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Transaction Cost Heterogeneity in the Interbank Market and Monetary Policy Implementation under alternative Interest Corridor Systems

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Abstract†

In the recent past, the ECB and other major central banks have used different interest corridor implementation schemes in order to exert control over the interest rate in the overnight interbank market. Typically, narrow corridor systems where the spread between the rates on the central bank’s standing facilities is relatively small are considered to feature relatively low levels of interbank rate volatility. However, as argued in this paper, the relationship between corridor width and volatility might be inverted if the redistribution of liquidity in the interbank market is impaired and especially if banks differ in the degrees they are impaired in trading on the interbank market. In particular, volatility control under a floor operating system, as currently employed by the ECB, might be achieved by implementing a wide interest corridor. In order to shed light on these insights, this paper introduces a theoretical model of an interbank market with frictions in the form of broadly defined transaction costs (which might stem from asymmetric information about counterparty credit risks or result from new regulatory capital rules affecting interbank exposures) and a central bank that chooses between a standard corridor system or a floor operating system.

JEL classification: E52, E58, G21

Keywords: interbank market, monetary policy implementation, interest corridor, floor operating system, transaction costs, excess reserves

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1 Introduction

As a matter of monetary policy implementation, interest corridor systems have gained increasing attention in recent years: A central bank is able to exert tight control over the interest rate in the unsecured overnight segment of the interbank market by setting the rates on a deposit and a lending facility where banks can take recourse to - at anytime and on their own initiatives - in order to place or obtain overnight liquidity as outside option to the interbank market. The corridor formed by the facility rates thereby bounds fluctuations and is an additional instrument that is designed for reducing volatility of the interbank rate supplementing or replacing reserve requirements with averaging provision.\(^2\)

In principle, several approaches for controlling the level and volatility of the interbank rate by implementing a corridor system come into consideration for a central bank, as exemplarily shown by Figure 1 for the euro area.\(^3\)

![Figure 1: EONIA: Distribution of market rates under (a) standard and (b) floor system. Vertical axis: #obs. Horizontal axis: EONIA in %. Data: ECB.](image)

Figure 1 shows the distribution of EONIA rates during two different periods, when the ECB’s key policy rates remained unchanged, respectively: in Figure 1(a) for the period between June 2003 and December 2005, when the ECB ran a “standard corridor system”, meaning that the targeted interbank rate was lying in the middle of the corridor formed by the rates on its deposit and marginal lending facility; in Figure 1(b) for the period between

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\(^3\)For an overview see Federal Open Market Committee (2015).
July 2009 and July 2010, when the ECB *de facto* operated a “floor system”, meaning that by an ample provision of liquidity the EONIA was driven down to its lower bound given by the deposit rate. During both periods, the dispersion of EONIA rates has been relatively low and the EONIA traded within a fairly narrow range on most days, indicating that volatility during these periods has been relatively moderate (with the exception of several extreme values).

A crucial policy variable for the control of volatility under an interest corridor regime is the corridor width, i.e. the spread between the central bank’s facility rates. Conventional wisdom is, that the dispersion of market rates should be the lower, that is volatility should be the lower, the narrower the interest corridor is. However, with regard to the control of volatility through adjustments in the corridor width the *sources* of volatility as well as the *type* of the corridor system implemented are of vital importance: Generally, a significant driver of volatility are interest rate effects produced by temporary liquidity imbalances in the banking sector when banks’ actual liquidity needs differ from the central bank’s liquidity provision through open market operations. If interbank rate volatility can be traced back to aggregate liquidity imbalances caused by stochastic liquidity factors (such as currency in circulation or government deposits with the central bank) a narrow *standard interest corridor* as well as a *floor operating system* generally should feature relatively low levels of volatility. However, as argued in this paper, the relationship between corridor width and volatility might be inverted when the interbank market is not frictionless such that the redistribution of liquidity in the banking sector is impaired.

In fact, the interbank market for unsecured overnight loans was regarded as almost frictionless until the outbreak of the financial crisis in 2007. But since then, the notion of a frictionless interbank market has been called into question. Problems of asymmetric information about counterparty credit risk became highly relevant during the crisis. In the presence of asymmetric information transactions in the interbank market become more expensive as lending involves costly checks of creditworthiness and borrowing is combined with signalling costs. The fact that banks across the banking sector may differ substantially in the costly effort they have to exert in order to (re-) obtain market access especially became apparent in the euro area during the sovereign debt crisis. Borrowing or lending
conditions for banks in core and periphery countries differed to a large extent, reflecting a high degree of cross-border fragmentation in formerly well integrated euro money markets. Furthermore, as discussed in Jackson and Noss (2015), market frictions of a different kind might even become more relevant in the future. Jackson and Noss (2015) consider possible costs stemming from interbank market exposures when banks have to comply with new regulatory capital rules. Risk-based capital requirements and a regulatory leverage ratio might increase the marginal costs of an interbank market transaction but each regulatory rule does affect a different market side. Whereas the former capital rule might increase the marginal costs of lending to the interbank market, the latter might increase the marginal costs of borrowing from that market. Cost heterogeneity in this respect reflects the differing degrees of binding capital constraints among banks.

Against this background, this paper introduces a theoretical model of bank reserve management and an interbank market in order to analyze in how far interbank market frictions in the form of broadly defined transaction costs influence banks’ liquidity demand and the equilibrium interbank market outcome. We discuss the implications for a central bank in forecasting the banking sector’s liquidity demand in the presence of broadly defined interbank market transaction costs in general and bank transaction costs heterogeneity in particular. In our analysis and discussion, we account for differing levels of transaction costs for lenders and borrowers arising from interbank market transactions and distinguish explicitly between the effects of transaction cost heterogeneity in a standard corridor system and in a floor operating system. Eventually, we discuss the special role attached to the width of the corridor in controlling interbank rate volatility in each implementation scheme in dependence on the sources of volatility.

We argue that in the presence of transaction cost heterogeneity in a standard corridor system it is no longer sufficient for a central bank that targets the corridor midpoint rate to arrange balanced aggregate liquidity conditions in order to implement the targeted rate. Dependent on which market side is more severely affected by transaction costs, the central bank either has to generate an aggregate liquidity deficit or an aggregate liquidity surplus to reach its target. Nevertheless, which banks will be active on which side on the interbank market is unknown to the central bank in advance of the conduct of open
market operations. Therefore, transaction cost heterogeneity brings about an additional factor of uncertainty about banks’ aggregate liquidity demand at the target rate and constitutes an additional source of interbank rate volatility. However, we argue that in a standard corridor system under balanced aggregate liquidity conditions, volatility induced by transaction cost heterogeneity cannot be attenuated by adjustments in the corridor width.

In contrast, our findings suggest that under a floor operating system an increase in the corridor width might reduce volatility of the interbank rate if the latter can be traced back to transaction cost heterogeneity. Moreover, we show that under a floor operating system transaction costs in the interbank market might lead to a concentration of liquidity in the banking sector at a particular set of banks. This uneven liquidity distribution implies that transaction costs of lending to the interbank market produce significantly larger interest rate effects than transaction costs of borrowing from the interbank market. With regard to newly introduced capital requirements to be imposed on banks, this result suggests that especially in a floor operating system, as actually implemented by the Eurosystem, capital constraints for lending banks in the interbank market might have a significantly larger impact on the interbank rate than capital constraints for borrowers in this market. Consequently, the liquidity provision by the central bank needed to implement a floor operating system has to be the more expansionary, the more pronounced transaction costs of lending to the interbank market are. In order to shed light on these insights, we conduct an in-depth analysis of the factors determining banks’ liquidity demand and the interbank market equilibrium. The theoretical framework we develop might serve as a basis for various further analyses of monetary policy implementation under interest corridor regimes in the presence of interbank market frictions.

The rest of this paper is structured as follows. Section 2 provides a brief overview of related literature. Section 3 presents the model setup. Section 4 derives optimal bank behavior with respect to the banks’ use of the central bank’s standing facilities and their interbank market activities. This allows us to analyze in-depth the determinants of banks’ liquidity demand, of interbank loan supply and demand and to identify the equilibrium interbank market outcome in Section 5. Section 6 discusses the implications for monetary
policy implementation distinguishing explicitly between a scenario in which the standing facilities form a standard corridor system and a scenario in which the central bank implements a floor operating system. Section 7 summarizes the paper.

2 Related Literature

Our paper contributes to the literature on bank reserve management, excess reserves, the interbank market, and monetary policy implementation. Although a large body of research within this literature discussed the implications of various forms of frictions in the interbank market, bank heterogeneity in this respect has received only little attention. We address this shortcoming by considering transaction cost heterogeneity in the interbank market for central bank reserves.

A seminal theoretical model of a commercial bank’s reserve management in money markets is presented in Poole (1968): A representative bank makes daily adjustments to its current account balances with the central bank (i.e. reserves) by trading in the interbank (money) market or by borrowing from the central bank at a penalty rate. Adjustments are necessary in order to avoid a costly under-/over-fulfillment of reserve requirements (resp. costly deviations from a targeted account balance) resulting from reserve drains or inflows during the day. Crucially, imperfect knowledge of daily reserve fluctuations on the part of the bank is the central argument in Poole (1968): The bank learns about the actual extent of the latter only after the interbank market is “effectively closed” when a “late” liquidity/payment shock finally determines the bank’s end-of-day account balances. Thus, uncertainty about daily reserve flows creates a precautionary demand for liquidity and therewith is the rationale for the bank’s demand for excess reserves.

In this vein, interest-elasticity of banks’ demand for excess reserves is introduced, what allows for an analysis of the equilibrium in the interbank market.\(^4\) In particular, several models evolving from Poole (1968) seek to explain stylized facts or observed patterns of the overnight interbank rate (pre-eminent in the federal funds market). The latter works include models that provide explanations for intra-reserve maintenance period patterns or settlement day volatility of the interbank rate: Clouse and Dow (1999) show i.a. how a

\(^4\text{See also Clouse and Dow (1999).}\)
fixed cost incurred when taking recourse to the discount window in conjunction with a 
decrease of banks’ reserve position uncertainty towards the end of a reserve maintenance 
period can increase the volatility of the federal funds rate during this stage and especially 
on the last day of a reserve maintenance period.\footnote{This fixed cost might stem from administrative barriers or the Fed’s implicit discouragement of banks to seek recourse to the discount window \citep[see][]{ClouseDow1999}, p. 1019.}. \citet{Furfine2000} finds that intra-reserve 
maintenance period patterns of the federal funds rate partially can be traced back to 
patterns in interbank payment flows. He reveals a positive relationship between the volume 
and volatility of interbank payment flows and the level and volatility of the federal funds 
rate. Crucially, the theoretical basis for these results is the assumption that banks’ daily 
reserve account uncertainty depends positively on the volume of daily interbank payments 
flows.\footnote{Therewith, an increase in the interbank payment volume implies an increase in reserve account uncertainty which leads to an increase in a bank’s precautionary liquidity demand, i.e. excess reserves, putting an “upward pressure” on the interbank rate. \citet{Furfine2000} also gives an empirical estimate for the relative magnitude of reserve account uncertainty for a panel of banks in the federal funds market.} \citet{BartoliniBertolaPrati2002} attribute the observed U-shaped intra-reserve 
maintenance period pattern and the increase in the volatility of the federal funds rate 
towards the end of the reserve maintenance period to two factors. They argue (similarly 
to \citet{ClouseDow1999}) that the decrease in reserve position uncertainty towards the 
end of a reserve maintenance period leads to a decrease in the interest-elasticity of banks’ 
reserve demand which produces larger movements of the federal funds rate in response 
to systemic liquidity shocks/imbalances. Moreover, the authors argue that federal funds 
rate volatility is the higher, the more impaired (or unwilling) the Federal Reserve is in 
itself ability to offset systemic liquidity imbalances through open market operations (be it 
actually or perceived by market participants).

Models that help to explain patterns of the interbank rate also building up on \citet{Poole1968} and explicitly considering a central bank that implements an interest corridor 
system are included in \cite{Woodford2001, Moschitz2004, BindseilCambaMendezHirschWeller2006, Whitesell2006, PerezQuirosRodriguezMendizabal2006}. By considering the width of the interest corridor, which is set by the central bank, the latter authors take into account a policy variable which crucially influences the volatility 
of the interbank rate and determines the banks’ demand for excess reserves in an interest 
corridor regime.
As pointed out by Woodford (2001) and Whitesell (2006), a standard interest corridor system has an advantageous property for the central bank’s liquidity management operations: In order to implement the target interbank rate being exactly the midpoint of the corridor, the central bank has to arrange a zero supply of excess reserves. As banks aggregate demand for excess reserves at the midpoint rate of the interest corridor is constantly zero over time and unaffected by the absolute level of interest rates or the corridor width, the central bank can avoid the permanent re-estimation of this liquidity factor. Ultimately, this property eliminates a potential source of forecast errors in the banking sector’s liquidity demand and therewith volatility of the interbank rate. However, this theoretically appealing property requires a frictionless interbank market. After the outbreak of the financial crisis in 2007, interbank market frictions have received increasing attention in the academic debate: Most papers dealing with interbank market frictions focus on asymmetric information about credit risks. Freixas and Jorge (2008) show that private information regarding credit risks may not only result in a rationing of firms in credit markets but may impose both a magnitude and a liquidity effect on the transmission mechanism of monetary policy. Heider, Hoerova, and Holthausen (2009) focus on asymmetric information in counterparty credit risks. Banks having an informational disadvantage hold more liquidity which might result in adverse selection or a dry-up in the interbank market depending on the extent of risk dispersion. A dry-up in the interbank market may also occur if banks are unable to hedge idiosyncratic liquidity shocks so that they start to hoard liquidity (Allen, Carletti, and Gale, 2009). Banks’ ability to access liquidity might furthermore depend on the extent of relationship lending, as Bräuning and Fecht (2012) find for the German unsecured overnight money market. The consequences of interbank market frictions in the form of transaction costs are already discussed by Poole (1968). He argues that the existence of transaction costs can explain why banks hold excess reserves instead of lending them in the interbank market, and why banks prefer to pay a relatively high interest rate at the discount window instead of borrowing from the interbank market. The more recent literature dealing explicitly with interbank market transaction costs starts with Bartolini, Bertola, and Prati (2001). They argue that uncertain liquidity flows and transaction costs are responsible for relatively high interbank rates.

Our paper contributes to these strands of literature by introducing lender and borrower specific transaction costs in a bank reserve management framework. We build up on Whitesell (2006) and Gaspar, Pérez Quirós, and Rodríguez Mendizábal (2008) and add to the literature on interbank market friction by considering the effects of transaction cost heterogeneity on the equilibrium in the interbank market. Thereby, we explicitly consider liquidity conditions in the banking sector, the interest corridor of the central bank’s standing facilities, and uncertainty about the banks’ actual liquidity needs.

3 Model Setup

The model presented in this paper builds on the framework introduced in Whitesell (2006) of a representative commercial bank’s reserve management under an interest corridor system.7 A one-period economy consists of a large number of commercial banks and a central bank that provides settlement accounts for banks and operates two standing facilities. Each commercial bank can use the lending facility to borrow (unlimitedly) liquidity at the rate $i^{LF}$ and the deposit facility to place (unlimitedly) liquidity at the rate $i^{DF}$ with $i^{DF} < i^{LF}$. A bank’s reserve holdings with the central bank at the beginning of the period under consideration (henceforth called ‘day’), are denoted by $\xi \in \mathbb{R}$. With respect to these beginning-of-day reserve holdings there are two types of commercial banks $i \in \{1, 2\}$. Letting $\xi_1 > 0$ and $\xi_2 < 0$, it is assumed that bank 1 has a liquidity surplus whereas bank 2 has a liquidity deficit. The central bank tolerates intraday overdrafts, but negative, as well as positive end-of-day reserve holdings, have to be balanced.8

7Among the set of models building up on Whitesell (2006), the two works closest to the model introduced in this paper are presented in Bech and Klee (2012) and Jackson and Noss (2015). In both works, the framework presented in Whitesell (2006) is extended by introducing interbank market transaction costs.

8In fact, the banks’ reserve holdings at the beginning of the day $\xi_i$, for $i = 1, 2$, actually represent the banks’ account balances after the conduct of central bank open market operations, after stochastic
At one point in time during the day, at ‘noon’, a commercial bank may use an interbank market to trade central bank reserves. A bank’s position in this market is \( b_i \). If \( b_i > 0 \) \((b_i < 0)\), the bank will borrow (lend) the amount \(|b_i|\) at the rate \(i^{BM} \). In both cases, transaction costs \( \gamma_i |b_i| \) accrue, with \( \gamma_i \geq 0 \). A bank trades on the interbank market in order to achieve a target central bank account balance given by \( T_i := \xi_i + b_i \).

After the interbank market is closed, a ‘late payment/liquidity shock’ to the commercial banks’ reserve holdings occurs. The late payment shock hitting bank \( i \) is the realization of a normally distributed random variable \( \epsilon_i \sim \mathcal{N}(0, \sigma^2_i) \) with the publicly observable probability density function \( f_i : \mathbb{R} \to \mathbb{R} \), with \( f_i(\varepsilon) = f_i(-\varepsilon) = F'_i(\varepsilon) \forall \varepsilon \in \mathbb{R} \) and \( \mathbb{E}[\epsilon_i] = 0 \). By an abuse of notation, in the following, \( \epsilon_i \) will denote the random variable as well as its realization. An inflow of funds in the evening is described by \( \epsilon_i \) taking a positive value. Moreover, it is assumed that \( \epsilon_1 \) and \( \epsilon_2 \) are identically distributed with probability density function \( f = f_1 = f_2 = F' \). Due to the late payment shock, the actual end-of-day central bank account balance is \( T_i + \epsilon_i \). If \( T_i + \epsilon_i < 0 \) \((T_i + \epsilon_i > 0)\), bank \( i \) faces a negative (positive) end-of-day account balance, henceforth end-of-day deficit (surplus).

The timing of events in the model is crucial. At the beginning of the day, the central bank determines the banking sector’s aggregate liquidity position, \( \tilde{\Xi} \), by making outright purchases/sales of securities in the open market. Subsequently, an autonomous factor shock, which is the realization of the random variable \( \alpha \sim \mathcal{N}(0, \sigma^2_{AF}) \), adds/drains reserves to/from the banking system (again, by an abuse of notation \( \alpha \) denotes the random variable as well as its realization). The banking sector’s aggregate liquidity position then is \( \Xi := \tilde{\Xi} + \alpha \). Thereafter, interbank payment flows (other than those attributable to new interbank market loans, e.g. customer payments) reshuffle reserves within the banking sector so that each commercial bank \( i \) ends up with reserve account holdings \( \xi_i \). Until the end of the day, there are no further reserve drains/inflows from/to the banking system, implying
that $\Xi = \xi_1 + \xi_2$. At ‘noon’, banks have the possibility to trade reserves on the interbank market. After the closure of this market, each bank is hit by a late payment/liquidity shock which determines its end-of-day reserve account balance. At last, banks take recourse to the central bank’s standing facilities in order to balance their reserve accounts overnight if necessary.\footnote{In part, this setup is following Bindseil and Jablecki (2011).}

Figure 2: Sequence of events within the period under consideration (filled circles). $\xi_1$ and $\xi_2$ are introduced as exogenous model variables and can, in principle, be considered as result of central bank open market operations, autonomous factor shocks, and subsequent interbank payment flows which redistribute reserves within the system in the beginning of the day (blank circles).

For analytical traceability, $\Xi$ as well as $\xi_i$ are introduced as exogenous variables and the factors determining the latter are not explicitly considered. In addition to the factors described above, these might be factors stemming from previous periods, as autonomous liquidity factors (e.g. cash withdrawals), and possible minimum reserve requirements. Therewith, the period under consideration might be interpreted as an arbitrary day in a setting where banks are subject to reserve requirements which have to be fulfilled exactly each day (with end-of-day required reserves being normalized to zero). Alternatively, the period might be interpreted as the last day of a reserve maintenance period if banks were allowed to make use of averaging provisions over the reserve maintenance period. Accordingly, with hypothetical reserve requirements bank 1 with positive reserve balances would have over-fulfilled reserve requirements at the beginning of the day and would have accumulated excess reserves in the amount of $\xi_1$. On the other hand, bank 2 with negative reserve balances would not have met reserve requirements and would exhibit a reserve deficiency of $|\xi_2|$ in ‘the morning’. Hence, the sign of $\Xi$ indicates whether the banking sector as a whole exhibits a liquidity deficit/surplus towards the central bank. Whereas the autonomous factor shock captures the central bank’s inability to perfectly control the banking sector’s liquidity position, the late payment shocks hitting the banks at the end
of the day capture banks’ inabilities to perfectly forecast their actual liquidity needs for the period under consideration. Any factors that determine banks’ beginning-of-day account holdings $\xi_i$ are outside the banks’ control, and thus are considered as “exogenous” liquidity factors from the banks’ perspectives. In the following, liquidity needs which arise from exogenous liquidity factors are termed “exogenous liquidity needs”. In the introduced setup with mandatory end-of-day current account balances of zero, the sign of a bank’s current account balance (at the beginning of the day the sign of $\xi_i$) then indicates whether bank $i$’s exogenous liquidity needs are partly/exactly/excessively covered. In addition to exogenous liquidity needs, a bank has a precautionary liquidity demand which results from its optimizing behavior:

Due to possible reserve account imbalances after the realization of the late payment shock bank $i$ eventually will face actual funding costs, $K_i$, given by

$$K_i = \left(i^{IBM} + \gamma_i \cdot \frac{b_i}{|b_i|} \right) \cdot (b_i) - \left(i^{LF} \cdot (T_i + \epsilon_i) \right) \cdot 1_{\epsilon_i \leq -T_i(\epsilon_i)}$$

$$- \left(i^{DF} \cdot (T_i + \epsilon_i) \right) \cdot 1_{\epsilon_i > -T_i(\epsilon_i)}$$  (1)

with $T_i = \xi_i + b_i$.

The first line of equation (1) captures bank $i$’s interest costs (resp. revenues) and its transaction costs accruing when using the interbank market. The second (third) line reveals the interest costs (revenues) from taking recourse to the central bank’s lending (deposit) facility in the case of an end-of-day deficit (surplus). Whether the bank will use the lending or the deposit facility ultimately depends on the late payment shock which determines the bank’s end-of-day account balance. Formally, this finds expression in the values the indicator functions $1_{\epsilon_i \leq -T_i(\epsilon_i)}$ and $1_{\epsilon_i > -T_i(\epsilon_i)}$ take.

In the absence of uncertainty about actual liquidity needs, with perfectly known liquidity needs, a bank could always minimize the funding costs that actually accrued. If funding conditions in the interbank market were more favorable than at the central bank, i.e. if $i^{DF} < \left(i^{IBM} + \gamma_i \cdot \frac{b_i}{|b_i|} \right) < i^{LF}$, the cost-minimizing bank would strictly avoid taking
recourse to any of the standing facilities, choose a trading position in the interbank market
such that \( b_i = -\xi_i - \epsilon_i \), and thus target a balanced reserve account, i.e. \( T_i = 0 \).

With uncertain liquidity needs, the bank decides over its target reserve account balance, \( T_i \), (and therewith over its interbank trading position \( b_i \)) such as to minimize expected funding costs, given by:

\[
\min_{T_i} \mathbb{E}[K_i] = - \left( \frac{b_i}{|b_i|} \right) \cdot \xi_i \\
- \int_{-\infty}^{-T_i} \left( i^{LF} - \left( i^{IBM} + \gamma_i \cdot \frac{b_i}{|b_i|} \right) \right) \cdot (T_i + \epsilon_i) dF(\epsilon_i) \\
+ \int_{-T_i}^{\infty} \left( i^{IBM} + \gamma_i \cdot \frac{b_i}{|b_i|} - i^{DF} \right) \cdot (T_i + \epsilon_i) dF(\epsilon_i)
\]

with \( b_i := T_i - \xi_i \).

The minimum funding costs the bank expects, as revealed by the first line of equation (2), arise from its beginning-of-day reserve account imbalance, which is known to the bank in advance of interbank market transactions. The second and third line capture additional expected cost components that arise from the uncertainty and liquidity risk the late payment shock creates. These are costs of having to take recourse to the standing facilities at the end of the day relative to using the interbank market ‘at noon’. The second line captures the expected relative costs of taking recourse to the lending facility in case of an end-of-day deficit. The third line reveals the expected opportunity costs of an end-of-day surplus faced by a bank that lends to the interbank market, respectively the relative costs of “overfunding” in the interbank market faced by a bank that borrows from this market. Thus, the bank aims at minimizing the costs that arise from the reserve account uncertainty.

\[12^\text{See also Whitesell (2006).}\]

\[13^\text{Of course, optimizing over } T_i \text{ is equivalent to the optimization over } b_i. \text{ However, of special interest in the following is } T_i \text{ which captures a bank’s precautionary liquidity demand.}\]

\[14^\text{See also Whitesell (2006).}\]
4 Optimal Bank Behavior

4.1 Optimal Target Level of Reserve Balances

The first-order condition of bank $i$ for cost minimization is stated by equation (3). With probability $F(-T_i)$, which is decreasing in $T_i$, (respectively $1 - F(-T_i)$, which is increasing in $T_i$) the bank faces an end-of-day liquidity deficit (surplus) and has to take recourse to the lending facility (deposit facility). The first-order condition thus implies that expected marginal revenues of an increase in the target level of reserve balances, $T_i$, in the form of avoided illiquidity costs (given by the LHS of (3)) must equal expected marginal opportunity costs of an increase in $T_i$ (given by the RHS of (3)). The bank reacts to any imbalances between the latter by making appropriate adjustments in $T_i$.

\[
\left[ i^{LF} - \left( i^{IBM} + \frac{T_i - \xi_i}{|T_i - \xi_i|} \cdot \gamma_i \right) \right] \cdot F(-T_i) = \frac{1}{2} \left[ i^{IBM} + \frac{T_i - \xi_i}{|T_i - \xi_i|} \cdot \gamma_i - i^{DF} \right] \cdot (1 - F(-T_i)).
\]

(3)

For the rest of the paper, it is assumed that the liquidity surplus bank always acts as lender, whereas the liquidity deficit bank always acts as borrower in the interbank market. Accordingly, bank 1 increases its target level of reserve balances, $T_1$, by cutting down its liquidity supply to the interbank market. Respectively, bank 2 increases $T_2$ by increasing its interbank liquidity demand. This yields a lower, respectively an upper bound for the interbank rate:

\[
i^{IBM}_1 := i^{LF} \cdot F(-\xi_1) + i^{DF} \cdot (1 - F(-\xi_1)) + \gamma_1,
\]

(4)

\[
i^{IBM}_2 := i^{LF} \cdot F(-\xi_2) + i^{DF} \cdot (1 - F(-\xi_2)) - \gamma_2.
\]

(5)

For bank 1, interbank rates below $i^{IBM}_1$ imply that expected marginal revenues of targeting a higher level of reserve balances are always exceeding expected marginal opportunity costs. In this case, bank 1 has no incentive to place any liquidity at all in the interbank market. Analogously, for interbank rates exceeding $i^{IBM}_2$, bank 2 is not willing to borrow any liquidity from the interbank market because in this case its expected marginal costs of an increase in its target reserve balance always exceed expected marginal revenues.
4.2 Optimal Precautionary Demand for Reserves in a Frictionless World

In the absence of transaction costs ($\gamma_1 = \gamma_2 = 0$), as discussed in Whitesell (2006), the target reserve account balance that minimizes a bank’s expected funding costs is a function of the interbank rate, the rates on the standing facilities, and the parameters of the distribution underlying the late payment shock. $T_i$ is derived from the first-order condition (3) and has the following representation:\(^\text{15}\)

$$T_i(\cdot) = \begin{cases} -F^{-1} \left( \frac{i^{BM} - i^{DF}}{i^{DF} - i^{IBM}} \right) & \text{if } \{ i = 1 \land i^{IBM} > i^{IBM} \} \lor \{ i = 2 \land i^{IBM} < i^{IBM} \} \\ \xi_i & \text{otherwise.} \end{cases}$$

(6)

Crucially, as expected liquidity needs due to the late payment shock are zero ($\epsilon \sim \mathcal{N}(0, \sigma^2)$), $T_i$ represents a bank’s precautionary demand for central bank reserves and reflects whether it is more favorable for the bank to accept an end-of-day liquidity surplus or a deficit. Under the assumption of $\epsilon_i$ being distributed symmetrically around zero, the sign of $T_i$ depends only on the ratio of marginal costs of using the lending facility to marginal costs of using the deposit facility at the end of the day relative to using the interbank market at noon. In the absence of transaction costs, whether the bank has a positive or a negative precautionary demand for liquidity, i.e. a positive or a negative demand for excess reserves, is determined only by the relative position of the interbank rate within the interest corridor. This a crucial result in Whitesell (2006): If $i^{IBM}$ corresponds to the midpoint of the interest corridor, banks’ marginal avoided illiquidity costs and marginal opportunity costs of targeting a higher level of precautionary liquidity are equal and thus banks have a demand for precautionary liquidity of zero.\(^\text{16}\)

In order to hit its target reserve account balance, bank $i$ chooses an adequate trading position $b_i$ in the interbank market. Thereby, the bank’s interbank liquidity demand (resp. supply) is composed of a precautionary component, capturing the bank’s precautionary

\(^{15}\)This is the result in Whitesell (2006) who discusses the demand for excess reserves and its determinants for one representative bank. In order to explicitly capture interbank market transactions and transaction cost heterogeneity across banks, the framework presented in Whitesell (2006) is extended by separately considering the behavior of lenders and borrowers in the interbank market.

demand for central bank reserves, and an exogenous component, captured by the bank’s beginning-of-day liquidity imbalances:

\[ b_i(\cdot) = T_i(\cdot) - \xi_i \]  

(7)

As \( \xi_i \) only captures the extent of liquidity imbalances of and divergences between individual banks and has no impact on individual banks’ demand for precautionary liquidity (in the case of interior solutions of (2)), the behavior of lenders and borrowers in the interbank market can be derived from their individual precautionary liquidity demand. The determinants of the latter are discussed in the following.

Equation (6) reveals that in the case of an active interbank market, a bank’s precautionary liquidity demand will decrease in \( i^{IBM} \). An increase in \( i^{IBM} \) makes precautionary liquidity holdings relatively less attractive as it implies relative marginal costs of using the lending facility to decrease, and opportunity costs of using the deposit facility to increase, so that the bank reduces its target reserve account balance in order to balance expected marginal revenues and costs of an increase in \( T_i \). Formally, this relationship is captured by equation (3) which states that expected marginal revenues (costs) of an increase in \( T_i \) are a decreasing (increasing) function of the interbank rate as well as of \( T_i \). Hence, in order to adjust its target reserve account balance to an increase in \( i^{IBM} \), bank 1 starts to increase its interbank liquidity supply, whereas bank 2 starts to decrease its interbank liquidity demand. Formally, this reads:

\[
\frac{\partial T_i}{\partial i^{IBM}} = \frac{\partial b_i}{\partial i^{IBM}} = -\frac{1}{f(-T_2)(i^{LF} - i^{DF})} \leq 0. 
\]  

(8)

The first-order condition (3) also reveals that with respect to the interest sensitivity of a bank’s demand for precautionary liquidity it is crucial, how strongly the probability of facing an end-of-day deficit \( F(-T_i) \) reacts to changes in \( T_i \) (resp. to changes in \( b_i \)). If there is only a weak response, interest sensitivity will be high because then, there must be a relatively strong increase or decrease in \( T_i \) to have a sufficiently high impact on \( F(-T_i) \) to restore optimality after a change in \( i^{IBM} \). Under the assumption of a normally distributed late payment shock with mean zero, the impact of a change in \( T_i \) on \( F(-T_i) \) is the lower,
the more $T_i$ deviates from 0 in either direction. Consequently, the interest sensitivity (in absolute value) of bank $i$’s precautionary liquidity demand, and therewith the interest sensitivity (in absolute value) of interbank loan demand (for $i = 2$) and supply (for $i = 1$), is the higher, the more $T_i$ deviates from 0. Formally, this is reflected by

$$\frac{\partial^2 T_i}{\partial (i^{IBM})^2} = \frac{\partial^2 b_i}{\partial (i^{IBM})^2} = \left\{\begin{array}{ll}
< 0 & \text{if } T_i < 0 \\
= 0 & \text{if } T_i = 0 \\
> 0 & \text{if } T_i > 0.
\end{array}\right.$$

(9)

However, for the interest sensitivity of a bank’s precautionary liquidity demand it is not only decisive how strongly $F(-T_i)$ reacts to changes in $T_i$ (resp. in $b_i$) but also how strongly expected marginal revenues and expected marginal costs of an increase in $T_i$ react to changes in $F(-T_i)$. This is determined by the width of the interest corridor formed by $i^{DF}$ and $i^{LF}$. The wider the interest corridor is, the more pronounced will expected marginal revenues and expected marginal costs of an increase in $T_i$ react to changes in $F(-T_i)$, hence the lower is the interest sensitivity of $T_i$. This is because the wider the interest corridor, the larger the spreads between the interbank rate and the facility rates might possibly become. For an individual bank, such large spreads imply relatively high marginal costs of using the standing facilities at the end of the day relative to using the interbank market at noon for reserve account balance adjustments. Accordingly, only a relatively small change in $T_i$, and therewith in the probabilities of using the facilities, is needed to have a sufficiently strong effect on expected marginal revenues and expected marginal costs of an increase in $T_i$ to restore optimality after a change in $i^{IBM}$. Considering symmetric changes of the interest corridor around some given corridor midpoint rate $i^{MR}$, with $i^{DF} \equiv i^{MR} - w$ and $i^{LF} \equiv i^{MR} + w$, it is

$$\frac{\partial^2 T_i}{\partial w \partial i^{IBM}} = \frac{\partial^2 b_i}{\partial w \partial i^{IBM}} = \frac{1}{2w^2} \cdot f(-T_i) - \frac{f'(T_i) \cdot (2 \cdot F(-T_i) - 1)}{4w^2 \cdot (f(-T_i))^3} \geq 0$$

(10)

which shows formally that the interest sensitivity of bank $i$’s precautionary liquidity demand (in absolute value), and therewith the interest sensitivity of loan demand (for $i = 2$) and supply (for $i = 1$), decreases in the width of the corridor.
In general, the width of the interest corridor is a crucial determinant of a bank’s precautionary liquidity demand and thus its interbank liquidity demand/supply: A symmetric increase in the corridor width leads to an increase in a bank’s relative marginal costs of using the standing facilities (with respect to using the interbank market) and therewith to an increase in expected marginal revenues and expected marginal costs of an increase in \( T_i \). The increase in expected marginal revenues of holding more precautionary liquidity will outweigh the increase in expected marginal costs, if the bank targets a negative reserve account balance \( (T_i < 0) \), which implies that the probability of using the lending facility at the end of the day is greater than 0.5. Consequently, the bank will increase the level of its precautionary liquidity holdings. Analogously, if the bank initially targets a positive reserve account balance, an increase in the corridor width will induce the bank to decrease its target reserve account balance. Also yielding the respective implications for interbank liquidity demand/supply, this formally reads

\[
\frac{\partial T_i}{\partial w} = \frac{\partial b_i}{\partial w} = \begin{cases} 
> 0 & \text{for } T_i < 0 \\
= 0 & \text{for } T_i = 0 \\
< 0 & \text{for } T_i > 0.
\end{cases}
\]

The effect of a change in the width of the corridor on a bank’s precautionary liquidity demand (resp. on interbank liquidity demand/supply) is the stronger, the more \( T_i \) deviates from zero. The more \( T_i \) deviates from zero, the higher is the probability that one of the facilities will be used after the occurrence of the late payment shock, hence the larger is the difference in the changes in expected marginal revenues and expected marginal costs of holding more precautionary liquidity implied by a change in \( w \). Consequently, as formally reflected by (10), a relatively pronounced change in \( T_i \) is needed to restore optimality after a change in the corridor width.

Furthermore, a bank’s precautionary liquidity demand is crucially determined by the extent of reserve account uncertainty the bank faces, captured by the standard deviation \( \sigma \) of the late payment shock. A change in reserve account uncertainty has opposing effects on a bank’s expected marginal revenues and expected marginal costs of holding more precautionary liquidity. The direction of these effects depends on whether the bank
targets a positive or a negative reserve account balance and therewith on the respective probabilities of using the facilities: An increase in $\sigma$ implies that $F(-T_i)$ will increase if $T_i > 0$, whereas $F(-T_i)$ will decrease if $T_i < 0$. Hence, an increase in $\sigma$ implies that expected marginal revenues (costs) of holding more precautionary liquidity will increase (decrease) if $T_i > 0$, and decrease (increase) if $T_i < 0$. Consequently, bank $i$ will increase its precautionary liquidity demand in response to an increase in uncertainty if $T_i > 0$, and vice versa:

$$\frac{\partial T_i}{\partial \sigma} = \frac{\partial b_i}{\partial \sigma} = \begin{cases} < 0 & \text{if } T_i < 0 \\ 0 & \text{if } T_i = 0 \\ > 0 & \text{if } T_i > 0. \end{cases}$$

(12)

As a bank’s precautionary liquidity demand is independent from its beginning-of-day reserve account balance per construction ($\frac{\partial T_i}{\partial \xi_i} = 0$), a change in the bank’s beginning-of-day reserve holdings is reflected completely in its interbank liquidity demand/supply:

$$\frac{\partial b_i}{\partial \xi_i} = -1.$$ 

(13)

Equations (8) - (13) illustrate that the surplus bank’s precautionary liquidity demand as well as the deficit bank’s precautionary liquidity demand are qualitatively affected in the same way by changes in $i^{BM}$, $w$, $\sigma$, and $\xi$. Crucially, as discussed in the next section, interbank market transaction costs will have opposing effects on $T_i$, for $i = 1, 2$.

### 4.3 Optimal Precautionary Demand for Reserves in the Presence of Transaction Costs

The impact of interbank market transaction costs on an individual bank’s precautionary demand for central bank reserves depends on the bank’s role as lender or borrower in the
interbank market. Explicit forms of banks’ target reserve account balances that minimize their expected funding costs in the presence of transaction costs are:

\[
T_1(\cdot) = \begin{cases} 
-F^{-1} \left( \frac{i^{BM} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) & \text{if } i^{BM} > i \\
\xi_1 & \text{if } i^{BM} \leq i 
\end{cases}
\] (14)

\[
T_2(\cdot) = \begin{cases} 
-F^{-1} \left( \frac{i^{BM} + \gamma_2 - i^{DF}}{i^{LF} - i^{DF}} \right) & \text{if } i^{BM} < \xi \\
\xi_2 & \text{if } i^{BM} \geq \xi 
\end{cases}
\] (15)

For the surplus bank 1, an increase in \(\gamma_1\) implies relative marginal central-bank-funding costs to increase, and marginal opportunity costs of using the deposit facility to decrease. For the deficit bank 2, an increase in \(\gamma_2\) implies marginal costs of using the lending facility relative to using the interbank market for reserve account adjustments to decrease, and marginal costs of using the deposit facility to increase. Hence, for the surplus bank, an increase in transaction costs implies expected marginal revenues of holding more precautionary liquidity to increase and expected marginal costs to decrease, inducing the bank to increase \(T_1\) and thus to decrease its interbank liquidity supply. For the deficit bank, transaction costs will have opposing effects on expected marginal revenues and costs of targeting a higher level of precautionary liquidity, respectively, inducing the bank to decrease \(T_2\) as well as its interbank liquidity demand. Formally, this reads:

\[
\frac{\partial T_1}{\partial \gamma_1} = \frac{\partial b_1}{\partial \gamma_1} = \frac{1}{f(-T_1) \cdot (i^{LF} - i^{DF})} > 0,
\] (16)

\[
\frac{\partial T_2}{\partial \gamma_2} = \frac{\partial b_2}{\partial \gamma_2} = \frac{-1}{f(-T_2) \cdot (i^{LF} - i^{DF})} < 0.
\] (17)

Analogously to the interest sensitivity, the transaction costs sensitivity of individual banks’ precautionary liquidity demand is the higher (in absolute value) the less \(F(-T_i)\) reacts to changes in \(T_i\) (resp. to changes in \(b_i\)) and the less expected marginal costs/revenues of demanding more precautionary liquidity react to changes in \(F(-T_i)\). Thus, transaction costs sensitivity of banks’ precautionary liquidity demand (in absolute value) is the higher, the more \(T_i\) deviates from zero, the higher the extent of reserve account uncertainty (as
captured by $\sigma$), and the narrower the interest corridor is. Formally, this is captured by equations (18) - (23):

$$\frac{\partial^2 T_1}{\partial i_{IBM} \partial \gamma_1} = \frac{\partial^2 b_1}{\partial i_{IBM} \partial \gamma_1} = \frac{-f'(-T_1)}{(iL_F - iD_F)^2 \cdot (f(-T_1))^3}$$

$$\begin{cases} > 0 & \text{if } T_1 < 0 \\ = 0 & \text{if } T_1 = 0 \\ < 0 & \text{if } T_1 > 0, \end{cases} (18)$$

$$\frac{\partial^2 T_2}{\partial i_{IBM} \partial \gamma_2} = \frac{\partial^2 b_2}{\partial i_{IBM} \partial \gamma_2} = \frac{f'(-T_2)}{(iL_F - iD_F) \cdot (f(-T_2))^2}$$

$$\begin{cases} < 0 & \text{if } T_2 < 0 \\ = 0 & \text{if } T_2 = 0 \\ > 0 & \text{if } T_2 > 0, \end{cases} (19)$$

$$\frac{\partial^2 T_1}{\partial \sigma \partial \gamma_1} = \frac{\partial^2 b_1}{\partial \sigma \partial \gamma_1} = \frac{1}{\sigma} \cdot \frac{\partial b_1}{\partial \gamma_1} > 0, (20)$$

$$\frac{\partial^2 T_2}{\partial \sigma \partial \gamma_2} = \frac{\partial^2 b_2}{\partial \sigma \partial \gamma_2} = \frac{1}{\sigma} \cdot \frac{\partial b_2}{\partial \gamma_2} < 0, (21)$$

$$\frac{\partial^2 T_1}{\partial w \partial \gamma_1} = \frac{\partial^2 b_1}{\partial w \partial \gamma_1} = \frac{-1}{2w^2 \cdot f(-T_1)} + \frac{f'(-T_1) \cdot (2 \cdot F(-T_1) - 1)}{4w^2 \cdot (f(-T_1))^3} \leq 0, (22)$$

$$\frac{\partial^2 T_2}{\partial w \partial \gamma_2} = \frac{\partial^2 b_2}{\partial w \partial \gamma_2} = \frac{2}{2w^2 \cdot f(-T_2)} - \frac{f'(-T_2) \cdot (2 \cdot F(-T_2) - 1)}{4w^2 \cdot (f(-T_2))^3} \geq 0. (23)$$

### 5 Interbank Market Equilibrium

Indicating the equilibrium variables with the superscript $^*$, the interbank market clearing condition and the equilibrium condition read:

$$B^* \overset{!}{=} 0, \text{ with } B := b_1 + b_2$$

$$\implies T^* = \Xi, \text{ with } T := T_1 + T_2. (25)$$
Together with the first-order condition given by (3) for \( i = 1, 2 \), the equilibrium interbank transaction volume \( b^* := b_1^* = -b_1^* \) and therewith the equilibrium level of individual banks’ precautionary liquidity demand \( T_i^* \) is implicitly given by

\[
F(-T_1^*) - F(-T_2^*) + \frac{\gamma_1 + \gamma_2}{i_{LF} - i_{DF}} = 0,
\]

the equilibrium interbank rate \( i^{IBM^*} \) is implicitly given by

\[
F^{-1}\left(\frac{i^{IBM^*} - \gamma_1 - i_{DF}}{i_{LF} - i_{DF}}\right) + \xi_1 + F^{-1}\left(\frac{i^{IBM^*} + \gamma_2 - i_{DF}}{i_{LF} - i_{DF}}\right) + \xi_2 \overset{\dagger}{=} 0.
\]

In principle, equation (25) describes the main task for the central bank’s liquidity management in controlling the interbank rate. The interbank market will clear at the targeted rate if banks’ aggregate liquidity demand at \( i^{\text{target}} \) is perfectly met, that is if the banking sector’s liquidity position at ‘noon’ is \( \Xi = T(i^{\text{target}}) \). However, as discussed in Whitesell (2006), there are two main impediments for the central bank in exerting perfect interest rate control: (1) forecasting the banking sector’s precautionary liquidity demand at the targeted interbank rate, and (2) creating adequate liquidity conditions in the banking system. In the model presented in this paper, and as argued in the next chapter, (1) is problematic in the presence of transaction costs, and (2) is complicated by the autonomous factor shock hitting the banking system after the central bank has adjusted the banking sector’s liquidity position. Basically, this autonomous factor shock replicates a major source of liquidity imbalances not captured by the model: errors in the central bank’s estimate of the banking sector’s “non-precautionary liquidity demand”. Typically, this “exogenous” liquidity demand would arise from minimum reserve requirements or from autonomous liquidity factors as cash withdrawals by non-banks or government deposits with the central bank.

In the model, an aggregate liquidity position of zero indicates, that the banking sector’s exogenous liquidity needs are covered exactly. However, at the end of the day, its actual liquidity position is \( \tilde{\Xi} + \alpha + \epsilon_1 + \epsilon_2 \). Relevant for the central bank in steering the interbank rate is the banking sector’s liquidity position when interbank market trading takes place, which is \( \Xi = \tilde{\Xi} + \alpha \). Accordingly, at noon, the banking sector’s exogenous liquidity needs
would be over-fulfilled (under-fulfilled) if \( \Xi > 0 \) (\( \Xi < 0 \)). Hence, in order to meet the banking sector’s exogenous liquidity needs at noon exactly, the central bank would have to arrange \( \tilde{\Xi} = -\alpha \) at the beginning of the day. With \( \alpha \sim N(0, \sigma_{AF}^2) \), the central bank’s estimate (formed at the beginning of the day) of the banking sector’s exogenous liquidity needs at noon is zero.

Ultimately, liquidity imbalances resulting from (1) or (2) might cause deviations of the equilibrium interbank rate from the central bank’s target. However, as argued in the next chapter, these interest rate effects as well as the policy measures to attenuate the latter crucially depend on the implemented corridor system.

6 Implications for Monetary Policy Implementation

6.1 Standard Corridor System

6.1.1 Interbank Rate and Volatility Control in a Frictionless World

In a standard corridor system the central bank’s standing facility rates symmetrically surround the targeted interbank rate, i.e. \( i_{DF} = i_{\text{target}} - w \) and \( i_{LF} = i_{\text{target}} + w \) or equivalently \( i_{\text{target}} = i_{MR} := \frac{1}{2} \cdot (i_{DF} + i_{LF}) \). As argued by Whitesell (2006) and Woodford (2001), by following such an approach a central bank can employ a crucial property of interest corridor systems that eliminates a potential source of errors in estimating the banking sector’s liquidity needs at the target rate: At the corridor midpoint rate \( i_{MR} \) banks’ precautionary liquidity demand is zero (given that there are no interbank market transaction costs, i.e. \( \gamma_1 = \gamma_2 = 0 \)). This property holds independently of the absolute level of the facility rates, the width of the interest corridor, and the degree of uncertainty banks have about their actual end-of-day reserve account balances (as captured by the standard deviation of the late payment shock, \( \sigma \)). As illustrated by the first-order condition (3), avoided illiquidity costs and opportunity costs of holding more precautionary liquidity are equal (at the margin) if \( i_{IBM} = i_{MR} \). Hence, optimality requires a bank to balance its reserve account exactly by trading in the interbank market such that it will face an end-of-day liquidity deficit and surplus with the same probability.\(^{17}\)

\(^{17}\)See Whitesell (2006), p. 1180 - 1181.
In the absence of transaction costs, the interbank market will clear at $i_{\text{target}} = i^{MR}$ if
the banking sector’s aggregate liquidity position when interbank trading takes place, $\Xi$, is
zero (as illustrated by equation (25)). As discussed in Whitesell (2006), where a frictionless
interbank market is considered, the only source of deviations of the interbank rate from
its target then is the central bank’s inability to perfectly control the liquidity conditions in
the banking system. Formally, the autonomous factor shock introduces uncertainty at the
time of the conduct of central bank open market operations about banks’ actual exogenous
liquidity needs at noon. With $\alpha \sim \mathcal{N}(0, \sigma_{AF}^2)$ and a central bank that chooses $\tilde{\Xi} = 0$, the
banking sector’s liquidity position after the shock, at noon, is $\Xi = \alpha$ (by an abuse of
notation letting $\alpha$ also denote the realization of the random variable). Ultimately, any
liquidity imbalances will be reflected in deviations of the interbank rate from its target
and comparative statics formally yield:

$$\frac{\partial i^{IBM}\ast}{\partial \Xi} < 0. \quad (28)$$

In accordance with conventional wisdom, this effect is the weaker, the narrower the interest
corridor is. Equation (11) illustrates this property: The narrower the interest corridor is,
the larger is the interest sensitivity of precautionary liquidity demand of an individual
bank and therefore (if $\gamma_1 = \gamma_2 = 0$) of the banking sector as a whole. Consequently, in
order to establish an equilibrium if $\Xi \neq 0$, the interbank rate has to move less under a
relatively narrow corridor to induce the banking sector to hold precautionary liquidity
in the amount of $\Xi$; i.e. the banking sector is relatively willing to absorb any exogenous
liquidity imbalances by building up precautionary liquidity buffers (which might as well be of negative sign). Hence, the central bank is able to attenuate the interest rate effects
of exogenous liquidity shocks, i.e. to control interbank rate volatility, by establishing a
relatively narrow interest corridor. Formally, this reads:

$$\frac{\partial^2 i^{IBM}\ast}{\partial w \partial \Xi} < 0. \quad (29)$$

Figure 3 illustrates the negative relationship between corridor width and volatility, if the
latter can be traced back to exogenous liquidity shocks. For a specific set of parameter
Figure 3: Simulated distribution of interbank market rates under (a) wide and (b) narrow standard corridor: Reserve account uncertainty: $\epsilon_i \sim \mathcal{N}(0, 1)$; certain precautionary liquidity demand: $\Gamma = 0$; stochastic exogenous liquidity needs: $\Xi = \alpha \sim \mathcal{N}(0, 0.5^2)$. 10000 draws of $\alpha$.

values, the model was solved for 10000 draws of the autonomous factor shock. The resulting dispersion of interbank rates is higher under a relatively wide interest corridor (Figure 3a) than it is under a relatively narrow (Figure 3b) interest corridor.\(^{18}\)

### 6.1.2 Interbank Rate and Volatility Control in the Presence of Transaction Cost Heterogeneity

In the presence of transaction costs the reallocation of reserves within the banking sector on the interbank market is impaired. Banks are induced to substitute away from the use of interbank loans in order to balance their reserve accounts at noon towards an increased reliance on the central bank’s standing facilities at the end of the day. Bank heterogeneity with respect to the degree banks are impaired in using the interbank market (i.e. $\gamma_1 \neq \gamma_2$) thereby might lead to erratic fluctuations of banks’ precautionary liquidity demand and therewith of interbank liquidity demand and supply. Eventually, these unpredictable demand and supply conditions might impair the central bank in establishing aggregate liquidity conditions adequate to hit the targeted interbank rate and might constitute a source of interbank rate volatility in addition to the autonomous factor shock.

\(^{18}\)The simulation methodology draws on Whitesell (2006).
Equations (16) and (17) show, that the banking sector’s aggregate precautionary liquidity demand $T$ is an increasing function of transaction costs of lending to and a decreasing function of transaction costs of borrowing from the interbank market. Dependent on which market side would be affected most by impaired trading conditions, the central bank would have to establish either an aggregate liquidity deficit ($\tilde{\Xi} < 0$) or a surplus ($\tilde{\Xi} > 0$) in order to reach its target. The necessary size of a deficit resp. surplus thereby depends on the width of the interest corridor and the degree of uncertainty banks have about their actual liquidity needs. Both a narrow corridor as well as high uncertainty about actual end-of-day reserve account balances would promote relatively large variations in banks’ precautionary liquidity demand at $i^{MR}$ (as discussed in Section 4.3 and formally captured by equations (20) - (23)). However, in order to forecast $T(i^{target})$ exactly, the central bank needed to know which banks would be active on which side on the interbank market before liquidity management operations were conducted. Under the assumption that interbank payment flows arbitrarily redistribute reserves within the banking sector after the conduct of open market operations and before interbank trading takes place, that information is the harder to acquire, the greater the extent of transaction cost heterogeneity is.

Without any differences between lending and borrowing transaction costs (i.e. $\gamma_1 = \gamma_2$) banks’ aggregate precautionary liquidity demand at the target rate is zero (for $\gamma_1 = \gamma_2$ equations 14 and 15 imply $T(i^{MR}) = 0$) and hence perfectly predictable. In this case, the only source of deviations of the interbank rate from its target is the autonomous factor shock and the same implications for the central bank in exerting control over the interbank rate and volatility as in Section 6.1.1 hold. However, in the presence of transaction cost heterogeneity (i.e. $\gamma_1 \neq \gamma_2$), $T(i^{target})$ is not yet determined at the time of central bank liquidity management operations and could at most be considered as random variable by this time. In this case, information about the distribution of the average lending and borrowing transaction costs would allow the central bank forming an estimate of an amount of liquidity most likely adequate to accommodate banks’ actual precautionary liquidity demand at noon. Fluctuations of banks’ actual precautionary liquidity demand around this estimate then were reflected in fluctuations of the equilibrium interbank rate around the central bank’s target rate. Eventually, the dispersion of equilibrium interbank rates,
i.e. volatility of the interbank rate, would be directly linked to the distribution of average transaction costs. Hence, transaction cost heterogeneity and therewith changing degrees to which the liquidity redistribution on the interbank market is impaired (from day to day) might constitute an additional source of interbank rate volatility: If actual transaction costs at noon deviate from the central bank’s early-morning estimate of expected average transaction costs, i.e. if banks’ actual precautionary liquidity demand deviates from the central bank’s forecast, the effects on the equilibrium interbank rate are formally captured by:

\[
\frac{\partial i_{IBM}}{\partial \gamma_1} = \frac{f(-T_2^*)}{f(-T_1^*) + f(-T_2^*)} > 0, \quad (30)
\]

\[
\frac{\partial i_{IBM}}{\partial \gamma_2} = \frac{-f(-T_1^*)}{f(-T_1^*) + f(-T_2^*)} < 0. \quad (31)
\]

Thereby, the magnitude of these effects depends on the width of the interest corridor and on aggregate exogenous liquidity conditions:

\[
\frac{\partial^2 i_{IBM}}{\partial w \partial \gamma_1} = \frac{\partial b^*}{\partial w} \cdot \left( \frac{-\xi_1 - \xi_2 \cdot f(-\xi_1 + b^*) \cdot f(-\xi_2 - b^*)}{(f(-\xi_1 + b^*) + f(-\xi_2 - b^*))^2} \right) \begin{cases} > 0 & \text{for } \Xi < 0 \\ = 0 & \text{for } \Xi = 0 \\ < 0 & \text{for } \Xi > 0, \end{cases} \quad (32)
\]

\[
\frac{\partial^2 i_{IBM}}{\partial w \partial \gamma_2} = \frac{\partial b^*}{\partial w} \cdot \left( \frac{-\xi_1 - \xi_2 \cdot f(-\xi_1 + b^*) \cdot f(-\xi_2 - b^*)}{(f(-\xi_1 + b^*) + f(-\xi_2 - b^*))^2} \right) \begin{cases} > 0 & \text{for } \Xi < 0 \\ = 0 & \text{for } \Xi = 0 \\ < 0 & \text{for } \Xi > 0, \end{cases} \quad (33)
\]

Hence, the width of the interest corridor is no instrument to generally attenuate the interest rate effects of unforeseen fluctuations of banks’ precautionary liquidity demand. In contrast to the dampening effect a narrow corridor has on the interest rate effects of exogenous liquidity imbalances (caused by the autonomous factor shock), the relationship between the width of the interest corridor and the interest rate effects described by (30) and (31) (which are produced by “endogenously” evolved liquidity imbalances) depends on the constellation of exogenous liquidity conditions and the extent of impaired interbank trading conditions.
Formally, these insights can be derived from (18), (19), (22), and (23), which back the following intuition: Generally, transaction costs lead to a drop in either interbank liquidity demand or supply which then exerts a down-/upward pressure on the equilibrium interbank rate, respectively. At that, in principle, adjustments in the corridor width could exert a stimulus on demand or supply being adequate to attenuate the ultimate transaction costs effects. However, whether corridor adjustments encourage or discourage lending to or borrowing from the interbank market depends on whether banks target positive or negative reserve account balances $T_i$, respectively (see equation (11)). Moreover, the magnitudes of these effects depend on the absolute values of $T_1$ and $T_2$, respectively (see equation (10)). As individual banks’ target account balances might differ both in their signs as well as in their absolute values under unbalanced exogenous liquidity conditions ($\Xi \neq 0$) and in the presence of transaction costs, whether a wide or a narrow corridor is suited to exert an adequate stimulus on interbank liquidity demand and supply to attenuate the respective transaction costs effect on $i^{IBM}$ ultimately depends on the constellation of $\Xi$, $\gamma_1$, and $\gamma_2$.

If the banking sector’s aggregate liquidity position at noon is balanced ($\Xi = 0$), the degree to which unforeseen transaction costs bias the equilibrium interbank rate is even independent of the corridor width: If $\Xi = 0$, the banking sector’s aggregate precautionary liquidity demand in equilibrium, $T^*$, necessarily is zero as well, and hence $T_1^* = -T_2^*$. In this case, any adjustments in the corridor width exert an impact on interbank liquidity demand and supply which is oriented in the same direction and of the same magnitude, respectively, so that the overall effect of corridor adjustments is neutral with respect to the equilibrium interbank rate (see equations (10) and (11)). Figure 4 illustrates the neutral relationship between corridor width and the dispersion of equilibrium interbank rates caused by unforeseen transaction costs at the time interbank trading takes place for $\Xi = 0$ and a specific set of parameter values chosen to solve the model for 10000 draws of $\gamma_1$ and $\gamma_2$. 

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Figure 4: Simulated distribution of interbank market rates under (a) wide and (b) narrow standard corridor: Stochastic precautionary liquidity demand: \( \gamma_1 \sim N(0, 0.1^2) \mid 0.4 \); reserve account uncertainty: \( \epsilon_i \sim N(0, 1) \); certain exogenous liquidity needs: \( \Xi = 0 \). 10000 draws.

6.2 Floor Operating System

6.2.1 Interbank Rate and Volatility Control in a Frictionless World

The implementation of a floor operating system allows for an extremely tight control over the level and volatility of the interbank rate - provided the interbank market is frictionless. Through an ample provision of liquidity the interbank rate is driven down to its lower bound given by the rate on the deposit facility which then corresponds to the central bank’s target rate (for analytical traceability, let \( i_{\text{target}} = i^{DF} + \delta \) for some small \( \delta > 0 \)).\(^{19}\)

Formally, this is shown in Section 4.2 and by equation (8) which states that banks’ precautionary liquidity demand is a decreasing function of the interbank rate. Thus, the interbank market clears at the targeted rate if the banking sector’s liquidity position (at noon) sufficiently exceeds banks’ exogenous liquidity needs, i.e. if \( \Xi = T(i^{\text{target}}) > 0 \). However, in contrast to the zero-precautionary-demand property in a standard corridor system, the banking sector’s demand for precautionary liquidity at the target rate in a floor system crucially depends on the extent of uncertainty banks have about their actual

\(^{19}\)See, for instance, Federal Open Market Committee (2015).
exogenous liquidity needs and on the width of the interest corridor. Comparative statics yield:

\[
\frac{\partial T(i_{\text{target}})}{\partial \sigma} > 0 \text{ if } T_i > 0 \text{ for } i = 1, 2, \quad (34)
\]

\[
\frac{\partial T(i_{\text{target}})}{\partial w} > 0 \text{ if } T_i > 0 \text{ for } i = 1, 2. \quad (35)
\]

Equation (34) directly follows from equation (12) and reflects that an increase in uncertainty (captured by \(\sigma\)) reduces the probability of facing an end-of-day liquidity surplus and therewith individual banks’ expected marginal opportunity costs of holding more precautionary liquidity in the scenario under consideration. The intuition behind equation (35) is the following: At \(i^{IBM*} = i_{\text{target}}\), an increase in the corridor width leads to an increase in the ratio of marginal avoided illiquidity costs \((i^{LF} - i_{\text{target}})\) to marginal opportunity costs \((i_{\text{target}} - i^{DF})\) of holding more precautionary liquidity. Hence, at \(i_{\text{target}}\), optimality requires an individual bank to increase its demand for precautionary liquidity in order to increase the probability of ending the day with a positive reserve account balance such as to equate expected marginal avoided illiquidity and opportunity costs again. Eventually, the central bank’s provision of liquidity in order to hit the targeted interbank rate in a floor operating system has to be the more expansionary, the higher banks’ uncertainty about their actual liquidity needs is and the wider the interest corridor is. Crucially, under the assumption of a zero-mean symmetrically distributed late payment shock \(\epsilon_i\) for \(i = 1, 2\), at interbank rates \(i^{IBM} < i^{MR}\) both banks necessarily target a positive reserve account balance, i.e. \(T_1^*, T_2^* > 0\), and the excess liquidity within the banking sector is distributed equally among banks on the interbank market. Hence, when the liquidity redistribution within the banking sector is unimpaired also the deficit bank builds up precautionary liquidity holdings. Thus, the conditions in equations (34) and (35) are satisfied at the targeted interbank rate in a floor system for both types of banks. Consequently, the overall effect of an increase in \(\sigma\) or \(w\) on banks’ aggregate precautionary liquidity demand is unambiguous.

As the implementation of a floor system per se can be used as an approach to minimize the dispersion of interbank rates resulting from autonomous factor shocks and to control volatility of the interbank rate, the width of the interest corridor in a frictionless world is
of minor importance in this regard. When the aggregate liquidity surplus generated by the central bank is sufficiently large, the banking sector’s liquidity demand is that highly interest-sensitive (as discussed in Section 4.2 and formally captured by equation (9)), that autonomous factor shocks produce only marginal interest rate effects - to a certain degree even under a relatively wide interest corridor. Figure 5 illustrates the relatively small importance of the corridor width with regard to the control of volatility of the interbank rate for a specific set of parameter values.

6.2.2 Interbank Rate and Volatility Control in the Presence of Transaction Cost Heterogeneity

A potential weakness of a floor operating system is that even theoretically it might be impracticable through the adoption of liquidity management measures to hit the targeted interbank rate when the redistribution of liquidity within the banking sector is impaired. With regard to the implementation of a low interbank rate lying close to the corridor floor, impaired borrowing conditions which reduce interbank liquidity demand are less problematic (at most they might lead to a significant decline in market activity). However, impaired lending conditions might create “liquidity sinks” in the banking sector with a particular set of banks that retreats from interbank lending, accumulates excess reserves, and which thereby drives up the interbank rate. As it was the case in a standard corridor
system, bank heterogeneity with respect to the degrees banks are impaired in trading in the interbank market possibly engenders a more volatile interbank rate. However, in a floor system, the latter might pre-eminently emanate from differing trading conditions for lenders in the interbank market. Ultimately, against conventional wisdom, the control of volatility in a floor system might be exerted by implementing a relatively wide interest corridor.

Equation (4) that defines a lower bound $i^{IBM}$ for the interbank rate formally captures the first statement above. Even if lending conditions are impaired only to a relatively moderate extent, i.e. even if lending transaction costs $\gamma_1$ are relatively small, the effective floor to the interbank rate might soon be located significantly above the rate on the deposit facility. For $i^{IBM} < \bar{i}$, positive end-of-day balances have zero marginal opportunity costs for surplus banks and hence the latter have no incentive to place any liquidity in the interbank market and thus build up precautionary liquidity buffers.

The other statements crucially depend on specific assumptions regarding the rationale behind banks' precautionary liquidity demand which has been the uncertainty about their actual exogenous liquidity needs. In the model, the latter is captured by the late payment/liquidity shock $\epsilon_i$ for bank $i$ under the assumption that both types of banks face the same kind and extent of uncertainty, i.e. $\epsilon_1 = \epsilon_2$. Ultimately, the distribution parameters of $\epsilon_i$ crucially determine the interest sensitivity of bank $i$'s precautionary liquidity demand $T_i$ and as discussed in Section 4.2, $\frac{\partial T_i}{\partial i^{IBM}}$ is the larger in absolute value, the more $T_i$ deviates from zero. Now, when a floor system with $\tilde{\Xi}, \Xi > 0$ is considered, ultimate implications of the assumption $\epsilon_1 = \epsilon_2$ are that in any feasible equilibrium it is $|T_1^*| \geq |T_2^*|$ and the interest sensitivity of $T_1$ is always greater or equal than the one of $T_2$, i.e. interbank liquidity supply reacts more sensitive to interest rate movements than interbank liquidity demand (as long as $\xi_1 > \xi_2$, this even holds independently of the sign of $\xi_2$). Although this is a strong assumption, intuition suggests that it is a plausible one especially in a setup for a floor operating system where $\Xi$ is sufficiently large and under the assumption of plausible transaction costs $\Gamma < w$ (implying $T_1, T_2 > 0$): If the opposite would be the case, that is if interbank liquidity demand would react more sensitive to interest rate movements than interbank liquidity supply, this would involve for bank 2
to be confronted with a higher variance of late liquidity outflows than bank 1 (i.e. the left tail of the distribution of $\epsilon_2$ needed to be fatter than the left tail of the distribution of $\epsilon_1$). Intuitively, if there would be any asymmetry in the distribution of late liquidity shocks among banks, the extent to which late liquidity outflows from a set of banks vary probably would be positively correlated with the liquidity endowment of this set of banks.

In the model, with $\xi_1 > \xi_2$, this would imply that the left tail of the distribution of $\epsilon_1$ needed to be fatter or equally fat than/as the left tail of the distribution of $\epsilon_2$ and thus that interbank liquidity supply would be more or equally interest-sensitive than interbank liquidity demand - exactly this is the case under the assumptions in the model setup.

The starting point to shed light on the effects of impaired interbank trading conditions in a floor system is equation (3) which illustrates that in the absence of transaction costs it is $|T_1^*| = |T_2^*|$ and in a floor system this implies that $T_1 = T_2 > 0$. Now, impaired interbank trading conditions ($\Gamma > 0$) result in a decline in market activity and therewith in the concentration of excess reserves at a particular set of banks. Comparative statics yield:

$$\frac{\partial b^*}{\partial \gamma_1} = \frac{\partial b^*}{\partial \gamma_2} = -\frac{1}{(i^{LF} - i^{DF}) \cdot (f(-T_1^*) + f(-T_2^*))} < 0,$$

which implies that

$$\frac{\partial T_1^*}{\partial \gamma_i} > 0 \text{ for } i = 1, 2,$$

$$\frac{\partial T_2^*}{\partial \gamma_i} < 0 \text{ for } i = 1, 2.$$ (37) (38)

Together with equation (9), which states that the interest sensitivity (in absolute value) of bank $i$’s precautionary liquidity demand is the higher, the more $T_i$ deviates from zero, and together with equations (18) and (19), which state that the transaction cost sensitivity of a bank’s precautionary liquidity demand (in absolute value) is the higher, the more $T_i$ deviates from zero, equations (37) and (38) imply that for any equilibrium interbank rate $i^{IBM^*}$ it is:

$$\left| \frac{\partial T_1^*}{\partial i^{IBM}} \right|_{i^{IBM^*}} > \left| \frac{\partial T_2^*}{\partial i^{IBM}} \right|_{i^{IBM^*}},$$

(39)
\[
\left| \frac{\partial h^{BM^*}}{\partial \gamma_1} \right| > \left| \frac{\partial h^{BM^*}}{\partial \gamma_2} \right|.
\] (40)

Accordingly, impaired lending conditions have a relatively more pronounced negative impact on interbank liquidity supply than impaired borrowing conditions have on interbank liquidity demand. Eventually, this implies that under a floor operating system, the pass-through rate of lending transaction costs on the equilibrium interbank rate is larger than the pass-through rate of borrowing transaction costs. Regarded over time, if banks differ in the degrees they are impaired in trading in the interbank market, i.e. if average transaction costs change from day to day, the interbank rate would trade on elevated levels and show increased volatility pre-eminently if lending conditions were impaired.

![Simulated distribution of interbank market rates under (a) narrow and (b) wide floor system](image)

Figure 6: Simulated distribution of interbank market rates under (a) narrow and (b) wide floor system: Stochastic precautionary liquidity demand: $\gamma_1 \sim N(0, 0.1^2) |_{0.4}, \gamma_2 = 0$; reserve account uncertainty: $\epsilon_i \sim N(0, 1)$; Certain exogenous liquidity needs: $\Xi = 5$; 10000 draws.

In contrast to a standard corridor system, adjustments in the corridor width of a floor system might be effective in attenuating the interest rate effects of erratic fluctuations of interbank liquidity supply brought about by daily variations in average lending transaction costs. Regarded over time, the dispersion of equilibrium interbank rates caused by changing average lending transaction costs would be lower under a relatively wide interest corridor: Analogously to the case of a standard corridor discussed in Section 6.1.2, adjustments in the corridor width exert a stimulus on interbank liquidity demand and supply. However, in a floor system, an increase in the corridor width always encourages lending to
the interbank market since it leads to an increase in opportunity costs of holding precautionary liquidity for surplus banks. Moreover, a wider corridor enhances market activity as the stimulus on interbank liquidity supply always exceeds the stimulus on interbank liquidity demand (due to $|T_1^*| \geq |T_2^*|$). Thus, the pass-through rate of lending transaction costs on the interbank rate decreases in the corridor width. While formally captured by equation (32) for the case of $\Xi > 0$, figure 6 illustrates the relationship between the width of the interest corridor and the dispersion of equilibrium interbank rates caused by varying average lending transaction costs.

7 Concluding Remarks

Regarding newly introduced regulatory capital rules to be imposed on banks and persistent counterparty credit risks in the euro area banking sector, interbank market frictions might be of concern for monetary policy implementation for an indefinite period of time. Although interest rate control under a floor operating system, as currently implemented by the Eurosystem, with low volatility of the interbank rate can be, in principle, exerted by supplying a sufficiently large amount of excess reserves, the theoretical model we introduced yields some important implications: When marginal costs of lending in the interbank market are elevated (e.g. due to risk-based capital requirements or counterparty monitoring efforts), the central bank’s provision of excess reserves needed to bring the interbank rate close to the rate on the deposit facility needs to be significantly more expansive. However, with regard to practical matters in the euro area where the amount of assets eligible for outright purchases by the Eurosystem is limited, the question arises whether the ECB is able to generate an aggregate liquidity surplus to the extent necessary to stabilize the interbank rate at constant low levels. We showed that in this case increasing the width of the interest corridor might be an appropriate policy action in order to put a downward pressure on the interbank rate, attenuate the interest-rate effects produced by friction costs that affect lenders in the interbank market (i.e. an elevated and more volatile interbank rate above the central bank’s target rate), enhance market activity and facilitate the redistribution of excess reserves in the banking sector. Regarding a possible future return to a standard corridor system by the Eurosystem with a benchmark allotment policy,
our model suggests that the central bank’s ability to implement a target interbank rate lying in the corridor midpoint with low volatility over time will be significantly impaired as long as interbank market frictions are present. In this case, interbank market frictions like counterparty credit risks or stricter regulatory capital requirements ultimately are an additional source of forecast errors in the central bank’s liquidity management as the banking sector’s aggregate demand for excess reserves might change erratically and to a significant extent from day to day.

A Appendix

Comparative Statics - Overview

\[
\frac{\partial i^{IBM^*}}{\partial \gamma_1} = \frac{f \left( F^{-1} \left( \frac{i^{IBM^*} + \gamma_2 - i^{DF}}{i^{LF} - i^{DF}} \right) \right) + f \left( F^{-1} \left( \frac{i^{IBM^*} + \gamma_2 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}{f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right) + f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}
\]

\[
= \frac{f (-\xi_2 - b_2)}{f (-\xi_1 - b_1) + f (-\xi_2 - b_2)} \in (0, 1)
\]

\[
\frac{\partial i^{IBM^*}}{\partial \gamma_2} = -\frac{f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}{f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right) + f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}
\]

\[
= -\frac{f (-\xi_1 - b_1)}{f (-\xi_1 - b_1) + f (-\xi_2 - b_2)} \in (-1, 0)
\]

\[
\frac{\partial b^*}{\partial \gamma_1} = -\frac{1}{(i^{LF} - i^{DF}) \cdot (f (-\xi_1 + b^*) + f (-\xi_2 - b^*))} < 0
\]

\[
\frac{\partial b^*}{\partial \gamma_2} = -\frac{1}{(i^{LF} - i^{DF}) \cdot (f (-\xi_1 + b^*) + f (-\xi_2 - b^*))} < 0
\]

\[
\frac{\partial i^{IBM^*}}{\partial \xi_1} = -\frac{f \left( F^{-1} \left( \frac{i^{IBM^*} + \gamma_2 - i^{DF}}{i^{LF} - i^{DF}} \right) \right) \cdot f \left( F^{-1} \left( \frac{i^{IBM^*} + \gamma_2 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}{f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right) + f \left( F^{-1} \left( \frac{i^{IBM^*} - \gamma_1 - i^{DF}}{i^{LF} - i^{DF}} \right) \right)}
\]

\[
= -\frac{f (-\xi_1 - b_1)}{f (-\xi_1 - b_1) + f (-\xi_2 - b_2)} \in (-1, 0)
\]

\[
\frac{\partial i^{IBM^*}}{\partial \xi_2} = \frac{\partial i^{IBM^*}}{\partial \xi_1} < 0
\]

\[
\frac{\partial b^*}{\partial \xi_1} = \frac{f (-\xi_1 + b^*)}{f (-\xi_1 + b^*) + f (-\xi_2 - b^*)} > 0
\]

\[
\frac{\partial b^*}{\partial \xi_2} = -\frac{f (-\xi_2 - b^*)}{f (-\xi_1 + b^*) + f (-\xi_2 - b^*)} < 0
\]
\[
\frac{\partial I_{BM^*}}{\partial \sigma} = - \frac{1}{\sigma} \cdot \left( F^{-1} \left( \frac{I_{BM^*} - \gamma_1 - M_{DF}}{L_{DF} - \gamma_1 - M_{DF}} \right) + F^{-1} \left( \frac{I_{BM^*} + \gamma_2 - M_{DF}}{L_{DF} - \gamma_2 - M_{DF}} \right) \right) \\
= \frac{\xi_1 + \xi_2}{\frac{1}{f(-\xi_1 - b^*)} + \frac{1}{f(-\xi_2 - b^*)}} \begin{cases} 
> 0 & \text{for } |\xi_1| > |\xi_2| \\
= 0 & \text{for } |\xi_1| = |\xi_2| \\
< 0 & \text{for } |\xi_1| < |\xi_2| 
\end{cases} 
\frac{\partial I_{BM^*}}{\partial w} = \frac{\frac{I_{BM^*} - \gamma_1 - M_{DF}}{2w}}{f^{-1}(\frac{I_{BM^*} - \gamma_1 - M_{DF}}{2w})} + \frac{\frac{I_{BM^*} + \gamma_2 - M_{DF}}{2w}}{f^{-1}(\frac{I_{BM^*} + \gamma_2 - M_{DF}}{2w})} \begin{cases} 
> 0 & \text{for } |\xi_1| < |\xi_2| \\
= 0 & \text{for } |\xi_1| = |\xi_2| \\
< 0 & \text{for } |\xi_1| > |\xi_2| 
\end{cases} \\
= \frac{\xi_1 + \xi_2}{\frac{1}{f(-\xi_1 - b^*)} + \frac{1}{f(-\xi_2 - b^*)}} \begin{cases} 
> 0 & \text{for } |\xi_1| < |\xi_2| \\
= 0 & \text{for } |\xi_1| = |\xi_2| \\
< 0 & \text{for } |\xi_1| > |\xi_2| 
\end{cases} 
\frac{\partial b^*}{\partial w} = \frac{2(\gamma_1 + \gamma_2)}{(2w)^2 (f(-\xi_1 + b^*) + f(-\xi_2 - b^*))} \geq 0 \\
\frac{\partial^2 I_{BM^*}}{\partial \gamma_1 \partial \xi_1} = - \left( \frac{\frac{\partial I_{BM^*}}{\partial \gamma_1}}{f^{-1}(\frac{I_{BM^*} - \gamma_1 - M_{DF}}{L_{DF} - \gamma_1 - M_{DF}})} \right)^2 + \left( \frac{\frac{\partial I_{BM^*}}{\partial \gamma_1}}{f^{-1}(\frac{I_{BM^*} + \gamma_2 - M_{DF}}{L_{DF} - \gamma_2 - M_{DF}})} \right)^2 \\
\left( \frac{\frac{1}{f^{-1}(\frac{I_{BM^*} - \gamma_1 - M_{DF}}{L_{DF} - \gamma_1 - M_{DF}})} + \frac{1}{f^{-1}(\frac{I_{BM^*} + \gamma_2 - M_{DF}}{L_{DF} - \gamma_2 - M_{DF}})}} \right)^2
\right)
\]

(52) (53) (54) (55) (56) (57) (58) (59)
\begin{align}
&= - \frac{\partial_i \mathbb{IBM}^* f'(-\xi_2 - b^*)}{f(-\xi_2 - b^*)} + \frac{\partial_i \mathbb{IBM}^* (-1) f'(-\xi_1 + b^*)}{f(-\xi_1 + b^*)} \\
&= - \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_2 - b^*}{\sigma^2} \right) f(-\xi_2 - b^*) + \partial_i \mathbb{IBM}^* \left( - \frac{-\xi_1 + b^*}{\sigma^2} \right) f(-\xi_1 + b^*)}{f(-\xi_2 - b^*) + f(-\xi_1 + b^*)} > 0
\end{align}

with $f'(x) \mu = -\frac{x}{\sigma^2} \cdot f(x)$ we obtain

\begin{align}
&= - \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_1 + b^*}{\sigma^2} \right) f(-\xi_1 + b^*)}{f(-\xi_1 + b^*)} + \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_2 - b^*}{\sigma^2} \right) f(-\xi_2 - b^*)}{f(-\xi_2 - b^*)} > 0
\end{align}

\begin{align}
\frac{\partial^2 \mathbb{IBM}^*}{\partial \gamma_2 \partial \xi_1} &= - \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_1 + b^*}{\sigma^2} \right) f(-\xi_1 + b^*)}{f(-\xi_1 + b^*)} + \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_2 - b^*}{\sigma^2} \right) f(-\xi_2 - b^*)}{f(-\xi_2 - b^*)} > 0 \quad \text{for } |\xi_1| < |\xi_2| \\
&= 0 \quad \text{for } |\xi_1| = |\xi_2| \quad (63)
\end{align}

\begin{align}
\frac{\partial^2 \mathbb{IBM}^*}{\partial \gamma_1 \partial \xi_1} &= - \frac{\partial_i \mathbb{IBM}^* \left( - \frac{-\xi_2 - b^*}{\sigma^2} \right) f(-\xi_2 - b^*)}{f(-\xi_2 - b^*)} > 0 \quad \text{for } |\xi_1| > |\xi_2| \\
&= 0 \quad \text{for } |\xi_1| = |\xi_2| \quad (64)
\end{align}

\begin{align}
\frac{\partial^2 \mathbb{IBM}^*}{\partial w \partial \gamma_2} &= \frac{\partial b^*}{\partial w} \left( \frac{-\xi_1 - \xi_2}{\sigma^2} \cdot f(-\xi_1 + b^*) \cdot f(-\xi_2 - b^*) \right) (f(-\xi_1 + b^*) + f(-\xi_2 - b^*)^2) > 0 \quad \text{for } |\xi_1| < |\xi_2| \\
&= 0 \quad \text{for } |\xi_1| = |\xi_2| \quad (65)
\end{align}

\begin{align}
\frac{\partial^2 \mathbb{IBM}^*}{\partial \gamma_2 \partial w} &= \frac{\partial b^*}{\partial w} \left( \frac{-\xi_1 - \xi_2}{\sigma^2} \cdot f(-\xi_1 + b^*) \cdot f(-\xi_2 - b^*) \right) (f(-\xi_1 + b^*) + f(-\xi_2 - b^*)^2) > 0 \quad \text{for } |\xi_1| < |\xi_2| \\
&= 0 \quad \text{for } |\xi_1| = |\xi_2| \quad (66)
\end{align}
\[
\frac{\partial^2 b^*}{\partial \gamma_1 \partial w} = \frac{f(-\xi_1 + b^*) + f(-\xi_2 - b^*) + w \cdot \frac{2\delta^*}{\partial w} \cdot (f'(-\xi_1 + b^*) - f'(-\xi_2 - b^*))}{2w^2 \cdot (f(-\xi_1 + b^*) + f(-\xi_2 - b^*))^2} > 0
\]

(67)

\[
\frac{\partial^2 b^*}{\partial \gamma_2 \partial w} = \frac{f(-\xi_1 + b^*) + f(-\xi_2 - b^*) + w \cdot \frac{2\delta^*}{\partial w} \cdot (f'(-\xi_1 + b^*) - f'(-\xi_2 - b^*))}{2w^2 \cdot (f(-\xi_1 + b^*) + f(-\xi_2 - b^*))^2} > 0
\]

(68)

(69)

**Loan Demand**

\[
\frac{\partial b_2}{\partial \xi^*} = -\frac{1}{f(-\xi_2 - b_2) \cdot (iLF - iDF)} < 0
\]

(70)

\[
\frac{\partial b_2}{\partial \gamma_2} = -\frac{1}{f(-\xi_2 - b_2) \cdot (iLF - iDF)} < 0
\]

(71)

\[
\frac{\partial b_2}{\partial \xi_2} = -1
\]

(72)

\[
\frac{\partial b_2}{\partial w} = \frac{2 \cdot F'(-\xi_2 - b_2) - 1}{f(-\xi_2 - b_2) \cdot (iLF - iDF)} \begin{cases} > 0 & \text{for } |\xi_2| > |b_2| \\ = 0 & \text{for } |\xi_2| = |b_2| \\ < 0 & \text{for } |\xi_2| < |b_2| \end{cases}
\]

(73)

\[
\frac{\partial b_2}{\partial \sigma} = \frac{\xi_2 + b_2}{\sigma} \begin{cases} > 0 & \text{for } |\xi_2| < |b_2| \\ = 0 & \text{for } |\xi_2| = |b_2| \\ < 0 & \text{for } |\xi_2| > |b_2| \end{cases}
\]

(74)

\[
\frac{\partial^2 b_2}{\partial \gamma_2 \partial \xi^*} = \frac{f'(-\xi_2 - b_2)}{(iLF - iDF)^2 \cdot (f(-\xi_2 - b_2))^3} \begin{cases} > 0 & \text{for } |\xi_2| < |b_2| \\ = 0 & \text{for } |\xi_2| = |b_2| \\ < 0 & \text{for } |\xi_2| > |b_2| \end{cases}
\]

(75)

\[
\frac{\partial^2 b_2}{\partial w \partial \xi^*} = \frac{2}{(2w)^2 \cdot f(-\xi_2 - b_2)} - \frac{f'(-\xi_2 - b_2) \cdot (2 \cdot F(-\xi_2 - b_2) - 1)}{2w \cdot (f(-\xi_2 - b_2))^3} > 0
\]

(76)

\[
\frac{\partial^2 b_2}{\partial \sigma \partial \xi^*} = -\frac{1}{\sigma \cdot (iLF - iDF) \cdot f(-\xi_2 - b_2)} < 0
\]

(77)

**Loan Supply**
\[
\frac{\partial b_1}{\partial i^*} = -\frac{1}{(i^{LF} - i^{DF}) \cdot f(-\xi_1 - b_1)} < 0 \tag{78}
\]
\[
\frac{\partial b_1}{\partial \gamma_1} = \frac{1}{f(-\xi_1 - b_1) \cdot (i^{LF} - i^{DF})} > 0 \tag{79}
\]
\[
\frac{\partial b_1}{\partial \xi_1} = -1 \tag{80}
\]
\[
\frac{\partial b_1}{\partial w} = \frac{2 \cdot F(-\xi_1 - b_1) - 1}{2w \cdot f(-\xi_1 - b_1)} \begin{cases} > 0 & \text{for } |\xi_1| < |b_1| \\ = 0 & \text{for } |\xi_1| = |b_1| \\ < 0 & \text{for } |\xi_1| > |b_1| \end{cases} \tag{81}
\]
\[
\frac{\partial b_1}{\partial \sigma} = \frac{\xi_1 + b_1}{\sigma} \begin{cases} > 0 & \text{for } |\xi_1| > |b_1| \\ = 0 & \text{for } |\xi_1| = |b_1| \\ < 0 & \text{for } |\xi_1| < |b_1| \end{cases} \tag{82}
\]
\[
\frac{\partial^2 b_1}{\partial i^{IBM} \partial \sigma} = -\frac{1}{\sigma \cdot (i^{LF} - i^{DF}) \cdot f(-\xi_1 - b_1)} < 0 \tag{84}
\]
\[
\frac{\partial^2 b_1}{\partial w \partial i^{IBM}} = -\frac{1}{2w^2 \cdot f(-\xi_1 - b_1)} \frac{f'(-\xi_1 - b_1) \cdot (2 \cdot F(-\xi_1 - b_1) - 1)}{4w^2 \cdot (f(-\xi_1 - b_1))^3} \tag{85}
\]
\[
\frac{\partial^2 b_1}{\partial i^{IBM} \partial \gamma_1} = -\frac{f'(-\xi_1 - b_1)}{(i^{LF} - i^{DF})^2 \cdot (f(-\xi_1 - b_1))^3} \begin{cases} > 0 & \text{if } |\xi_1| < |b_1| \\ = 0 & \text{if } |\xi_1| = |b_1| \\ < 0 & \text{if } |\xi_1| > |b_1| \end{cases} \tag{86}
\]
\[
\frac{\partial^2 b_1}{\partial \sigma \partial \gamma_1} = \frac{1}{\sigma \cdot f(-\xi_1 - b_1) \cdot (i^{LF} - i^{DF})} > 0 \tag{87}
\]
\[
\frac{\partial^2 b_1}{\partial w \partial \gamma_1} = \frac{1}{2w^2 \cdot f(-\xi_1 - b_1)} + \frac{f'(-\xi_1 - b_1) \cdot (2 \cdot F(-\xi_1 - b_1) - 1)}{4w^2 \cdot (f(-\xi_1 - b_1))^3} < 0 \tag{88}
\]
\[
\frac{\partial^2 b_1}{\partial w \partial \sigma} = \frac{1}{\sigma} \cdot \frac{2 \cdot F(-\xi_1 - b_1) - 1}{2w \cdot f(-\xi_1 - b_1)} \begin{cases} > 0 & \text{if } |\xi_1| < |b_1| \\ = 0 & \text{if } |\xi_1| = |b_1| \\ < 0 & \text{if } |\xi_1| > |b_1| \end{cases} \tag{89}
\]
Bibliography


