behavior is a feasible option in the end (due to falling board size, the set of potential representatives is smaller after each delegation decision) – then the median citizen has an incentive to stay in her pivotal position to ensure her preferred type of representative. However, under some circumstances it is necessary for median members to install a median member of a different type than then themselves in the next board. Outcomes B and C resemble such scenarios, start with B: If the ideal representative of region j 's median

citizen is not a member of the last board, given the median member of this last board is the median citizen, then this median citizen has initiate an appropriate shift in the composition of the boards at some point in the chain of delegation. In the presentation given in figure 7, this shift happens when board two's median member (which is still the median citizen) has to determine board three's median member. Yet, since any type lower than the median citizen will favor a representative of even lower type than $\lambda_j^*(m_j^1)$, the median citizen has to prevent this by establishing her optimal representative as the lowest type in the created board. Further, since this newly created board's median member is of a lower type than the median citizen, there will be a continuing downward trend in the superordinate boards median members. This is caused by the fact that the lowest type $\lambda_j^*(m_j^1)$ is already a second-best solution for to median members of all superordinate boards which has to be maintained. Therefore only board members with the highest preferences for the public good are not transmitted to the next boards and the median citizen has ensured that the realized representative is of the type $\lambda_j^*(m_j^1)$. Outcome C is the most extreme version of such a pattern: Here, the median citizen has to leave her pivotal position immediately and will install a median member of board two who is of a lower preference than herself.

Note that there are mainly two factors which influence the emergence of these different outcomes: The position of $\lambda_j^*(m_j^1)$ in the overall population of region j – which is determined by d , the degree of spillovers – and the sizes of the boards. The depiction according to figure 7 is only based on the first of these two driving factors. If the median citizen's ideal representative is rather far away from the median citizen itself, what resembles large spillovers (a low value of d), the emergence of outcome A becomes more unlikely. The same effect can be achieved if the boards' size is very small, especially with respect to the last board, since this immediately lowers the chance that the median citizen's ideal representative is a viable option in the last board.

9 Conclusion

By arguing that local public goods which are provided in different regions can be seen as perfect substitutes, this paper provides an explanation for recent findings of empirical evaluations of efforts of decentralization around the world. The most important finding in this context is the one that efforts of decentralization were overall unable to improve the provision of local public goods. An explanation for this equivalence of centralized and decentralized provision structures is found in the interplay between the decision to delegate political power to a representative and the behavior of this representative: Citizens of a region engage in strategic delegation when it comes to electing the region's representative – meaning they delegate power to a representative who is not the median citizen in terms of preferences for the public good. The reasoning behind this behavior is the attempt to put the own region's representative in a more favorable bargaining position relative to the other region's representative to be better able to free-ride on the efforts of the other region. Importantly, the degree of strategic delegation is different in centralized and decentralized settings. This difference leads to the equivalence of the two political structures as it accounts for fiercer

(decentralized) or less fierce (centralized) competition between the representative. Independent from the mechanism of provision, strategic delegation always leads to representatives who favor lower provision levels than the median citizens. This is not surprising and caused by the substitutability of the local public goods. Introducing complementarity would likely lead to distortions to above median representatives (concerning the preference for the public good). However, as it would be the case for complementary public goods, strategic delegation always leads to welfare reductions. Lastly, the paper took a look on multistage governments. This was mainly motivated by the previous findings on strategic delegation concerning the delegation of power to non-median citizens: Multistage structures with multiple delegation decisions might exponentially increase the degree of strategic delegation and thus greatly increase welfare losses. However, this concern was not confirmed.

From a different perspective, the main findings of this work can be interpreted in an additional way: The distorting effects which arise in the provision of regional public goods – e.g. underprovision for decentralized provision, overprovision for cost sharing and so on – were thought to be caused by the specific designs of the decision mechanisms, cost schemes or else. This work suggests that there are inherent characteristics of goods which alone and quite independently from the surrounding political structure, create distortions or incentives for distorting behavior. Here this was the perfect substitutability of the local public goods which rendered representatives (or rather the preferences of the representative for the public good) to be substitutes and thus led to free riding not only on the representatives' stage but also in the appointment of the representatives. Next, as argued in section 7, the distinction between the concepts of centralized and decentralized provision of public goods appears to be unclear: Here, there was none. However, this can be explained by the lack of differences between the two formulations. An assumed change in behavior of the representatives is not sufficient to lead to different outcomes as is absorbed by changes in the delegation decisions. As the last section for centralized provision has shown, different political structures can be created (in the sense of different outcomes) and defined by the different competences they incorporate.

The main point of critique concerning the presented model might easily be the assumption of perfect substitutability of local public goods. The following two points can be made to address this issue: First, I do not claim that my formulation of the model is more general than previous attempts. Whether it is better suited to describe reality can be questioned. Furthermore, any explication why the proposed model is more appropriate than existing ones would be vulnerable to criticism because of the following reason: There exists a convincing story or example for nearly every imaginable kind of public good or service which illustrates why these goods – when provided in regionally distinct places – can be seen as perfect substitutes. However, nearly without effort, this story can be retold such that the understanding of imperfect substitutes seems more accurate. One of the few examples where this might not be possible is the case of global carbon dioxide reduction – as has been argued beforehand. The base for this ambivalence lays in the intuitive similarity of the two concepts of substitutability. Despite the difference between the concepts is clear from a technical point of view (constant marginal rate of substitution for perfect substitutes and a diminishing rate for imperfect substitutes), the two concepts appear to be fairly similar when placed in the context of real world scenarios. The second answer is related to the first one: Given the literature's stark focus on the

formulation of local public goods as imperfect substitutes – due to whatever reasons – this approach constitutes a counterbalance. This argument becomes even more appealing given the fact that the model used in this paper is able to reproduce well established knowledge stemming from the complementary formulation of the model. Hence when it comes to the richness of insights which can be drawn from the model, the present formulation can be considered to be superior. However, this comes at the price of mathematical complexity

Regarding future research, there are some points to start from. The empirical assessment of the driving mechanism of the paper (equivalence of centralization and decentralization due to the leveling effect of strategic delegation) would be very greatly complicated by the need for data on types or preferences of politicians and representatives. Furthermore, the following question would be whether delegates exhibit constant preferences such that they have to be replaced by more appropriate persons or if they are able to adjust their preferences (or rather their behavior) to electoral desires – one can easily agree to the latter. Due to these complications, a promising start might be the focus on the aspect of strategic delegation alone: Works like the one of [Hamman et al. \(2011\)](#) provide an experimental framework which can easily be extended to pursue the question under which circumstances voters engage in strategic delegation. Lastly, there could be an investigation which settings enhance this incentive for strategic delegation and which dissolve it.

APPENDIX

A Benchmark scenario – social planner

Optimization of the social planner:

$$\max_{g_1, g_2} W = m_1 \ln \left(g_1 + \frac{g_2}{1+d} \right) + m_2 \ln \left(g_2 + \frac{g_1}{1+d} \right) - (g_1 + g_2)$$

$$(1.1) \quad \frac{\partial W}{\partial g_1} = \frac{m_1}{g_1 + \frac{g_2}{1+d}} + \frac{m_2}{g_2 + \frac{g_1}{1+d}} \frac{1}{1+d} - 1 \stackrel{!}{\leq} 0 \quad (1.2) \quad g_1 \stackrel{!}{\geq} 0 \quad (1.3) \quad \frac{\partial W}{\partial g_1} g_1 \stackrel{!}{=} 0$$

$$(2.1) \quad \frac{\partial W}{\partial g_2} = \frac{m_1}{g_1 + \frac{g_2}{1+d}} \frac{1}{1+d} + \frac{m_2}{g_2 + \frac{g_1}{1+d}} - 1 \stackrel{!}{\leq} 0 \quad (2.2) \quad g_2 \stackrel{!}{\geq} 0 \quad (2.3) \quad \frac{\partial W}{\partial g_2} g_2 \stackrel{!}{=} 0$$

The second order conditions are negative such that every solution is a maximum:

$$\frac{\partial^2 W}{\partial g_1^2} = -\frac{m_1}{\left(g_1 + \frac{g_2}{1+d}\right)^2} - \frac{m_2}{\left(g_2 + \frac{g_1}{1+d}\right)^2} \frac{1}{(1+d)^2} < 0$$

$$\frac{\partial^2 W}{\partial g_2^2} = -\frac{m_1}{\left(g_1 + \frac{g_2}{1+d}\right)^2} \frac{1}{(1+d)^2} - \frac{m_2}{\left(g_2 + \frac{g_1}{1+d}\right)^2} < 0$$

Case 1: $g_1, g_2 > 0$

$$(1.1) \stackrel{!}{=} 0 \quad \rightarrow \quad \frac{m_1(1+d)}{g_1(1+d) + g_2} + \frac{m_2}{g_2(1+d) + g_1} = 1$$

$$(2.1) \stackrel{!}{=} 0 \quad \rightarrow \quad \frac{m_1}{g_1(1+d) + g_2} + \frac{m_2(1+d)}{g_2(1+d) + g_1} = 1$$

$$(1.1)=(2.1) \quad \rightarrow \quad \frac{m_1(1+d)}{g_1(1+d) + g_2} + \frac{m_2}{g_2(1+d) + g_1} = \frac{m_1}{g_1(1+d) + g_2} + \frac{m_2(1+d)}{g_2(1+d) + g_1}$$

$$\frac{m_1 d}{g_1(1+d) + g_2} = \frac{m_2 d}{g_2(1+d) + g_1}$$

$$[g_2(1+d) + g_1] \frac{m_1}{m_2} = g_1(1+d) + g_2$$

$$g_2 = g_1 \frac{(1+d) - \mu}{(1+d)\mu - 1} \equiv g_1 \Psi$$

$$\text{in (1.1)} \quad \rightarrow \quad 1 = \frac{m_1(1+d)}{g_1(1+d) + g_1 \Psi} + \frac{m_2}{g_1 \Psi(1+d) + g_1}$$

$$\begin{aligned}
g_1 &= \frac{m_1(1+d)}{1+d + \frac{(1+d)-\mu}{(1+d)^{\mu-1}}} + \frac{m_2}{\frac{(1+d)-\mu}{(1+d)^{\mu-1}}(1+d) + 1} \\
g_1 &= \frac{m_1(1+d)}{\frac{(1+d)[(1+d)\mu-1]+1+d-\mu}{(1+d)^{\mu-1}}} + \frac{m_2}{\frac{[(1+d)-\mu](1+d)+(1+d)\mu-1}{(1+d)^{\mu-1}}} \\
g_1 &= \left(\frac{m_1(1+d)}{\mu[(1+d)^2-1]} + \frac{m_2}{(1+d)^2-1} \right) [(1+d)\mu-1] \\
g_1 &= \frac{m_2(1+d) + m_2}{(2+d)d} [(1+d)\mu-1] \\
g_1 &= \frac{m_2[(1+d)\mu-1]}{d} \\
g_1 &= \frac{(1+d)m_1 - m_2}{d}
\end{aligned}$$

$$\begin{aligned}
g_1 \text{ in } g_2 = g_1 \Psi &\quad \rightarrow \quad g_2 = g_1 \frac{(1+d) - \mu}{(1+d)\mu - 1} = \frac{(1+d)m_1 - m_2}{d} \frac{(1+d) - \mu}{(1+d)\mu - 1} \\
&\quad g_2 = \frac{m_2[(1+d)\mu - 1]}{d} \frac{(1+d) - \mu}{(1+d)\mu - 1} \\
&\quad g_2 = \frac{m_2}{d} [(1+d) - \mu] \\
&\quad g_2 = \frac{(1+d)m_2 - m_1}{d}
\end{aligned}$$

The conditions (1.1) to (2.3) are all fulfilled. g_1 is positive if $(1+d)m_1 > m_2$ what is always true. g_2 is positive for $(1+d)m_2 > m_1$ what can be simplified to $1+d > \mu$.

Case 2: $g_1 > 0, g_2 = 0$

$$\begin{aligned}
(1.1) \stackrel{!}{=} 0 &\quad \rightarrow \quad \frac{m_1}{g_1} + \frac{m_2}{g_1} = 1 &\quad \rightarrow \quad g_1 = m_1 + m_2 \\
(2.1) \stackrel{!}{\leq} 0 &\quad \rightarrow \quad \frac{m_1}{g_1(1+d)} + \frac{m_2(1+d)}{g_1} \leq 1 \\
&\quad m_1 + (1+d)^2 m_2 \leq (1+d)(m_1 + m_2) \\
&\quad \mu + (1+d)^2 \leq (1+d)(\mu + 1) \\
&\quad (1+d)^2 - (1+d) \leq (1+d)\mu - \mu \\
&\quad d(1+d) \leq d\mu &\quad \rightarrow \quad 1+d \leq \mu
\end{aligned}$$

All conditions are fulfilled. Thus the above solution is true for $1+d \leq \mu$.

Case 3: $g_1 = 0, g_2 > 0$

$$(2.1) \stackrel{!}{=} 0 \quad \rightarrow \quad \frac{m_1}{g_2} + \frac{m_2}{g_2} = 1 \quad \rightarrow \quad g_2 = m_1 + m_2$$

$$\begin{aligned}
(1.1) \stackrel{!}{\leq} 0 & \quad \rightarrow \quad \frac{m_1(1+d)}{g_2} + \frac{m_2}{g_2(1+d)} \leq 1 \\
& \quad m_1(1+d)^2 + m_2 \leq (1+d)(m_1 + m_2) \\
& \quad \mu(1+d)^2 + 1 \leq (1+d)(\mu + 1) \\
& \quad \mu(1+d)[(1+d) - 1] \leq (1+d) - 1 \\
& \quad \mu(1+d) \leq 1 \quad \rightarrow \quad \zeta
\end{aligned}$$

Since $\mu > 1$ and $d \geq 0$, the above condition can never be true such that the present assumption does not lead to a solution.

Case 4: $g_1 = g_2 = 0$

Due to logarithmic utility from the public good, this is never optimal. For $g_1, g_2 \rightarrow 0$, the left hand sides of both (1.1) and (2.1) converge towards infinity and therefore violate these conditions.

Solution:

$$g_1^*(m_1, m_2) = \begin{cases} \frac{(1+d)m_1 - m_2}{d} & \text{for } 1+d > \mu \\ m_1 + m_2 & \text{for } 1+d \leq \mu \end{cases} \quad \text{and} \quad g_2^*(m_1, m_2) = \begin{cases} \frac{(1+d)m_2 - m_1}{d} & \text{for } 1+d > \mu \\ 0 & \text{for } 1+d \leq \mu \end{cases}$$

B Decentralization with representatives

B.1 Behavior of competitive representatives

Exchange λ_j by m_j (and analogously $\lambda_{-j} = m_{-j}$) to obtain the behavior of the median citizens, if they are assumed to be the representatives (for decentralized provision without representatives if the median citizens' votes are pivotal). The optimization problem of region j 's representative who is of type λ_j given g_{-j} is

$$\begin{aligned}
\max_{g_j} u_j &= \lambda_j \ln \left(g_j + \frac{g_{-j}}{1+d} \right) - g_j & \text{such that} & & g_j \geq 0. \\
(1.1) \quad \frac{\partial u_j}{\partial g_j} &= \frac{\lambda_j}{g_j + \frac{g_{-j}}{1+d}} - 1 \stackrel{!}{\leq} 0 & (1.2) \quad g_j &\geq 0 & (1.3) \quad \frac{\partial u_j}{\partial g_j} g_j \stackrel{!}{=} 0
\end{aligned}$$

Case 1: $g_j > 0$

$$(1.1) \stackrel{!}{=} 0 \quad \rightarrow \quad \frac{\lambda_j}{g_j + \frac{g_{-j}}{1+d}} - 1 \stackrel{!}{=} 0 \quad \rightarrow \quad \frac{\lambda_j(1+d)}{g_j(1+d) + g_{-j}} = 1 \quad \rightarrow \quad g_j = \lambda_j - \frac{g_{-j}}{1+d}$$

All conditions are satisfied. g_j is positive for $\lambda_j > \frac{g_{-j}}{1+d}$ or better $\lambda_j(1+d) > g_{-j}$.

Case 2: $g_j = 0$

$$(1.1) \stackrel{!}{\leq} 0 \quad \rightarrow \quad \frac{\lambda_j}{\frac{g_{-j}}{1+d}} - 1 \stackrel{!}{\leq} 0 \quad \rightarrow \quad \frac{\lambda_j(1+d)}{g_{-j}} \leq 1 \quad \rightarrow \quad \lambda_j(1+d) \leq g_{-j}$$

For $\lambda_j(1+d) \leq g_{-j}$, the assumption $g_j = 0$ satisfies all conditions and is a solution.

Best response function of region j ' representative:

$$g_j(g_{-j}, \lambda_j, d) = \begin{cases} \lambda_j - \frac{g_{-j}}{1+d} & \text{for } \lambda_j(1+d) > g_{-j} \\ 0 & \text{for } \lambda_j(1+d) \leq g_{-j} \end{cases}$$

Next, consider the intersections of the two best response functions $g_j(g_{-j})$ ($j = 1, 2$) to determine the Nash equilibria in the stage of the representatives:

Case 1: $g_1, g_2 > 0$

$$\begin{aligned} g_j = \lambda_j - \frac{g_{-j}}{1+d} & \quad \rightarrow \quad g_j = \lambda_j - \left(\lambda_{-j} - \frac{g_j}{1+d} \right) \frac{1}{1+d} \\ g_j & = \lambda_j - \frac{\lambda_{-j}}{1+d} + \frac{g_j}{(1+d)^2} \\ g_j \left(1 - \frac{1}{(1+d)^2} \right) & = \frac{\lambda_j(1+d) - \lambda_{-j}}{(1+d)} \\ g_j & = \frac{\lambda_j(1+d) - \lambda_{-j}}{(1+d)} \times \frac{(1+d)^2}{(2+d)d} \quad \rightarrow \quad g_j = \frac{[\lambda_j(1+d) - \lambda_{-j}](1+d)}{(2+d)d} \end{aligned}$$

These values g_j are strictly positive for $\lambda_j(1+d) > \lambda_{-j}$. Inserting $j = 1, 2$ and defining $L \equiv \frac{l_1}{l_2}$ leads to the condition $1+d > L > \frac{1}{1+d}$.

Case 2: $g_j > 0, g_{-j} = 0$

$$g_j = \lambda_j \quad , \quad g_{-j} = 0 \quad \text{for} \quad \lambda_{-j}(1+d) \leq g_j \quad \rightarrow \quad \lambda_{-j}(1+d) \leq \lambda_j$$

For $j = 1$, the condition which ensures that region 2 will provide nothing is $1+d \leq L$. For $j = 2$, region 1 will not provide the public good for $L \leq \frac{1}{1+d}$.

Case 3: $g_1 = g_2 = 0$

This would only be possible for $l_1 = l_2 = 0$. However, as this has been excluded, this can never constitute a Nash equilibrium.

Solution:

Dependent on λ_j and λ_{-j} , region j can either be *idle*, can provide the public good *jointly* with the other

region or provide the public good *alone*:

$$g_j(\lambda_j, \lambda_{-j}) = \begin{cases} g_j^{\text{idle}} = 0 & \text{for } \lambda_j \leq \frac{\lambda_{-j}}{1+d} \\ g_j^{\text{joint}} = \frac{[\lambda_j(1+d) - \lambda_{-j}](1+d)}{(2+d)d} & \text{for } \frac{\lambda_{-j}}{1+d} < \lambda_j < \lambda_{-j}(1+d) \\ g_j^{\text{alone}} = \lambda_j & \text{for } \lambda_{-j}(1+d) \leq \lambda_j \end{cases}$$

B.2 Utility of median citizens (lemma 7)

The utility of region j 's median citizen in the three scenarios obtained in the last section is the following:

$$u_j(\lambda_j, \lambda_{-j}) = \begin{cases} u_j^{\text{idle}} = u_j(g_j^{\text{idle}}, g_{-j}^{\text{alone}}) = m_j \ln\left(\frac{\lambda_{-j}}{1+d}\right) & , \lambda_j \leq \frac{\lambda_{-j}}{1+d} \\ u_j^{\text{joint}} = u_j(g_j^{\text{joint}}, g_{-j}^{\text{joint}}) = m_j \ln(\lambda_j) - \frac{[\lambda_j(1+d) - \lambda_{-j}](1+d)}{(2+d)d} & , \frac{\lambda_{-j}}{1+d} < \lambda_j < \lambda_{-j}(1+d) \quad (\text{UD}) \\ u_j^{\text{alone}} = u_j(g_j^{\text{alone}}, g_{-j}^{\text{idle}}) = m_j \ln(\lambda_j) - \lambda_j & , \lambda_{-j}(1+d) \leq \lambda_j \end{cases}$$

The first-best representatives of region j 's median citizen are found with the help of the following (three) maximization problem(s):

$$\begin{aligned} \max_{\lambda_j} u_j^s(\lambda_j, \lambda_{-j}) \quad \text{s.t.} \quad & \lambda_j \text{ is feasible,} \quad \text{for } s = \text{idle, joint, alone} \\ \rightarrow \quad \lambda_j^{\text{idle}} = \text{any } \lambda_j \quad , \quad & \rightarrow \quad \lambda_j^{\text{joint}} = m_j \frac{(2+d)d}{(1+d)^2} \quad , \quad \rightarrow \quad \lambda_j^{\text{alone}} = m_j \end{aligned}$$

One has to refer to these optimal representatives as first-best, since they do not necessarily (in combination with λ_{-j}) induce the assumed scenario. Therefore, whenever the representative λ_j^s does not induce scenario s , the second-best representative (if one wants to induce scenario s) is given by the representative whose type is as close as possible to the first-best type λ_j^s but does still induce scenario s (a corner solution). This is correct since $u_j(\lambda_j, \lambda_{-j})$ is concave in λ_j (or linear respectively “flat” for region j being idle). The ideal representatives (IR) for region j 's median citizen given λ_{-j} under the assumption that a certain scenario has to be established are the following:

$$\lambda_j^* = \begin{cases} s \stackrel{!}{=} \text{idle} \rightarrow \lambda_j^{\text{idle}*} = \text{any } \lambda_j \quad \text{with } \lambda_j \leq \frac{\lambda_{-j}}{1+d} \\ s \stackrel{!}{=} \text{joint} \rightarrow \lambda_j^{\text{joint}*} = \begin{cases} \frac{\lambda_{-j}}{1+d} & \text{for } m_j \frac{(2+d)d}{(1+d)^2} \leq \frac{\lambda_{-j}}{1+d} \quad (\text{second-best, } m_j \text{ too low}) \\ m_j \frac{(2+d)d}{(1+d)^2} & \text{for } \frac{\lambda_{-j}}{1+d} < m_j \frac{(2+d)d}{(1+d)^2} < \lambda_{-j}(1+d) \quad (\text{first-best}) \\ \lambda_{-j}(1+d) & \text{for } \lambda_{-j}(1+d) \leq m_j \frac{(2+d)d}{(1+d)^2} \quad (\text{second-best, } m_j \text{ too high}) \end{cases} \\ s \stackrel{!}{=} \text{alone} \rightarrow \lambda_j^{\text{alone}*} = \begin{cases} \lambda_{-j}(1+d) & \text{for } \lambda_{-j}(1+d) > m_j \quad (\text{second-best, } m_j \text{ too low}) \\ m_j & \text{for } \lambda_{-j}(1+d) \leq m_j \quad (\text{first-best}) \end{cases} \end{cases} \quad (\text{IR})$$

To somehow approach a more general best-response function $\lambda_j(\lambda_{-j})$, the question is which scenario should be induced given λ_{-j} . The resulting median utility levels for the above representatives are the following (the same separating conditions as above are true):

$$u_j^* = \begin{cases} s \stackrel{!}{=} \text{idle} \rightarrow & u_j^{\text{idle}^*}(\lambda_{-j}, m_j) = m_j \ln \left(\frac{\lambda_{-j}}{1+d} \right) \quad \text{with } \lambda_j \leq \frac{\lambda_{-j}}{1+d} \\ s \stackrel{!}{=} \text{joint} \rightarrow & u_j^{\text{joint}^*}(\lambda_{-j}, m_j) = \begin{cases} m_j \ln \left(\frac{\lambda_{-j}}{1+d} \right) \\ m_j \ln \left(m_j \frac{(2+d)d}{(1+d)^2} \right) - m_j + \lambda_{-j} \frac{1+d}{(2+d)d} \\ m_j \ln (\lambda_{-j}(1+d)) - \lambda_{-j}(1+d) \end{cases} \\ s \stackrel{!}{=} \text{alone} \rightarrow & u_j^{\text{alone}^*}(\lambda_{-j}, m_j) = \begin{cases} m_j \ln (\lambda_{-j}(1+d)) - \lambda_{-j}(1+d) \\ m_j \ln (m_j) - m_j \end{cases} \end{cases} \quad (\text{UD}^*)$$

This representation allows the finding that optimal median utility is – despite the partially defined structure – continuous at the points of transition. This can be further extended to two important mechanisms. First, whenever a region is the sole provider of the public good but cannot elect the first-best representative of this scenario ($\lambda_{-j}(1+d) > m_j$ is true), this region can strictly increase median utility by switching to joint provision. Due to $\lambda_{-j}(1+d) > m_j$, $\lambda_{-j}(1+d) \leq m_j \frac{(2+d)d}{(1+d)^2}$ cannot be true. This means that the upper second-best representative of the joint scenario cannot even be second-best. However, electing a representative of this type leads to the same median utility. Due to this knowledge one can conclude that the optimal representative of the joint provision scenario – which is either the lower second-best type of the unconstrained first-best type – must lead to a strictly higher median utility¹⁴.

Second, the utility of region j 's median citizen from joint provision cannot be smaller than the utility if this citizen's region is idle. Similar to the previous mechanism, this is due to the fact that the (lower) corner solution $\lambda_j^{\text{joint}^*} = \frac{\lambda_{-j}}{1+d}$ which is always feasible, leads to the same utility as in the idle scenario. Furthermore, this implies that whenever an idle region can induce joint provision with the first-best representative of the joint scenario, it can strictly raise median utility by doing so (since the unconstrained optimum yields strictly higher utility than the corner solution). These findings are captured by lemma 7:

Lemma 7 *A pair of representatives which induces provision of the public good by one region alone cannot be a pair of mmp representatives, if the providing region's representative is not of the first-best type of the alone scenario, or if the idle region can induce joint provision with the first-best representative of the joint scenario.*

¹⁴Given λ_{-j} and $\lambda_j = \lambda_{-j}(1+d)$ ($\lambda_{-j}(1+d) > m_j$ is true), a more mathematical formulation of this property is the following: Region j sets $\lambda_j = \varepsilon \lambda_{-j}(1+d)$ with ε close to but below zero. Joint provision is implied and the following is true:

$$u_j^{\text{joint}} = m_j \ln (\varepsilon \lambda_{-j}(1+d)) - \frac{[\varepsilon \lambda_{-j}(1+d)^2 - \lambda_{-j}](1+d)}{(2+d)d} \quad \text{and} \quad \left. \frac{\partial u_j^{\text{joint}}}{\partial \varepsilon} \right|_{\varepsilon=1} = m_j - \overbrace{\lambda_{-j}(1+d)}^{> m_j} \frac{(1+d)^2}{(2+d)d} < 0$$

Hence (around $\varepsilon = 1$, the point of transition from alone to joint) further lowering the representative's type leads to higher utility.

B.3 Proof of lemma 1

Lemma 1 *There exists no pair of mmp representatives such that region two provides the public good alone. Or equivalent: There exists no pair of mmp representatives such that region one is idle.*

If region two is the sole provide by electing the second-best representative of type $\lambda_2^{\text{alone}^*} = \lambda_1(1+d)$, we know due to lemma 1 that this representative can never be an element of a pair of mmp representatives. Hence consider the scenario in which region two is the sole provider with the first-best representative $\lambda_2^{\text{alone}^*} = m_2$. Region one obtains a median utility of $u_1^{\text{idle}^*} = m_1 \ln\left(\frac{m_2}{1+d}\right)$. By electing a representative of type $\lambda_1^{\text{alone}^*} = \lambda_{-2}(1+d) = m_2(1+d)$, region one realizes a median utility of $u_1^{\text{alone}^*} = m_1 \ln(m_2(1+d)) - m_2(1+d)$. This figure is always strictly larger than the utility given region one is idle:

$$\begin{aligned} \Delta u_1 &= u_1^{\text{alone}^*} - u_1^{\text{idle}^*} = m_1 \ln(m_2(1+d)) - m_2(1+d) - m_1 \ln\left(\frac{m_2}{1+d}\right) \\ &= m_1 \ln\left((1+d)^2 \exp\left(\frac{\mu}{1+d}\right)\right) > 1 \end{aligned}$$

Hence for the case of region two being the sole provider with representative $\lambda_2^{\text{alone}^*} = m_2$, region one always faces an incentive to deviate – note that the deviation shown here is not necessarily the best deviation available since it is based on a second-best representative.

B.4 Proof of theorem 1

Theorem 1 – mmp representatives for rather similar regions and/or small spillovers:

Given $\mu < \frac{(2+d)d}{1+d}$, the unique pair of (pure) mmp representatives is given by $\lambda_j^{\text{joint}^} = m_j \frac{(2+d)d}{(1+d)^2}$ (for $j = 1, 2$) such that both regions provide a positive amount of the public good. [Proof: B.4]*

Theorem 1 is proven with the help of three lemmas. Lemmas 2 and 3 prove that the pair of representatives is a pair of mmp representatives and lemma 4 shows this pair's uniqueness in the considered parameter space.

B.4.1 Proof of lemma 2

Lemma 2 *Given $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ (for $i = 1, 2$) and $\mu < 1+d$, the regions provide the public good jointly and no median citizen has an incentive to lower her representative's type such that the other region will become the sole supplier of the public good.*

The representatives considered in lemma 2 imply joint provision: First, $L = \mu$ is true, furthermore $\frac{1}{1+d} <$

$L < 1 + d$ is true by assumption¹⁵. Both regions obtain a median utility of

$$u_j^{\text{joint}} = m_j \ln \left(m_j \frac{(1+d)^2 - 1}{(1+d)^2} \right) - \frac{m_j(1+d) - m_{-j}}{1+d}.$$

By sufficiently lowering their representative's type, a region can force the other region to become the sole supplier of the public good and obtain a median utility of u'_j . The difference in region j 's median utility when engaging in such an action is then given by

$$\begin{aligned} \Delta'_j &= \overbrace{m_j \ln \left(m_{-j} \frac{(1+d)^2 - 1}{(1+d)^3} \right)}^{u'_j, \text{ region } j \text{ idle}} - \overbrace{\left(m_j \ln \left(m_j \frac{(1+d)^2 - 1}{(1+d)^2} \right) - \frac{m_j(1+d) - m_{-j}}{1+d} \right)}^{u_j^{\text{joint}}} \\ &= m_j \ln \left(\underbrace{\frac{m_{-j}}{m_j} \frac{1}{1+d} \exp \left[\frac{(1+d)m_j - m_{-j}}{(1+d)m_j} \right]}_{\delta'_j} \right). \end{aligned}$$

Let δ'_j denote the argument of the logarithm. Δ'_j will be positive if this argument is larger than one. From the point of view of the two regions, δ'_j takes on the forms

$$\delta'_1 = \frac{1}{\mu(1+d)} \exp \left[1 - \frac{1}{\mu(1+d)} \right] \quad \text{and} \quad \delta'_2 = \frac{\mu}{1+d} \exp \left[1 - \frac{\mu}{1+d} \right].$$

Maximizing these expressions or using computational methods reveals that δ'_1 is maximized at $\mu = 1$ and $d = 0$ and δ'_2 at $\mu = 1 + d$. However, the value of δ'_1 and δ'_2 at these extrema is one, thus Δ'_j is bounded above by zero for both regions, see figure 8 for an illustration. Figure 8 also shows that despite no region has an incentive to make the other region the sole supplier, this is less harmful for region 2 to do – what is obviously due to the lower preferences for the public good in this region.

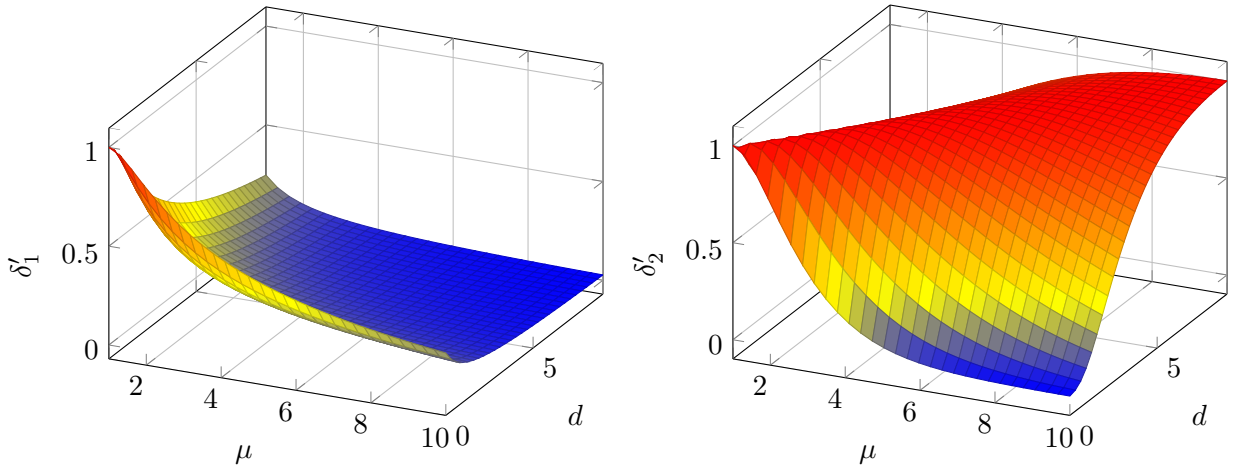


Figure 8 Curvature of δ'_1 and δ'_2 . Both never exceeding one.

¹⁵The restriction imposed by the lower bound is always fulfilled since $\mu > 1$.

B.4.2 Proof of lemma 3

Lemma 3 Given $\lambda_j = m_j \frac{(2+d)d}{(1+d)^2}$ (for $i = 1, 2$) and $\mu < \frac{(2+d)d}{1+d}$, the regions provide the public good jointly and no median citizen has an incentive to increase her representative's type such that her home region will become the sole supplier of the public good.

The condition $\mu < \frac{(2+d)d}{1+d}$ is slightly stricter than $\mu < 1 + d$ from lemma 2 and defines a subset of the parameter combinations allowed in lemma 2¹⁶. Thus the representatives lead to joint provision. Both regions can increase their representative's type such that they become the sole provider of the public good and obtain utility u_j'' . Assume that both regions can achieve this with the respective first-best representative ($\lambda_1^{\text{alone}^*} = m_1$ is high enough to make region one the sole provider and $\lambda_2^{\text{alone}^*} = m_2$ makes region two the sole provider). For this assumption to be true the following needs to hold:

$$\text{region one becoming the sole provider (with } m_1): \quad L \geq 1 + d \quad \rightarrow \quad \mu \geq \frac{(2+d)d}{1+d} \quad (\text{B.1})$$

$$\text{region two becoming the sole provider (with } m_2): \quad L \leq \frac{1}{1+d} \quad \rightarrow \quad \mu \leq \frac{1+d}{(2+d)d} \quad (\text{B.2})$$

Both of these inequalities constitute upper bounds for μ given d (or lower bounds for d given μ). Furthermore, B.2 is fulfilled whenever B.1 is true for all $\mu \geq 1$ and can be dropped. The difference in region j 's median utility from becoming the sole provider is given by

$$\begin{aligned} \Delta_j'' &= \overbrace{m_j \ln(m_j) - m_j}^{u_j'', \text{ region } j \text{ sole provider}} - \overbrace{\left(m_j \ln \left(m_j \frac{(1+d)^2 - 1}{(1+d)^2} \right) - \frac{m_j(1+d) - m_{-j}}{1+d} \right)}^{u_j^{\text{joint}}} \\ &= m_j \ln \left(\underbrace{\frac{(1+d)^2}{(1+d)^2 - 1} \exp \left[-\frac{m_{-j}}{m_j(1+d)} \right]}_{\delta_j''} \right). \end{aligned}$$

Δ_j'' is positive if δ_j'' exceeds one. In contrast to the proof of lemma 2, there actually exist parameter combinations which yield an incentive to become the sole supplier for both regions. Refer to figures 9 and 10 for the curvature of δ_1'' and δ_2'' as well as the implied incentives.

The parameter combinations which imply $\delta_1'' > 1$ or $\delta_2'' > 1$ are colored red in the right parts of figures 9 and 10. First consider the incentives of region two. Above the black line (equation B.2) region two cannot become the sole provider with the first-best representative m_2 – however, the region does not even have an incentive to do so if it could do this. Thus – by considering this deviation which is actually too good and infeasible – one can discard any motive of region two to become the sole provider above the black line. Second, look at region one's incentives. Again, region one cannot become the sole provider with the first-best representative m_1 above the black line (equation B.1) and has – like region two – not even an incentive to do

¹⁶Note: $\frac{(2+d)d}{1+d} < 1 + d$ since $(1+d)^2 - 1 < (1+d)^2 \rightarrow (2+d)d > (1+d)^2 \rightarrow \frac{(2+d)d}{1+d} < 1 + d$.

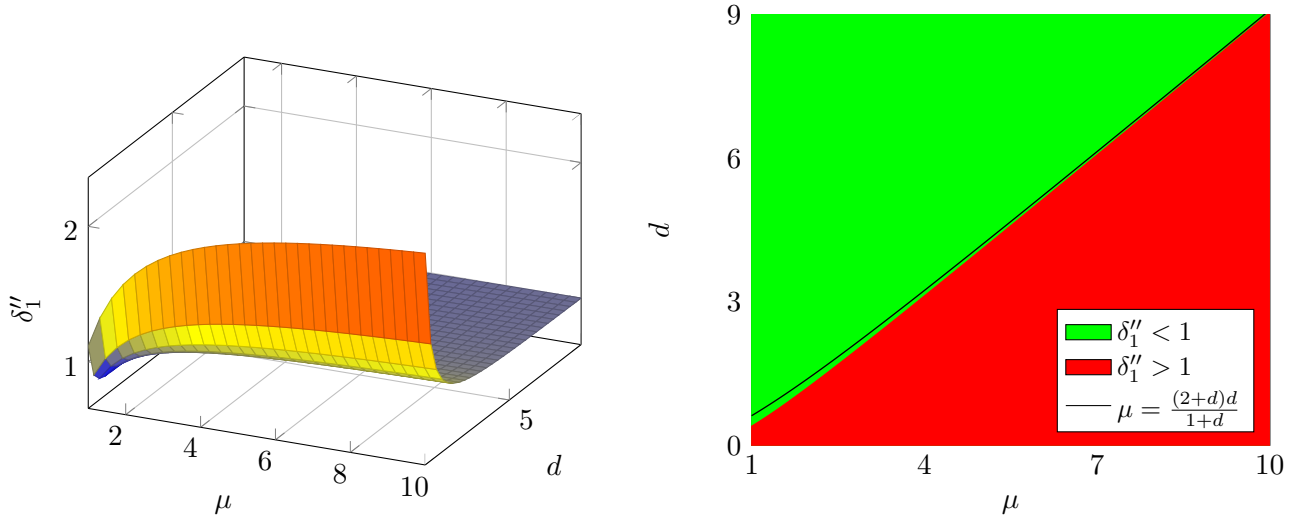


Figure 9 Curvature of δ_1'' (left) and the implied incentives via the top-view (right).

so if it could become the sole provider by electing the representative m_1 . To rule out all scenarios in which region one actually faces an incentive to become the sole provider and this is achievable with the first-best representative m_1 , lemma 3 only considers the parameter combinations $\mu < \frac{(2+d)d}{1+d}$ (above the black line). This assumption also eliminates all incentives of region two to become the sole provider.

Note that the assumption $\mu < \frac{(2+d)d}{1+d}$ is a conservative approximation of the parameter space where the considered representatives constitute a pair of mmp representatives: Since there exists no closed form solution of the boundary between the red and green are in figure 9, and due to the perfect match to the requirement of lemma 4, I take $\mu < \frac{(1+d)^2-1}{1+d}$ as a strict approximation of this boundary.

B.4.3 Proof of lemma 4

Lemma 4 *Given $\mu < \frac{(2+d)d}{1+d}$, there exists no pair of mmp representatives which implies that a region provides the public good alone.*

Due to lemma 1 it is clear that region two cannot be the sole provider in any possible pair of mmp representatives. Thus consider the scenario in which region one is the sole provider of the public good. Whenever $\lambda_1 \geq \lambda_2(1+d)$ is true, region one is the sole provider. The first-best representative for region one's median citizen whenever the region provides the public good alone is $\lambda_1^{\text{alone}^*} = m_1$. However, if λ_2 is too large (larger than $\frac{m_1}{1+d}$), the optimal representative (assuming that region one wants to be the sole provider) is given by the second-best solution $\lambda_1^{\text{alone}^*} = \lambda_2(1+d)$ (see equation IR).

At first, assume $\lambda_2 < \frac{m_1}{1+d}$ such that $\lambda_1^{\text{alone}^*} = m_1$ is feasible. Given these representatives, region two has an incentive to induce joint provision by electing a candidate of type $\lambda_2^{\text{joint}^*} = m_2 \frac{(2+d)d}{(1+d)^2}$. This representative leads to joint provision whenever $\frac{1}{1+d} < L < 1+d$ is true. This condition can be reduced in the following

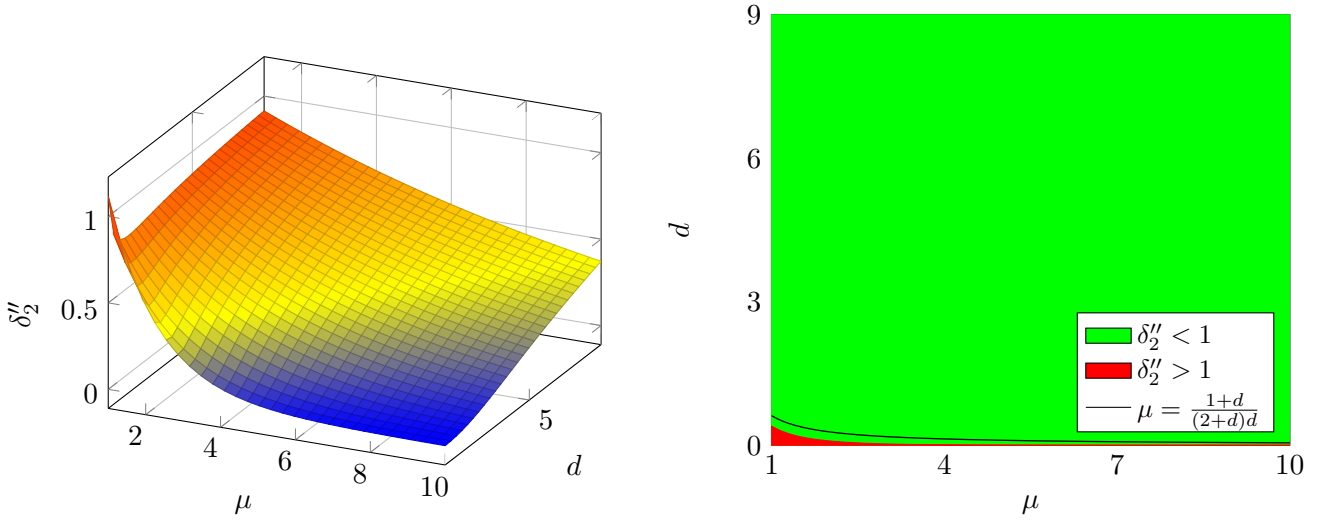


Figure 10 Curvature of δ_2'' (left) and the implied incentives via the top-view (right).

way:

$$\frac{1}{1+d} < L < 1+d \quad \rightarrow \quad \frac{1}{1+d} < \mu \frac{(1+d)^2}{(2+d)d} < 1+d$$

Since $\mu \frac{(1+d)^2}{(2+d)d}$ is larger than one, the left inequality is always fulfilled. Additionally, the right inequality can be rewritten as $\mu < \frac{(2+d)d}{(1+d)^2}$. Note that this is the same requirement as in lemma 3 and that this condition is always true since $\mu < \frac{(2+d)d}{1+d}$ is assumed in lemma 4. The rise in region two's median utility from inducing joint provision with the first-best representative is given by

$$\begin{aligned} \Delta_2''' &= \overbrace{u_2\left(\lambda_1 = m_j, \lambda_2 = m_2 \frac{(2+d)d}{(1+d)^2}\right)}^{\text{joint provision}} - \overbrace{u_2\left(\lambda_1 = m_1, \lambda_2 < \frac{m_1}{1+d}\right)}^{\text{provision solely by region one}} \\ &= m_2 \ln\left(m_2 \frac{(2+d)d}{(1+d)^2}\right) - m_2 + m_1 \frac{1+d}{(2+d)d} - m_2 \ln\left(\frac{m_1}{1+d}\right) \\ &= m_2 \ln\left(\underbrace{\frac{(2+d)d}{\mu(1+d)} \exp\left[\frac{\mu(1+d)}{(2+d)d} - 1\right]}_{\delta_2'''}\right) \end{aligned}$$

Using a numerical approach or minimizing reveals that δ_2''' approaches the value of one for growing values of d and falling values of μ (see figure 11). However, it is always larger than one. Therefore – whenever region one is the sole provider with a representative of type $\lambda_1 = m_1$ – region two always faces an incentive to induce joint provision by setting $\lambda_2 = m_2 \frac{(2+d)d}{(1+d)^2}$ (what requires $\mu < \frac{(2+d)d}{(1+d)^2}$ to hold).

Now assume $\lambda_2 \geq \frac{m_1}{1+d}$. Accordingly, the second-best representative of type $\lambda_1^{\text{alone}^*} = \lambda_2(1+d)$ makes region one the sole supplier while maximizing the utility of the region's median citizen (within the scenario of sole provision). Yet, due to lemma 1, whenever a region is the sole provider with the second-best representative,

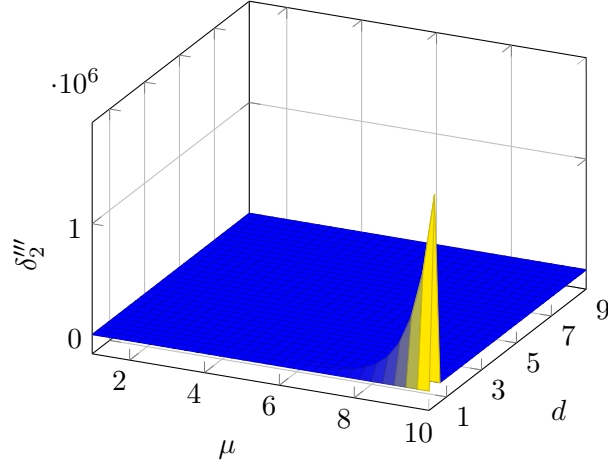


Figure 11 Curvature of δ_1''' and δ_2''' . Both never falling below one.

this region faces an incentive to induce a transition towards joint provision.

B.5 Proof of theorem 2

Theorem 2 *Given $\mu > \frac{(2+d)d}{1+d}$, there exists a continuum of mmp representatives characterized by $\lambda_1 = m_1$ and $\lambda_2 \in [0, \varepsilon_2^* m_2]$ which leads to sole provision by region one for every pair of mmp representatives in the continuum. ε_2^* depends on the parameters μ and d such that there exists a continuum of mmp representatives of different size for all possible combinations of μ and d .*

Due to lemma 1 we know that region two will never provide the public good alone. Lemma 7 states that any region providing the public good alone must do this with the first-best representative of this scenario to eliminate incentives to deviate. Hence the last possible pair of mmp representatives is $\lambda_1^{\text{alone}^*} = m_1$ and $\lambda_2^{\text{idle}^*} < \frac{m_1}{1+d}$ – what actually constitutes a continuum of mmp pairs or representatives. Lemmas 5 and 6 establish this continuum as pairs of mmp representatives by ruling out incentives for deviation from it (lemma 5 from the perspective of region two and lemma 6 from the perspective of region one). Furthermore, note that the outcomes of these pairs of mmp representatives are identical and given by sole provision by region one. Additionally, this outcome is unique for all parameter combinations of μ and d “below” the border for which there existed no closed form solution and was obtained in the proof of lemma 3 (see subsection B.4.2).

B.5.1 Proof of lemma 5

Lemma 5 *Given $\lambda_1 = m_1$, $\lambda_2 \leq \frac{m_1}{1+d}$ and $\mu > \frac{(2+d)d}{1+d}$, region one is the sole provider of the public good and region two’s median citizen has no incentive to induce a different scenario.*

Referring to IR immediately shows that region one provides the public good solely while region two is idle. Region two's median citizen can either force a transition towards joint provision or even make region two the sole provider of the public good itself.

First consider the first option: The first-best representative for joint provision of region two is $\lambda_2^{\text{joint}^*} = m_2 \frac{(2+d)d}{(1+d)^2}$. Given the representative of region one, this type of representative induces joint provision if $\frac{m_1}{1+d} < m_2 \frac{(2+d)d}{(1+d)^2} < m_1(1+d)$ holds (see IR). The inequalities can be reduced to $\mu < \frac{(2+d)d}{1+d} < \mu(1+d)^2$. Obviously, the right inequality is always true – region two's median citizen's type can never be too high such that the ideal representative of joint provision is the upper, second-best representative. Yet it might be too small. For $\frac{(2+d)d}{1+d} > \mu$ – what has been assumed – region two can only induce joint provision with the second-best representative. However – see UD* – this yields the same utility as in the scenario where region two is idle. Thus region two has no incentive to induce joint provision.

Now consider the second option, region two moving to sole provision and rendering region one idle. Given $\lambda_1 = m_1$, the first-best representative of region two of the alone scenario m_2 is obviously too small to actually induce sole provision by region two¹⁷. Hence region two would have to elect the second-best representative $\lambda_2^{\text{alone}^*} = m_1(1+d)$. This option can be ruled out too: Assume that region two elects this representative and becomes the sole supplier. According to UD*, region two obtains the same utility as when moving to joint provision by marginally lowering its representative's type. However, the utility obtained after this deviation is not the maximally achievable in the joint scenario: Since $\mu > \frac{(2+d)d}{1+d}$, the best representative for the joint case is the (lower) corner solution $\lambda_2^{\text{joint}^*} = \frac{m_1}{1+d}$. As argued above, this representative leads to the same utility as in the initial, idle scenario. Thus, thinking backward, the utility in the idle scenario is strictly larger than the utility region two achieves in joint provision when electing the (not even second-best) representative $\lambda_2 = m_1(1+d)$ – and this level of utility is the same as the one region two obtains when providing the public good alone. Thus region two cannot increase median utility by providing the public good alone.

B.5.2 Proof of lemma 6

Lemma 6 *Given $\mu > \frac{(2+d)d}{1+d}$ and $\lambda_1 = m_1$, there exists a critical representative of region two $\lambda_2 = \varepsilon_2^* m_2$ characterized by the parameter ε_2^* (which is conditional on μ and d) such that region one will be the sole provider of the public good and does not face an incentive to induce a different scenario given $\lambda_2 \leq \varepsilon_2 m_2$.*

For $\lambda_1 = m_1$ and $\lambda_2 \leq \frac{m_1}{1+d}$ the representatives lead to sole provision of region one. Assume that this is the case. Furthermore, let λ_2 be given by $\varepsilon_2 m_2$ with $\varepsilon_2 \in [0, 1]$ and obviously $\varepsilon_2 m_2 \leq \frac{m_1}{1+d}$. Hence the type of region two's representative is characterized as a fraction of the region's median citizen's type. Now, region one's median citizen can either force a transition to joint provision or even make region two the sole provider. First consider the first option: Given $\lambda_2 = \varepsilon_2 m_2$, region one can either induce joint provision with

¹⁷Since $m_2 > m_1(1+d)$ can never be true.

the first-best representative of this scenario or has to elect one of the two second-best representatives of joint provision:

$$\begin{aligned}
(1): \quad & \varepsilon_2 m_2 (1+d) \leq \frac{m_1 (2+d)d}{(1+d)^2} \quad \rightarrow \quad \text{transition to joint by } \lambda_1^{\text{joint}^*} = \varepsilon_2 m_2 (1+d) \quad (\text{upper second-best rep.}) \\
(2): \quad & \frac{\varepsilon_2 m_2}{1+d} < \frac{m_1 (2+d)d}{(1+d)^2} < \varepsilon_2 m_2 (1+d) \quad \rightarrow \quad \text{transition to joint by } \lambda_1^{\text{joint}^*} = m_1 \frac{(2+d)d}{(1+d)^2} \quad (\text{first-best rep.}) \\
(3): \quad & \frac{m_1 (2+d)d}{(1+d)^2} \leq \frac{\varepsilon_2 m_2}{1+d} \quad \rightarrow \quad \text{transition to joint by } \lambda_1^{\text{joint}^*} = \frac{\varepsilon_2 m_2}{1+d} \quad (\text{lower second-best rep.})
\end{aligned}$$

Note that the inequality which characterizes case (1) implies the initially assumed inequality $\varepsilon_2 m_2 \leq \frac{m_1}{1+d}$. For the cases (2) and (3), this inequality is not implied and has to be kept. The utility of region one's median citizen which is implied by inducing joint provision with the obtained ideal representatives is the following:

$$\begin{aligned}
(1): \quad & u_1^{\text{joint}(1)^*} = m_1 \ln(\varepsilon_2 m_2 (1+d)) - \varepsilon_2 m_2 (1+d) \\
(2): \quad & u_1^{\text{joint}(2)^*} = m_1 \ln\left(m_1 \frac{(2+d)d}{(1+d)^2}\right) - m_1 + \varepsilon_2 m_2 \frac{1+d}{(2+d)d} \\
(3): \quad & u_1^{\text{joint}(3)^*} = m_1 \ln\left(\frac{\varepsilon_2 m_2}{1+d}\right)
\end{aligned}$$

In line with previous findings, the median utility in region one of the second option – forcing region two to provide the public good alone – is the same as the one of case (3) (region two will provide the public good alone for any $\lambda_1 \leq \frac{\varepsilon_2 m_2}{1+d}$). Subtracting the current level of utility from these figures leads to the increase in median utility if region one would induce joint provision according to the representatives of cases (1) to (3):

$$\begin{aligned}
\Delta u_1^{(1)} &= u_1^{\text{joint}(1)^*} - \overbrace{m_1 \ln(m_1) - m_1}^{u_1^{\text{alone}^*}} = m_1 \ln\left(\varepsilon_2 \frac{1+d}{\mu} \exp\left[1 - \varepsilon_2 \frac{1+d}{\mu}\right]\right) \quad \text{for } \mu \frac{(2+d)d}{(1+d)^3} \geq \varepsilon_2 \\
\Delta u_1^{(2)} &= \dots = m_1 \ln\left(\frac{(2+d)d}{(1+d)^2} \exp\left[\varepsilon_2 \frac{1+d}{\mu(2+d)d}\right]\right) \quad \text{for } \mu \frac{(2+d)d}{1+d} > \varepsilon_2 > \mu \frac{(2+d)d}{(1+d)^3} \text{ and } \frac{\mu}{1+d} \geq \varepsilon_2 \\
\Delta u_1^{(3)} &= \dots = m_1 \ln\left(\varepsilon_2 \frac{1}{\mu(1+d)} \exp[1]\right) \quad \text{for } \varepsilon_2 \geq \mu \frac{(2+d)d}{1+d} \text{ and } \frac{\mu}{1+d} \geq \varepsilon_2
\end{aligned}$$

Denote the arguments of the logarithms as $\delta_1^{(k)}$ with $k = 1, 2, 3$. Whenever these values are larger than one, region one can strictly increase median utility when switching to joint provision (according to the respective case). Note that all these arguments are rising in ε_2 . Hence they can take on values smaller than one for sufficiently small values of ε_2 ¹⁸. Seemingly troubling appears the fact that the values of ε_2 which render the respective $\delta_1^{(k)}$ negative are not in the allowed range of ε_2 for this specific scenario. However, note that – despite this finding – it is always possible to dissolve all incentives for region one to deviate to joint provision: Falling values of ε_2 do not only render the three different alternatives of joint provision

¹⁸This argument is backed by the findings $\delta_1^{(1)}(\varepsilon_2 = 0) = 0$, $\delta_1^{(2)}(\varepsilon_2 = 0) = \frac{(2+d)d}{(1+d)^2} < 1$ and $\delta_1^{(3)}(\varepsilon_2 = 0) = 0$.

less attractive (what might be insufficient to dissolve region one's incentives for joint provision), but do also make it less likely that region one can switch to joint provision by the first-best or lower second-best representative. Therefore – since a transition to joint provision is only possible with the upper second-best representative $\lambda_1^{\text{joint}^*} = \varepsilon_2 m_2 (1 + d)$ for sufficiently low values of ε_2 and $\delta_1^{(1)} \rightarrow 0$ for $\varepsilon \rightarrow 0$ – there exists always a critical value of ε_2 such that region one faces no incentive to induce joint provision.

This critical value of ε_2 is referred to as ε_2^* and, given that $E_2^{(k)}$ is the set of allowed values for ε_2 of scenario k (including $\varepsilon_2 m_2 \leq \frac{m_1}{1+d}$ for scenarios 2 and 3), defined the following way:

$$\varepsilon_2^* = \min_k \left\{ \varepsilon_2^{(k)*} \right\} \quad \text{where} \quad \varepsilon_2^{(k)*} = \begin{cases} \varepsilon_2^{(k)\text{inner}} & \text{from } \delta_1^{(k)} \stackrel{!}{=} 1 \quad \text{if } \varepsilon_2^{(k)\text{inner}} \text{ in } E_2^{(k)} \\ \infty & \text{if } \varepsilon_2^{(k)\text{inner}} \text{ not in } E_2^{(k)} \end{cases}$$

The intuition behind this definition is straightforward and resembles the reasoning above: Whenever $\delta_1^{(k)}$ is positive for all $\varepsilon_2 \in E_2^{(k)}$ ¹⁹, the critical value ε_2^* must be such that this scenario k is not feasible for region one.

C Centralization with representatives

C.1 Behavior of cooperative representatives

The optimization problem of the regions' representatives who are of types λ_1 and λ_2 is the following:

$$\max_{g_1, g_2} \Sigma = u_1 + u_2 = \lambda_1 \ln \left(g_1 + \frac{g_2}{1+d} \right) + \lambda_2 \ln \left(g_2 + \frac{g_1}{1+d} \right) - (g_1 + g_2) \quad \text{such that } g_j \geq 0 \text{ for } j = 1, 2$$

$$(1.1) \quad \frac{\partial \Sigma}{\partial g_1} = \frac{\lambda_1(1+d)}{g_1(1+d) + g_2} + \frac{\lambda_2}{g_2(1+d) + g_1} - 1 \stackrel{!}{\leq} 0 \quad (1.2) \quad g_1 \stackrel{!}{\geq} 0 \quad (1.3) \quad \frac{\partial u_j}{\partial g_1} g_1 \stackrel{!}{=} 0$$

$$(2.1) \quad \frac{\partial \Sigma}{\partial g_2} = \frac{\lambda_1}{g_1(1+d) + g_2} + \frac{\lambda_2(1+d)}{g_2(1+d) + g_1} - 1 \stackrel{!}{\leq} 0 \quad (2.2) \quad g_2 \stackrel{!}{\geq} 0 \quad (2.3) \quad \frac{\partial u_j}{\partial g_2} g_2 \stackrel{!}{=} 0$$

Note that this optimization problem as well as the emerging first order conditions are identical to the ones obtained in the benchmark scenario (section A in the appendix) if one exchanges m_1 by λ_1 and m_2 by λ_2 . Furthermore, to adjust the calculus, exchange μ by L (recall that $L \equiv \frac{\lambda_1}{\lambda_2}$). From this follows that concerning what was called case 3 in the benchmark scenario, region two can actually become the sole provider of the public good as $L(1+d) \leq 1$ is actually possible. The solution of the cooperative representatives' problem

¹⁹As argued above, this cannot be the case for $k = 1$ such that there exists always a ε_2 low enough to dissolve all incentives of region one to induce joint provision.

reads

$$g_1^*(\lambda_1, \lambda_2) = \begin{cases} \lambda_1 + \lambda_2 & \text{for } L \geq 1 + d \\ \frac{(1+d)\lambda_1 - \lambda_2}{d} & \text{for } 1 + d > L > \frac{1}{1+d} \\ 0 & \text{for } \frac{1}{1+d} \geq L \end{cases} \quad \text{and} \quad g_2^*(\lambda_1, \lambda_2) = \begin{cases} 0 & \text{for } L \geq 1 + d \\ \frac{(1+d)\lambda_2 - \lambda_1}{d} & \text{for } 1 + d > L > \frac{1}{1+d} \\ \lambda_1 + \lambda_2 & \text{for } \frac{1}{1+d} \geq L \end{cases} .$$

C.2 Uniform provision of the public good

The representatives maximize the sum of their utility by choosing the uniform level of public good provision g . The Accessible amount of the public good in both regions is thus given by $G = g + \frac{g}{1+d} = g \frac{2+d}{1+d}$.

$$\max_g \Sigma = u_1 + u_2 = [\lambda_1 + \lambda_2] \ln \left(g \frac{2+d}{1+d} \right) - 2g \quad \text{such that} \quad g \geq 0$$

$$\frac{\partial \Sigma}{\partial g} = \frac{\lambda_1 + \lambda_2}{g} - 2 \stackrel{!}{=} 0 \quad \left(\frac{\partial^2 \Sigma}{\partial g^2} = -\frac{\lambda_1 + \lambda_2}{g^2} < 0 \right) \quad \rightarrow \quad g^* = \frac{\lambda_1 + \lambda_2}{2} \quad \rightarrow \quad G = \frac{[\lambda_1 + \lambda_2](2+d)}{2(1+d)}$$

The median citizen of region j maximizes:

$$\max_{\lambda_j} u_j = m_j \ln \left(\frac{[\lambda_j + \lambda_{-j}](2+d)}{2(1+d)} \right) - \frac{\lambda_j + \lambda_{-j}}{2} \quad \text{such that} \quad \lambda_j \geq 0$$

$$\frac{\partial u_j}{\partial \lambda_j} = \frac{m_j}{\lambda_j + \lambda_{-j}} - \frac{1}{2} \stackrel{!}{=} 0 \quad \left(\frac{\partial^2 u_j}{\partial \lambda_j^2} = -\frac{m_j}{[\lambda_j + \lambda_{-j}]^2} < 0 \right) \quad \rightarrow \quad \lambda_j^* = 2m_j - \lambda_{-j}$$

Due to $m_1 \geq m_2$, the sum of both representatives' types will always be given by $2m_1$. Hence $g = m_1$ and $G = m_1 \frac{2+d}{1+d}$ will be true.

Rest of section C proofs yet to come.

D Provision in a multistage government

Theorem 3 *Given uniformity of preferences, the degree of strategic delegation in multistage governments as defined in section 8 is neither affected by the number of boards in the two regions (T_j) nor the sizes of these boards.*

As the proof of theorem 3 builds on the solution of the multistage model – which is obtained by backward induction – consider the incentives of the last board in region j , board $B_j^{T_j}$. The median member of this board maximizes private utility by choosing an appropriate representative. The deciding median member takes the outcome of the game between the representatives into account. By assumption this outcome is given by joint

provision²⁰. Furthermore, note that we are considering the two scenarios of decentralized and centralized provision at the same time. Conditional on these scenarios²¹, the optimal (first-best) representatives from the perspective of the median member of region j 's last board ($m_j^{T_j}$) are given by (see tables 2 and 3 and exchange m_j by $m_j^{T_j}$)

$$\lambda_j^{\text{FB}} = \begin{cases} m_j^{T_j} \frac{(2+d)d}{(1+d)^2} & \text{for decentralization} \\ m_j^{T_j} \frac{d}{1+d} & \text{for centralization} \end{cases} .$$

These representatives might be infeasible since they might not be a member of the last board and are thus not available as a representative. First, assume this is not the case. Board $B_j^{T_j-1}$'s median member optimizes over the type of the last board's median member (in anticipation of this median member's delegation decision) and faces the following optimization problem:

$$\max_{m_j^{T_j}} u_j^{T_j-1} = \begin{cases} m_j^{T_j-1} \ln \left(m_j^{T_j} \frac{(2+d)d}{(1+d)^2} \right) - m_j^{T_j} + f(\lambda_{-j}) & \text{for decentralization} \\ m_j^{T_j-1} \ln \left(m_j^{T_j} \frac{(2+d)d}{(1+d)^2} \right) - m_j^{T_j} + g(\lambda_{-j}) & \text{for centralization} \end{cases} \quad \text{s.t.} \quad m_j^{T_j} \in \left[\underline{m}_j^{T_j}, \overline{m}_j^{T_j} \right]$$

Due to the phenomenon discussed in section 7, the utility of the board's median member is identical for centralization and decentralization (ignoring the residuals f and g dependent on λ_{-j}). Furthermore, the restriction on feasible median members is stricter than the one before at the decision of the type of the representative since the set of possible median members of the next superordinate board is a subset of the members of the delegating board. However, since the solution to both formulations of the maximization problem is $m_j^{T_j} = m_j^{T_j-1}$, this condition is not binding as a board's median member is always able to install herself as the next board's median member. The solution of the multistage structure – if the last board's median member is able to select her first-best representative – is straightforward since the last maximization problem is the same for all boards B_j^{t-1} determining the median member of the next board, m_j^t . Hence, all boards will create superior boards which are subsets of themselves with identical median types. Thus for $\lambda_j^{\text{FB}} \in B_j^{T_j}$ (the last board can select the first-best representative), $m_j = m_2 = \dots = m_j^{T_j}$ and $\lambda_j = \lambda_j^{\text{FB}}$ holds.

Now assume that the first-best representative (for the last board's median member) is not viable pick. Since this first-best representative constitutes a discount on the delegating board's median member, only the lower boundary of this board is important. Hence for $\lambda_j^{\text{FB}} \notin B_j^{T_j}$, the last board selects the second-best representative who is given by the lower corner solution: $\lambda_j^{\text{SB}} = \underline{\lambda}_j^{T_j} = m_j^{T_j} - s_j^{T_j} m_j$. The second last board

²⁰Since we are interested in the effects on strategic delegation which does only occur for joint provision.

²¹The optimization problem of the median member reads (see equations UD and UC)

$$\max_{\lambda_j} u_j^{T_j} = \begin{cases} m_j^{T_j} \ln(\lambda_j) - \frac{\lambda_j(1+d)^2}{(2+d)d} + f(\lambda_{-j}) & \text{for decentralization} \\ m_j^{T_j} \ln \left(\lambda_j \frac{2+d}{1+d} \right) - \frac{\lambda_j(1+d)}{d} + g(\lambda_{-j}) & \text{for centralization} \end{cases} \quad \text{s.t.} \quad \lambda_j \in B_j^{T_j} ,$$

where f and g are functions solely dependent on λ_{-j} .

then faces the following maximization problem:

$$\max_{m_j^{T_j}} u_j^{T_j-1} = \begin{cases} m_j^{T_j-1} \ln \left(m_j^{T_j} - s_j^{T_j} m_j \right) - \frac{(m_j^{T_j} - s_j^{T_j} m_j)(1+d)^2}{(2+d)d} + f(\lambda_{-j}) & \text{(dec.)} \\ m_j^{T_j-1} \ln \left(\frac{(m_j^{T_j} - s_j^{T_j} m_j)(1+d)}{d} \right) - \frac{(m_j^{T_j} - s_j^{T_j} m_j)(1+d)}{d} + g(\lambda_{-j}) & \text{(cen.)} \end{cases} \quad \text{s.t. } m_j^{T_j} \in \left[\underline{m}_j^{T_j}, \overline{m}_j^{T_j} \right]$$

The first-best solution of the problem above is given by

$$m_j^{T_j, \text{FB}} = \begin{cases} m_j^{T_j-1} \frac{(2+d)d}{(1+d)^2} + s_j^{T_j} m_j & \text{for decentralization (implies } \lambda_j = m_j^{T_j} - s_j^{T_j} m_j = m_j^{T_j-1} \frac{(2+d)d}{(1+d)^2} \text{)} \\ m_j^{T_j-1} \frac{d}{1+d} + s_j^{T_j} m_j & \text{for centralization (implies } \lambda_j = m_j^{T_j} - s_j^{T_j} m_j = m_j^{T_j-1} \frac{d}{1+d} \text{)} \end{cases}.$$

Given the second last board is able to induce these first-best median types in the last board, it somehow fixes the last board's second-best decision since the types of the representatives become the first-best ones. Hence – if this fix is feasible (meaning if $m_j^{T_j, \text{FB}} \in \left[\underline{m}_j^{T_j}, \overline{m}_j^{T_j} \right]$) – all boards which are situated below the second last board face the same optimization problem as above and create median preserving subsets. Hence despite we assumed $\lambda_j^{\text{FB}} \notin B_j^{T_j}$ ex ante, whenever $m_j^{T_j, \text{FB}} \in \left[\underline{m}_j^{T_j}, \overline{m}_j^{T_j} \right]$ holds, the first best representative will be given by the least caring member in the last board and will actually be a viable representative ex post: $\lambda_j^{\text{SB}} = \underline{\lambda}_j^{T_j} = \lambda_j^{\text{FB}}$. The solution is then given by $m_j = m_2 = \dots = m_j^{T_j-1}$, $m_j^{T_j}$ as defined above and $\lambda_j = \lambda_j^{\text{FB}}$.

For $m_j^{T_j, \text{FB}} \notin \left[\underline{m}_j^{T_j}, \overline{m}_j^{T_j} \right]$, the second last board has to install a second-best median member in the last board. This second-best solution is given by the lowest possible median member: $m_j^{T_j, \text{SB}} = \underline{m}_j^{T_j}$ ²². The type of the representative – given the two second-best decisions – is then given by $\lambda_j^{\text{SB}} = \underline{m}_j^{T_j} - s_j^{T_j} m_j = m_j^{T_j-1} - s_j^{T_j-1} m_j$.

Now assume the following situation: All the boards B_j^τ with $\tau = t, \dots, T_j$ – thus we are looking at the $T_j - t + 1$ last boards – are unable to play first-best strategies. Hence the requirements $\lambda_j^* \notin B_j^{T_j}$ and $m_j^{\tau+1, \text{FB}} \notin \left[\underline{m}_j^{\tau+1}, \overline{m}_j^{\tau+1} \right]$ (for $\tau = t, \dots, T_j - 1$) have to hold. These requirements imply the following inequalities (let A be $\frac{(2+d)d}{(1+d)^2}$ for decentralization and $\frac{d}{1+d}$ for centralization) and lead to the denoted types of the representative:

$$B_j^{T_j} : \quad \lambda_j^{\text{FB}} = m_j^{T_j} A < \underline{\lambda}_j^{T_j} \quad \rightarrow \quad \lambda_j = \underline{\lambda}_j^{T_j}$$

²²This lowest feasible median member of board $B_j^{T_j}$ is given by $\underline{m}_j^{T_j} = \underline{\lambda}_j^{T_j-1} + s_j^{T_j} m_j = m_j^{T_j-1} - s_j^{T_j-1} m_j + s_j^{T_j} m_j$. The upper bound of the set of feasible median members is $\overline{m}_j^{T_j} = \overline{\lambda}_j^{T_j-1} - s_j^{T_j} m_j = m_j^{T_j-1} + s_j^{T_j-1} m_j - s_j^{T_j} m_j$. Fiddling around with the first-best median types and this upper bound reveals that it is never striking such that only the lower bound may prevent the second last board from selecting the first-best and fixing median member:

$$m_j^{T_j, \text{FB}} = m_j^{T_j-1} A + s_j^{T_j} m_j < m_j^{T_j-1} + s_j^{T_j-1} m_j - s_j^{T_j} m_j = \overline{m}_j^{T_j} \quad \text{with} \quad A = \begin{cases} \frac{(2+d)d}{(1+d)^2} & \text{dec.} \\ \frac{d}{1+d} & \text{cen.} \end{cases}$$

since $\underbrace{m_j^{T_j-1} [A - 1]}_{<0} < m_j \underbrace{[s_j^{T_j-1} - s_j^{T_j}]}_{>0}$ due to $A < 1$








$$\begin{aligned}
B_j^{T_j-1} : m_j^{T_j, \text{FB}} &= m_j^{T_j-1} A + s_j^{T_j} m_j < \underline{m}_j^{T_j} & \rightarrow m_j^{T_j} &= \underline{m}_j^{T_j} & \rightarrow \lambda_j &= m_j^{T_j-1} - s_j^{T_j-1} m_j \\
B_j^{T_j-2} : m_j^{T_j-1, \text{FB}} &= m_j^{T_j-2} A + s_j^{T_j-1} m_j < \underline{m}_j^{T_j-1} & \rightarrow m_j^{T_j-1} &= \underline{m}_j^{T_j-1} & \rightarrow \lambda_j &= m_j^{T_j-2} - s_j^{T_j-2} m_j \\
&\vdots & & & & \\
B_j^\tau : m_j^{\tau+1, \text{FB}} &= m_j^\tau A + s_j^{\tau+1} m_j < \underline{m}_j^{\tau+1} & \rightarrow m_j^{\tau+1} &= \underline{m}_j^{\tau+1} & \rightarrow \lambda_j &= m_j^\tau - s_j^\tau m_j
\end{aligned}$$

Consider the requirement of the arbitrary board B_j^τ which guarantees that this board cannot play the first-best strategy:

$$m_j^{\tau+1, \text{FB}} = m_j^\tau A + s_j^{\tau+1} m_j < \underline{m}_j^{\tau+1} = \lambda_j^\tau + s_j^{\tau+1} m_j = m_j^\tau - s_j^\tau m_j + s_j^{\tau+1} m_j \quad \rightarrow \quad m_j^\tau (1 - A) > s_j^\tau m_j$$

Two main insights can be drawn from the obtained inequality. At first, notice that the assumed scenario is realistic and constitutes an exhaustive approach: Boards only engage in strategic delegation to below median median types – with respect to superordinate boards’ median members – or create median preserving superordinate boards, therefore $m_j^\tau \leq m_j$ holds for all $\tau = t, \dots, T_j$. Additionally, by the design of the multistage setting, s_j^τ becomes smaller for higher values of τ . Thus, if $m_j^t (1 - A) > s_j^t m_j$ is true, then $m_j^\tau (1 - A) > s_j^\tau m_j$ is implied for all $\tau = t - 1, \dots, T_j$. This means that whenever a board B_j^t cannot install the first-best median member in the next board, then all higher boards B_j^τ (with $\tau = t - 1, \dots, T_j$) cannot play first-best strategies as well. The second insight is more powerful and yields the solution of the model for a multistage government: Due to the reasoning above, the inequality $m_j^\tau (1 - A) > s_j^\tau m_j$ becomes harder to fulfill for lower boards in the multistage government, meaning for lower values of τ . And whereas it can still be true for $\tau = 2$, it can never be fulfilled for $\tau = 1$. Due to $m_j^1 = m_j$ and $s_j^1 = 1$ follows $m_j (1 - A) \not> m_j$. Hence at least the first board B_j^1 – the overall population of region j – will be able to play the first-best strategy and by this compensate the second-best behavior of all superordinate boards such that the type of the representative will be $\lambda_j = \lambda_j^{\text{FB}} = m_j A$.

References

- Bergstrom, T., Blume, L. and Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, 29, 25–49.
- Besley, T. and Coate, S. (2003). Centralized versus decentralized provision of local public goods: a political economy approach. *Journal of Public Economics*, 87(12), 2611–2637. 
- Brennan, G. and Buchanan, J. M. (1980). *The Power to Tax: Analytical Foundations of a Fiscal Constitution* (Vol. 9). 
- Buchholz, W., Haupt, A. and Peters, W. (2005). International environmental agreements and strategic voting. *Scandinavian Journal of Economics*, 107(1), 175–195. 
- Dur, R. and Roelfsema, H. (2005). Why does centralisation fail to internalise policy externalities? *Public Choice*, 122(3-4), 395–416. 
- Hamman, J. R., Weber, R. A. and Woon, J. (2011). An Experimental Investigation of Electoral Delegation and the Provision of Public Goods. *American Journal of Political Science*, 55(4), 737–751. 
- Hirshleifer, J. (1983). From weakest-link to best-shot: The voluntary provision of public goods. *Public Choice*, 41(3), 371–386.
- Malesky, E. J., Tran, A. and Viet, C. N. (2014). The Impact of Recentralization on Public Services: A Difference-in-Differences Analysis of the Abolition of Elected Councils in Vietnam. *American Political Science Review*, 108(1), 144–168. 
- Mansuri, G. and Rao, V. (2013). *Localizing Development - Does Participation Work?* Washington, DC: The World Bank. (Vol. 1, pp. 15-48).
- Oates, W. E. (1972). *Fiscal Federalism*. New York: Harcourt Brace.
- Oates, W. E. (2005). Toward A Second-Generation Theory of Fiscal Federalism. *International Tax and Public Finance*, 12(4), 349–373.
- Qian, Y. and Weingast, B. R. (1997). Federalism as a Commitment to Preserving Market Incentives. , 11(4), 83–92.
- Roelfsema, H. (2007). Strategic delegation of environmental policy making. *Journal of Environmental Economics and Management*, 53(2), 270–275. 
- Treisman, D. (2007). *The Architecture of Government: Rethinking Political Decentralization*. New York: Cambridge University Press.