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Conference Paper

Mobile telephony in emerging markets

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel -
Session: Market Structure and Firm Strategy in Oligopoly, No. D03-V1

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Göller, Daniel; Andersson, Kjetil; Hansen, Bkørn (2016) : Mobile telephony in emerging markets, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Market Structure and Firm Strategy in Oligopoly, No. D03-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/145839>

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Abstract

Rapidly increasing sales of multi-SIM phones, mobile penetration rates above 100% and reported customer behavior all point to the fact that a significant share of mobile customers in emerging markets tend to use more than one SIM card. A primary motive for this is to avoid making expensive off-net calls. We add a segment of flexible prepaid customers, who choose to "multi-sim" in equilibrium to the seminal model of competing telephone networks á la Laffont, Rey and Tirole (1998b). In equilibrium, the networks choose to set a very high prepaid off-net price to achieve segmentation. This incentive prevails, even if termination rates are set to marginal costs.

Keywords: Mobile networks, multihoming, dual-SIM phones, termination rates

JEL-Classification: D43, L13, L96

1 Introduction

In many mobile markets the total number of mobile subscriptions is significantly larger than the number of individuals that has a mobile subscription. Apparently, some individuals own more than one mobile subscription. An illustration of multiple SIM ownership is given in international statistics on mobile subscription penetration, i.e. the number of mobile subscriptions in percent of the population. About half of the countries included in the International Telecommunication Union's database have mobile subscription penetration above 100 percent in 2013, and one third of the countries have had so for five years or more.¹

To our knowledge, all theoretical papers on competing telephone networks, including, and building on, the seminal works of Laffont, Rey and Tirole (1998a, 1998b) and Armstrong (1998), assume that customers single-home, i.e. subscribe to one and only one network. Armstrong and Wright (2009) and Hoernig (2014) are two recent papers that summarize and generalize this literature. We add to the existing literature by studying competition between mobile networks by relaxing the assumption that all customer single-home. In the following we will refer to a customer that actively uses more than one mobile subscription as a *multi-simmer*. The presence of multi-simming customers will induce the networks to set a high off-net price, i.e. the price of calling a customer at another network, to achieve segmentation.

Part of the difference between the number of subscriptions and the number of subscribers referred to above can be attributed to inactive subscriptions. This is especially true for emerging economies where mobile operators run frequent promotions and price sensitive customers, as a result, tend to pile up prepaid SIM cards while only using the SIM connected to the latest offer, see Gilett (2012). Furthermore, mobile network operators, and regulators, have a varying practice

¹The data comprises mobile penetration rates in 229 countries measured yearly in the period from 2000 to 2013. 117 countries had penetration rates above 1 in 2013, and 75 countries had had it for 5 or more years. The metrics are calculated from the data published at <http://www.itu.int/en/ITU-D/Statistics/Pages/stat/default.aspx>.

regarding how they define, or whether they report, the number of active subscriptions. Thus, it is hard to determine to which degree mobile customers tend to actually use more than one SIM from reported statistics alone. Wireless Intelligence (Gillet 2012) combines reported number on mobile subscriptions with survey data to shed light on this issue. The markets surveyed comprise 22 developed and 17 developing countries in the period from 2009 to 2012, and respondents were asked how many SIM cards they use.² A key finding is that multi-simming seems to be commonplace: the average customer in developed countries reports using 1.57 SIM cards while the average customer in developing countries reports using as much as 1.97 SIM cards³.

While the models in the literature on single-simming telephone customers plausibly capture essential mechanisms in markets for mobile calls in developed economies, they seem less suited for emerging economies. The primary reason for the former claim is that in developed economies a large share of existing multi-simmers are customers that buy separate SIM cards for their various mobile broadband devices. Thus, even if the average developed market customer reports using as much as 1.57 SIMs, this does not necessarily imply that the single-sim assumption, in models for mobile *calls*, is ill-suited.

In emerging markets, however, mobile calls are still the most important mobile service. A key mechanism that induces multi-simming here, is the frequently observed price differential between on-net and off-net calls. For instance, the telecom regulator in India reports that off-net prices are 5 times higher than on-net prices, see Sarma (2011). A customer can avoid making expensive off-net calls by subscribing to several networks, and select the SIM from the same network as the recipient when originating a call. This, of course, may involve a hassle- or time cost, since a caller has to keep track of which networks her friends subscribe to and actively select the right SIM card when making a call. The very high reported number of SIMs used in developing countries suggests that a significant share of the customers does not consider this hassle as a major obstacle.

In this paper we analyze a theoretical model where we relax the assumption that all customers single-home. In particular, we assume two types of customers, high income and high time cost customers (H-type), and low income and low time cost customers (L-type). There are two mobile networks that each offer a

²For customers connected to a mobile network using the GSM technology, every subscription is aligned with a physical sim card (inserted in the mobile phone) which contain the subscriber identity and authenticate the subscriber on the network. Consequently, there is a one-to-one connection between the number of SIM cards issued and the number of subscriptions. GSM is, globally, the dominating mobile technology, especially in emerging markets, which is the focus of our paper. For this reason, and because it is term used in the industry, we adopt the term *multi-simmer* for subscribers actively using more than one SIM. For customers subscribing to the main competing wireless standard, i.e. CDMA, a mobile subscription is not necessarily connected to a physical SIM-card.

³Behind this average there is a distribution consisting of a segment of single simmers, a segment of dualsimmers, and segments with 3 or more SIMs. Internal data from a mobile network operator present in several South East Asian countries finds that dual-simming is the most frequent multi-simming mode. Moreover, on average about 30 % of the customers are actively using more than one SIM card.

postpaid and a prepaid tariff designed for the two types. The H-type customers are modeled as in Laffont, Rey and Tirole (1998b), i.e. they are uniformly distributed on the Hotelling line and choose one of the networks tariffs. The L-type on the other hand, have no brand preferences and low or zero time costs and may choose to multi-sim on the networks' tariffs to avoid high off-net prices when calling single-simming H-types. Thus, the model fits key market characteristics in emerging economies. In particular, we analyze how, and whether, results in the standard model of competing telephone networks are affected by departing from the universally employed single-homing assumption. As such, we also contribute to the literature on multi-homing in two-sided markets, see e.g. Rochet and Tirole (2003) and Doganoglu and Wright (2006).

Our main results are as follows: Independent of the termination rate, the networks set the off-net prepaid calling price sufficiently high, such that a symmetric equilibrium emerges in which the high-income customers single-sim on the networks' postpaid tariffs and the low-income customers multi-sim on the networks' prepaid tariffs. Intuitively, the existence of multi-simmers makes the off-net prepaid calling price a "free" segmentation parameter. By setting a high prepaid off-net price, the network makes the prepaid tariff unattractive for the high income customers, who do not multi-sim due to high hassle costs. This does not distort any traffic of the multi-simming low-income customers, who do not make off-net calls in equilibrium. Thus, our model provides an explanation for high off-net prices that is not driven by call externalities, i.e. the recipient of a call derives utility. In our model, high off-net prices do not lead to a communication breakdown as in Jeon, Laffont and Tirole (2004), even if no prepaid off-net calls are made. Contrary to Hoernig (2007), high off-net prices do not depend on asymmetric market shares or predatory pricing.

The prepaid on-net pricing also differs markedly from existing models since a network's on-net prepaid price must cater to two types of calls: i) from the multi-simming prepaid customers to the network's single-simming postpaid customers and ii) from the multi-simmers to other multi-simmers. For the first type of calls the network has a monopoly, whereas the second type is subject to competition from the other network's SIM. If using different SIMs for the second type of calls are not too strong substitutes, the networks set the on-net prepaid price according to a Lerner rule using the aggregate of these calls. If the SIMs are strong substitutes the networks choose a mixing strategy.

The rest of our paper is organized as follows: In the next section we present a model with multi-simming customers. In section 3, we describe the networks' equilibrium pricing. In section 4, we provide brief summary and some concluding remarks.

2 A model with multi-simming customers

Mobile calls are offered by two competing, interconnected networks (i, j) with full coverage. Following Becker (1965), we assume that customers differ with respect to their opportunity cost of time. Individuals with high time costs are

typically less willing to spend time searching for the lowest priced consumption alternatives, and are more concerned about the convenience aspect of services compared to those with a lower time cost. Moreover, as noted by Becker, time costs vary positively with income. To formalize this idea, we assume that there are two types of customers, H-types and L-types. The H-types have high time cost and high income, whereas the L-types have low or zero time cost and low income. The number of H-types and L-types is normalized to 1 and λ , respectively, both with perfectly divisible mass.

Each network may offer a postpaid and a prepaid tariff. The tariffs are denoted by

$$T_i = \{r_i, p_i, p_{ij}\} \text{ and } \bar{T}_i = \{\bar{p}_i, \bar{p}_{ij}\},$$

respectively. Here, p_i and \bar{p}_i are the prices to make on-net calls, whereas p_{ij} and \bar{p}_{ij} denote the prices of off-net calls, i.e., calls from a customer connected to network i to a customer connected to network j . Finally, r_i is the subscription fee of the the postpaid tariff. As is common in the literature, we assume that:

Assumption 1 (price-structure) *A network may not choose a tariff in which the off-net price is below the on-net price.*

In the following, to avoid unnecessary complicated notation, we specify the customers' utility functions under the presumption that the networks' prepaid off-net prices are very high, i.e. $\bar{p}_{ij} \gg \bar{p}_i$. We explain below what we mean by "very high" and discuss under which conditions this price-setting and its corresponding consumer configuration is indeed an equilibrium.

The timing of events is as follows. At date 0, a reciprocal termination charge, a , is determined. That is, a is either set by the regulator or agreed upon by the networks. At date 1, the networks each offer a postpaid and a prepaid tariff. The customers observe the tariffs at date 2 and sign up to one or more of them. We make the following simplifying assumptions:

Assumption 2 (H-type) *H-type customers do not multi-sim.*

Assumption 3 (L-type) *L-type customers cannot subscribe to a postpaid tariff.*

The first assumption is, to our knowledge, ubiquitously adopted in the previous literature on competing telephone networks. In our model, multi-simming is induced by high off-net call prices, i.e., a multi-simmer can reach all customers on-net and can hence avoid making expensive off-net calls. A customer chooses to multi-sim whenever these calling benefits exceed the costs of multi-simming. These costs include the hassle of keeping track of which networks friends subscribe to and selecting the right SIM card when originating a call. To keep the analysis simple, we assume that the time costs of the H-types are sufficiently high so that they do not multi-sim.⁴ The second assumption roots back to the financial

⁴Note that a situation in which all customers multi-sim cannot be an equilibrium. To see this, note that any one customer could choose to single-sim, save the multi-simming costs and still call all other customers on-net. This point is well known in the literature on multihoming, see e.g. Doganoglu and Wright (2006).

situation of low-income customers in emerging economies: Incomes are not only low, but also irregular and uncertain, see e.g. Collins et.al. (2009). Hence, signing up to a tariff with a (recurring) subscription fee is unattractive, or even unfeasible.

In theory, a H-type customer has four tariff choices, i.e., to single-sim on either of the four tariffs offered by the networks. An L-type customer has three choices: to single-sim on either of the networks' prepaid tariffs or to multi-sim on both of the networks' prepaid tariffs. In the equilibrium we are going to establish, the H-types single-sim on the networks' postpaid tariffs and the L-types multi-sim on the prepaid tariffs. Below we present the primitives of our model and the necessary conditions and assumptions for the aforementioned to be an equilibrium consumer configuration. In the next section, we then derive the networks' optimal pricing.

Utilities of the H-types

As is standard in the literature on competing telephone networks, the H-types have idiosyncratic brand preferences. Following Laffont et al. (1998b), Armstrong (1998) and the subsequent literature, we model this by assuming that the H-types are uniformly distributed on a Hotelling line with support $[0, 1]$. The networks are situated at the extremes, i.e., network 1 is located at point 0 and network 2 is located at point 1. When an H-type customer located at x subscribes to a tariff offered by network i , he or she incurs transportation costs of $t|x - x_i|$, where $x_i \in \{0, 1\}$ is the location of networks 1 and 2, respectively.

All customers, H-types and L-types, incur a fixed benefit, v_0 , of being connected to a network. Throughout, we assume that v_0 is sufficiently large such that all customers choose to be connected to at least one network. The customers derive utility from making calls with indirect utility $v(p)$, $p \in \{p_i, p_{ij}\}$, of a particular call. The demand for calls, $q(p)$, is a decreasing function for which it holds that $v'(p) = -q(p)$. In line with the literature, calls are made according to a balanced calling pattern, that is, each customer calls all other customers (not SIM cards) with equal probability.⁵ Recall that we work under the presumption that the L-type customers multi-sim, which we prove to be the case in Section 3. We assume that multi-simmers have both SIM cards active for receiving calls. That is, they either use dual-sim phones or carry two active single-sim phones. Let n_i denote the number of H-types that subscribe to network i , which means that $1 - n_i$ customers subscribe to network j . Hence, a H-type subscribed to network i calls $n_i + \lambda$ customers on-net, and $1 - n_i$ customers off-net. This allows us to write the utility of a H-type located at x on the Hotelling line, and who uses network i 's postpaid tariff, as

$$U^H(T_i) = v_0 + (n_i + \lambda)v(p_i) + (1 - n_i)v(p_{ij}) - r_i - t|x - x_i|. \quad (1)$$

If he or she uses network i 's prepaid tariff, her or his utility amounts to

$$U^H(\bar{T}_i) = v_0 + (n_i + \lambda)v(\bar{p}_i) + (1 - n_i)v(\bar{p}_{ij}) - t|x - x_i|.$$

⁵Dessein (2003, 2004) and Hoernig et al. (2014) consider network competition with non-balanced calling patterns.

In the separating equilibrium we are going to establish, the networks use the off-net prepaid price as a segmentation device to prevent the H-types from using the prepaid tariff. Hence, given equilibrium tariffs, we assume that the H-types "cannot" take the prepaid tariffs.⁶ As a deviation, a network may, however, consider to offer a low off-net price. In that case, the IC constrained is that $U^I(T_i) \geq U^I(\bar{T}_i)$, or equivalently,

$$(n_i + \lambda)(v(p_i) - v(\bar{p}_i)) + (1 - n_i)(v(p_{ij}) - v(\bar{p}_{ij})) \geq r_i,$$

which implies that the utility gain from calling at lower postpaid prices must exceed the utility loss of paying the subscription fee, r_i .

Utilities of the L-types

Recall that L-type customers are financially constrained and may only choose prepaid tariffs. Contrary to the H-types, brand preferences etc. are a luxury the L-types cannot afford and, hence, we assume that they have travel costs $t = 0$. They may either multi-sim using both networks or single-sim on either of the networks' prepaid tariffs. The utility of a multi-simming L-type customer amounts to

$$U^L(\bar{T}_1, \bar{T}_2) = v_0 + n_i v(\bar{p}_i) + (1 - n_i) v(\bar{p}_j) + \lambda v^m(\bar{p}_1, \bar{p}_2) - h, \quad (2)$$

whereas it is given by

$$U^L(\bar{T}_i) = v_0 + (n_i + \lambda) v(\bar{p}_i) + (1 - n_i) v(\bar{p}_{ij}),$$

if he or she single-sims using network $i \in \{1, 2\}$. As before, v_0 is the intrinsic utility of being connected, which does not depend on whether the customer is a multi or a single-simmer. The L-types time costs of multi-simming are represented by the constant $h \geq 0$. This constant reflects the hassle of keeping track of which network the customer's friends are subscribed to, and/or possible additional expenses of buying a dual-SIM phone. Like the H-types, the L-types exhibit a balanced calling pattern and obtain indirect utility, $v(p)$, $p \in \{\bar{p}_i, \bar{p}_j\}$, from calling the H-types.

Since off-net prepaid calling prices are set very high, in order to make the tariffs unattractive for the H-types, it is clear that a multi-simming L-type never makes off-net calls. Hence, $n_i v(\bar{p}_i)$, is the utility of using SIM i to call the H-type customers connected to network i at on-net price \bar{p}_i . Similarly, the third term of (2), $(1 - n_i) v(\bar{p}_j)$, is the utility of using SIM j to call the H-type customers connected to network j at on-net price \bar{p}_j . The fourth term, $\lambda v^m(\bar{p}_1, \bar{p}_2)$, is the utility of calling the other multi-simmers. Because the other multi-simmers can be reached on-net using either of the two SIM cards, the networks' call offerings are substitutes. In the most extreme case, in which the multi-simmers use dual-SIM hand-sets that automatically select the lowest price for the call, SIM 1 and SIM

⁶In principle, the networks could set the off-net prepaid price sufficiently high such that it becomes virtually impossible to make off-net calls. In that case, the H-types cannot reach a large part of the customer base, which makes it unattractive to use the tariff. This dis-utility is not sufficiently captured by the fact that $v(\bar{p}_{ij})$ approaches zero for large \bar{p}_{ij} .

2 would be perfect substitutes for these types of calls.⁷ More realistically, multi-simmers have to select a default SIM, so that there is a small hassle involved in making calls with the other SIM. We formalize this by assuming that the utility of a call to another multi-simmer, $v^m(\bar{p}_1, \bar{p}_2)$, is a well behaved, continuously differentiable indirect utility function. The demand for using SIM 1 to call another multi-simmer is denoted by $q_1(\bar{p}_1, \bar{p}_2) = -\partial v^m(\bar{p}_1, \bar{p}_2)/\partial \bar{p}_1$, whereas the demand for using SIM 2 to call another multi-simmer is $q_2(\bar{p}_1, \bar{p}_2) = -\partial v^m(\bar{p}_1, \bar{p}_2)/\partial \bar{p}_2$. For symmetric prices, $\bar{p}_1 = \bar{p}_2 = \bar{p}$, the customer is indifferent with regards to which SIM card to use. Her or his utility thus is identical to calling a single-simmer, i.e.

$$v^m(\bar{p}, \bar{p}) = v(\bar{p}). \quad (3)$$

Moreover, we assume that, in case of equal on-net prices, the multi-simmer splits his calls evenly between the two SIM cards such that $q_1(\bar{p}, \bar{p}) = q_2(\bar{p}, \bar{p}) = q(\bar{p})/2$.⁸

A L-type customer prefers to multi-sim if $U^L(\bar{T}_1, \bar{T}_2) > U^L(\bar{T}_i)$, $i \in \{1, 2\}$, or equivalently if

$$(1 - n_i)(v(\bar{p}_j) - v(\bar{p}_{ij})) + \lambda(v^m(\bar{p}_1, \bar{p}_2) - v(\bar{p}_i)) > h.$$

In a symmetric equilibrium the last term on the left hand side of the inequality is zero, and hence multi-simming takes place when

$$\frac{1}{2}(v(\bar{p}_{on}^*) - v(\bar{p}_{off}^*)) > h. \quad (4)$$

Since \bar{p}_{off}^* is set very high, in order to prevent the H-types to use the pre-paid tariff, multi-simming always takes place in the equilibrium we are going to establish. That is, we consider cases in which h is not prohibitively high.

Market shares and firms' profit

Let us consider our candidate equilibrium in which the H-types single-sim on the postpaid tariff and the L-types multi-sim using the prepaid tariffs. Network i 's share of inflexible customers are found by solving $U^H(T_i) = U^H(T_j)$ for the indifferent customer. After rearranging, the market share of network i can be written as

$$n_i = \frac{t + r_j - r_i + v(p_{ij}) - v(p_j) + \lambda(v(p_i) - v(p_j))}{2t - v(p_i) - v(p_j) + v(p_{ij}) + v(p_{ji})}, \quad (5)$$

which simplifies to $n_1 = n_2 = 1/2$ in a symmetric equilibrium.⁹ Let us now derive the networks' equilibrium profits. The fixed cost of serving a customer is f per

⁷This would require a dual-sim hand-set that continuously monitors the networks' prices and automatically selects the SIM-card offering the lowest price for each particular call. At the time of writing, to our knowledge, this technology is not available.

⁸In the appendix we show that, for example, the Shubik-Levitan utility function, see Shubik and Levitan (1980), satisfies (3).

⁹Note that we are working under the presumption that hotelling is stable, i.e., we only consider parameter constellations in which the hotelling game does not have corner solutions, in which one network has all customers in the H-type segment. This assumption makes sense in equilibrium, but not when checking for possible deviations. For details, see the discussion below (12) in the appendix.

contract. This also includes the cost of issuing a SIM card for the customer's chosen tariff. A network has a cost of c per call it originates or terminates. Hence, the marginal cost of an on-net call, i.e., a call that originates and terminates on the same network, is $2c$. Since off-net calls terminate on the rival's network, the originating network pays a termination fee, a , to the other network so that the perceived total cost of the call is $a + c$. It follows that a network's margin on an incoming call from the rival is $a - c$. Consequently, network i 's profit, π_i , amounts to

$$\pi_i = \lambda\pi_i^L + n_i\pi_i^H, \quad (6)$$

where

$$\begin{aligned} \pi_i^L &= -f + (n_i q(\bar{p}_i) + \lambda q_i(\bar{p}_1, \bar{p}_2))(\bar{p}_i - 2c), \text{ and} \\ \pi_i^H &= r_i - f + (n_i + \lambda)q(p_i)(p_i - 2c) + (1 - n_i)(q(p_{ij})(p_{ij} - c - a) + q(p_{ji})(a - c)). \end{aligned}$$

Here, π_i^H and π_i^L are the profits made on a H-type and L-type customer, respectively.

3 Equilibrium pricing and consumer configuration

In this section, we derive the networks' optimal pricing. Throughout, we focus on symmetric equilibria in which the prepaid and the postpaid tariffs of both networks are identical. When checking for possible deviations from our equilibrium candidate, we allow a network to deviate to a different prepaid *or* postpaid tariff but not both at the same time. Before presenting our first result, let us denote the on-net price after the deviation by \bar{p}_i^d and the off-net price by \bar{p}_{ij}^d , respectively. Moreover, let \bar{p}_{on}^* denote the equilibrium prepaid on-net price, which we are going to derive below.

Theorem 1 *If*

$$\begin{aligned} & \left(\frac{1}{2} + \lambda\right)(v(2c) - v(\bar{p}_i^d)) + \frac{1}{2}(v(a + c) - v(\bar{p}_{ij}^d)) \leq r^* \\ & \Leftrightarrow f + t \geq \left(\frac{3}{2} + \lambda\right)v(2c) - \left(\frac{1}{2} + \lambda\right)v(\bar{p}_i^d) \\ & \quad - \frac{1}{2}(v(a + c) + v(\bar{p}_{ij}^d)) + (\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*) \end{aligned} \quad (7)$$

holds for any $\bar{p}_i^d \leq \bar{p}_{ij}^d \leq \bar{p}_{on}^$, the networks set their off-net prepaid prices sufficiently high so that a symmetric equilibrium emerges in which the H-types single-sim on the networks' postpaid tariffs and the L-types multi-sim on the networks' prepaid tariffs.*

Note that (7) implies that if a network deviates to a new prepaid tariff, in order to induce the L-types to single-sim on it, its own H-types also switch to the new tariff. Here, \bar{p}_{on}^* denotes the equilibrium prepaid on-net price, which we derive in

Proposition 1. (7) can readily be derived from (??) by inserting the equilibrium subscription fee, r^* , which we calculate in Proposition 2. At the end of the proof of Proposition 3, we demonstrate that there exist sensible parameter constellations, in which the aforementioned deviation is unprofitable. This roots back to the fact that the H-types' high traveling costs, t , allow the networks to set high subscription fees, which gives a high profit in the H-type segment. Consequently, the deviator loses a lot in the H-type segment compared to what it gains in the L-type segment.

We perform the proof of Theorem 1 in several steps. First, we characterize the networks' optimal pricing strategies under the presumption that they set their respective prepaid off-net prices sufficiently high in order to achieve the consumer configuration described in Theorem 1. Second, we prove that this is indeed optimal for the networks. Intuitively, the existence of multi-simmers makes the off-net prepaid calling price a "free" segmentation parameter. By setting a high prepaid off-net price, the networks make the prepaid tariff unattractive for the H-type customers, who do not multi-sim due to high time costs. Note that the high off-net price does not distort traffic of the multi-simming L-type customers, since multi-simmers do not make off-net calls in equilibrium. Let us now derive optimal pricing. The networks compete for the multi-simming L-types using the on-net prices \bar{p}_i . We establish the following result:

Proposition 1 *There is a critical value $k(\lambda)$, such that if $|\partial q_i(\bar{p}_i, \bar{p}_{on}^*)/\partial \bar{p}_i| \leq k(\lambda)$, there exists a symmetric pure strategy equilibrium, in which the equilibrium on-net price of network i , $i \in \{1, 2\}$, is $\bar{p}_i^* = \bar{p}_{on}^*$, where \bar{p}_{on}^* is implicitly defined by*

$$\frac{\bar{p}_{on}^* - 2c}{\bar{p}_{on}^*} = -\frac{(1 + \lambda)q(\bar{p}_{on}^*)}{\bar{p}_{on}^*(q'(\bar{p}_{on}^*) + 2\lambda\partial q_i(\bar{p}_{on}^*, \bar{p}_{on}^*)/\partial \bar{p}_i)} > 0. \quad (8)$$

If $|\partial q_i(\bar{p}_{on}^, \bar{p}_{on}^*)/\partial \bar{p}_i| > k(\lambda)$ there exists a symmetric mixed strategy equilibrium, in which the networks, $i \in \{1, 2\}$, choose \bar{p}_i from a probability distribution with support $[\bar{p}_{on}^*, \bar{p}_{on}^M]$, where \bar{p}_{on}^M is defined below.*

Proof. The proposition states that if $|\partial q_i(\bar{p}_{on}^*, \bar{p}_{on}^*)/\partial \bar{p}_i| \leq k(\lambda)$, it is a best reply for network i to charge \bar{p}_{on}^* , given that network j also charges \bar{p}_{on}^* . After taking the derivative of (6) with respect to \bar{p}_i , plugging in \bar{p}_{on}^* for both \bar{p}_i and \bar{p}_j and some reorganization of terms, we obtain (8). Note that to set $\bar{p}_i^* = \bar{p}_{on}^*$ is only a local maximum of network i 's profit function, given that network j also charges \bar{p}_{on}^* . There may exist a second local maximum, which may emerge when network i sets a very high price as a best reply to $\bar{p}_j^* = \bar{p}_{on}^*$. In that case, $q_i(\bar{p}_i, \bar{p}_j) = 0$ and hence $\partial q_i(\bar{p}_i, \bar{p}_{on}^*)/\partial \bar{p}_i = 0$. Less formally, this means that all L-types use network j 's tariff to call one another. However, by setting a high on-net price, network i capitalizes on the $\lambda/2q(p_i)$ number of calls from the L to the H-types, where it has a monopoly. The second local maximum (best reply candidate) is found by maximizing π_i^L under the constraints that $q_i(\bar{p}_i, \bar{p}_j) = 0$ and $\bar{p}_j^* = \bar{p}_{on}^*$. After reorganizing terms, the optimal price \bar{p}_{on}^M is given by

$$\frac{\bar{p}_{on}^M - 2c}{\bar{p}_{on}^M} = -\frac{q(\bar{p}_{on}^M)}{\bar{p}_{on}^M(q'(\bar{p}_{on}^M))} > \frac{\bar{p}_{on}^* - 2c}{\bar{p}_{on}^*}.$$

Note, however, that to set $\bar{p}_j^* = \bar{p}_{on}^*$ cannot be a best reply for network j , given that network i charges $\bar{p}_j^* = \bar{p}_{on}^M$. In that case, network j could increase its on-net price by a small amount to increase the profit it derives from the calls from L-types to L-types, hence increasing its total profit. Likewise, $\bar{p}_i = \bar{p}_{on}^M$ cannot be a best reply to $\bar{p}_j^* = \bar{p}_{on}^M$, because a small decrease in price would yet again increase network i 's profit. Consequently, a mixing equilibrium candidate emerges in which the networks choose their prices from a probability distribution with support $[\bar{p}_{on}^*, \bar{p}_{on}^M]$. We can conclude that the pure strategy equilibrium emerges if

$$\begin{aligned} \pi_i^L(\bar{p}_{on}^*, \bar{p}_{on}^*) &\geq \pi_i^L(\bar{p}_{on}^M, \bar{p}_{on}^*) \Leftrightarrow \\ \left(\frac{1}{2}q(\bar{p}_{on}^*) + \lambda q_i(\bar{p}_{on}^*, \bar{p}_{on}^*)\right)(\bar{p}_{on}^* - 2c) &\geq \frac{1}{2}q(\bar{p}_{on}^M)(\bar{p}_{on}^M - 2c). \end{aligned}$$

Whether $\pi_i^L(\bar{p}_{on}^*, \bar{p}_{on}^*) \geq \pi_i^L(\bar{p}_{on}^M, \bar{p}_{on}^*)$ or not depends on how elastic the call demand $q_i(\bar{p}_i, \bar{p}_{on}^*)$ reacts to changes in \bar{p}_i . For instance, when $|\partial q_i(\bar{p}_i, \bar{p}_{on}^*)/\partial \bar{p}_i|$ approaches infinity, the margin and profits from the L-type segment approach zero, if network i sets its on-net prepaid price according to (8). Consequently, there exists a critical value $k(\lambda)$ above which \bar{p}_{on}^M and not \bar{p}_{on}^* is a best reply to \bar{p}_{on}^* . ■

Multi-simmers make two types of calls that are relevant for network i , (i) to network i 's H-types and (ii) to other multi-simmers. For the first type of calls the network has a monopoly, whereas the second type is subject to competition from the other SIM. If using SIM i or SIM j for the second type of calls is perceived as "not too close substitutes", i.e., when $|\partial q_i(\bar{p}_i, \bar{p}_{on}^*)/\partial \bar{p}_i| \leq k(\lambda)$, the network sets \bar{p}_i according to a Lerner rule using the aggregate of these calls, i.e., (8). When the substitutability is strong, a mixed strategy equilibrium similar to as in Varian (1980) emerges. To shed more light on the magnitude of the critical value, $k(\lambda)$, we assume an explicit functional form for call utilities. In the appendix, we show that the Shubik-Levitan (SL) utility function, see Shubik and Levitan (1980), satisfies our critical call utility assumption (3). Assuming SL utility functions, the demand function for the L-types' calls of type (i) is given by $q(\bar{p}_i) = b - \bar{p}_i$, $b > 0$, and for type (ii) calls it amounts to $q_i(\bar{p}_i, \bar{p}_j, z) = (b - \bar{p}_1)/2 + (z/4(1 - z))(\bar{p}_j - \bar{p}_i)$. Here, $z \in [0, 1)$ is a parameter that measures the degree of substitutability between using SIM i and SIM j for type (ii) calls. When $z = 0$, the SIM cards are independent and when $z \rightarrow 1$ the SIM cards are perfect substitutes.

It can readily be shown that, given that network j sets $\bar{p}_j^* = \bar{p}_{on}^*$, that

$$\pi_i^L(\bar{p}_{on}^M, \bar{p}_{on}^*) - \pi_i^L(\bar{p}_{on}^*, \bar{p}_{on}^*) = v[2c]\delta(z, \lambda),$$

where $v[2c] = (b - 2c)^2/2$ and

$$\delta(z, \lambda) = -\lambda^2(16(1+\lambda)^2 - 8z(4+7\lambda+3\lambda^2) + z^2(16+23\lambda+8\lambda^2))/4(-4(1+\lambda) + z(4+3\lambda))^2.$$

Thus, since $v[2c]$ is a positive constant, whether we have a pure or a mixed strategy equilibrium depends on $\delta(z, \lambda)$ only. It can be shown that for $0 < z \lesssim 0.94$, $\delta(z, \lambda)$ is always negative.¹⁰ In this case, network i 's best response is to

¹⁰The details are available upon request.

set $\bar{p}_i = \bar{p}_{on}^*$, and we arrive at the pure strategy equilibrium of Proposition 1. For $0.95 \lesssim z < 1$, $\delta(z, \lambda)$ is always positive. In this case, network i 's best response is to set $\bar{p}_i = \bar{p}_{on}^M$, and we arrive at the mixed strategy equilibrium.¹¹

Even though we have no empirical data on z (i.e. the degree of substitutability between using SIM i or SIM j for type (ii) calls), our SL-result suggests that the pure strategy equilibrium may exist even when substitutability is quite high. We therefore focus on the pure strategy equilibrium in the rest of the paper.¹²

Let us now turn to the H-type segment. We can adopt the following well known result from the literature:

Corollary 1 *In equilibrium, the postpaid calling prices are $p_1^* = p_2^* = 2c$, and $p_{12}^* = p_{21}^* = a + c$.*

The rationale is straightforward. A situation where one or more of the calling prices in the two-part tariff are above perceived marginal cost cannot be part of a symmetric equilibrium. In that case, a network can keep market shares, in the H-type segment, constant by simultaneously increasing the subscription fee and decreasing a calling price. As is well known in the literature, see e.g. Laffont et al. (1998b), this is a profitable deviation because the increase in profit due to the higher subscription fee *and* the higher call volume outweighs the loss in profit that accrues due to the lower calling price. Because this argument holds for any calling prices above perceived marginal cost, we can directly conclude that $p_1^* = p_2^* = 2c$ and $p_{12}^* = p_{21}^* = a + c$.

Whereas the calling prices in the H-type segment do not depend on what is going on in the L-type segment, this is not so for the subscription fee, r_i . Let us establish that

Proposition 2 *In equilibrium, the networks set $r_1^* = r_2^* = r^* = f + t - v(2c) + v(a + c) - (\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*)$.*

PROOF: See Appendix.

Except for the last term, the expression for r^* is identical to Laffont et al.(1998b). Similarly to their model, the networks use r as an instrument to attract the H-type customers. The last term, $-(\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*)$ is negative and decreasing in the number of L-types. It represents that, compared to Laffont et al.(1998b), the networks have an additional incentive to lower the subscription fee, trying to attract more H-types. More precisely, the additional incentive arises due to the calls that are being made from the multi-simming L-types to the H-types. Thus, the

¹¹For $0.94 \lesssim z \lesssim 0.95$, the sign depends on λ in a complicated way. Details are available upon request.

¹²We conjecture that our segmentation equilibrium is even more stable when the networks mix in the prepaid segment. To see this, note that the networks mix between p_{on}^* and the monopoly price p_{on}^M . Hence, profits in the mixing equilibrium are higher than profits in the pure strategy equilibrium, and to deviate from it is less attractive than to deviate from the pure strategy equilibrium.

existences of multi-simmers increases competition for market shares in the H-type segment, which results in a lower equilibrium r .

After inserting equilibrium prices, we can write network i 's profit as follows:

$$\pi_i^* = \frac{t}{2} + (a-c) \left(\frac{1}{2}\right)^2 q(a+c) - \frac{1}{2}(v(2c) - v(a+c)) + \lambda((\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*, \bar{p}_{on}^*) - f).$$

The first three terms are identical to the profit in Laffont et al. (1998). The last term is new and stems from the existence of the multi-simming segment. More precisely, it amounts to net revenues from the calls between the multi-simmers minus the fixed cost of serving them, λf . The profit from the calls from the multi-simmers to the H-types does not appear in the equilibrium profit expression, since it is competed away due to the lower subscription fee, see Proposition 2. So far, we have derived the optimal tariffs given that the H-types use the postpaid tariffs and given that the L-types multi-sim on the prepaid tariffs. What remains to be shown is that

Proposition 3 *The networks have no incentive to deviate from the equilibrium described in Theorem 1.*

PROOF: See Appendix.

In the proof of Proposition 3, we show that there exist sensible parameter constellations so that the networks have no incentive to deviate to a new tariff that induces a different customer-tariff matching than described in Theorem 1. It is clear that to offer a new postpaid tariff cannot change the customer-tariff matching, since the equilibrium postpaid tariff is optimal given that only the H-types can use it. The customer-tariff matching may change when a network deviates to a new prepaid tariff. The following possibilities exist:

- (i) The H-types migrate to the new prepaid tariff and the L-types single-sim on the deviator's prepaid tariff.
- (ii) The H-types use the postpaid tariff and the L-types single-sim on the deviator's prepaid tariff.
- (iii) The H-types use the postpaid tariffs and the L-types multi-sim on the prepaid tariffs.
- (iv) The H-types migrate to the new prepaid tariff and the L-types continue to multi-sim.
- (v) The H-types use the postpaid tariff and the L-types single-sim on the competitor's prepaid tariff.

The intuition behind why (i) is not a profitable deviation, unless t is “small” and/or λ is “very large”, is as follows. Due to the L-types flexibility and the H-types high t , the profits in the H-type segment exceed those in the L-type segment.

Hence, the gain that arises because the L-types single-sim on the deviator’s tariff is smaller than the loss that stems from the fact that the H-types switch from the expensive post-paid to the cheap prepaid tariff. Note that this argument holds, of course, only if the L-type segment is not “too large” or t is not “too small”. In the latter case the equilibrium subscription fee is low and so are the profits in the H-type segment. (ii) is a profitable deviation if the condition that is necessary for Theorem 1 to hold is violated, see (7). In that case, the deviator increases its profit in the L-type segment without losing profit in the H-type segment.¹³ Note that, similar to (i), the condition holds if t is “not too small” and/or λ is not “too large”. A deviation that induces (iii) does not lead to a new customer-tariff matching and is thus, as explained above, not profitable. (iv) is no profitable deviation because a prepaid tariff is a special case of a postpaid tariff where $r = 0$. Finally, a tariff that induces (v) is an unprofitable deviation since profits in the L-type segment are reduced and so are the termination revenues.¹⁴

4 Discussion

A key market characteristic of mobile telephony markets in emerging economies is that a significant share of the customers actively use different SIM-cards from different networks, i.e. they multi-sim. A main motivation for this is to avoid making expensive off-net calls. To our knowledge, this is the first paper to set up a model, within the standard framework of competing telephone networks, that explains this phenomenon. We show that, in equilibrium, the networks set the off-net prepaid prices sufficiently high such that a symmetric equilibrium emerges in which the H-types single-sim on the networks’ postpaid tariffs and the L-types multi-sim on the networks’ prepaid tariffs. We explain that the existence of multi-simmers makes the off-net prepaid calling price a “free” segmentation parameter. By setting a high prepaid off-net price, the network makes the prepaid tariff unattractive for the H-types. The crucial point is that it can do so without distorting any traffic of the multi-simming L-types, who do not make off-net calls in equilibrium.

The equilibrium postpaid pricing (towards the H-types) is similar to what we know from the literature: In particular, calling prices are set equal to marginal cost, and the networks extract rents from the subscription fee. The subscription fee is, in our model, however, lower than in Laffont et al. (1998b), since the existence of multi-simmers induces the networks to compete harder for market shares in the H-type segment.

The prepaid tariff is markedly different compared to the equilibrium linear tariff, under network discrimination, of Laffont et al. (1998b). Firstly, in our model, since the multi-simmers do not make off-net calls, the off-net price is used purely as a segmentation device and can be set totally independent of the termination rate. Secondly, the on-net price must cater to two types of calls: (i) from the

¹³Termination revenues are, however, lower after the deviation because they are maximized in equilibrium, in which both networks have half of the H-types.

¹⁴To see this note that the off-net prepaid price of the non-deviator is set so high so that the L-types do not make off-net calls.

multi-simmers to network i 's H-types and (ii) from the multi-simmers to other multi-simmers. For the first type of calls, the network has a monopoly, whereas the second type of calls is subject to competition from the other SIM. If using SIM i or SIM j for the second type of calls are not too strong substitutes, the networks set the on-net prepaid price according to a Lerner rule using the aggregate of these calls. If they are strong substitutes, the networks choose a mixing strategy. Increased deployment of multi-SIM phones is likely to increase the substitutability. This may explain the prevalence of frequent "on-net campaigns" in mobile markets in emerging economies.

Intriguingly, the networks' incentives to set a high off-net prepaid price is independent of the termination rate. Indeed, the proof of Proposition 3 is derived for $a = c$, i.e., the networks will set high off-net prepaid prices even for cost based termination rates.

Our results certainly rests on a number of strict assumptions, and the model, of course, captures only selected aspects of mobile telephony markets in emerging economies. We believe the mechanisms focused on in this paper are important ones though. Further research should certainly explore effects of relaxing these assumptions. As multi-homing opens for a large set of consumer-tariff configurations, we expect that this will generate other types of equilibria than the one we focused on. Hopefully our paper sets the stage for further research on multi-simming, which is prevalent in huge markets and deserves more attention than it has gained so far.

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Appendix

Shubik Levitan call utility

The point of this subsection is to demonstrate that the Shubik-Levitan utility function, see Shubik and Levitan (1980), is a prominent example that satisfies both (3) and $q_1(\bar{p}, \bar{p}) = q_2(\bar{p}, \bar{p}) = q(\bar{p})/2$. Recall that a multi-simmer makes two types of calls: calls to single-simmers and calls to other multi-simmers. Suppose that a multi-simmer’s utility of a call to a singlesimmer at network i is given by

$bq - q^2/2$, where b is a positive constant so that the indirect utility amounts to $v(\bar{p}_1) = (b - \bar{p}_1)^2/2$ and the call demand is $q(\bar{p}_1) = b - \bar{p}_1$. Moreover, suppose that for calls to other multi-simulers the L-types have a Shubik Levitan utility function, or more precisely $u(q_1, q_2) = b(q_1 + q_2) - (z(q_1 + q_2)^2 + 2(1 - z)(q_1^2 + q_2^2))$. Here, q_1 and q_2 are the calls made with SIM 1 and SIM 2, respectively, and z is a constant satisfying $0 \leq z < 1$. The parameter z reflects to which degree a L-type considers the two SIMs as substitutes: Calling with SIM 1 and SIM 2 are independent when $z = 0$, and perfect substitutes when $z \rightarrow 1$. Indirect utility is given by $\text{Max}_{q_1, q_2} \{u(q_1, q_2) - \bar{p}_1 q_1 - \bar{p}_2 q_2\}$, which after some manipulation can be written as

$$v^m(\bar{p}_1, \bar{p}_2, z) = \frac{2b^2 + \bar{p}_1^2 + \bar{p}_2^2 - 2b(\bar{p}_1 + \bar{p}_2) - \frac{z}{2}(2b - \bar{p}_1 - \bar{p}_2)^2}{4(1 - z)}.$$

Hence, the demand for calls with the two SIMs amounts to

$$q_1(\bar{p}_1, \bar{p}_2, z) = -\frac{\partial v^m(\bar{p}_1, \bar{p}_2, z)}{\partial \bar{p}_1} = \frac{b - \bar{p}_1}{2} + \frac{z}{4(1 - z)}(\bar{p}_2 - \bar{p}_1), \text{ and}$$

$$q_2(\bar{p}_1, \bar{p}_2, z) = -\frac{\partial v^m(\bar{p}_1, \bar{p}_2, z)}{\partial \bar{p}_2} = \frac{b - \bar{p}_2}{2} + \frac{z}{4(1 - z)}(\bar{p}_1 - \bar{p}_2).$$

We have thus shown that for symmetric prices these call utilities satisfy (3) and that $q_1(\bar{p}, \bar{p}, z) = q_2(\bar{p}, \bar{p}, z) = q(\bar{p})/2$ for all $z \in [0, 1)$.

Proof of Proposition 2

Given that the networks set the calling prices in the postpaid tariffs equal to perceived marginal cost, the profit of network i can be written as

$$n_i(r_i - f) + (a - c)(1 - n_i)n_i q(a + c) + \lambda(-f + (\bar{p}_i^* - 2c)(n_i q(\bar{p}_i^*) + \lambda q_i(\bar{p}_i^*, \bar{p}_j^*)).$$

Let us insert the right-hand side of (5) for n_i and maximize the above equation with respect to r_i . Then, we obtain the following best response function:

$$r_i = \frac{1}{D}[(t - v(2c) + v(a + c) + (a - c)q(a + c))r_j + (t - v(2c) + v(a + c))(f + t - (\bar{p}_i^* - 2c)\lambda q(\bar{p}_i^*) - v(2c) + v(a + c))],$$

where $D = 2(t - v(2c) + v(a + c)) + (a - c)q(a + c) > 0$. The best response is upward sloping when $a \geq c$ since stability in the H-type segment requires $t - v(2c) + v(a + c) > 0$. Solving the networks' best responses yields the equilibrium subscription fees

$$r_1^* = r_2^* = t + f - v(2c) + v(a + c) - (\bar{p}_i^* - 2c)\lambda q(\bar{p}_i^*).$$

■

Proof of Proposition 3

In Propositions 1 and 2, we derived optimal prices under the presumption that the H-types single-sim on the postpaid tariff and the L-types multi-sim on the prepaid tariffs. Recall that the prepaid off-net price is used as a segmentation instrument. Hence, the H-types opt for the postpaid tariff because, otherwise, making off-net calls would be prohibitively expensive. The L-types avoid the high off-net prices by multi-simming, i.e., by using both networks' tariffs. What remains to be shown is that the networks have no incentive to deviate from our symmetric segmentation equilibrium. Possible deviations are to introduce either (a) a new postpaid tariff or (b) a new prepaid tariff. We can readily establish that

(a) to offer a new postpaid tariff cannot be a profitable deviation. To see this, recall that the L-types, being financially constrained, cannot use it. Moreover, the equilibrium candidate postpaid tariffs are already locally optimal. That is, they are optimal given that only the H-types use them. Hence, there is no scope to introduce a new postpaid contract.

(b) to offer a new prepaid tariff could, in theory, be a profitable deviation. By offering an attractive tariff, a network could, for example, try to induce the L-types to switch from multi-simming to single-simming on its own network. Let us denote the on-net price after the deviation \bar{p}_i^d and the off-net price \bar{p}_{ij}^d , respectively. After deviating to a new prepaid tariff, the customer-tariff matching may change. The following possibilities exist:

- (i) The H-types migrate to the new prepaid tariff and the L-types single-sim on the deviator's prepaid tariff.
- (ii) The H-types use the postpaid tariff and the L-types single-sim on the deviator's prepaid tariff.
- (iii) The H-types use the postpaid tariffs and the L-types multi-sim on the prepaid tariffs.
- (iv) The H-types migrate to the new prepaid tariff and the L-types continue to multi-sim.
- (v) The H-types use the postpaid tariff and the L-types single-sim on the competitor's prepaid tariff.

Case (i)

Let us first analyze the most intricate case. The deviator's own H-types switch to the new prepaid tariff if and only if:

$$\begin{aligned}
& \left(\frac{1}{2} + \lambda\right)(v(2c) - v(\bar{p}_i^d)) + \frac{1}{2}(v(a+c) - v(\bar{p}_{ij}^d)) \leq r^* \\
& \Leftrightarrow f + t \geq \left(\frac{3}{2} + \lambda\right)v(2c) - \left(\frac{1}{2} + \lambda\right)v(\bar{p}_i^d) \\
& \quad - \frac{1}{2}(v(a+c) + v(\bar{p}_{ij}^d)) + (\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*)
\end{aligned} \tag{9}$$

after inserting equilibrium r^* . Moreover, the L-types prefer single-simming on the deviator's prepaid tariff over multi-simming iff:

$$\begin{aligned}
& v(\bar{p}_{ij}^d) \leq v(\bar{p}_{on}^*) \text{ and} \\
& v(\bar{p}_i^d) \leq v(\bar{p}_{off}^*).
\end{aligned} \tag{10}$$

Note that due to Assumption 1, (10) implies that $\bar{p}_i^d \leq \bar{p}_{ij}^d \leq \bar{p}_{on}^*$. The idea of the proof is to show that, under reasonable parameter constellations, the deviation is non-profitable, if the deviator deviates to the flat tariff $\bar{p}_i^d = \bar{p}_{ij}^d = \bar{p}_{on}^*$ and manages to attract all H-types to the new tariff. This hypothetical result is an upper bound to the deviator's profit after the deviation since this flat tariff constitutes the highest pair of prices that do not violate (10). The deviator's profit function can be written as

$$\begin{aligned}
\Pi^d &= (n_i^d + \lambda)(n_i^d + \lambda)q(\bar{p}_i^d)(\bar{p}_i^d - 2c) \\
& \quad + (n_i^d + \lambda)(1 - n_i^d)q(\bar{p}_{ij}^d)(\bar{p}_{ij}^d - a - c) \\
& \quad + (n_i^d + \lambda)(1 - n_i^d)q(p_{a+c})(a - c) \\
& \quad + (n_i^d + \lambda)(-f),
\end{aligned}$$

where the first line represents that $n_i^d + \lambda$ customers call one another on-net at the calling price \bar{p}_i^d . The second line constitutes the calls from the deviator's to the non-deviator's network, which, after the deviation has $(1 - n_i^d)$ customers. Moreover, the third line is made up of the calls from the non-deviator's network to the deviator, from which the deviator gets terminations revenues amounting to a . Let us now calculate the deviator's market share in the H-type segment, n_i^d . The utility of a H-type, who uses the deviator's prepaid tariff is given by:

$$v_0 + (n_i^d + \lambda)v(\bar{p}_i^d) + (1 - n_i^d)v(\bar{p}_{ij}^d) - tx,$$

whereas the utility of a H-type customer who uses the non-deviator's postpaid tariff amounts to:

$$v_0 + (1 - n_i^d)v(2c) + (n_i^d + \lambda)v(a+c) - r_j^* - t(1-x).$$

Solving for the indifferent customer yields us the deviator's market share in the H-type segment:

$$n_i^d = \frac{r_j^* + t - v(2c) - \lambda v(a+c) + v(\bar{p}_{ij}^d) + \lambda v(\bar{p}_i^d)}{2t - v(2c) + v(a+c) + v(\bar{p}_{ij}^d) - v(\bar{p}_i^d)}.$$

As we will show, hotelling is very unstable after the deviation and corner solutions are possible. For n_i^d to constitute the deviator's market share after the deviation, the customer located at $x = 1$ must prefer network j 's tariff over network i 's tariff, given that all other customers use network i 's tariff. Likewise the customer located at $x = 0$ must prefer network i 's tariff over network j 's tariff, given that all other customer use network j 's tariff. Otherwise, corner equilibria exist, in which all H-types use the same network. The hotelling stability conditions are as follows:¹⁵

$$t - r_j^* > (1 + \lambda)(v(\bar{p}_i^d) - v(a + c)) \quad (11)$$

$$t + r_j^* > v(2c) - v(\bar{p}_{ij}^d) + \lambda(v(a + c) - v(\bar{p}_i^d)). \quad (12)$$

In particular (11) may be violated. To see this, we insert for equilibrium r_j^* and obtain

$$f < v(2c) + v(a + c) + (\bar{p}_{on}^* - 2c)\lambda q(\bar{p}_{on}^*) - (1 + \lambda)v(\bar{p}_i^d),$$

which does not depend on t . This counter-intuitive result stems from the fact that t is part of equilibrium r_j^* and thus cancels out the H-types' brand preference parameter (or traveling cost) t . Hence, even if t is very large, (11) may be violated, in particular if \bar{p}_{on}^* is close to $2c$ and λ is large. When (11) is violated, both the corner solution, in which the deviator gets all H-types and the one where he gets n_i^d H-types constitute equilibria of the hotelling problem. Moreover, it is straightforward that also a mixing equilibrium exists. To avoid these problems, we show that even if the deviator gets all the H-types, there exist sensible parameter constellations in which the deviation is non-profitable. Then, no other deviation within Case (i) can be profitable either.¹⁶ If the deviator sets $\bar{p}_i^d = \bar{p}_{ij}^d = \bar{p}_{on}^*$, and attracts all H-types, his equilibrium profit simplifies to

$$\Pi^d = (1 + \lambda)(1 + \lambda)q(\bar{p}_i^d)(\bar{p}_i^d - 2c) - (1 + \lambda)f. \quad (13)$$

Assuming that $a = c$, we can write the difference between the equilibrium profit and the profit after the deviation as

$$\Pi_i^* - \Pi^d = f + \frac{t}{2} - (\bar{p}_{on}^* - 2c)q(\bar{p}_{on}^*)(1 + 2\lambda + \lambda^2/2). \quad (14)$$

Using the Shubik-Levitan specification, we can show that

$$(\bar{p}_{on}^* - 2c)q(\bar{p}_{on}^*) = \frac{2(b - 2c)^2(z - 1)(1 + \lambda)(z(2 + \lambda) - 2(1 + \lambda))}{(z(4 + 3\lambda) - 4(1 + \lambda))^2}.$$

Using this, it is straightforward that (14) is positive for all $f \geq 0$ if:¹⁷

$$\frac{t}{v[2c]} > \frac{8(z - 1)(1 + \lambda)(z(2 + \lambda) - 2(1 + \lambda))(1 + 2\lambda + \lambda^2/2)}{(z(4 + 3\lambda) - 4(1 + \lambda))^2}. \quad (15)$$

¹⁵Recall that standard hotelling models only have one stability condition. In contrast, after the deviation, we arrive at two distinct stability conditions due to the fact that we are in an asymmetric situation, in which one network offers a prepaid and the other offers a postpaid tariff.

¹⁶That is, unless f is very large, in which case (12) is violated such that $n_i^d = 1$ is an equilibrium of the hotelling problem.

¹⁷The details are available upon request.

This implies that if the ratio of transportation costs, t , to the utility derived from a single call priced at marginal cost, $v[2c]$, is larger than the right hand side of (15), the deviation is not profitable. Our claim is that (15) is true for reasonable parameter configurations. For example, if $\lambda \leq 2$, i.e., if the L-types constitute up to two-thirds of all customers, the condition holds for any $z \in [0, 1)$ if $\frac{t}{v[2c]} > 8$.¹⁸

Case (ii), Case(iii), Case(iv)

Case (ii) does not constitute an equilibrium because (10) is violated. That is, our equilibrium only holds for parameter constellations, in which it is impossible to attract the L-types to single-sim without inducing the H-types to take the prepaid tariff. Case (iii) cannot be a profitable deviation since the customer-tariff matching does not change and because the proposed equilibrium tariffs are locally optimal. It is straightforward that Case (iv) cannot be a profitable deviation: Given that the L-types multisim on the prepaid tariff, our proposed equilibrium postpaid tariff is optimal. This is so because a prepaid tariff is just a special case of a postpaid tariff with $r = 0$.

Case(v)

Finally, a tariff that induces (v) is an unprofitable deviation since profits in the L-type segment are reduced and so are the termination revenues. To see this note that the off-net prepaid price of the non-deviator is set so high so that the L-types do not make off-net calls. ■

¹⁸The normalization of the number of H-types to 1 may confuse the interpretation of this condition. We emphasize that the denominator in (15) is to be interpreted as the utility of a *single* call, not all calls made to H-types, hence the claim that (15) is true for reasonable parameter values. We have confirmed this by running the model without the normalization. Results are available upon request.