

Hansen, Emanuel

Conference Paper

Political Competition with Endogenous Party Formation and Citizen Activists

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel -
Session: Political Processes: Theory and Evidence, No. G22-V1

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Hansen, Emanuel (2016) : Political Competition with Endogenous Party Formation and Citizen Activists, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Political Processes: Theory and Evidence, No. G22-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/145923>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Political Competition with Endogenous Party Formation and Citizen Activists

October, 2015
Preliminary Version

Abstract

The paper studies political competition between endogenously formed parties instead of independent candidates. Party formation allows policy-motivated citizens to nominate one of their fellow party members as their candidate for a general election and to share the cost of running in this election. Thus, like-minded citizens are able to coordinate their political behavior in order to improve the policy outcome. The paper focuses on political equilibria with two active parties, and investigates the properties of stable parties and the policy platforms offered in equilibrium. The platforms of both parties can neither be fully convergent as in the median voter model (Downs 1957) nor extremely polarized as in the citizen candidate model (Besley & Coate 1997). In the benchmark case of full electoral certainty, a unique political equilibrium with positive platform distance exists. Endogenous party formation thus eliminates a major weakness of the citizen candidate model, the extreme multiplicity of equilibria. The model remains tractable, and the qualitative results are shown to be robust under the assumption of electoral uncertainty, where vote results cannot be perfectly predicted.

Keywords: Elections, Party Formation, Platform Choice, Electoral Uncertainty

JEL classification: D72

1 Introduction

This paper studies electoral competition between two endogenously formed political parties. The agents in this model are policy-motivated citizens who are not only entitled to vote but can also join political parties. Parties serve as twofold coordination devices. First, the members of each party make monetary contributions to the parties in order to finance an exogenous (campaign) cost of running in the general election. Second, they jointly decide about the party's presidential candidate in primary elections. Parties can commit to policy platforms by nominating one of their party fellows with appropriate policy preferences as their presidential candidate. As party membership is costly, the agents will only become active if the induced policy gains resulting from this activity are sufficiently large to outweigh the cost of political activity. In this model, party platforms can be interpreted as local public goods that have to be provided and agreed on by the party members. Agents make their membership decision on the basis of the same policy preferences that also govern their voting behavior. There are two exogenous parameters, the cost of party membership and the degree of electoral uncertainty.

Most of the existing literature on political competition studies the policy platforms proposed by a set of independent candidates that do not engage in party formation. This paper instead simultaneously investigates the characteristics and platform choices of stable political parties. In a political equilibrium, no citizen has an incentive to change his party affiliation, taking into account the effect of his deviation on the party platforms. Political equilibria can be characterized by the tuple of policy platforms offered by the parties and a partition of the set of agents into the set of independents and the membership sets of both parties. I concentrate on political equilibria with two active parties, which exist for all combinations of the exogenous parameters.

The focus of this paper is on the effect of endogenous party formation on the equilibrium policy platforms and the implied degree of policy convergence or polarization, respectively. The main contribution is to show that the equilibrium distance between party platforms is bounded from below as well as from above. This is in contrast to the results of the citizen candidate model by Besley & Coate (1997). Intuitively, parties can only attract citizens that are willing to incur the membership cost if their platforms are sufficiently different. Thus, there can never be too much (or even full) policy convergence. On the other hand, political polarization is limited by the desire to offer competitive platforms and by the coordination enabled by political parties. If both platforms were too polarized, the members of each party would prefer to nominate a more moderate presidential candidate in order to

increase the probability of winning the general election. In this situation, independent citizens with moderate policy preferences would indeed benefit from becoming politically active as the achievable policy gains would outweigh the membership cost.

The properties of political equilibria depend on the degree of electoral uncertainty and on the membership cost. As the electoral risk increases, the attractiveness of moderate platforms is weakened, and more extreme platforms can be supported in equilibrium. Put differently, if the electoral outcome becomes less predictable, the upper bound on the platform distance becomes larger while the lower bound remains unchanged. In the limiting case of full electoral certainty, both bounds coincide and a unique pair of policy platforms can be offered in equilibrium. With respect to the second exogenous parameter, both boundaries on the platform distance increase as the membership cost gets larger. Intuitively, citizens ask for more difference in the policy platforms and higher policy gains in order to be willing to engage politically. Combining these comparative statics, it can be shown that the classical prediction of full policy convergence to the median voter is only sustainable for the twofold limiting case of full electoral certainty and zero costs of political activity.

The paper proceeds as follows. After sketching the related literature in section 2, the model will be presented in section 3. In sections 4 to 6, the game is analyzed and the main results for a given pair of the exogenous parameters are derived. In section 7, I present comparative statics with respect to membership costs and the degree of electoral uncertainty. For the special case of electoral certainty, the existence of a unique political equilibrium is derived. Section 8 concludes.

2 Related literature

The model builds on the citizen candidate framework introduced by Besley & Coate (1997) and Osborne & Slivinski (1996). In both versions of this model, the set of candidates is determined endogenously from the set of citizens who are not only entitled to vote in a democratic election, but can also decide to run as (individual) candidates, facing an exogenous cost of candidacy. There are no parties, and citizens cannot coordinate their political behavior. The models do not deliver a unique theoretical prediction but a multiplicity of political equilibria with either one or two candidates. Their main insight is that the endogeneity of the candidate set eliminates the possibility of completely convergent platforms in two-candidate equilibria. This impossibility result is in sharp contrast to the classical prediction of the median voter model by Downs (1957) and the probabilistic voting model by Lindbeck & Weibull (1987), but is in line with empirical observations. In both ver-

sions of the citizen candidate model, there may however be equilibria with arbitrarily polarized candidates. In the model by Besley & Coate (1997), the platform distance in two-candidate equilibria is only bound by the extremes of the policy space.¹

A number of papers extend the basic citizen candidate framework to accommodate political parties. For example, Rivière (1999) studies the formation of parties as cost-sharing devices and provides a game-theoretical explanation for Duverger’s law, i.e., the prevalence of two-party systems under the plurality rule. The same result is derived in a different environment by Osborne & Tourky (2008), who analyze the incentives to form parties within a group of legislators under the assumptions of costly participation and economies of party size. In contrast, Levy (2004) examines whether the formation of political parties can be effective in the sense that it enables offering platforms that would not be feasible without parties. Morelli (2004) studies the implications of alternative electoral systems for the formation of parties by agents with heterogeneous policy preferences. Snyder & Ting (2002), as well as Poutvaara & Takalo (2007), show that parties may serve as brand names or screening devices, which provide superior information about the candidates’ preferences or quality, respectively.

In contrast to this paper, these papers do not examine the effects of endogenous formation of political parties on political polarization. Directly related to this issue, they do not show that party formation alleviates the (often criticized) indeterminacy of the basic citizen candidate model. Furthermore, these papers either consider only the case of electoral uncertainty or strongly restrict the type space. In this paper, I will instead study the implications of endogenous policy formation on platform choice in a general setting, allowing for different degrees of electoral uncertainty as well as a continuum of agents without restrictions on the location of bliss points.²

To my knowledge, only one previous paper investigates the effect of political parties on platform choice within the citizen candidate framework. Cadigan & Janeba (2002) study party competition in a US-style presidential election with primary elections and identify a strong connection between membership structures and party platforms. Instead of endogenizing membership decisions, however, they assume exogenous party affiliations of the citizens. The drawback of this model is that any combination of platforms represents a political equilibrium for some membership

¹In the version of Osborne & Slivinski (1996), there is large set of equilibria with potentially large, but limited polarization. In contrast to the analysis in this paper, however, the upper bound on the platform distance results from the assumption of sincere instead of strategic voting and is not related to the candidates’ behavior or coordination.

²Dhillon (2004) provides an overview over the existing theoretical models with pre-election as well as post-election party formation, with a particular focus on papers that extend the citizen candidate model.

structures. As they cannot distinguish between stable and unstable membership structures, the model only delivers very limited insights into the effects of party formation. Furthermore, Cadigan & Janeba (2002) do not consider the general case of electoral uncertainty.

In addition, there is a small number of papers on the formation of political parties outside the citizen candidate framework. Most closely related, Roemer (2006) studies the effects of endogenous party formation and campaign contributions by policy-motivated citizens. Similar to my model, the unique political equilibrium of Roemer’s model features positive but limited platform distance. However, both models differ considerably in many aspects. Most importantly, Roemer applies the cooperative notion of “Kantian equilibrium” in which agents consider joint (proportional) deviations of all party members at the contribution stage. The implications of this equilibrium concept differ strongly from the non-cooperative notion of Nash equilibrium that I will apply in my model.³ Furthermore, the platforms are chosen through a Nash bargaining process in which the agents’ influence is proportional to their individual contributions in his model. In my model, in contrast, there are primary elections wherein each party member has exactly one vote.

In other papers, citizens only decide whether to support exogenously given political parties by contributing to their electoral campaigns (Herrera et al. 2008, Campante 2011, Ortuño-Ortín & Schultz 2005). Although there is no endogenous party formation in these models, citizens have an indirect influence on policy platforms, which are chosen by the parties, taking into account the induced contribution behavior. Poutvaara (2003) also models endogenous party formation and predicts a positive but limited platform distance. However, the results are mainly driven by the assumption that agents make their membership decisions based on expressive objectives while, in my model, they follow from strategic membership decision and cooperation between like-minded citizens.⁴

Finally, this paper also relates to the literature on probabilistic voting and electoral uncertainty, beginning with the seminal paper of Lindbeck & Weibull (1987). Eguia (2007) studies the effect of electoral uncertainty in the citizen candidate model. Without party formation, electoral uncertainty has the effect of increasing the set of political equilibria with two candidates by allowing for asymmetric equilibria.

³For example, every citizen is member of one party in the model of Roemer (2006) while there is a (large) set of independents in any equilibrium of my model.

⁴Besides, there exist a few models on endogenous formation of political parties under proportional electoral systems in which the implemented policy is given as a weighted sum of the party platforms (e.g. Gøromberg et al. 2004, Gerber & Ortuño-Ortín 1998). Due to the incentives given by this electoral system, these models typically predict an extremely high level of political polarization.

However, electoral uncertainty per se does not lead to additional centripetal forces and does not limit political polarization. Both models focus on the behavior of individual agents and do not examine the effects of party formation.

3 The model

There is a continuum of citizens N of mass one. The policy space X is one-dimensional and given by the real line $(-\infty, +\infty)$. The citizens have linear Euclidean preferences and heterogeneous bliss points w_i . Thus, if policy $x \in X$ is implemented, citizen i receives a policy payoff of

$$v_i(x) = -|x - w_i|. \quad (1)$$

The distribution of bliss points in the population has full support on \mathbb{R} , but is not known ex ante. The population median m is commonly perceived to be the realization of a random variable with twice continuously differentiable cdf Φ and pdf ϕ . In particular, I assume that m is perceived as normally distributed with mean zero and standard deviation σ .⁵ As the median voter will be decisive in the general election, this assumption induces electoral uncertainty.

A general election with plurality (“winner-takes-all”) rule takes place to choose a president who is entitled to implement policy. There are two parties, the leftist party L and the rightist party R . The election is party-based in the sense that only the two parties have the right to nominate presidential candidates who run against each other in the general election. In order to nominate a candidate, however, each party is required to pay an exogenous cost C of candidacy, which must be financed jointly by the members and supporters of each party. The presidential candidate of each party is determined in a series of pairwise primary elections in which all party members are entitled to stand for office and to vote. Neither a party nor a candidate is able to make a binding policy commitment prior to the general election. As will become clear later on, the bliss point of the leftist (rightist) party’s candidate can consequently be interpreted as policy platform l (r).

The membership structures of both parties are not given exogenously. Instead, they follow endogenously from the citizens’ optimizing behavior. Specifically, citizens choose their affiliation by making contributions $\alpha_i^P \in [0, \infty)$ to the parties

⁵The assumption of a normally distributed population mean is motivated by an extension of the central limit theorem. This theorem states that, for a sample with a sufficient number of independent and identically distributed random variables, the distribution of the sample mean approximates a normal distribution. Ma et al. (2011) discuss the conditions under which the same result applies for the distribution of the sample median and other sample quantiles.

$P \in \{L, R\}$. The utility of citizen $i \in N$ is linearly decreasing in his contributions, and given by

$$v_i(x) - \alpha_i^L - \alpha_i^R \quad (2)$$

if policy x is implemented. Agent i becomes a member of party $P \in \{L, R\}$ if and only if he contributes $\alpha_i^P \geq c$. Thus, c represents the cost of political activity, which may correspond to monetary costs, but can also be interpreted as hours worked and effort spent for the electoral campaign and party meetings. To rule out that only degenerate parties are formed in equilibrium, I assume that $c < C/2$ is satisfied. Each citizen can join one party at most.⁶ The result of the party formation game is a partition of the set N into the member sets of each party (M^L, M^R) and the set of independents (I) such that $N = M^L \cup M^R \cup I$.⁷

The political process consists of four stages. At the first stage, all agents $i \in N$ simultaneously choose their party affiliation by making contributions to both parties $\alpha_i^L, \alpha_i^R \geq 0$. Party P becomes active and is entitled to nominate a presidential candidate if and only if $\sum_{i \in N} \alpha_i^P \geq C$. At the second stage, a series of pairwise primary elections is conducted in each active party to select the presidential candidate. In the pairwise elections of each party, only the respective party members are entitled to vote. In the subsequent general election, the Condorcet winners of each party's primaries run as presidential candidates.⁸

At the third stage, the population median is drawn and the general election between the nominated presidential candidates takes place. All citizens observe the identities, i.e., the bliss points, of both presidential candidates and simultaneously cast their votes. The winner is determined according to the plurality rule and becomes president. If there is only one active party and presidential candidate, he directly enters the presidential office. At the last stage of the political process, the elected president implements some policy $x \in X$. The candidates are not able to make binding policy commitments at earlier stages of the political process. If there is no active party, a default policy $x_0 \in \mathbb{R}$ is implemented.

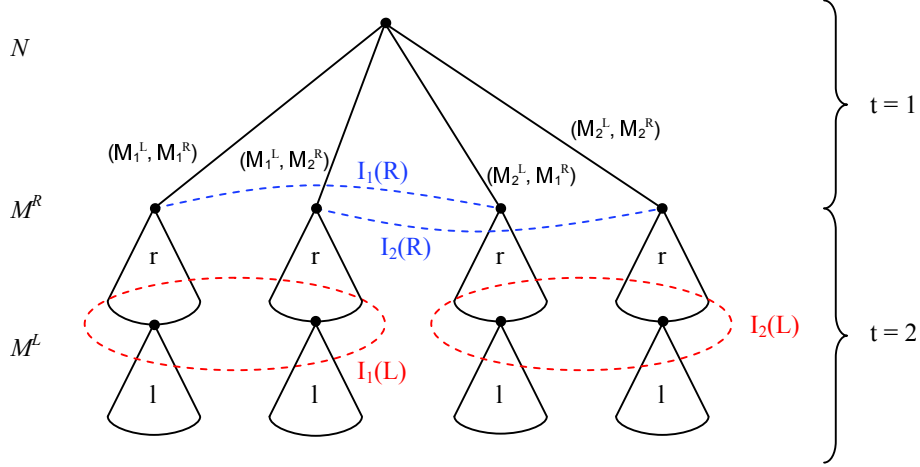
Figures 1 and 2 depict the timing of the game and its information structure graphically. At the first stage, the citizens simultaneously choose their contributions

⁶This assumption simplifies the following analysis without affecting the results. It can be shown that no citizen wants to be a member of both parties in any political equilibrium. Note also that it is possible to support a party without becoming its member (for $\alpha_i^P < c$) or to contribute more than the exogenous membership cost. The additional generality of this financing structure has no effect on the result of the model.

⁷As I will show in the following sections, the member sets of both parties are finite in any political equilibrium.

⁸As shown in the following section, the existence of a Condorcet winner is guaranteed for each finite membership set.

Figure 1: The party formation subgame



(α^L, α^R) , which induce a partition of the agent set N into the membership sets of both parties M^L , M^R and the set of independents. Note that figure 1 only depicts two possible membership structures for each party (e.g. M_1^L , M_2^L) in order to illustrate the basic structure, although there is an infinite number of possible membership structures in general. At the first stage, each agent $i \in N$ must hold beliefs about the resulting membership structures and the platforms that would arise in case of his membership in any party as well as in case of his independence. These beliefs determine the expected effect of his political activity on his individual payoff and must be consistent in equilibrium.

At the second stage, the members of each party jointly choose their presidential candidate and the policy platform, respectively. With respect to the information structure, I assume that at the time of candidate nomination, the members of party $P \in \{L, R\}$ can observe the set of their party fellows (M^P) and their bliss points, but not the membership set of the competing party (M^{-P}). For the members of party R , all nodes involving membership set M_1^R are thus contained in the same information set $I_1(R) = \{(M^L, M^R) \mid M^R = M_1^R\}$. Similarly, the members of party L can neither observe the membership set M^R nor the chosen platform r of the rightist party when they decide about their own platform l . Rather, the information set $I_1(L) = \{(M^L, M^R) \mid M^L = M_1^L\}$ consists of all nodes involving the same membership set M_1^L , but different sets M^R and platforms r .

Thus, a specific form of updating takes place at the beginning of the second stage: Members of the leftist party can perfectly update their previous belief about the leftist party's membership structure M^L , while their beliefs about M^L remain constant. Consequently, the members of party L must hold a belief about M^R and the finally chosen platform r in each information set (see figure 1). In the following,

I will only consider the belief \hat{r} about the competing party's platform r explicitly, as this is the only payoff-relevant variable (in contrast to M^R). After the primary election stage, the nominated candidates and the associated platforms of both parties become public information, and all citizens update their beliefs \hat{r} as well as \hat{l} . The remaining stages of the game are depicted graphically in figure 2 below.

This information structure simplifies the analysis while it does not change the qualitative results of the model. In particular, lower and upper bounds on the platform distance in political equilibria could also be identified under the alternative assumption that all agents can observe both member sets M^L and M^R at the primary election stage.⁹

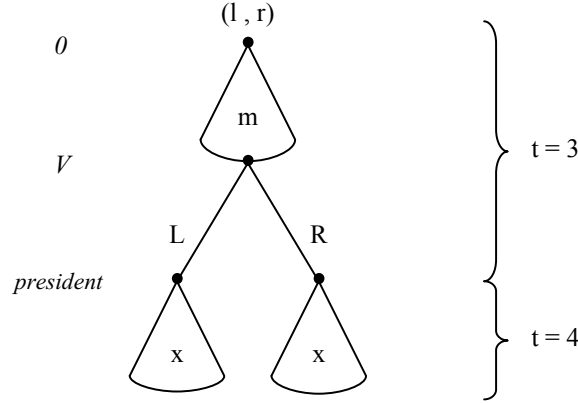
An allocation is given by a tuple of party platforms (r, l) (the presidential candidates' bliss points) and a partition of the population into the sets of party members and independents. A Perfect Bayesian equilibrium of this game is given by a strategy profile and a belief system such that, first, the strategies are sequentially rational given the belief system and, second, the belief system is derived from the optimal strategies everywhere on the equilibrium path. Additionally, I assume that agents do not play weakly dominated strategies at the candidate selection stage and vote sincerely at the general election stage.¹⁰ The goal of this paper is to identify the set of equilibrium platform combinations and the corresponding set of stable membership structures. I concentrate on political equilibria in pure strategies with two active parties.¹¹ In the following, I will solve the model backwards starting with the policy implementation stage.

⁹Given this information structure, the analysis of deviations from equilibrium is simplified considerably. If a previously independent agent deviates by joining party L , this will in general induce a change of platform l . The members of party R cannot react to this deviation by changing their platform r , however, since they are not able to observe the deviation. In the alternative case of fully observable membership sets at the second stage, the same change in the party affiliation of one agent might induce platform changes in both parties. Due to the finite set of feasible platforms, however, the implied reaction function of the competing party would in general be discontinuous and depend strongly on the specific composition of M^R . Accounting for these best responses would thus require a large number of case distinctions.

¹⁰At the general election stage, the assumption of sincere voting seems innocuous. With any finite set of voters and only two alternatives, sincere voting would be the weakly dominant strategy. With a continuum of voters, the notion of weak dominance is not properly defined since no voter can ever be pivotal. The economic intuition however does not change, leaving sincere voting as the only reasonable equilibrium behavior.

¹¹In general, there may also exist political equilibria in mixed strategies and equilibria in which there is only one active party with a sufficiently moderate platform (see appendix).

Figure 2: The general election subgame



4 Policy implementation and general election

The last two stages of the game can be solved straightforwardly. At the last stage, the elected president decides which policy to implement. Assume agent i with bliss point w_i is the president. Recall that he was unable to commit to any policy before. He can thus maximize his individual payoff $v_i(x) = -|x - w_i|$ by implementing his bliss point $x = w_i$. This policy choice is anticipated by all agents at the previous stages. Thus, the nomination of agent i as presidential candidate by party L implies a (credible) commitment to his individual ideal policy w_i . In the following, I will thus interpret the ideal policies of both presidential candidates as the parties' policy platforms l and r .

At the general election stage, all citizens vote for one of the parties or one of the nominated presidential candidates, respectively. The bliss points of both candidates are known. For clarity, we denote these bliss points by l and r , as they represent the platforms offered by both parties L and R . As a convention, the party with a more leftist platform will be called party L , and its platform will be denoted by l such that $l \leq r$ holds.

All citizens vote sincerely in the general election. Thus, citizen $i \in N$ votes for the party whose platform is closer to his own bliss point w_i , and the median voter's preference prevails. Thus, the leftist party L will win the election if and only if its platform l is located more closely to the median voter's bliss point m (the population median) than platform r , i.e., if $m < \frac{l+r}{2}$ holds.

Ex ante, however, the agents do not know the exact location of the population median $m \in \mathbb{R}$, but only its probability distribution. Thus, the winning probability $p(l, r)$ of party L is equal to the value of the distribution function at the arithmetic

mean of both platforms,

$$p(l, r) = \Phi\left(\frac{l + r}{2\sigma}\right). \quad (3)$$

Obviously, the winning probability is increasing both in l and r (for $l < r$). Besides, note that the random distribution of m induces electoral uncertainty as all agents assign positive winning probabilities to both parties for any combination of l and r ex ante. As I will show in the following section, this electoral uncertainty implies a smooth trade-off between the subjective desirability and the winning probabilities of alternative party platforms, which is in line with the economic intuition and often referred to in political discussions. To simplify notation, we focus on the case of a standard normal distribution with $\sigma = 1$ in the following.¹²

5 Candidate selection

At the candidate selection stage, the members of both parties simultaneously nominate their presidential candidates. As the nomination process in both parties is completely symmetric, I will only consider intra-party decision making in the leftist party L . To avoid case distinctions, I impose the simplifying assumption that each party has an odd number of members.¹³

At this stage, both member sets M^L and M^R have been determined as the outcome of the party formation game at the first stage. By the assumed information structure, the members of party L can only observe the composition M^L of their own party (see figure 1). For each information set $I_k(L)$, however, they hold a belief \hat{r} about the resulting platform of the rightist party.

The presidential candidate is selected by the members of party L in a series of pairwise elections. This procedure will lead to a clear-cut decision if and only if one member represents a Condorcet winner, i.e., if a majority of member prefer one agent $i \in M^L$ to all other potential candidates. Lemma 1 states that a Condorcet winner exists for any combination of member set M^L and belief \hat{r} .

Lemma 1. *Let M^L be the set of members of party L , m^L the party median and $\hat{r} \geq m^L$ the commonly held belief about party R 's platform. The selected candidate of party L is given by the member with bliss point $l(M^L, \hat{r}) = \max\{m^L, l_M(\hat{r}, M^L)\}$, where $l_M(\hat{r}, M^L) \equiv \arg \max_{\{w_i: i \in M^L\}} (\hat{r} - w_i)p(w_i, \hat{r})$*

First, note that candidate selection serves only as a device to commit to the

¹²In section 7, I study the effects of variations in electoral risk, as captured by σ .

¹³For an even number of members, only minor changes occur, while the qualitative results remain valid.

preferred platform, as the agents' utilities do not depend on the identity of the candidates. Conditional on platform l and belief \hat{r} , the expected policy payoff to member i of party L is given by

$$\begin{aligned}\tilde{v}_i(l, \hat{r}) &\equiv p(l, \hat{r})(-|l - w_i|) + [1 - p(l, \hat{r})](-|\hat{r} - w_i|) \\ &= p(l, \hat{r})\{|\hat{r} - w_i| - |l - w_i|\} - |\hat{r} - w_i|.\end{aligned}\quad (4)$$

Each member would like to choose l in order to maximize his individual policy payoff, given the expected platform of the competing party \hat{r} . For illustration, look at the preferences of a leftist citizen such that $w_i < \hat{r}$ holds. Obviously, he would never choose a platform $l > \hat{r}$ as this would imply an even lower policy payoff than a certain implementation of policy \hat{r} . Furthermore, no platform to the left of a member's bliss point can be individually optimal, since any platform $l < w_i$ leads to a lower winning probability $p(l, \hat{r})$ as well as a lower policy payoff in case of winning (compared to w_i). For platforms in the remaining interval $[w_i, \hat{r}]$, the policy payoff function simplifies to

$$\tilde{v}_i(l, \hat{r}) = p(l, \hat{r})(\hat{r} - l) - (\hat{r} - w_i).$$

In this interval, the platform preferences involve a trade-off between the probability of winning $p(l, r)$ and the subjective desirability $(l - w_i)$. As platform l approaches \hat{r} , member i benefits from an increasing winning probability of party L , but receives a lower payoff in case of electoral success. Each member prefers the platform which induces the largest shift of the expected policy towards his bliss point. In order to measure this shift, I define the policy effect function

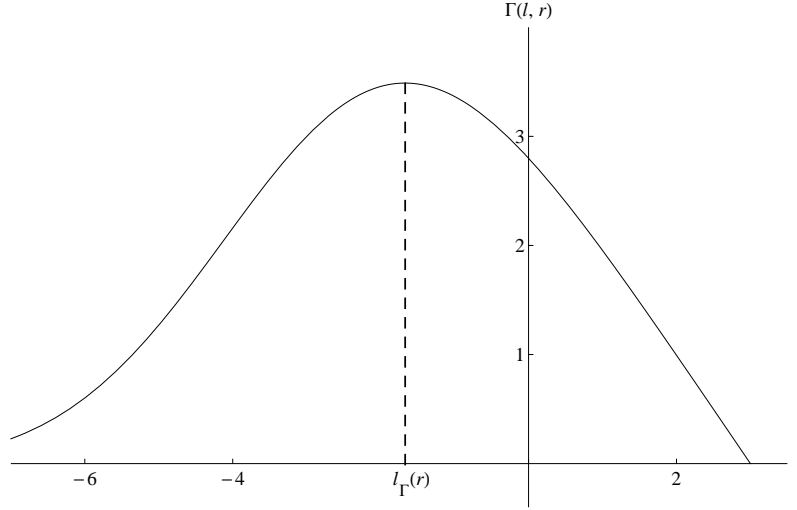
$$\Gamma(l, \hat{r}) \equiv (\hat{r} - l)p(l, \hat{r}) = (\hat{r} - l)\Phi\left(\frac{r + l}{2}\right). \quad (5)$$

In the appendix, I show that this function is strictly quasi-concave for the case of a normally distributed population median m . I denote its unique maximizer by $l_\Gamma(\hat{r}) = \arg \max_{l \in \mathbb{R}} \Gamma(l, \hat{r})$. Figure 3 depicts the policy effect function graphically.

As the party platform must equal the bliss point of some party member $j \in M^L$, however, this platform may not be feasible. Taking this restriction into account, the feasible platform with the highest policy effect is given by $l_M(\hat{r}, M^L) = \arg \max_{\{w_i: i \in M^L\}} \Gamma(l, \hat{r})$. By the quasi-concavity of the policy effect function, the policy payoff of agent i is maximized by the platform $l_M(\hat{r}, M^L)$ if this is more moderate than w_i , and by his own bliss point w_i otherwise.

Second, I show in the appendix that the platform preferences satisfy the single-

Figure 3: The policy effect function



Horizontal axis: Platform l of the leftist party. Vertical axis: Policy effect function $\Gamma(l, r)$ for $r = 3$, $\sigma = 1$.

crossing property (see Lemma 3). Thus, voting behavior is monotonic in each pairwise election. The preferred candidate of the median party member consequently represents a Condorcet winner and is nominated as presidential candidate. As explained above, the median member prefers the maximum of his own bliss point and platform $l_M(\hat{r})$.

Note that pairwise elections are not the only decision procedure leading to the nomination of the Condorcet winner as presidential candidate. For example, the same platforms arise under the formal rule that the median party member is entitled to nominate his preferred candidate.¹⁴ Furthermore, one could think of a richer model with US-style primary elections in which all party members are entitled to vote and to run as candidates. In such a model, the unchallenged candidature of the Condorcet winner identified above would represent a subgame equilibrium, too.¹⁵

6 Political equilibria

Political equilibria can be characterized by membership structures M^L , M^R and the resulting platforms l , r . In the previous section, I identified the presidential candidates that are nominated by the members of party L in each information set, i.e., for any combination of member set M^L and belief \hat{r} . In a political equilibrium, the

¹⁴This decision rule is applied in the model of Poutvaara (2003).

¹⁵With such a primary election stage, there may be additional equilibria with two winning candidates between whom the median member m^L is indifferent.

platform beliefs must be consistent. This implies that the equilibrium platform l must be the Condorcet winner in set M^L , given the correct belief $\hat{r} = r$ (accordingly for platform r). If membership structures were given exogenously by some partition (M^L, M^R) , then this condition would already pin down the unique equilibrium combination of policy platforms.

At the first stage of the game studied here, however, policy-motivated citizens choose their party affiliation endogenously. In a political equilibrium, membership structures must therefore be stable in the sense that

- (I) no member of any party can profitably deviate by becoming politically independent,
- (II) no independent citizen can profitably deviate by joining any party,
- (III) no member of any party can profitably deviate by changing his party affiliation.

Conditions (I) to (III) are necessary and sufficient conditions for any political equilibrium. However, they do not give many insights by themselves, as the effects of the mentioned deviations depend in a non-trivial way on the complete vector of contributions α^L, α^R and the implied membership sets M^L, M^R . In the following, I will examine the implications of these conditions on the set of policy platforms that can be supported in equilibrium. After deriving necessary conditions for political equilibria, I prove equilibrium existence.

Consider some vector of contribution decisions (α_0^L, α_0^R) and the induced membership structure M_0^L, M_0^R . Let the resulting policy platforms be given by l_0 and r_0 . This constellation can only represent an equilibrium if there is no profitable deviation at the party formation stage, i.e., if no agent would benefit from changing his party affiliation. Party L is active if and only if the sum of its contributions is larger than the exogenous cost of running: $\sum_{i \in N} \alpha_i^L \geq C$. It is *efficient* if $\sum_{i \in N} \alpha_i^L \in [C, C + c)$ holds, which implies that there is no wasteful over-contribution and that the withdrawal of any member would induce the inactivity of its former party. Conditions (I) and (II) jointly lead to the following lemma.

Lemma 2. *In any equilibrium with two active parties, both parties are efficient, i.e., $\sum_{i \in N} \alpha_i^P \in [C, C + c)$ for $P \in \{L, R\}$.*

Lemma 2 can be proven by contradiction. In order to do this, assume that there is a political equilibrium with non-efficient contributions. Let the party platforms be given by l_0 and r_0 . In equilibrium, the members of both parties hold correct

beliefs $\hat{r} = r_0$ and $\hat{l} = l_0$. Now, consider two specific deviations. First, the exit of the most leftist member j of party L would not induce L 's inactivity but shift its party median to a more rightist position $m_1^L > m_0^L$. As party R cannot react to this deviation and belief \hat{r} remains unchanged, the withdrawal of agent j will induce the nomination of a weakly more moderate candidate $l_1 \geq l_0$ by Lemma 1. Agent j will prefer to maintain his membership in L if and only if the shift from l_0 to l_1 is so large that the reduction in his policy payoff outbalances the saved membership cost.

Next, consider a more rightist, independent agent k with bliss point $w_k \in (l_1, r)$. If he would join party L , this would have the same effects on the party median and, consequently, on the nominated candidate as the previously considered exit of the leftist member j . Thus, the policy platform shifts from l_0 to l_1 once again, inducing an increase of k 's policy payoff. Agent k profits from this deviation if this effect outweighs the cost c of joining party L . In the appendix, I show that the payoff increase to the entrant k is strictly larger than the payoff decrease to j from leaving party L (in absolute values). Thus, whenever agent j prefers not to become independent, it is profitable for k to join party L . Since either j or k will always have an incentive to change his party affiliation, there cannot be a political equilibrium with inefficient parties.

Lemma 2 implies the number of party members will be smaller than $\frac{C}{c} + 1$ in any political equilibrium. Consequently, the sets of members of both parties will always be finite, and there will be independent agents in any equilibrium.

Party structures can thus only be stable if the exit of any member of L causes the inactivity of his party and guarantee the implementation of the opposing platform r . Given this pivotality, agent i prefers to stay a party member if the policy gains induced by his activity outweigh the cost c of his membership. In equilibrium, this can only be true for some party members if the policy effect $\Gamma(l, r)$ of each party is sufficiently large. Furthermore, membership structures can only be stable if no independent agent has an incentive to join one of the parties. By the following proposition, each party's platform has to satisfy a set of necessary conditions, conditional on the platform of the opposing party.

Proposition 1. *In every political equilibrium in which party R offers platform r , the leftist platform l satisfies the following two conditions:*

(i) *Moderate and extreme boundary: $l \in [\eta_1(r, c), \eta_2(r, c)]$, where both thresholds are given by both roots of function $A(l, r, c) = \Gamma(l, r) - c$ in l and satisfy $\eta_1(r, c) \leq l_\Gamma(r) \leq \eta_2(r, c)$.*

(ii) *Extreme boundary: $l > \lambda(r, c)$, where the threshold $\lambda(r, c)$ is given by the unique*

root of function $B(l, r, c) = \Gamma(l_\Gamma(r), r) - \Gamma(l, r) + 2p(l, r)(l_\Gamma(r) - l) - c$ in l and satisfies $\lambda(r, c) < l_\Gamma(r)$.

Proposition 1 implies that the leftist party's platform must be located in some well-defined interval, which depends on the opposing platform r . Part i is a consequence of the efficiency of parties derived in Lemma 2 and condition (I) on party members. No member (including the presidential candidate himself) would be willing to maintain his political activity if the activity of party L would not increase its policy payoff sufficiently strong. For every party member, the induced policy gain is weakly smaller than the policy effect function $\Gamma(l, r) = (r - l)p(l, r)$, which must exceed the membership cost c , thus. The moderate bound $\eta_2(r, c)$ follows from the necessity to have a sufficiently large platform difference $(r - l)$. No agent would be willing to bear the cost of c if the offered platforms were too similar. In particular, the positive costs of political activity eliminate the possibility of full policy convergence, the classical result due to Downs (1957). Additionally, there is an extreme boundary $\eta_1(r, c)$ since the members of party L would not be willing to support a party with negligible electoral prospects. By the quasi-concavity of the policy effect function $\Gamma(l, r)$, both boundaries are well-defined (see figure 3).

The second part of Proposition 1 follows from condition (II), according to which no independent agent must have an incentive to join a party. The extreme boundary $\lambda(r, c)$ is derived in two steps. Consider an allocation in which platform l is located to the left of the maximum effect platform $l_\Gamma(r)$. By Lemma 1, this platform will be chosen if and only if (a) it provides a higher policy effect than any other available platform and (b) the party median is even more extreme: $l = l_M(M^L, r) \geq m^L$. If available, the median party member would prefer to offer the platform with maximum policy effect $l_\Gamma(r)$.

If an independent agent with bliss point $w_i = l_\Gamma(r)$ were to join party L , he would thus become presidential candidate. Thus, an equilibrium with platform l' only exists if this agent cannot benefit from joining party L . On the one hand, he can clearly achieve a policy gain by joining. On the other hand, he can save the cost c and free-ride on the provision of party L by other leftist citizens by staying independent. The net gain from entering party L is given by

$$\begin{aligned} B(l, r, c) &= \tilde{v}_i(l_\Gamma(r), r) - c - \tilde{v}_i(l, r) \\ &= \underbrace{\Gamma[l_\Gamma(r), r] - p(l, r)[r + l - 2l_\Gamma(r)]}_{\text{policy gain}} - c \end{aligned} \quad . \quad (6)$$

In any political equilibrium, $B(l, r, c)$ must be negative. Thus, platform l has to

be sufficiently moderate. For values of l close enough to $l_\Gamma(r)$, the membership cost dominates the achievable policy gain. If platform l becomes more extreme, the net gain will however strictly increase for two reasons. First, as the distance between l and $l_\Gamma(r)$ increases, platform l becomes less attractive to the potential entrant. Second, the probability of party L 's victory in the general election becomes smaller. Consequently, there is a unique cut-off value $\lambda(r, c)$ such that there is an incentive to deviate whenever $l \leq \lambda(r, c)$ holds. Thus, the function $\lambda(r, c)$ represents an extreme boundary for platform l , conditional on the platform of party R .¹⁶

As the game is completely symmetric between both political parties, corresponding necessary conditions have to be fulfilled for the equilibrium platform of the rightist party R . The following corollary recapitulates the analysis so far and identifies a set of potential political equilibria.

Corollary 1. *In any political equilibrium, the party platforms l and r satisfy the following necessary conditions:*

1. *Platform l of the leftist party L is located in the interval*

$$B^L(r, c) = [\max \{\eta_2(r, c), \lambda(r, c)\}, \eta_1(r, c)]$$

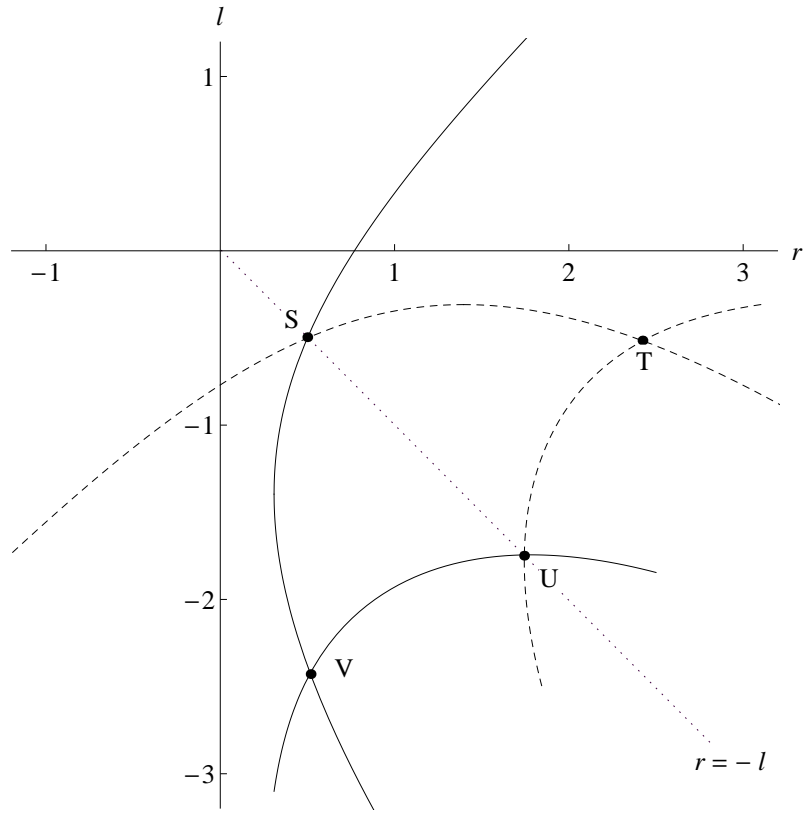
2. *Platform r of the rightist party R is located in the interval*

$$B^R(l, c) = [-\eta_1(-l, c), \min \{\eta_2(-l, c), -\lambda(-l, c)\}]$$

Note that for any given membership structure, there is a unique reaction function $l(M^L, r)$ with respect to the platform of the competing party R . Since the party structures are not given exogenously, however, the correspondences $B^L(r, c)$ and $B^R(l, c)$ represent the collection of all reaction functions for the complete set of stable membership structures. Figure 6 depicts these correspondences for both parties in a diagram with platform r on the horizontal and platform l on the vertical axis. The upper and lower bounds for platform l are given by the solid lines, the bounds for platform r by the dashed lines. Consider an allocation with any pair of platforms l and r . If the point (r, l) is not located in the area between both solid lines, platform l cannot be supported in any equilibrium, i.e., by any membership structure.

¹⁶Note that $l > \lambda(r, c)$ is a necessary but not a sufficient condition for the stability of party L 's membership structure. More exactly, one can show that agents with slightly more moderate bliss point $w_i > l_\Gamma(r)$ have an even larger incentive to join party L and still prefer to join party L in constellations with a slightly more moderate platform $l = \lambda(r, c) + \varepsilon$. While the construction of a sufficient condition is possible, it does not provide additional economic insights.

Figure 4: Stable parties and supportable platforms



Horizontal axis: Rightist party platform r . Vertical axis: Leftist party platform l . Region $STUV$: Potential equilibrium platforms for $c = 0.5$, $\sigma = 1$.

In figure 6, region $STUV$ represents the intersection of the correspondences $B^L(r, c)$ and $B^R(l, c)$ for the parameter values $c = 0.5$ and $\sigma = 1$. It contains the set of all tuples (l, r) that satisfy the necessary platform conditions established in Proposition 1. The set of political equilibria is a subset of this intersection, as the conditions identified in Proposition 1 are necessary, but not sufficient for equilibrium. For any combination of platforms outside this interval, in contrast, there is a profitable deviation for at least one agent.

Figure 6 graphically shows that the distance between both party platforms is bounded from above as well as from below for the considered example. Formally, upper and lower boundaries for the platform distance can be derived from the conditional boundary functions $\eta_2(r, c)$ and $\lambda(r, c)$ for all parameter values. In the *minimal distance equilibrium* S , both parties offer the most moderate platforms that can be supported against each other. This implies that the policy effect delivered by both platforms is exactly sufficient to cover the membership cost c . In the *maximum distance equilibrium* U , both parties nominate the most extreme presidential candidates for which the necessary conditions in Proposition 1 hold. By the symmetry between both parties, the rightist party's platforms in both constellations is a fixed point of the conditional boundary function: $\underline{r}(c) = -\eta_2(\underline{r}(c), c)$ and $\bar{r}(c) = -\lambda(\bar{r}(c), c)$.

Proposition 2. *In every political equilibrium, the platform distance $r - l$ is*

- (i) *weakly larger than $2c > 0$, and*
- (ii) *smaller than $2\bar{r}(c)$, where $\bar{r}(c) > c$ is defined as the unique root of $G(r, c) = \lambda(r, c) + r$ in r .*

For part (i), note that the function $-\eta_2(r, c)$ is strictly decreasing in r . Thus, it has at most one fixed point. It is easy to show that this fixed point is given by the exogenous membership cost c .¹⁷

The proof for part (ii) of Proposition 2 is slightly more complicated. First, I show that the derivative of the maximum effect platform $l_\Gamma(r)$ (the best answer) with respect to r is always larger than -1 . Intuitively, whenever the platform of party R becomes more extreme, party L would achieve a higher winning probability *ceteris paribus*. While the members of party L might prefer to change their platform as well, their best response will never involve a more extreme shift that would eliminate this advantage. Second, the incentives for the potential entrant with bliss point $l_\Gamma(r)$ change: his policy payoffs both in case of joining party L and in

¹⁷More concretely, it can be shown that either $\eta_2(r, c)$ has a unique fixed point in r or that the associated boundary $\eta_1(r, c)$ has a unique fixed point in r . In both cases, the fixed point is given at $r = c$.

case of staying independent increase because the rightist platform r becomes less competitive. Altogether, the derivative of the extreme boundary function $-\lambda(r, c)$ in r is smaller than 1 such that there can be at most one fixed-point. Exploiting this fixed-point property of \bar{r} , it can finally be shown that the defining function $G(r, c) = \lambda(r, c) + r$ has a unique root for any $c \geq 0$.

Proposition 2 establishes the main result of this paper and represents a qualification of the insights provided in the basic citizen candidate model (Besley & Coate 1997). As in the citizen candidate model, there can only be limited policy convergence due to the costs of political activity. In contrast, there can only be limited polarization in my model because of the coordination possibilities provided by political parties. The following proposition establishes the existence of equilibria for all parameter constellations, ensuring the relevance of these insights.

Proposition 3. *The set of political equilibria is non-empty for all levels of the membership cost $c \geq 0$.*

By this proposition, platform tuples (l, r) and stable membership structures exist such that the sufficient conditions (I)-(III) are satisfied. To give an intuition for this result, consider a political constellation where $l = m^L \in [l_\Gamma(r), \eta_2(r, c)]$ and party L is efficient according to Lemma 2. In this situation, the policy platform l is given by the bliss point of the median party member who prefers this constellation to any other platform (see Lemma 1). Consequently, the offered platform will not change as long as the party median is constant. Clearly, it is possible to construct membership structures (with multiple party members that share the party median's bliss point m^L) such that m^L does not change due to the entry of any independent agent. This implies that neither an independent agent nor a current member of party R has an incentive to join party L . Moreover, if party L is efficient and the bliss points of all its member are sufficiently leftist, no member of L would benefit from becoming independent (as the moderate boundary condition $l < \eta_2(r, c)$ is satisfied). If platform r and membership set M^R satisfy equivalent conditions, no agent $i \in N$ can profitably change his party affiliation. Thus, the existence of a political equilibrium with policy platforms l and r is guaranteed.

7 Comparative statics

In the previous sections, I have established the existence of political equilibria and their properties given some fixed membership cost c . Moreover, I have focused on the specific case of the standard normal distribution, $\sigma = 1$. This section investigates

the effects of changes with respect to both exogenous parameters, c and σ . In particular, I am interested in the effects on the boundary functions $\underline{r}(c, \sigma)$, $\bar{r}(c, \sigma)$ and the implications for equilibrium platform distance.¹⁸ First, I consider variations in the membership cost c , a crucial ingredient of the citizen candidate framework.

Proposition 4. *The minimal distance boundary $\underline{r}(c, \sigma)$ and the maximal distance boundary $\bar{r}(c, \sigma)$ are strictly increasing in c :*

$$\frac{d\underline{r}(c, \sigma)}{dc} = 1 > 0, \quad \frac{d\bar{r}(c, \sigma)}{dc} > 0.$$

For c approaching zero, the limits of both boundaries are given by

$$\lim_{c \rightarrow 0} \underline{r}(c, \sigma) = 0, \text{ and } \lim_{c \rightarrow 0} \bar{r}(c, \sigma) = \frac{0.5\sigma}{\phi(0)}.$$

In equilibrium, party members are only willing to maintain their activity if each party's activity has a sufficiently large effect on expected policy, i.e., if the platform distance is large enough. As the cost of political activity becomes larger, party members demand increasing policy effects and platform distances. Thus, the minimal distance boundary increases. If, however, the membership cost approaches zero, the members will be willing to accept increasing convergence. In the limit, party membership is costless and is even consistent with full policy convergence.

With respect to the maximal distance boundary, increasing membership costs tighten the combined coordination and free-riding problem faced by potential activists. Whenever platform l is located to the left of the maximal effect position $l_\Gamma(r)$, all party members unanimously prefer to have a presidential candidate with bliss point $l_\Gamma(r)$ instead. As long as l does not exceed the extreme boundary, however, agents with this bliss point prefer to free-ride on the current party members, because the feasible policy gain is outweighed by the membership cost. With increasing c , an even larger policy gain is required to make political activity profitable. Thus, more extreme platforms can be supported in equilibrium and the maximal distance between both parties increases. When the membership cost converges to zero, on the other hand, this coordination problem vanishes and an agent with a desirable bliss point $w_i = l_\Gamma(r)$ will be willing to join party L whenever he is sure that he will become presidential candidate, i.e. whenever the initial platform is more extreme.

¹⁸In the previous sections, σ was set equal to one in order to simplify notation, and all boundaries were written as functions of c only. In the following, I will allow for variations in σ . With some abuse of notation, I redefine the boundaries η_1 , η_2 , λ , \underline{r} and \bar{r} to be the corresponding functions of c and σ .

With $c \rightarrow 0$, an independent agent with bliss point $l_\Gamma(r)$ benefits from entering the party whenever this has an effect on the party's platform. Thus, the party median members can always recruit their preferred candidates. Proposition 4 gives the mutually best platform choices, $l = -\frac{0.5\sigma}{\phi(0)}$ and $r = \frac{0.5\sigma}{\phi(0)}$, that would be chosen by party medians with extreme policy preferences, $m^L \rightarrow -\infty$ and $m^R \rightarrow \infty$.

Proposition 5. *The minimal distance boundary $\underline{r}(c, \sigma)$ is independent of the degree of electoral uncertainty while the maximal distance boundary $\bar{r}(c, \sigma)$ is strictly increasing in σ :*

$$\frac{d\underline{r}(c, \sigma)}{d\sigma} = 0, \quad \frac{d\bar{r}(c, \sigma)}{d\sigma} > 0.$$

In the case of full electoral certainty, both boundaries coincide:

$$\lim_{\sigma \rightarrow 0} \underline{r}(c, \sigma) = \lim_{\sigma \rightarrow 0} \bar{r}(c, \sigma) = c$$

In figure 6, both the *minimal* and the *maximal distance equilibrium* involve symmetric platforms $l = -r < 0$, and equal winning probabilities for both parties (independently of σ). In the minimal distance case, the platform distance must be large enough so that party members do not benefit from leaving their party, and causing its inactivity. Thus, the membership cost c must not outweigh the policy effect $\Gamma(-r, r, \sigma) = [r - (-r)]_{\frac{1}{2}} = r$, which is not affected by increasing uncertainty in this symmetric constellation. The same policy effect is even given for $\sigma = 0$, the case of a perfectly known population median.¹⁹

In contrast, the maximal distance boundary is derived by considering a shift from an extreme to a more centrist platform, i.e., a deviation from a symmetric to an asymmetric allocation. This platform shift is profitable to the party members and the potential entrant if and only if the winning probability increases sufficiently. Higher electoral risk however reduces the increase in winning probability and the incentive for independent agents to join a political party. Overall, increasing electoral risk diminishes the inherent centripetal forces of platform choice in endogenous parties, and more polarized platforms can be supported in equilibrium.

Corollary 2. *With electoral uncertainty, $\sigma = 0$, the platforms of both parties are given by $r = c$ and $l = -c$ in every two-party equilibrium.*

For the case of electoral certainty, all voters know the median voter position $m = 0$ ex ante. The uniqueness of party platforms for this case, $\sigma = 0$, is directly

¹⁹Note, however, that the moderate boundary $\eta_2(r, c, \sigma)$ changes for all values $r \neq \underline{r}$. Specifically, the moderate boundary function rotates clockwise with increasing σ .

implied by the limits of both boundaries in Proposition 5. With σ approaching zero, the lower and upper boundaries $\underline{r}(c, \sigma)$, $\bar{r}(c, \sigma)$ converge and, in the limit, coincide. The economic intuition for this case is however simple, and can be provided directly.

If the bliss point of the population median is ex ante known, the political equilibrium can only involve two active parties if, first, those offer symmetric platforms $l = -r$, giving rise to identical winning probabilities. For any other constellation, one party would inevitably lose the general election and have no effect on the implemented policy. Thus, no agent would be willing to bear the cost of engaging in this party. Second, there cannot be an equilibrium with $r = -l < c$, as the distance between both parties and the implied policy effect would be too small for any agent to be willing to bear the cost of political activity.

Finally, platform polarization is limited by the possibility to recruit and nominate moderate independent citizens. Under electoral certainty, if any entrant with bliss point $w_i \in (l, 0)$ were to join party L and to be nominated as presidential candidate, he would certainly win the general election against platform $r = -l$. Since this would induce a shift of the expected policy $E(x) = 0$ to $w_i \in (l, 0)$, all members of party L would strictly prefer his nomination. Thus, an equilibrium with divergent platforms exists if and only if no independent agent can benefit from this deviation. For the potential entrant, entering party L improves the policy payoff by r . For $r > c$, joining party L would clearly be a profitable deviation. Thus, there is a unique political equilibrium with $r = c$ and $l = -c$.

Consequently, the effect of endogenous party formation is most obvious in the case of electoral certainty, which is the case on the basic citizen candidate model concentrates. The first two arguments also apply in the model by Besley & Coate (1997), implying that the platform distance must exceed a lower bound. Without party formation, however, there is no mechanism limiting policy polarization in equilibrium. Consequently, every symmetric allocation with platform distance beyond the lower bound represents a political equilibrium.²⁰ The resulting multiplicity of equilibria contrasts sharply with the unique determination of equilibrium platforms derived in Corollary 2.²¹

²⁰In the model by Besley & Coate (1997), the lower bound on the platform distance depends on the cost of running in the general election, which has to be paid by a single candidate. Here, the lower bound instead depends on the cost of party membership. Intuitively, the latter cost should be considerably smaller.

²¹Note that, in the Osborne & Slivinski (1996) version of the citizen candidate model, the platform distance is not uniquely determined, but is nevertheless bound from below and from above. The upper bound follows from the assumption of sincere voting. Intuitively, extreme polarization is prevented by the assumption that voters are able to coordinate in Osborne & Slivinski (1996), while it is hindered (more completely) by the coordination of party members in my model.

8 Conclusion

Building on the citizen candidate framework, this paper has investigated political competition between endogenously formed parties. There seems to be little doubt that modeling political competition between parties instead of individual candidates brings theory closer to real-world politics. The model possesses a number of compelling properties. The analysis has focused on equilibria with two active parties, which are shown to exist for all levels of membership costs and electoral uncertainty. In contrast to the median voter model (Downs 1957), there can never be full convergence of party platforms in equilibrium. Thus, the party formation model reproduces one of the main results of the basic citizen candidate model without parties (Besley & Coate 1997). At the same time, allowing for party formation alleviates the major drawback of the citizen candidate model, the extreme multiplicity of equilibria. This becomes most obvious in the benchmark case of full electoral certainty, i.e., perfect information about the median voter's preferences. In this case, infinitely many equilibria with two running candidates exist in the basic citizen candidate model. In contrast, the party formation model possesses a unique equilibrium with two active parties.

This paper has concentrated on a particularly simple framework to enhance the clarity of the arguments. A richer model could allow for, e.g., a larger number of potential parties, a multi-dimensional policy space, more general rules with respect to intra-party decision-making, more general policy preferences, or different modeling of electoral uncertainty. Further analyses show that the economic intuition and the main results are robust with respect to all these modifications.

Appendix

Proof of Lemma 1

Lemma 1 identifies the optimal choice of party platform l in the primary election of party L , conditional on the membership structure M^L and belief \hat{r} . It is proven through a series of lemmas.

Lemma 3. *Given any platform belief \hat{r} , the platform preferences of party members over the set of potential platforms fulfill the single crossing property.*

Proof. The single-crossing property implies that the preferences of agent i with respect to pairwise comparisons between two alternatives are monotonic in his bliss point ω_i . Consider the case $l_1 < l_2 < \hat{r}$. An agent with bliss point w_i prefers l_1 to be the platform of party L instead of l_2 if and only if the following condition holds:

$$\begin{aligned} F(l_1, l_2, \hat{r}, w_i) &= \tilde{v}_i(l_1, \hat{r}) - \tilde{v}_i(l_2, \hat{r}) \\ &= p(l_1, \hat{r})(|w_i - \hat{r}| - |w_i - l_1|) - p(l_2, \hat{r})(|w_i - \hat{r}| - |w_i - l_2|) \\ &> 0 \end{aligned}$$

First, note that $F(l_1, l_2, \hat{r}, w_i) \big|_{w_i < l_1} = p(l_1, \hat{r})(\hat{r} - l_1) - p(l_2, \hat{r})(\hat{r} - l_2) = \Gamma(l_1, \hat{r}) - \Gamma(l_2, \hat{r}) = -F(l_1, l_2, \hat{r}, w_i) \big|_{w_i > \hat{r}}$. Thus, agents with bliss points at both extremes of the policy space will always have conflicting preferences.

Second, the derivative of function F with respect to w_i is given as

$$\frac{dF(\cdot)}{dw_i} = \begin{cases} 0 & \text{for } w_i \leq l_1 \\ -2p(l_1, \hat{r}) < 0 & \text{for } w_i \in (l_1, l_2] \\ 2[p(l_2, \hat{r}) - p(l_1, \hat{r})] > 0 & \text{for } w_i \in (l_2, \hat{r}] \\ 0 & \text{for } w_i \geq \hat{r} \end{cases}$$

As long as platforms l_1 and l_2 provide different policy effects, there is exactly one cut-off value $\psi(l_1, l_2, \hat{r})$ such that $F(l_1, l_2, \hat{r}, \psi(\cdot)) = 0$ holds.

For $\Gamma(l_1, \hat{r}) > \Gamma(l_2, \hat{r})$, the cut-off is located in the interval (l_1, l_2) . All agents with bliss points to the left of l_1 prefer l_1 and we get the following version of the single-crossing property:

$$\begin{aligned} F(l_1, l_2, \hat{r}, w_i) \leq 0 &\Rightarrow F(l_1, l_2, \hat{r}, w_j) < 0 \quad \forall w_j > w_i, \text{ and} \\ F(l_1, l_2, \hat{r}, w_i) \geq 0 &\Rightarrow F(l_1, l_2, \hat{r}, w_k) > 0 \quad \forall w_k < w_i \end{aligned}$$

For $\Gamma(l_1, \hat{r}) < \Gamma(l_2, \hat{r})$, the cut-off is located in the interval (l_2, \hat{r}) . This time, all

agents to the left of l_1 prefer platform l_2 and the preferences exhibit the following monotonicity:

$$\begin{aligned} F(l_1, l_2, \hat{r}, w_i) \leq 0 &\Rightarrow F(l_1, l_2, \hat{r}, w_j) < 0 \quad \forall w_j < w_i, \text{ and,} \\ F(l_1, l_2, \hat{r}, w_i) \geq 0 &\Rightarrow F(l_1, l_2, \hat{r}, w_k) > 0 \quad \forall w_k > w_i \end{aligned}$$

For the special case of identical policy effects $\Gamma(l_1, \hat{r}) = \Gamma(l_2, \hat{r})$, all agents with bliss points to the left of l_1 as well as to the right of \hat{r} are indifferent between both platforms while the moderate agents in the interval (l_1, \hat{r}) strictly prefer the moderate platform l_2 . Trivially, the preferences satisfy the single-crossing property in the following sense:

$$F(l_1, l_2, \hat{r}, w_i) \geq 0 \Rightarrow F(l_1, l_2, \hat{r}, w_k) \geq 0 \quad \forall w_k \in \mathbb{R}$$

Similar arguments apply for other constellations, e.g. $l_1 < \hat{r} < l_2$. \square

Lemma 4. *For any member set M^L and platform belief \hat{r} , there is a Condorcet winner in the primary election of party L .*

Proof. Let the finite set of feasible platforms, i.e., the set of bliss points of party L 's members, be given by A . Denote by l^* the platform in A that maximizes the utility of the median party member with platform $w_i = m^L$:

$$l^* = \arg \max_{l \in A} \tilde{v}_i(l, \hat{r}) = -p(l, \hat{r}) |\hat{r} - m^L| - [1 - p(l, \hat{r})] |l - m^L|$$

By the single-crossing property established in Lemma 3, platform l^* is preferred by a majority of party members (the median member plus either all members with $w_j \leq m^L$ or all members with $w_j \geq m^L$) to any other available platform $l' \in A$. Consequently, l^* wins any pairwise election and represents a Condorcet winner. \square

Lemma 5. *On $(-\infty, r)$, the policy effect function $\Gamma(l, r) = p(l, r)(r - l)$ is strictly quasi-concave in l and has a unique maximizer $l_\Gamma(r)$.*

Proof. For $l < r$, the policy effect function and its first and second derivatives with respect to w_i are given as

$$\begin{aligned} \Gamma(l, r) &= (r - l)\Phi\left(\frac{r + l}{2}\right), \\ \Gamma_1(l, r) &= \frac{d\Gamma(l, r)}{dl} = -\Phi\left(\frac{r + l}{2}\right) + \frac{r - l}{2}\phi\left(\frac{r + l}{2}\right), \text{ and} \\ \Gamma_{11}(l, r) &= \frac{d^2\Gamma(l, r)}{dl^2} = -\frac{1}{2}\phi\left(\frac{r + l}{2}\right) - \frac{1}{2}\phi\left(\frac{r + l}{2}\right) + \frac{r - l}{4}\phi'\left(\frac{r + l}{2}\right) \end{aligned}$$

$$= - \left(1 + \frac{r^2 - l^2}{8} \right) \underbrace{\phi \left(\frac{r+l}{2} \right)}_{>0}.$$

As l approaches $-\infty$ and r , respectively, the first derivative goes to:

$$\begin{aligned} \lim_{l \rightarrow -\infty} \Gamma'(l, r) &= \lim_{l \rightarrow -\infty} \frac{r-l}{2} \phi \left(\frac{r+l}{2} \right) = \lim_{l \rightarrow -\infty} \frac{(r-l)/2}{1/\phi \left(\frac{r+l}{2} \right)} \\ &= \lim_{l \rightarrow -\infty} \frac{\phi \left(\frac{r+l}{2} \right)^2}{\phi \left(\frac{r+l}{2} \right)} = \lim_{l \rightarrow -\infty} \frac{\phi \left(\frac{r+l}{2} \right)^2}{-\frac{r+l}{2} \phi \left(\frac{r+l}{2} \right)} \\ &= \lim_{l \rightarrow -\infty} \frac{\phi \left(\frac{r+l}{2} \right)}{-\frac{r+l}{2}} = 0, \text{ and} \\ \lim_{l \rightarrow r} \Gamma'(l, r) &= -\Phi \left(\frac{2r}{2} \right) < 0. \end{aligned}$$

The second derivative $\Gamma''(l, r)$ is negative if and only if

$$\begin{aligned} 1 + \frac{r^2 - l^2}{8} &> 0 \\ \Rightarrow l &\in (-\sqrt{r^2 + 8}, +\sqrt{r^2 + 8}) \end{aligned}$$

For $l < -\sqrt{r^2 + 8}$, the policy effect function is thus strictly convex. Moreover, it is strictly increasing in this region, since $\lim_{l \rightarrow r} \Gamma_1(l, r) = 0$. In the interval $(-\sqrt{r^2 + 8}, r)$, the function is strictly concave. Combining these results we know that $\Gamma(l, r)$ is strictly quasi-concave on $(-\infty, r)$.

As $\Gamma_1(l, r)$ is positive for $l < -\sqrt{r^2 + 8}$ and negative for $l \rightarrow r$, the policy effect function must have a unique maximum on $l \in (-\infty, r)$. This maximum must be located in the interval $(-\sqrt{r^2 + 8}, r)$. \square

Lemma 6. *For any membership set M^L and belief \hat{r} , the policy payoff of the median party member with bliss point $m^L < \hat{r}$ is maximized by platform $l(M^L, \hat{r}) = \max \{m^L, l_M(\hat{r}, M^L)\}$, where $l_M(\hat{r}, M^L) = \arg \max_{l \in A} \Gamma(l, r)$.*

Proof. The party median's policy payoff is given by $\tilde{v}_{m^L}(l, r) = p(l, r) \{ |r - m^L| - |l - m^L| \} - |r - m^L|$. For $l \leq 2m^L - \hat{r}$ and $l \geq r$, the payoff is smaller than in the case of certain implementation of policy r , while it is strictly larger for any platform in the interval $(2m^L - \hat{r}, \hat{r})$. We can thus focus on this interval, where the derivative of $\tilde{v}_{m^L}(l, r)$ with respect to l is given by:

$$\frac{d\tilde{v}_{m^L}(l, r)}{dl} = \begin{cases} \frac{dp(l, r)}{dl}(r + l - 2m^L) + p(l, r) > 0 & \text{for } l < m^L \\ \Gamma_1(l, r) & \text{for } l \in (m^L, \hat{r}) \end{cases}$$

Consequently, the median member prefers its own bliss point m^L to any more leftist platform, independently of the implied policy effects. Furthermore, he will prefer a more moderate platform l' to his own bliss point if and only if l' provides a larger policy effect $\Gamma(l, r)$. Thus, \tilde{v}_{m^L} will be maximized by the maximum of m^L and the maximum effect platform $l_M(\hat{r}, M^L)$. \square

Proof of Lemma 2

Assume there is a two-party equilibrium with membership structures M_0^L, M_0^R , party medians m_0^L, m_0^R and platforms $l_0 = l(M_0^L, r_0)$, $r_0 = r(M_0^R, l_0)$ such that party L is not efficient, i.e., $\sum_{i \in N} \alpha_i^L \geq C + c$. Then, neither any member $j \in M_0^L$ nor any independent citizen with arbitrary bliss point w_k must have an incentive to deviate. In particular, this must be true for the most extreme member j with bliss point $w_j = \min \{w_i : i \in M^L\} \leq m^L$. Assume he deviates by reducing his contribution by c , which has two effects. First, the agent saves the membership cost of c . Second, if this deviation implies he leaves party L , the party median changes and becomes more moderate $m_1^L \geq m_0^L$. Since the party is not efficient by assumption, it will still be active in the general election. However, the adopted platform changes to $l_1 \geq l_0$. The deviation is non-profitable for agent j if and only if the induced policy loss would be larger than the membership cost. Otherwise he could profitably deviate by leaving party L , implying that the initial allocation cannot represent an equilibrium. Thus, the following condition must hold:

$$\tilde{v}_j(l_0, r_0) - \tilde{v}_j(l_1, r_0) = \Gamma(l_0, r_0) - \Gamma(l_1, r_0) > c \quad (7)$$

Next, consider the incentives for an independent with bliss point $w_k \in (l_1, r_0)$ to join party L . If he would enter party L , this deviation would have exactly the same effect on the party median as the exit of the extreme member j . Again, we have the new party median $m_1^L \geq m_0^L$. Furthermore, the new platform will either be given by the bliss point of the entrant w_k or by the platform adopted after j 's exit, $l_1 \geq l_0$, once again.

Consider the latter is true, and l_1 is the newly adopted platform. The change in agent k 's policy payoff is given by

$$\begin{aligned} \tilde{v}_k(l_1, r_0) - c - \tilde{v}_k(l_0, r_0) &= p(l_1, r_0) [r_0 + l_1 - 2w_k] - p(l_0, r_0) [r_0 + l_0 - 2w_k] - c \\ &= \Gamma(l_0, r_0) - \Gamma(l_1, r_0) + 2(r - w_k)[p(l_1, r_0) - p(l_0, r_0)] - c \\ &> \Gamma(l_0, r_0) - \Gamma(l_1, r_0) - c. \end{aligned}$$

The last expression is positive whenever condition (7) above holds. If the new platform is instead given by the bliss point of the entrant w_k , the induced increase of the policy payoff to k will be even larger.

Thus, if the extreme member j cannot deviate profitably by leaving party L , joining the party will be a profitable deviation for agent k . In other words, if party L is not efficient, there is always a profitable deviation for at least one of these two agents, which is a contradiction to the equilibrium assumption.

Proof of Proposition 1

Part (i) For the first part of Proposition 1, assume the policy effect associated with platform l does not exceed the membership cost: $\Gamma(l, r) < c$. By Lemma 2, parties are efficient in every political equilibrium. Whenever one member of L deviates by choosing $\alpha_i^L = 0$ and leaving party L , the sum of contributions to party L falls below the amount required cost of running C . Thus, there will no presidential candidate nominated by L in the general election, and the rightist candidate wins certainly. For the presidential candidate or any more extreme member with $\omega_i < l$, this deviation induces a utility change of

$$v_i(r) - (\tilde{v}_i(l, r) - \alpha_i^L) \geq v_i(r) - \tilde{v}_i(l, r) + c = -\Gamma(l, r) + c > 0$$

Thus, leaving party L would be profitable to agent i and, in equilibrium, platform l must fulfill the condition $\Gamma(l, r) \geq c$.

By Lemma 5 in the appendix, the policy effect function is strictly quasi-concave and approaches 0 for $l \rightarrow -\infty$ and $l \rightarrow r$. If $\Gamma(l_\Gamma(r), r) < c$, function A has no root. If $\Gamma(l_\Gamma(r), r) = c$, the maximum effect platform $l_\Gamma(r) = \eta_1(r, c), \eta_2(r, c)$ represents the unique root; the only equilibrium with platform r also involves $l_\Gamma(r)$. If finally $\Gamma(l_\Gamma(r), r) > c$, function $A(x, r, c)$ has two roots in x , denoted by $\eta_1(r, c) < l_\Gamma(r)$ and $\eta_2(r, c) > l_\Gamma(r)$. Then, $A(x, r, c) \geq 0$ holds if and only if $x \in [\eta_1(r, c), \eta_2(r, c)]$. Consequently, the condition stated in the first part of Lemma 3 is only fulfilled for platforms l in this interval.

Part (ii) For the second part of Proposition 1, consider an allocation with platforms r and $l < l_\Gamma(r)$. This position l can represent the outcome of a primary subgame equilibrium if and only if it provides higher policy effect than the bliss point of any other member in party L and if the party median is even more extreme: $m^L(M^L) \leq l$. The allocation can only represent an equilibrium, if an independent agent with bliss point $w_i = l_\Gamma(r)$ cannot profitably deviate by joining party L . Given

this deviation, agent i would certainly win the primary of party L and run as its presidential candidate in the general election since his bliss point is preferred to any other available platform by the party median. On the one hand, this change in L 's platform increases the policy payoff to the entrant i . On the other hand, he has to pay the membership cost c . Overall, the induced change of utility for agent i is given by

$$B(l, r, c) = \tilde{v}_i(l_\Gamma(r), r) - c - \tilde{v}_i(l, r) = \Gamma(l_\Gamma(r), r) - p(l, r)[r + l - 2l_\Gamma(r)] - c$$

Agent i benefits from joining party L if and only if $B(l, r, c) > 0$ holds. The function has a unique root in l which is located in the interval $[2l_\Gamma(r) - r, l_\Gamma(r)]$. First, the deviation is profitable for any $l < 2l_\Gamma(r) - r$ since

$$\begin{aligned} B(l, r, c) &= \Gamma(l_\Gamma(r), r) - p(l, r) \underbrace{[r + l - 2l_\Gamma(r)]}_{<0} - c \\ &> \Gamma(l_\Gamma(r), r) - c > \Gamma(l, r) - c \geq 0 \end{aligned}$$

The second inequality holds by the definition of $l_\Gamma(r)$, the last one must hold in any potential equilibrium by part (i). Next, joining party L is obviously not profitable for i if l approaches $l_\Gamma(r)$: $\lim_{l \rightarrow l_\Gamma(r)} B(l, r, c) = -c < 0$. Finally, the incentive for political activity is strictly decreasing in l in this interval:

$$\frac{dB(l, r, c)}{dl} = -\frac{dp(l, r)}{dl} \underbrace{(r + l - 2l_\Gamma(r))}_{>0} - p(l, r) < 0$$

Thus, function $B(l, r, c)$ has a unique root in l in $(-\infty, l_\Gamma(r))$, which we define as the extreme boundary function $\lambda(r, c)$. For all platforms $l < \lambda(r, c)$, joining party L and becoming its presidential candidate at the cost of c is a profitable deviation for agent i . Consequently, there cannot be an equilibrium with platforms r and $l < \lambda(r, c)$.

By the symmetry of both parties, the platform of party R must satisfy $r \leq -\lambda(-l, c)$ in every political equilibrium.

Proof of Proposition 2

Proposition 2 builds on the following lemma with respect to derivative of function $l_\Gamma(r)$.

Lemma 7. *The derivative of the maximum effect platform $l_\Gamma(r)$ with respect to platform r is given by $\frac{dl_\Gamma(r)}{dr} > -1$.*

Proof. The maximum policy effect platform $l_\Gamma(r)$ is implicitly (and uniquely) defined by the equation

$$\frac{d\Gamma(l_\Gamma(r), r)}{dl} = \frac{r - l_\Gamma(r)}{2} \phi\left(\frac{r + l_\Gamma(r)}{2}\right) - \Phi\left(\frac{r + l_\Gamma(r)}{2}\right) = 0$$

Substituting in $x = \frac{r + l_\Gamma(r)}{2}$, we can the following dependence:

$$\begin{aligned} l_\Gamma(r) = f(x) &= x - \frac{\Phi(x)}{\phi(x)}, \text{ and} \\ r = g(x) &= x + \frac{\Phi(x)}{\phi(x)}. \end{aligned}$$

Making use of these function, we can rewrite the function $l_\Gamma(r) = f(x(r)) = f(g^{-1}(r))$.

For being able to use the inverse function g^{-1} , function g must be monotonic in x . For the standard normal distribution, $\frac{\phi(x)}{\phi(x)} = -x$ holds, so that the derivative of g equals $g'(x) = 2 + x \frac{\Phi(x)}{\phi(x)}$. We show that $x \frac{\Phi(x)}{\phi(x)} > -1$, which is a sufficient condition for $g'(x) > 0$ everywhere on \mathbb{R} . We do this by considering the auxiliary function $a(x) = x\Phi(x) + \phi(x)$, and proving that this function is positive for all $x \in \mathbb{R}$. We can derive the limit of $a(x)$ for $x \rightarrow -\infty$ by making use of l'Hopital's rule several times:

$$\begin{aligned} \lim_{x \rightarrow -\infty} a(x) &= \lim_{x \rightarrow -\infty} [x\Phi(x) + \phi(x)] = \lim_{x \rightarrow -\infty} \frac{x}{1/\Phi(x)} + 0 \\ &= \lim_{x \rightarrow -\infty} \frac{1}{-\phi(x)/[\Phi(x)]^2} = \lim_{x \rightarrow -\infty} -\frac{\Phi(x)^2}{\phi(x)} = \lim_{x \rightarrow -\infty} -\frac{2\Phi(x)\phi(x)}{\phi(x)} \\ &= \lim_{x \rightarrow -\infty} -\frac{2\Phi(x)\phi(x)}{-x\phi(x)} = \lim_{x \rightarrow -\infty} \frac{2\Phi(x)}{x} = 0 \end{aligned}$$

Moreover, $a(x)$ is strictly increasing in x , since $a'(x) = \Phi(x) + x\phi(x) + \phi(x) = \Phi(x) > 0$ for any $x \in (-\infty, +\infty)$. Thus, we have shown that $a(x) > 0$ for every $x \in (-\infty, +\infty)$, which is equivalent to

$$\begin{aligned} x\Phi(x) &> -\phi(x) \\ \Leftrightarrow x \frac{\Phi(x)}{\phi(x)} &> -1 \end{aligned}$$

Consequently, $g'(x) = 2 + x \frac{\Phi(x)}{\phi(x)} > 0$ holds for all x , and the inverse function $x(r) = g^{-1}(r)$ is well-defined. Its derivative is given by $x'(r) = [g'(g^{-1}(r))]^{-1} > 0$.

Second, consider the maximum effect platform function $l_\Gamma(r) = f(g^{-1}(r))$. Its

derivative is given by

$$\begin{aligned}\frac{dl_{\Gamma}(r)}{dr} &= f'(x)x'(r) = \left[-x \frac{\Phi(x)}{\phi(x)} \right] x'(r) \\ &= - \frac{x\Phi(x)/\phi(x)}{2 + x\Phi(x)/\phi(x)}\end{aligned}$$

As shown above, the denominator of this fraction is strictly positive. The sign of the numerator, and thus the complete expression, equals the sign of x . For all $x < 0$, it follows that $\frac{dl_{\Gamma}(r)}{dr} > 0$ (note that $x < 0$ results for $r < \frac{0.5}{\phi(0)}$). For $x > 0$, the numerator is positive, but strictly smaller than the denominator, implying that $\frac{dl_{\Gamma}(r)}{dr} \in (-1, 0)$ holds. Thus, the derivative of $l_{\Gamma}(r)$ is larger than -1 on its whole domain. \square

Making use of Lemma 7, Proposition 2 can be proven in the following.

Part (i): Lower bound By Lemma 3, the moderate platform boundary is defined as the larger root of $A(l, r, c) = \Gamma(l, r) - c$. Making use of this implicit definition, the derivative of $\eta_2(r, c)$ with respect to r is given by

$$\frac{d\eta_2(r, c)}{dr} = - \frac{\partial A / \partial r}{\partial A / \partial \eta_2} = - \frac{\Phi(\frac{\eta_2+r}{2}) + \frac{r-\eta_2}{2}\phi(\frac{\eta_2+r}{2})}{-\Phi(\frac{\eta_2+r}{2}) + \frac{r-\eta_2}{2}\phi(\frac{\eta_2+r}{2})}$$

The numerator of this expression is positive. As η_2 is the larger root of $A(l, r, c)$, the denominator is negative by the strict quasi-concavity of the policy effect function. Thus, $\frac{d\eta_2(r, c)}{dr} > 0$ holds for all $r \in \mathbb{R}$. Thus, the equation $r = -\eta_2(r, c)$ has at most one solution in r . Denote this solution by $\underline{r}(c)$. In this intersection, both platform are equal to their conditional minimal boundaries: $l = \eta_2(r, c)$ and $r = -\eta_2(-l, c)$. The intersection exists if and only if $\eta_2(r, c) < -r$ holds for some $r \in \mathbb{R}$.

On the other hand, the extreme boundary value $\eta_1(r, c)$ (also defined in Lemma 7) is strictly decreasing in r , and its derivative is given by $\frac{d\eta_1(r, c)}{dr} < -1$. Thus, the equation $r = -\eta_1(r, c)$ has at most one solution, too. However, this solution exists if and only if $\eta_1(r, c) > -r$ for some $r \in \mathbb{R}$ which is equivalent to $\eta_2(r, c) > -r$ for all $r \in \mathbb{R}$. Thus, the 45° line has either a unique intersection with the function $\eta_2(r, c)$ or a unique intersection with the function $\eta_1(r, c)$.

Finally, plugging in $r = \underline{r}$, $l = -\underline{r}$ gives

$$A(-\underline{r}, \underline{r}, c) = \Gamma(-\underline{r}, \underline{r}) - c = \underline{r} - c$$

Obviously, this function has value zero if and only if $\underline{r} = c$. Thus, the minimal

distance boundary \underline{r} exists. By the values of the derivatives, the constellation $l = -\underline{r}$, $r = \underline{r}$ in fact constitutes the equilibrium with smallest platform distance $r - l$.

Part (ii): Upper bound The extreme boundary $\lambda(r, c)$ is defined as the unique root of the function $B(l, r, c) = \Gamma(l_\Gamma(r), r) - p(l, r)[r + l - 2l_\Gamma(r)] - c$ in l . The partial derivatives of $B(\lambda, r, c)$ with respect to λ and r are given by

$$\begin{aligned}\frac{\partial B(\lambda, r, c)}{\partial \lambda} &= -\frac{1}{2}\phi\left(\frac{\lambda+r}{2}\right)(r+\lambda-2l_\Gamma(r)) - \Phi\left(\frac{\lambda+r}{2}\right), \text{ and} \\ \frac{\partial B(\lambda, r, c)}{\partial r} &= 2\Phi\left(\frac{l_\Gamma(r)+r}{2}\right) + 2\Phi\left(\frac{\lambda+r}{2}\right)\frac{dl_\Gamma(r)}{dr} \\ &\quad - \frac{1}{2}\phi\left(\frac{\lambda+r}{2}\right)\underbrace{(r+\lambda-2l_\Gamma(r))}_{>0} - \Phi\left(\frac{\lambda+r}{2}\right) < 0\end{aligned}$$

Thus, the derivative of $\lambda(r, c)$ with respect to r follows as

$$\begin{aligned}\frac{d\lambda}{dr} &= -\frac{\partial B(\lambda, r, c)/\partial r}{\partial B(\lambda, r, c)/\partial \lambda} \\ &= \frac{2\Phi\left(\frac{l_\Gamma(r)+r}{2}\right) + 2\Phi\left(\frac{\lambda+r}{2}\right)\frac{dl_\Gamma(r)}{dr} - \frac{1}{2}\phi\left(\frac{\lambda+r}{2}\right)\underbrace{(r+\lambda-2l_\Gamma(r))}_{>0} - \Phi\left(\frac{\lambda+r}{2}\right)}{\frac{1}{2}\phi\left(\frac{\lambda+r}{2}\right)(r+\lambda-2l_\Gamma(r)) + \Phi\left(\frac{\lambda+r}{2}\right)}\end{aligned}$$

This derivative is larger than -1 if

$$\Phi\left(\frac{l_\Gamma(r)+r}{2}\right) + \Phi\left(\frac{\lambda+r}{2}\right)\frac{dl_\Gamma(r)}{dr} > 0.$$

Every term in this expression except $\frac{dl_\Gamma(r)}{dr}$ is strictly positive. For $r \leq \frac{0.5}{\phi(0)}$, the condition above holds because $\frac{dl_\Gamma(r)}{dr} \geq 0$, as shown in the proof of Lemma 7. For $r > \frac{0.5}{\phi(0)}$, we have $\frac{dl_\Gamma(r)}{dr} > -1$. Making use of the fact that $\lambda(r, c) < l_\Gamma(r)$, we then get

$$\Phi\left(\frac{l_\Gamma(r)+r}{2}\right) + \Phi\left(\frac{\lambda+r}{2}\right)\frac{dl_\Gamma(r)}{dr} > \Phi\left(\frac{\lambda+r}{2}\right)\left(1 + \frac{dl_\Gamma(r)}{dr}\right) > 0$$

Thus, $\frac{d\lambda(r, c)}{dr} > -1$ holds for all $r \in \mathbb{R}$.

Consequently, there can be at most one intersection between the boundary function $\lambda(r, c)$ and the 45° line ($l = -r$). Looking at the defining function of $\bar{r}(c)$, this statement is equivalent to the existence of a unique root in the function

$\tilde{B}(r, c) = B(-r, r, c)$. This function can be expressed as

$$\begin{aligned}\tilde{B}(r, c) &= \Gamma(l_\Gamma(r), r) - p(-r, r)(r + (-r) - 2l_\Gamma(r)) - c \\ &= \Gamma(l_\Gamma(r), r) + l_\Gamma(r) - c\end{aligned}$$

For $r = \frac{0.5}{\phi(0)}$, we have $l_\Gamma(r) = -\frac{0.5}{\phi(0)}$ and $\tilde{B}(r, c) = -c < 0$. Moreover, $\tilde{B}(r, c)$ is strictly increasing in r :

$$\begin{aligned}\frac{d\tilde{B}(r, c)}{dr} &= \frac{d\Gamma(l_\Gamma(r), r)}{dl} \frac{dl_\Gamma(r)}{dr} + \frac{d\Gamma(l_\Gamma(r), r)}{dr} + \frac{dl_\Gamma(r)}{dr} \\ &= 2\Phi\left(\frac{l_\Gamma(r) + r}{2}\right) + \frac{dl_\Gamma(r)}{dr} > 0\end{aligned}$$

Once again, this statement holds for all $r \in \mathbb{R}$. Furthermore, it can be shown that there are $\delta > 0$, $\varepsilon \in (0, 1)$ such that $\frac{d\tilde{B}(r, c)}{dr} > \varepsilon > 0$ holds for all $r > \frac{0.5}{\phi(0)} + \delta$. Thus, it is guaranteed that $\lim_{r \rightarrow \infty} \tilde{B}(r, c) > 0$. We can conclude that there exists a unique threshold $\bar{r}(c) > \frac{0.5}{\phi(0)}$ such that $\tilde{B}(\bar{r}, c) = 0$ and $\tilde{B}(r, c) > 0$ if and only if $r > \bar{r}$. In the maximum distance equilibrium, the party platforms are given by $(l, r) = (-\bar{r}, \bar{r})$.

Finally, the threshold \bar{r} strictly exceeds c , the rightist party's platform in the minimum distance equilibrium. At $r = c$, we have

$$\tilde{B}(c, c) = \Gamma(l_\Gamma(c), c) + l_\Gamma(c) - c = [p(l_\Gamma(c), c) - 1](c - l_\Gamma(c)).$$

For all $\sigma > 0$, we have $p(l_\Gamma(c), c) < 1$. Moreover, $l_\Gamma(c) < c$ holds generally. Thus, $\tilde{B}(c, c) < 0$, which implies that $\bar{r} > c$ by the arguments above.

Proof of Proposition 3

We prove the existence of symmetric equilibria with $l = -d$ and $r = d$, where $d \in [c, c + \varepsilon]$ for some $\varepsilon > 0$. Consider an allocation in which both parties are efficient, party medians are given by $m^L = -d$, $m^R = d$, and all party members are weakly more extreme, i.e., $\omega_i \leq -d$ for all $i \in M^L$ and $\omega_j \geq d$ for all $j \in M^R$. Let all members contribute only the membership fee c to their party, so that each party consists of exactly $C/c + 1 \geq 3$ members. Thus, the entry of a more moderate agent would not cause a shift in the party medians.

Recall that the platform bounds specified in Proposition 1 are only necessary, but not sufficient conditions. The allocation represents an equilibrium if and only if conditions (I), (II) and (III) are satisfied. Conditions (I) and (III) are satisfied for all $d \geq c$, because leaving his party would induce a policy loss of d to each party

member, but only save the cost of activity c .

Condition (II) ensures that no independent agent can profitably join one of the parties. First, consider the case $c < \frac{0.5}{\phi(0)}$. For all $d \leq \frac{0.5}{\phi(0)}$, we have $l_\Gamma(d) \leq -d$. Thus, the party medians prefer their own bliss point to any more moderate platform. If $c < \frac{0.5}{\phi(0)}$, the allocation characterized above represents an equilibrium for all $d \in \left[c, \frac{0.5}{\phi(0)}\right]$, consequently. Now, consider case $c \geq \frac{0.5}{\phi(0)} > 0$, and let $d \geq c$. Then, some independent agents with $\omega_i \in (-d, d)$ could indeed enter party L and be nominated as presidential candidate. Taking into account the membership cost c , their utility would however change by

$$\begin{aligned} \Gamma(\omega_i, r) - \Gamma(-d, d) + 2p(-d, d)(\omega_i - l) - c &= \Gamma(\omega_i, r) + \omega_i - c \\ &= [p(\omega_i, d) - 1](d - \omega_i) + d - c < d - c. \end{aligned}$$

Because $p(\omega_i, d) < 1$, the first term in the last line is strictly negative. Thus, there is a $\varepsilon > 0$ such that the utility change of the joining agent is negative for all $d \in [c, c + \varepsilon]$. If the symmetric allocation satisfies this condition, it consequently represents an equilibrium.

Proof of Proposition 4

The minimal distance boundary is given by $\underline{r}(c) = c$, which directly gives the derivative provided in Proposition 3.

The maximal distance boundary $\bar{r}(c)$ is defined implicitly as the root of function $\tilde{B}(r, c) = \Gamma(l_\Gamma(r), r) + l_\Gamma(r) - c$ in r . Thus, its derivative is given as

$$\frac{d\bar{r}(c)}{dc} = -\frac{\partial \tilde{B}(\bar{r}, c)/\partial c}{\partial \tilde{B}(\bar{r}, c)/\partial r} = \frac{1}{2\Phi\left(\frac{l_\Gamma(r)+r}{2}\right) + \frac{dl_\Gamma(r)}{dr}} > 0$$

Note that the positive sign of the denominator has already been proven for Proposition 2.

For the limit, $\bar{r}(0)$ is given by the root of $\Gamma(l_\Gamma(r), r) + l_\Gamma(r)$. This equation is satisfied for $\frac{0.5}{\phi(0)}$. As there is at most one root as shown above, this must be the limit $\bar{r}(0)$.

Proof of Proposition 5

First, note that $\underline{r}(c, \sigma) = c$ for all $\sigma \geq 0$. Next, look at the derivative of $\bar{r}(c, \sigma)$ with respect to σ . The maximum distance boundary is defined by $\bar{r}(c, \sigma) = r \in$

$\mathbb{R} : f(r, \sigma) = \Gamma(l_\Gamma(r, \sigma), r, \sigma) + l_\Gamma(r, \sigma) - c = 0$. By Proposition 2, function f has a unique root in r . The derivative follows as $\frac{d\bar{r}(c, \sigma)}{d\sigma} = -\frac{\partial f(\bar{r}, \sigma)/\partial \sigma}{\partial f(\bar{r}, \sigma)/\partial r}$.

The partial derivative of f in r is given by:

$$\begin{aligned} \frac{\partial f}{\partial r} &= \underbrace{\Gamma_1(l_\Gamma(r, \sigma), r, \sigma)}_{=0} \frac{dl_\Gamma(\bar{r}, \sigma)}{d\bar{r}} + \Gamma_2(l_\Gamma(r, \sigma), r, \sigma) + \frac{dl_\Gamma(r, \sigma)}{dr} \\ &= 2 \Phi\left(\frac{r + l_\Gamma(r, \sigma)}{2\sigma}\right) + \frac{dl_\Gamma(r, \sigma)}{dr} \end{aligned}$$

As $\bar{r} + l_\Gamma(\bar{r}, \sigma) > 0$ holds in general, the induced winning probability of party L is strictly larger than one half. Thus, the partial derivative in \bar{r} is strictly positive.

$$\frac{\partial f}{\partial \bar{r}} > 1 + \underbrace{\frac{dl_\Gamma(\bar{r}, \sigma)}{d\bar{r}}}_{>-1} > 0$$

With respect to the partial derivative of f in σ , we get the following expression where I omit the arguments of $l_\Gamma(r, \sigma)$ in order to simplify notation.

$$\begin{aligned} \frac{\partial f}{\partial \sigma} &= \underbrace{\frac{\partial \Gamma(l_\Gamma, r, \sigma)}{\partial l_\Gamma}}_{=0} \frac{dl_\Gamma}{d\sigma} + \frac{\partial \Gamma(l_\Gamma, r, \sigma)}{\partial \sigma} + \frac{dl_\Gamma}{d\sigma} \\ &= -\frac{r^2 - l_\Gamma^2}{2\sigma^2} \phi\left(\frac{r + l_\Gamma}{2\sigma}\right) + \frac{1}{\sigma} \frac{8\sigma^2 l_\Gamma + (r - l_\Gamma)(r + l_\Gamma)^2}{8\sigma^2 + r^2 - l_\Gamma^2} \end{aligned}$$

It can be shown that this expression is negative if and only if the following condition holds:

$$\left[1 - \Phi\left(\frac{r + l_\Gamma}{2\sigma}\right)\right] (r - l_\Gamma)(r + l_\Gamma)^2 < 8\sigma^2(r + l_\Gamma)\Phi\left(\frac{r + l_\Gamma}{2\sigma}\right) - 8\sigma^2 l_\Gamma$$

Making use of the fact that $p(l_\Gamma(r, c, \sigma), r, \sigma) > \frac{1}{2}$ once again, the following sufficient condition for $\frac{\partial f}{\partial \sigma} < 0$ can be derived:

$$\begin{aligned} &\left[1 - \Phi\left(\frac{r + l_\Gamma}{2\sigma}\right)\right] (r - l_\Gamma)(r + l_\Gamma)^2 < 4\sigma^2(r - l_\Gamma) \\ &\Leftrightarrow \left[1 - \Phi\left(\frac{r + l_\Gamma}{2\sigma}\right)\right] \left(\frac{r + l_\Gamma}{2\sigma}\right)^2 < 1 \end{aligned}$$

Substituting $b = \frac{\bar{r} + l_\Gamma}{2\sigma} > 0$, this condition is given as

$$\hat{f}(b) = b^2 [1 - \Phi(b)] < 0$$

It can be shown that this condition holds on the relevant interval (for all $b > 0$). Thus, we have established $\frac{\partial f}{\partial \sigma} < 0$.

Consequently, the maximum distance boundary $\bar{r}(c, \sigma)$ is strictly increasing in σ :

$$\frac{d\bar{r}(c, \sigma)}{d\sigma} = - \frac{\overbrace{\frac{\partial f(\bar{r}, \sigma)}{\partial \sigma}}^{<0}}{\underbrace{\frac{\partial f(\bar{r}, \sigma)}{\partial r}}_{>0}} > 0$$

Finally, consider the limit of $\bar{r}(c, \sigma)$ for σ converging to zero. Look at platform $\tilde{l} = -r + \varepsilon$ with arbitrarily small $\varepsilon > 0$. The policy effect of this platform is given by

$$\Gamma(\tilde{l}, r, \sigma) = (r - \tilde{l})\Phi\left(\frac{r + \tilde{l}}{2\sigma}\right) = (2r - \varepsilon)\Phi\left(\frac{\varepsilon}{2\sigma}\right)$$

For $\sigma \rightarrow 0$, this policy effect converges to $\lim_{\sigma \rightarrow 0} \Gamma(\tilde{l}, r, \sigma) = 2r - \varepsilon$. Thus, $\tilde{l} = l_\Gamma(r, 0)$ holds for $\varepsilon \rightarrow 0$. Then, looking at the defining function of $\bar{r}(c, \sigma)$ for general r , we get:

$$\lim_{\sigma \rightarrow 0} f(r, \sigma) = \Gamma(\tilde{l}, r, \sigma) + \tilde{l} - c = 2r - \varepsilon - r + \varepsilon - c$$

Since $\bar{r}(c, \sigma)$ is defined to be the root of function $f(r, \sigma)$ in r , we obviously have the limiting result: $\lim_{\sigma \rightarrow 0} \bar{r}(r, \sigma) = c$.

References

- Besley, T. & Coate, S. (1997), ‘An economic model of representative democracy’, *Quarterly Journal of Economics* **112**(1), 85–114.
- Cadigan, J. & Janeba, E. (2002), ‘A citizen-candidate model with sequential elections’, *Journal of Theoretical Politics* **14**(4), 387–407.
- Campante, F. R. (2011), ‘Redistribution in a Model of Voting and Campaign Contributions’, *Journal of Public Economics* **95**(7), 646–656.
- Dhillon, A. (2004), Political Parties And Coalition Formation, Warwick Economics Research Paper Series No. 697. URL: <http://ideas.repec.org/p/wrk/warwec/697.html>.
- Downs, A. (1957), *An Economic Theory of Democracy*, Harper and Row, New York.
- Eguia, J. (2007), ‘Citizen candidates under uncertainty’, *Social Choice and Welfare* **29**(2), 317–331.
- Gerber, A. & Ortuño-Ortín, I. (1998), ‘Political compromise and endogenous formation of coalitions’, *Social Choice and Welfare* **15**(3), 445–454.
- Gomberg, A., Marhuenda, F. & Ortuño-Ortín, I. (2004), ‘A model of endogenous political party platforms’, *Economic Theory* **24**(2), 373–394.
- Herrera, H., Levine, D. K. & Martinelli, C. (2008), ‘Policy platforms, campaign spending and voter participation’, *Journal of Public Economics* **92**(3-4), 501 – 513.
- Levy, G. (2004), ‘A model of political parties’, *Journal of Economic Theory* **115**(2), 250–277.
- Lindbeck, A. & Weibull, J. W. (1987), ‘Balanced-budget redistribution as the outcome of political competition’, *Public Choice* **52**, 273–297.
- Ma, Y., Genton, M. & Parzen, E. (2011), ‘Asymptotic properties of sample quantiles of discrete distributions’, *Annals of the Institute of Statistical Mathematics* **63**(2), 227–243.
- Morelli, M. (2004), ‘Party formation and policy outcomes under different electoral systems’, *Review of Economic Studies* **71**(3), 829–853.

- Ortuño-Ortín, I. & Schultz, C. (2005), ‘Public funding of political parties’, *Journal of Public Economic Theory* **7**(5), 781–791.
- Osborne, M. J. & Slivinski, A. (1996), ‘A model of political competition with citizen-candidates’, *Quarterly Journal of Economics* **111**(1), 65–96.
- Osborne, M. J. & Tourky, R. (2008), ‘Party Formation in Single-Issue Politics’, *Journal of the European Economic Association* **6**(5), 974–1005.
- Poutvaara, P. (2003), ‘Party platforms with endogenous party membership’, *Public Choice* **117**(1-2), 79–98.
- Poutvaara, P. & Takalo, T. (2007), ‘Candidate quality’, *International Tax and Public Finance* **14**(1), 7–27.
- Rivière, A. (1999), Citizen candidacy, party formation and duverger’s law, Working Paper No. dpe00/1. URL: <http://idei.fr/doc/conf/eco/riviere.pdf>.
- Roemer, J. E. (2006), ‘Party competition under private and public financing: A comparison of institutions’, *Advances in Theoretical Economics* **6**(1), 1229.
- Snyder, J. M. & Ting, M. M. (2002), ‘An informational rationale for political parties’, *American Journal of Political Science* **46**(1), 90–110.