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## Commercial Policies in the Presence of Input-Output Linkages

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# Commercial Policies in the Presence of Input-Output Linkages

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## Abstract

How do input-output linkages modify countries' incentives to conduct commercial policies? We address this question in a version of the Melitz (2003) model where the production-side of the economy is enriched by input-output linkages. The bundle of intermediate inputs used in production in addition to labor is a composite good governed by the same CES aggregator as the final good. Cooperative policies correct for an *input distortion* generated by the fact that firms' markups translate into the price of the composite good. In the analysis of non-cooperative trade policy, the input distortion stemming from domestic markups counteracts the standard terms-of-trade externality, resulting in a lower optimal tariff and potentially an optimal import subsidy.

*JEL-Classification:* F12, F13, D60, L52

*Keywords:* International trade, monopolistic competition, input-output linkages, optimal commercial policy, efficiency

# 1 Introduction

It is well known that the decentralized equilibrium in the Melitz (2003) monopolistic competition model of international trade with heterogeneous firms is efficient (Dhingra and Morrow, 2014). This claim holds for the closed or perfectly integrated economy. It also holds in the case of costly trade, but only if welfare is jointly maximized. From a unilateral perspective, an import tariff is optimal.<sup>1</sup> In this paper, we analyze *optimal commercial policies* in a version of the Melitz (2003) where the production-side of the economy is enriched by input-output linkages as in Costinot and Rodríguez-Clare (2014) and Caliendo et al. (2015). By input-output linkages we mean that a sector uses its output as an input into production.

Trade in intermediate goods has increased dramatically over the last decades. Imported inputs account for 30% of world exports in 1995 (Hummels et al., 2001). The value-added to gross-value ratio of exports, an inverse measure of vertical specialization has fallen since 1970 (Johnson and Noguera, 2012). Antràs et al. (2012) document that the “upstreamness” of industries has risen.

Costinot and Rodríguez-Clare (2014) show that conditional on observed trade shares and the estimated trade elasticity, implied welfare gains from trade are larger by an order of magnitude if one accounts for input-output linkages compared to a situation where one ignores them. Yi (2003) argues that input-output linkages magnify the effects of trade policy.<sup>2</sup>

We assume that monopolistically competitive intermediate input producers produce differentiated varieties using a Cobb-Douglas technology that combines intermediate inputs and labor. The bundle of intermediate inputs is a composite good governed by the same CES aggregator as the final good, which is assembled by perfectly competitive firms. In order to rule out inter-sectoral reallocation of resources, we focus on a single-sector economy.<sup>3</sup>

Compared to the standard Melitz (2003) mode, intermediate input producers face an ad-

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<sup>1</sup>Demidova and Rodríguez-Clare (2009) prove this for a “small open” economy. Felbermayr et al. (2013) generalize the result to two large countries. Jung (2012) shows for a small open economy, subsidies on operating fixed costs are also welfare improving.

<sup>2</sup>Goldberg and Pavcnik (2016) discuss this point in a recent manuscript prepared for the new *Handbook of Commercial Policy*.

<sup>3</sup>Caliendo et al. (2015) show that trade taxes generate entry effects in the presence of multiple sectors.

ditional problem. They choose labor and material input in a cost-minimizing way. The price of each input into the composite good is subject to a constant markup over marginal cost, which translates into the aggregate price of the aggregate good. Intermediate input producers therefore use too much labor and too little material input into production, which constitutes what we call an input distortion.<sup>4</sup>

We show that in such an economy, in contrast to the standard version model where input-output linkages are not accounted for, the laissez-faire equilibrium is not socially optimal. The optimal cooperative policy that addresses this distortion is a subsidy on domestically and imported intermediate goods that exactly offsets the markup.<sup>5</sup> It is independent of the degree of firm-level heterogeneity in productivities and of the labor cost share in production and also holds with asymmetric countries. When governments only have trade policy at their disposal, the joint-welfare-maximizing policy is also a subsidy, but at a lower rate. We also consider uncooperative trade policy. From a unilateral perspective, domestic goods come at a cost above social (production) cost, as in the case of cooperative policies. Imported goods, however, come at social cost, namely their price at the border. This observation has two implications. First, the input distortion prevails, but its relevance depends on labor cost share and the weight of domestic goods in composite good. An import subsidy may lower the price index, but is not the first-best policy to address the markups of domestic goods. Second, governments will find it optimal to exploit the terms-of-trade externality by means of an import tariff (or domestic subsidy) increasing the relative price of imported goods. Clearly, the input distortion and the terms-of-trade externality interfere. We find that the input distortion dominates, if the labor cost share is sufficiently low.

We are not the first who address (trade) policy implications of input-output linkages. One strand of the literature assumes contractual imperfections. The resulting hold-up problem is

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<sup>4</sup>Note that this distortion is not present with perfectly competitive intermediate input producers as in a version of a Ricardian quantitative trade model à la Caliendo and Parro (2015).

<sup>5</sup>The model is flexible enough to allow for entry effects (which are different from selection effects). It turns out that while the mass of potential entrants depends on the degree of input-output linkages, it is not affected by the consumption subsidy. This finding suggests that the mass of potential entrants is socially optimal also in the version of the Melitz (2003) model that allows for input-output linkages. We have also experimented with other policy instruments such as a subsidy on the use of the composite input into production. It turns out that this policy is not welfare enhancing. In particular, it affects the mass of potential entrants.

mitigated by import subsidies on intermediate goods in the model by Antràs & Staiger (2012) or discriminatory import tariffs, if organizational choice is taken into account (Ornelas and Turner, 2012). Díez (2014) also analyzes the effects of tariffs on offshoring and outsourcing by extending the Antràs and Helpman (2004) framework. We assume complete contracts and do not allow for vertical integration, but focus on the role of monopolistic competition on the market for intermediate inputs.

Blanchard et al. (2016) use a terms-of-trade model of trade policy with political economy motives. They allow for tariffs on final goods, but rule out trade taxes/subsidies for intermediate goods. They find that final good tariffs decrease in the domestic content of foreign-produced final goods. We work with a different framework and consider another polar case where governments cannot distinguish between intermediate and final use.

Caliendo et al. (2015) work with a multi-sector version of the framework, but only consider the welfare consequences of import tariffs. We discuss the underlying distortion and characterize optimal cooperative and un-cooperative policies.

From a more broader perspective, we contribute to the literature on efficiency of market outcomes when one deviates from the standard assumptions of a monopolistic competition model. Dhingra and Morrow (2014) analyze deviations on the demand-side of the economy. They postulate a demand function that generates variable markups and discuss the implications for efficiency. Jung (2015) analyzes allocational efficiency in a Melitz-type model with CES-Benassy preferences. He shows that when love of variety is below CES, variety is too large and productivity is too low, and the first-best policy is a tax on production fixed costs. In this paper, we stick to CES demand, and analyze the implications of modifications on the production-side of the economy for efficiency and optimal policies.

The remainder of the paper is structured as follows. Section 2 introduces commercial policies into a version of the Melitz (2003) model that allows for intermediate inputs and input-output-linkages and derives a generalized welfare formula. Section 3 derives optimal cooperative policies of potentially asymmetric countries. Section 4 characterizes welfare-maximizing un-cooperative trade policy. The final section concludes.

## 2 Model

In this section we generalize Caliendo et al. (2015) by introducing a tax-cum-subsidy on the “consumption” of domestically produced and imported intermediate inputs.<sup>6</sup> The model features monopolistic competition among differently productive firms, as in Melitz (2003), the key novel feature being that production requires intermediate inputs in addition to labor. For the sake of tractability and easier notation, we assume that the bundle of intermediate inputs used in production is a composite good governed by the same CES aggregator as the final good. This captures, albeit in a stylized way, the fact a large class of goods are used in consumption as well as intermediate inputs in production. We assume, plausibly, that when imposing taxes or subsidies governments cannot distinguish between “final use” and “input use” of a good.<sup>7</sup>

There is a world with  $M$  countries, indexed by  $i$  and  $j$ . Within each country, the final good is assembled from intermediate goods originating in all countries, and production of any intermediate uses that country's labor as well as the aforementioned composite of intermediate inputs. We use the term intermediate goods when referring to *assembly* of final goods, and the term intermediate input when referring to *production* of intermediate goods. Labor markets in all countries as well as final goods markets are perfect whereas markets for intermediate goods are characterized by monopolistic competition. The “number” of intermediate goods produced in each country is determined by entry of firms, subject to a fixed entry cost and the usual productivity draw from a distribution function  $G(\varphi)$  - with corresponding density  $g(\varphi)$  - which is assumed Pareto and the same for all countries. We use  $N_j$  to measure the mass of entrants, i.e. potential producers, in country  $j$ . In addition to a fixed cost  $f_{ji} > 0$  of taking up shipments of a good produced in country  $j$  to country  $i$ , there are “iceberg costs” of trade,  $\tau_{ji} \geq 1$ .

Our paper focuses on optimal government intervention in order to correct distortions that

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<sup>6</sup>Caliendo et al. (2015) focus on the role of import tariffs, ruling out domestic consumption subsidies. They present a version of the model with two sectors of the kind presented here. A subsidy on the “consumption” of imported varieties is equivalent to an import subsidy.

<sup>7</sup>Blanchard et al. (2016) take a different stance by assuming that only input trade is subject to trade policy.

are present in this model. We shall look at efficient policies, formed in a cooperative way by all countries, as well as non-cooperative policies. The policy instruments considered will be taxes/subsidies as well as tariffs.

## 2.1 Households

We assume that a representative household in any one country  $i$  consumes a composite good that is assembled from tradable intermediates originating in all of the  $M$  countries. Utility is linear in consumption of this good, denoted by  $C_i$ :

$$U_i(C_i) = C_i. \quad (1)$$

Assembly is governed by a CES aggregator with elasticity of substitution  $\sigma > 1$ . Given that in each country the market for this good is perfectly competitive, the good is sold at a price  $\tilde{P}_i$ , which is equal to the minimum unit cost, given prices of the intermediates, inclusive of iceberg-type trade costs and taxes or subsidies.

Households derive income from two sources. They in-elastically supply one unit of labor, earning a wage equal to  $w_i$ . In addition, they receive a lump-sum redistribution of any revenue generated by government policies, or face a lump-sum tax such as might be required to finance the fiscal cost of such policies. Denoting the lump-sum transfer/tax in country  $i$  by  $T_i$ , aggregate income by households in country  $i$  is

$$I_i = w_i L_i + T_i, \quad (2)$$

where  $L_i$  denotes the mass of households/consumers in country  $i$ . Given perfect labor markets, firms as well as households assume  $w_i$  to be given. Moreover, we assume households to spend all income on consumption of the final good.

## 2.2 Assembly of the aggregate good

We use  $Q_i$  to denote the sum of  $C_i$  plus demand for this same type of aggregate good to be used as an intermediate input in the production of all firms located in country  $i$ . As indicated above, we assume that assembly of this good is governed by a CES production function with elasticity of substitution  $\sigma > 1$ . Thus, we have

$$Q_i = \left[ \sum_{j=1}^M N_j \int_{\varphi_{ji}^*}^{\infty} \tilde{q}_{ji}(\varphi)^{\frac{\sigma-1}{\sigma}} g(\varphi) d\varphi \right]^{\frac{\sigma}{\sigma-1}}. \quad (3)$$

In this expression,  $\tilde{q}_{ji}(\varphi)$  denotes the quantity of an intermediate good originating in country  $j$  and produced by a firm with productivity  $\varphi$ , available for use in assembly of the aggregate good in country  $i$ . Note that we assume a uniform elasticity of substitution for all countries. Assuming costless product differentiation and modeling firm heterogeneity with a continuum of firms, we may use  $\varphi$  to index varieties of intermediates, or firms, whereby  $\varphi_{ji}^*$  denotes the threshold that a firm operating in country  $j$  needs to surpass to profitably sell its product in country  $i$ , given iceberg trade costs and fixed cost of exporting.

Cost-minimizing assembly requires

$$\min_{\{\tilde{q}_{ji}(\varphi)\} \geq 0} \sum_{j=1}^M N_j \int_{\varphi_{ji}^*}^{\infty} \tilde{p}_{ji}(\varphi) \tilde{q}_{ji}(\varphi) g(\varphi) d\varphi \quad \text{s.t. equation (3)}. \quad (4)$$

In this expression,  $\tilde{p}_{ji}(\varphi)$  denotes the price of a good originating in country  $j$ , produced by a firm with productivity  $\varphi$ , and sold country  $i$ , inclusive of iceberg trade costs and wedges introduced by government policies of country  $i$ . Given perfect competition on all national markets for the aggregate good, the value function corresponding to this minimization problem, per unit of  $Q_i$ , is equal to the price of this good which we denote by  $\tilde{P}_i$ :

$$\tilde{P}_i = \left( \sum_{j=1}^M N_j \int_{\varphi_{ji}^*}^{\infty} \tilde{p}_{ji}(\varphi)^{1-\sigma} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}}. \quad (5)$$

Using  $Y_i := \tilde{P}_i Q_i$  to denote the value of aggregate demand in country  $i$ , conditional demand



of country  $i$  for a variety  $\varphi$  from country  $j$  follows as

$$\tilde{q}_{ji}(\varphi) = \left( \frac{\tilde{p}_{ji}(\varphi)}{\tilde{P}_i} \right)^{-\sigma} \frac{Y_i}{\tilde{P}_i}. \quad (6)$$

### 2.3 Production

Producers of goods used for assembly use two types of inputs: labor, denoted by  $l$ , and a bundle of intermediate inputs, denoted by  $m$ .<sup>8</sup> For simplicity, we assume technology to be symmetric across all firms and countries. As indicated above, the bundle  $m$  is composed of intermediates according the exact same CES aggregate that governs final assembly and is given in (3). We thus assume that any producer can source intermediate inputs from abroad without paying additional fixed costs, over and above the variable iceberg cost which also govern imports for the purpose of final goods assembly.<sup>9</sup> Labor and the intermediate input bundle are combined using the following constant returns to scale Cobb-Douglas production function

$$\tilde{q}_j(\varphi) = \varphi l_j(\varphi)^\gamma m_j(\varphi)^{1-\gamma}. \quad (7)$$

In this expression,  $l_j(\varphi)$  denotes labor input and  $m_j(\varphi)$  denotes the quantity of the aggregate bundle of intermediate inputs used by firm  $\varphi$  located in country  $j$ . Note that the production function (7) nests the standard Melitz (2003) case without input-output linkages for  $\gamma = 1$ .

Cost minimization by firm  $\varphi$  requires

$$\min_{(l_i(\varphi), m_i(\varphi)) \geq 0} \left\{ w_j l_j(\varphi) + \tilde{P}_j m_j(\varphi) \right\} \quad \text{s.t. equation (7)}. \quad (8)$$

Conditional input demands emerge as

$$l_j(\varphi) = \gamma \frac{x_j}{w_j} \frac{\tilde{q}_j(\varphi)}{\varphi} \quad \text{and} \quad m_j(\varphi) = (1 - \gamma) \frac{x_j}{\tilde{P}_j} \frac{\tilde{q}_j(\varphi)}{\varphi}, \quad (9)$$

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<sup>8</sup>To avoid confusion, we use the term intermediates when referring to final goods *assembly*, and the term intermediate *inputs* when referring to production.

<sup>9</sup>Imbruno (2014) considers more complex situations where sourcing from abroad also implies fixed costs of importing.

where  $x_j$  denotes the value function corresponding to (8), per unit of  $\frac{\tilde{q}_i(\varphi)}{\varphi}$ . It is straightforward to show that

$$x_j = Aw_j^\gamma \tilde{P}_j^{1-\gamma}, \quad \text{where} \quad A := \gamma^{-\gamma}(1-\gamma)^{\gamma-1}. \quad (10)$$

Marginal cost of firm  $\varphi$  in country  $j$  is equal to  $x_j/\varphi$ . In the absence of input-output linkages,  $\gamma = 1$ , the input cost index  $x_j$  boils down to the wage rate  $w_j$ .

Turning to profit maximizing output levels, we must now specify policy and trade cost wedges. Government policies introduce price wedges, and real trade costs give rise to quantity wedges. Thus, if  $t_{ji}$  is the ad valorem tax (subsidy if negative) on the use (sale) of a good originating in country  $i$  and sold in country  $j$ , and if  $\tau_{ji} \geq 1$  is the iceberg-type real trade cost caused by shipping a good from  $j$  to  $i$ , then we have

$$p_{ji}(\varphi) = \frac{\tilde{p}_{ji}(\varphi)}{1+t_{ji}} \quad \text{and} \quad q_{ji}(\varphi) = \tau_{ji}\tilde{q}_{ji}(\varphi), \quad (11)$$

where  $p_{ji}(\varphi)$  is the net of tax (ex factory) price of a good produced by firm  $\varphi$  located in  $j$  and sold in  $i$ , and  $q_{ji}(\varphi)$  is the quantity that firm  $\varphi$  in country  $j$  has to produce in order to deliver  $\tilde{q}_{ji}(\varphi)$  units of its variety in country  $i$ .

As usual, we assume that firms take the prices of final goods,  $\tilde{P}_i$ , as given for all  $i$ . Moreover, we assume market segmentation, whence profit maximizing prices can be determined independently for all destinations  $j$ . The profit maximizing problem solved by firm  $\varphi$  in country  $i$  therefore is

$$\max_{p_{ji}(\varphi) \geq 0} \left\{ p_{ji}(\varphi) \tilde{q}_{ji}(\varphi) - \frac{x_j}{\varphi} q_{ji}(\varphi) - w_j f_{ji} \right\} \quad (12)$$

Remember that  $f_{ij}$  denotes the fixed cost that a firm located in country  $j$  has to incur in order to serve consumers or producers in country  $i$ . We assume that these costs are the same for goods shipped for the purpose of final goods assembly and for the purposes of intermediate input use, and that they are independent on the firms productivity. Note also that final assembly demand is governed by (6) above. The first order condition for this maximization problem implies the following pricing rule for a firm with productivity  $\varphi$ :

$$p_{ji}(\varphi) = \frac{\sigma}{\sigma-1} \frac{\tau_{ji} x_j}{\varphi}, \quad (13)$$

where  $\frac{\sigma}{\sigma-1}$  is the mark-up of prices over marginal cost, inclusive of iceberg trade cost  $\tau_{ji}$ . Recall that consumers and producers in country  $i$  face a price  $\tilde{p}_{ji}(\varphi) = (1 + t_{ji}) p_{ji}(\varphi)$ , which includes a tax (subsidy) at rate  $t_{ji}$  if  $t_{ij} > 0$  (if  $t_{ij} < 0$ ). Inserting (13) and (11) into conditional demand by country  $i$  in (6), we obtain

$$\tilde{q}_{ji}(\varphi) = \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ji} x_j}{\varphi} \right)^{-\sigma} \frac{Y_i \tilde{P}_i^{\sigma-1}}{(1 + t_{ji})^\sigma}. \quad (14)$$

Note that iceberg trade costs and the consumption tax-cum-subsidy are completely symmetric in their effect on demand. However, production  $q_{ji}(\varphi) = \tau_{ji} \tilde{q}_{ji}(\varphi)$ , differs as iceberg trade costs require the use of resources, whereas commercial policy does not.

Net of tax revenue of a firm with productivity  $\varphi$  located in  $j$  from selling to  $i$  is

$$r_{ji}(\varphi) = p_{ji}(\varphi) \tilde{q}_{ji}(\varphi) = \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ji} x_j}{\varphi} \right)^{1-\sigma} \frac{Y_i \tilde{P}_i^{\sigma-1}}{(1 + t_{ji})^\sigma}. \quad (15)$$

Maximum profits earned by firm  $\varphi$  located in country  $j$  from selling in country  $i$ , net of fixed costs of market access, are

$$\pi_{ji}(\varphi) = \frac{1}{\sigma-1} \left( \frac{\tau_{ji} x_j \tilde{q}_{ji}(\varphi)}{\varphi} - (\sigma-1) w_j f_{ji} \right). \quad (16)$$

## 2.4 Selection and entry

Firms first decide about entry, based on a fixed entry cost  $w_j f_j^e$  and *expected* productivity, and once they have entered they select themselves into different markets, based on fixed costs of market access  $w_j f_{ji}$  and *observed* productivity. In the spirit of backward induction, we first look at the selection effect, which determines the equilibrium threshold levels of productivity  $\varphi_{ji}^*$ , and then turn to entry.

The presence of a fixed cost of access to national markets implies threshold levels of productivity that firms in any one country need to surpass in order to take up selling in domestic as well as foreign markets. Having learned about its productivity subsequent to entry, a firm will sell to a given market  $i$  only if it earns a positive profit from doing so. The threshold productivity level for a firm in country  $j$  to select itself into selling to market  $i$  is denoted by  $\varphi_{ji}^*$

and determined by the condition  $\pi_{ji}(\varphi_{ji}^*) = 0$ . Solving this condition for  $\varphi_{ji}^*$  yields

$$\varphi_{ji}^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} (1+t_{ji}) x_j w_j^{\frac{1}{\sigma-1}} \tilde{P}_i^{-1} \left( \frac{Y_i}{1+t_{ji}} \right)^{-\frac{1}{\sigma-1}}. \quad (17)$$

This equation highlights a selection effect of trade costs and tax policy. A number of observations are worth pointing out. First, other things equal, a rise in real trade costs  $\tau_{ji}$  increases the threshold level of productivity that separates country  $j$  firms exporting to country  $i$  from those that do not, on the same footing as does a rise in marginal cost  $x_j$ ; selection into exporting becomes tougher. Secondly, an equiproportional increase in the fixed market access cost  $f_{ji}$  and the variable real trade cost  $\tau_{ji}$  affects the threshold  $\varphi_{ji}^*$  in the same way as does an equiproportional increase in the tax wedge  $1+t_{ji}$ . And finally, the second line decomposes the effect of the tax policy into a direct substitution effect, which works on the same footing as with real trade costs, and an effective market size effect that works through deflating total expenditure by country  $i$  by  $(1+t_{ji})^{-1}$ .<sup>10</sup> Clearly, being more expensive on account of  $t_{ji}$ ,  $\tau_{ji}$  or  $x_j$  makes it more difficult for firms from  $j$  to enter  $i$ , which in turn raises country  $i$ 's price index. Moreover, other things equal, covering the fixed entry cost  $f_{ji}$  is more difficult if the market is smaller (in terms of lower expenditure  $Y_i$ ). We will show below that the mass of potential entrants depends on the strength of input-output linkages, but is invariant to changes in iceberg trade costs and commercial policy.

It must be emphasized, however, that these are partial equilibrium effects since they ignore general equilibrium repercussions captured by the endogenous variables  $w_j$ ,  $\tilde{P}_i$  and  $Y_i$  on the right-hand side of (17). The presence of intermediate inputs,  $\gamma < 1$ , affects these repercussions. For instance, it effectively waters down the effect that an increase in country  $j$ 's wage rate, say in a scenario of falling real trade costs, has on the threshold level of exporting. The reduction in destination country  $i$ 's price index that occurs in the general equilibrium adjustment of such a scenario similarly has a watered down effect on the export threshold level

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<sup>10</sup>Caliendo et al. (2015) show that the mass of entrants is affected by changes in an import tariff, if tariff revenue is not fully redistributed to consumers. Equation (17) assumes full redistribution of revenues or, if relevant, lump-sum financing of the subsidy bill. Moreover, Costinot and Rodríguez-Clare (2014) and Caliendo et al. (2015) show that there are entry effects in the presence of multiple sectors. In order to focus on the implications of the relative cost distortion, we abstract from multiple sectors.

$\varphi_{ji}^*$ . Since the two feedback effects work in opposite directions, the net effect of them being watered down by the presence of intermediate inputs is ambiguous, as regards the general equilibrium adjustment of  $\varphi_{ji}^*$ .

Free entry implies that expected profits from selling to all markets is equal to the entry cost:

$$\sum_{i=1}^M \int_{\varphi_{ji}^*} \pi(\varphi) g(\varphi) d\varphi = w_j f_j^e. \quad (18)$$

This is an equilibrium condition stating that a potential entrant expects zero profits, given the distribution of the productivity level as captured by the density  $g(\varphi)$ , with an associated distribution function  $G(\varphi)$ . Plausibly, and for the sake of a closed form solution for the integral, we assume a Pareto distribution for  $\varphi$ , with a shape parameter denoted by  $\theta > 0$ . To guarantee a finite average productivity in equilibrium, we further assume  $\theta > \sigma - 1$ . We show in the appendix that with this additional assumption the zero profit equilibrium condition may be written as

$$\sum_{i=1}^M f_{ji} (\varphi_{ji}^*)^{-\theta} = \frac{\theta - (\sigma - 1)}{\sigma - 1} f_j^e \quad (19)$$

Although not directly evident from (19), the zero profit equilibrium condition is of key importance for the determination of  $N_j$ , the equilibrium mass of firms entering in order to take up production in country  $j$ . We assume that firms live for one period only, whence  $N_j$  also denotes the number of potential producers in country  $j$ . However, all firms with  $\varphi < \min_i \{\varphi_{ji}^*\}$  never start producing, although the entire mass of firms incur the entry cost  $w_j f_{ji}$ . The mass of firms located in country  $j$  and serving country  $i$  is given by

$$N_{ji} = N_j [1 - G(\varphi_{ji}^*)] = N_j (\varphi_{ji}^*)^{-\theta}, \quad (20)$$

where the second equality follows from the Pareto distribution with a shape parameter  $\theta$ .

It is instructive to see that entry-country's wage rate  $w_j$  drops from the zero profit condition (19) since it appears both, in the equations describing the selection into markets, as appearing in (17), and on the right-hand side of (18). The underlying assumption here is that fixed costs of foreign market entry in country  $i$  draw on resources from the sending country  $j$ . Moreover, the free entry equilibrium condition (19) implies that a policy that affects one

productivity threshold  $\varphi_{ij}^*$  has repercussions on at least one other threshold  $\varphi_{ik \neq j}^*$  in order to restore zero profits in expectations.

## 2.5 Goods market equilibrium

The remaining equilibrium conditions relate to goods and labor markets. Given goods market equilibrium for each variety produced by country labor market will be in equilibrium due to Walras' Law which implies balanced trade. We thus close our model by a balanced trade condition as well as a goods market equilibrium condition for each of our  $M$  countries. It proves convenient to first pin down the price of the final good which is equal to the unit cost of final goods assembly given in (5). Using the markup pricing condition for firm  $\varphi$  of country  $j$  when selling to country  $i$  as given in (11), it can be shown (see the appendix) that

$$\tilde{P}_i = \left( \sum_{j=1}^M N_j \chi_{ji} \xi_{ji} \right)^{\frac{1}{1-\sigma}}, \quad (21)$$

where  $\chi_{ji} := \left( \frac{\sigma}{\sigma-1} (1 + t_{ji}) \tau_{ji} x_j \right)^{1-\sigma}$  is an inverse measure of the freeness of exports from country  $j$  to country  $i$ . It represents the *intensive margin* component in the minimum cost of assembly in country  $i$  that would be present also without firm-heterogeneity. In turn,  $\xi_{ji} := \int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi$  represents the *extensive margin* component that derives from *selection* of firms into different markets. Assuming Pareto for  $g(\varphi)$  implies  $\xi_{ji} = \frac{\theta}{\theta - (\sigma-1)} \left( \varphi_{ji}^* \right)^{\sigma - \theta - 1}$ ; see (48) in the appendix. As regards the selection effect, we have emphasized above that it involves two channels, the freeness of trade channel and the market size channel; see (17) and the subsequent discussion above. And finally,  $N_j$  in (21) represents the extensive margin component that derives from firm *entry*.

It can be shown (see again the appendix) that the value of country  $i$  expenditure falling on goods from country  $j$ , evaluated at country  $i$ 's domestic prices, is equal to  $N_j \chi_{ji} \xi_{ji} \times Y_i \tilde{P}_i^{\sigma-1}$ . Denoting the share of country  $j$  goods in country  $i$ 's expenditure by  $\lambda_{ji}$ , we have

$$\lambda_{ji} = \frac{N_j \chi_{ji} \xi_{ji}}{\tilde{P}_i^{1-\sigma}} \quad (22)$$

The close relationship between the price index  $\tilde{P}_i$  and the expenditure shares  $\lambda_{ji}$  is helpful in that it will eventually allow us to express a country's aggregate welfare as a function of the share of expenditure on domestic goods.<sup>11</sup>

Demand for the good produced by firm  $\varphi$  located in country  $j$  is equal to  $\sum_{i=1}^M q_{ji}(\varphi)$ , where  $q_{ji}(\varphi) = \tau_{ji}\tilde{q}_{ji}(\varphi)$  is given from (14) above. Equilibrium requires that  $\sum_{i=1}^M q_{ji}(\varphi) = q_j(\varphi)$ , where  $q_j(\varphi)$  denotes gross output of this firm, inclusive of (iceberg) real trade costs. This condition may be written as

$$q_j(\varphi) = [C_{jj}(\varphi) + C_{j\cdot}(\varphi) + M_{jj}(\varphi) + M_{j\cdot}(\varphi)] \quad (23)$$

where  $C_{jj}(\varphi)$  is final consumption demand originating from country  $j$ 's own domestic households, while  $C_{j\cdot}(\varphi)$  is consumption demand originating from foreign countries' final consumption. Similarly,  $M_{jj}(\varphi)$  denotes demand for intermediate input use by domestic firms, and  $M_{j\cdot}(\varphi)$  is demand for intermediate input use in other countries. Consumption demands are governed by the demand function as given in (14), with  $Y_i$  being replaced by  $w_i L_i + T_i$ . In turn, intermediate input demands are governed by this same demand function (14), with  $Y_i$  being replaced by the value foreign firms' demand for the bundle of intermediate inputs, in line with the demand function for  $m_i(\varphi)$  as given in (9) above:

$$C_{jj}(\varphi) = \left( \frac{\sigma}{\sigma-1} \frac{x_j}{\varphi} \right)^{-\sigma} \frac{\tilde{P}_j^\sigma}{(1+t_{jj})^\sigma} \times \frac{(w_j L_j + T_j)}{\tilde{P}_j} \quad (24)$$

$$C_{j\cdot}(\varphi) = \sum_{i \neq j}^M \tau_{ji} \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ji} x_j}{\varphi} \right)^{-\sigma} \frac{\tilde{P}_i^\sigma}{(1+t_{ji})^\sigma} \times \frac{(w_i L_i + T_i)}{\tilde{P}_i} \quad (25)$$

$$M_{jj}(\varphi) = \left( \frac{\sigma}{\sigma-1} \frac{x_j}{\varphi} \right)^{-\sigma} \frac{\tilde{P}_j^\sigma}{(1+t_{jj})^\sigma} \times \frac{N_j}{\tilde{P}_j} \int_{\varphi_{jj}^*} m_j(\varphi_j) \tilde{P}_j g(\varphi) d\varphi \quad (26)$$

$$M_{j\cdot}(\varphi) = \sum_{i \neq j}^M \tau_{ji} \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ji} x_j}{\varphi} \right)^{-\sigma} \frac{\tilde{P}_i^\sigma}{(1+t_{ji})^\sigma} \times \frac{N_i}{\tilde{P}_i} \int_{\varphi_{ii}^*} m_i(\varphi_i) \tilde{P}_i g(\varphi) d\varphi \quad (27)$$

Note that demand is “factory gate demand” which is gross of the “iceberg cost” that will melt down on the way to a good's final delivery. Moreover, note that in each case demand has two

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<sup>11</sup>Indeed, implicitly defining a price index for country  $i$ 's imports from country  $j$  through  $\tilde{P}_{ij}^{1-\sigma} := N_j \chi_{ji} \xi_{ji}$ , it can be shown that  $\lambda_{ij} = \left( \tilde{P}_{ij} / \tilde{P}_j \right)^{1-\sigma}$ .

factors, the first term capturing allocation of some category of aggregate demand to firm  $\varphi$  in country  $j$ , and the second term specifying the type of aggregate demand considered. Notice that these equations look at quantities, not values. Aggregate intermediate input demand in any one country is demand by all firms in existence, whence we must aggregate over all firms, noting that the least productive firm in any country has productivity  $\varphi_{ii}^*$ . Aggregation is possible in quantity terms, since aggregate demand by different firms is demand for the same type of aggregate good.

Aggregating (23) over domestic firms is possible only in value terms, since firms are producing differentiated goods. Consistently with the above, we evaluate total production by the firm's producer, or "factory gate" price, which is  $p_{jj}$ :

$$N_j \int_{\varphi_{jj}^*} p_{jj}(\varphi) q_j(\varphi) g(\varphi) d\varphi = N_j \int_{\varphi_{jj}^*} p_{jj} [C_{jj}(\varphi) + C_{j\cdot}(\varphi) + M_{jj}(\varphi) + M_{j\cdot}(\varphi)] g(\varphi) d\varphi. \quad (28)$$

In the following, we use  $Z_j := N_j \int_{\varphi_{jj}^*} p_{jj}(\varphi) q_j(\varphi) g(\varphi) d\varphi$  to denote aggregate output of country  $j$ , or total revenue of all firms located in  $j$ , evaluated at country  $j$ 's producer prices  $p_{jj}(\varphi)$ . Of course, the value of output  $Z_j$  is linked to total expenditure  $Y_j$ . Consumption expenditure derives from household income, which includes labor income plus government revenue  $T_j$ , assumed to be redistributed in lump-sum fashion; see (2). Note that  $T_j$  can be negative (subsidy bill).

Cobb-Douglas technology implies that intermediate inputs command a share  $1 - \gamma$  of variable costs. In equilibrium, variable costs are a fraction  $\frac{\sigma-1}{\sigma}$  of total revenue; see (13). The goods market equilibrium condition (28) may therefore be written as

$$\begin{aligned} Z_j &= \frac{\lambda_{jj}}{1+t_{jj}} \left[ (w_j L_j + T_j) + (1-\gamma) \frac{\sigma-1}{\sigma} Z_j \right] \\ &+ \sum_{i \neq j} \frac{\lambda_{ji}}{1+t_{ji}} (w_i L_i + T_i) \\ &+ \sum_{i \neq j} \frac{\lambda_{ji}}{1+t_{ji}} (1-\gamma) \frac{\sigma-1}{\sigma} Z_i \end{aligned} \quad (29)$$

The right-hand side of this equation can be simplified by using  $w_j L_j + T_j + (1-\gamma) \frac{\sigma-1}{\sigma} Z_j = Y_j$ ,



which leads to

$$Z_j = \frac{\lambda_{jj}}{1 + t_{jj}} Y_j + \sum_{i \neq j} \frac{\lambda_{ji}}{1 + t_{ji}} Y_i \quad (30)$$

This states that gross domestic output is equal to domestic demand for this output plus exports. Balanced trade requires equal values for exports and imports at border prices, i.e.,

$$\sum_{i \neq j} \frac{\lambda_{ji}}{1 + t_{ji}} Y_i = \sum_{i \neq j} \frac{\lambda_{ij}}{1 + t_{ij}} Y_j \quad (31)$$

Observing balanced trade, the goods market equilibrium may therefore be written as

$$Z_j = Y_j \sum_i \frac{\lambda_{ij}}{1 + t_{ij}} \quad (32)$$

Next, we explore the link between the value of production,  $Z_j$ , and value added. Cobb-Douglas technology implies that intermediate inputs take up a share  $1 - \gamma$  of variable cost. In turn, variable cost are a fraction  $\frac{\sigma-1}{\sigma}$  of  $Z_j$ . The zero profit condition plus labor market equilibrium then lead to

$$\left[ 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right] Z_j = w_j L_j \quad (33)$$

This states that in value terms net output (i.e., net of domestic intermediate input use) must be equal to domestic value added, which is equal to households' net of tax labor income. Combining this with the above goods market equilibrium condition (32) we obtain

$$w_j L_j + (1 - \gamma) \frac{\sigma - 1}{\sigma} Y_j \sum_i \frac{\lambda_{ij}}{1 + t_{ij}} = \sum_i \frac{\lambda_{ij}}{1 + t_{ij}} Y_j, \quad (34)$$

which may be rewritten as

$$Y_j = \tilde{\mu}_j \times w_j L_j, \quad \text{where } \tilde{\mu}_j := \left( \left[ 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right] \sum_i \frac{\lambda_{ij}}{1 + t_{ij}} \right)^{-1} \quad (35)$$

We call the term  $\tilde{\mu}_j$  the “gross output multiplier”. It links value added  $w_j L_j$  to the gross of tax value of output, and – broadly speaking – it incorporates the input output linkage as well as

government tax policy. Looking at tax revenue in somewhat more detail, we have

$$I_j = w_j L_j + T_j = w_j L_j + Y_j \sum_i \frac{t_{ij} \lambda_{ij}}{1 + t_{ij}}$$

In the appendix we show that inserting (35) yields

$$I_j = \mu_j \times w_j L_j \quad \text{where } \mu_j := \tilde{\mu}_j \sum_i \left( 1 + t_{ij} - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) \frac{\lambda_{ij}}{1 + t_{ij}} \quad (36)$$

We shall henceforth refer to  $\mu$  as the “income multiplier”; it links value added to income, incorporating lump-sum redistribution (financing) of tax revenue (subsidy bill). Notice that for  $\gamma = 1$  we have  $\mu_j = \tilde{\mu}_j = \left( \sum_i \frac{\lambda_{ij}}{1 + t_{ij}} \right)^{-1}$ , which is intuitive.

## 2.6 Firm entry

We now use the equilibrium conditions derived above to solve for the mass of entrants. Employing the definition of the various margins and the zero cutoff profit condition (17), we can write aggregate sales of firms from country  $i$  in country  $j$  written as

$$\frac{\lambda_{ij}}{1 + t_{ij}} Y_j = \frac{\theta \sigma}{\theta - (\sigma - 1)} N_i (\varphi_{ij}^*)^{-\theta} w_i f_{ij}. \quad (37)$$

This relationship tells that aggregate sales can be written as the product of the sales of the average firm and the mass of entrants; compare (37) to the free entry condition (19).

Using the free entry condition to substitute out expected profits (sales), we obtain

$$N_i = \frac{\sigma - 1}{\theta \sigma w_i f_i^e} \sum_j \frac{\lambda_{ij}}{1 + t_{ij}} Y_j = \frac{\sigma - 1}{\theta \sigma f_i^e} \frac{Z_i}{w_i},$$

where the equality follows from (30). As in the standard case, the equation commands that the mass of entrants is proportional to the ratio of the value of production  $Z_i$  and the wage rate  $w_i$ . While in the standard model free entry implies that the value of production is used to pay workers, in our context firms also have to pay for material input, which drives a wedge between value of output and value added. Using the zero profit condition (33) to substitute

out  $Z_i$ , the equilibrium mass of entrants is given by

$$N_i = \frac{\sigma - 1}{\theta \sigma f_i^\varepsilon} \frac{L_i}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}}. \quad (38)$$

In the presence of input-output linkages ( $\gamma < 1$ ), the mass of entrants is larger than in their absence ( $\gamma = 1$ ). The intuition is that for a given wage rate, the value of production must be larger with input-output linkages than without.

Importantly, changes in commercial policy do not trigger entry effects. Hence, a constant fraction of workers is devoted to entry activity. Key drivers of this invariance result are that net government revenue is fully rebated to workers and that we consider a single-sector model. Our result is in line with Caliendo et al. (2015). Their findings suggest that the invariance result would not obtain, if there was no full rebate of net government tax revenue. It will turn out below that the optimal policy is a subsidy, which has to be financed by a lump-sum tax on labor income. We therefore ignore cases without full rebate (or financing).

## 2.7 Welfare

Having characterized industry equilibrium, we now turn to the derivation of a welfare formula. Real income of the representative agent in country  $i$  is given by  $W_i = \mu_i \times w_i / \tilde{P}_i$ , where  $\mu_i$  is the income multiplier from equation (36). In order to make the welfare formula comparable to that popularized by Arkolakis et al. (2012), we use the expression for the domestic expenditure share to substitute out the real wage.

Employing the definitions of the various margins, the domestic expenditure share can be written as

$$\lambda_{ii} = \frac{\theta (\sigma f_{ii})^{\frac{\sigma-1-\theta}{\sigma-1}}}{\theta - (\sigma - 1)} \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta} N_i (\tilde{\mu}_i L_i)^{\frac{\theta - (\sigma-1)}{\sigma-1}} \left( \frac{x_i}{\tilde{P}_i} \right)^{-\theta} (1 + t_{ii})^{1 - \frac{\sigma\theta}{\sigma-1}},$$

where we have used the zero cutoff profit condition to substitute out  $\varphi_{ii}^*$  and the definition of the gross output multiplier.

Using the definition of the input cost index, we can use the above expression to solve for

the real wage as

$$\frac{w_i}{\bar{P}_i} = \zeta_i \left( N_i \times \tilde{\mu}_i^{\frac{\theta-(\sigma-1)}{\sigma-1}} \times (1+t_{ii})^{1-\frac{\sigma\theta}{\sigma-1}} \times \lambda_{ii}^{-1} \right)^{\frac{1}{\gamma\theta}}, \quad \text{where} \quad (39)$$

$$\zeta_i := \left( \frac{\theta (L_i / (\sigma f_{ii}))^{\frac{\theta-(\sigma-1)}{\sigma-1}}}{\theta - (\sigma - 1)} \left( \frac{\sigma - 1}{\sigma} \right)^\theta A^{-\theta} \right)^{\frac{1}{\gamma\theta}} \quad (40)$$

**Welfare formula.** Combining the previous expressions, we can express welfare as

$$W_i = \zeta_i \times \mu_i \times \left( \tilde{\mu}_i^{1-\frac{\theta}{\sigma-1}} \times (1+t_{ii})^{\frac{\sigma\theta}{\sigma-1}-1} \times \lambda_{ii} \right)^{-\frac{1}{\gamma\theta}}. \quad (41)$$

This welfare formula generalizes the welfare formula in Arkolakis et al. (2012) straightforwardly. Note the effect of policy on the domestic expenditure share is translated into a welfare effect with elasticity  $1/(\gamma\theta)$ . For an *ex post* evaluation of welfare consequences of foreign policies and domestic trade policy, one additionally has to observe changes in the income multiplier and the gross output multiplier. Moreover, one needs information about an additional parameter, namely the elasticity of trade with respect to fixed costs  $\frac{\theta}{\sigma-1} - 1$ , which governs the welfare consequences of a change in the gross output multiplier. For the *ex post* evaluation of domestic commercial policy, one also has to back out the trade elasticity with respect to commercial policy,  $\frac{\sigma\theta}{\sigma-1} - 1$ .

**Domestic expenditure share.** In order to characterize optimal policies, we have to rely on the *ex ante* evaluation of welfare consequences of commercial policy, which involves the effect of commercial policy on welfare through changes in the domestic expenditure share. In order to pave the ground for this type of analysis, we derive an expression for the domestic expenditure share. Starting from the expression for the price index, we show in the appendix

that the domestic expenditure share can be written as

$$\lambda_{ii} = \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{x_j}{x_i} \right)^{-\theta} \right)^{-1}. \quad (42)$$

This expression highlights that with uniform policies  $t_{ii} = t_{ji}$  for all source countries  $j$ , commercial policy only has an indirect effect on the domestic expenditure share through the relative wage and the relative input cost index.

Making further progress, we employ the definition of the input cost share and use equation (39) to substitute the real wage:

$$\lambda_{ii} = \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{1-\frac{\theta\sigma}{\sigma-1}} \Gamma_{ij} \right)^{-1}, \quad \text{where} \quad (43)$$

$$\Gamma_{ij} := \left( \left( \frac{\xi_j}{\xi_i} \right)^{\theta\gamma} \frac{N_j}{N_i} \left( \frac{\tilde{\mu}_j}{\tilde{\mu}_i} \right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} \left( \frac{1+t_{jj}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{\lambda_{jj}}{\lambda_{ii}} \right)^{-1} \right)^{\frac{1-\gamma}{\gamma}}. \quad (44)$$

This expression highlights the implications of input-output linkages for the domestic expenditure share. In the absence of input-output linkages ( $\gamma = 1$ ),  $\Gamma_{ij} = 1$ , and the domestic expenditure share is simply a function of the relative mass of entrants, relative trade costs (real and policy wedges) and relative wages. With input-output linkages, it additionally depends on the relative gross output multiplier, relative domestic policy wedges, and relative domestic expenditure shares.

### 3 Cooperative commercial policies

In this section, we consider the case where potentially asymmetric countries *cooperatively* determine their commercial policies in order to maximize their joint welfare. This perspective prevents countries from conducting beggar-thy-neighbor policies. Hence, the scenario analyzed in this section is free of terms-of-trade considerations, which allows us to discuss the optimal policy implications of the input distortion inherent to a monopolistic competition model of international trade with input-output linkages. We explore the interaction of the

input distortion with the terms-of-trade externality in the next section.

In this first subsection of this section, we shall assume that domestically produced and imported inputs are treated uniformly by policy makers, restricting the policy choices to one per country. This greatly simplifies the analysis for the following reasons. Firstly, not discriminating against imported intermediate goods allows to determine income and gross output multipliers independently of the domestic expenditure share. Secondly, the domestic expenditure share, which is itself an determinant of welfare (see equation (41)), is not directly driven by commercial policy. With symmetric countries, the optimal policy therefore trades off changes in the income and gross output multipliers and the direct effect of commercial policy, which arises when conditioning on the domestic expenditure share. We shall consider the case of symmetric countries first. With asymmetric countries, however, commercial policy indirectly affects the domestic expenditure share through the general equilibrium adjustments of wages and input cost indices. We, in turn, consider the case of country asymmetry.

In the second subsection of this section, we turn to the case where governments only have one instrument at their disposal, referring to situations where either trade policy interventions are restricted, e.g., by international trade agreements, or domestic policy interventions are unwelcome for some reasons. In order to pave the ground for the analysis of optimal uncooperative trade policy, which we take up in the next section, we focus on the latter.<sup>12</sup>

### 3.1 Uniform treatment of domestically produced and imported intermediate goods

With uniform treatment of domestically and imported intermediate goods, i.e.,  $t_{ji} = t_{ii}$  for all source countries  $j$ , income and gross output multipliers are given by

$$\mu_i \equiv \kappa \left( 1 + t_i - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) \text{ and } \tilde{\mu}_i = \kappa (1 + t_i), \quad (45)$$

where  $\kappa$  is a constant defined by  $\kappa := 1 / [1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}]$ . Notice that the multipliers are determined independently of expenditure shares. The reason is that with uniform treatment of

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<sup>12</sup>It will become clear below that the policy addressing domestic rather than imported goods has almost symmetric effects, the difference being that in the welfare calculus domestic expenditure shares have to be replaced by import expenditure shares, and *vice versa*.

domestically produced and imported goods, we can factor out policy wedges in the summation over policy-adjusted expenditure shares and exploit that expenditure shares add up to unity.

Expression (45) implies that the direct effect of commercial policy on welfare and its effect through the gross output multiplier can be combined by a new variable  $\Omega_i(t_i)$  defined as

$$\Omega_i(t_i) := \left[ \tilde{\mu}_i(t_i)^{1-\frac{\theta}{\sigma-1}} \times (1+t_i)^{\frac{\sigma\theta}{\sigma-1}-1} \right]^{-\frac{1}{\gamma\theta}} \propto (1+t_i)^{-\frac{1}{\gamma}}. \quad (46)$$

In a closed-economy version of the model ( $\lambda = 1$ ),  $\Omega_i(t_i)$  represents the effect of commercial policy on welfare through the real wage  $w/\tilde{P}$ . We will see below that this intuition also carries over to the case of potentially asymmetric open economies, where domestic expenditure shares turn out to be determined independently of the stance of commercial policies.

**Symmetric countries.** With symmetric countries, commercial policy has no bearing on the domestic expenditure share; see equation (42). The joint welfare maximization problem can be stated as

$$\max_t \tilde{W} = \mu(t) \times \Omega(t);$$

see equations (41), and (46).

The optimal policy intervention  $t^*$  is obtained by setting  $\partial\tilde{W}/\partial t = 0$ . Notice that in the absence of input-output linkages ( $\gamma = 1$ ), the objective function  $\tilde{W}$  is invariant to changes in commercial policy; compare equations (45) and (46). Hence, the effect of commercial policy via the term  $\check{\mu}(t)$  is always exactly offset by changes in the income multiplier.

In the presence of input-output linkages, the first-order condition of the welfare-maximization problem implies<sup>13</sup>

$$1 + t^* = \frac{\sigma - 1}{\sigma}. \quad (47)$$

This result highlights that the effect of a subsidy on welfare through changes in the price index outweighs its effect through the income multiplier, if the subsidy is not too large. The

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<sup>13</sup>Formally, this result follows from noting that  $d\mu/dt = 1 / [(1+t_i - (1-\gamma)\frac{\sigma-1}{\sigma})]$ .

optimal policy that maximizes joint welfare of symmetric countries is a subsidy of rate  $|t^*| = 1/\sigma$ . In contrast to the standard Melitz (2003) case without input-output linkages, laissez-faire market outcomes are not socially optimal.<sup>14</sup>

The optimal policy exactly offsets the markup over marginal cost each intermediate good producer, which directly feeds into the price index (5) via the term  $\chi$  representing the *intensive margin*. The “correction” of the price of a good clearly has repercussions on demand for that good and on firms’ revenue and profits. While firms’ profits are in general decisive in the determination of cutoff productivity levels, we show in the appendix that with symmetric countries and uniform treatment of domestically produced and imported goods, commercial policy has no effect on *selection*. Recall from section 2 that commercial policy has no bearing on the extensive margin component that works through firm entry; see equation (38). As in our setting all firms charge the same markup, firm heterogeneity has no optimal policy implications. Moreover, the labor cost share  $\gamma$  does not interfere with the markup. Therefore, the optimal policy is also independent of the labor cost share  $\gamma$ .

The optimal subsidy is independent of the labor cost share  $\gamma$  and the degree of firm heterogeneity measured by  $\theta$ . The intuition for these observations is that variations in a labor cost share  $\gamma < 1$  and in firm heterogeneity have no implications for the markups charged by the monopolistically competitive intermediate good producers.

Optimal commercial policy raises the overall efficiency of the economy by correcting an input distortion. In the presence of markups, the price of the composite good is too high such that intermediate good producers use too little material input into production; see the conditional input demands (9). By using more of the composite good for production of inputs, each intermediate good producers produces more efficiently, which translates into an aggregate productivity gain. Hence, in our setting efficiency gains do not arise from reallocation of resources across firms, but from a more efficient allocation of the composite good to final consumption and intermediate input use.

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<sup>14</sup>Dhingra and Morrow (2012) prove that market outcomes in the standard Melitz (2003) model with CES preferences are efficient, while in the presence of preferences that lead to variable markups, the efficiency result breaks. In a similar vein, Jung (2015) proves that the efficiency result does not carry over to the case of CES-Benassy preferences. In this paper, we explore the sensitivity of the result to modifications on the production side of the economy.



In order to illustrate this point, consider the case of a closed economy with a fixed mass of  $N$  *symmetric* firms.<sup>15</sup> Let  $q$  denote output per firm. Aggregate output is given by  $Q = \left(N \times q^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = N^{\frac{\sigma}{\sigma-1}} \times q$ .

The aggregate good can be used for consumption ( $C$ ) or used as input into production  $N \times m$ , where  $m$  denotes demand for inputs of each firm. Production is given by  $q = l^\gamma m^{1-\gamma}$ , where the productivity is normalized to unity, and labor input per firm is a constant fraction of labor total endowment.

The *planner* chooses  $m$  in order to maximize real consumption  $C$  subject to the technologies  $Q$  and  $q$ :

$$\max_m C = Q - Nm = N^{\frac{\sigma}{\sigma-1}} l^\gamma m^{1-\gamma} - Nm.$$

The first-order condition of this maximization problem implies

$$\frac{l}{m} = (1 - \gamma)^{-\frac{1}{\gamma}} N^{-\frac{1}{\gamma(\sigma-1)}}.$$

In the decentralized equilibrium, relative input demand is  $l/m = \gamma \tilde{P} / [(1 - \gamma)w]$ , where the price index is given by  $\tilde{P} = \left(N^{-\frac{1}{\sigma-1}} \times \frac{\sigma}{\sigma-1} \times A \times (1+t)\right)^{\frac{1}{\gamma}}$ .<sup>16</sup> Hence, relative input demand is

$$\frac{l}{m} = (1 - \gamma)^{-\frac{1}{\gamma}} \times N^{-\frac{1}{\gamma(\sigma-1)}} \times \left(\frac{\sigma}{\sigma-1} \times (1+t)\right)^{\frac{1}{\gamma}}.$$

This expression highlights that with the optimal subsidy, the policy-driven decentralized equilibrium exactly resembles the planner outcome.

**Asymmetric countries.** With asymmetric countries, we have to back out the effect commercial policy has on domestic expenditure shares and relative wages. With uniform treatment, commercial policies exhibit no direct effect on domestic expenditure shares, but potentially affect them through general equilibrium adjustments in the relative wage and the  $\Gamma_{ij}$  terms; see equations (43) and (44). With uniform treatment, the  $\Gamma_{ij}$  terms collapse to functions of

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<sup>15</sup>As we have argued above, firm heterogeneity is not important in this section as all firms charge the same markup.

<sup>16</sup>Formally, this follows from considering the limiting case  $\theta \rightarrow \sigma - 1$  in equation (39).

relative domestic expenditure shares. Relative wages are determined by the balanced trade conditions

$$\sum_{i \neq j} \frac{\lambda_{ji}}{1+t_i} \tilde{\mu}_i w_i L_i = \frac{\tilde{\mu}_j w_j L_j}{1+t_j} \sum_{i \neq j} \lambda_{ij} = \frac{\tilde{\mu}_j w_j L_j}{1+t_j} (1 - \lambda_{jj}),$$

where we have used  $Y_i = \tilde{\mu}_i w_i L_i$  to replace gross outputs and  $\sum_i \lambda_{ij} = 1$ . Uniform treatment implies that balanced trade can be rewritten as

$$\sum_{i \neq j} \lambda_{ji} w_i L_i = w_j L_j (1 - \lambda_{jj}),$$

where we have used equation (45). This expression highlights that domestic expenditure shares and relative wages are determined independently of uniform commercial policies.

Joint welfare is maximized by choosing  $M$  potentially different policies  $t_j$

$$\max_{\{t_j\}} W \equiv \sum_j \zeta_j \times \mu_j \times \tilde{\mu}_j^{\frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)}} \times (1 + t_j)^{-\frac{1}{\gamma \theta} (\frac{\sigma \theta}{\sigma - 1} - 1)} \times \lambda_{jj}^{-\frac{1}{\gamma \theta}}.$$

Recall from our analysis above that with uniform commercial policies, domestic expenditure shares are invariant to changes in commercial policies. The first-order conditions of the joint welfare maximization problem therefore collapse to  $M$  symmetric first-order conditions of the same type as in the symmetric country case. Hence, the optimal policy result described above for symmetric countries carries over to the case of asymmetric countries.

### 3.2 Efficiency gains of moving from laissez-faire to social optimum

Before turning to the case of optimal cooperative trade policy, we want to gauge the importance of optimal policy. In order to obtain clear-cut results, we analyze the welfare gains of moving from laissez-faire equilibrium to social optimum reached by the implementation of the first-best consumption subsidy for the closed economy case. Let  $lf$  and  $*$  denote variables obtained under laissez-faire and optimal-policy driven equilibria, respectively. Moreover, let  $\rho := \frac{\sigma - 1}{\sigma} \in (0, 1)$  be an inverse measure of the mark-up. Then, the change in welfare

induced by moving from laissez-faire to a policy-driven equilibrium is given by

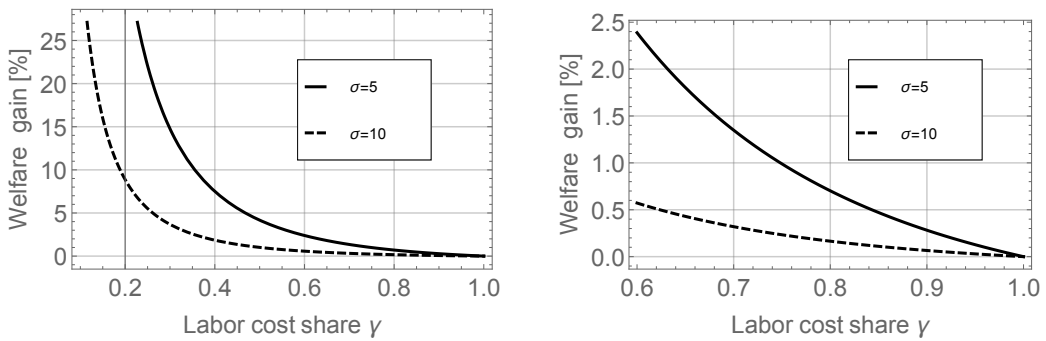
$$\frac{W^*}{W^{lf}} - 1 = \frac{\mu^*/\tilde{P}^*}{\mu^{lf}/\tilde{P}^{lf}} - 1 = \frac{\gamma\rho^{-\frac{1-\gamma}{\gamma}}}{1 - (1-\gamma)\rho} - 1,$$

where we have used  $\mu^{lf} = 1$  and  $\tilde{P}^*/\tilde{P}^{lf} = \rho^{\frac{1}{\gamma}}$ .

As discussed above, in the absence of input-output linkages we have  $W^*/W^{lf} = 1$ . In the limiting case where markups vanish ( $\rho \rightarrow 1$ ), we have  $W^*/W^{lf} \rightarrow 1$ . We show in the appendix that an increase in the mark-up (lower  $\rho$ ) magnifies the welfare gains of moving from laissez-faire to the optimal policy driven equilibrium. Intuitively, the gains from repairing the distortion are larger, the more severe the distortion. Moreover, we establish in the appendix that the welfare gains of moving from laissez-faire to social optimum is larger, the smaller  $\gamma$ . Clearly, the smaller  $\gamma$ , the more relevant is the input distortion.

Figure 1 graphically illustrates the welfare gains for different values of  $\gamma$  and  $\sigma$ . We let the labor cost share vary between 0.1 and 1 and depict the welfare gains for  $\sigma = 5$  and  $\sigma = 10$ , which are reasonable bounds in single-sector studies; see Anderson and van Wincoop (2003). The left panel shows that the welfare gains can be sizable, if input-output linkages are sufficiently important. In the right panel, we zoom in and take a closer look at cost shares in the interval  $\gamma \in (0.7, 1)$ .

Figure 1: Welfare gains of moving from laissez-faire to policy-driven equilibrium



### 3.3 Welfare-maximizing cooperative trade policy

We now assume that the governments are restricted to coordinating their trade policy, while domestic market interventions are ruled out.<sup>17</sup> For the sake of illustration, we focus on symmetric countries. Given symmetry, there is only a single policy instrument which is applied to imports from all countries. Let  $\vartheta$  denote this symmetric trade policy. The key difference to the scenarios considered above is that domestically produced goods are exempted from the policy. Commercial policy therefore drives a wedge between prices of domestically produced and imported goods and plays a role in the determination of the domestic expenditure share  $\lambda$ .

In this setting, the multipliers can be written as

$$\mu = \kappa \left[ \frac{1 + \vartheta}{\lambda\vartheta + 1} - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right] \text{ and } \tilde{\mu} = \kappa \frac{1 + \vartheta}{\vartheta\lambda + 1},$$

where, as above,  $\kappa := 1 / [1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}]$ . Notice that, in contrast to the scenario above, the multipliers do depend on the domestic expenditure share. It is given by

$$\lambda = \left( 1 + \eta (1 + \vartheta)^{1 - \frac{\sigma\theta}{\sigma - 1}} \right)^{-1},$$

where  $\eta := \tau^{-\theta} \left( \frac{f^x}{f^d} \right)^{1 - \frac{\theta}{\sigma - 1}}$  comprises variable trade costs and foreign over domestic market access costs ( $f^x / f^d$ ) and therefore can be viewed as a measure of the “freeness of trade”.

The joint welfare maximization problem can be stated as

$$\max_{\vartheta} \tilde{W} = \mu(\vartheta, \lambda(\vartheta)) \times \tilde{\mu}(\vartheta, \lambda(\vartheta)) \times \lambda(\vartheta)^{-\frac{1}{\gamma}}.$$

We show in the appendix that

$$\text{sign} \frac{dW/W}{d\vartheta} = \text{sign} \left[ \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \vartheta\lambda}{1 + \vartheta - (\lambda\vartheta + 1) (1 - \gamma) \frac{\sigma - 1}{\sigma}} - \frac{1}{\gamma} \left( 1 + \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \frac{\vartheta\lambda}{\sigma - 1} \right) \right].$$

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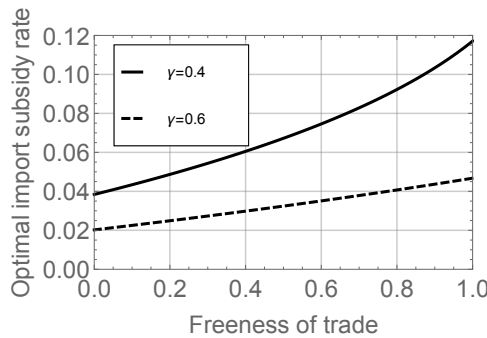
<sup>17</sup>Notice that results for the opposite assumption can be obtained by a clever re-interpretation of expenditure shares.

The optimal trade policy follows from setting this expression to zero and solving for  $\vartheta$ . The complication, however, is that the domestic expenditure share is itself a function of trade policy.

We formally prove in the appendix that the welfare-maximizing policy is to subsidize imported goods, but at a smaller rate than in the case of uniform treatment of domestically produced and imported goods. The reason is that the subsidy on imported goods has the unwelcome effect that it distorts the allocation of spending on domestically and imported goods, such that the economies spend too little on domestically produced goods and waste resources by importing too much.

Figure 2 illustrates the role of freeness of trade for the optimal import subsidy. We consider two symmetric countries. We follow Caliendo et al. (2015) in setting  $\sigma = 2$  and  $\theta = 4$ . The  $x$ -axis represents the freeness of trade  $\eta$ , which is allowed to vary between 0 and 1. The  $y$ -axis plots the welfare-maximizing import subsidy rates ( $|\vartheta|$ ) for different values of  $\gamma$ . In the standard Melitz (2003) model, we have  $\gamma = 1$ , and the optimal trade policy for symmetric countries would be laissez-faire (horizontal line at  $\vartheta = 0$ ). As argued above, with  $\gamma < 1$ , the optimal import subsidy rate is strictly above 0, and below  $|\vartheta| < 1/\sigma (= 1/2)$ .

Figure 2: Optimal import subsidy rates for symmetric countries



Intuitively, welfare-maximizing import subsidies are largest with  $\eta = 1$ . With  $\eta < 1$ , an import subsidy makes imports relatively more than attractive than domestically produced goods, but cause loss in transit due to trade costs. Moreover, lower labor cost shares  $\gamma$  make the input distortion more relevant, which calls for a higher import subsidy, if domestically produced and imported goods are not treated uniformly.

## 4 Non-cooperative trade policy

In this section, we consider policies that are optimal from a *unilateral* perspective, ignoring welfare effects on foreign economies. We focus on two countries,  $H$  and  $F$ , that are assumed to be symmetric in all dimensions but commercial policies. In order to relate to the literature on optimal trade policy, we rule out domestic commercial policy. The government sets the welfare-maximizing tax-cum-subsidy related to imports.

In the non-cooperative setting, the government does not take into account that imported intermediate goods are subject to markups over foreign social (production) costs. Rather, the social costs of imported intermediate goods are the prices at the border. From this perspective, in contrast to domestically produced goods, imported intermediate inputs are not too expensive and do not generate a problem in the cost-minimization problem of producers.

Another difference to the cooperative setting is that governments may follow beggar-thy-neighbor strategies. In fact, the different perception of prices of domestically produced and imported intermediate inputs, constitutes a rationale for import tariffs – or a subsidy on domestically produced goods – that exploit a terms-of-trade externality as in the standard Melitz case; see Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013).<sup>18</sup>

In general equilibrium, an import tariff not only addresses the terms-of-trade externality, but also affects the input distortion. On impact, an import subsidy lowers the price index, which is welcome from the perspective of the input distortion. An import subsidy, however, cannot be the first-best instrument as the distortion arises from domestically produced intermediate goods. Trade policy can only be a vehicle to imperfectly address the input distortion. We aim at figuring out under which conditions one or the other rationale dominates, resulting in either optimal import tariffs or subsidies.

One implication immediately stands out. The optimal tariff, if any, must be smaller than in the standard Melitz (2003) case without input-output linkages. Moreover, the “freeness of trade”  $\eta := \tau^{-\theta} (f^x/f^d)^{1-\frac{\theta}{\sigma-1}}$  which combines variable and fixed relative to domestic market

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<sup>18</sup>Note that although Demidova and Rodríguez-Clare (2009) assume the country to be small, the government can affect “world market prices” by means of trade policy as in a monopolistically competitive setting, each firm is a monopolist in the particular variety it produces; see also Gros (1987) for the case of homogeneous firms.

access costs will be key as it drives the ratio of domestically produced and imported intermediate inputs. As can be seen from the analysis in section 3.3, optimal policies also hinge on the labor cost share  $\gamma$  in the absence of uniform treatment of domestically produced and imported goods.

#### 4.1 Preliminaries

In our setting, the multipliers are given by

$$\begin{aligned}\mu_i &= \tilde{\mu}_i \frac{\kappa^{-1} (\lambda_{ii} t_{ji} + 1) + t_{ji} (1 - \lambda_{ii})}{1 + t_{ji}} \text{ and} \\ \tilde{\mu}_i &= \kappa \frac{1 + t_{ji}}{t_{ji} \lambda_{ii} + 1},\end{aligned}$$

where  $\kappa := 1 / [(1 - (1 - \gamma) \frac{\sigma-1}{\sigma})]$  is a constant. We simplify the analysis by assuming that only country  $H$  conducts trade policy, while country  $F$  is inactive. This assumption implies that  $\tilde{\mu}_F = \kappa$  and  $\mu_F = 1$ .

Domestic expenditure shares are determined by equations (43) and (44). In the two-country setting, balanced trade can be written as Balanced trade implies

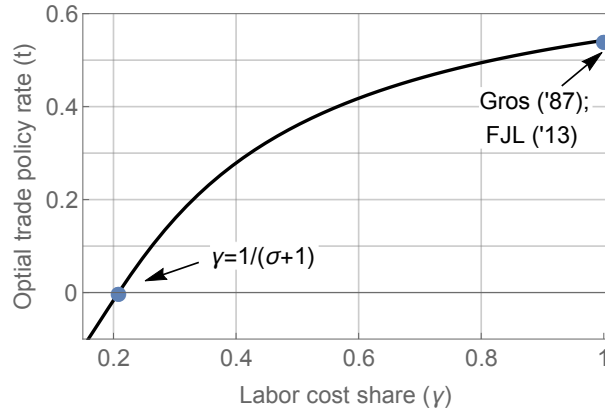
$$\frac{w_i}{w_j} = \frac{1 + t_{ji}}{1 + t_{ij}} \frac{1 - \lambda_{jj}}{1 - \lambda_{ii}} \frac{\tilde{\mu}_j L_j}{\tilde{\mu}_i L_i}.$$

The optimal policy analysis requires to back out the effect of trade policy on the domestic expenditure share. We show in the appendix that

$$\begin{aligned}\frac{d\lambda_{HH}}{\lambda_{HH}} &= \frac{\frac{1-\gamma}{\gamma} \left[ 1 - \lambda_{FF} + \frac{\theta-(\sigma-1)}{\theta\sigma-(\sigma-1)} \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \right] + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1}}{1 + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \left( \lambda_{FF} + \frac{1+t_{FH}}{t_{FH}\lambda_{HH}+1} \lambda_{HH} \right) + \frac{1-\gamma}{\gamma} \left( 2 - \lambda_{HH} - \lambda_{FF} + \frac{\theta-(\sigma-1)}{\sigma-1} \frac{t_{FH}\lambda_{HH}(1-\lambda_{HH})}{t_{FH}\lambda_{HH}+1} \right)} \\ &\times (1 - \lambda_{HH}) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} > 0.\end{aligned}$$

The expression highlights that a tariff increases the domestic expenditure share. Moreover, it shows that for given trade shares and initial policies, the percentage increase of Home's expenditure share induced by the same percentage increase in the tariff is larger in the presence of input-output linkages than in their absence. This is a general equilibrium effect.

Figure 3: The role of the labor cost share



## 4.2 Optimal trade policy

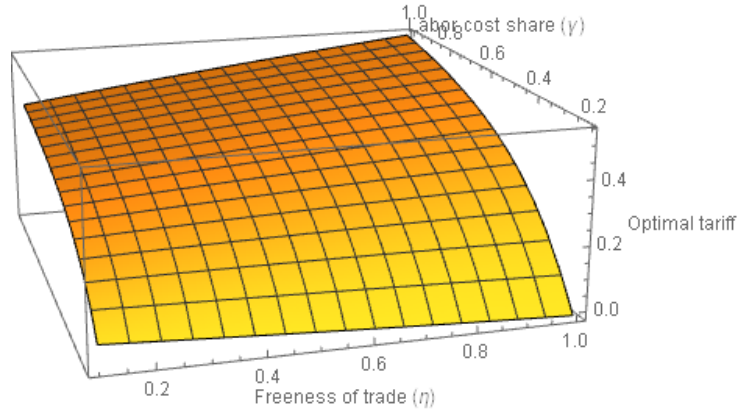
In the appendix, we derive the first-order condition of the welfare-maximization problem for the general case. Moreover, we show analytically that in the absence of trade cost ( $\eta = 1$ ) and with an inactive selection effect ( $\theta \rightarrow \sigma - 1$ ), the optimal policy is an import tariff if  $\gamma > 1/(\sigma + 1)$ , while it is an import subsidy if  $\gamma$  smaller than this threshold. This result implies that under the certain conditions, the input distortion dominates the terms-of-trade externality if the labor cost share is sufficiently small compared to the trade elasticity, which drives the terms-of-trade effect. Moreover, the result implies firm heterogeneity is not required for the input distortion to dominate the terms-of-trade externality.

Figure 3 draws on the first-order condition derived in the appendix and illustrates the welfare-maximizing trade policy as a function of the labor cost share  $\gamma$ . For  $\gamma = 1$ , our analysis resembles the optimal tariff characterized in Gros (1987) and Felbermayr et al. (2013). For the given parametrization ( $\eta = 1, \sigma = 3.8, \theta = \sigma - 1$ ), the critical  $\gamma$  is approximately  $\gamma \approx 0.21$ .

For the standard Melitz (2003) case ( $\gamma = 1$ ), Felbermayr et al. (2013) discuss the role of the freeness of trade for optimal trade policy. They find that the optimal tariff is increasing in the freeness of trade, such that a fall in non-tariff trade barriers commands a higher optimal tariff. We can, however, find cases where the opposite conclusion holds. We show in the appendix that with an inactive selection effect ( $\theta \rightarrow \sigma - 1$ ), for  $\gamma = 1/(\sigma + 1)$  a tariff would be



Figure 4: The role of the labor cost share and the freeness of trade



optimal when  $\eta < 1$ .<sup>19</sup> Hence, the optimal tariff can also decrease in the freeness of trade, if  $\gamma$  is sufficiently small. For intermediate values of  $\gamma$ , we find an inverted U-shape (the optimal tariff increases in  $\eta$  for small values of  $\eta$  and falls in  $\eta$  for large value of  $\eta$ ).

Figure 4 illustrates optimal tariffs as a function of  $\gamma$  and  $\eta$  for  $\sigma = 3.8$  and  $\theta = 2.8$ . Note that under this parametrization, the critical  $\gamma$  for which laissez-faire is optimal is  $\gamma = 1/(\sigma + 1) \approx 0.2$ . Clearly, the optimal tariff is always smaller in the presence of input-output linkages than in their absence.

## 5 Conclusion

In this paper we have analyzed optimal commercial policies in a version of the Melitz (2003) model with intermediate-input linkages. We find that input-output linkages in combination with monopolistic competition gives rise to an import distortion which is not present in models with perfect competition or models that ignore input-output linkages. In the laissez-faire equilibrium, firms use too much labor input and too little material input. From this perspective, the model calls for more use of intermediate inputs. One straightforward implication would be to allow for integration to circumvent the double marginalization problem. Given that in our setting production requires inputs from a continuum of suppliers located around

<sup>19</sup>Recall from above that under these assumptions, laissez-faire is optimal with  $\eta = 1$ .

the world, however, this does not seem to be promising strategy.

The optimal cooperative policy is a uniform subsidy on domestically produced and imported goods that exactly offsets the markup. We show that the welfare gains of moving from market outcomes to the efficient outcome can be substantial. Welfare gains come from a more efficient allocation of the composite good on final and intermediate use and not from reallocation of resources across firms. In the calculation of welfare losses from non-cooperation, the relevant benchmark seems to be a world with efficient levels of subsidies. Clearly, this statement requires qualification as most countries tax labor. Labor taxes would alleviate the pressure to subsidize intermediate inputs. We ignore labor income taxes, as they would not only be applied to production workers, but also to fixed labor input into production and innovation (entry) activity.

When coming to un-cooperative trade policy, we find that the input distortion counteracts the standard terms-of-trade externality, potentially resulting in an import subsidy. Whether an import subsidy or a tariff is optimal depends on the labor cost share and freeness of trade. It also will depend on the degree of firm heterogeneity, but we have not explored the role of heterogeneity yet. We find that optimal tariffs may fall in real trade costs, if the labor cost share is sufficiently small. This finding questions the importance of the World Trade Organization, as in world with falling trade costs tariff would have been reduced anyway.

Restricting the government to a single instrument (trade policy) in the presence of two distortions can certainly not lead to efficient outcomes. We will characterize the optimal policy mix in the future. Preliminary results suggests that the optimal policy mix is a subsidy on domestically produced intermediate goods and a tax-cum-subsidy on imported goods. Moreover, our analysis of un-cooperative trade policy so far ignores retaliation. We plan to characterize un-cooperative Nash tariffs and to compute welfare losses in the vein of Ossa (2016).

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## A Detailed derivations

### A.1 Zero profit condition

Taking (16), we insert for profits using (12) and for quantities  $\tilde{q}_{ji}(\varphi)$  from (14) which yields

$$\int_{\varphi_{ji}^*} \pi(\varphi) g(\varphi) d\varphi = \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} \frac{(\tau_{ji}x_j)^{1-\sigma}}{(1+t_{ji})^\sigma} Y_i \tilde{P}_i^{\sigma-1} \int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi - w_j f_{ji} \int_{\varphi_{ji}^*} g(\varphi) d\varphi.$$

Assuming a Pareto distribution for  $\varphi$ , we have  $G(\varphi) = 1 - \varphi^{-\theta}$  and  $g(\varphi) = \theta\varphi^{-1-\theta}$ , whereby we assume  $\theta > \sigma - 1$ . This implies

$$\int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi = \theta \int_{\varphi_{ji}^*} \varphi^{\sigma-\theta-2} d\varphi = \frac{\theta}{\theta - (\sigma - 1)} (\varphi_{ji}^*)^{\sigma-\theta-1}. \quad (48)$$

This term, which we simply refer to as  $\xi_{ji}$  in the text, represents the selection of firms located in country  $j$  into different markets, including the come market. We refer to this and

$\int_{\varphi_{ji}^*} g(\varphi) d\varphi = (\varphi_{ji}^*)^{-\theta}$ , whence we may write

$$\int_{\varphi_{ji}^*} \pi(\varphi) g(\varphi) d\varphi = \frac{\sigma^{-\sigma}}{(\sigma-1)^{1-\sigma}} \frac{(\tau_{ji}x_j)^{1-\sigma}}{(1+t_{ji})^\sigma} Y_i \tilde{P}_i^{\sigma-1} \frac{\theta}{\theta - (\sigma - 1)} (\varphi_{ji}^*)^{\sigma-\theta-1} - w_j f_{ji} (\varphi_{ji}^*)^{-\theta}.$$

Substituting for  $(\varphi_{ji}^*)^{\sigma-1}$  according to the first line of (17), we have

$$(\varphi_{ji}^*)^{\sigma-1} = \frac{\sigma^\sigma}{(\sigma-1)^{\sigma-1}} (x_j \tau_{ji})^{\sigma-1} (1+t_{ji})^\sigma \frac{Y_i}{\tilde{P}_i^{\sigma-1}} w_j f_{ji},$$

whence expected profits of selling from  $j$  to  $i$  reduce to

$$\int_{\varphi_{ji}^*} \pi(\varphi) g(\varphi) d\varphi = \frac{\theta}{\theta - (\sigma - 1)} w_j f_{ji} (\varphi_{ji}^*)^{-\theta} - w_j f_{ji} \varphi_{ji}^{*\theta} = \frac{\sigma - 1}{\theta - (\sigma - 1)} w_j f_{ji} (\varphi_{ji}^*)^{-\theta}.$$

Summing up over all markets (countries)  $i = 1, \dots, M$  and inserting into (16) leads to equation (19).

## A.2 Price index and expenditure shares

Using the markup pricing condition for firm  $\varphi$  of country  $j$  when selling to country  $i$  as given in (11), the price index for goods assembly given in (5) emerges as

$$\begin{aligned}
\tilde{P}_i &= \left( \sum_{j=1}^M N_j \int_{\varphi_{ji}^*} \left( \frac{\sigma}{\sigma-1} \frac{(1+t_{ji})\tau_{ji}x_j}{\varphi} \right)^{1-\sigma} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}} \\
&= \left( \sum_{j=1}^M N_j \left( \frac{\sigma}{\sigma-1} (1+t_{ji})\tau_{ji}x_j \right)^{1-\sigma} \int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}} \\
&= \left( \sum_{j=1}^M N_j \chi_{ji} \xi_{ji} \right)^{\frac{1}{1-\sigma}}, \tag{49}
\end{aligned}$$

where  $\chi_{ji} := \left( \frac{\sigma}{\sigma-1} (1+t_{ji})\tau_{ji}x_j \right)^{1-\sigma}$  and  $\xi_{ji} := \int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi$ . Using (48), we may write

$$\tilde{P}_i^{1-\sigma} = \frac{\theta}{\theta - (\sigma - 1)} \sum_{j=1}^M N_j \chi_{ji} (\varphi_{ji}^*)^{\sigma-\theta-1}$$

From (6) it follows that country  $i$ 's expenditure on goods originating in country  $j$  may be written as

$$N_j \int_{\varphi_{ji}^*} \tilde{p}_{ji}(\varphi) \tilde{q}_{ji}(\varphi) g(\varphi) d\varphi = N_j \int_{\varphi_{ji}^*} \tilde{p}_{ji}(\varphi)^{1-\sigma} g(\varphi) d\varphi \times Y_i \tilde{P}_i^{\sigma-1}$$

Inserting from (11) and (13), we have

$$\begin{aligned}
N_j \int_{\varphi_{ji}^*} \tilde{p}_{ji}(\varphi) \tilde{q}_{ji}(\varphi) g(\varphi) d\varphi &= N_j \left( \frac{\sigma}{\sigma-1} \tau_{ji} x_j (1+t_{ji}) \right)^{1-\sigma} \int_{\varphi_{ji}^*} \varphi^{\sigma-1} g(\varphi) d\varphi \times Y_i \tilde{P}_i^{\sigma-1} \\
&= N_j \chi_{ji} \xi_{ji} \times Y_i \tilde{P}_i^{\sigma-1}
\end{aligned}$$

Forming expenditures shares and using (49), we have

$$\lambda_{ji} := \frac{N_j \int_{\varphi_{ji}^*} \tilde{p}_{ji}(\varphi) \tilde{q}_{ji}(\varphi) g(\varphi) d\varphi}{\sum_{n=1}^I N_n \int_{\varphi_{ni}^*} \tilde{p}_{ni}(\varphi) \tilde{q}_{ni}(\varphi) g(\varphi) d\varphi} = \frac{N_j \chi_{ji} \xi_{ji}}{\tilde{P}_i^{1-\sigma}}$$

### A.3 The income multiplier

From equation (2) and  $T_j = Y_j \sum_i \frac{t_{ij}\lambda_{ij}}{1+t_{ij}}$ , and using (35), we have

$$I_j = \left(1 + \tilde{\mu}_j \sum_i \frac{t_{ij}\lambda_{ij}}{1+t_{ij}}\right) \times w_j L_j$$

This can be rewritten as

$$\begin{aligned} I_j &= \tilde{\mu}_j \left[ \left(1 - (1-\gamma) \frac{\sigma-1}{\sigma}\right) \sum_i \frac{\lambda_{ij}}{1+t_{ij}} + \sum_i \frac{t_{ij}\lambda_{ij}}{1+t_{ij}} \right] \times w_i L_i \\ &= \tilde{\mu}_j \sum_i \left(1 + t_{ij} - (1-\gamma) \frac{\sigma-1}{\sigma}\right) \frac{\lambda_{ij}}{1+t_{ij}} \times w_i L_i \end{aligned}$$

### A.4 Firm entry

We now use the equilibrium conditions derived above to solve for the mass of entrants. The zero cutoff profit condition can be written as

$$\begin{aligned} \varphi_{ij}^* &= \frac{\sigma}{\sigma-1} \left( \frac{\sigma w_i f_{ij} (1+t_{ij})}{Y_j} \right)^{\frac{1}{\sigma-1}} \frac{\tau_{ij} x_i (1+t_{ij})}{\tilde{P}_j} \Leftrightarrow \\ (\varphi_{ij}^*)^{\sigma-1} &= \left( \frac{\sigma}{\sigma-1} \right)^{\sigma-1} \left( \frac{\sigma w_i f_{ij} (1+t_{ij})}{Y_j} \right) \left( \frac{\tau_{ij} x_i (1+t_{ij})}{\tilde{P}_j} \right)^{\sigma-1} \Leftrightarrow \\ \left( \frac{\sigma}{\sigma-1} \frac{\tau_{ij} x_i (1+t_{ij})}{\tilde{P}_j \varphi_{ij}^*} \right)^{1-\sigma} &= \frac{\sigma w_i f_{ij} (1+t_{ij})}{Y_j} \end{aligned}$$

Using this expression in the expression for the expenditure share (22), we have

$$\begin{aligned} \lambda_{ij} &= \frac{\theta}{\theta - (\sigma-1)} N_i (\varphi_{ij}^*)^{-\theta} \left( \frac{\sigma}{\sigma-1} \frac{(1+t_{ij}) \tau_{ij} x_i}{\varphi_{ij}^* \tilde{P}_j} \right)^{1-\sigma} \\ &= \frac{\theta}{\theta - (\sigma-1)} N_i (\varphi_{ij}^*)^{-\theta} \frac{\sigma w_i f_{ij} (1+t_{ij})}{Y_j} \Leftrightarrow \\ \frac{\lambda_{ij}}{1+t_{ij}} Y_j &= \frac{\theta \sigma}{\theta - (\sigma-1)} N_i (\varphi_{ij}^*)^{-\theta} w_i f_{ij} \end{aligned}$$

Summing both sides over  $j$  and solving for  $N_i$ , we have

$$\begin{aligned}\sum_j \frac{\lambda_{ij}}{1+t_{ij}} Y_j &= \frac{\theta\sigma}{\theta-(\sigma-1)} N_i w_i \sum_j (\varphi_{ij}^*)^{-\theta} f_{ij} \\ &= \frac{\theta\sigma}{\theta-(\sigma-1)} N_i w_i \frac{\theta-(\sigma-1)}{\sigma-1} f_i^e \Leftrightarrow \\ N_i &= \frac{\sigma-1}{\theta\sigma w_i f_i^e} \sum_j \frac{\lambda_{ij}}{1+t_{ij}} Y_j\end{aligned}$$

where the second line follows from (19).

Using the trade balance condition (31) and (32), we obtain

$$N_i = \frac{\sigma-1}{\theta\sigma w_i f_i^e} Y_i \sum_j \frac{\lambda_{ji}}{1+t_{ji}} = \frac{\sigma-1}{\theta\sigma f_i^e} \frac{Z_i}{w_i}.$$

The expression in the text follows from using (33).

## A.5 Domestic expenditure share

In order to derive the expression for the domestic expenditure share, we compute  $\tilde{P}_i^{1-\sigma}$ :

$$\begin{aligned}\tilde{P}_i^{1-\sigma} &= \frac{\theta}{\theta-(\sigma-1)} \sum_{j=1}^M N_j (\varphi_{ji}^*)^{-\theta} \left( \frac{\sigma}{\sigma-1} \frac{(1+t_{ji}) \tau_{ji} x_j}{\varphi_{ji}^*} \right)^{1-\sigma} \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i (\varphi_{ii}^*)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \left( \frac{\varphi_{ji}^*}{\varphi_{ii}^*} \right)^{\sigma-1-\theta} \tau_{ji}^{1-\sigma} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\sigma} \left( \frac{x_j}{x_i} \right)^{1-\sigma} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i (\varphi_{ii}^*)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \left( \frac{\sigma w_j f_{ji} (1+t_{ji})}{\sigma w_i f_{ii} (1+t_{ii})} \frac{\tau_{ji} x_j (1+t_{ji})}{x_i (1+t_{ii})} \right)^{\sigma-1-\theta} \tau_{ji}^{1-\sigma} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\sigma} \left( \frac{x_j}{x_i} \right)^{1-\sigma} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i (\varphi_{ii}^*)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \left( \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{1}{\sigma-1}} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{\frac{1}{\sigma-1}+1} \tau_{ji} \frac{x_j}{x_i} \right)^{\sigma-1-\theta} \tau_{ji}^{1-\sigma} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\sigma} \left( \frac{x_j}{x_i} \right)^{1-\sigma} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i (\varphi_{ii}^*)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\sigma+\frac{\sigma(1-\theta)}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{x_j}{x_i} \right)^{-\theta} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i (\varphi_{ii}^*)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{x_j}{x_i} \right)^{-\theta} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}{\theta-(\sigma-1)} N_i \left( \frac{\sigma}{\sigma-1} \left( \frac{\sigma w_i f_{ii} (1+t_{ii})}{Y_i} \right)^{\frac{1}{\sigma-1}} x_i (1+t_{ii}) \right)^{\sigma-1-\theta} (1+t_{ii})^{1-\sigma} x_i^{1-\sigma} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{x_j}{x_i} \right)^{-\theta} \right) \\ &= \frac{\theta \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} (\sigma f_{ii})^{\frac{\sigma-1-\theta}{\sigma-1}} N_i \left( \frac{w_i}{Y_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} (1+t_{ii})^{\frac{\sigma(\sigma-1-\theta)}{\sigma-1}+1-\sigma} \tilde{P}_i^{\theta-(\sigma-1)} x_i^{-\theta} \left( 1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left( \frac{1+t_{ji}}{1+t_{ii}} \right)^{1-\frac{\sigma\theta}{\sigma-1}} \left( \frac{f_{ji}}{f_{ii}} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{w_j}{w_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left( \frac{x_j}{x_i} \right)^{-\theta} \right)\end{aligned}$$



Hence, the ratio of input cost to the price index is implied by

$$\begin{aligned} \left(\frac{x_i}{\tilde{P}_i}\right)^\theta &= \frac{\theta \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} (\sigma f_{ii})^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{w_i}{Y_i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} (1+t_{ii})^{1-\frac{\sigma\theta}{\sigma-1}}}{\theta - (\sigma-1)} N_i \left(\frac{w_i}{Y_i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} (1+t_{ii})^{1-\frac{\sigma\theta}{\sigma-1}} \\ &\quad \times \left(1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left(\frac{1+t_{ji}}{1+t_{ii}}\right)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{w_j}{w_i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{x_j}{x_i}\right)^{-\theta}\right). \end{aligned}$$

Using this expression to substitute out the ratio from the domestic expenditure share, we obtain

$$\lambda_{ii} = \left(1 + \sum_{j=1}^M \frac{N_j}{N_i} \tau_{ji}^{-\theta} \left(\frac{1+t_{ji}}{1+t_{ii}}\right)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{w_j}{w_i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{x_j}{x_i}\right)^{-\theta}\right)^{-1}.$$

Substituting out the input cost index, we obtain

$$\begin{aligned} \left(\frac{x_j}{x_i}\right)^{-\theta} &= \left(\frac{w_j}{w_i}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{w_j^\gamma \tilde{P}_j^{1-\gamma}}{w_i^\gamma \tilde{P}_i^{1-\gamma}}\right)^{-\theta} \\ &= \left(\frac{w_j}{w_i}\right)^{-\theta} \left(\frac{w_j^{\gamma-1} \tilde{P}_j^{1-\gamma}}{w_i^{\gamma-1} \tilde{P}_i^{1-\gamma}}\right)^{-\theta} \\ &= \left(\frac{w_j}{w_i}\right)^{-\theta} \left(\frac{\xi_j}{\xi_i} \left(\frac{N_j}{N_i}\right)^{\frac{1}{\gamma\theta}} \left(\frac{\tilde{\mu}_j}{\tilde{\mu}_i}\right)^{\frac{\theta-(\sigma-1)}{\gamma\theta(\sigma-1)}} \left(\frac{1+t_{jj}}{1+t_{ii}}\right)^{-\frac{1}{\gamma\theta}(\frac{\sigma\theta}{\sigma-1}-1)} \left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{-\frac{1}{\gamma\theta}}\right)^{(1-\gamma)\theta} \\ &= \left(\frac{w_j}{w_i}\right)^{-\theta} \left(\left(\frac{\xi_j}{\xi_i}\right)^{\theta\gamma} \frac{N_j}{N_i} \left(\frac{\tilde{\mu}_j}{\tilde{\mu}_i}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} \left(\frac{1+t_{jj}}{1+t_{ii}}\right)^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{-1}\right)^{\frac{1-\gamma}{\gamma}}. \end{aligned}$$

## A.6 Selection with uniform treatment of domestically and imported goods

We start from the zero cutoff profit condition

$$\varphi_{ji}^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} (1+t_{ji})^{\frac{\sigma}{\sigma-1}} x_j w_j^{\frac{1}{\sigma-1}} \frac{Y_i^{\frac{1-\sigma}{\sigma-1}}}{\tilde{P}_i}.$$

Substituting out gross output, we obtain

$$\varphi_{ji}^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}} (L_i)^{\frac{1}{1-\sigma}}}{\sigma-1} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} (1+t_{ji})^{\frac{\sigma}{\sigma-1}} x_j \frac{\tilde{\mu}_i^{\frac{1-\sigma}{\sigma-1}}}{\tilde{P}_i} \left(\frac{w_j}{w_i}\right)^{\frac{1}{\sigma-1}}.$$

Next, we employ the definition of the input cost index  $x_j$  :

$$\varphi_{ji}^* = A \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} (1+t_{ji})_j^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_i^{\frac{1}{1-\sigma}} \left( \frac{w_j}{\tilde{P}_j} \right)^\gamma \frac{\tilde{P}_j}{\tilde{P}_i} \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}}.$$

Using equation (39) to substitute out the real wage, we obtain

$$\varphi_{ji}^* = \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_j}{\tilde{P}_i} (1+t_{ji})^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_i^{\frac{1}{1-\sigma}} \left( \tilde{\mu}_j^{\frac{\theta-(\sigma-1)}{\sigma-1}} (1+t_{jj})^{1-\frac{\sigma\theta}{\sigma-1}} \right)^{\frac{1}{\theta}} \lambda_{jj}^{-\frac{1}{\theta}}.$$

With symmetric treatment of domestically produced and imported goods, we have

$$\begin{aligned} \varphi_{ji}^* &\propto \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_j}{\tilde{P}_i} (1+t_{ji})^{\frac{\sigma}{\sigma-1}} \tilde{\mu}_i^{\frac{1}{1-\sigma}} (1+t_j)^{-1} \lambda_{jj}^{-\frac{1}{\theta}} \\ &\propto \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_j}{\tilde{P}_i} \frac{1+t_i}{1+t_j} \lambda_{jj}^{-\frac{1}{\theta}}. \end{aligned}$$

This expression highlights that with symmetric countries, cutoffs are invariant to changes in commercial policy.

With asymmetric countries, we need some more steps to that the same claim holds under the additional restriction that all countries conduct the same policy. The condition above can be rewritten as

$$\varphi_{ji}^* \propto \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{1}{\sigma-1}-1} \left( \frac{w_j/\tilde{P}_j}{w_i\tilde{P}_i} \right) \frac{1+t_i}{1+t_j} \lambda_{jj}^{-\frac{1}{\theta}}.$$

Using again equation (39), we obtain

$$\begin{aligned} \varphi_{ji}^* &\propto \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{2-\sigma}{\sigma-1}} \frac{\zeta_j}{\zeta_i} \left( \frac{N_j}{N_i} \right)^{\frac{1}{\gamma\theta}} \left( \frac{\tilde{\mu}_j}{\tilde{\mu}_i} \right)^{\frac{\theta-(\sigma-1)}{\gamma\theta(\sigma-1)}} \left( \frac{1+t_j}{1+t_i} \right)^{\frac{1}{\gamma\theta} (1-\frac{\sigma\theta}{\sigma-1})} \\ &\quad \times \left( \frac{\lambda_{jj}}{\lambda_{ii}} \right)^{-\frac{1}{\gamma\theta}} \frac{1+t_i}{1+t_j} \lambda_{jj}^{-\frac{1}{\theta}} \\ &\propto \zeta_j \frac{\sigma^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{1-\sigma}}}{\sigma-1} N_j^{\frac{1}{\theta}} f_{ji}^{\frac{1}{\sigma-1}} \tau_{ji} \left( \frac{w_j}{w_i} \right)^{\frac{2-\sigma}{\sigma-1}} \frac{\zeta_j}{\zeta_i} \left( \frac{N_j}{N_i} \right)^{\frac{1}{\gamma\theta}} \left( \frac{1+t_j}{1+t_i} \right)^{-\frac{1}{\gamma}} \left( \frac{\lambda_{jj}}{\lambda_{ii}} \right)^{-\frac{1}{\gamma\theta}} \frac{1+t_i}{1+t_j} \lambda_{jj}^{-\frac{1}{\theta}}. \end{aligned}$$

Recognizing that with uniform treatment domestic expenditure shares and relative wages are determined independently of commercial policy, cutoff productivity levels are also de-

terminated independently of commercial policies in the presence of country asymmetries.

## A.7 Efficiency gains of moving from laissez-faire to social optimum

Taking the derivative of the welfare differential with respect to  $\rho := \frac{\sigma-1}{\sigma}$ , we obtain

$$\begin{aligned}\frac{\partial W^*}{\partial \rho} &= \gamma \frac{-\frac{1-\gamma}{\gamma} \rho^{-\frac{1-\gamma}{\gamma}-1} [1 - (1-\gamma)\rho] - \rho^{-\frac{1-\gamma}{\gamma}} (1-\gamma)}{[1 - (1-\gamma)]^2} \\ &= -(1-\gamma) \rho^{-\frac{1-\gamma}{\gamma}-1} \frac{1 - (1-\gamma)\rho + \rho\gamma}{[1 - (1-\gamma)]^2} \\ &= -\frac{2\gamma(1-\gamma) \rho^{-\frac{1-\gamma}{\gamma}}}{[1 - (1-\gamma)]^2} < 0.\end{aligned}$$

Taking the derivative with respect to  $\gamma$ , we obtain

$$\begin{aligned}\frac{\partial W^*}{\partial \gamma} &= \frac{\left( \rho^{-\frac{1-\gamma}{\gamma}} - \gamma \ln(\rho) \rho^{-\frac{1-\gamma}{\gamma} - \frac{\gamma-(1-\gamma)}{\gamma^2}} \right) [1 - (1-\gamma)\rho] - \gamma \rho^{-\frac{1-\gamma}{\gamma}} \rho}{[1 - (1-\gamma)\rho]^2} \\ &= \rho^{-\frac{1-\gamma}{\gamma}} \frac{\left( 1 + \frac{\ln(\rho)}{\gamma} \right) [1 - (1-\gamma)\rho] - \gamma \rho}{[1 - (1-\gamma)\rho]^2} \\ &= \rho^{-\frac{1-\gamma}{\gamma}} \frac{1 - (1-\gamma)\rho + \frac{\ln(\rho)}{\gamma} [1 - (1-\gamma)\rho] - \gamma \rho}{[1 - (1-\gamma)\rho]^2} \\ &= \rho^{-\frac{1-\gamma}{\gamma}} \frac{1 - \rho + \gamma \rho + \frac{\ln(\rho)}{\gamma} [1 - (1-\gamma)\rho] - \gamma \rho}{[1 - (1-\gamma)\rho]^2} \\ &= \rho^{-\frac{1-\gamma}{\gamma}} \frac{1 - \rho + \frac{\ln(\rho)}{\gamma} [1 - (1-\gamma)\rho]}{[1 - (1-\gamma)\rho]^2}.\end{aligned}$$

Evaluated at  $\gamma = 1$ , we have

$$\left. \frac{\partial W^*}{\partial \gamma} \right|_{\gamma=1} < 0 \Leftrightarrow 1 + \ln(\rho) < \rho,$$

which always holds. In general, we have

$$\frac{\partial W^*}{\partial \gamma} < 0 \Leftrightarrow 1 + \ln(\rho) \frac{1 - (1-\gamma)\rho}{\gamma} - \rho < 0.$$

We numerically check whether the inequality holds for all combinations of  $\gamma \in (0, 1)$  and  $\rho \in (0, 1)$ . We find that the inequality always holds. Hence, the welfare gains of moving from laissez-faire to social optimum is larger, the smaller  $\gamma$ .

## A.8 Welfare-maximizing cooperative trade policy

The gross output multiplier is given by

$$\begin{aligned}\tilde{\mu} &\equiv \left(1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}\right)^{-1} \left(\lambda + \frac{1 - \lambda}{1 + \vartheta}\right)^{-1} \\ &= \kappa \frac{1 + \vartheta}{(1 + \vartheta)\lambda + 1 - \lambda} = \kappa \frac{1 + \vartheta}{\vartheta\lambda + 1}.\end{aligned}$$

Totally differentiating this expression, we obtain

$$\begin{aligned}\frac{d\tilde{\mu}}{\tilde{\mu}} &= \frac{d\vartheta}{1 + \vartheta} - \frac{\lambda}{\vartheta\lambda + 1} d\vartheta - \frac{\vartheta\lambda}{\vartheta\lambda + 1} \frac{d\lambda}{\lambda} \\ &= \frac{\vartheta\lambda + 1 - (1 + \vartheta)\lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} d\vartheta - \frac{\vartheta\lambda}{\vartheta\lambda + 1} \frac{d\lambda}{\lambda} \\ &= \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} d\vartheta - \frac{\vartheta\lambda}{\vartheta\lambda + 1} \frac{d\lambda}{\lambda}\end{aligned}$$

Evaluated at free trade ( $\vartheta = 0$ ), we have

$$\left.\frac{d\tilde{\mu}/\tilde{\mu}}{d\vartheta}\right|_{\vartheta=0} = 1 - \lambda.$$

The income multiplier reduces is given by

$$\begin{aligned}
\mu &\equiv \tilde{\mu} \left[ \left( 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) \lambda + \left( 1 + \vartheta - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) \frac{1 - \lambda}{1 + \vartheta} \right] \\
&= \frac{\tilde{\mu}}{1 + \vartheta} \left[ \left( 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) \lambda (1 + \vartheta) + \left( 1 + \vartheta - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right) (1 - \lambda) \right] \\
&= \frac{\tilde{\mu}}{1 + \vartheta} \left[ \lambda (1 + \vartheta) - (1 - \gamma) \frac{\sigma - 1}{\sigma} \lambda (1 + \vartheta) + (1 + \vartheta) (1 - \lambda) - (1 - \gamma) \frac{\sigma - 1}{\sigma} (1 - \lambda) \right] \\
&= \frac{\tilde{\mu}}{1 + \vartheta} \left[ 1 + \vartheta - (1 - \gamma) \frac{\sigma - 1}{\sigma} \lambda (1 + \vartheta) - (1 - \gamma) \frac{\sigma - 1}{\sigma} (1 - \lambda) \right] \\
&= \frac{\tilde{\mu}}{1 + \vartheta} \left[ 1 + \vartheta - (1 - \gamma) \frac{\sigma - 1}{\sigma} \lambda - (1 - \gamma) \frac{\sigma - 1}{\sigma} \lambda \vartheta - (1 - \gamma) \frac{\sigma - 1}{\sigma} + (1 - \gamma) \frac{\sigma - 1}{\sigma} \lambda \right] \\
&= \tilde{\mu} \frac{\lambda \vartheta + 1}{1 + \vartheta} \left[ \frac{1 + \vartheta}{\lambda \vartheta + 1} - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right] \\
&= \kappa \left[ \frac{1 + \vartheta}{\lambda \vartheta + 1} - (1 - \gamma) \frac{\sigma - 1}{\sigma} \right].
\end{aligned}$$

Totally differentiating this expression, we obtain

$$\begin{aligned}
\frac{d\mu}{\mu} &= \frac{\frac{\lambda \vartheta + 1 - (1 + \vartheta) \lambda}{(\lambda \vartheta + 1)^2} d\vartheta - \frac{\vartheta}{(\lambda \vartheta + 1)^2} d\lambda}{\frac{1 + \vartheta}{\lambda \vartheta + 1} - (1 - \gamma) \frac{\sigma - 1}{\sigma}} \\
&= \frac{1}{(\lambda \vartheta + 1)^2} \frac{(1 - \lambda) d\vartheta - \vartheta d\lambda}{\frac{1 + \vartheta}{\lambda \vartheta + 1} - (1 - \gamma) \frac{\sigma - 1}{\sigma}}.
\end{aligned}$$

Evaluated at free trade ( $\vartheta = 0$ ), we have

$$\left. \frac{d\mu/\mu}{d\vartheta} \right|_{\vartheta=0} = \kappa (1 - \lambda).$$

The domestic expenditure share is given by

$$\lambda = \frac{1}{1 + \eta (1 + \vartheta)^{1 - \frac{\sigma \theta}{\sigma - 1}}}.$$

Totally differentiating, we obtain

$$\frac{d\lambda}{\lambda} = -(1 - \lambda) \left( 1 - \frac{\sigma \theta}{\sigma - 1} \right) \frac{d\vartheta}{1 + \vartheta}.$$

Notice that changes in  $\vartheta$  affect the domestic expenditure share through an effect on the price

(intensive margin) and selection (extensive margin). Evaluated at free trade ( $\vartheta = 0$ ), we obtain

$$\left. \frac{d\lambda/\lambda}{d\vartheta} \right|_{\vartheta=0} = (1 - \lambda) \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right).$$

In totally differentiated from, welfare is given by

$$\frac{dW}{W} = \frac{d\mu}{\mu} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{d\tilde{\mu}}{\tilde{\mu}} - \frac{1}{\gamma\theta} \frac{d\lambda}{\lambda}.$$

Evaluated at free trade ( $\vartheta = 0$ ), a subsidy on imported varieties increases welfare as

$$\begin{aligned} \left. \frac{dW/W}{d\vartheta} \right|_{\vartheta=0} &= \kappa(1 - \lambda) + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)}(1 - \lambda) - \frac{1}{\gamma\theta}(1 - \lambda) \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \\ &= (1 - \lambda) \left[ \kappa + \frac{1}{\gamma\theta} \left( \frac{\theta - (\sigma - 1)}{\sigma - 1} - \frac{\sigma\theta}{\sigma - 1} + 1 \right) \right] \\ &= (1 - \lambda) \left( \kappa - \frac{1}{\gamma} \right) < 0. \end{aligned}$$

Hence, an import subsidy is welfare enhancing.

In general, the change in welfare induced by trade policy is given by

$$\begin{aligned} \frac{dW/W}{d\vartheta} &= \frac{1}{(\lambda\vartheta + 1)^2} \frac{1 - \lambda - \vartheta d\lambda/d\vartheta}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \left( \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} - \frac{\vartheta\lambda}{\vartheta\lambda + 1} \frac{d\lambda/d\vartheta}{\lambda} \right) - \frac{1}{\gamma\theta} \frac{d\lambda/d\vartheta}{\lambda} \\ &= \left( \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \right) \frac{1 - \lambda}{(\vartheta\lambda + 1)(1 + \vartheta)} - \left( \frac{1}{(\lambda\vartheta + 1)^2} \frac{\vartheta\lambda}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{\vartheta\lambda}{\vartheta\lambda + 1} + \frac{1}{\gamma\theta} \right) \frac{d\lambda/d\vartheta}{\lambda} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} - \left( \frac{1}{\lambda\vartheta + 1} \frac{\vartheta\lambda}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \vartheta\lambda + \frac{\vartheta\lambda + 1}{\gamma\theta} \right) \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} - \left( \frac{1}{\lambda\vartheta + 1} \frac{\frac{1-\lambda}{(1+\vartheta)(\vartheta\lambda+1)}}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{1}{\gamma\theta} \left( \frac{\theta\vartheta\lambda}{\sigma - 1} - \vartheta\lambda + \vartheta\lambda + 1 \right) \right) \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} - \frac{\frac{\sigma\theta}{\sigma-1} - 1}{\lambda_{ii}\vartheta + 1} \frac{\vartheta\lambda}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{\frac{\sigma\theta}{\sigma-1} - 1}{\gamma\theta} \left( \frac{\theta\vartheta\lambda}{\sigma - 1} + 1 \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma-1} - 1 \right) \vartheta\lambda}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{1}{\gamma\theta} \left( \frac{\theta}{\sigma - 1} - 1 - \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \left( \frac{\theta\vartheta\lambda}{\sigma - 1} + 1 \right) \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1}{\lambda_{ii}\vartheta + 1} \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma-1} - 1 \right) \vartheta\lambda}{\frac{1+\vartheta}{\lambda_{ii}\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{1}{\gamma\theta} \left( \frac{\theta}{\sigma - 1} - 1 - \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \frac{\theta\vartheta\lambda}{\sigma - 1} - \frac{\sigma\theta}{\sigma - 1} + 1 \right) \right] \frac{1 - \lambda_{ii}}{(1 + \vartheta)(\vartheta\lambda_{ii} + 1)} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma-1} - 1 \right) \vartheta\lambda}{\frac{1+\vartheta}{\lambda_{ii}\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{1}{\gamma\theta} \left( \frac{\theta}{\sigma - 1} - \frac{\sigma\theta}{\sigma - 1} - \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \frac{\theta\vartheta\lambda}{\sigma - 1} \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1}{\lambda\vartheta + 1} \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma-1} - 1 \right) \vartheta\lambda}{\frac{1+\vartheta}{\lambda\vartheta+1} - (1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{\gamma} \left( 1 + \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \frac{\vartheta\lambda}{\sigma - 1} \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \\ &= \left[ \frac{1 + \vartheta - \left( \frac{\sigma\theta}{\sigma-1} - 1 \right) \vartheta\lambda}{1 + \vartheta - (\lambda_{ii}\vartheta + 1)(1 - \gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{\gamma} \left( 1 + \left( \frac{\sigma\theta}{\sigma - 1} - 1 \right) \frac{\vartheta\lambda}{\sigma - 1} \right) \right] \frac{1 - \lambda}{(1 + \vartheta)(\vartheta\lambda + 1)} \end{aligned}$$

Evaluating the welfare change at  $\vartheta = -1/\sigma$ , we obtain

$$\begin{aligned} \left. \frac{dW/W}{d\vartheta} \right|_{\vartheta=-\frac{1}{\sigma}} &\propto \frac{\frac{\sigma-1}{\sigma} + \left(\frac{\sigma\theta}{\sigma-1} - 1\right) \frac{\lambda}{\sigma}}{\frac{\sigma-1}{\sigma} - \left(-\frac{\lambda}{\sigma} + 1\right) (1-\gamma) \frac{\sigma-1}{\sigma}} + \frac{1}{\gamma} \left(1 + \left(\frac{\sigma\theta}{\sigma-1} - 1\right) \frac{\lambda}{\sigma(\sigma-1)}\right) \\ &= \frac{1 + \left(\frac{\sigma\theta}{\sigma-1} - 1\right) \frac{\lambda}{\sigma-1}}{1 - \left(1 - \frac{\lambda}{\sigma}\right) (1-\gamma)} + \frac{1}{\gamma} \left(1 + \left(\frac{\sigma\theta}{\sigma-1} - 1\right) \frac{\lambda}{\sigma(\sigma-1)}\right) > 0. \end{aligned}$$

Hence, starting from  $\vartheta = -1/\sigma$ , an increase in  $\vartheta$ , i.e., lowering the rate of the import subsidy, raises welfare. The optimal policy is an import subsidy with a rate  $|\vartheta| < 1/\sigma$ .

## A.9 Un-cooperative trade policy

**Income multiplier.** The income multiplier is given by

$$\begin{aligned} \mu_H &= \frac{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) \lambda_{HH} + (1 + t_{FH} - (1 - \gamma) \frac{\sigma-1}{\sigma}) \frac{1 - \lambda_{HH}}{1 + t_{FH}}}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) \left(\lambda_{HH} + \frac{1 - \lambda_{HH}}{1 + t_{FH}}\right)} \\ &= \frac{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) \lambda_{HH} (1 + t_{FH}) + (1 + t_{FH} - (1 - \gamma) \frac{\sigma-1}{\sigma}) (1 - \lambda_{HH})}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (\lambda_{HH} (1 + t_{FH}) + 1 - \lambda_{HH})} \\ &= \frac{\lambda_{HH} (1 + t_{FH}) - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} (1 + t_{FH}) + (1 + t_{FH}) (1 - \lambda_{HH}) - (1 - \gamma) \frac{\sigma-1}{\sigma} (1 - \lambda_{HH})}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (\lambda_{HH} + \lambda_{HH} t_{FH} + 1 - \lambda_{HH})} \\ &= \frac{- (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} (1 + t_{FH}) + 1 + t_{FH} - (1 - \gamma) \frac{\sigma-1}{\sigma} (1 - \lambda_{HH})}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (\lambda_{HH} t_{FH} + 1)} \\ &= \frac{- (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} t_{FH} + 1 + t_{FH} - (1 - \gamma) \frac{\sigma-1}{\sigma} + (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (\lambda_{HH} t_{FH} + 1)} \\ &= \frac{t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + 1 - (1 - \gamma) \frac{\sigma-1}{\sigma}}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (\lambda_{HH} t_{FH} + 1)} \end{aligned}$$

Totally differentiating this expression, we obtain

$$\begin{aligned}
\frac{d\mu_H}{\mu_H} &= \frac{[1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] (\lambda_{HH} t_{FH} + 1) - (t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + \kappa^{-1}) \lambda_{HH}}{(\lambda_{HH} t_{FH} + 1) (t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + \kappa^{-1})} dt_{FH} \\
&\quad - \left( \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} + \frac{(1 - \gamma) \frac{\sigma-1}{\sigma} t_{FH} \lambda_{HH}}{t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + \kappa^{-1}} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}}{t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + \kappa^{-1}} - \frac{\lambda_{HH}}{\lambda_{HH} t_{FH} + 1} \right) dt_{FH} \\
&\quad - \left( 1 + \frac{(1 - \gamma) \frac{\sigma-1}{\sigma}}{\mu_H (1 - (1 - \gamma) \frac{\sigma-1}{\sigma})} \right) \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}) (\lambda_{HH} t_{FH} + 1)}{t_{FH} [1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}] + \kappa^{-1}} - \lambda_{HH} \right) \frac{dt_{FH}}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \kappa \frac{\mu_H + (1 - \gamma) \frac{\sigma-1}{\sigma} (1 - \mu_H)}{\mu_H} \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \kappa \frac{1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH}}{\mu_H} - \lambda_{HH} \right) \frac{dt_{FH}}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \kappa \left( 1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \kappa \frac{1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} - \lambda_{HH} \mu_H (1 - (1 - \gamma) \frac{\sigma-1}{\sigma})}{\mu_H} \frac{dt_{FH}}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \kappa \left( 1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\lambda_{HH} t_{FH} + 1}{\kappa} \frac{d\mu_H}{\mu_H} &= \frac{1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} - \lambda_{HH} \mu_H (1 - (1 - \gamma) \frac{\sigma-1}{\sigma})}{\mu_H} dt_{FH} \\
&\quad - \left( 1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \lambda_{HH} t_{FH} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \frac{1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \lambda_{HH} - \lambda_{HH} \mu_H + \lambda_{HH} \mu_H (1 - \gamma) \frac{\sigma-1}{\sigma}}{\mu_H} dt_{FH} \\
&\quad - \left( 1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \lambda_{HH} t_{FH} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{1}{\mu_H} - \lambda_{HH} + (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \lambda_{HH} \right) dt_{FH} \\
&\quad - \left( 1 - (1 - \gamma) \frac{\sigma-1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \lambda_{HH} t_{FH} \frac{d\lambda_{HH}}{\lambda_{HH}}
\end{aligned}$$



**Gross output multiplier.** The gross output multiplier is given by

$$\begin{aligned}
\tilde{\mu}_H &\equiv \frac{1}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) \left( \lambda_{HH} + \frac{1-\lambda_{HH}}{1+t_{FH}} \right)} \\
&= \frac{1 + t_{FH}}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) ((1 + t_{FH}) \lambda_{HH} + 1 - \lambda_{HH})} \\
&= \frac{1 + t_{FH}}{(1 - (1 - \gamma) \frac{\sigma-1}{\sigma}) (t_{FH} \lambda_{HH} + 1)}.
\end{aligned}$$

Totally differentiating this expression, we obtain

$$\begin{aligned}
\frac{d\tilde{\mu}_H}{\mu_H} &= \left( \frac{1}{1 + t_{FH}} - \frac{\lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \right) dt_{FH} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{t_{FH} \lambda_{HH} + 1}{1 + t_{FH}} - \lambda_{HH} \right) \frac{dt_{FH}}{t_{FH} \lambda_{HH} + 1} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{t_{FH} \lambda_{HH} + 1}{1 + t_{FH}} - \lambda_{HH} \right) \frac{dt_{FH}}{t_{FH} \lambda_{HH} + 1} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= (t_{FH} \lambda_{HH} + 1 - \lambda_{HH} (1 + t_{FH})) \frac{1}{1 + t_{FH}} \frac{dt_{FH}}{t_{FH} \lambda_{HH} + 1} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= (t_{FH} \lambda_{HH} + 1 - \lambda_{HH} - \lambda_{HH} t_{FH}) \frac{1}{1 + t_{FH}} \frac{dt_{FH}}{t_{FH} \lambda_{HH} + 1} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \frac{1 - \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{dt_{FH}}{1 + t_{FH}} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}}
\end{aligned}$$

**Balanced trade (symmetric countries).** With two symmetric countries, balanced trade reads

$$\frac{w_H}{w_F} = (1 + t_{FH}) \frac{1 - \lambda_{FF} \tilde{\mu}_F}{1 - \lambda_{HH} \tilde{\mu}_H}.$$

Totally differentiating this expression exploiting the facts that  $t_{FF} = t_{HF} = 0$ , we obtain

$$\begin{aligned}
\frac{d\frac{w_H}{w_F}}{w_H/w_F} &= \frac{1}{1+t_{FH}} dt_{FH} + \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} - \frac{d\tilde{\mu}_H}{\mu_H} \\
&= \frac{1}{1+t_{FH}} dt_{FH} + \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} \\
&\quad - \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{dt_{FH}}{1+t_{FH}} + \frac{t_{FH}\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left(1 - \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1}\right) \frac{dt_{FH}}{1+t_{FH}} + \left(1 + \frac{t_{FH}(1-\lambda_{HH})}{t_{FH}\lambda_{HH}+1}\right) \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} \\
&= \frac{t_{FH}\lambda_{HH}+1-1+\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{dt_{FH}}{1+t_{FH}} + \frac{t_{FH}\lambda_{HH}+1+t_{FH}-t_{FH}\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&\quad - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} \\
&= \frac{t_{FH}+1}{t_{FH}\lambda_{HH}+1} \lambda_{HH} \frac{dt_{FH}}{1+t_{FH}} + \frac{1+t_{FH}}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} \\
&= \frac{\lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} + \frac{1+t_{FH}}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}}
\end{aligned}$$

**Home's domestic expenditure share.** With two symmetric countries, Home's domestic expenditure share is given by

$$\lambda_{HH} = \frac{1}{1 + \left(\frac{\tilde{\mu}_F}{\tilde{\mu}_H}\right)^{\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1}} \tau^{-\theta} (1+t_{FH})^{1-\frac{\sigma\theta}{\sigma-1}} \left(\frac{f_{FH}}{f_{HH}}\right)^{\frac{\sigma-1-\theta}{\sigma-1}} \left(\frac{w_H}{w_F}\right)^{\frac{\theta\sigma}{\sigma-1}-1} \left(\frac{\lambda_{HH}}{\lambda_{FF}}\right)^{\frac{1-\gamma}{\gamma}}}.$$

Totally differentiating this expression, we obtain

$$\begin{aligned}
-\frac{1}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} &= -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \left(1 - \frac{\sigma\theta}{\sigma-1}\right) \frac{dt_{FH}}{1+t_{FH}} + \left(\frac{\theta\sigma}{\sigma-1} - 1\right) \frac{d\frac{w_H}{w_F}}{w_H/w_F} \\
&\quad + \frac{1-\gamma}{\gamma} \left(\frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{d\lambda_{FF}}{\lambda_{FF}}\right).
\end{aligned}$$

**Foreign expenditure share.** By analogy, the change in Foreign's expenditure share is

$$\begin{aligned}
-\frac{1}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}} &= \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{d\frac{w_H}{w_F}}{w_H/w_F} \\
&\quad + \frac{1-\gamma}{\gamma} \left(\frac{d\lambda_{FF}}{\lambda_{FF}} - \frac{d\lambda_{HH}}{\lambda_{HH}}\right) \Leftrightarrow \\
-\left(\frac{1}{1-\lambda_{FF}} + \frac{1-\gamma}{\gamma}\right) \frac{d\lambda_{FF}}{\lambda_{FF}} &= \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{d\frac{w_H}{w_F}}{w_H/w_F} - \frac{1-\gamma}{\gamma\theta} \frac{d\lambda_{HH}}{\lambda_{HH}}.
\end{aligned}$$

Using balanced trade to substitute out the change in the relative wage, we obtain

$$\begin{aligned}
-\left(\frac{1}{1-\lambda_{FF}} + \frac{1-\gamma}{\gamma}\right) \frac{d\lambda_{FF}}{\lambda_{FF}} &= \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{\lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} \\
&\quad - \frac{1-\gamma}{\gamma} \frac{d\lambda_{HH}}{\lambda_{HH}} + \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{1+t_{FH}}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&\quad - \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}}.
\end{aligned}$$

Collecting terms, we obtain

$$\begin{aligned}
\left[\frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) \lambda_{FF} - 1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}\right] \frac{d\lambda_{FF}}{\lambda_{FF}} &= \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) \lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} \\
&\quad + \left(\frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) (1+t_{FH})}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma}\right) \frac{d\lambda_{HH}}{\lambda_{HH}}
\end{aligned}$$

Hence,

$$\frac{d\lambda_{FF}}{\lambda_{FF}} = \frac{\frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) \lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} + \left(\frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) (1+t_{FH})}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma}\right) \frac{d\lambda_{HH}}{\lambda_{HH}}}{\frac{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) \lambda_{FF} - 1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}}$$

**Home's domestic expenditure share, c'd.** Substituting out the change in the relative wage, we obtain

$$\begin{aligned}
\left(-\frac{1}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma}\right) \frac{d\lambda_{HH}}{\lambda_{HH}} &= -\frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
&\quad + \left(1 - \frac{\theta\sigma}{\sigma-1}\right) \frac{dt_{FH}}{1+t_{FH}} + \left(\frac{\theta\sigma}{\sigma-1} - 1\right) \frac{\lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} \\
&\quad + \left(\frac{\theta\sigma}{\sigma-1} - 1\right) \frac{1+t_{FH}}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&\quad - \frac{1-\gamma}{\gamma} \frac{d\lambda_{FF}}{\lambda_{FF}} - \left(\frac{\theta\sigma}{\sigma-1} - 1\right) \frac{\lambda_{FF}}{1-\lambda_{FF}} \frac{d\lambda_{FF}}{\lambda_{FF}}
\end{aligned}$$

Collecting terms, we obtain

$$\begin{aligned}
& \left( -\frac{1}{1-\lambda_{HH}} - \frac{(1+t_{FH})\left(\frac{\theta\sigma}{\sigma-1}-1\right)}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
= & -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} + \left( \frac{\theta\sigma}{\sigma-1}-1 \right) \left( \frac{\lambda_{HH}}{t_{FH}\lambda_{HH}+1} - \frac{1}{1+t_{FH}} \right) dt_{FH} \\
& - \left( \frac{1-\gamma}{\gamma} + \frac{\left(\frac{\theta\sigma}{\sigma-1}-1\right)\lambda_{FF}}{1-\lambda_{FF}} \right) \frac{d\lambda_{FF}}{\lambda_{FF}}
\end{aligned}$$

where

$$\left( \frac{\lambda_{HH}}{t_{FH}\lambda_{HH}+1} - \frac{1}{1+t_{FH}} \right) dt_{FH} = -\frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{dt_{FH}}{1+t_{FH}}.$$

Hence,

$$\begin{aligned}
& \left( -\frac{1}{1-\lambda_{HH}} - \frac{(1+t_{FH})\left(\frac{\theta\sigma}{\sigma-1}-1\right)}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
= & -\frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} - \left( \frac{\theta\sigma}{\sigma-1}-1 \right) \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{dt_{FH}}{1+t_{FH}} \\
& - \left( \frac{1-\gamma}{\gamma} + \frac{\left(\frac{\theta\sigma}{\sigma-1}-1\right)\lambda_{FF}}{1-\lambda_{FF}} \right) \frac{d\lambda_{FF}}{\lambda_{FF}}
\end{aligned}$$

Substituting out the change in Foreign's domestic expenditure share, we obtain

$$\begin{aligned}
& \left( -\frac{1}{1-\lambda_{HH}} - \frac{(1+t_{FH})\left(\frac{\theta\sigma}{\sigma-1}-1\right)}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
= & -\frac{\frac{1-\gamma}{\gamma} + \frac{\left(\frac{\theta\sigma}{\sigma-1}-1\right)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{\left(1-\frac{\theta\sigma}{\sigma-1}\right)\lambda_{FF}-1} {1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \left( \frac{\left(1-\frac{\theta\sigma}{\sigma-1}\right)(1+t_{FH})}{t_{FH}\lambda_{HH}+1} \frac{\lambda_{HH}}{1-\lambda_{HH}} - \frac{1-\gamma}{\gamma} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
& - \left( \frac{\theta\sigma}{\sigma-1}-1 \right) \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH}+1} \frac{dt_{FH}}{1+t_{FH}} - \frac{\frac{1-\gamma}{\gamma} + \frac{\left(\frac{\theta\sigma}{\sigma-1}-1\right)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{\left(1-\frac{\theta\sigma}{\sigma-1}\right)\lambda_{FF}-1} {1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \frac{\left(1-\frac{\theta\sigma}{\sigma-1}\right)\lambda_{HH}}{t_{FH}\lambda_{HH}+1} dt_{FH} \\
& - \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} - \frac{\frac{1-\gamma}{\gamma} + \frac{\left(\frac{\theta\sigma}{\sigma-1}-1\right)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{\left(1-\frac{\theta\sigma}{\sigma-1}\right)\lambda_{FF}-1} {1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta-(\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H}
\end{aligned}$$

Collecting terms for the change in the domestic expenditure share, we obtain



and

$$\begin{aligned}
& \frac{1 + \left(\frac{\theta\sigma}{\sigma-1} - 1\right) (\lambda_{FF} + \mu_H \lambda_{HH})}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1} \frac{1}{1 - \lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
= & \left( -\frac{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF}}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1} \lambda_{HH} - \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{\left(\frac{\theta\sigma}{\sigma-1} - 1\right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \Leftrightarrow \\
& \frac{1 + \left(\frac{\theta\sigma}{\sigma-1} - 1\right) (\lambda_{FF} + \mu_H \lambda_{HH})}{1 - \lambda_{HH}} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \lambda_{HH} + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + 1 \right) \right) \frac{\left( \frac{\theta\sigma}{\sigma-1} - 1 \right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \lambda_{HH} + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{\left( \frac{\theta\sigma}{\sigma-1} - 1 \right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \left( \lambda_{HH} + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{\left( \frac{\theta\sigma}{\sigma-1} - 1 \right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \frac{\lambda_{HH} + \lambda_{HH} t_{FH} + 1 - \lambda_{HH}}{1 + t_{FH}} + \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{\left( \frac{\theta\sigma}{\sigma-1} - 1 \right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \frac{\lambda_{HH} t_{FH} + 1 + 1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{\left( \frac{\theta\sigma}{\sigma-1} - 1 \right)}{t_{FH} \lambda_{HH} + 1} dt_{FH} \\
= & \left( \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + \frac{1 - \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} \\
= & \left( 1 - \lambda_{HH} \mu_H + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + \frac{1 - \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} - 1 + \lambda_{HH} \mu_H \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} \\
= & \left( 1 - \lambda_{HH} \mu_H + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + \frac{1 - \lambda_{HH} - t_{FH} \lambda_{HH} - 1}{t_{FH} \lambda_{HH} + 1} + \lambda_{HH} \mu_H \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} \\
= & \left( 1 - \lambda_{HH} \mu_H + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} - \frac{1 + t_{FH}}{t_{FH} \lambda_{HH} + 1} \lambda_{HH} + \lambda_{HH} \mu_H \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} \\
= & \left( 1 - \lambda_{HH} \mu_H + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} - \mu_H \lambda_{HH} + \lambda_{HH} \mu_H \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}} \\
= & \left( 1 - \lambda_{HH} \mu_H + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} \right) \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{dt_{FH}}{1 + t_{FH}}
\end{aligned}$$

Collecting terms for the gross output multiplier, we obtain

$$\begin{aligned}
& -\frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} - \frac{\frac{1-\gamma}{\gamma} + \frac{(\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = -\left(1 + \frac{\frac{1-\gamma}{\gamma} + \frac{(\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}}\right) \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = -\frac{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma} + \frac{1-\gamma}{\gamma} + \frac{(\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = -\frac{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1 + (\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}-1}{1-\lambda_{FF}} - \frac{1-\gamma}{\gamma}} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = \frac{1}{\left(1 - \frac{\theta\sigma}{\sigma-1}\right) \lambda_{FF} - 1 - \frac{1-\gamma}{\gamma} (1 - \lambda_{FF})} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = -\frac{1}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1 + \frac{1-\gamma}{\gamma} (1 - \lambda_{FF})} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H}
\end{aligned}$$

Combining direct and gross output multiplier effects

$$\begin{aligned}
& \left( -\frac{\frac{1-\gamma}{\gamma} + \frac{(\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}+1}{1-\lambda_{FF}}} \lambda_{HH} - \frac{1-\lambda_{HH}}{1+t_{FH}} \right) \frac{\left(\frac{\theta\sigma}{\sigma-1} - 1\right)}{t_{FH}\lambda_{HH} + 1} dt_{FH} \\
& - \frac{1}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1 + \frac{1-\gamma}{\gamma} (1 - \lambda_{FF})} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{d\tilde{\mu}_H}{\mu_H} \\
& = \left( -\frac{\frac{1-\gamma}{\gamma} + \frac{(\frac{\theta\sigma}{\sigma-1}-1)\lambda_{FF}}{1-\lambda_{FF}}}{\frac{(1-\frac{\theta\sigma}{\sigma-1})\lambda_{FF}+1}{1-\lambda_{FF}}} \lambda_{HH} - \frac{1-\lambda_{HH}}{1+t_{FH}} \right) \frac{\left(\frac{\theta\sigma}{\sigma-1} - 1\right)}{t_{FH}\lambda_{HH} + 1} dt_{FH} \\
& - \frac{1}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1 + \frac{1-\gamma}{\gamma} (1 - \lambda_{FF})} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{1-\lambda_{HH}}{t_{FH}\lambda_{HH} + 1} \frac{dt_{FH}}{1+t_{FH}} \\
& + \frac{1}{\left(\frac{\theta\sigma}{\sigma-1} - 1\right) \lambda_{FF} + 1 + \frac{1-\gamma}{\gamma} (1 - \lambda_{FF})} \frac{1-\gamma}{\gamma} \frac{\theta - (\sigma-1)}{\sigma-1} \frac{t_{FH}\lambda_{HH}}{t_{FH}\lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}}
\end{aligned}$$

Collecting direct effects







$$\begin{aligned}
\frac{dW_H}{W_H} &= \frac{d\mu_H}{\mu_H} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{d\tilde{\mu}_H}{\tilde{\mu}_H} - \frac{1}{\gamma\theta} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \frac{1}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} \frac{1}{\lambda_{HH} t_{FH} + 1} \left[ \left( \frac{1}{\mu_H} - \lambda_{HH} + (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \lambda_{HH} \right) dt_{FH} - \left( 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right) \right. \\
&\quad \left. + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \left( \frac{1 - \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{dt_{FH}}{1 + t_{FH}} - \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} \right) - \frac{1}{\gamma\theta} \frac{d\lambda_{HH}}{\lambda_{HH}} \right] \\
&= \frac{1}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} \frac{\frac{1}{\mu_H} - \lambda_{HH} + (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \lambda_{HH}}{\lambda_{HH} t_{FH} + 1} dt_{FH} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{1 - \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{dt_{FH}}{1 + t_{FH}} \\
&\quad - \frac{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{1}{\gamma\theta} \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{\frac{1}{\mu_H} - \lambda_{HH} + (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \lambda_{HH}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{dt_{FH}}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \left( \frac{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{t_{FH} \lambda_{HH}}{t_{FH} \lambda_{HH} + 1} + \frac{1}{\gamma\theta} \right) \frac{d\lambda_{HH}}{\lambda_{HH}} \\
&= \left( \frac{\frac{1}{\mu_H} - \lambda_{HH} \left( 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right)}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{dt_{FH}}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \left( \left( \frac{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \right) \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} + \frac{1}{\gamma\theta} \right) \frac{d\lambda_{HH}}{\lambda_{HH}}.
\end{aligned}$$

**The role of  $\gamma$  in the absence of selection.** Evaluated at  $t_{FH} = 0$  (and therefor  $\mu_H = 1$ ), we have

$$\begin{aligned}
\left. \frac{dW_H/W_H}{dt_{FH}} \right|_{t_{FH}=0} &= \left( \frac{\frac{1}{\mu_H} - \lambda_{HH} \left( 1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H} \right)}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \frac{1 - \lambda_{HH}}{1 + t_{FH}} \right) \frac{1}{\lambda_{HH} t_{FH} + 1} \\
&\quad - \left( \left( \frac{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma} \frac{\mu_H - 1}{\mu_H}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \right) \frac{\lambda_{HH} t_{FH}}{\lambda_{HH} t_{FH} + 1} + \frac{1}{\gamma\theta} \right) \frac{d\lambda_{HH}/\lambda_{HH}}{dt_{FH}} \\
&= \left( \frac{1 - \lambda_{HH}}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} (1 - \lambda_{HH}) \right) - \frac{1}{\gamma\theta} \frac{d\lambda_{HH}/\lambda_{HH}}{dt_{FH}} \\
&= \left( \frac{1}{1 - (1 - \gamma) \frac{\sigma - 1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma\theta(\sigma - 1)} \right) (1 - \lambda_{HH}) - \frac{1}{\gamma\theta} \frac{d\lambda_{HH}/\lambda_{HH}}{dt_{FH}}
\end{aligned}$$

and

$$\frac{d\lambda_{HH}/\lambda_{HH}}{dt_{FH}} = \frac{\frac{1-\gamma}{\gamma} \left[ 1 - \lambda_{FF} + \frac{\theta - (\sigma - 1)}{\theta\sigma - (\sigma - 1)} (1 - \lambda_{HH}) \right] + \left( \frac{\theta\sigma}{\sigma - 1} - 1 \right) \lambda_{FF} + 1 - \lambda_{HH}}{1 + \left( \frac{\theta\sigma}{\sigma - 1} - 1 \right) (\lambda_{FF} + \lambda_{HH}) + \frac{1-\gamma}{\gamma} (2 - \lambda_{HH} - \lambda_{FF})} (1 - \lambda_{HH}) \left( \frac{\theta\sigma}{\sigma - 1} - 1 \right)$$

Hence,

$$\begin{aligned} \frac{1}{1 - \lambda_{HH}} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} &= \frac{1}{1 - (1 - \gamma) \frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)} - \\ &\frac{\frac{\theta\sigma}{\sigma-1} - 1 \frac{1-\gamma}{\gamma} \left[ 1 - \lambda_{FF} + \frac{\theta - (\sigma-1)}{\theta\sigma - (\sigma-1)} (1 - \lambda_{HH}) \right] + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda_{FF} + 1 - \lambda_{HH}}{\gamma \theta \left[ 1 + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) (\lambda_{FF} + \lambda_{HH}) + \frac{1-\gamma}{\gamma} (2 - \lambda_{HH} - \lambda_{FF}) \right]} \end{aligned}$$

Assuming symmetry in the initial situation, we have

$$\begin{aligned} \frac{1}{1 - \lambda} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} &= \frac{1}{1 - (1 - \gamma) \frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)} - \\ &\frac{\frac{\theta\sigma}{\sigma-1} - 1 \frac{1-\gamma}{\gamma} (1 - \lambda) \left[ 1 + \frac{\theta - (\sigma-1)}{\theta\sigma - (\sigma-1)} \right] + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda + 1 - \lambda}{\gamma \theta \left[ 1 + 2\lambda \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) + \frac{1-\gamma}{\gamma} 2(1 - \lambda) \right]} \end{aligned}$$

Additionally assuming absence of trade costs, we have  $\lambda = 0.5$ :

$$\begin{aligned} \frac{1}{1 - \lambda} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} &= \frac{1}{1 - (1 - \gamma) \frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)} \\ &- \frac{\frac{\theta\sigma}{\sigma-1} - 1 \frac{1-\gamma}{\gamma} \frac{1}{2} \left[ 1 + \frac{\theta - (\sigma-1)}{\theta\sigma - (\sigma-1)} \right] + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \frac{1}{2} + \frac{1}{2}}{\gamma \theta \left[ 1 + 2\frac{1}{2} \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) + \frac{1-\gamma}{\gamma} 2\frac{1}{2} \right]} \\ &= \frac{1}{1 - (1 - \gamma) \frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)} \\ &- \frac{1}{2} \frac{\frac{\theta\sigma}{\sigma-1} - 1 \frac{1-\gamma}{\gamma} \left[ 1 + \frac{\theta - (\sigma-1)}{\theta\sigma - (\sigma-1)} \right] + \frac{\theta\sigma}{\sigma-1} - 1 + 1}{\gamma \theta \left[ 1 + \frac{\theta\sigma}{\sigma-1} - 1 + \frac{1-\gamma}{\gamma} \right]} \\ &= \frac{1}{1 - (1 - \gamma) \frac{\sigma-1}{\sigma}} + \frac{\theta - (\sigma - 1)}{\gamma \theta (\sigma - 1)} \\ &- \frac{1}{2} \frac{\frac{\theta\sigma}{\sigma-1} - 1 \frac{1-\gamma}{\gamma} \left[ 1 + \frac{\theta - (\sigma-1)}{\theta\sigma - (\sigma-1)} \right] + \frac{\theta\sigma}{\sigma-1}}{\gamma \theta \left[ \frac{\theta\sigma}{\sigma-1} + \frac{1-\gamma}{\gamma} \right]} \end{aligned}$$

Further assume  $\theta \rightarrow \sigma - 1$

$$\begin{aligned}
\frac{1}{1-\lambda} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} &= \frac{1}{1-(1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{2} \frac{\frac{(\sigma-1)\sigma}{\sigma-1} - 1}{\gamma(\sigma-1)} \frac{\frac{1-\gamma}{\gamma} + \frac{\theta\sigma}{\sigma-1}}{\frac{\theta\sigma}{\sigma-1} + \frac{1-\gamma}{\gamma}} \\
&= \frac{1}{1-(1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{2} \frac{\sigma-1}{\gamma(\sigma-1)} \\
&= \frac{1}{1-(1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{2\gamma} = \frac{2\gamma-1+(1-\gamma)\frac{\sigma-1}{\sigma}}{2\gamma(1-(1-\gamma)\frac{\sigma-1}{\sigma})} \\
&= \frac{2\gamma-1+\frac{\sigma-1}{\sigma}-\gamma\frac{\sigma-1}{\sigma}}{2\gamma(1-(1-\gamma)\frac{\sigma-1}{\sigma})} = \frac{\gamma(2-\frac{\sigma-1}{\sigma})-\frac{1}{\sigma}}{2\gamma(1-(1-\gamma)\frac{\sigma-1}{\sigma})} \\
&= \frac{\gamma(\frac{2\sigma-\sigma+1}{\sigma})-\frac{1}{\sigma}}{2\gamma(1-(1-\gamma)\frac{\sigma-1}{\sigma})} = \frac{\gamma(\frac{\sigma+1}{\sigma})-\frac{1}{\sigma}}{2\gamma(1-(1-\gamma)\frac{\sigma-1}{\sigma})} \\
&= \frac{\gamma(\sigma+1)-1}{2\gamma\sigma(1-(1-\gamma)\frac{\sigma-1}{\sigma})}
\end{aligned}$$

Hence,

$$\frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} \geq 0 \Leftrightarrow \gamma(\sigma+1) \geq 1 \Leftrightarrow \gamma \geq \frac{1}{\sigma+1}.$$

**The role of  $\eta$ .** Recall that

$$\begin{aligned}
\frac{1}{1-\lambda} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} &= \frac{1}{1-(1-\gamma)\frac{\sigma-1}{\sigma}} + \frac{\theta-(\sigma-1)}{\gamma\theta(\sigma-1)} \\
&\quad - \frac{\frac{\theta\sigma}{\sigma-1} - 1}{\gamma\theta} \frac{\frac{1-\gamma}{\gamma}(1-\lambda) \left[ 1 + \frac{\theta-(\sigma-1)}{\theta\sigma-(\sigma-1)} \right] + \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) \lambda + 1 - \lambda}{1 + 2\lambda \left( \frac{\theta\sigma}{\sigma-1} - 1 \right) + \frac{1-\gamma}{\gamma} 2(1-\lambda)}
\end{aligned}$$

Consider the limiting case  $\theta \rightarrow \sigma - 1$

$$\frac{1}{1-\lambda} \frac{dW_H/W_H}{dt_{FH}} \Big|_{t_{FH}=0} = \frac{1}{1-(1-\gamma)\frac{\sigma-1}{\sigma}} - \frac{1}{\gamma} \frac{\frac{1-\gamma}{\gamma}(1-\lambda) + (\sigma-1)\lambda + 1 - \lambda}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)}$$

Moreover, consider  $\gamma = 1/(\sigma + 1)$ . Then,

$$\begin{aligned}
\frac{1}{1-\lambda} \left. \frac{dW_H/W_H}{dt_{FH}} \right|_{t_{FH}=0} &= \frac{1}{1 - \left(1 - \frac{1}{\sigma+1}\right) \frac{\sigma-1}{\sigma}} - (\sigma+1) \frac{\frac{1-\gamma}{\gamma} (1-\lambda) + (\sigma-1)\lambda + 1 - \lambda}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
&= \frac{\sigma+1}{2} \left( 1 - 2 \frac{\frac{1-\gamma}{\gamma} (1-\lambda) + (\sigma-1)\lambda + 1 - \lambda}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)} \right) \\
&= \frac{\sigma+1}{2} \frac{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda) - \frac{1-\gamma}{\gamma} 2(1-\lambda) - 2(\sigma-1)\lambda - 2(1-\lambda)}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
&= \frac{\sigma+1}{2} \frac{1 - 2(1-\lambda)}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)} \\
&= \frac{(\sigma+1)(\lambda - 0.5)}{1 + 2\lambda(\sigma-1) + \frac{1-\gamma}{\gamma} 2(1-\lambda)}.
\end{aligned}$$

This expression implies that with  $\gamma = 1/(\sigma + 1)$ , which implies that laissez-faire is optimal with  $\eta = 1$ , with  $\eta < 1 \Leftrightarrow \lambda > 0.5$ , a tariff would be optimal.