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Conference Paper

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: International Climate Policy, No. D15-V1

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Hillebrand, Marten; Hillebrand, Elmar (2016) : Optimal Climate Policies in a Dynamic Multi-Country Equilibrium Model, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: International Climate Policy, No. D15-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/145903>

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Optimal Climate Policies in a Dynamic Multi-Country Equilibrium Model*

March 1, 2016

Abstract

This paper develops a dynamic general equilibrium model with an arbitrary number of different regions to study alternative climate policies and the consequences of climate change. Countries differ with respect to their state of economic development, factor endowments, and climate damages and trade on global markets for capital and exhaustible resources. Our main theoretical result derives an optimal climate policy consisting of an optimal emissions tax and an optimal transfer policy. The optimal climate tax can be determined explicitly in our framework and is independent of any weights attached to the interests of different countries. These weights only determine optimal transfers which distribute tax revenues across countries. We infer that the real political issue is not the amount of taxation required to reduce global warming but how the burden of climate change should be shared via transfer payments between different countries. To offer some guidance on this matter, we conduct a numerical simulation study which analyzes the optimal transfers between OECD and Non-OECD countries.

JEL classification: E10, E61, H21, H23, Q43, Q54

Keywords: Climate change, Multi-country equilibrium model, Optimal climate tax, Optimal transfers, Exhaustible resources, Emissions trading system.

*Acknowledgements. We would like to thank Hans-Georg Buttermann, Philipp Harms, Tomoo Kikuchi, and Klaus Wälde for helpful discussions and participants of the brownbag seminar at JGU for their comments.

1 Introduction

Global warming and its consequences for the world economy constitute one of the biggest challenges of the twenty-first century. While the fact that climate change is caused by man-made emissions of carbon-dioxide (CO_2) is now largely undisputed, political measures required to reduce these emissions and limit climate damages remain controversial and are also difficult to implement. An obvious reason for this is that climate change is a global problem and climate policies must be coordinated and implemented by different countries with heterogeneous characteristics each acting in their own economic interests.

These general insights assign an important role to economic theory in providing guidance to the political process by evaluating the economic consequences of alternative climate policies. They also suggest that any theoretical evaluation of these effects should be derived within a framework with multiple countries or regions which pursue their own economic interests and can differ with respect to their economic and various other characteristics. Such a setting then permits to quantify the consequences of alternative policy proposals at the level of individual countries and can, therefore, uncover ways and potential obstacles to implement such a policy globally.

Developing a theoretical framework with these properties and applying it to determine an optimal climate policy is the general aim of the present paper. Specific questions to be addressed are the following. First, what is the optimal growth path in a multi-country world which incorporates the interrelations between economic activity and climate conditions? Second, which policy implements this solution as an economic equilibrium and to what extent does it depend on the weights attached to the interests of different countries? Third, how do heterogeneities across countries such as different levels of economic development, stock of natural resources, or climate damages affect the optimal policy?

The model developed in this paper uses dynamic general equilibrium theory to obtain an internally consistent economic framework with explicit market structures and price formation and parameter values disciplined by empirical observations. Our world economy decomposes into an arbitrary number of different countries or regions each of which is politically autonomous and pursues its own interest. Regions interact on global markets for capital and exhaustible resources and can differ along various dimensions including population size, productivity, stocks of natural resources, and climate damages.

The economic part is complemented by a climate model describing how fossil emissions determine atmospheric concentration of carbon dioxide and the resulting damages on the economy. With these features, our model falls into the class of integrated assessment models which incorporate the full interactions between climate variables and the production process. Such models were first introduced in Nordhaus (1977) and further developed in Nordhaus & Yang (1996). Since then, many extensions and refinements have been developed which are comprehensively surveyed in Nordhaus (2011). The model closest to ours is Golosov et al. (2014) who develop a single-region model for the

world economy that takes full advantage of dynamic general equilibrium theory. Their main result is an explicit formula for the optimal tax on fossil emissions. Employing the same climate model as in Golosov et al. (2014), our analysis delivers an intuitive and simple generalization of this formula to a multi-country setting.

An extension of Golosov et al. (2014) to a multi-region framework is developed in Hassler & Krusell (2012). To preserve analytical tractability, they impose strong restrictions on trade between regions which are only allowed to trade fossil energy inputs. To avoid such restrictions, we propose a different model of the production process which explicitly separates the resource stage and the energy stage. Trade at the resource stage is unrestricted in our model while trading of energy outputs is confined to the domestic market. If these energy outputs are interpreted as electricity and heat or services like fuel-based transportation, we view this as an adequate description of reality. In addition, our model includes an international capital market which permits intertemporal borrowing and lending between countries. While capital is perfectly mobile, labor can only be allocated within each country, which is the traditional Ricardian assumption in trade theory.

With these features, our model is general enough to incorporate various types of heterogeneity which are potentially important in the political discussion on climate policy. At the same time, it retains the virtue of analytical tractability and offers various theoretical insights which constitute the first part of our analysis. Our main theoretical result shows that the *efficiency* of allocations can strictly be separated from the *distributional issue* how the burden of climate change should be shared across countries via transfers. The efficient solution is unique and completely determines the allocation of production factors and resources across countries together with an optimal climate path. It can be implemented by a global tax on CO₂ emissions which can be characterized explicitly in our model. As a consequence, the optimal climate tax is completely independent of the weights attached to the interests of different countries. Such weights are only relevant in the distributional of tax revenue across countries via transfers. We infer that the real political issue is not the amount of taxation required to reduce global warming but how the burden of climate change should be shared via transfer payments between different countries. To offer some guidance on this matter, the second part of our analysis conducts a numerical simulation study based on calibrated parameter values to analyze the size of optimal transfers between OECD and Non-OECD countries.

The paper is organized as follows. Section 2 introduces the model. The decentralized equilibrium solution and its properties under different climate policies are studied in Section 3. Section 4 studies efficient allocations obtained as solutions to a planning problem. The existence and form of optimal climate policies which implement the efficient solution as an equilibrium allocation are studied in Section 5. Section 6 presents quantitative results from numerical simulations based on calibrated parameter values. Section 7 concludes, mathematical results and proofs can be found in the Appendix.

2 The Model

2.1 The world economy

The world economy is divided into $L \geq 1$ regions indexed by $\ell \in \mathbb{L} := \{1, \dots, L\}$. Although these regions typically represent unions or groups of different countries, we will nevertheless refer to region $\ell \in \mathbb{L}$ as a country. Each country $\ell \in \mathbb{L}$ pursues its own interests and takes autonomous political decisions. They are geographically or institutionally separated, which imposes certain restrictions on trade between them.

In each country $\ell \in \mathbb{L}$ we decompose the production process into three stages. The *final stage* produces a consumable output commodity based a set of inputs including energy goods and services. The *energy stage* produces these goods and services based on renewable and exhaustible resources. The *resource stage* described the extraction of exhaustible resources. These different stages and the *consumption sector* in each country will be described in detail in this section together with a global *climate model*.

2.2 Production sectors

Production sectors in each country $\ell \in \mathbb{L}$ are identified by index $i \in \mathbb{I}_0 := \{0, 1, \dots, I\}$. Sector $i = 0$ is the *final sector* which produces a consumption good that can also be invested to become capital in the following period. Sectors $i \in \mathbb{I} := \mathbb{I}_0 \setminus \{0\}$ are *energy sectors* which supply energy goods and services as inputs to final good production. All sectors $i \in \mathbb{I}_0$ use labor $N_{i,t}^\ell \geq 0$ and capital $K_{i,t}^\ell \geq 0$ as production inputs in period t .

Final sector

Sector $i = 0$ in country $\ell \in \mathbb{L}$ consists of a single representative firm which produces consumption good output in period t using the technology

$$Y_t^\ell = (1 - D_t^\ell) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, E_t^\ell) \quad (1)$$

where

$$E_t^\ell = G((E_{i,t}^\ell)_{i \in \mathbb{I}}) \quad (2)$$

is an index of energy inputs determined by some aggregator G . The term $Q_{0,t}^\ell > 0$ in (1) is an exogenous, possibly time- and country-specific productivity parameter which is diminished by damages due to climate change. The latter is measured by a (time-varying and potentially country-specific) damage index $D_t^\ell \in [0, 1[$ which will depend on CO₂ concentration in the atmosphere and will be specified below.

Energy sectors

Each energy sector consists of a single representative firm. Their outputs should be broadly interpreted as energy goods (like electricity, or heat) or services (like transportation). These goods and services are either based on an exhaustible resource (like

coal or oil) which generates emissions during the production process (like coal-fired power generation or fuel-based transportation services) or provide substitutes for energy services which do (like nuclear power or renewable energy production). Energy sectors thus represent the production stage at which emissions are potentially generated.

Let $\mathbb{I}_x \subset \mathbb{I}$ be the subset of exhaustible energy sectors which base their production on an exhaustible resource like coal, gas, or uranium which is an essential input to their production. We identify each exhaustible resource by the (unique) energy sector $i \in \mathbb{I}_x$ which uses this resource in production and denote by $X_{i,t}^\ell > 0$ the amount of the resource used at time t . All other energy sectors $i \in \mathbb{I} \setminus \mathbb{I}_x$ use renewable sources like wind or solar energy that are non-exhaustible and, therefore, do not enter as production inputs.

With the previous distinction, output produced by *exhaustible energy sector* $i \in \mathbb{I}_x$ in period t is

$$E_{i,t}^\ell = Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell). \quad (3)$$

By contrast, output of a *renewable energy producer* $i \in \mathbb{I} \setminus \mathbb{I}_x$ in period t is given by

$$E_{i,t}^\ell = Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell). \quad (4)$$

Similar to (1), both specifications (3) and (4) allow for time- and country-specific productivity $Q_{i,t}^\ell$. In general, a higher productivity $Q_{i,t}^\ell > Q_{i,t}^{\ell'}$ may reflect a more developed technology in country ℓ relative to ℓ' or the fact that conditions to produce energy of type $i \in \mathbb{I}$ are more favorable in country ℓ than in ℓ' due to geographic conditions, etc. For example, a solar energy plant located in the Sahara seems likely to produce more electricity output than an identical plant located in a northern European country like Norway while the opposite holds in the case with hydroelectric power generation.

Technology and productivity

The remainder imposes the following standard restrictions on the production technologies (1), (3), (4), and the aggregator (2).

Assumption 1

- (i) Each $F_i : \mathbb{R}_+^{n_i} \rightarrow \mathbb{R}_+$, $i \in \mathbb{I}_0$ and aggregator $G : \mathbb{R}_+^I \rightarrow \mathbb{R}_+$ are C^2 on the interior of their domain, linear homogeneous, strictly increasing, and concave.
- (ii) F_i satisfies the Inada-condition $\lim_{z_m \searrow 0} \partial_{z_m} F_i(z) = \infty \forall z \in \mathbb{R}_{++}^{n_i}$, $m = 1, \dots, n_i$.

Denoting by $\mathbf{Q}_t := (Q_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_0}$ the world productivity vector in period $t \geq 0$, we assume that the evolution of the productivity sequence $(\mathbf{Q}_t)_{t \geq 0}$ is determined exogenously.

Resource sectors

An exhaustible resource is identified by the unique energy sector $i \in \mathbb{I}_x$ which uses this resource in production. In each country $\ell \in \mathbb{L}$, there exists a resource sector represented by a single firm which extracts these resources and supplies them to the global resource market. The amount of resource of type $i \in \mathbb{I}_x$ extracted and supplied in period t is

denoted $X_{i,t}^{\ell,s} \geq 0$ (to be distinguished from the amount $X_{i,t}^\ell$ demanded by energy sector $i \in \mathbb{I}_x$ in that country). Resource firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R_{i,0}^\ell \geq 0$ as a given parameter. Feasible extraction plans are non-negative sequences $(X_{i,t}^{\ell,s})_{t \geq 0}$ which respect the feasibility constraint

$$\sum_{t=0}^{\infty} X_{i,t}^{\ell,s} \leq R_{i,0}^\ell. \quad (5)$$

To avoid trivialities, we impose the initial condition $\sum_{\ell \in \mathbb{L}} R_{i,0}^\ell > 0$, i.e., initial world resources are strictly positive for all $i \in \mathbb{I}_x$. It may be, however, that $R_{i,0}^\ell = 0$ in which case country ℓ does not own any resources of type i .

2.3 Climate model

Emissions of CO₂ are generated by using ('burning') fossil resources like coal, gas, etc. to produce energy services. As a consequence, only energy sectors $i \in \mathbb{I}_x$ which use exhaustible resources can potentially cause emissions.¹ The amount of CO₂ generated by using one unit of exhaustible resource $i \in \mathbb{I}_x$ is physically determined by its carbon-content represented by the coefficient $\zeta_i \geq 0$. In particular, $\zeta_i = 0$ if the resource does not generate emissions, like uranium in the case of nuclear energy production.

It follows that total emissions in period t measured in units of CO₂ are given by

$$Z_t := \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell. \quad (6)$$

As in Golosov et al. (2014), the climate state in period t is represented by $\mathbf{S}_t = (S_{1,t}, S_{2,t})$ which consists of permanent and non-permanent CO₂ in the atmosphere. Given the sequence of emissions $\{Z_t\}_{t \geq 0}$ determined by (6), the climate state evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \quad (7a)$$

$$S_{2,t} = (1 - \phi) S_{2,t-1} + (1 - \phi_L) \phi_0 Z_t \quad (7b)$$

Total concentration of CO₂ is the sum of permanent and non-permanent CO₂, i.e.,

$$S_t = S_{1,t} + S_{2,t}. \quad (8)$$

Climate damage in country ℓ is determined by a differentiable, strictly increasing damage function $D^\ell : [\bar{S}, \infty[\rightarrow [0, 1[$ which depends on total CO₂ in the atmosphere, i.e.,

$$D_t^\ell = D^\ell(S_t). \quad (9)$$

¹This abstracts from emissions generated from using renewable resources like biomass, etc.

A specific functional form to be used in our simulation study is

$$D^\ell(S) = 1 - \exp\{-\gamma^\ell(S - \bar{S})\}, \quad \gamma^\ell > 0 \quad (10)$$

which corresponds to the choice in Golosov et al. (2014).² Differences in climate damages thus enter via country specific parameters γ^ℓ , $\ell \in \mathbb{L}$. The parameter $\bar{S} > 0$ in (10) represents the pre-industrial level of CO₂ in the atmosphere.

2.4 Consumption sector

The consumption sector in country $\ell \in \mathbb{L}$ consists of a single representative household who supplies labor and capital to the production process and decides about consumption and capital formation taking factor prices as given. In addition, the consumer is entitled to receive all profits from domestic firms and transfers from the government. Let K_0^ℓ denote initial capital supply in $t = 0$ and $N_t^\ell > 0$ the labor supplied in period t . The sequence $(\mathbf{N}_t^s)_{t \geq 0}$ of world labor supply $\mathbf{N}_t^s := (N_t^\ell)_{\ell \in \mathbb{L}}$ is exogenously given.

The household's preferences over non-negative consumption sequences $(C_t^\ell)_{t \geq 0}$ are represented by a standard time-additive utility function

$$U((C_t^\ell)_{t \geq 0}) = \sum_{t=0}^{\infty} \beta^t u(C_t^\ell). \quad (11)$$

The subsequent analysis imposes the following restrictions on U .

Assumption 2

The discount factor in (11) satisfies $0 < \beta < 1$ while u is of the form

$$u(C) = \begin{cases} \frac{C^{1-\sigma} - 1}{1-\sigma} & \text{for } \sigma > 0, \sigma \neq 1 \\ \log(C) & \text{for } \sigma = 1. \end{cases} \quad (12)$$

Specification (12) is precisely the class of utility functions consistent with balanced growth. This form is key to obtain our separation result in Section 4. It includes the case with logarithmic utility studied in Golosov et al. (2014) as a special case.

2.5 Summary of the economy

The economy \mathcal{E} introduced in the previous sections can be summarized by its regional and sectoral structure $\langle \mathbb{L}, \mathbb{I}_0, \mathbb{I}_x \rangle$, the production technologies $\langle (\mathbf{Q}_t)_{t \geq 0}, (F_i)_{i \in \mathbb{I}_0}, G, (c_i)_{i \in \mathbb{I}_x} \rangle$, consumer characteristics $\langle (\mathbf{N}_t^s)_{t \geq 0}, u, \beta \rangle$, and climate parameters $\langle (\zeta_i)_{i \in \mathbb{I}_x}, \phi, \phi_0, \phi_L \rangle$. In addition, initial values for capital supply $\mathbf{K}_0^s = (K_0^\ell)_{\ell \in \mathbb{L}}$, exhaustible resource stocks $\mathbf{R}_0 = (R_{i,0}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_x}$, and the climate state \mathbf{S}_{-1} are given.

²The general version of GHKT allows for γ to be time- and state-dependent. Here, we assume that it is constant, as they do in their numerical simulations, too.

3 The Decentralized Equilibrium

The following sections derive the competitive equilibrium of the economy where producers and consumers behave optimally and markets clear. The equilibrium variables are determined for a given climate policy which imposes a tax on each unit of CO₂ emitted into the atmosphere and distributes the revenues as lump-sum transfers to consumers.

While all transactions take place in $t = 0$ and the consumption good in this period is chosen as the numeraire, prices in period t are denominated in time t consumption. Let $r_t > 0$ denote the rental price of capital at time t and $q_t > 0$ the price of the time t consumption good measured in units of time zero consumption. Since the economy is deterministic and capital depreciates fully after one period, the price of time t consumption can be expressed as ³

$$q_t = \prod_{s=1}^t r_s^{-1} \quad (13)$$

for $t \geq 0$ where $q_0 = 1$. As labor and energy outputs will be immobile across countries, their prices will, general, be country-specific. Let $w_t^\ell > 0$ denote the wage and $p_{i,t}^\ell > 0$ the price per unit of energy type $i \in \mathbb{I}$ in country ℓ and period t .

3.1 Climate policies

Carbon tax policies

To reduce CO₂ emissions, the following equilibrium analysis considers a carbon tax policy which levies a (possibly time and country-specific) tax per unit of CO₂ emitted in country ℓ . Formally, such a policy can be defined as follows.

Definition 1

A Carbon Tax Policy (CTP) is a non-negative sequence $\tau = (\tau_t)_{t \geq 0}$ where $\tau_t = (\tau_t^\ell)_{\ell \in \mathbb{L}}$ are the (country-specific) tax rates to be paid per unit of CO₂ emitted in period $t \geq 0$.

An alternative scenario to be explored in Section 5 is an *Emissions Trading System (ETS)* which issues emission allowances traded on a global carbon market. Below we will show that such a system can achieve the same objective as a CTP.

Transfer policies

Carbon taxes paid by dirty energy sectors are collected by governmental authorities and

³This holds because the model is deterministic and $1/r_t$ is the price of a bond traded in period $t - 1$ that pays-off one unit of the consumption good at time t . In the stochastic case, one would choose the stochastic discount factor $q_t = \beta^t u'(C_t)/u'(C_0)$ and apply the expectations operator in (24). It follows from the Euler equation (34) derived below that this approach is identical to using (13) in the present case.

passed on as a lump-sum transfer T_t^ℓ to consumers. A natural restriction would be to assume that transfers equal tax revenue in each country. We will, however, adopt a more general setting which does not require revenues and transfers to balance at the national but only at the world level. Thus, we impose the following feasibility condition of world transfers in each period $t \geq 0$:

$$\sum_{\ell \in \mathbb{L}} T_t^\ell = T_t := \sum_{\ell \in \mathbb{L}} \tau_t^\ell \underbrace{\sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell}_{\text{Emissions in country } \ell} . \quad (14)$$

Here, T_t are global tax revenues in period t . If $T_t^\ell > \tau_t^\ell \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell$, country ℓ receives a net transfer from the other countries in period t and contributes a net transfer otherwise. In fact, even $T_t^\ell < 0$ is not excluded, in which case consumers in country ℓ are taxed to finance transfers received by other countries.

A transfer policy now specifies the share of global tax revenue transferred to country ℓ .

Definition 2

A transfer policy is a mapping $\theta : \mathbb{L} \rightarrow \mathbb{R}$, $\ell \mapsto \theta^\ell$ satisfying $\sum_{\ell \in \mathbb{L}} \theta^\ell = 1$ which determines the transfers in (14) received by country $\ell \in \mathbb{L}$ as $T_t^\ell = \theta^\ell T_t$ for all $t \geq 0$.

The assumption that transfer shares are constant over time is without loss of generality, as the behavior of consumers will exclusively depend on their lifetime transfer income

$$T^\ell := \sum_{t=0}^{\infty} q_t T_t^\ell . \quad (15)$$

Thus, defining total discounted tax revenue

$$T := \sum_{t=0}^{\infty} q_t T_t , \quad (16)$$

lifetime transfers satisfy

$$T^\ell = \theta^\ell T = \theta^\ell \sum_{t=0}^{\infty} q_t \sum_{\ell' \in \mathbb{L}} \tau_t^{\ell'} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^{\ell'} . \quad (17)$$

and we simply could have defined θ^ℓ as the ratio T^ℓ/T .

3.2 Producer behavior

All firms in sectors $i \in \mathbb{I}_0$ choose their productions plans to maximize the discounted stream of profits. As there is no intertemporal linkage between these decisions, the decision problems of final and energy sectors can be formulated and solved on a period-by-period basis. This, however, is not possible in the case of resource sectors which solve intertemporally dependent problems.

Final sector

For notational convenience, we will identify the composition of F_0 and the aggregator G by the same symbol F_0 and write $F_0(K, N, (E_i)_{i \in \mathbb{I}})$.⁴ Given damage-adjusted productivity and factor prices for labor $w_t^\ell > 0$, capital $r_t > 0$, and the list of energy prices $p_t^\ell = (p_{i,t}^\ell)_{i \in \mathbb{I}} \gg 0$, the firm in the final sector solves the following decision problem in period $t \geq 0$:

$$\max_{(K, N, E_1, \dots, E_I) \in \mathbb{R}_+^{2+I}} \left\{ (1 - D_t^\ell) Q_{0,t}^\ell F_0(K, N, (E_i)_{i \in \mathbb{I}}) - w_t^\ell N - r_t K - \sum_{i \in \mathbb{I}} p_{i,t}^\ell E_i \right\} \quad (18)$$

A solution to (18) satisfies the following first order conditions which equate prices and marginal products of each production factor for all $t \geq 0$:

$$(1 - D_t^\ell) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) = r_t \quad (19a)$$

$$(1 - D_t^\ell) Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) = w_t^\ell \quad (19b)$$

$$(1 - D_t^\ell) Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) = p_{i,t}^\ell \quad \forall i \in \mathbb{I}. \quad (19c)$$

Exhaustible energy sectors

A firm in the exhaustible energy sector $i \in \mathbb{I}_x$ in country $\ell \in \mathbb{L}$ takes productivity $Q_{i,t}^\ell > 0$, the wage $w_t^\ell > 0$ and capital return $r_t > 0$, and the domestic selling price $p_{i,t}^\ell > 0$ as given. In addition, the global price $v_{i,t} > 0$ for resource $i \in \mathbb{I}_x$ and the tax $\tau_t^\ell \geq 0$ to be paid per unit of CO₂ emitted enter the decision problem as given parameters. Recall that the total emissions generated by using $X_{i,t}^\ell \geq 0$ units of resources in production are simply given by $Z_{i,t}^\ell = \zeta_i X_{i,t}^\ell$. Thus, the decision problem solved in period t reads:

$$\max_{(K, N, X) \in \mathbb{R}_+^3} \left\{ p_{i,t}^\ell Q_{i,t}^\ell F_i(K, N, X) - w_t^\ell N - r_t K - (v_{i,t} + \zeta_i \tau_t^\ell) X \right\}. \quad (20)$$

The first order optimality conditions with respect to labor, capital and resource type i in production of energy type i in country ℓ are given by:

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) = r_t \quad (21a)$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) = w_t^\ell \quad (21b)$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) = v_{i,t} + \zeta_i \tau_t^\ell. \quad (21c)$$

Clearly, the solution to (20) becomes independent of τ_t^ℓ if $\zeta_i = 0$, i.e., the firm employs a clean technology.

Renewable energy sectors

A renewable energy firm $i \in \mathbb{I} \setminus \mathbb{I}_x$ in country $\ell \in \mathbb{L}$ takes sector specific productivity

⁴The partial derivative $\partial_{E_i} F_0$ is therefore a shorthand for $\partial_E F_0 \partial_{E_i} G$. The reason for initially keeping F_0 and G separate is that Assumption 1 does not require G to satisfy the Inada conditions. This allows for energy services to be complements, substitutes, or even perfect substitutes in which case G would be a linear function.

$Q_{i,t}^\ell > 0$, the domestic wage w_t^ℓ , global capital return $r_t > 0$, and the domestic energy price $p_{i,t}^\ell > 0$ as given and solves the following decision problem in each period $t \geq 0$:

$$\max_{(K,N) \in \mathbb{R}_+^2} \left\{ p_{i,t}^\ell Q_{i,t}^\ell F_i(K, N) - w_t^\ell N - r_t K \right\}. \quad (22)$$

A solution to (22) is characterized by the following first order conditions

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i(K_{i,t}^\ell, N_{i,t}^\ell) = r_t \quad (23a)$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell) = w_t^\ell. \quad (23b)$$

Resource sectors

The resource sector $i \in \mathbb{I}_x$ in country $\ell \in \mathbb{L}$ chooses a non-negative extraction sequence $(X_{i,t}^{\ell,s})_{t \geq 0}$ which satisfies the resource constraint (5). Resources extracted in period t are supplied to a global resource market at price $v_{i,t} \geq 0$. Given sequences of world resource prices $(v_{i,t})_{t \geq 0}$ and discount factors $(q_t)_{t \geq 0}$, the resource sector maximizes the discounted sum of future profits. The decision problem reads

$$\max_{(X_{i,t}^{\ell,s})_{t \geq 0}} \left\{ \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_{i,t}^{\ell,s} \mid X_{i,t}^{\ell,s} \geq 0 \forall t \geq 0, (5) \text{ holds} \right\}. \quad (24)$$

By (5), existence of a non-trivial solution to (24) requires $R_{i,0}^\ell > 0$. Using this restriction, the following results characterizes solutions to (24).

Lemma 1

If $R_{i,0}^\ell > 0$, solutions to problem (24) can be characterized as follows:

(i) An interior solution $(X_t^*)_{t \geq 0} \gg 0$ exists only if resource prices evolve as

$$v_{i,t} - c_i = (v_{i,0} - c_i)/q_t \geq 0 \quad \forall t \geq 0. \quad (25)$$

(ii) If (25) holds, there are two cases:

(a) If $v_{i,0} = c_i$, any sequence $(X_t^*)_{t \geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^\ell$ is a solution.

(b) If $v_{i,0} > c_i$, any sequence $(X_t^*)_{t \geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* = R_{i,0}^\ell$ is a solution.

Only in Case (b) where $v_{i,t} = c_i$ for all $t \geq 0$ is it optimal not to exhaust the entire stock of resources. Below, however, we will demonstrate that this case can only occur at equilibrium if $R_{i,0}^\ell = \infty$. In either case, (25) permits maximum profits $\Pi_i^\ell := \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_t^*$ to be written as

$$\Pi_i^\ell = (v_{i,0} - c_i) R_{i,0}^\ell. \quad (26)$$

Intuitively, the discounted profit stream (26) of resource sector $i \in \mathbb{I}_x$ is the excess value of the initial stock of resources valued at time-zero prices net of extraction costs. Also

note that given an optimal extraction plan $(X_{i,t}^{\ell,s})$ determined as a solution to (24), the period profit of resource sector $i \in \mathbb{I}_x$ in country $\ell \in \mathbb{L}$ is given by

$$\Pi_{i,t}^{\ell} = (v_{i,t} - c_i)X_{i,t}^{\ell,s} \geq 0. \quad (27)$$

In general, however, the quantity in (27) will be indeterminate at equilibrium due to the multiplicity of solutions to (24).

Equilibrium profits

A direct consequence of Assumption 1 and the first order conditions derived in (19), (23), and (21) is that profits in final production and all energy sectors are zero. Thus, only firms in the resource sector potentially make profits. We will assume that resource firms in country ℓ are completely owned by domestic consumers. Thus, by (26) the total lifetime profit income of consumers in country $\ell \in \mathbb{L}$ is

$$\Pi^{\ell} = \sum_{i \in \mathbb{I}_x} \Pi_i^{\ell} = \sum_{i \in \mathbb{I}_x} (v_{i,0} - c_i)R_{i,0}^{\ell}. \quad (28)$$

In words, profit income in country ℓ equals the total value of domestic exhaustible resources net of extraction costs.

3.3 Consumer behavior

Budget constraints

In each period $t \geq 0$, consumers in country $\ell \in \mathbb{L}$ receive labor income $w_t^{\ell}N_t^{\ell} > 0$, the return r_t on their current net asset holdings K_t^{ℓ} , profit income $\Pi_t^{\ell} \geq 0$, and transfers T_t^{ℓ} . Their choices of current consumption $C_t^{\ell} \geq 0$ and next period's capital holding K_{t+1}^{ℓ} satisfy the period budget constraint

$$C_t^{\ell} + K_{t+1}^{\ell} = r_t K_t^{\ell} + w_t^{\ell}N_t^{\ell} + T_t^{\ell} + \Pi_t^{\ell} \quad \forall t \geq 0. \quad (29)$$

At the individual level, capital investment may be negative⁵ but must satisfy the No-Ponzi game condition requiring consumers to ultimately repay any outstanding debt:

$$\lim_{t \rightarrow \infty} q_t K_{t+1}^{\ell} \geq 0. \quad (30)$$

Using (29) and (30) one can recursively eliminate asset holdings to obtain the consumer's lifetime budget constraint:

$$\sum_{t=0}^{\infty} q_t C_t^{\ell} \leq K_0^{\ell} + \sum_{t=0}^{\infty} q_t w_t^{\ell} N_t^{\ell} + \Pi^{\ell} + T^{\ell} \quad (31)$$

⁵This assumption can be interpreted as a shorthand for assuming that an international bond market coexists with the capital market on which borrowing and lending takes place. Then, the consumer in period t chooses investment in capital K_{t+1}^{ℓ} and bonds B_{t+1}^{ℓ} subject to the budget constraint (29). Market clearing on the bond market requires $\sum_{\ell \in \mathbb{L}} B_t^{\ell} = 0$.

where Π^ℓ is the consumer's lifetime profit income determined by (28) and T^ℓ her lifetime transfer income defined in (15). Equation (31) shows that the existence of a solution to (33) requires the solvency condition

$$W^\ell := K_0^\ell + \sum_{t=0}^{\infty} q_t w_t^\ell N_t^\ell + \Pi^\ell > T^\ell. \quad (32)$$

Here, W^ℓ is the consumer's lifetime non-transfer income.

Optimal consumption plans

Given strictly positive sequences of wages $(w_t^\ell)_{t \geq 0}$ and returns $(r_t)_{t \geq 0}$ (defining the discount factors $(q_t)_{t \geq 0}$ by (13)), lifetime profit income $\Pi^\ell \geq 0$ and transfers T^ℓ , and initial financial wealth K_0^ℓ satisfying (32), the consumer chooses a consumption sequence $(C_t^\ell)_{t \geq 0}$ to maximize lifetime utility (11). The decision problem then reads:

$$\max_{(C_t^\ell)_{t \geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t^\ell) \mid C_t^\ell \geq 0 \ \forall t \geq 0, (31) \text{ holds} \right\}. \quad (33)$$

Standard (variational) arguments imply that any solution $(C_t^{\ell*})_{t \geq 0}$ to (33) must satisfy the Euler equations

$$r_{t+1} \beta u'(C_{t+1}^\ell) = u'(C_t^\ell) \quad \forall t \geq 0. \quad (34)$$

Further, any solution satisfies the lifetime budget constraint (31) with equality which is equivalent to the transversality condition

$$\lim_{t \rightarrow \infty} q_t K_{t+1}^\ell = 0. \quad (35)$$

In fact, the restriction imposed by Assumption 2 allows us to characterize the unique solution to (33) in the following lemma.

Lemma 2

Let Assumption 2 hold and the solvency condition (32) be satisfied. Then, problem (33) has a unique solution $(C_t^{\ell})_{t \geq 0}$ given by*

$$C_t^{\ell*} = \frac{(\beta^t / q_t)^{\frac{1}{\sigma}} [W^\ell + T^\ell]}{\sum_{s=0}^{\infty} q_s (\beta^s / q_s)^{\frac{1}{\sigma}}} \quad t \geq 0. \quad (36)$$

3.4 Market clearing

Restrictions on trade

Trade between countries occurs on global markets for capital/output and exhaustible resources of each type $i \in \mathbb{I}_x$. That is, both the final/capital good and exhaustible resources can freely be exported without any additional costs. As there is no sign restriction on capital investment, consumers can also take loans in the capital market.

By contrast, labor supply is immobile and can only be employed in domestic production sectors. Likewise, energy goods and services can only be used in domestic final good production.⁶ Thus, there are domestic markets for labor and energy of all types $i \in \mathbb{I}$ in each country $\ell \in \mathbb{L}$.

Domestic markets

Market clearing on the domestic labor market in country ℓ requires

$$\sum_{i \in \mathbb{I}_0} N_{i,t}^\ell \stackrel{!}{=} N_t^\ell. \quad (37)$$

Since energy is non-tradable across countries, market clearing on the domestic energy market in country $\ell \in \mathbb{L}$ requires that energy demanded in final production coincide with energy supplied by sector $i \in \mathbb{I}$. This is already embodied in the previous notation by using the same symbol $E_{i,t}^\ell$ for energy demand and supply.

International resource markets

Market clearing on the world market for exhaustible resource $i \in \mathbb{I}_x$ requires

$$\sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell,s} \stackrel{!}{=} \sum_{\ell \in \mathbb{L}} X_{i,t}^\ell \quad (38)$$

in each period t . If $X_{i,t}^\ell < X_{i,t}^{\ell,s}$ country ℓ is a net exporter of resource $i \in \mathbb{I}_x$ in period t and a net importer otherwise. An immediate consequence of Lemma 1 is that the equilibrium amount of resources $X_{i,t}^{\ell,s}$ supplied by country ℓ and, as a consequence, period profits (27) will, in general, be indeterminate.

Summing (5) over all countries and using (38), the allocation of resources across production sectors must satisfy the world resource constraint

$$\sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^\ell \leq R_{i,0} \quad (39)$$

where $R_{i,0} := \sum_{\ell \in \mathbb{L}} R_{i,0}^\ell$ is the total initial stock of resource i . As resources extracted in different countries are perfect substitutes, the equilibrium allocation of exhaustible resources $(X_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_x, t \geq 0}$ satisfying the feasibility constraint (39) can always be chosen compatible with the individual resource constraints in each country.

International capital market

Let $K_t := \sum_{\ell \in \mathbb{L}} K_t^\ell$ be the aggregate capital stock supplied to the world capital market in period t . Market clearing on the world capital in period t requires

$$\sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_0} K_{i,t}^\ell \stackrel{!}{=} K_t \quad \forall t \geq 0. \quad (40)$$

⁶In fact, this is the reason why we disentangle the energy stage and the resource stage in our model, compared to GHKT.

At equilibrium, the consumers' period budget constraints (29), the zero profit conditions for all sectors $i \in \mathbb{I}_0$ in conjunction with (27) and the market clearing conditions (37), (14), (38), and (40) can be used to obtain the following equilibrium condition:

$$K_{t+1} + \sum_{\ell \in \mathbb{L}} C_t^\ell + \sum_{i \in \mathbb{I}_x} c_i \sum_{\ell \in \mathbb{L}} X_{i,t}^\ell = \sum_{\ell \in \mathbb{L}} Y_t^\ell \quad \forall t \geq 0. \quad (41)$$

Since period profit income (27) is, in general, indeterminate at equilibrium, so is individual capital supply K_t^ℓ , $\ell \in \mathbb{L}$.

3.5 Equilibrium

For purposes of a compact notation, we employ the following vector notation for the variables introduced in the previous sections for each t :

$$\begin{aligned} \mathbf{Q}_t &:= (Q_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_0} & \mathbf{N}_t^s &:= (N_t^\ell)_{\ell \in \mathbb{L}} & \mathbf{C}_t &:= (C_t^\ell)_{\ell \in \mathbb{L}} \\ \mathbf{Y}_t &:= (Y_t^\ell)_{\ell \in \mathbb{L}} & \mathbf{E}_t &:= (E_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}} & \mathbf{X}_t &:= (X_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_x} \\ \mathbf{N}_t &:= (N_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_0} & \mathbf{K}_t &:= (K_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_0} & \mathbf{S}_t &:= (S_{t,1}, S_{t,2}) \\ \mathbf{w}_t &:= (w_t^\ell)_{\ell \in \mathbb{L}}, & \mathbf{p}_t &:= (p_{i,t}^\ell)_{(\ell,i) \in \mathbb{L} \times \mathbb{I}} & \mathbf{v}_t &:= (v_{i,t})_{i \in \mathbb{I}_x}. \end{aligned} \quad (42)$$

All variables defined in (42) take values in the appropriate positive orthant of \mathbb{R}^n . Note that $\mathbf{N}_t \in \mathbb{R}_+^{L(I+1)}$ refers to the allocation of labor across sectors in each country while $\mathbf{N}_t^s \in \mathbb{R}_+^L$ is the vector of labor supplied by consumers in each country.

Let the climate tax policy τ and transfer policy θ defined as above be given. The following definition of equilibrium is standard.

Definition 3

Given tax policy τ and transfer policy θ , an equilibrium of \mathcal{E} is an allocation $\mathbf{A}^* = (\mathbf{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*)_{t \geq 0}$ and prices $\mathbf{P}^* = (r_t^*, \mathbf{w}_t^*, \mathbf{p}_t^*, \mathbf{v}_t^*)_{t \geq 0}$ such that:

- (i) The allocation is consistent with the production technologies (1), (4), and (3), and the market clearing conditions/resource constraints (37), (39), (40), and (41).
- (ii) Producers behave optimally, i.e., equations (19), (21), and (23) hold for all $t \geq 0$. Profits of resource firms are given by (26) while resource prices evolve as in (25).
- (iii) Consumers behave optimally as described in Lemma 2 with profit incomes determined by (28) and transfers satisfying (17) and the solvency condition (32).
- (iv) Climate variables evolve according to (7) with emissions given by (6) and climate damages in (1) determined by (8) and (9).

If we want to emphasize the dependence of the equilibrium on the policy variables, we will write $\mathbf{A}^*(\tau, \theta)$, etc. The separate impact of the tax policy τ and the transfer policy θ on the equilibrium allocation will be revealed by Proposition 1 in the next section. A special case of Definition 3 is the Laissez faire equilibrium with no taxation in which $\tau \equiv 0$. The induced equilibrium allocation $\mathbf{A}_{LF}^* := \mathbf{A}^*(0, \theta)$ constitutes an important benchmark in the subsequent discussion. It is clear that this solution will, in general not constitute a Pareto optimal outcome due to the climate externality in production.

3.6 Properties of equilibrium

The following results establishes additional properties of equilibrium under our restrictions on technologies and preferences.

Lemma 3

Under Assumptions 1 and 2, any equilibrium allocation has the following properties:

- (i) The equilibrium allocation is interior, i.e., $\mathbf{A}^* \gg 0$.
- (ii) Consumption $C_t^{\ell*}$ is a constant share of world consumption $\bar{C}_t^* := \sum_{\ell \in \mathbb{L}} C_t^{\ell*}$, i.e.,

$$C_t^{\ell*} = \mu^\ell C_t^* \quad \forall t \geq 0. \quad (43)$$

for all $\ell \in \mathbb{L}$ and $t \geq 0$ where $\mu^\ell > 0$ and $\sum_{\ell \in \mathbb{L}} \mu^\ell = 1$.

- (iii) Each exhaustible resource $i \in \mathbb{I}_x$ which has $R_{i,0}^\ell < \infty$ is fully exploited, i.e., for all $\ell \in \mathbb{L}$,

$$\sum_{t=0}^{\infty} X_{i,t}^{\ell*} = R_{i,0}^\ell. \quad (44)$$

Based on these properties, one can disentangle the impact of the tax policy τ and the transfer policy θ on the equilibrium allocation \mathbf{A}^* . This result will play a major role in Section 5 when we study optimal policies which implement the social optimum as an equilibrium allocation.

Proposition 1

Given a tax policy τ and transfer policy θ , let $\mathbf{A}^* = (\mathbf{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*)_{t \geq 0}$ be the equilibrium allocation. Define aggregate equilibrium consumption $\bar{C}_t^* = \sum_{\ell \in \mathbb{L}} C_t^{\ell*}$ and total tax revenues $T^* := \sum_{t=0}^{\infty} q_t^* \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \tau_t^\ell \zeta_i X_{i,t}^{\ell*}$. Then, the following holds:

- (i) The tax policy τ determines the aggregate equilibrium allocation

$$\bar{\mathbf{A}}^* = (\bar{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*)_{t \geq 0} \quad (45)$$

which is independent of the transfer policy θ .

(ii) The transfer policy θ determines consumption in country $\ell \in \mathbb{L}$ as a constant share of world consumption, i.e., $C_t^{\ell*} = \mu^{\ell*} \bar{C}_t^{*}$ for all $t \geq 0$ where

$$\mu^{\ell*} = \frac{W^{\ell*} + \theta^{\ell} T^*}{\sum_{\hat{\ell} \in \mathbb{L}} W^{\hat{\ell}*} + T^*}. \quad (46)$$

with non-transfer lifetime income $W^{\ell*}$ defined as in (32) for all $\ell \in \mathbb{L}$.

4 Optimality

In this section we determine an optimal allocation as the solution to a social planning problem (SPP) which maximizes a weighted utility index of consumers in different countries subject to the constraints imposed by technology, resources, and climate change. In particular, the SPP incorporates the climate externality. A major advantage of our previous restriction on preferences is that it permits to compute an *optimal allocation* in two steps. First, we determine an *efficient allocation* which maximizes utility of a world representative consumer. This solution completely specifies the optimal climate path and the entire allocation of production factors and resources across countries and, most importantly, is independent of the weights attached to the interests of different countries. Second, we distribute consumption across countries to obtain an *optimal allocation* which maximizes a weighted utility index reflecting the trade-off between the interests of different countries.

4.1 Optimal allocations

Consider a social planner who chooses a feasible world allocation subject to restrictions imposed by technology, factor mobility, and resource constraints. Formally, using the notation introduced in the previous section, the planner takes the sequences of productivity $(\mathbf{Q}_t)_{t \geq 0}$ and labor supply $(\mathbf{N}_t^s)_{t \geq 0}$ as given. Further, initial world capital $K_0 > 0$, the initial world stock $R_{i,0} > 0$ of exhaustible resource $i \in \mathbb{I}_x$, and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ are given. While final output, capital, and exhaustible resources can freely be allocated across countries, labor and energy outputs can only be used within each country. Thus, the planner faces essentially the same restrictions as firms/consumers in the decentralized solution including constraints (40) and (37) when allocating capital and labor. Further, the given initial stock of world resources imposes the restrictions (39) on the use of exhaustible resource $i \in \mathbb{I}_x$ in production.⁷ The major

⁷As resources extracted in different countries are perfect substitutes, the solution to the planning problem does not determine where these resources are extracted. However, any allocation of exhaustible resources $(X_{i,t}^{\ell})_{(\ell,i) \in \mathbb{L} \times \mathbb{I}_x, t \geq 0}$ satisfying the feasibility constraint (39) can always be chosen compatible with the individual resource constraints (5) in each country.

difference is that the planning problem incorporates the link between productivity in final production, damage, and climate change. Thus, the decision involves the choice of a feasible allocation defined next.

Definition 4

- (i) A feasible allocation of \mathcal{E} is a sequence $\mathbf{A} = (\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0}$ which satisfies equations (1), (4), (3), (6), (7), (8), (37), (39), (40), (41) for all t .
- (ii) The set of feasible allocations of \mathcal{E} is denoted \mathbb{A} .

The distribution of consumption across countries necessarily induces a trade-off between the utility levels attained by consumers in different countries. Assume that the planner uses a weighting scheme corresponding to a list of utility weights $\omega = (\omega^\ell)_{\ell \in \mathbb{L}}$ where $\omega^\ell \geq 0$ represents the weight attached to consumer utility in country ℓ in the planner's decision. Formally, we have

Definition 5

A utility weighting scheme is a map $\omega : \mathbb{L} \rightarrow \mathbb{R}_+$, $\ell \mapsto \omega^\ell$ which satisfies $\sum_{\ell \in \mathbb{L}} \omega^\ell = 1$.

Let $\Omega \subset \mathbb{R}_+^{\mathbb{L}}$ denote the set of all weighting schemes. Each weighting scheme $\omega \in \Omega$ defines the following weighted utility index

$$V((\mathbf{C}_t)_{t \geq 0}; \omega) := \sum_{t=0}^{\infty} \beta^t \sum_{\ell \in \mathbb{L}} \omega^\ell u(C_t^\ell) \quad (47)$$

that depends on disaggregated world consumption $(\mathbf{C}_t)_{t \geq 0}$ where $\mathbf{C}_t = (C_t^\ell)_{\ell \in \mathbb{L}}$ for all t . Given $\omega \in \Omega$, define the following **Weighted Social Planning Problem (WSPP)**

$$\max_{\mathbf{A}} \left\{ V((\mathbf{C}_t)_{t \geq 0}; \omega) \mid \mathbf{A} = (\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0} \in \mathbb{A} \right\}. \quad (48)$$

Assuming it exists and is unique, denote the solution to (48) as

$$\mathbf{A}^{\text{opt}} = (\mathbf{C}_t^{\text{opt}}, K_{t+1}^{\text{opt}}, \mathbf{Y}_t^{\text{opt}}, \mathbf{E}_t^{\text{opt}}, \mathbf{K}_t^{\text{opt}}, \mathbf{N}_t^{\text{opt}}, \mathbf{X}_t^{\text{opt}}, \mathbf{S}_t^{\text{opt}})_{t \geq 0}. \quad (49)$$

It is clear that, in general, the solution to (48) will depend on the weighting scheme ω . Thus, we will write $\mathbf{A}^{\text{opt}}(\omega)$ as a way of emphasizing this dependence.

4.2 Efficient aggregate allocations

Consider now a modified planning problem which faces the same restrictions as before but does not specify the distribution of consumption across different countries. Denoting aggregate world consumption in period t by $\bar{C}_t \geq 0$, we modify the resource constraint (39) to become

$$K_{t+1} + \bar{C}_t + \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} c_i X_{i,t}^\ell = \sum_{\ell \in \mathbb{L}} Y_t^\ell. \quad (50)$$

These restrictions lead to the following definition of a feasible aggregate allocation.

Definition 6

- (i) A feasible aggregate allocation is a sequence $\bar{\mathbf{A}} = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0}$ which satisfies equations (1), (4), (3), (6), (7), (8), (37), (39), (40), (50) for all t .
- (ii) The set of feasible aggregate allocations of \mathcal{E} is denoted $\bar{\mathbf{A}}$.

The terminology 'aggregate' is used because such an allocation only involves aggregate world consumption but does not specify how this consumption is distributed across countries. In particular, the aggregate equilibrium allocation in (45) is feasible, i.e., $\bar{\mathbf{A}}^* \in \bar{\mathbf{A}}$. The planner's objective is to maximize lifetime utility of a fictitious world representative consumer who consumes \bar{C}_t in period t and has the same utility function as consumers in each country. This leads to the following **Aggregate Social Planning Problem (ASPP)**:

$$\max_{\bar{\mathbf{A}}} \left\{ \sum_{t=0}^{\infty} \beta^t u(\bar{C}_t) \mid \text{s.t. } \bar{\mathbf{A}} = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0} \in \bar{\mathbf{A}} \right\}. \quad (51)$$

Assuming it exists and is unique, the solution to (51) is

$$\bar{\mathbf{A}}^{\text{eff}} = (\bar{C}_t^{\text{eff}}, K_{t+1}^{\text{eff}}, \mathbf{Y}_t^{\text{eff}}, \mathbf{E}_t^{\text{eff}}, \mathbf{K}_t^{\text{eff}}, \mathbf{N}_t^{\text{eff}}, \mathbf{X}_t^{\text{eff}}, \mathbf{S}_t^{\text{eff}})_{t \geq 0} \quad (52)$$

and referred to as the *efficient aggregate allocation*. The efficient solution $\bar{\mathbf{A}}^{\text{eff}}$ specifies the entire allocation of production factors and resources across countries but leaves undetermined the distribution of consumption. It is therefore independent of any weights attached to the interests of different countries.

4.3 Optimal distribution of consumption

The main result of this section shows that a solution to the WSPP (48) can be obtained by optimally distributing aggregate consumption determined by the efficient allocation (52). To prepare this result, let a weighting scheme ω be arbitrary but fixed. Suppose in an arbitrary period, aggregate consumption $C > 0$ is pre-determined. Then, an optimal distribution of C across countries in this period based on the weighting scheme ω obtains as the solution to the problem

$$\max_{\mu = (\mu^\ell)_{\ell \in \mathbb{L}}} \left\{ \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell C) \mid \mu^\ell \geq 0, \sum_{\ell \in \mathbb{L}} \mu^\ell = 1 \right\}. \quad (53)$$

The following result shows that (53) has a unique solution which, crucially, is independent of C . This result forms the basis for our subsequent separation result.

Lemma 4

Let u satisfy Assumption 2. Then, for any weighting scheme $\omega = (\omega^\ell)_{\ell \in \mathbb{L}}$ and any $C > 0$ the following holds:

(i) Problem (53) has a unique solution $\mu_{\text{opt}}(\omega) = (\mu_{\text{opt}}^\ell(\omega))_{\ell \in \mathbb{L}}$ which is independent of C .

(ii) The maximum value (53) takes the form

$$\sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu_{\text{opt}}^\ell C) = \begin{cases} (1 - \sigma)m(\omega)u(C) & \sigma \neq 1 \\ m(\omega) + u(C) & \sigma = 1 \end{cases} \quad (54)$$

where $m(\omega) := \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu_{\text{opt}}^\ell) > 0$.

4.4 From efficiency to optimality - a separation result

Lemma 4 allows us to state the main result of this section in the following theorem.

Theorem 1

Let Assumptions 1 and 2 be satisfied and define the efficient allocation \mathbf{A}^{eff} as in (52). Given a weighting scheme ω , let $\mu_{\text{opt}}(\omega) = (\mu_{\text{opt}}^\ell(\omega))_{\ell \in \mathbb{L}}$ solve (53). Then, the allocation

$$\mathbf{A} = (\mu^{\text{opt}}(\omega) \bar{C}_t^{\text{eff}}, K_{t+1}^{\text{eff}}, \mathbf{Y}_t^{\text{eff}}, \mathbf{E}_t^{\text{eff}}, \mathbf{K}_t^{\text{eff}}, \mathbf{N}_t^{\text{eff}}, \mathbf{X}_t^{\text{eff}}, \mathbf{S}_t^{\text{eff}})_{t \geq 0} \quad (55)$$

solves the WSPP (48), i.e., $\mathbf{A} = \mathbf{A}^{\text{opt}}(\omega)$.

In words, this result says that a solution to the WSPP can simply be obtained from the efficient solution (52) by distribution aggregate consumption in an optimal fashion determined by the weighting scheme ω . In particular, the entire allocation of production factors, resources, and emissions remains exactly the same as in the efficient solution.

As its major implication, the previous result allows to compute a unique efficient allocation of production factors, resources, emissions, climate damage, etc. which is completely independent of the weights that the interests of different countries receive in the decision. The weighting scheme is therefore irrelevant for answering the question what the optimal climate path is and *where* and *how* emissions should be reduced.

4.5 Computing the efficient allocation

The previous results show that the social optimum is essentially characterized by the efficient solution (52). Adopting a standard infinite-dimensional Lagrangian approach, it is now straightforward to obtain explicit conditions which completely characterize this solution. Detailed computations can be found in Section A.4 in the appendix. The main findings are as follows.

First, the total costs Λ_t of emitting one additional unit of CO₂ in period t (measured in units of time t consumption) equals the discounted sum of all future world climate

damage caused by this emission. Formally,

$$\Lambda_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n})}{dS} \frac{Y_{t+n}^\ell}{1 - D^\ell(S_{t+n})} \left(\phi_L + (1 - \phi_L)\phi_0(1 - \phi)^n \right). \quad (56)$$

Equation (56) is a multi-country version of the result in Golosov et al. (2014). Note that Λ_t is independent of ℓ and i and depends on the structural parameters of the model and endogenous model variables in a complicated way, unless stronger restrictions similar to those in Golosov et al. are imposed. The term (56) is the key quantity to incorporate the climate externality into the (shadow) price of exhaustible resources.

The following optimality conditions essentially ensure *intratemporal* and *intertemporal efficiency* in production in each period $t \geq 0$. Denote by

$$\hat{p}_{i,t}^\ell = (1 - D_t^\ell) Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell). \quad (57)$$

the time t shadow price of energy type $i \in \mathbb{I}$ in country $\ell \in \mathbb{L}$ (measured in units of time t consumption units). Using (57), marginal products of capital are equalized across all countries and sectors, i.e., for all $\ell, \ell' \in \mathbb{L}$ and all $i \in \mathbb{I}_0$:⁸

$$(1 - D^\ell(S_t)) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) = \hat{p}_{i,t}^{\ell'} Q_{i,t}^{\ell'} \partial_K F_i(K_{i,t}^{\ell'}, N_{i,t}^{\ell'}, X_{i,t}^{\ell'}) \quad (58)$$

Second, in each country $\ell \in \mathbb{L}$ marginal products of labor are equalized across all sectors $i \in \mathbb{I}_0$ (although not across countries due to labor immobility), i.e.,

$$(1 - D^\ell(S_t)) Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) = \hat{p}_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell). \quad (59)$$

Third, for all $i \in \mathbb{I}_x$, the constraint (39) is binding and resource extraction is intratemporally efficient in each period $t \geq 0$, i.e., for all $\ell \in \mathbb{L}$:

$$\hat{p}_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \zeta_i \Lambda_t = \hat{v}_{i,t}. \quad (60)$$

Equation (60) defines the true shadow price of resources as the marginal product in production minus the cost of emissions defined in (56). This is the key difference to the equilibrium equations which fail to take this cost into account.

Intertemporal efficiency of final good allocation (consumption vs. capital formation) is ensured by the standard Euler equation which holds for all $\ell \in \mathbb{L}$ and $t \geq 1$:

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t) (1 - D^\ell(S_t)) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}). \quad (61)$$

Condition (61) in conjunction with (58) also equates (implicit) capital returns across countries in each period. Defining the shadow price of resource extraction as in (60), intertemporally efficient extraction of resource $i \in \mathbb{I}_x$ is ensured by the condition:

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)} \left(\hat{v}_{i,t+1} - c_i \right). \quad (62)$$

⁸Here and in the sequel, we incur a slight abuse of notation by including $X_{i,t}^\ell$ as a 'dummy' argument of F_i even if $i \notin \mathbb{I}_x$ to save some notation.

Finally, standard arguments also require the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t u'(\bar{C}_t) K_{t+1} = 0. \quad (63)$$

Summarizing, any non-negative sequence $\bar{\mathbf{A}} = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0}$ which satisfies the feasibility conditions (1), (4), (3), (6), (7), (8), (37), (39), (40), (50), and the optimality conditions (56), (57), (58), (60), (61), and (62) for all $t \geq 0$ as well as (63) is a solution to (51). Compared to the laissez-faire equilibrium allocation, the social planner includes a wedge between the marginal product of dirty resource i and its (shadow) price which accounts for the externality cost of an additional unit of emissions. Specifically, if $D^\ell \equiv 0$, the efficient solution coincides with the aggregate equilibrium allocation (45).

5 Optimal Climate Policies

In this section, we determine climate policies which implement the ω -optimal allocation defined as the solution to (48) as an equilibrium allocation in the sense of Definition 3. The properties of equilibrium stated in Proposition 1 and the separation result from Section 4 allow us to achieve this goal in two steps. In a first step, an efficient climate tax policy τ^{eff} is computed which implements the efficient allocation as an aggregate equilibrium allocation defined as in Proposition 1, i.e., $\bar{\mathbf{A}}^*(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$. In a second step, an optimal transfer scheme θ^{opt} is computed based on a given weighting scheme ω which together with τ^{eff} implements the ω -optimal allocation as an equilibrium allocation, i.e., $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{opt}}) = \mathbf{A}^{\text{opt}}(\omega)$. The transfer policy specifies equilibrium transfers $(T^{\ell*})_{\ell \in \mathbb{L}}$ such that each country attains the optimal consumption share $\mu^{\text{opt}}(\omega)$ at equilibrium.

5.1 The optimal climate tax policy

Using the efficient allocation (52), define the climate tax policy $\tau^{\text{eff}} = (\tau_t^{\text{eff}})_{t \geq 0}$ as

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n}^{\text{eff}})}{u'(\bar{C}_t^{\text{eff}})} \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n}^{\text{eff}})}{dS} \frac{Y_{t+n}^{\ell, \text{eff}}}{1 - D^\ell(S_{t+n}^{\text{eff}})} \left(\phi_L + (1 - \phi_L) \phi_0 (1 - \phi)^n \right), \quad t \geq 0. \quad (64)$$

Thus, optimal climate taxes $\tau_t^\ell \equiv \tau_t^{\text{eff}}$ are uniform across dirty sectors in all countries and incorporate the total damage from emitting one unit of CO₂ in period t in the efficient allocation. The following results shows that this policy does implement the efficient solution (52) as an aggregate equilibrium allocation defined as in (45). Recall from Proposition 1 that this allocation is independent of the transfer policy.

Theorem 2

Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). Then, the induced aggregate equilibrium allocation defined in (45) is efficient, i.e., $\bar{\mathbf{A}}^(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$.*

With the specific functional form (10) for climate damage, the optimal tax formula (56) takes the more specific form

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n}^{\text{eff}})}{u'(\bar{C}_t^{\text{eff}})} \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_{t+n}^{\ell, \text{eff}} \left(\phi_L + \phi_0(1 - \phi_L)(1 - \phi)^n \right). \quad (65)$$

In general, the expression can not be computed explicitly, as it involves the entire path of future output and discount factors determined by aggregate consumption. However, if consumption and output along the efficient solution are approximately constant over time (as will be the case in our simulation study in Section 6), a convenient approximation to (65) is

$$\tau_t^{\text{eff}} \approx \hat{\tau}_t^{\text{eff}} := \sum_{\ell \in \mathbb{L}} \gamma^\ell Y_t^{\ell, \text{eff}} \left(\frac{\phi_L}{1 - \beta} + \phi_0 \frac{1 - \phi_L}{1 - \beta(1 - \phi)} \right). \quad (66)$$

In fact, if $L = 1$, the approximation (66) is precisely the tax-formula derived in Golosov et al. (2014) under a set of additional restrictions (log utility, Cobb-Douglas production, all capital used in the final sector).

5.2 The optimal transfer policy

Given the efficient climate tax policy τ^{eff} defined in (64), the aggregate equilibrium allocation uniquely determines all equilibrium prices and profits. Let $W^{\ell, \text{eff}}$ denote lifetime non-transfer income of consumer ℓ defined by (32) along this equilibrium and T^{eff} the total tax revenue defined as in (16). For an arbitrary weighting scheme ω , let the induced optimal consumption shares $\mu^{\text{opt}} = (\mu^{\ell, \text{opt}})_{\ell \in \mathbb{L}}$ be the unique solution to (53). Consider the following transfer policy $\theta^{\text{opt}} = (\theta^{\ell, \text{opt}})_{\ell \in \mathbb{L}}$ defined for each $\ell \in \mathbb{L}$ as

$$\theta^{\ell, \text{opt}} = \frac{\mu^{\ell, \text{opt}} \left[\sum_{\hat{\ell} \in \mathbb{L}} W^{\hat{\ell}, \text{eff}} + T^{\text{eff}} \right] - W^{\ell, \text{eff}}}{T^{\text{eff}}}. \quad (67)$$

This transfer policy determines consumer ℓ 's lifetime cum-transfer income $W^\ell + T^\ell$ to be a share $\mu^{\ell, \text{opt}}$ of world cum-transfer income $\sum_{\hat{\ell} \in \mathbb{L}} [W^{\hat{\ell}} + T^{\hat{\ell}}]$. We now have the following result.

Theorem 3

Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). For any weighting scheme ω , define the transfer policy θ^{opt} as in (67). Then, the induced equilibrium allocation is ω -optimal, i.e., $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{opt}}) = \mathbf{A}^{\text{opt}}(\omega)$.

5.3 Emissions trading systems (ETS)

An alternative climate policy is to introduce a global *emissions trading system (ETS)*. Such a system issues a number of emission allowances in each period which put an

upper bound on Carbon emissions in that period. Allowances are time-specific, i.e., can not be transferred across different periods.⁹ The possession of one emission allowance for period t allows the owner to emit one unit of fossil energy/carbon dioxide into the atmosphere in that period. As the total number of allowances issued in period t puts an upper bound ('cap') on CO₂ emissions in that period, such a system is also called a 'cap-and trade system'.

The parameters of the ETS are fully described by the sequence of allowances issued in each period. Formally, we have:

Definition 7

An Emissions Trading System (ETS) is a sequence $\{\bar{Z}_t\}_{t \geq 0}$ where $\bar{Z}_t \geq 0$ is the number of emission allowances issued in period t .

In each period $t \geq 0$, a dirty sector $i \in \mathbb{I}_x$ in country ℓ needs to cover its emissions by purchasing an equal number of emission allowances from the ETS at the prevailing carbon price $p_t^Z \geq 0$ which is determined endogenously.¹⁰ Thus, the demand of sector i in country ℓ for allowances in period t is $Z_{i,t}^\ell = \zeta_i X_{i,t}^\ell$. Market clearing on the Carbon market requires

$$\sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} Z_{i,t}^\ell \leq \bar{Z}_t. \tag{68}$$

Condition (68) holds with equality if $p_t^Z > 0$ in which case total emissions (6) coincide with the cap \bar{Z}_t set by the ETS. Conversely, if $\sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} Z_{i,t}^\ell < \bar{Z}_t$, then $p_t^Z = 0$.

One can now easily define an equilibrium with an ETS $\bar{Z} = (\bar{Z}_t)_{t \geq 0}$ which replaces the tax system τ in Definition 3. In addition to the variables listed in Definition 3, the equilibrium determines a price sequence $(p_t^{Z*})_{t \geq 0}$ such that (68) holds for all $t \geq 0$. Here, we omit such a formal definition which is straightforward. Denote by $\mathbf{A}^*(\bar{Z}, \theta)$ the resulting equilibrium allocation with the transfer policy θ defined as before.

5.4 Optimal emissions trading systems

Defining the efficient aggregate allocation \mathbf{A}^{eff} as in (52), let $(Z_t^{\text{eff}})_{t \geq 0}$ denote the induced emissions determined by (6). We now have the following result.

⁹This assumption seems justified by the time scale adopted in the quantitative exercise below in which the length of one time period corresponds to 10 years.

¹⁰The assumptions that dirty sectors must purchase all emission rights and do not receive any initial endowments is without loss of generality. One can easily show that such an endowment enters the profit maximization problems as an additive constant and, therefore, has no effect on the behavior of the firm. The revenue received by consumers as a transfer from the ETS in the present case are replaced one-for one by profits from dirty sectors corresponding to the value of their endowment.

Theorem 4

Let Assumptions 1 and 2 hold. Then, the ETS $\bar{Z}^{\text{eff}} = (Z_t^{\text{eff}})_{t \geq 0}$ induces an aggregate equilibrium allocation which is efficient, i.e., $\bar{\mathbf{A}}^*(\bar{Z}^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$.

Thus, the efficient allocation can be implemented by choosing aggregate emission allowances ('caps') equal to the optimal path determined by the efficient allocation. Clearly, by defining the transfer policy as in (67), the resulting equilibrium allocation $\mathbf{A}^*(\bar{Z}^{\text{eff}}, \theta^{\text{opt}})$ is also ω -optimal. Further, the equilibrium Carbon price sequence $(p_t^{Z^*})_{t \geq 0}$ coincides with the optimal tax defined in (56). Thus, we can directly interpret this tax as the price for CO₂.

6 The Quantitative Model

This section presents a numerical case study which serves to illustrate and quantify our previous theoretical results. Our aim is to compare the consequences of climate change for OECD and Non-OECD countries. Specifically, we compare the laissez faire and the optimal solution and their differences with respect to output and climate damage, energy production, and resource extraction. We also study the size of optimal transfers between OECD and Non-OECD countries under different scenarios for climate damages.

6.1 Calibration

OECD vs. Non-OECD countries

The world economy is divided into $L = 2$ regions. Region 1 represents the members of the OECD ('*Organization for Economic Cooperation and Development*'). Region 2 comprises the rest of the world.¹¹ Using data from the World Bank we calibrate our parameter set to match the following stylized facts:

- the world population share of OECD-countries is 18%¹²
- GDP in OECD countries makes up 69% of world GDP¹³
- OECD countries own 68,5% of the global capital stock¹⁴

¹¹The following countries are OECD member states: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Israel, Italy, Japan, South Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States. Major Non-OECD countries are China, India, Russia, and Brazil.

¹²<http://data.worldbank.org/indicator/SP.POP.TOTL>.

¹³<http://data.worldbank.org/indicator/NY.GDP.MKTP.CD>.

¹⁴Berlemann & Wesselhoeft (2014) reported estimates of capital stocks for 103 countries for the period 1970-2011 using the perpetual inventory method. Their estimates imply a world capital stock

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- only 16% of global crude oil reserves are located in OECD countries.¹⁵

The following sections detail our parameter choices.

Productivity and labor supply

Following Golosov et al. (2014) and Acemoglu et al. (2012), we take each model period to represent ten years starting in $t = 2015$. We abstract from explicit growth of productivities and labor force by setting $Q_{i,t}^\ell \equiv Q_i^\ell$ and $N_t^\ell \equiv N^\ell$ for all $\ell \in \mathbb{L}$ and $i \in \mathbb{I}_0$. As all technologies are linear homogeneous and our specification of preferences is consistent with balanced growth, these restrictions are without loss of generality, as we can interpret all variables as intensive form quantities relative to the exogenously determined growth path of the economy.

Total labor supply and the world population size are normalized to unity. Based on the stylized facts reported above, we choose labor supply $N^1 = 0.18$ in OECD and $N^2 = 0.82$ in Non-OECD countries. Productivities in final good production are set to $Q_0^1 = 2.59$ and $Q_0^2 = 0.62$ in order to match the observed GDP shares reported above in the initial modelling period. By contrast, productivities in energy sectors are homogeneous and chosen to obtain a plausible energy mix in the initial period.¹⁶

Sectoral structure

There are three energy sectors ($I = 3$) in each region which are broadly interpreted as follows. Sector $i = 1$ produces all energy outputs and services based on various fuel types derived from crude *oil* which ranges from oil refineries (which produce petroleum products) to traffic and transportation services based on fossil fuel such as motorvehicles, cargo aircrafts, railroad cargo etc. Sector $i = 2$ produces energy based on *coal*, although the underlying resource comprises various kinds of fossil fuel (anthracite coal, lignite, and natural gas). Its output includes coal-based power generation and heat. Finally, sector $i = 3$ subsumes all energy services which do not produce emissions including nuclear energy. We also assume that production in sector $i = 3$ is completely based on renewable energy sources like wind, solar, water.¹⁷ With our previous notation we thus

of 64 499 Billion US-\$₂₀₀₀ of which 44 208 Billion US-\$₂₀₀₀ (68,5%) is located in OECD-member states (and the remaining 31,5% in Non-OECD countries).

¹⁵According to the EIA, global crude oil reserves are 1423 bn bbl (Billion barrel) of which 1194 bn barrel or 84% are located in Non-OECD-countries, see <https://www.eia.gov/cfapps/ipdbproject/IEDIndex3.cfm?tid=5&pid=57&aid=6>.

¹⁶In principle we also could have allowed for heterogeneity at the energy stage, too. However, empirical observations based on IEA data show that the energy mix of OECD and Non-OECD regions is somewhat similar: The IEA reports a share of oil in total primary energy supply of 36% (26%) for OECD (Non-OECD) countries, 44% (54%) for coal and gas combined and 19% (19%) for clean energy. We believe that introducing heterogeneity at the energy stages across regions leads to minor changes in the quantitative results.

¹⁷As nuclear energy would also be included in sector 3, this abstracts from the fact that uranium is an exhaustible resource, too. This seems justified, however, because the existing stocks of uranium are abundant relative to fossil reserves.

have $\mathbb{I} = \{1, 2, 3\}$ and $\mathbb{I}_x = \{1, 2\}$.

Production technologies

The production technologies F_i , $i \in \mathbb{I}_0$ in (1),(3), and (4) and the aggregator G in (2) are specified as follows. The production function in the final sector takes the CES-form

$$F_0(K, N, E) = \left[\alpha_{0,K} K^{\varrho_0} + \alpha_{0,N} N^{\varrho_0} + (1 - \alpha_{0,K} - \alpha_{0,N}) E^{\varrho_0} \right]^{\frac{1}{\varrho_0}} \quad (69)$$

where $\alpha_{0,k} > 0$, $\alpha_{0,l} > 0$, $\alpha_{0,k} + \alpha_{0,l} < 1$, and $\varrho_0 > 0$.

Energy sectors $i \in \mathbb{I}_x = \{1, 2\}$ use a technology of the form

$$F_i(K, N, X) = X^{\alpha_{x,i}} \left[\alpha_{i,K} K^{\varrho_i} + (1 - \alpha_{i,K}) N^{\varrho_i} \right]^{\frac{1 - \alpha_{x,i}}{\varrho_i}} \quad (70)$$

where $0 < \alpha_{x,i} < 1$. This specification captures the fact that exhaustible resources are an essential input to production in these sectors.

The technology used by sector $i = 3$ is

$$F_3(K, N) = \left[\alpha_{3,K} K^{\varrho_3} + (1 - \alpha_{3,K}) N^{\varrho_3} \right]^{\frac{1}{\varrho_3}}. \quad (71)$$

As a first approximation, we set all parameters ϱ_i in (69), (70), and (71) equal to zero which induces a Cobb-Douglas technology. This allows us to interpret the α 's as cost shares of production factors in the respective sector.

With this interpretation in mind, we set $\alpha_{0,K} = 0.3$ and $\alpha_{0,N} = 0.6$ in (69) which are commonly used in the literature.¹⁸ For sector $i = 1$, we set $\alpha_{1,K} = 0.85$ and $\alpha_{1,N} = 1 - \alpha_{1,K} = 0.15$ and $\alpha_{1,X} = 0.33$ based on data constructed from the the Bureau of Economic Analysis (2007).¹⁹

As the remaining two energy sectors produce electricity, we can base our parameter choices on the nominal electricity generation costs for Germany reported in Hillebrand (1997). This implies cost shares $\alpha_{2,K} = 0.69$ and $\alpha_{2,X} = 0.26$ for sector $i = 2$ and $\alpha_{3,K} = 0.7$ for nuclear power plants. However, sector $i = 3$ also includes renewable energies like wind or solar power in our framework for which Loeschel & Otto (2009) reports an even higher share of capital costs. For this reason, we choose a slightly

¹⁸The resulting cost share of energy is equal to 10% which is higher than the value of 4% used in Golosov, Hassler, Krusell & Tsyvinski (2014). However, our greater value can be justified, since the energy stage in Golosov, Hassler, Krusell & Tsyvinski (2014) corresponds to our resource extraction stage and has therefore a lower share in aggregate GDP.

¹⁹The data is use-table highlights inter-sectoral linkage for 389 industries/commodities for the United States. The table contains detailed data for industries that can be aggregated to specific sectors which are relevant for this study, for instance “Refineries” or “Transportation”. Especially for these three isolated industry groups, which represent our energy type $i = 1$ “oil based energy goods and services”, we then calculated the parameters representing the cost shares for capital, labor and resources. Additional details are available upon request.

higher capital share setting $\alpha_{3,K} = 0.75$. The previous choices are also in line with the general observation made by the Department of Energy & Climate Change (2013) that electricity generated from nuclear as well as wind and hydro power plants is relatively more capital intensive compared to conventional or thermal power generation.²⁰

The aggregator function G in (2) is the composition of two aggregators G_1 and G_2 . The function G_2 aggregates outputs produced in sectors $i = 2$ and 3 (electricity and heat) to an intermediate composite

$$EL_t^\ell = G_2(E_{2,t}^\ell, E_{3,t}^\ell) := \left[\kappa_2 (E_{2,t}^\ell)^{\rho_2^E} + (1 - \kappa_2) (E_{3,t}^\ell)^{\rho_2^E} \right]^{\frac{1}{\rho_2^E}}. \quad (72)$$

The function G_1 aggregates this intermediate composite with oil-based energy services produced in sector $i = 1$ to a final energy composite

$$E_t^\ell = G_1(E_{1,t}^\ell, EL_t^\ell) := \left[\kappa_1 (E_{1,t}^\ell)^{\rho_1^E} + (1 - \kappa_1) (EL_t^\ell)^{\rho_1^E} \right]^{\frac{1}{\rho_1^E}}. \quad (73)$$

There is some considerable degree of freedom to set the parameter ρ_1^E in (73) which determines the elasticity of substitution between electricity and fossil fuel. We choose a moderately positive value of $\rho_1^E = 0.2$. This ensures that oil is not an essential input to production and the model has a well-defined steady state.

The elasticity of substitution between CO₂-intensive and clean electricity/heat goods and services is higher. Setting $\rho_2^E = 0.6$ in (72) we follow Loeschel & Otto (2009) who report an elasticity of substitution equal to 2.5.

Finally, we use the industry data from the Bureau of Economic Analysis (2007) to approximate the κ 's in (72) and (73) by the corresponding relative cost shares in final output production. We set $\kappa_1 = 0.3818$, which corresponds to the cost of electricity and heat production relative to the cost of transportation per unit GDP in 2007. The second choice $\kappa_2 = 0.5$ is in line with the observations in Golosv et al. (2014) who choose a relative price between dirty and clean electricity generation equal to unity.

Stocks of exhaustible resources

Global resource reserves are crucial for the production of energy goods and services based on exhaustible resources. Our specification of initial world resources of oil is based on the concept of proven reserves which allows for changes in resource prices but assumes that firms can fully exploit these resources without any change in the applied technology. The U.S. Energy Information Administration (2011) reported that proven global reserves of fossil fuels amount to 231 Gt of crude oil, 889 Gt of coal (443 Gt of Anthracite and 537 Gt of Lignite), and 6972 Bill.m³ natural gas.²¹ Thus, physically

²⁰We see no problem in using data sources for different countries (here: Germany and the U.S.) to calibrate our production parameters as the underlying technologies should be similar enough such that the resulting cost shares for capital and labor are roughly the same.

²¹<http://www.eia.gov/totalenergy/data/annual/index.cfm>.

measured coal reserves are almost four times larger than those of crude oil. While we set initial reserves for oil equal to this value, we adopt the same arguments as in Golosov et al. (2014) to assume that there is no scarcity rent on coal such that the stock of coal is not exploited. This can be justified by a backstop technology which will replace current coal usage in the future. Formally, this corresponds to a choice $R_{2,0} = \infty$ in our simulations.

In accordance with the stylized facts reported above, initial oil resources are set to $R_{1,0}^1 = 229$ bn bbl in OECD countries vs. $R_{1,0}^2 = 1195$ bn barrel in the rest of the world. As coal extraction generates zero profits in our simulation, the world distribution of coal reserves is irrelevant.

Extraction costs

The operating costs of crude oil extraction reported in the World Economic Outlook 2015 by the International Monetary Fund (2015) vary considerably across different countries ranging from 4.4\$ per blue barrel (bbl) in Kuwait up to 12\$ per bbl in Venezuela. Average extraction cost weighted by domestic relative to global crude oil production for the years 2000-2010 is equal to 8.1\$ per bbl. These figures, however, represent short term marginal production cost. Since we want to consider the total cost of production (including, for instance, capital costs), we add a cost component and set the extraction cost for oil to 26 \$ per bbl. As one metric ton contains approximately 7.1475 bbl., this implies an average extraction cost parameter $c_1 = 0.00018168$ Mio.\$ per t crude oil. The average cost of extracting coal from the wells are 43\$ per ton of coal (see International Energy Agency (2010) p. 212), so we set $c_2 = 0.00043$ Mio.\$ per t of coal.

Resource prices and energy mix

The average prices for crude oil and coal from 2005-2014 reported by the World Bank (2015) were 83,59 \$/bbl for oil and 60 \$/t for coal.²² We choose energy productivity levels $Q_{i,0}^\ell$ such that initial resource prices in the simulations are close to these values. At the same time, they imply an empirically plausible mix of global primary energy demand of and 18% clean energy (nuclear plus renewables), 31% oil and 50% coal and gas in the laissez faire equilibrium.²³

Climate dynamics and damages

As our model of the Carbon cycle (7) and the damage function (9) are identical to Golosov et al. (2014), we also use their parameter values setting $\phi_0 = 0.393$, $\phi_L = 0.2$, $\phi = 0.0228$, and $\gamma_1 = \gamma_2 = 5.3 \times 10^{-5}$ in the benchmark case with homogeneous climate damages. Regional differences in γ^ℓ will be explored below. Further, the pre-industrial CO₂-level is $\bar{S} = 581$ and the initial values for permanent and non-permanent CO₂ are $S_{1,-1} = 726$ and $S_{2,-1} = 123$ GtC so that global carbon concentration in the first simulation period matches empirically observed CO₂-concentration of 849 GtC in 2015.

²²<http://databank.worldbank.org/data/databases/commodity-price-data>.

²³See International Energy Agency, World Energy Outlook 2000-2014, primary energy demand by fuel.

Emissions

To capture the CO₂ emissions from burning fossil fuels, we need the specific CO₂ factors per unit of oil and coal, respectively. For oil, we take $\zeta_1 = 3.089$ Gt CO₂ per Gt crude oil. For coal, we take the average CO₂ factor of lignite (1.3957 Gt CO₂ per Gt lignite) and anthracite (2.843 Gt CO₂ per Gt anthracite) weighted by the respective shares of total global coal reserves. This gives a specific CO₂ content of $\zeta_2 = 2.0498$ Gt CO₂ per Gt coal.²⁴

Consumption sector

In accordance with Assumption 2, we restrict consumer utility as in (12). Our benchmark calibration uses $\theta = 1$ which gives a logarithmic utility function and allows us to compare our findings directly to Golosov et al. (2014). The annual discount factor is $\beta = 0.985$, so for the model we set $\beta = 0.985^{10}$. Further, consumers in region 1 own 68.5% of the world capital stock which is chosen close to its long-run value.

Standard parameter set

The following table summarizes the parameter values used in the benchmark simulation.

Simulation parameters			
Final sector			
$\rho_0 = 0$	$\alpha_{0,K} = 0.3$	$\alpha_{0,N} = 0.6$	$Q_0^\ell = 1$
$\rho_1^E = 0.2$	$\rho_2^E = 0.6$	$\kappa_1 = 0.3818$	$\kappa_2 = 0.5$
Energy sectors			
$\rho_1 = 0$	$\alpha_{1,K} = 0.85$	$\alpha_{1,X} = 0.33$	$Q_1^\ell = 650$
$\rho_2 = 0$	$\alpha_{2,K} = 0.69$	$\alpha_{2,X} = 0.26$	$Q_2^\ell = 1000$
$\rho_3 = 0$	$\alpha_{3,K} = 0.75$	$\alpha_{3,X} = 0$	$Q_3^\ell = 5400$
Resource sectors			
$c_1 = 0.00018168$	$c_2 = 0.00043$		
Climate parameters			
$\zeta_1 = 0.84168$	$\zeta_2 = 0.596$	$\bar{S} = 581$	$\phi_L = 0.2$
$\phi_0 = 0.393$	$\phi = 0.0228$	$\gamma_1 = 0.000053$	$\gamma_2 = 0.000053$
Consumption sector			
$\beta = 0.985^{10}$	$\sigma = 1$		
Initial values			
$K_0 = 0.35$	$R_{1,0} = 331$	$S_{1,-1} = 726$	$S_{2,-1} = 123$

Table 1: Standard Parameter Set.

²⁴Since one ton of carbon equals 44/12 tons of CO₂, our ζ_1 implies a specific carbon content of 843 KgC/ ton oil, which is almost identical to the carbon content of 844 KgC/t oil given in Golosov et al. (2014). However, this relation does not hold exactly in the case of coal, since we choose a weighted average of anthracite and lignite with corresponding different carbon contents, while Golosov et al. (2014) set their carbon content of coal equal to the value of anthracite. Expressed in carbon units, we assume ζ_2 has 560 Kg carbon per ton of coal, Golosov et al (2014) assume 716 KgC/t coal.

6.2 Simulation results

Using the parametrization listed in Table 1, we compare the optimal and the laissez faire equilibrium. The following figures compare the effects at the resource stage, the energy stage, and the climate stage for both scenarios.

The resource stage

Figure 1 shows the extraction paths of oil (picture (a)) and coal (picture (b)) in the laissez-faire and the optimal equilibrium. The model predicts an annual extraction of 3.2 Gt crude oil over the next five decades in scenario 1, which is close to currently observed quantities: according to the EIA, about 3.7 Gt crude oil were extracted in 2013. Compared to this, firms extract less crude oil in the optimal policy scenario over the next five decades with average yearly extraction equal to 2.7 Gt. Interestingly, extraction of oil reserves in the optimal equilibrium is only initially lower than in the laissez faire case but becomes higher after $t = 2055$. Intuitively, the tax on emissions reduces the demand for oil which results in a decrease of resource prices over time to ensue that the resource is completely exhausted. This is precisely the 'forgotten supply side' argument advanced by Sinn (2012). In our case, the initially curbed extraction of oil in the optimal equilibrium is shifted into the future with the total amount extracted unchanged.

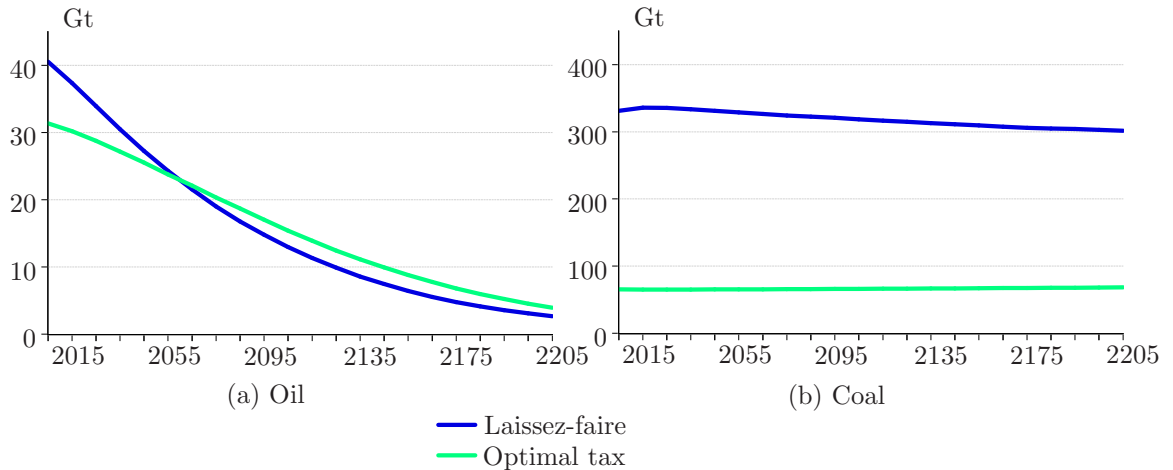


Figure 1: Exhaustible resource extraction: optimal tax versus laissez-faire

Regarding the extraction of coal, matters are different because this resource is not fully extracted. Thus, its price is constant and equals extraction costs. As a consequence, there is no price adjustment on the supply side and the optimal policy reduces the use of coal dramatically and also immediately: Introducing the tax on fossil fuel in $t = 2015$ reduces coal extraction instantaneously by 88%. In the laissez-faire equilibrium, the extraction of coal grows over the next thirty years and then slowly declines due to lower

demand from final output producers. After 100 years, the annual extraction rates are equal to initial levels. By contrast, coal extraction remains almost constant and even increases slightly over time along the optimal equilibrium. Thus, we can conclude that although the carbon tax is the same for coal and oil, the effects on the two types of fossil fuel are very different. Precisely the same effects are present in the quantitative study of Golosv et al. (2014).

The energy stage

Figure 2 compares the world energy mix, i.e., the relative shares of energy sectors in global energy production for both scenarios. The figures in the laissez faire equilibrium are broadly consistent with the energy shares in total primary energy demand reported in the World Energy Outlook 2014 International Energy Agency (2014). According to the observations in 2012, the fraction of oil based primary energy demand was 31% (model: 32%), the fraction of coal and natural gas was 51% (model: 55%) and the remaining 18% (model: 13%) was primary energy demand based on clean energy carriers. The

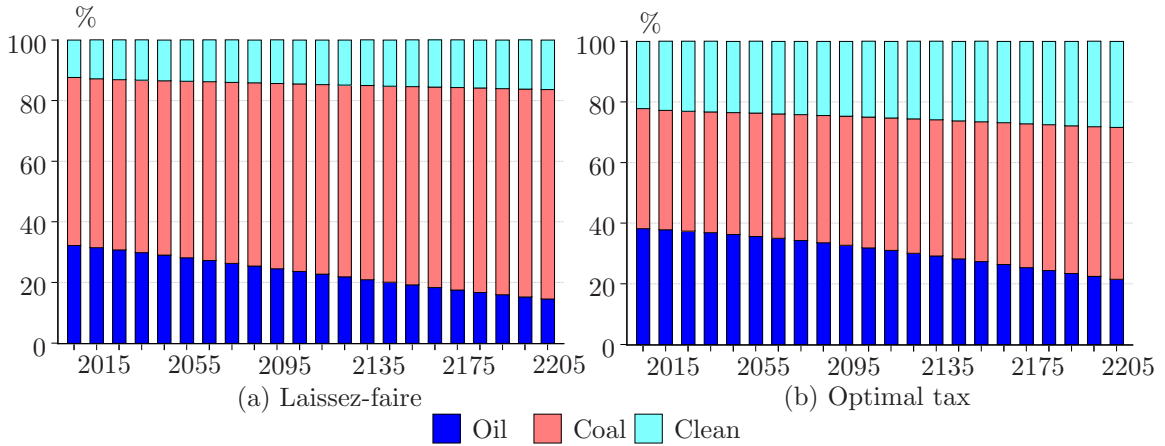


Figure 2: Energy Mix: optimal tax versus laissez-faire

introduction of a carbon tax immediately affects the energy mix and increases the share of clean energies to almost one fourth. Further, it increases the share of oil-based energy production while reducing the share of coal. These substitution effects seem primarily driven by the price adjustments at the resource stage described previously. It is also noteworthy that even under laissez faire the energy mix shifts slightly towards clean energies over the simulation horizon. In this scenario, the model predicts a decreasing share of oil based energy consumption to 24.5% over the next 100 years while the share of coal based production rises to 61% and the share of clean production to 15%. Thus, the weight of fossil fuel based energy production slightly decreases to 85% compared to initially 87%.

Qualitatively, changes in the energy mix under the optimal policy are similar: While

the share of oil decreases, coal and clean energy increase their shares. Nevertheless, the quantitative changes are noticeably stronger and the tax on emissions leads to much lower use of fossil energy types: In 2115 (2215), the fossil fuel share equals 75% (70%) while emissions-free energy production constitutes the remaining 25% (30%).

The climate stage

The shift in energy production towards clean technologies suggests that the optimal tax policy also has significant effects on emissions and climate damage. Figure 3(a) confirms that a climate tax leads to a significant reduction of fossil emissions which reduces climate damages (measured as a percentage of world GDP) significantly, as depicted in Figure 3(b). The gains from the optimal policy are notable already in the short run: 50 years from now, global damages are 7.3% of GDP in the laissez faire scenario rather than 3.2% under optimal taxation. This gap becomes even wider in later periods such that at the end of the simulation period (year 2215), damages in the laissez-faire regime exceed 20% of world GDP in the laissez faire case compared to 6.1% in the optimal case.

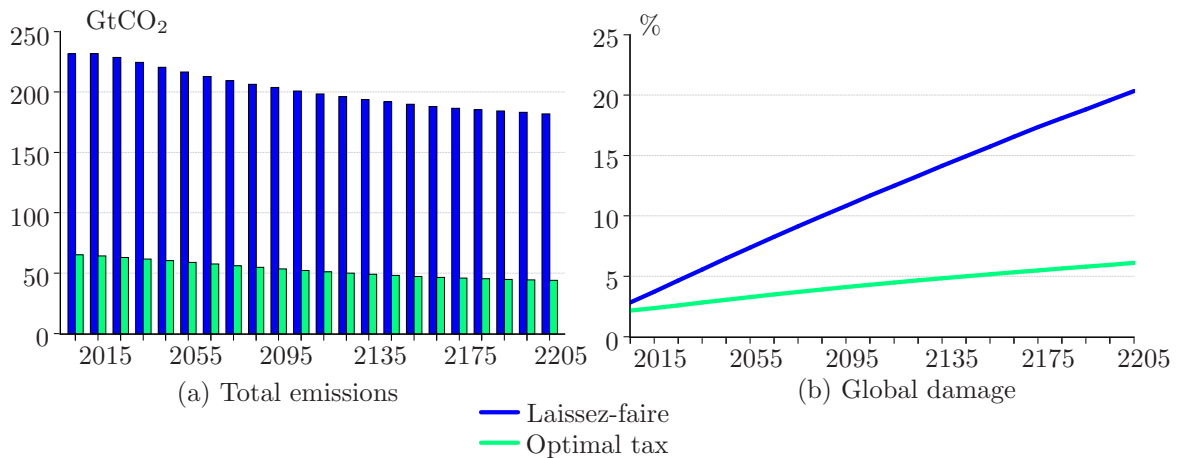


Figure 3: Global emissions and damage per World GDP: Optimal tax versus laissez-faire

Comparing the evolution of final output, we find that initial GDP in non-OECD countries is about 45% of GDP in OECD countries in the Laissez faire equilibrium, which is close to current empirical observations. Moreover, the introduction of a global tax on CO₂ emissions can help to decrease damages to GDP due to climate change in the long run: at the end of the simulation horizon, GDP in the OECD and the non-OECD is 11% higher under the optimal tax regime compared to the laissez-faire scenario. Interestingly, we find that policy intervention leads to a period of lower aggregate output compared to laissez-faire in both regions: the introduction of a carbon tax costs 1% of GDP over the next twenty five years. This period of adjustment prevails until 40 years

from now, while afterwards, the effect of a carbon tax on GDP is positive. The short term negative effect of a carbon tax is due to the fact that damages based on climate change are comparatively small over the next 50 to 60 years, combined with the fact that the a tax on arelatively more productive factor leads to lower aggregate output.

Figure 4 displays climate damages for both regions as a percentage of GDP in each country. If no climate tax is imposed, OECD countries suffer large and increasing climate damages which reach up to 15% of domestic GDP over the next 200 years. A carbon tax policy limits these damages to less than 5% of GDP. In both cases, damages are dramatically larger in non-OECD countries which exceed 30% of GDP in 2200 in the laissez faire allocation. A carbon tax limits these damages significantly to less than 10% of GDP over the next 200 years.

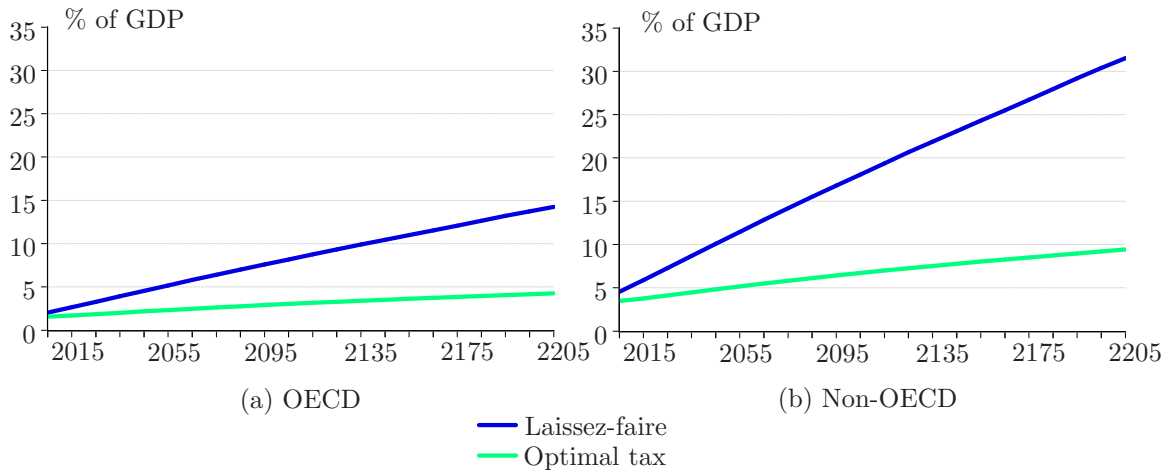


Figure 4: Climate damages in both regions: Optimal tax versus laissez-faire

To see how these damages translate into a change of global mean temperature, we hypothesize a logarithmic relationship between atmospheric CO₂ concentration and global mean surface temperature, as proposed in Nordhaus & Yang (1996). For the exact mapping from atmospheric CO₂-concentration S_t to global mean surface temperature $TEMP_t$, we follow Golosov et al.(2014) by assuming that

$$TEMP_t = 3 \log\left(\frac{S_t}{\bar{S}}\right) / \log 2$$

Figure 5 confirms that temperature increases dramatically along the laissez faire equilibrium which increases by 2.3 degrees over the next 50 years and will be more than 3 larger than currently observed temperature levels by the end of 2115. Introducing the (optimal) Carbon tax can significantly attenuate this problem and limit the increase to

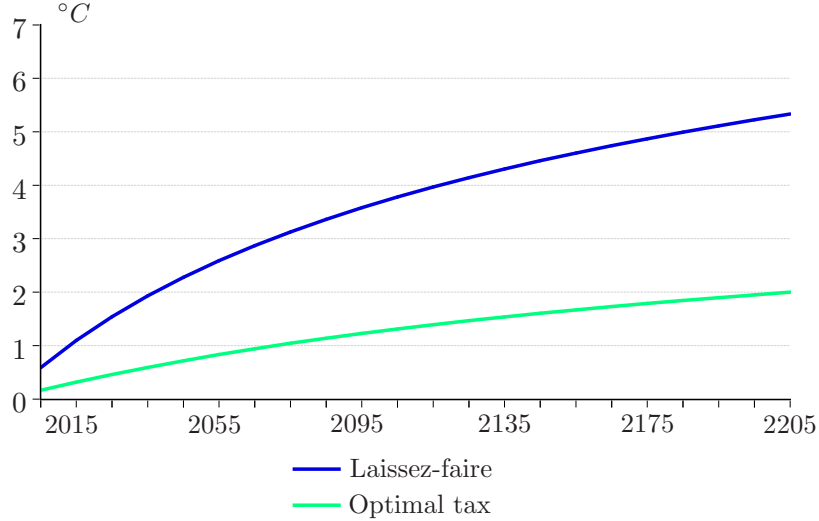


Figure 5: Change in global mean temperature: optimal tax versus laissez-faire

0.83 degree Celsius over the next fifty years and to just slightly more than two degrees until 2115. In fact, the latter is the target value for global mean surface temperature in the year 2100 that is typically discussed in the ongoing political debate about climate change.

6.3 Optimal transfer policies

Utility weights

In this section we study the size of optimal transfers between countries. Recall that any optimal transfer policy depends on some weighting scheme ω which is equivalent to a certain (constant) consumption share of each country out of world consumption. As the following analysis is not about fairness and redistribution of world income but about how the cost of climate change and the burden of implementing climate policies should be shared, we choose the consumption share $\mu^{\ell*}$ of country ℓ in the Laissez faire allocation as our target value. The Laissez faire case corresponds to the extreme case where all countries agree not to take any measures against climate change and is therefore a natural benchmark and also a threat point. We then investigate how transfers in the optimal equilibrium must be chosen to achieve the same distribution of world consumption. Thus, transfers in the optimal equilibrium ensure that each country ℓ attains the same relative wealth as in laissez faire case. A justification for this approach is provided in the following result which shows that the optimal climate tax leads to a Pareto improvement, i.e., both countries attain higher utility relative to the laissez faire equilibrium allocation. Formally, let $\mu_{LF} = (\mu_{LF}^1, \mu_{LF}^2)$ denote the consumption

shares along the laissez faire equilibrium which are constant by Lemma 3(ii). Define the transfer policy $\theta_{LF} = (\theta_{LF}^1, \theta_{LF}^2)$ by letting

$$\theta_{LF}^\ell = \frac{\mu_{LF}^\ell [W^{\text{eff}} + T^{\text{eff}}] - W^{\ell, \text{eff}}}{T^{\text{eff}}}, \quad \ell \in \{1, 2\}. \quad (74)$$

Here, T^{eff} are total tax revenues defined as in (16) and $W^{\text{eff}} := \sum_{\ell \in \mathbb{L}} W^{\ell, \text{eff}}$ represent aggregate non-transfer lifetime incomes of consumers defined as in (32) along the efficient equilibrium allocation. Clearly, transfer policy (74) induces the consumption share μ_{LF}^ℓ for country ℓ along the efficient equilibrium allocation. We now have the following result.

Lemma 5

The equilibrium allocation $\mathbf{A}^(\tau^{\text{eff}}, \theta_{LF})$ Pareto-improves the laissez faire allocation \mathbf{A}_{LF}^* .*

Heterogeneous climate damages

A common argument is that poorer countries are more affected by climate damages than rich countries. To incorporate this argument in our study, we include three additional scenarios where climate damages in Non-OECD countries are slightly (B), significantly (C), and dramatically (D) higher than in OECD countries, respectively. The parameter variations are displayed in the following table together with the resulting consumption shares in the laissez faire equilibrium. In the benchmark scenario A, these target con-

Scenario	Damage parameters			Consumption shares (LF)	
	$\gamma_1 \cdot 10^5$	$\gamma_2 \cdot 10^5$	Mean	OECD	Non-OECD
A: No differences	5.3	5.3	5.3	68.16%	31.84%
B: Small differences	4.4	6.2	5.3	68.66%	31.34%
C: Medium differences	3.6	7.0	5.3	69.08%	30.92%
D: Large differences	2.1	8.5	5.3	69.88%	30.12%

Table 2: Parameter variations and target consumption shares.

sumption shares essentially coincide with the shares of world GDP produced in each region. As Non-OECD countries are increasingly exposed to climate damages, their share of world consumption is gradually reduced till up to 2% in the extreme case of Scenario D.

Transfer payments

The results of our simulation experiment are reported in Table 3 which displays tax revenue in each country as a percentage of global revenue together together with the consumption shares θ^1 and θ^2 required to induce the target consumption shares from Table 2. The net transfer displayed in the last column is the difference between tax revenue collected in and transfers received by OECD countries (both expressed as percentages of global tax revenue).

Scenario	Tax revenue		Transfer shares		Net Transfer
	OECD	Non-OECD	OECD	Non-OECD	OECD → Non-OECD
A	69.28%	30.72%	53.16%	46.84%	16.12%
B	69.50%	30.50%	73.56%	26.44%	-4.06%
C	69.70%	30.30%	89.33%	10.67%	- 19.63%
D	70.11%	29.89%	115.49%	-15.49%	- 45.38%

Table 3: Tax revenue and optimal transfers between OECD and Non-OECD countries.

In the benchmark scenario, OECD countries collect slightly more than 69% of global tax revenue but only receive 53% as transfers resulting in a net transfer of 16% to Non-OECD countries. In the additional three scenarios where Non-OECD countries are increasingly exposed to climate damages, OECD countries acquire an increased share of world consumption in the laissez faire equilibrium. Non-OECD countries benefit much more from the climate tax than OECD countries which continue to collect most of the tax revenues. To distribute this benefit equally across the two regions, OECD countries receive an increasing share of transfer payments. This effect become most extreme in Scenario D where they collect 70% of global tax revenue and receive 115% of global tax revenue resulting in a net transfer of 45% from Non-OECD countries. As this share exceeds tax revenue in Non-OECD countries (which amounts to only about 30%), these countries must tax their consumers to finance transfer payments. The intuition for this somewhat surprising result is that climate effects in OECD countries are small if not negligible in this scenario while Non-OECD countries suffer dramatically. Thus, these countries benefit much more from a globally imposed climate tax and must share his benefit with OECD countries via transfers. The most extreme version of this effect would be where only one poor countries are affected by climate damages and rich countries must be incentivized to take action against climate change via transfers.

7 Conclusions

Our main theoretical result shows that the *efficiency problem* of choosing an optimal policy which internalizes the climate externality and implements an efficient allocation can strictly be separated from the *distributional issue* how the burden of climate change should be shared among countries. Thus, the optimal climate policy is uniquely determined and can either be implemented by a uniform global tax on carbon emission or a globally organized emissions trading system where the caps are chosen to achieve the optimal climate path. The real political issue is therefore how transfer payments between countries should be chosen to share the burden of climate change. This choice is subject to the weight attached to the interests of different countries and is therefore crucially

determined by heterogeneities across countries. To offer some preliminary guidance on the matter, our numerical study explored the consequences of heterogeneities in climate damages for optimal transfers between OECD and Non-OECD countries.

The framework developed in this paper can be extended in various directions. One such extension is to replace the deterministic setup by a stochastic environment with random perturbations. A second extension is a setup with endogenous growth and directed technical change as in Daron Acemoglu & Hemous (2012). Both modifications were considered in Golosov et al. (2014) and we believe that our framework is also amendable to them. A final avenue of future research is to model the political process between different regions as non-cooperative game as is done in Dutta & Radner (2006). Within the framework of this paper, \mathbb{L} would be the set of players each of which chooses their climate tax policy τ (alternatively, the caps \bar{Z} of their domestic emissions trading system) as their strategies and receives utility of domestic consumers as their pay-off.

A Mathematical Appendix

A.1 Proof of Lemma 1

Using standard Lagrangian arguments, a non-negative sequence $(X_t^*)_{t \geq 0}$ is a solution to (24) if $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^\ell$ and there exist non-negative Lagrangian variables $(\sigma_t^*)_{t \geq 0}$ and $\mu_{\text{opt}} \geq 0$ such that $((X_t^*, \sigma_t^*)_{t \geq 0}, \mu_{\text{opt}})$ solves

$$q_t(v_{i,t} - c_i) + \sigma_t - \mu = 0 \quad \forall t \geq 0 \quad (\text{A.1a})$$

$$\sigma_t X_t = 0 \quad \forall t \geq 0 \quad (\text{A.1b})$$

$$\mu \left(\sum_{t=0}^{\infty} X_t - R_{i,0}^\ell \right) = 0. \quad (\text{A.1c})$$

If $X_t^* > 0$ for all $t \geq 0$, then, $\sigma_t = 0$ by (A.1b) and $v_{i,t} \geq c_i$ by (A.1a) for all t . Using $q_0 = 1$ and $q_t/q_{t-1} = r_t^{-1}$ for all $t > 0$ in (A.1a), resource prices must evolve as in (25). The remaining assertions follow immediately. \blacksquare

A.2 Proof of Lemma 2

By Assumption 2, Euler equation (34), and (13), optimal consumption evolves as

$$C_t^{\ell*} = C_{t-1}^{\ell*} (\beta r_t)^{\frac{1}{\sigma}} = C_0^{\ell*} \prod_{s=1}^t (\beta r_s)^{\frac{1}{\sigma}} = C_0^{\ell*} \left(\frac{\beta^t}{q_t} \right)^{\frac{1}{\sigma}} \quad t \geq 1. \quad (\text{A.2})$$

Using (A.2), the l.h.s. in the lifetime budget constraint (31) can be written as

$$\sum_{t=0}^{\infty} q_t C_t^{\ell*} = \sum_{t=0}^{\infty} q_t C_0^{\ell*} (\beta^t/q_t)^{\frac{1}{\sigma}} = C_0^{\ell*} \sum_{t=0}^{\infty} (\beta^t q_t^{\sigma-1})^{\frac{1}{\sigma}}. \quad (\text{A.3})$$

Using (A.3) in (31) - which holds with equality- gives

$$C_0^{\ell*} = \frac{W^\ell + T^\ell}{\sum_{t=0}^{\infty} (\beta^t q_t^{\sigma-1})^{\frac{1}{\sigma}}}. \quad (\text{A.4})$$

Using (A.4) in (A.2) yields (36). ■

A.3 Proof of Lemma 3

(i) This is a direct consequence of the equilibrium conditions (19), (23), (21) and the boundary behavior of F_i and u imposed by Assumptions 1 and 2.

(ii) Set $\mu^\ell := C_0^{\ell*}/C_0^*$ for $\ell \in \mathbb{L}$. By Lemma 2, the growth rates of each sequence $(C_t^{\ell*})_{t \geq 0}$ are independent of ℓ and equal to the growth rates of aggregate consumption $(C_t^*)_{t \geq 0}$. Induction then implies that (43) holds for all $t \geq 0$.

(iii) Let $i \in \mathbb{I}_x$ be arbitrary. It is clear that (5) requires $\lim_{t \rightarrow \infty} X_{i,t}^{\ell,s} = 0$ for all $\ell \in \mathbb{L}$ which implies $\lim_{t \rightarrow \infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell,s} = 0$. Further, by (38), $\lim_{t \rightarrow \infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^\ell = 0$ which implies

$$\lim_{t \rightarrow \infty} X_{i,t}^\ell = 0 \quad (\text{A.5})$$

for all $\ell \in \mathbb{L}$. If (43) were non-binding in some (in fact, each) country ℓ , this would require $v_{i,t} = c_i$ for all t by Lemma 1. But this is incompatible with (21c), (A.5), and the Inada conditions which imply $\lim_{t \rightarrow \infty} \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) = \infty > v_{i,t} = c_i$. In this regard, note the Inada conditions and (21a,b) exclude that either $K_{i,t}^\ell$ or $N_{i,t}^\ell$ converge to zero for $t \rightarrow \infty$. ■

A.4 Proof of Proposition 1

(i) Using the result from Lemma 2, aggregate consumption in period t satisfies

$$\bar{C}_t^* = \frac{(\beta^t/q_t)^{\frac{1}{\sigma}} [\sum_{\ell \in \mathbb{L}} W^\ell + T]}{\sum_{s=0}^{\infty} q_s (\beta^s/q_s)^{\frac{1}{\sigma}}} \quad t \geq 0 \quad (\text{A.6})$$

and is, therefore, independent of θ . By (41), so is K_{t+1} and all other equilibrium quantities in $\bar{\mathbf{A}}^*$.

(ii) Using Lemma 2 and (15) in conjunction with (46) and (A.6) gives

$$C_t^{\ell*} = \frac{(\beta^t/q_t)^{\frac{1}{\sigma}} [W^\ell + \theta^\ell T]}{\sum_{s=0}^{\infty} q_s (\beta^s/q_s)^{\frac{1}{\sigma}}} = \mu^{\ell*} \bar{C}_t^*$$

for all $t \geq 0$ and $\ell \in \mathbb{L}$, proving the claim. ■

A.5 Proof of Lemma 4

Let ω and $C > 0$ be given. Using (12), direct computations reveal that the objective function in (53) can be written as

$$\sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell C) = a \left(u(C) + \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell) \right) + (1 - \sigma) u(C) \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell) \quad (\text{A.7})$$

where $a = 0$ if $\sigma \neq 1$ and $a = 1$ otherwise. In either case, the solution to (53) can be computed by solving

$$\max_{\mu = (\mu^\ell)_{\ell \in \mathbb{L}}} \left\{ \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell) \mid \mu^\ell \geq 0, \sum_{\ell \in \mathbb{L}} \mu^\ell = 1 \right\} \quad (\text{A.8})$$

and is, therefore, independent of C .

To see that (A.8) admits a unique solution, suppose first that $\omega \gg 0$. The boundary behavior of u implies that any solution to (A.8) is bounded away from zero, by, say $\underline{\mu} = (\underline{\mu}^\ell)_{\ell \in \mathbb{L}} \gg 0$. This and the constraints in (A.8) define a compact, convex subset of \mathbb{R}_{++}^L on which the map $\mu \mapsto \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu^\ell)$ is continuous and strictly concave, which ensures that a solution μ_{opt} exists and is unique. Second, if, say, $N < L$ components of ω are zero, set $\mu_{\text{opt}}^\ell = 0$ for these components and repeat the previous argument for the problem (A.8) restricted to an obvious subset of \mathbb{R}^{L-N} .

Finally, using (A.7), the maximum value can be expressed as in (54). \blacksquare

A.6 Proof of Theorem 1

Let weighting scheme ω be arbitrary but fixed and μ_{opt} be the unique solution to (53). Let $(\hat{\mathbf{C}}_t, \hat{\Gamma}_t)_{t \geq 0}$ where $\hat{\mathbf{C}}_t = (\hat{C}_t^\ell)_{\ell \in \mathbb{L}}$, $t \geq 0$ be an arbitrary feasible allocation which satisfies the constraints in (48). Defining $(\mathbf{C}_t^*, \Gamma_t^*)_{t \geq 0}$ as in the lemma, we have to show that

$$V((\hat{\mathbf{C}}_t)_{t \geq 0}; \omega) \leq V((\mathbf{C}_t^*)_{t \geq 0}; \omega). \quad (\text{A.9})$$

Define aggregate consumption $(\hat{C}_t)_{t \geq 0}$ induced by $(\hat{\mathbf{C}}_t)_{t \geq 0}$ as $\hat{C}_t := \sum_{\ell \in \mathbb{L}} \hat{C}_t^\ell$, $t \geq 0$. Then, $(\hat{C}_t, \hat{\Gamma}_t)_{t \geq 0}$ satisfies the constraints in (51). Since $(C_t^*, \Gamma_t^*)_{t \geq 0}$ solves this problem

$$\sum_{t=0}^{\infty} \beta^t u(\hat{C}_t) \leq \sum_{t=0}^{\infty} \beta^t u(C_t^*). \quad (\text{A.10})$$

Let $a = 1$ if $\sigma = 1$ and $a = 0$ otherwise. By (53) and Lemma 4, we have for all $t \geq 0$

$$\sum_{\ell \in \mathbb{L}} \omega^\ell u(\hat{C}_t^\ell) \leq \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu_{\text{opt}}^\ell \hat{C}_t) = a \left(m(\omega) + u(\hat{C}_t) \right) + (1 - \sigma) m(\omega) u(\hat{C}_t). \quad (\text{A.11})$$

and

$$\sum_{\ell \in \mathbb{L}} \omega^\ell u(C_t^{\ell*}) = \sum_{\ell \in \mathbb{L}} \omega^\ell u(\mu_{\text{opt}}^\ell C_t^*) = a\left(m(\omega) + u(C_t^*)\right) + (1 - \sigma)m(\omega)u(C_t^*). \quad (\text{A.12})$$

Equations (A.11) and (A.12) being true for all $t \geq 0$ and (A.10) then gives

$$\begin{aligned} V((\hat{\mathbf{C}}_t)_{t \geq 0}; \omega) &= \sum_{t=0}^{\infty} \beta^t \sum_{\ell \in \mathbb{L}} \omega^\ell u(\hat{C}_t^\ell) \\ &\leq a\left(\frac{m(\omega)}{1 - \beta} + \sum_{t=0}^{\infty} \beta^t u(\hat{C}_t)\right) + (1 - \sigma)m(\omega) \sum_{t=0}^{\infty} \beta^t u(\hat{C}_t) \\ &\leq a\left(\frac{m(\omega)}{1 - \beta} + \sum_{t=0}^{\infty} \beta^t u(C_t^*)\right) + (1 - \sigma)m(\omega) \sum_{t=0}^{\infty} \beta^t u(C_t^*) \\ &= \sum_{t=0}^{\infty} \beta^t \sum_{\ell \in \mathbb{L}} \omega^\ell u(C_t^{\ell*}) = V((\mathbf{C}_t^*)_{t \geq 0}; \omega) \end{aligned}$$

This proves (A.9) and the claim. ■

A.7 Computing the efficient solution (52)

For brevity, set $\mathbf{E}_t^\ell := (E_{i,t}^\ell)_{i \in \mathbb{I}}$ and $D_t^\ell := D^\ell(S_{1,t} + S_{2,t})$. We adopt a standard Lagrangian-type approach also used in Golosov et al. (2014) to characterize the solution to (51). Define the Lagrangian function

$$\begin{aligned} &\mathcal{L}\left(\left(C_t, K_{t+1}, \left((K_{i,t}^\ell, N_{i,t}^\ell)_{i \in \mathbb{I}_0}, (E_{i,t}^\ell)_{i \in \mathbb{I}}, (X_{i,t}^\ell)_{i \in \mathbb{I}_x}\right)_{\ell \in \mathbb{L}}, S_{1,t}, S_{2,t}\right)_{t \geq 0}\right) \quad (\text{A.13}) \\ := &\sum_{t=0}^{\infty} \beta^t \left[u(C_t) + \lambda_{0,t} \left(\sum_{\ell \in \mathbb{L}} (1 - D^\ell(S_{1,t} + S_{2,t})) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}) \right. \right. \\ &\left. \left. - C_t - K_{t+1} - \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} c_i X_{i,t}^\ell \right) \right. \\ &+ \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \lambda_{i,t}^\ell \left(Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - E_{i,t}^\ell \right) + \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_n} \lambda_{i,t}^\ell \left(Q_{i,t}^\ell F_i(K_{i,t}^\ell, N_{i,t}^\ell) - E_{i,t}^\ell \right) \\ &+ \sum_{\ell \in \mathbb{L}} \lambda_{N,t}^\ell \left(N_t^\ell - \sum_{i \in \mathbb{I}_0} N_{i,t}^\ell \right) + \lambda_{K,t} \left(K_t - \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_0} K_{i,t}^\ell \right) \\ + &\lambda_{S_1,t} \left(S_{1,t} - S_{1,t-1} - \phi_L \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell \right) \\ + &\lambda_{S_2,t} \left(S_{2,t} - (1 - \phi) S_{2,t-1} - (1 - \phi_L) \phi_0 \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t}^\ell \right) \left. \right] \\ &+ \sum_{i \in \mathbb{I}_x} \mu_i \left(R_{i,0} - \sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^\ell \right) \end{aligned}$$

Standard arguments imply that $A = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t \geq 0}$ is a solution to (51), if there exist non-negative Lagrangian multipliers $\lambda = (\lambda_t)_{t \geq 0}$ and $\mu = (\mu_i)_{i \in \mathbb{I}}$, $\lambda_t := (\lambda_{0,t}, ((\lambda_{i,t}^\ell)_{i \in \mathbb{I}_0}, \lambda_{N,t}^\ell)_{\ell \in \mathbb{L}}, \lambda_{K,t}, \lambda_{S_1,t}, \lambda_{S_2,t})$ such that (A, λ, μ) solve the resulting first order and complementary slackness conditions. For $t \geq 0$, define

$$\Lambda_t := \phi_L \frac{\lambda_{S_1,t}}{\lambda_{0,t}} + (1 - \phi_L) \phi_0 \frac{\lambda_{S_2,t}}{\lambda_{0,t}}. \quad (\text{A.14})$$

After eliminating as many Lagrangian variables as possible, these conditions can be summarized for all $t \geq 0$, $\ell \in \mathbb{L}$, and $i \in \mathbb{I}$ (suppressing quantifiers when convenient) as:

$$u'(C_t) = \lambda_{0,t} = \beta \lambda_{K,t+1} \quad (\text{A.15a})$$

$$\lambda_{K,t} = \lambda_{0,t} (1 - D_t^\ell) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) \quad (\text{A.15b})$$

$$\lambda_{N,t}^\ell = \lambda_{0,t} (1 - D_t^\ell) Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) \quad (\text{A.15c})$$

$$\lambda_{i,t}^\ell = \lambda_{0,t} (1 - D_t^\ell) Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) \quad (\text{A.15d})$$

$$\mu_i = \beta^t \left[\lambda_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \lambda_{0,t} c_i - \lambda_{0,t} \zeta_i \Lambda_t \right] \quad (\text{A.15e})$$

$$\lambda_{S_1,t} = \lambda_{0,t} \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_t)}{dS} \frac{Y_t^\ell}{1 - D^\ell(S_t)} + \beta \lambda_{S_1,t+1} \quad (\text{A.15f})$$

$$\lambda_{S_2,t} = \lambda_{0,t} \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_t)}{dS} \frac{Y_t^\ell}{1 - D^\ell(S_t)} + (1 - \phi) \beta \lambda_{S_2,t+1}. \quad (\text{A.15g})$$

Note that $\lambda_{0,t}$ can be interpreted as a shadow price of time t consumption. Thus, the time t shadow price of energy of type $i \in \mathbb{I}$ produced in country $\ell \in \mathbb{L}$ measured in time t consumption goods can be defined as

$$\hat{p}_{i,t}^\ell := \frac{\lambda_{i,t}^\ell}{\lambda_{0,t}}. \quad (\text{A.16})$$

Assuming that $\lim_{n \rightarrow \infty} \beta^{n+1} \lambda_{S_1,t+n} = \lim_{n \rightarrow \infty} ((1 - \phi)\beta)^{n+1} \lambda_{S_2,t+n} = 0$, (A.15f) and (A.15g) can be solved forward to obtain a result similar to GHKT:

$$\frac{\lambda_{S_1,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(C_{t+n})}{u'(C_t)} \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n})}{dS} \frac{Y_{t+n}^\ell}{1 - D^\ell(S_{t+n})} \quad (\text{A.17a})$$

$$\frac{\lambda_{S_2,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(C_{t+n})}{u'(C_t)} (1 - \phi)^n \sum_{\ell \in \mathbb{L}} \frac{dD^\ell(S_{t+n})}{dS} \frac{Y_{t+n}^\ell}{1 - D^\ell(S_{t+n})}. \quad (\text{A.17b})$$

Using (A.17a) and (A.17b) in (A.14) gives precisely the condition (56).

Combining (A.15a) and (A.15b) gives the familiar Euler equation

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t) (1 - D^\ell(S_t)) Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i \in \mathbb{I}}). \quad (\text{A.18})$$

As the l.h.s in (A.15e) is independent of ℓ and t , we obtain using (A.16)

$$\hat{p}_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \zeta_i \Lambda_t = \hat{p}_{i,t}^{\ell'} Q_{i,t}^{\ell'} \partial_X F_i(K_{i,t}^{\ell'}, N_{i,t}^{\ell'}, X_{i,t}^{\ell'}) - \zeta_i \Lambda_t =: \hat{v}_{i,t} \quad (\text{A.19})$$

for all $\ell, \ell' \in \mathbb{L}$ and $t \geq 0$ and

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)} (\hat{v}_{i,t+1} - c_i) \quad (\text{A.20})$$

for all $t \geq 0$. Essentially, (A.19) ensures intratemporal and (A.20) intertemporally efficiency of resource extraction. Analogously to the proof of Lemma 3 (iii), one can easily show that (39) in conjunction with Assumption 1 exclude a solution $\mu_i = 0$. Thus, $\mu_i > 0$ and the resource constraint (39) is always binding.

A.8 Proof of Lemma 5

Let $\ell \in \mathbb{L}$ be arbitrary and $(\bar{C}_t^{LF})_{t \geq 0}$ and $(\bar{C}_t^{\text{eff}})_{t \geq 0}$ be the aggregate consumption sequences along the efficient and laissez faire allocation, respectively. By Lemma 3(ii) and Assumption 2, utility of country ℓ along the LF allocation is $U((\mu_{LF}^\ell \bar{C}_t^{LF})_{t \geq 0}) = a + bU((\bar{C}_t^{LF})_{t \geq 0})$ where a and $b > 0$ are constants that depend only on μ^{LF} . Further, by construction and Lemma 3(ii), utility of country ℓ along the allocation $\mathbf{A}^*(\tau^{\text{eff}}, \theta_{LF})$ is $U((\mu_{LF}^\ell \bar{C}_t^{\text{eff}})_{t \geq 0}) = a + bU((\bar{C}_t^{\text{eff}})_{t \geq 0})$. Thus, $U((\mu_{LF}^\ell \bar{C}_t^{LF})_{t \geq 0}) < U((\mu_{LF}^\ell \bar{C}_t^{\text{eff}})_{t \geq 0})$ if and only if $U((\bar{C}_t^{LF})_{t \geq 0}) < U((\bar{C}_t^{\text{eff}})_{t \geq 0})$ which follows directly from the definition of the efficient allocation (52). ■

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