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Conference Paper

Firm Dynamics and Occupational Choice

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel -
Session: Labor Economics: Theory, No. E20-V2

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Koelle, Michael (2016) : Firm Dynamics and Occupational Choice, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Labor Economics: Theory, No. E20-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

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FIRM DYNAMICS AND OCCUPATIONAL CHOICE

[This version: February 2016]

Abstract

Abstract: I look at the dynamics of firms when investment decisions interact with occupational choice. To model the implications for firm survival and growth, I extend a neoclassical growth model by an endogenous shutdown condition that is driven by the reservation wage in alternative employment. This model is able to generate multiple steady-state equilibria that arise through convexities in the optimal growth path of a firm. I provide empirical evidence consistent with the model predictions using panel data from urban Colombia. I also structurally estimate the model to identify the wage function in a way that is robust to occupational choices being driven by particular wage offers that are observed as subsequent outcomes. The findings are useful for understanding heterogeneous economic decisions of the large number of self-employed small firm owners in developing countries.

Keywords: Occupational choice, search frictions, entrepreneurship,

JEL Codes: J24, J64, L26, O17.

1 Introduction

Small firms, owned and operated by an entrepreneur, unite several economic elements. They are firms, because they turn capital and labour inputs into output. But they are also a source of employment for their self-employed owner. Given their small size and profits, the benefits of separation of ownership and control (Fama (1980), Diamond (1984)) do not outweigh transaction costs.

This is particularly true in developing countries, where a large share of the workforce are self-employed owners of small firms, often referred to as microenterprises. A puzzle in the recent literature has been why microenterprises do not grow, even if returns to capital are as high as 15 percent per month in Ghana (Fafchamps, McKenzie, Quinn, and Woodruff (2014)) or 20 percent in Mexico (McKenzie and Woodruff (2008)).

In this paper, I provide a possible explanation for this apparent puzzle. When ownership and control are not separable, then entrepreneurs base their investment policies not only on the outside option of capital, but also on the outside option of their labour. In urban markets for differentiated labour, search frictions imply that prospective workers are only offered a small sample of all wages that prevail in the market at a particular point in time. At the same time, search frictions allow for a dispersion in wages, even conditional on worker productivity. Forward-looking entrepreneurs anticipate the possibility that, in future periods, they might receive an attractive wage offer, shut down their business, and take up the offer. Rational investors discount the value of future returns by this shutdown probability. Then, the possibility of a future exit to wage-employment provides an incentives for small firm owners to underinvest into their business. This suggest that, if we want to understand the returns to capital in small firms, and in particular the returns documented for microenterprises in developing countries, we should look at the outside option of wage-employment in a labour market with search frictions.

In an empirical application of the model, I measure the value of job opportunities in wage employment, relative to being self-employed, for owners of small businesses in urban Colombia between 2007-2010. Exploiting the US financial crisis as an external shock to the Colombian economy and its labour market, I am able to identify the joint distribution of earnings in wage employment and self-employment. I structurally estimate the model

to identify the joint wage distribution in a way that is robust to the presence of unobserved earnings components that are potentially correlated across sectors, and to occupational choices being driven by random components of particular wage offers. With this, I can analyse how differences in alternative wage employment opportunities affect the choices small firm owners make about their business. In line with the model predictions, I find that firms with owners that have attractive opportunities in wage employment are more likely to shut down the firm, and less likely to grow.

Several stylised facts from the data suggest the search frictions are present in developing countries, including in Colombia. Self-employment and wage-employment coexist in urban labour markets of all developing countries, and transitions between the two sectors are quite common. On the other side, this means that individuals switch between occupations quite frequently. This kind of behaviour is consistent with frequent updates on the relative attractiveness of sectors. At the same time, there does not seem to be a clear dominance of one sector over another. This is evident from Figure 1, which plots the wage and business income distributions in the urban labour markets of Ghana and Colombia. It is clear that neither employment sector offers incomes that make it always strictly preferable. Rather, occupational choice is based on the relative earnings possibilities at the individual level and depends both on the initial position of a worker in the earnings distribution, and on the earnings she is subsequently offered.

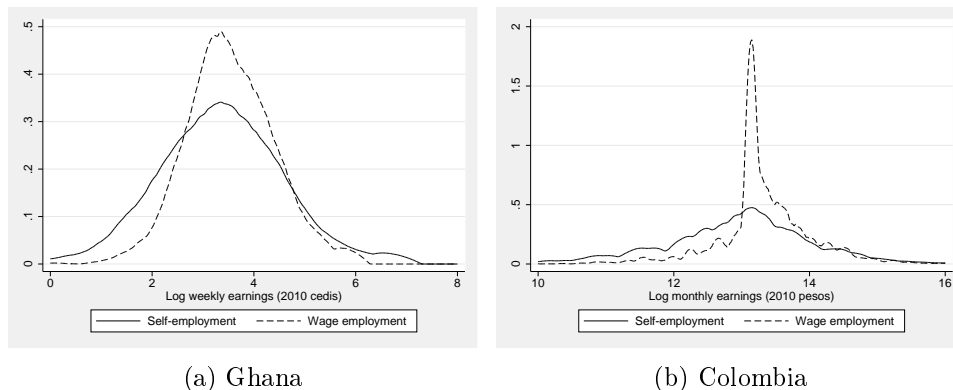


FIGURE 1
EARNINGS DISTRIBUTIONS IN WAGE AND SELF-EMPLOYMENT

I propose a simple but general two-period model that captures this environment and how it can affect investment decisions of small firm owners. I nest a standard model of optimal investment under uncertainty within a two-sector labour market search model. Self-employed individuals can engage in on-the-job search for a wage job. When a sufficiently attractive job offer arrives, agents will take it and shut down their firm. Effectively, the expectation about the shutdown probability acts as an additional discount factor in the intertemporal asset allocation problem. This endogenous effective discount factor depends on the reservation wage, and serves as the critical transmission channel between occupational choice and optimal capital accumulation.

In isolation – when attractive wage employment opportunities are absent – the model collapses to a neoclassical investment model. When I include occupational choice, and therefore the outside option of employment, the resulting path of optimal investment can be convex. This implies that there can be multiple steady states. Then, the initial asset endowment determines which equilibrium is reached – what in development economics is described as a ‘poverty trap’ (Kraay and McKenzie (2014)). Whether convexities arise, and whether they change the law of motion sufficiently to create multiple equilibria, depends critically on the relative location of business incomes and the wage distribution. While an improvement in the wage distribution decreases investment throughout the firm size distribution, the effect is strongest where the wage profile at the reservation wage is particularly steep. Given the earnings distributions in Figure 1 that are typically found in developing countries, this applies particularly in areas towards the middle of the firm size distribution.

I provide empirical evidence that supports the relevance of this model. I analyse the model’s testable predictions with panel data on labour force participants in urban Colombia. An empirical challenge arises because in a model of job search, employment transitions are endogenous to both unobserved heterogeneity of the worker and random components of a wage offer. I overcome these challenges to identification of the full wage distribution by imposing the selection mechanism suggested by the model. I implement this by structurally estimating the theoretical model. I find that business owners with relatively better prospects in wage employment are more likely to switch from self-employment to wage employment. They are also less likely to grow their firms in terms of employment. The size of both of these effects follows an inverse U-shaped pattern: within the group of small owner-operated

firms, they are strongest for business owners in the middle of the firm size distribution. I also document evidence for substantial search frictions in occupational choice in Colombia. Only a third of the self-employed receive a job offer in wage employment every year.

With this, my paper makes several contributions to the literature. Firstly, I provide a framework that is able to explain a number of puzzles documented by a recent literature on small firms and microenterprises in developing countries. The evidence on the effect of policies aimed at alleviating constraints to the business performance and environment of microenterprises is mixed (De Mel, McKenzie, and Woodruff (2008), (2012); Karlan and Valdivia (2011); Fafchamps, McKenzie, Quinn, and Woodruff (2014); Karlan, Knight, and Udry (2015)). Some programmes show impacts on the performance of the average firm, while others do not. The introduction of microcredit is not a transformative policy for the average microenterprise (Banerjee, Duflo, Glennerster, and Kinnan (2015)). Business training programmes may improve business practices, but not firm size or capital stock (McKenzie and Woodruff (2014)). Still, marginal returns to capital are very high (De Mel, McKenzie, and Woodruff (2008), Fafchamps, McKenzie, Quinn, and Woodruff (2014)). Beyond looking at the simple average, the literature documents a very heterogeneous distribution of treatment effects. The largest absolute returns typically accrue to the ex ante largest firms. Programmes targeted at particularly promising entrepreneurs also find large positive effects of capital grants. (Fafchamps and Woodruff (2015), McKenzie (2015), Fafchamps and Quinn (2015)).

The convex paths of asset choices that arise from my model are able to rationalise many of these findings. Shocks to capital only have lasting effects if they allow firms to move to a path towards a higher equilibrium. Otherwise they merely speed up an accumulation process that would have happened in the absence of a programme; or they have no impact at all for firms already at their equilibrium size. Productivity improvements similarly can have lasting effects for the ex ante most successful firms if they lead to the emergence of higher equilibria, and if firms start to move towards them.

On a theoretical level, my paper contributes to the literature on occupational choice and self-employment (Lucas (1978), Kihlstrom and Laffont (1979), Jovanovic (1982), Hopenhayn (1992), Evans and Jovanovic (1989)). While the literature has considered capital constraints, human capital accumulating, and learning about comparative advantage, I add to this literature

by considering the role of uncertainty and search frictions in the wage labour market. I do so by combining elements of models of job search (Mortensen (1970), Lippman and McCall (1976), Mortensen and Pissarides (1999); Manning (2011)) with models of occupational choice and investment behaviour. These channels provide another rationale for individuals to revise their initial occupational choices. The model goes even further in providing a mechanism where also the *anticipation* of such revisions will have effects on behaviour of business owners.

Conceptually, the mechanisms that the model describes are applicable to any labour market. There is ample evidence for search frictions and earnings dispersion developed (van den Berg (1990), Ridder and van den Berg (2003), Cahuc, Postel-Vinay, and Robin (2006), Adda, Dustmann, Meghir, and Robin (2013)) and developing countries (Meghir, Narita, and Robin (2015), Narita (2014), Lopez-Garcia (2013)) alike. Empirically though, this model is probably most relevant for labour markets of developing countries, especially in emerging middle-income countries where there is a high incidence of wage jobs next to a large self-employment sector. The model and the empirical findings supporting it, speak to a large and long body of literature on the nature of labour markets in developing countries (Lewis (1954), Harris and Todaro (1970), Fields (1975), Magnac (1991)), in particular the role of self-employment (Bosch and Maloney (2010), Günther and Launov (2012), Haywood and Falco (2016)).

Finally, my model provides a natural basis for a simple structural approach that enables identification of two-sector earnings distributions, when unobserved permanent *and* transitory components of earnings – that is, permanent worker heterogeneity and random variation in wages offers – are correlated with the sector choice. My approach addresses the empirical challenge that arises from the central idea of a search model based, namely that idiosyncratic draws from the wage distribution determine whether a job offer is accepted or not. Alternative approaches rely on the assumption that transitions between sectors are driven by random variation, even if they allow for comparative advantage modeled as permanent unobserved heterogeneity that attracts different, though potentially correlated, remuneration across sectors of employment (Card (1996), Lemieux (1998), Suri (2011)) A data-generating process that follows from a search model would violate such assumptions.

The paper is structured as follows. In section 2, I present the model and

conditions for its equilibrium. In section 3, I derive comparative statics. I discuss their implications in section 4 and I summarise the model's testable implications. In section 5 I present the empirical strategy. I implement this on a dataset that I discuss in section 6 and present and discuss the empirical results in section 7. Finally, I conclude in section 8.

2 Conceptual Framework

This framework starts with the idea that owners of small businesses in urban areas have access to an active market for wage jobs. Most of their daily decisions about the business may be on the intensive margin - decisions about buying, selling, and investments. Business owners will periodically consider the extensive margin too - that is, whether to continue or shut down their business. For large firms, exit decisions may be based on alternative uses of capital.¹ For small owner-managed firms and self-employed workers, however, alternative employment opportunities are perhaps really the binding constraints to firm survival. That is, availability and relative attractiveness of wages compared to business income decide whether firm owners shut down the firm and take up a wage job. In the model that I propose, the value from present and future incomes, conditional on frictions, is indeed the only determinant of occupational choice for individuals.

Both the return on assets and the 'return' to labour, in the form of wages earned in the market, are uncertain. What is crucially different, however, is how offers are made and when information is revealed. Financial institutions make *anonymous* offers of financial contracts to the general public. Investors' knowledge of the value of a financial contract does not change between the pre-contracting stage and the moment when the contract is signed.² Employers, on the other hand, make job offers only to *specific* applicants. Workers then only know the *distribution* of wage contracts in the pre-contracting stage, and learn about the value of a specific contract only when an *personal offer* has been made to them. Workers, in the canonical partial equilibrium search model (Stigler (1962), Mortensen (1970), McCall

¹In the classical literature on firms (e.g. Evans (1987), Jovanovic (1982)) exit decisions follow a learning story: firms exit the industry when they have observed their performance sufficiently to conclude that capital is not used in the firm more efficiently than in the market.

²Depending on the kind of contract, this value can be deterministic or stochastic (and then only known in expectation). The principle of anonymous, public contract offers holds for both types

(1970)) go around searching for the optimal contract. During that stage, they never know precisely at which moment in time an acceptable offer will be made to them.

I convey this idea in a simple, but general two-period model. I nest an otherwise standard model of investment behaviour under uncertainty within a simple two-sector labour market search model. Self-employed individuals can engage in on-the-job search for a wage job.³ If they do, they face a distribution of wages and only learn about the value of a particular possible wage offer if and when it arrives.⁴ A decision about investing into the business has to be made before an eventual job offer arrival. The reservation wage summarises the optimal exit strategy, and the two-period model allows for a particularly tractable, closed-form solution of the reservation wage. This allows me to keep the algebra simple and focus on how occupational choice interacts with investment behaviour. The reservation wage determines the endogenous shutdown condition. Effectively, this acts as an additional discount factor when the forward-looking agent makes her investment decision.

The model is able to generate two key insights. Firstly, it can generate non-concavities in the optimal path of capital even with an underlying neo-classical production function. Multiple stationary firm size equilibria may arise as a result, and the initial capital stock determines which equilibrium is reached. Secondly, an improvement in the wage offer distribution decreases incentives for accumulating capital. Individuals with better prospects in wage employment will tend towards a smaller capital stock, in anticipation of the possibility that they may shut down the firm – and lose their investment – when an attractive job offer arrives.

³I model search as an exogenous process. Koelle and Quinn (2016) include both a costly search effort choice and capital accumulation into an occupational choice model between self-employment, wage employment, and non-employment. In such a model, agents have to trade off two potential investments: investments into capital to increase future earning capacity in self-employment, and investment into costly search to increase the probability of receiving a job offer.

⁴For simplicity, I do not model uncertainty in business profits. Instead, incomes from self-employment are assumed to be deterministic. This implies that agents know about their entrepreneurial ability, and that there is no market risk to running a business. I relax the latter assumption in the empirical part.

2.1 A model of investment under on-the-job-search

There are two periods, and two occupations: self-employment and wage employment. Consider an individual who, in the first period, is self-employed as the owner of a small business. The timing of the model is as following: In period 1, the agent owns assets k_1 and earns income $\pi(k_1)$. Still in period 1, she has to decide how much to invest into future assets k_2 . Credit constraints prevent her from borrowing, but she can invest up to the total of her wealth: $k_2 \leq k_1 + \pi(k_1)$. The only use of future assets is to generate business income in the second period, which is known with certainty. She consumes what she not invests: $c_1 = \pi(k_1) + k_1 - k_2$ and receives utility $u(c_1)$ from consumption. At the start of period two, wage offers are drawn from a compound lottery. There is a probability λ that the agent receives a wage offer. This allows for the presence of search frictions. If an offer is made, it is itself drawn from a distribution $F(w)$. This distribution is known to the agent. The agent consumes her wage and earns utility $u(w)$. Otherwise she will remain self-employed and have utility $u(\pi(k_2))$ from consumption of her income. The agent discounts future consumption with discount factor β .

I make the following assumptions on functions and parameters:

(A). *Assumptions on model primitives*

A1. CONCAVE UTILITY: *The utility function $u(c)$ is three times continuously differentiable, with*

$$u'(c) > 0 \quad u''(c) < 0 \quad u'''(c) > 0.$$

A2. NEOCLASSICAL PRODUCTION FUNCTION: *The production function $\pi(k)$ is twice continuously differentiable and obeys the Inada conditions:*

$$\begin{aligned} \pi(0) &= 0 & \lim_{k \rightarrow 0} \pi'(k) &= +\infty \\ \pi'(k) &> 0 & \lim_{k \rightarrow \infty} \pi'(k) &= 0 \\ \pi''(k) &< 0. \end{aligned}$$

A3. CUMULATIVE DISTRIBUTION FUNCTION: *The function $F(w; a)$ is a cumulative distribution function of w parameterised by a that admits*

first order stochastic dominance in a :

$$\begin{aligned} F(w, a) &= \int_{-\infty}^w f(x, a) dx \\ \text{with } \int_{-\infty}^{+\infty} f(x, a) dx &= 1 \\ F_a(w, a) &= \frac{\partial F(w, a)}{\partial a} < 0. \end{aligned}$$

A4. DISCOUNTING: *The discount factor is $0 < \beta \leq 1$.*

A5. SEARCH FRICTIONS: *A wage offer arrives with probability $0 \leq \lambda \leq 1$.*

The assumption of credit constraints is necessary to make the problem interesting, else all firms would immediately reach their optimal size. The assumption that capital in the second period is not available outside the production function is made for convenience. It is important that *some* return from capital is lost when agents exit self-employment.⁵ I parameterise the wage distribution in assumption A3 so that I can discuss an improvement in the wage distribution by means of comparative statics calculus. The strictly positive discount factor in assumption A4 ensures an interior solution.

The value function summarises the decision problem:

$$\begin{aligned} V(z, k_2; k_1) = \max_{z, k_2} & \left\{ u(\pi(k_1) + k_1 - k_2) + \right. \\ & \left. \beta \cdot \left[\lambda \cdot \int_z \left(u(w) - u(\pi(k_2)) \right) dF(w, a) + u(\pi(k_2)) \right] \right\} \end{aligned} \quad (1)$$

⁵It is possible to extend the model to allow agents to use assets for intertemporally shifting consumption. However, with such an extension, a precautionary savings motive provides an additional mechanism for asset accumulation. By shutting down this possibility, the model focusses on business profits as the rationale for asset accumulation.

2.2 Equilibrium

The first order conditions are the necessary conditions for an optimal choice of z and k_2 , given the state variable k_1 and parameters:

$$\begin{aligned} V_1(z, k_2; k_1) &= -\beta \lambda f(z, a) \cdot [u(z) - u(\pi(k_2))] \\ &= 0; \end{aligned} \quad (2)$$

$$\begin{aligned} V_2(z, k_2; k_1) &= -u'(\pi(k_1) + k_1 - k_2) + \\ &\quad \beta \cdot [1 - \lambda \cdot (1 - F(z, a))] \cdot u'(\pi(k_2)) \cdot \pi'(k_2) \\ &= 0. \end{aligned} \quad (3)$$

where V_1 denotes the partial derivative with respect to the first argument. We can solve condition (2) for z for an explicit expression of the reservation wage as a function of k_2 :

$$z(k_2) = \pi(k_2) \quad (4)$$

Plugging this into the other first-order condition (3) reduces the problem to a univariate model with k_2 as the unique choice variable:

$$\begin{aligned} V_2(k_2; k_1) &= -u'(\pi(k_1) + k_1 - k_2) + \\ &\quad \beta \cdot [1 - \lambda \cdot (1 - F(z(k_2), a))] \cdot u'(\pi(k_2)) \cdot \pi'(k_2) \\ &= 0. \end{aligned} \quad (5)$$

I retain an explicit expression for z to facilitate a comparison of my model with the neoclassical investment model. Condition (5) expresses the Euler equation that characterises the condition for an optimal asset allocation across periods. It equates the marginal utility of consumption in period 1 with the discounted, expected marginal utility of consumption in period 2. Since the marginal value of capital in wage-employment is zero, the expected marginal utility of capital is really only the marginal utility in self-employment, times the probability of staying self-employed. Agents discount future marginal utility from investment by the expected probability shutting down the firm. I refer to

$$\delta(z(k_2)) = \beta \cdot [1 - \lambda \cdot (1 - F(z(k_2), a))] \quad (6)$$

as the *effective discount factor*. When this effective discount factor is equal to one, the Euler equation just describes a basic neoclassical savings-investment model in the tradition of Ramsey (1928). My model reduces to a Ramsey model when $\lambda = 0$ – when the agent never receives a wage offer – or when

$F(z(k_2, a) = 1$ – when the reservation wage is higher than any wage offered in the market. The effective discount factor reaches its lower bound $\beta \cdot (1 - \lambda)$ when the agent accepts job offers at any wage. Then, $F(z(k_2, a) = 0$ and the effective discount factor is determined only by the degree of search friction. Somewhat contrary to intuition, occupational choice affects the investment behaviour more if labour markets are more efficient, and the self-employed frequently receive job offers.

The concavity of the utility function and a strictly positive discount factor ensure that an inner solution of the first order condition is always reached. Further, a second order condition is required to ensure that the first order decision indeed describes a maximum. The second-order, sufficient condition for an optimal investment choice is given by:

$$\begin{aligned}
 V_{22}(k_2; k_1) = & u''(\pi(k_1) + k_1 - k_2) + \delta(z(k_2)) \cdot \\
 & \left[u''(\pi(k_2)) \cdot \pi'(k_2) + u'(\pi(k_2)) \cdot \pi''(k_2) \right] \\
 & + \beta \lambda f(z(k_2), a) \cdot u'(\pi(k_2)) \cdot \pi'(k_2) \cdot \frac{dz}{dk_2}
 \end{aligned} \tag{7}$$

where V_{22} denotes the second derivative of the value function, twice with respect to the second argument. The sign of (7) is ambiguous. The value function is not globally concave. This is not necessary for a solution to exist - we only require the value function to be *locally* concave at an optimum. Therefore, we can appeal to the result that the first and second order conditions are necessary and sufficient to describe a maximum of the value function. That is, at any optimum, the first order condition (5) and the second order condition $V_{22} < 0$ *must* hold.

3 Comparative statics of optimal investment choices

In the following I characterise how the equilibrium is affected by changes in the economic environment that an individual decision maker faces. Two comparative statics results are of special interest here. Firstly, I analyse how optimal asset choices k_2 vary with initial capital stock k_1 . In analogy to models with many periods, I will refer to this policy function that maps initial into future capital stock as the *law of motion of assets*. This also allows me to characterise steady state equilibria. It turns out that the policy function of this model can be locally convex, and that there can be multiple, stable steady state equilibria. A key determinant of the shape of the policy function is the location of the wage distribution relative to the profit function.

This leads me to, secondly, analyse how optimal asset choices, and by extension the evolution of firm size, are affected by changes to the value of wage employment. In particular, I derive comparative statics with respect to a first order stochastic improvement in the wage offer distribution.

3.1 Policy function for investment

The slope of the policy function $k_2(k_1)$ is characterised by the derivative of the first order condition for optimal investment (5):

$$\frac{dk_2}{dk_1} = \frac{-V_{21}}{V_{22}} = \frac{u''(\pi(k_1) + k_1 - k_2) \cdot (1 + \pi'(k_1))}{V_{22}} > 0. \quad (8)$$

The sign follows from the numerator being negative by concavity of the utility function, and the denominator 7 being negative at an optimum.

From the first order condition, it is straightforward to verify that a steady state in assets must exist. Since $F(z, a)$ is a cumulative density function it is bounded in the unit interval. By concavity of $\pi(k)$ there always exists a k such that $-1 + \beta \cdot [(1 - \lambda) + \lambda \cdot F(z, a)] \cdot \pi'(k) = 0$. However, unlike in the standard neoclassical investment model, this steady state need not be unique because the law of motion of assets may be locally non-concave.

In order to show this, I take the second derivative of the law of motion:

$$\begin{aligned} \frac{d}{dk_1} \left(\frac{dk_2}{dk_1} \right) &= \frac{-V_{211} \cdot V_{22} + V_{21} \cdot V_{221}}{V_{22}^2} \\ &= \frac{1}{V_{22}^2} \cdot \left\{ u'''(\pi(k_1) + k_1 - k_2) \cdot (1 + \pi'(k_1))^2 \cdot \right. \\ &\quad \left[\delta(z(k_2)) \cdot \left[u''(\pi(k_2)) \cdot \pi'(k_2) + u'(\pi(k_2)) \cdot \pi''(k_2) \right] \right. \\ &\quad \left. \left. + \beta \lambda \cdot f(z(k_2), a) \cdot u'(\pi(k_2)) \cdot \pi'(k_2) \cdot \frac{dz}{dk_1} \right] \right. \\ &\quad \left. + V_{21} \cdot \left[u''(\pi(k_1) + k_1 - k_2) \cdot \pi''(k_1) \right] \right\} \end{aligned} \quad (9)$$

The sign of (9) is ambiguous. The third, and fourth lines are negative, while the third line is positive; the first line is a positive constant multiplied

with the terms in lines 2 and 3. When the expression in the third line dominates, (9) is positive and the law of motion is convex. I discuss further in section 4.2 when we can expect this to be the case, and when to expect multiple equilibria.

From condition (9), we can check that the law of motion is always concave (and therefore convergence to a unique steady state is guaranteed) whenever my model reduces to the neoclassical investment model. This is the case if the second summand of (9) reduces to zero. This happens under any of the following conditions: if there are no wage offers and $\lambda = 0$; if firm owners' investment strategy does not consider alternative for their labour and $\frac{dz}{dk_2} = 0$; or finally, if the reservation wage does not overlap with wages offered in the market (for example, if it is higher than any market wage) and $f(z(k_2), a) = 0$.

3.2 Stochastic improvement in the wage offer distribution

Next I describe how optimal investments change with improvement in the wage offer distribution. Through their effect on $\frac{dz}{dk_2}$, differences in the location of the wage distribution are give rise to very different laws of motion of assets, holding productive characteristics in self-employment constant. Here I discuss the comparative statics effects of such differences on the equilibrium path.

I model an improvement in the wage distribution as a first order stochastically-dominant shift in the wage distribution (see Assumption 3). Increases of the parameter a induce such a shift of the distribution function. I obtain the comparative static

$$\frac{dk_2}{da} = - \frac{\beta \lambda \cdot \frac{\partial F(w;a)}{\partial a} \cdot u'(\pi(k_2)) \pi'(k_2)}{V_{22}} < 0 \quad (10)$$

Since $V_{22} < 0$ at an optimum, the sign of (10) is globally negative. A stochastic improvement in the wage offer distribution reduces investment everywhere. Such a shock to the wage distribution will act as a shock to the effective discount factor. For a given reservation wage, it is now more likely that a wage offer is accepted. This brings down the optimal investments, and in turn the reservation wage, until a new equilibrium is reached.

4 Discussion

I start the discussion of the intuition of the model and its implications by illustrating the law of motion for a situation with a unique equilibrium, and for one with multiple equilibria, in Figure 2. Like all graphical representations of the model in this paper, this graph has been obtained by computer simulation of a parametric version of the model. I simulate using a Cobb-Douglas production function, a CRRA utility function and a lognormal wage distribution.⁶ The left-hand graph depicts a unique steady state; the law of motion is globally concave. The right-hand graph shows a law of motion which is both concave and convex, and which gives rise to multiple stable steady state equilibria. Stable equilibria are located where the law of motion crosses the 45 degree line and is locally concave at the point of crossing. These points are labelled A and C in the graph. The steady state at point B is unstable: a small perturbation of k_1 causes assets to move away from the steady state.

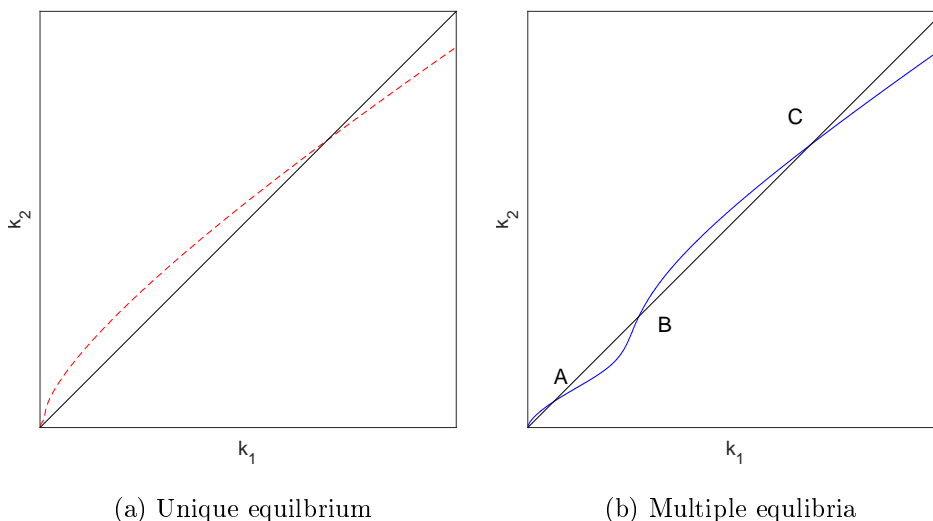


FIGURE 2

LAW OF MOTION FOR ASSETS

For any level of initial assets above B, firm size will converge to the high capital equilibrium C whereas for entrepreneurs with initial assets below B

⁶ Appendix A lists the details of parameter assumption for each figure.

it will tend towards the low capital equilibrium A. This happens because those individuals decrease their asset holdings in expectation of leaving self-employment for a better paying wage job. They will eventually improve their incomes if they end up accepting a job offer. Those who do not switch because they do not receive a sufficiently attractive wage offer, or no job offer at all, will have a lower capital stock and income.

4.1 Effective discount factor as transmission channel

The effective discount factor (6) relates occupational choices of the firm owner to shutdown probabilities of the business. This becomes evident from the Euler equation (5). The presence of the effective discount factor is what changes the Euler equation compared to the neoclassical investment model. Here, I provide some further intuition into the mechanisms behind changes in the effective discount factor.

Given the closed-form solution (13), the reservation wage increases monotonically with initial assets. This does not automatically lead to large changes in the effective discount factor. For that, we also require sufficient probability mass of the wage offer distribution to be located in the neighbourhood of the reservation wage:

$$\frac{\partial \delta(z(k_2))}{\partial k_1} = \beta \lambda \cdot f(z(k_2), a) \cdot \frac{\partial z}{\partial k_2} \cdot \frac{\partial k_2}{\partial k_1} \quad (11)$$

Only when $f(z(k_2), a)$ is positive does a change in initial assets, through a change in future assets and the reservation wage, translate into a change of the effective discount factor.

This mechanism, with effective discount factors corresponding to figures 2a and 2b, respectively, is illustrated in the left-hand panel of Figure 3. The graph plots the effective discount factor for each level of initial capital stock. In the unique equilibrium case, the effective discount factor rises very sharply. Moreover, this rise happens to coincide with an area of low initial assets, where the marginal return is high and assets are below any steady state level. With multiple equilibria, the increase in the discount factor occurs more slowly, and at a higher level of initial assets.

How the effective discount factor rises along the asset distribution depends on the density of the wage distribution at the reservation wage. The right-hand panel of Figure 3 shows these densities, again corresponding to

figures 2a and 2b. It plots initial capital stock k_1 against $f(z)$.⁷ The discount factor increases as more and more mass of the wage distribution lies below the reservation wage. The slope of the effective discount factor is proportional to the slope of the wage distribution profile. The effective discount factor remains constant where this is flat.

In fact, the effective discount factor that gives rise to figure 2b rises from its lower asymptote at $\beta(1 - \lambda)$ – when all jobs are accepted – to its higher asymptote at β – when no wages are accepted. As the effective discount factor rises, the asset evolution moves from the lower to the upper asymptote path. Figure 4 illustrates this. The upper and lower limiting equilibrium paths correspond to the situations where none or all of the job offers are accepted, respectively. The production function remains unchanged throughout.

The figure illustrates how a low assets, low income equilibrium can arise even if the (technical) marginal returns to capital are high. Starting from low levels of k_1 , the reservation wage is lower than the lower bound of market

⁷ Since $z = \pi k_2$ and $k_2(k_1)$ is monotonically increasing, there is a one-to-one mapping between z and k_1 .

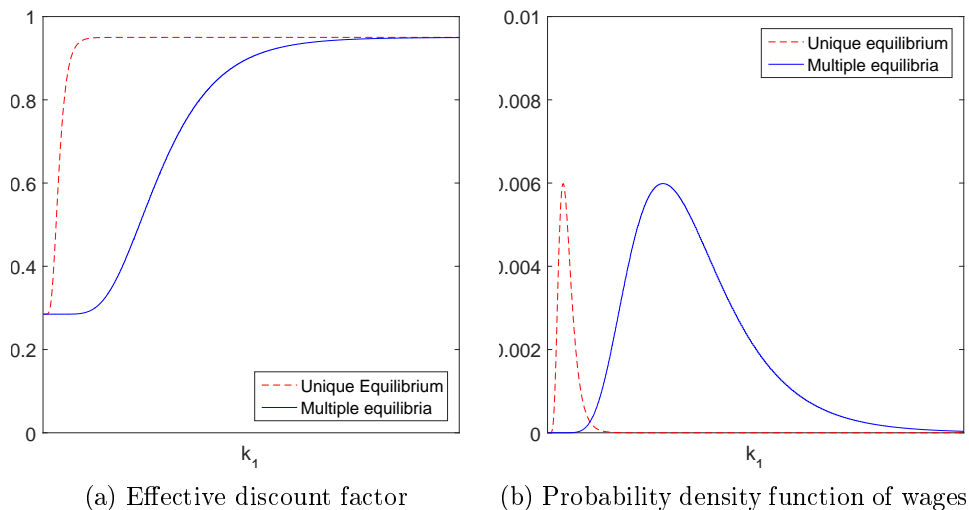


FIGURE 3

MECHANISMS DETERMINING THE SHAPE OF THE LAW OF MOTION

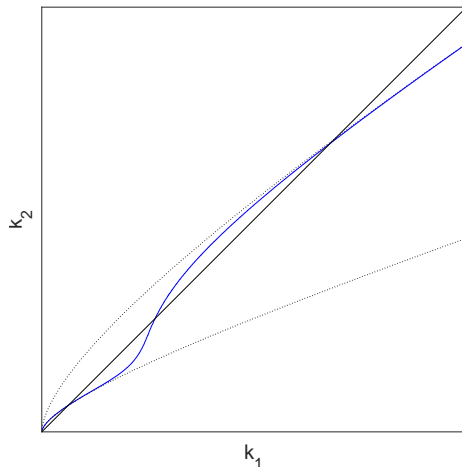


FIGURE 4

MULTIPLE EQUILIBRIA AND LIMITING INVESTMENT PROFILES

wages. All job offers are accepted, firms very frequently shut down, and the remaining firms are on a path to a low equilibrium. As we move up on the horizontal axis and initial assets increase, the law of motion changes from concave to convex when business profits start to become competitive, and the lowest-paid wage offers cease to be attractive. The effective discount factor rises as a result. As k_1 increases even further, wages cannot keep up with business profits, and the reservation wage rises above the highest wages in the market. Discounting now only occurs with β , and the firm is on the same equilibrium path that would occur in isolation.

As a result, the model sustains both a low-asset, low-income equilibrium with a high probability of shutdown and exit to wage employment, and a high-asset, high-income equilibrium with a zero probability of a voluntary shutdown.

4.2 When should we expect multiple equilibria?

Convexities in the law of motion of assets are a necessary, but not a sufficient condition for multiple equilibria to occur. In this section I provide further insight and discussion into both kind of conditions for multiple equilibria.

Recall from the discussion following equation (9) that convexities in the law of motion of assets arise when the second term in that equation dominates. From (11), we can see that this obtains most easily when the slope effective discount factor is steep. In addition, a convexity is also more likely when the utility and production functions are relatively flat. Then, the derivatives $u'()$, $u''()$, $\pi'()$ and $\pi''()$ are all relatively small in absolute value. This makes it more likely for the positive summand, the discount factor channel to dominate in (9).

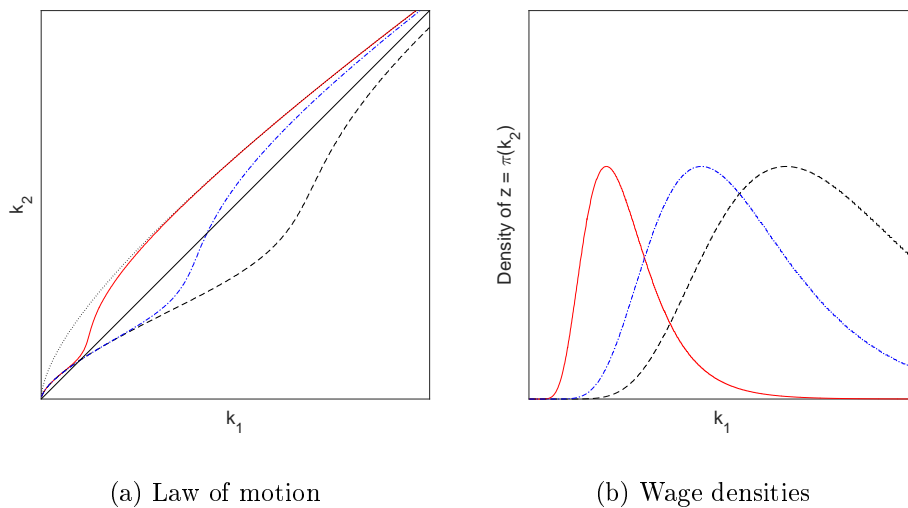


FIGURE 5

CONVEXITIES WITH AND WITHOUT MULTIPLE EQUILIBRIA

Of course, convexities may occur without triggering multiple equilibria. Figure 5 shows the three possible laws of motion that convexities may give rise to. The right-hand panel documents the corresponding wage distributions. If wage densities pick up at initial asset levels sufficiently *below* the steady state, then a local convexity may simply push up the investment profile towards a higher equilibrium path. In a sense, the firm changes course with its investment policy at a stage before the eventual target (the steady state capital stock) has been reached. This scenario is drawn in red solid lines. In fact, the policy function converges to the investment profile that would arise in isolation, a reference point drawn as a thin dotted line.

If instead, wage densities pick up *beyond*, but sufficiently close to, a steady state, then multiple equilibria can arise. This is the constellation of Figure (2), and is repeated here for comparison, again drawn in blue with a dashed-dotted line. Under this scenario, firms with initial assets above the steady state capital stock at some critical level of k_1 revise their policies of decreasing capital stock. They eventually start accumulating capital again until they reach a higher steady state.

Finally, if wage densities pick up at initial assets far beyond the steady state, firms react in a manner similar to the second case, and slow down the decline in their capital stock. However, if this occurs at high levels of k_1 where capital stock already decreases rapidly, this revision of policy may not be strong enough such as to lead to a new expansion of capital stock. This scenario is draw in dashed black lines.

Note that as we move up the initial asset distribution, the ‘dents’ in the laws of motion that convexities create become more and more pronounced, even though wage densities increase more slowly with the reservation wage. This illustrates the observation that the investment profile is more likely to turn convex when marginal productivity and utilities themselves are flatter. By implication, convexities are most likely to be found in precisely the kind of scenario that results in multiple equilibria - in the shallow parts of the policy function beyond the first steady state.

4.3 Testable Hypotheses

This model of investment behaviour interacted with behavioural choice yields a number of empirical predictions that we can test for.

1. *The probability of switching from self-employment into wage-employment is higher when the probability of receiving an attractive job offer is higher.*

This hypothesis is a basic test of whether the fundamental premise of this model is supported by empirical evidence: owners of small businesses do take opportunities in wage employment into account when they decide whether to continue in business or shut down their firm. In the model, the probability of switching is given by

$$\lambda \cdot (1 - F(z, a)).$$

In a sense, this test also gives us an idea of whether wage employment opportunities affect the effective discount factor, which of course we cannot observe. But theoretically, it is proportional to $1 - \lambda \cdot (1 - F(z, a))$.

2. *Investments into the firm are lower when the probability of receiving an attractive job offer is higher.*

This hypothesis is predicted by the comparative static of optimal investment with respect to an improvement in the wage distribution (10). The model predicts a decrease of investment at any point of the distribution of business incomes.

3. *The effect sizes in Hypotheses 1 and 2 follow an inverse U-shaped pattern. Effects are largest for individuals with medium business incomes.*

Figure 5 illustrates this prediction. If there are convexities in the path of optimal investment, then investment decreases most sharply in the middle of the capital (and therefore business income) distribution. The effect is less pronounced for very small and very large firms. Owners of small firms will already take up most job offers they manage to receive. For owners of large firms, wage employment does not constitute a competitive alternative to running their own business.

5 Empirical Framework

In this section I present the identification strategy and the equations I estimate to test the predictions of the model. The main concern is finding an adequate expression for the counterfactual wage distribution that each individual who is currently self-employed faces. Wages are only observed for those individuals who are actually wage-employed, and business incomes are only observed for those who are actually self-employed. The predictions of the model, however, crucially depend on the relative position of wage offers and business incomes. We need to make counterfactual predictions for both of these objects.

This exercise is complicated by the fact that in economic terms, the model is one of individual choice, and should be thought of as conditional on a given comparative advantage in productive characteristics wage and self-employment, respectively. We can condition on a vector of observable characteristics \mathbf{x} when predicting counterfactual wages. Clearly, there will be still a large part of the wage distribution that is determined by unobservable characteristics which may be correlated between the two employment sectors.

In this section, I first present two empirical measures of the attractiveness of wage employment relative to self-employment based on earnings differen-

tials. These measures can be identified from data on earnings distributions and sectors of employment. I then discuss identification of the counterfactual wage distribution under alternative sets of assumptions about the cross-sector dependance of unobservable wage determinants. I take advantage of the transition structure implied by the model and propose a simple structural method to identify comparative advantage in a two-sector model when transitions depend on random shocks that are not independent from the observed outcome, which in this case are draws from the random wage distribution.

5.1 Expressing counterfactual employment opportunities

In order to test the hypotheses summarised in section 4.3 we need to have an empirical measure of the relative attractiveness of wage employment. The model suggests a simple reservation wage strategy:

$$z = \pi(k_2)$$

However, business incomes in the second period are only observed for those who are actually self-employed in that period. But the law of motion of capital (8) suggests that future business incomes are a function of current incomes, and that future business incomes equal the reservation wage:

$$z_i = \pi(k_{i2}) = g(\pi(k_{i1})) \quad (12)$$

where $g(k_1)$ is some increasing function of individual i 's business incomes in the first period. The simplest functional form is

$$g(\pi(k_{i1})) = \pi(k_{i1}) \equiv \pi_{i1}. \quad (13)$$

and is the one assumed in most specifications that I estimate.

I express the relative attractiveness of wage-employment in two ways: firstly, by a dummy of whether the expected wage in period 2 is higher than the reservation wage:

$$\mathbf{1}(E(w_{i,2}) > \pi_{i,1} | \Psi_i) \quad (14)$$

and secondly by the probability that individual i receives a wage offer higher than his or her reservation wage:

$$\Pr(w_{i,2} > \pi_{i,1} | \Psi_i) = 1 - F(\pi_{i,1} | \Psi_i) \quad (15)$$

which can be calculated as a quantile of i 's wage distribution. Both measures are conditional on an information set Ψ_i . This expresses the fact the each

individual faces a specific wage distribution $F(w, a)$, where Ψ_i empirically proxies for a . The information set generally contains observable characteristics \mathbf{x}_i , but potentially also other information about the self-employed individual, for example unobservable components of their income. I now discuss identification of $F(w, a)$ under different sets Ψ_i .

5.2 Identification of counterfactual wage distributions

The central challenge to identification is that we only ever observe an individual in a single sector at a time. In order to assess the relationship between alternative employment opportunities and outcomes for the self-employed, we need to be able to construct the measures (15) and (14) for *all* self-employed. It is therefore necessary to predict the unobserved, counterfactual wage, and its distribution, for the self-employed. To this I now turn.

First, I set up the problem. Suppose that the log of potential wages w_{it} , and incomes from self-employment, π_{it} , of individual i in time period t are determined according to the following process:

$$\ln w_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + \epsilon_{it} \quad (16)$$

$$\ln \pi_{it} = \mathbf{x}'_{it}\boldsymbol{\gamma} + \nu_{it} \quad (17)$$

Earnings (wages and profits) contain two components: an observable ($\mathbf{x}'_{it}\boldsymbol{\beta}$ and $\mathbf{x}'_{it}\boldsymbol{\gamma}$, respectively) and an unobservable part (ϵ_{it} and ν_{it} , respectively).⁸ In order to identify (14) and (15), we need to identify both the observable and the unobservable part. For the observable part, this means correctly identifying the *predictive* coefficient $\boldsymbol{\beta}$ from earnings functions of the actually wage-employed only, which is possible under weak conditions.

However, we observe incomes only for those individuals actually employed in sector $S_{it} \in \{0, 1\}$:

$$w_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + \epsilon_{it} \quad \text{iff } S_{it} = 1 \quad (18)$$

$$\pi_{it} = \mathbf{x}'_{it}\boldsymbol{\gamma} + \nu_{it} \quad \text{iff } S_{it} = 0 \quad (19)$$

The main threat to identification arises from a correlation between ϵ_{it} and ν_{it} . This poses two concerns: firstly, the earnings residual in self-employment, ν_{it} , may contain some information about the counterfactual ϵ_{it}

⁸It is straightforward to extend x_i to contain different predictors for earnings in wage-employment and self-employment.

and therefore the mean of the individual-specific wage distribution. Wage predictions are systematically biased towards the mean that is only conditional on observables. Ignoring this causes us to underestimate wage employment attractiveness for high-ability individuals, and to overestimate for low-ability individuals. Secondly, correlation may lead to selection on unobservables. This biases wage predictions that we make for individuals that are self-employed with coefficients that we have identified from data on the wage-employed.

5.2.1 Conditional Independence

As a benchmark, consider first the case of conditional independence between unobservables in wage employment and self-employment.

(B1). *Assumption: Conditional Independence*

CONDITIONAL INDEPENDENCE: $\nu_{it} \perp\!\!\!\perp \epsilon_{is} \mid \mathbf{x}_{it} \quad \forall s, t.$

If Assumption B1 holds, then $E(\epsilon_{it} \mid \mathbf{x}_{it}) = E(\epsilon_{it} \mid \mathbf{x}_{it}, \nu_{it})$. In this case, the Mincerian wage regression is straightforwardly identified with OLS. It also follows that the only informative information set is $\Psi_i = \{\mathbf{x}_i\}$. The measure (14) is then identified by a linear prediction.

In order to identify (15), we need to know the conditional distribution of ϵ_{i2} , given $\mathbf{x}_{i,2}$. If this conditional distribution is normal, then we can express (15) by

$$\Pr(w_{i,2} > \pi_{i,1} \mid \Psi_i) = \Phi\left(\frac{\mathbf{x}'_{i,2} - \pi_{i,1}}{\sigma_e}\right) \quad (20)$$

Under conditional independence, the wage distribution is also nonparametrically identified by

$$\Pr(w_{i,2} > \pi_{i,1} \mid \Psi_i) = F(\pi_{i,1} \mid \Psi_i) \quad (21)$$

and can in principle be estimated without imposing strong parametric assumptions using a nonparametric or semiparametric estimator for $F()$.

5.2.2 Endogenous Switching Model

If unobservable components of business incomes and wages are correlated, selection into wage employment and self-employment is based on comparative advantage (Roy (1951)). This implies two things: firstly, conditional independence is violated if the average level of the unobserved component

(say, ability) for wage-employed is different from its average level in the population. Further, the fact that someone is self-employed is informative about the level of ϵ_i in the cross-section. If the sector of employment, but not the exact income in that sector, is informative about the wage distribution of an individual, then counterfactual wages can be identified with an endogenous switching model (Lee (1978), Maddala (1983)), here abbreviated as ESM. This is what Assumption B2 states.

(B2). **Assumption: Endogenous Switching Model**

B2.1. SELECTION MECHANISM: *Employment in the wage sector ($S_i = 1$) is chosen iff*

$$\mathbf{x}_i' \boldsymbol{\xi} + \mathbf{q}_i' \boldsymbol{\zeta} + \eta_i > 0 \quad (22)$$

where η_i is random variable, \mathbf{q}_i is a vector of variables and $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ denote parameter vectors of corresponding dimension.

B2.2. JOINT NORMALITY: *The error terms of (16), (17) and (22) are jointly distributed as*

$$\begin{pmatrix} \eta_i \\ \epsilon_i \\ \nu_i \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \sigma_{\eta\epsilon} & \sigma_{\eta\nu} \\ \sigma_{\epsilon\eta} & \sigma_\epsilon^2 & \sigma_{\epsilon\nu} \\ \sigma_{\eta\nu} & \sigma_{\epsilon\nu} & \sigma_\eta^2 \end{pmatrix} \right)$$

B2.3. INFORMATION SET: $\boldsymbol{\Psi}_i = \{\mathbf{x}_i, S_i\}$ *is the sufficient information set for the counterfactual wage distribution.*

The selection mechanism (Assumption B2.1.) includes an unobserved taste shifter η_i . This allows for unobserved, non-pecuniary preference components that influence selection. Such a selection mechanism is sometimes referred to as a ‘generalised Roy Model’ (?). Assumption B2.3. on the information set is plausible if individuals do not have sufficient knowledge about predictive power of their comparative advantage in self-employment – including, for example, the case when they cannot separately identify the level of their comparative advantage from other determinants of their business income. They might still be able to know the wage distribution that individuals face who are similar to themselves on observable characteristics, including the initial sector of employment.

As Lee (1978) and Maddala (1983) explain, all coefficients but the correlation between incomes in the two sectors, $\sigma_{\epsilon\nu}$ are identified with cross-sectional data if there is an excludable variable \mathbf{q}_i that shifts the latent switching variable, but that are excludable from either outcome equation.

Furthermore, the model allows us to identify the mean and variance of the wage distribution, conditional on the information set in B2.3. The linear prediction for the mean is identified by exploiting the information content of the inverse Mills ratio:

$$E(w_i | S_i = 0) = E(\mathbf{x}_i' \boldsymbol{\beta} | S_i = 0) + E(\epsilon_i | S_i = 0) \quad (23)$$

$$= \mathbf{x}_i' \boldsymbol{\beta} - \sigma_\epsilon \rho_{\epsilon\eta} \frac{\phi(\mathbf{x}_i' \boldsymbol{\xi} + \mathbf{q}_i' \boldsymbol{\zeta})}{1 - \Phi(\mathbf{x}_i' \boldsymbol{\xi} + \mathbf{q}_i' \boldsymbol{\zeta})} \quad (24)$$

The ESM relaxes the assumption of independance, but is still quite restrictive. In particular, with cross-sectional data we cannot identify $\rho_{\epsilon\nu}$, and therefore cannot condition wage distributions on any information set that includes unobservable determinants of an individual's specific business income.

5.2.3 Dynamic Selection Model (DSM)

We can exploit the panel structure of the data in order to relax the restrictive assumption on the information set that identification with a static endogenous switching model requires. I propose a simple structural model, the *dynamic selection model* (DSM), that follows the transition structure suggested by the theoretical search model.

With the DSM, the switching probability can be disentangled into two parts: the component of selection based on comparative advantage, that is $Pr(w_{i2} > \pi_{i1} | \boldsymbol{\Psi}_i)$, and the component of selection associated with the search friction, λ . The endogenous switching model is not able to distinguish between those two components. Instead, it rationalises selection into self-employment ($S_i = 0$) by a negative shock η_i to selection. On the other hand, the parameter $\rho_{\eta\epsilon}$ in (24) expresses the correlation between this error term, and wages. Therefore, the ESM might underpredict counterfactual wages for the self-employed. By modelling dynamic and not just static selection, the DSM allows us to identify the parameter $\rho_{\epsilon\nu}$, and therefore to include individual-specific components of unobservables ν_{i1} in the information set.

(B3.). *Dynamic Selection Model*

B3.1. SELECTION MECHANISM *Transitions between sectors of employment*

occur according to the following structure:

<i>Case</i>	$(\mathbf{S}_{i1}, \mathbf{S}_{i2})$	<i>Transition probability</i>
1	(0, 1)	$\lambda \cdot \Pr(w_{i2} > \pi_{i1} \Psi_i)$
2	(0, 0)	$(1 - \lambda) + \lambda \cdot (1 - \Pr(w_{i2} > \pi_{i1} \Psi_i))$
3	(1, 1)	$(1 - \delta)$
4	(0, 0)	δ

B3.2. JOINT NORMALITY The error terms of (18), (19) are jointly distributed as

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \nu_1 \\ \nu_2 \end{pmatrix} \sim N \begin{pmatrix} \sigma_\epsilon^2 & \rho_\epsilon \sigma_\epsilon^2 & \rho \sigma_\epsilon \cdot \sigma_\nu & \rho \cdot \sigma_\epsilon \cdot \sigma_\nu \\ \cdot & \sigma_\epsilon^2 & \rho \cdot \sigma_\epsilon \cdot \sigma_\nu & \rho \cdot \sigma_\epsilon \cdot \sigma_\nu \\ \cdot & \cdot & \sigma_\nu^2 & \rho_\nu \cdot \sigma_\nu^2 \\ \cdot & \cdot & \cdot & \sigma_\nu^2 \end{pmatrix}$$

B3.3. INFORMATION SET The relevant information set to predict the counterfactual wage distribution is $\Psi_i = \{\mathbf{x}_i, v_{i1}\}$

Assumption B3.1. imposes the structure of the search model from section 2, and extends it by an exogenous transition probability δ from wage-employment to self-employment, which should be thought of as the probability of job loss. There is no unobserved taste shifter, and therefore assumption B3.2. is based on a quadrivariate normal distribution of two variables, ϵ and ν , in two periods, 1 and 2. The parameters of the model are identified under Assumption B3. The model can be estimated using maximum likelihood. Derivation and statement of the likelihood function is relegated to Appendix B.

By exploiting the discrete dynamic choice structure implied by the theoretical model, the DSM is able to uncover the full earnings distribution in a two-sector model under an endogenous switching mechanism. An alternative approach (Lemieux (1998); Suri (2011)) identifies correlated, and sector-specific returns to individual unobserved heterogeneity using linear panel methods.⁹ However, identification requires that only permanent and not transitory components of the relative attractiveness of sectors matters for switching. By imposing the structure of the model, I am able to allow

⁹Applications of the model to occupational choice and earnings distributions in developing countries include Haywood and Falco (2016), and Imbert (2013)

for this correlation of entry into wage-employment with specific draws from the wage distribution.¹⁰

5.3 Identification of effect on outcomes

The ultimate goal of the empirical analysis is to identify the effect of alternative wage employment opportunities on observable outcomes of choices made by business owners. The outcomes of interest are transitions out of self-employment into wage-employment, and firm growth. These outcomes are measured as binary variables, and the latent variable is a function of alternative employment opportunities (15) and (14) and other variables:

$$y_{i,c,t}^m = \begin{cases} 1 & \text{if } \alpha_0 + M_{i,c,t}^j \alpha_1^j + \mathbf{\Gamma}'_{i,c,t} \alpha_2 + \tau_{ct} + \tau_{i,c,t} > 0 \\ 0 & \text{if } \alpha_0 + M_{i,c,t}^j \alpha_1^j + \mathbf{\Gamma}'_{i,c,t} \alpha_2 + \tau_{ct} + \tau_{i,c,t} \leq 0 \end{cases} \quad (25)$$

where $y_{i,c,t}^m$ denotes the outcome m for individual i in city c and period t , $M_{i,c,t}^j$ the j th measure of wages relative to business incomes, and τ_i an error term. The main parameter of interest is α_1^j , the coefficient on the alternative employment opportunity measure j . Throughout, I control for local labour market conditions at the city-year level $\tau_{c,t}$ as a potential driver of relative wages, transitions, and firm growth. In different specifications I also include other control variables $\mathbf{\Gamma}_{i,c,t}$.

I also estimate a model that allows me to test for heterogeneous effects:

$$y_{i,c,t}^m = \begin{cases} 1 & \text{if } \alpha_0 + M_{i,c,t}^j \cdot \sum_{k=1}^K h_{it}^k \alpha_{1,k}^j + \mathbf{\Gamma}'_{i,c,t} \alpha_2 + \sum_{k=1}^K h_{it}^k \alpha_{3,k}^j + \tau_{ct} + \tau_{i,c,t} > 0 \\ 0 & \text{if } \alpha_0 + M_{i,c,t}^j \cdot \sum_{k=1}^K h_{it}^k \alpha_{1,k}^j + \mathbf{\Gamma}'_{i,c,t} \alpha_2 + \sum_{k=1}^K h_{it}^k \alpha_{3,k}^j + \tau_{ct} + \tau_{i,c,t} \leq 0 \end{cases} \quad (26)$$

where I interact the measures with dummy variables indicating the levels $k = 1, \dots, K$ of some discrete variable h_{it} , and control for these dummies directly.

5.4 Procedure

I combine all of this in a procedure that consists of three steps.

1. In the first step, I estimate earnings equations and predict counterfactual wages.

¹⁰This identification strategy is then much closer to ‘semi-structural’ approaches of selection correction (Lee (1978); Heckman (1979))

2. In the second step, I construct empirical counterparts to the alternative employment opportunities measures (15) and (14). These take the following forms :

$$\hat{\mathbf{1}}() = \mathbf{1} \left(\hat{E}(w_{it}|\Psi_{it}) > \pi_{it} \right) \quad \hat{\text{Pr}}() = \Phi \left(\frac{\hat{E}(w_{it}|\Psi_{it}) - \pi_{it}}{\hat{SD}(w_{it})} \right) \quad (27)$$

$$\text{where} \quad \hat{E}(w_{it}|\Psi_{it}) = \mathbf{x}'_{it}\hat{\beta} \quad (\text{OLS})$$

$$\hat{E}(w_{it}|\Psi_{it}) = \mathbf{x}'_{it}\hat{\beta} + \left(\hat{\rho}_{\epsilon\eta} \frac{\hat{\sigma}_{\epsilon}}{\hat{\sigma}_{\eta}} \right) \cdot \lambda_{it} \quad (\text{ESM})$$

$$\hat{E}(w_{it}|\Psi_{it}) = \mathbf{x}'_{i2}\hat{\beta} + \left(\hat{\rho} \frac{\hat{\sigma}_{\epsilon}}{\hat{\sigma}_{\nu}} \right) \cdot \nu_{i1} \quad (\text{DSM})$$

with λ_{it} the inverse Mills ratio. The corresponding standard distribution estimates are obtained from the sum of squared residuals (OLS), and maximum likelihood estimates (ESM, DSM), respectively.

3. Finally, I estimate binary outcome equations using constructed measures as a regressor. In particular, I estimate the probit model (26) using as outcomes:

- *Switch from SE to WE*: A dummy equal one if a business owner transits to wage-employment between periods 1 and 2, zero if stays self-employed.
- *Grow Firm*: A dummy equal one a business owner adds salaried employees from outside the household to the firm between periods 1 and 2, zero otherwise.

Since the outcome regressions include regressors that are constructed using coefficients from first-step regressions, standard errors are obtained with a non-parametric bootstrap procedure.

6 Data

6.1 Background

The data used in this analysis consist of a panel survey of urban households in Colombia, the Encuesta Social Longitudinal de Fedesarrollo (Fedesarrollo Social Longitudinal Survey; ESLF).¹¹ It was collected by the Bogotá-

¹¹For recent empirical work on occupational choice in Colombia, see [Magnac \(1991\)](#) [Bosch and Maloney \(2010\)](#) [Mondragón-Vélez and Peña \(2010\)](#), and [Cuesta and Bohórquez \(2011\)](#).

based foundation Fedesarrollo, supported by the Inter-American Development Bank (IADB) and the Bogotá, Bucamaranga and Cali Chambers of Commerce. The 2007-2010 waves include a unified module on the labour market. The 2007 wave covers only the Metropolitan Areas of Bogotá, Bucamaranga and Cali. In 2008, the survey was enlarged to include ten additional cities and Metropolitan Areas.¹²

The sampling frame consists of the 1993 Census population list of dwellings, households and individuals. Census lists are updated using information from the Colombian National Statistical Office DANE and municipal planning departments. Households are selected into the survey using a thrice-stratified random sampling design: the primary sampling units are Census enumeration areas, secondary sampling units are housing blocks (*‘manzanas’*), and the final and tertiary sampling units are households. Households attriting from the panel are replaced by new ones.

Generally, for every regression I use the largest sample possible in order to maximise statistical power. For a rough overview, that means pooling cross-sections for OLS estimations, pooling adjacent-year panels for probit, and for ESM and DSM wage equation estimations.

6.2 Descriptive Statistics

The universe of the survey from 2008 onwards comprises of the 13 cities and metropolitan areas constituting the ‘total national urban population’ as defined for statistical purposes by the Colombian National Statistical Office DANE. Table 1 illustrates some general statistics of the urban labour market in Colombia. The labour force participation rate, that is the proportion of the working age population in the labour force, is fairly constant at about 55 percent. A little over 50% of the labour force are wage employed, about a third self-employed, and about 10% are classified as unemployed. Over the years, there is a slight but significant trend of decreasing rates of wage-employment, and increasing rates of unemployment and self-employment.

The period covered by the sample corresponds to the global financial crisis. GDP growth in 2007 was 6.9 percent, then dropped stepwise to 3.5 percent in 2008 and 1.7 percent in 2009 before recovering in 2010 with 4

¹²Barranquilla, Cartagena, Cúcuta, Ibagué, Manizales, Medellín, Montería, Pasto, Pereira, and Villavicencio.

TABLE 1
COMPOSITION OF THE URBAN LABOUR MARKET IN COLOMBIA
2007-2010

	2007	2008	2009	2010
LF Participation Rate	54.4	55.4	54.7	55.4
% of Labour Force				
Wage-Employed	58.3	54.7	53.0	50.9
Self-Employed	31.3	33.7	33.9	35.9
Unemployed	8.8	10.0	11.7	12.1

Source: Encuesta Social Longitudinal de Fedesarrollo (ESLF).

Ratios are representative for the total urban national population of Colombia. *Wage-employed* includes private and government sector employees, domestic and casual workers. *Self-Employed* includes own-account workers and employers.

percent and 2011 with 5.9 percent. This adverse macroeconomic environment may explain some of the trends. The global financial crisis constitutes a purely external shock to the Colombian economy. Like other South American economies, it had been on a high growth trajectory for a number of years. The global recession that ensued after the financial crisis in developed countries led to a sharp reduction in GDP growth, with a quick recovery.

I exploit the fact that the economic downturn brought with it a higher number of involuntary job separations. Labour market transitions across all years are shown in Table 2.¹³ The vertical dimension corresponds to the employment status of an individual in a given year t , and the horizontal dimension to the employment status in the following year $t + 1$. The table should be read row-wise. Every entry in the main matrix shows the percentage of those in the row category in year t that are in the column category in year $t + 1$. For example, 6.6 percent of those out of the labour force switched to wage-employment in the following year. The main diagonal corresponds to inertia: individuals who stay in the same form of employment in two adjacent years.

¹³Transition matrices separated by year are available upon request. They show a 20% increase in transitions from wage employment to self-employment in 2008-2009, compared to the previous and following year. The percentage of involuntary job separations exhibits the same pattern.

TABLE 2
LABOUR MARKET TRANSITIONS IN URBAN COLOMBIA 2007-2010

Employment Status t	Employment Status $t+1$					Total %
	Out of LF	WE	SE	Other	UE	
Out of LF	81.5	6.6	6.5	0.6	4.7	100
WE	8.3	72.9	11.9	0.5	6.4	100
SE	15.7	16.7	62.3	1.0	4.3	100
Other	44.6	20.1	23.4	5.4	6.5	100
UE	28.6	29.8	16.5	1.1	24.0	100
Total $t + 1$	45.1	29.0	18.9	0.7	6.3	100

Source: Encuesta Social Longitudinal de Fedesarrollo (ESLF).

WE includes private and government sector employees, domestic and casual workers. *SE* includes own-account workers and employers. *Other* includes unpaid labour and unspecified.

Several stylised facts emerge from the analysis of transitions. First, there is large flexibility in the labour market. Fewer than 75 percent of the wage-employed, and fewer than two thirds of the self-employed remain in their respective occupations. Second, bilateral movements between self-employment and wage employment are more prevalent than movements with other sectors, for example unemployment. This supports the idea of on-the-job-search of the self-employed as an important economic mechanism in urban labour markets of developing countries.

Summary statistics of the variables used in the analysis, and precise variable definitions, can be found in Appendix C. On average, self-employed individuals are almost a decade older, have more than a year less of formal education, and are more likely to be married as well as head of their household. Interestingly, more than three quarters of both wage-employed and self-employed report being satisfied with their employment, although 41 percent report reasons for self-employment that imply involuntary self-employment back lack of opportunities. Just over 20 percent of those self-employed in a given year switch into wage employment in the following year.¹⁴ A fifth of those who stay self-employed in consecutive periods add employees to their firm. Figure 6 reveals that more than half the firms in the sample (56 percent) are owner-operator firms without other employees. Approximately 21 percent have a single employee, 8 percent have two employees. The ninety-

¹⁴ These transition rates are higher than in Table 2 because here I only consider individuals who are either wage-employed or self-employed in t and $t + 1$.

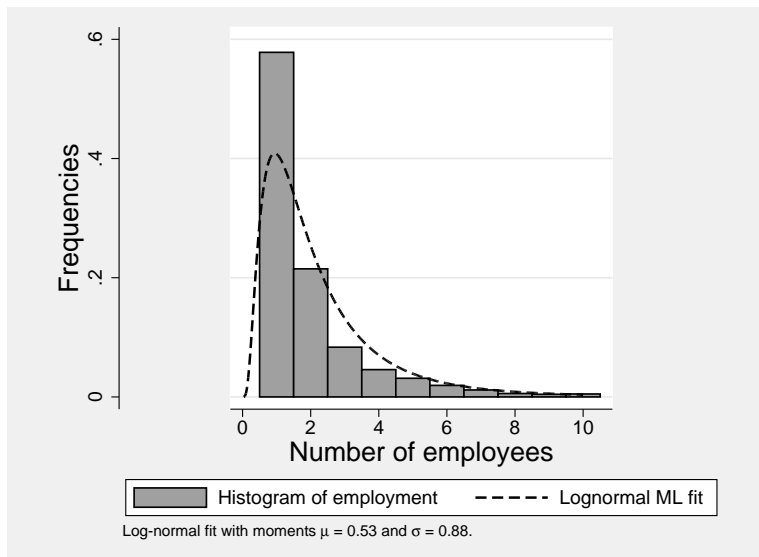


FIGURE 6
FIRM SIZE DISTRIBUTION

eth percentile is at five workers. Less than three percent of the firms in the sample have more than ten workers.

7 Results

This section presents the results from the empirical application and test of the model presented in this paper. I first discuss results of wage distribution estimation, including from a structural estimation of the theoretical model. Then I use the estimated wage distributions to construct counterfactual measures of alternative employment opportunities in wage employment. These proxy for the relative attractiveness of wage employment versus self-employment for all owners of small businesses. Finally, I present empirical evidence that tests for, and finds evidence supporting, the empirical predictions from the model. I conclude this section with robustness checks, and a placebo test.

7.1 First Step: Wage Equations and Predicted Wages

I estimate the conditional mean of log earnings as Mincer equations, dependant on educational attainment, a second-order polynomial of potential exper-

ience, and on gender. All of these explanatory variables are pre-determined with respect to the sector of employment. I then estimate three alternative models of wage distributions: model B1.1. with a simple OLS wage regression, model B1.2. by maximum likelihood estimation of the endogenous switching model on cross-sectional data on earnings and employment sectors, and the dynamic search model B1.3. on a balanced panel over two periods of earnings, sector, and transition data.

Table 3 presents the results. Column (1) shows the coefficients from a single wage equation using OLS. Columns (2)-(4) show the coefficients of the wage, business income, and selection equations of the estimated ESM. Finally, columns (5)-(6) show the parameter estimates of the structural dynamic selection model.

The coefficients the earnings equations are reported in the upper panel of the table. The coefficients are very similar between OLS and the ESM, though the coefficient on education is somewhat lower in the DSM. This suggests that DSM seems indeed to do well at controlling for correlated unobservables that are likely to bias this coefficient upwards. It seems that the ESM, by essentially controlling for the *average* unobserved heterogeneity in a sector, is not sufficient for doing this. The main differences between the models, however, consist in the shape of the wage distribution that they estimate. In particular, the structural DSM estimate suggest a substantial degree of correlation of unobservables $\sigma_{\epsilon\nu}$, and of search frictions captured by λ .

7.2 Second step: Measures of alternative employment opportunities

With the predicted wages I construct the sample counterparts to the measures of alternative employment opportunities. Table 4 summarises, and shows that many self-employed have a good chance of receiving a favourable wage draw, though there is substantial heterogeneity across individuals. A majority of them can expect to increase their income if they switch to wage-employment. The ESM produces somewhat lower predictions of wages compared to other other two methods. As remarked when I discussed identification, this might potentially be an underestimate.

TABLE 3
ESTIMATES OF THE WAGE AND BUSINESS INCOME DISTRIBUTIONS

	(1)	(2)	(3)	(4)	(5)	(6)
	OLS		ESM			DSM
	WE	WE	SE	Selection	WE	SE
Parameters of conditional earnings mean						
Years of Schooling	0.102*** (-0.002)	0.100*** (0.002)	0.092*** (0.004)	-0.008*** (0.003)	0.086*** (0.002)	0.083*** (0.003)
Experience	0.033*** (-0.002)	0.032*** (0.001)	0.014*** (0.003)	-0.028*** (0.002)	0.030*** (0.002)	0.041*** (0.004)
Experience ² /100	-0.041*** (-0.003)	-0.042*** (0.003)	-0.049*** (0.005)	-0.001 (0.004)	-0.040*** (0.003)	-0.057*** (0.005)
Male	0.245*** (-0.013)	0.244*** (0.011)	0.525*** (0.029)	-0.014 (0.019)	0.239*** (-0.014)	0.425*** (0.030)
Constant	11.673*** (-0.033)	11.676*** (0.026)	13.040*** (0.084)	0.038*** (0.004)	12.700*** (-0.032)	11.908*** (0.074)
Household size				0.904*** (0.051)		
Parameters of earnings distribution						
Std. Dev. σ_{\bullet}		0.589*** (0.004)	1.432*** (0.022)		0.567*** (0.005)	0.927*** (0.010)
Correlation $\rho_{\bullet\eta}$		0.110** (0.055)	0.861*** (0.009)			
Autocorrelation ρ_{\bullet}					0.575*** (0.011)	0.489*** (0.016)
Cross-correlation $\rho_{e\nu}$						0.385*** (0.005)
Search friction parameters						
Wage offer prob. λ					0.328*** (0.012)	
Job loss prob. δ					0.132*** (0.005)	
Observations N_{\bullet}	12,880	12,063	7,090	19,153	5,510	2,623

*** p<0.01 ** p<0.05 * p<0.10. Column 1 reports OLS regression on log wages. Columns (2) - (4) report maximum likelihood estimation (MLE) of the ESM, with Household size as exclusion restriction. Columns (5) - (6) report MLE of the DSM on a balanced 2-period panel of SE and WE individuals.

TABLE 4
PREDICTED VARIABLES SUMMARY

		N	Mean	SD	I	Quartiles II	III
OLS	$\widehat{\mathbf{1}}(E(w_i) > \pi_i)$	8083	0.652	0.476	0	1	1
	$\widehat{\text{Pr}}(w_i > \pi_i)$	8083	0.630	0.311	0.379	0.679	0.931
ESM	$\widehat{\mathbf{1}}(E(w_i) > \pi_i)$	7090	0.577	0.494	0	1	1
	$\widehat{\text{Pr}}(w_i > \pi_i)$	7090	0.571	0.341	0.259	0.606	0.910
DSM	$\widehat{\mathbf{1}}(E(w_i) > \pi_i)$	2623	0.683	0.465	0	1	1
	$\widehat{\text{Pr}}(w_i > \pi_i)$	2623	0.638	0.301	0.416	0.687	0.911

Source: Sample distribution of predicted measures as defined in (27).

7.3 Third step: Outcome regressions

With these intermediate results, I can now proceed to testing whether alternative wage employment opportunities matter for decisions of business owners. I first test whether variation in these measures can predict transitions out of self-employment and firm growth. I then proceed to testing for whether there is evidence of a inverse U-shaped pattern of the effect sizes as we move along the firm size distribution.

7.3.1 Main result: Transitions and firm growth

Table 5 reports the main result. I run probit regressions of transitions from self-employment to wage employment, and of the incidence of firm growth measure in number of employees, on the measures of alternative employment opportunities.¹⁵ I report mean marginal effects. I control throughout for city-year dummies to capture shocks to the local labour market that drive earnings, switching, and hiring decisions. The top panel reports results obtained with measures based on wage function estimations based on OLS regressions, the ESM, and structural DSM estimation.

I find evidence that supports the both the first and the second prediction from the model: better relative opportunities in wage employment increase the likelihood of shutting down the firm and taking up a wage job, and

¹⁵I tried logit and LPM models, and obtained very similar results.

TABLE 5
THE EFFECT OF ALTERNATIVE EMPLOYMENT OPPORTUNITIES ON FIRM
EXIT AND GROWTH

	(1)	(2)	(3)	(4)
Dependent Variable	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm
OLS wage distribution estimates				
$\hat{\mathbf{I}}()$	0.024 (0.016)		-0.038** (0.018)	
$\widehat{\text{Pr}}()$		0.066*** (0.023)		-0.075*** (0.025)
<i>Alternative Levels of Clustering</i>				
None	(0.017)	(0.024)***	(0.016)**	(0.023)***
Household	(0.015)	(0.022)***	(0.016)**	(0.023)***
City	(0.009)***	(0.017)***	(0.016)**	(0.023)***
Year	(0.008)***	(0.008)***	(0.008)***	(0.011)***
City-year	(0.009)***	(0.017)***	(0.016)**	(0.023)***
N_W	12,880	12,880	12,880	12,880
N_S	—	—	—	—
N_2	2,993	2,993	2,368	2,368
ESM wage distribution estimates				
$\hat{\mathbf{I}}()$	0.013 (0.021)		-0.048** (0.019)	
$\widehat{\text{Pr}}()$		0.064 (0.118)		-0.085** (0.040)
N_W	12,063	12,063	12,063	12,063
N_S	7,090	7,090	7,090	7,090
N_{Out}	2,577	2,577	2,034	2,034
DSM wage distribution estimates				
$\hat{\mathbf{I}}()$	0.062*** (0.027)		-0.023** (0.019)	
$\widehat{\text{Pr}}()$		0.102*** (0.034)		-0.047** (0.026)
N_W	5,510	5,510	5,510	5,510
N_S	2,623	2,623	2,623	2,623
N_{Out}	2,052	2,052	2,052	2,052

*** p<0.01 ** p<0.05 * p<0.10. Probit Regressions of model (26), average marginal effects. Dependent variables in column header. All specifications include a constant and city-year dummies. Bootstrapped SE with 200 replications clustered at the individual level. N_W : sample size wages; N_S : sample size SE incomes; N_{Out} : sample size outcome regression.

they reduce the incentives to invest into firm growth. These results are robust to different models of unobserved heterogeneity in earnings. In all specifications, the effect of alternative employment opportunities on firm growth is negative and statistically significant at the 5% level. I report a number of alternative ways of clustering in the first set of regressions based on OLS. Empirically, clustering at the individual level yields the most conservative standard errors. Therefore, I maintain this level of clustering.

7.3.2 Heterogeneous effects

The theoretical model predicts an inverse U-shaped pattern for effect size if convexities in the law of motion of capital are present. I test for this by interacting the alternative employment opportunities measures with initial firm size. For power reasons, I restrict this to firms with up to 4 hired workers. I additionally report interactions with levels of educational attainment.

The results in Table 6 indeed show an inverse U-shape of effect sizes. The effects are the strongest for firms with two hired workers, and become lower for smaller and larger firms. This is precisely the subset of firms for which we would alternative employment opportunities expect to have an effect at all. To further investigate this possibility, I also interact the measures with educational attainment instead. Effects are strongest for individuals with a university degree, and for those who have completed *media*, an upper secondary educational qualification that is a pre-requisite for entry to university. Effects are not significant for other levels of education.

This result, while no conclusive test of the shape of the investment function itself, is certainly consistent with the presence of convexities in investments along the firm size distribution.

7.4 Robustness

I have found that a higher comparative advantage in wage-employment can explain transitions from self- to wage-employment, as well as firm growth. The results are robust to including a number of controls, which I present in Tables 11 and 12 in the appendix. I discuss them briefly here.

I first control for firm size in levels, without interaction terms. Coef-

TABLE 6
HETEROGENEOUS EFFECTS BY FIRM SIZE AND EDUCATION

Dependent Variable	(1) Switch SE to WE	(2) Switch SE to WE	(3) Grow Firm	(4) Grow Firm
Interaction	$\hat{\mathbf{1}}()$	$\widehat{\text{Pr}}()$	$\hat{\mathbf{1}}()$	$\widehat{\text{Pr}}()$
Panel A: Interaction Firm Size				
Measure \times No employees	0.001 (0.022)	0.019 (0.031)	-0.032 (0.026)	-0.086** (0.039)
Measure \times 1 employee	0.021 (0.028)	0.062* (0.036)	-0.074*** (0.025)	-0.125*** (0.032)
Measure \times 2 employees	0.063 (0.043)	0.149*** (0.058)	-0.124*** (0.035)	-0.191*** (0.045)
Measure \times 3 employees	0.055 (0.061)	0.124 (0.082)	-0.069 (0.059)	-0.132 (0.080)
Measure \times 4 employees	0.007 (0.058)	0.127* (0.077)	-0.039 (0.066)	-0.127* (0.074)
N_2	2,581	2,581	2,058	2,058
Test: Jointly	0.735	0.121	0.005***	0.000***
Test: Firm size (0 = 1)	0.483	0.248	0.164	0.266
Test: Firm size (1 = 2)	0.349	0.139	0.135	0.088*
Test: Firm size (2 = 3)	0.896	0.765	0.341	0.371
Test: Firm size (3 = 4)	0.535	0.975	0.613	0.939
Test: Firm size (0 = 2)	0.182	0.047**	0.017**	0.040**
Test: Firm size (0 = 3)	0.415	0.227	0.560	0.609
Test: Firm size (0 = 4)	0.922	0.209	0.928	0.647
Panel B: Interaction Educational Attainment				
Measure \times Primary	-0.029 (0.031)	0.001 (0.047)	0.011 (0.035)	0.019 (0.051)
Measure \times Secondary	0.019 (0.038)	0.094 (0.058)	-0.019 (0.032)	-0.064 (0.050)
Measure \times <i>Media</i>	0.037 (0.038)	0.082 (0.053)	-0.064* (0.036)	-0.120** (0.049)
Measure \times Technical	0.060 (0.048)	0.154** (0.071)	-0.024 (0.051)	-0.072 (0.072)
Measure \times University	0.099** (0.061)	0.133** (0.061)	-0.113*** (0.055)	-0.185*** (0.055)
N_2	2,807	2,807	2,214	2,214
Test: Jointly	0.067*	0.004***	0.020**	0.000***
Test: Primary = Secondary	0.329	0.204	0.522	0.239
Test: Primary = Media	0.157	0.222	0.110	0.037**
Test: Primary = Technical	0.138	0.080*	0.590	0.324
Test: Primary = University	0.014**	0.078*	0.001***	0.003***

Probit Regressions, average marginal effects. Dependent variables in column header. All specifications include a constant, city-year dummies and control for each level of the interaction variable. Firm size is number of hired workers not from the owner's household. Education is highest level of degree completed. SE obtained by bootstrapping OLS, predictions, and probit with 200 replications each. SE clustered at the individual level. 'Test' lines report p-values of multiple coefficients tests. 'Jointly' tests joint significance of interaction terms. 'A = B' tests equality of coefficients on A and B.

ficients become larger in absolute magnitude. Similarly, I also control for income in self-employment. In other words, I compare two individuals that earn the same income in self-employment, but differ in their opportunities to earn income in wage employment. The magnitude of coefficients changes little but standard errors are slightly higher. I next control for a number of household variables. Coefficients on switching are lower, but those of growth similar to before and significant. Finally, I confirm that the preference measure do not just pick up the direct impacts of wage predictors by additionally controlling for the latter.

Next, I construct two subjective measures of job satisfaction and stated sectorial preference, the dummies *Dissatisfied* and *SE by Constraint* (the details are in Appendix C). They proxy for compensating differentials, that is non-income determinants of occupational choice (Roback (1982)). The stated preference measures, when included as controls, predict outcomes with the expected negative sign: if they increase, switching becomes more likely and growth less likely. But they have little effects on the coefficients of interest. I also change the form of the wage prediction in the first step. I include city and city-year effects in the wage prediction, and I allow for nonlinear wage-schooling profiles by expressing years of schooling in terms of linear spline functions.

Finally, I to check that the effects of attrition from the panel survey are not sufficient to explain my results. I drop data from cities for which attrition was particularly high. I exclude the first survey wave, which has a slightly different design. I verify that attrition is not correlated with the sector of employment - in particular, it is not higher for the self-employed than it is for the wage-employed. I predict the attrition probability and re-estimate, weighting observations by the inverse of their predicted attrition probability. None of these issues affect the main results and the conclusions drawn from them.

7.5 Placebo test: Workers from within the household

I rely on firm growth as an empirical measure of investment, since my data, like most labour force surveys, lacks information on the capital stock of firms run by household members. I define firm growth as an indicator whether a firm adds paid employees to their workforce. I argue that this is the right measure to use. Even though firing workers is relatively easy for small employers in the urban informal sector, hiring new workers comes with a certain

cost (searching, screening, on-the-job training, etc.) that is especially significant in labour markets with large informational frictions. Conceptually, the fact of bearing a hiring cost in all states of the world, but having a benefit from the worker only in some states (when the business is continued) is quite similar to making a capital investment that is to some degree irreversible.

TABLE 7
PLACEBO TEST: WORKERS FROM WITHIN THE HOUSEHOLD

	(1)	(2)	(3)	(4)
Dependent Variable	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm
$\widehat{\mathbf{1}}(E(w_i) > \pi_i)$			-0.019 (0.013)	
$\widehat{\text{Pr}}(w_i > \pi_i)$				-0.037 (0.033)
N_1			12,880	12,880
N_2			2,368	2368

*** p<0.01 ** p<0.05 * p<0.10. Probit Regressions, average marginal effects. Dependent variables in column header. All specifications include a constant, and city-year dummies. SE obtained by bootstrapping OLS, predictions, and probit with 200 replications each. SE clustered at the individual level.

I provide evidence for these claims by running a placebo exercise where I replace the definition of firm growth by an indicator whether the firm hires an additional member of the household or not to work in the firm. Family members are quite a different source of labour. They are often unpaid, can be ‘hired’ and ‘fired’ for no cost, and generally provide a flexible source of labour to cover spikes in demand, temporary absence of the business owner, etc. With the results in Table 7, I show that this alternative definition of firm growth, which does not share the features of an investment with irreversible components, is not affected by alternative employment opportunities of the firm owner.

8 Conclusion

In this paper, I propose a model in which differences in employment opportunities impact the investment decisions of small business owners. The effective discount factor serves as the central transmission channel: occupational choice affects the shutdown conditions of firms, and this lowers *ex ante* incentives for investments.

This interaction of occupational choice with investment decisions opens up the possibility of non-concave capital growth paths and multiple firm size equilibria in an otherwise neoclassical intertemporal choice model. I discuss conditions under which convexities and multiple equilibria arise. Generally, they are most likely to arise in the middle of the size distribution of small, owner-operated firms. For these owners, differences in the likelihood to shut down the firm and take up a wage offer really matter for whether they will grow the firm and add employees, or whether they will keep it small and eventually shut it down.

Using panel data on self-employment and wage employment in urban Colombia, and structurally identifying the wage function in the presence of correlated heterogeneity that is endogenous to choices, I find evidence that is consistent with the model. Owners of firms with two employees, and who have higher academic qualifications, are most susceptible to be affected by the relative attractiveness of wage employment. Firms that have fewer or more employees are affected to a much lesser degree by those outside opportunities, and effects are not significant for them. Presumably, the first group of individuals have the appropriate skills, education, and experience to run a fairly successful firm – but with these endowments, they can at the same time expect to find well-paid wage employment.

More generally, my model stresses the importance of distinguishing between *effective* and *technical* marginal returns to capital when thinking about optimal investment policies of firms. When discounted by the probability of exiting self-employment, decision-relevant returns to capital may be much lower than what time preferences and production technology alone suggest.

From a policy perspective, it might seem troubling to find that especially the owners of fairly successful small firms, who might be thought of as the pool from which the future generation of entrepreneurs is drawn, are the ones with the strongest incentives to take up wage jobs, and in turn leave their

firms small. However, those who take up wage jobs generally do so because it makes them better off. Optimal policies would support both those businesses who want to grow (and that create jobs as they do), and to help those firm owners who stay small to find a wage job where they can make the most productive use of their skills. The challenge is clearly how to separately identify one group from the other, which is an issue that calls for further research.

References

- ADDA, J., C. DUSTMANN, C. MEGHIR, AND J.-M. ROBIN (2013): “Career Progression, Economic Downturn, and Skills,” *NBER Working Paper No. 18832*.
- BANERJEE, A., E. DUFLO, R. GLENNERSTER, AND C. KINNAN (2015): “The miracle of microfinance? Evidence from a randomized evaluation,” *American Economic Journal: Applied Economics*, 7(1), 22–53.
- BOSCH, M., AND W. MALONEY (2010): “Comparative analysis of labor market dynamics using markov processes: An application to informality,” *Labour Economics*, 17(4), 621–631.
- CAHUC, P., F. POSTEL-VINAY, AND J.-M. ROBIN (2006): “Wage bargaining with on-the-job search: Theory and evidence,” *Econometrica*, 74(2), 323–364.
- CARD, D. (1996): “The effects of unions on the structure of wages: A longitudinal analysis,” *Econometrica*, 64(4), 957–979.
- CUESTA, J., AND C. BOHÓRQUEZ (2011): “Labour market transitions and social security in Colombia,” Discussion paper, World Bank Policy Research Working Paper No. 5650.
- DE MEL, S., D. MCKENZIE, AND C. WOODRUFF (2008): “Returns to capital in microenterprises: Evidence from a field experiment,” *The Quarterly Journal of Economics*, 123(4), 1329–1372.
- (2012): “One-time transfers of cash or capital have long-lasting effects on microenterprises in Sri Lanka,” *Science*, 335(6071), 962–966.
- DIAMOND, D. W. (1984): “Financial Intermediation and Delegated Monitoring,” *The Review of Economic Studies*, 51(3), 393–414.

- EVANS, D., AND B. JOVANOVIĆ (1989): “An Estimated Model of Entrepreneurial Choice under Liquidity Constraints,” *Journal of Political Economy*, 97(4), 808–827.
- EVANS, D. S. (1987): “Tests of alternative theories of firm growth,” *Journal of Political Economy*, 95(4), 657–674.
- FAFCHAMPS, M., D. MCKENZIE, S. QUINN, AND C. WOODRUFF (2014): “Microenterprise Growth and the Flypaper Effect: Evidence from a Randomized Experiment in Ghana,” *Journal of Development Economics*, 116, 211–226.
- FAFCHAMPS, M., AND S. QUINN (2015): “Aspire,” *NBER Working Paper No. 21084*.
- FAFCHAMPS, M., AND C. WOODRUFF (2015): “Identifying Gazelles: Expert Panels vs. Surveys as a Means to Identify Firms with Rapid Growth Potential,” *CEPR Discussion Paper no. 10597*.
- FAMA, E. F. (1980): “Agency Problems and the Theory of the Firm,” *Journal of Political Economy*, pp. 288–307.
- FIELDS, G. (1975): “Rural-urban Migration, Urban Unemployment and Underemployment, and Job-Search Activity in LDCs,” *Journal of Development Economics*, 2(2), 165–187.
- GÜNTHER, I., AND A. LAUNOV (2012): “Informal Employment in Developing Countries: Opportunity or Last Resort?,” *Journal of Development Economics*, 97(1), 88–98.
- HARRIS, J., AND M. TODARO (1970): “Migration, Unemployment and Development: A Two-Sector Analysis,” *American Economic Review*, 60(1), 126–142.
- HAYWOOD, L., AND P. FALCO (2016): “Entrepreneurship vs Joblessness: Explaining the Rise in Self-Employment,” *Journal of Development Economics*, 118, 245–265.
- HECKMAN, J. (1979): “Sample Selection Bias as a Specification Error,” *Econometrica*, 47(1), 153–161.
- HOPENHAYN, H. (1992): “Entry, Exit, and Firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60(5), 1127–1150.

- IMBERT, C. (2013): “Decomposing the Labor Market Earnings Inequality: The Public and Private Sectors in Vietnam, 1993-2006,” *World Bank Economic Review*, 27(1), 55–79.
- JOVANOVIĆ, B. (1982): “Selection and the Evolution of Industry,” *Econometrica*, 50(3), 649–670.
- KARLAN, D., R. KNIGHT, AND C. UDRY (2015): “Consulting and Capital Experiments with Microenterprise Tailors in Ghana,” *Journal of Economic Behaviour and Organization*, 118, 281–302.
- KARLAN, D., AND M. VALDIVIA (2011): “Teaching Entrepreneurship: Impact of Business Training on Microfinance Clients and Institutions,” *Review of Economics and Statistics*, 93(2), 510–527.
- KIHLSTROM, R., AND J.-J. LAFFONT (1979): “A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion,” *Journal of Political Economy*, 87(4), 719–748.
- KOELLE, M., AND S. QUINN (2016): “Self-Employment, Capital and Job Search in Urban Ghana: Evidence from a Structural Model,” *Mimeo*.
- KRAAY, A., AND D. MCKENZIE (2014): “Do Poverty Traps Exist? Assessing the Evidence,” *Journal of Economic Perspectives*, 28(3), 127–148.
- LEE, L.-F. (1978): “Unionism and Wage Rates: A Simultaneous Equations Model with Qualitative and Limited Dependent Variables,” *International Economic Review*, 19(2), 415–433.
- LEMIEUX, T. (1998): “Estimating the Effects of Unions on Wage Inequality in a Panel Data Model with Comparative Advantage and Nonrandom Selection,” *Journal of Labor Economics*, 16(2), 261–291.
- LEWIS, W. A. (1954): “Economic Development with Unlimited Supplies of Labour,” *The Manchester School*, 22(2), 139–191.
- LIPPMAN, S., AND J. MCCALL (1976): “Job Search in a Dynamic Economy,” *Journal of Economic Theory*, 12(3), 365–390.
- LOPEZ-GARCIA, I. (2013): “Human Capital and Labor Informality in Developing Countries: A Life-Cycle Approach,” Job Market Paper.
- LUCAS, R. (1978): “On the Size Distribution of Business Firms,” *The Bell Journal of Economics*, 9(2), 508–523.

- MADDALA, G. (1983): *Limited-Dependent and Qualitative Variables in Econometrics*. Cambridge University Press.
- MAGNAC, T. (1991): “Segmented or Competitive Labor Markets,” *Econometrica*, 59(1), 165–187.
- MANNING, A. (2011): “Imperfect Competition in the Labor Market,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and R. Layard, pp. 973–1041. Elsevier.
- MCCALL, J. (1970): “Economics of Information and Job Search,” *The Quarterly Journal of Economics*, 84, 113–126.
- MCKENZIE, D. (2015): “Identifying and Spurring High-Growth Entrepreneurship: Experimental Evidence from a Business Plan Competition,” *BREAD Working Paper no. 462*.
- MCKENZIE, D., AND C. WOODRUFF (2008): “Experimental Evidence on Returns to Capital and Access to Finance in Mexico,” *The World Bank Economic Review*, 22(3), 457–482.
- (2014): “What Are We Learning from Business Training and Entrepreneurship Evaluations around the Developing World?,” *The World Bank Research Observer*, 29(1), 48–82.
- MEGHIR, C., R. NARITA, AND J.-M. ROBIN (2015): “Wages and Informality in Developing Countries,” *American Economic Review*, 105(4), 1509–46.
- MONDRAGÓN-VÉLEZ, C., AND X. PEÑA (2010): “Business Ownership and Self-Employment in Developing Economies: The Colombian Case,” in *International Differences in Entrepreneurship*, ed. by J. Lerner, and A. Schoar, pp. 89–127. University of Chicago Press.
- MORTENSEN, D. (1970): “Job search, the Duration of Unemployment, and the Phillips Curve,” *American Economic Review*, 60(5), 847–862.
- MORTENSEN, D., AND C. PISSARIDES (1999): “New Developments in Models of Search in the Labor Market,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and R. Layard, pp. 2567–2627. Elsevier.
- NARITA, R. (2014): “Self-Employment in Developing Countries: A Search-Theoretic Approach,” *Mimeo*.
- RAMSEY, F. (1928): “A Mathematical Theory of Saving,” *The Economic Journal*, 38, 543–559.

REFERENCES

- RIDDER, G., AND G. VAN DEN BERG (2003): “Measuring Labor Market Frictions: A Cross-Country Comparison,” *Journal of the European Economic Association*, 1(1), 224–244.
- ROBACK, J. (1982): “Wages, Rents, and the Quality of Life,” *Journal of Political Economy*, pp. 1257–1278.
- ROY, A. (1951): “Some Thoughts on the Distribution of Earnings,” *Oxford Economic Papers*, 3(2), 135–146.
- STIGLER, G. (1962): “Information in the Labor Market,” *Journal of Political Economy*, 70, 94–105.
- SURI, T. (2011): “Selection and Comparative Advantage in Technology Adoption,” *Econometrica*, 79(1), 159–209.
- VAN DEN BERG, G. (1990): “Nonstationarity in Job Search Theory,” *The Review of Economic Studies*, 57(2), 255–277.

Appendix

A Parametric assumptions in model simulations

I assume very standard functional forms. The production function is assumed to be Cobb-Douglas, with only capital as an input. The utility function is CRRA. The wage distribution is log-normal. Search frictions are binary variables that necessarily follow a Bernoulli distribution with a unique parameter, the mean λ . I maintain these functional forms throughout:

$$\pi(k) = A \cdot k^\alpha \quad (28)$$

$$u(c) = \frac{c^{1-\theta}}{1-\theta} \quad (29)$$

$$\ln w \sim N(\mu_w, \sigma_w^2) \quad (30)$$

I also maintain the parameter values chosen for the initial simulation, and only vary the mean of the log-wage distribution:

TABLE 8
PARAMETRIC ASSUMPTIONS IN MODEL SIMULATIONS

Figure	2a	2b	5 (solid)	5 (dash-dot)	5 (dash)
β	0.95	0.95	0.95	0.95	0.95
θ	1.5	1.5	1.5	1.5	1.5
A	1.6	1.6	1.6	1.6	1.6
α	0.5	0.5	0.5	0.5	0.5
μ_w	1	2	1.6	2	2.2
σ_w	0.2	0.2	0.2	0.2	0.2
λ	0.7	0.7	0.7	0.7	0.7

B Derivation of Likelihood Function

The likelihood function is derived as follows. First, it should be noted that the likelihood is not separable over time, but it is separable between individuals. The likelihood for every individual, which comprises an observation in each of the two periods, is dependent on the combinations of states, or labour market sectors, the individual is observed in. With 2 periods and 2 states, there are 4 possible cases.

Case 1: SE in period 1, WE in period 2

With $\pi_{it} = \mathbf{z}'_{it}\gamma + \nu_{it}$, the density of SE income is $f(\nu_{i1})$. All densities are conditional (on x_{it}), with the conditioning suppressed for easier reading. Similarly, with $w_{it} = \mathbf{x}'_{it}\beta + \epsilon_{it}$, and noting that the earnings observation in the second period is conditional on the error term of the first period, the density of wage in period 2 is $f(\epsilon_{i2}|\nu_{i1})$. Together, the product $f(\nu_{i1}) \cdot f(\epsilon_{i2}|\nu_{i1})$ is the joint density $f(\nu_{i1}, \epsilon_{i2})$ by Bayes' Rule. The probability mass of observing SE in period 1 is $(1 - \omega)$ and the probability mass of observing WE in period 2 conditional on SE in period 1 and a residual ν_{i1} is $\lambda \cdot \Pr(w_{i2} > \pi_{i1}|\nu_{i1})$. This implies that the joint likelihood for the individual is $(1 - \omega) \cdot f(\nu_{i1}) \cdot \lambda \cdot \Pr(w_{i2} > \pi_{i1}|\nu_{i1}) \cdot f(\epsilon_{i2}|\nu_{i1})$. It is manipulated in the following manner:

$$\begin{aligned} L_i^1 &= (1 - \omega) \cdot f(\nu_{i1}) \cdot \lambda \cdot \Pr(w_{i2} > \pi_{i1}|\nu_{i1}) \cdot f(\epsilon_{i2}|\nu_{i1}) \\ &= f(\nu_{i1}) \cdot f(\epsilon_{i2}|\nu_{i1}) \cdot \Pr(\mathbf{x}'_{i2}\beta + \epsilon_{i2} > \mathbf{z}'_{i1}\gamma + \nu_{i1}|\nu_{i1}) \cdot \lambda(1 - \omega) \\ &= f(\nu_{i1}, \epsilon_{i2}) \cdot \Pr(-\epsilon_{i2} < \mathbf{x}'_{i2}\beta - \mathbf{z}'_{i1}\gamma - \nu_{i1}|\nu_{i1}) \cdot \lambda(1 - \omega) \\ &= f(\nu_{i1}, \epsilon_{i2}) \cdot \Pr\left(-\epsilon_{i2} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \nu_{i1} < \mathbf{x}'_{i2}\beta - \mathbf{z}'_{i1}\gamma - \nu_{i1} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \cdot \nu_{i1} \middle| \nu_{i1}\right) \lambda(1 - \omega) \\ &= f(\nu_{i1}, \epsilon_{i2}) \Pr\left(\frac{-\epsilon_{i2} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \nu_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}} < \frac{\mathbf{x}'_{i2}\beta - \pi_{i1} + \left(\rho \frac{\sigma_\epsilon}{\sigma_\nu}\right) \cdot \nu_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}} \middle| \nu_{i1}\right) \lambda(1 - \omega) \end{aligned}$$

Under the normality assumption (5.2.3) the conditional density of $\epsilon_{i2}|\nu_{i1}$ is normal with mean $E(\epsilon_{i2}|\nu_{i1}) = \rho \frac{\sigma_\epsilon}{\sigma_\nu} \nu_{i1}$ and variance $Var(\epsilon_{i2}|\nu_{i1}) = \sigma_\epsilon^2(1 - \rho^2)$. The variable

$$z = \frac{-\epsilon_{i2} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \nu_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}} \sim N(0, 1)$$

then follows a standard normal distribution. Furthermore, the marginal bivariate distribution of ν_{i1} and ϵ_{i2} is a bivariate normal distribution. I denote the joint distribution of standardized normal variables $z_1 = \frac{x_1 - \mu_1}{\sigma_1}$ and $z_2 = \frac{x_2 - \mu_2}{\sigma_2}$ with correlation ρ as

$$\phi(z_1, z_2; \rho) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1 - \rho^2}} \exp\left\{-\frac{1}{2(1 - \rho^2)}[z_1^2 - 2\rho \cdot z_1 z_2 + z_2^2]\right\}.$$

The likelihood under normality in Case 1 is then

$$L_i^1 = \phi\left(\frac{\nu_{i1}}{\sigma_\nu}, \frac{\epsilon_{i2}}{\sigma_\epsilon}; \rho\right) \cdot \Phi\left(\frac{\mathbf{x}'_{i2}\beta - \pi_{i1} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \cdot \nu_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}}\right) \cdot \lambda(1 - \omega)$$

Case 2: SE in period 1 and 2

The reasoning follows exactly Case 1. The density of SE income in period 1 is $f(\nu_{i1})$ and in period 2 $f(\nu_{i2}|\nu_{i1})$, and the joint density $f(\nu_{i1}, \nu_{i2})$. The probability mass of observing SE in period 1 is $(1 - \omega)$, the probability mass of observing SE in

period 2 conditional on SE in period 1 and a residual ν_{i1} is $1 - \lambda \cdot \Pr(w_{i2} > \pi_{i1} | \nu_{i1}) = (1 - \lambda) + \lambda \cdot (1 - \Pr(w_{i2} > \pi_{i1} | \nu_{i1}))$. Then, the derivation for the likelihood of an individual in Case 2 is

$$\begin{aligned} L_i^2 &= (1 - \omega) \cdot f(\nu_{i1}) \cdot [(1 - \lambda) + \lambda \cdot (1 - \Pr(w_{i2} > \pi_{i1} | \nu_{i1}))] \cdot f(\nu_{i2} | \nu_{i1}) \\ &= f(\nu_{i1}, \nu_{i2}) \left[(1 - \lambda) + \lambda \Pr \left(\frac{-\epsilon_{i2} + \rho \frac{\sigma_\epsilon}{\sigma_\nu}}{\sigma_\epsilon \sqrt{1 - \rho^2}} < \frac{\mathbf{x}_{i2}' \beta - \pi_{i1} + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \nu_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}} \middle| \nu_{i1} \right) \right] (1 - \omega) \\ &= \phi \left(\frac{\nu_{i1}}{\sigma_\nu}, \frac{\nu_{i2}}{\sigma_\nu}; \rho_\nu \right) \left[(1 - \lambda) + \lambda \cdot \Phi \left(- \left(\frac{\mathbf{x}_{i2}' \beta + \rho \frac{\sigma_\epsilon}{\sigma_\nu} \cdot \nu_{i1} - \pi_{i1}}{\sigma_\epsilon \sqrt{1 - \rho^2}} \right) \right) \right] (1 - \omega) \end{aligned}$$

noting again the implications of the quadrivariate normality assumption in (5.2.3).

Case 3: WE in period 1 and 2

The density of the wage in period 1 is $f(\epsilon_{i1})$ and in period 2 $f(\epsilon_{i2} | \epsilon_{i1})$, jointly $f(\epsilon_{i1}, \epsilon_{i2})$. The probability mass of observing WE in period 1 is ω , the probability mass of observing WE in period 2 conditional on WE in period 1 and a residual ϵ_{i1} is $(1 - \delta)$. The likelihood of an individual in Case 3 is

$$\begin{aligned} L_i^3 &= \omega \cdot f(\epsilon_{i1}) \cdot [1 - \delta] \cdot f(\epsilon_{i2} | \epsilon_{i1}) \\ &= f(\epsilon_{i1}, \epsilon_{i2}) \cdot [1 - \delta] \cdot \omega \\ &= \phi \left(\frac{\epsilon_{i1}}{\sigma_\epsilon}, \frac{\epsilon_{i2}}{\sigma_\epsilon}; \rho_\epsilon \right) \cdot [1 - \delta] \cdot \omega \end{aligned}$$

Case 4: WE in period 1, and SE in period 2

The density of the wage in period 1 is $f(\epsilon_{i1})$ and the density of income in period 2 $f(\nu_{i2} | \epsilon_{i1})$, jointly $f(\nu_{i1}, \epsilon_{i2})$. The probability mass of observing WE in period 1 is ω , the probability mass of observing SE in period 2 conditional on WE in period 1 and a residual ϵ_{i1} is δ . The likelihood of an individual in Case 4 is

$$\begin{aligned} L_i^4 &= \omega \cdot f(\epsilon_{i1}) \cdot \delta \cdot f(\nu_{i2} | \epsilon_{i1}) \\ &= f(\epsilon_{i1}, \nu_{i2}) \cdot \delta \cdot \omega \\ &= \phi \left(\frac{\epsilon_{i1}}{\sigma_\epsilon}, \frac{\nu_{i2}}{\sigma_\nu}; \rho \right) \cdot \delta \cdot \omega \end{aligned}$$

Joint Likelihood

Under assumption of independence across individuals, the joint likelihood is the product of the individual likelihoods:

$$\mathcal{L}(\beta, \gamma, \delta, \lambda, \omega, \sigma_\epsilon, \sigma_\nu, \rho, \rho_\epsilon, \rho_\nu) = \prod_{i=1}^N [L_i^1]^{D_i^1} [L_i^2]^{D_i^2} [L_i^3]^{D_i^3} [L_i^4]^{D_i^4}$$

D_i^j is a dummy variable indicating the case. The log-likelihood is then obtained by taking the natural logarithm of the respective expressions.

C Data definitions and summary statistics

Table 9: DEFINITIONS OF ALL VARIABLES USED

Variable	Description
A: Main Variables	
Male	Dummy equal one if male, zero if female.
Age	Age of the individual in years.
Years of Schooling	Years of formal education. Constructed by combining information on highest level of education attended, and highest grade completed.
Experience	Labour market experience in years, constructed as $Age - Years\ of\ Schooling - 6$.
HH head	Dummy equal one if household head.
Married	Dummy equal one for married and cohabitating.
HH Size	Number of individuals reported to be living in the household.
Monthly income	Real monthly income (in 2010 COP, inflated with CPI) for self-employed; real monthly wage for wage-employed, including overtime and extra pay, excluding fringe benefits
Dissatisfied	Dummy equal one if said dissatisfied with current job.
SE by Constraint	<i>(SE only)</i> Dummy equal one if self-employed by constraint. Constructed from list of ten options to question about main reason of being self-employed.
Firm Size	<i>(SE only)</i> Number of employees that are not household members.
Registered	<i>(SE only)</i> Dummy equal one if business registered with any authority or official organism.
C: Outcome Variables	
Switch WE to SE	<i>(WE only)</i> Dummy equal one if a wage-employed individual transits to self-employment in the next period, zero if stays wage-employed.

CONTINUED ON NEXT PAGE

TABLE 9 – CONTINUED FROM PREVIOUS PAGE

Variable	Description
Switch SE to WE	(<i>SE only</i>) Dummy equal one if a self-employed individual transits to wage-employment in the next period, zero if stays wage-employed.
Grow Firm	(<i>Non-switching SE only</i>) Dummy equal one if self-employed individual adds employees to firm, zero otherwise.

TABLE 10
SUMMARY STATISTICS OF ALL VARIABLES USED

	N	WE Mean	Stdev	N	SE Mean	Stdev	T-test p-value
A							
Male	12,880	0.55	.	8,083	0.57	.	0.001
Age	12,880	35.93	11.84	8,082	44.33	13.65	0.000
Years of Schooling	12,880	10.74	4.07	8,083	9.52	4.46	0.000
Experience	12,880	19.19	13.31	8,083	28.81	15.37	0.000
Household head	12,880	0.36	.	8,083	0.50	.	0.000
Married	12,857	0.48	.	8,075	0.61	.	0.000
HH size	12,063	4.87	1.97	7,545	4.52	1.96	0.000
Monthly Income †	12,880	803.86	1093.1	8,083	763.32	3958.6	0.368
Monthly Income (log)	12,880	13.32	0.71	8,083	12.15	3.27	0.000
Dissatisfaction	12,880	0.21	.	7,589	0.23	.	0.004
SE by Constraint	.	.	.	7,856	0.41	.	.
Firm Size	.	.	.	7,255	1.89	14.79	.
Registered	.	.	.	7,398	0.29	.	.
B							
Switch WE to SE	5,178	0.14
Switch SE to WE	.	.	.	2,993	0.21	.	.
Grow Firm	.	.	.	2,368	0.20	.	.

Source: Encuesta Social Longitudinal de Fedesarrollo (ESLF)

† In thousands of Colombian Pesos (COP) of 2010. In 2010 the exchange rate was COP 1,898 for one US dollar. Standard Deviation of binary variables not reported.

D Additional tables and figures

TABLE 11
ROBUSTNESS CHECKS (1)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent Variable	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm
$\hat{\mathbf{I}}()$	Control: Firm Size				Control: SE income			
$\widehat{\text{Pr}}()$	0.026 (0.017)		-0.050*** (0.019)		0.024 (0.017)		-0.031 (0.020)	
Control		0.069*** (0.024)		-0.097*** (0.027)		0.079*** (0.025)		-0.070** (0.030)
	0.003*** (0.001)	0.004*** (0.001)	-0.012*** (0.004)	-0.013*** (0.004)	-0.000 (0.002)	0.003 (0.002)	0.003 (0.003)	0.001 (0.003)
N_1	12,880	12,880	12,880	12,880	12,880	12,880	12,880	12,880
N_2	2,789	2,789	2,210	2,210	2,701	2,701	2,154	2,154
$\hat{\mathbf{I}}()$	Controls: HH size, head, married, children				Controls: Wage predictors			
$\widehat{\text{Pr}}()$	0.011 (0.017)		-0.041** (0.018)		0.027* (0.016)		-0.035** (0.017)	
		0.036 (0.024)		-0.079*** (0.026)		0.065*** (0.023)		-0.068*** (0.025)
N_1	12,880	12,880	12,880	12,880	12,880	12,880	12,880	12,880
N_2	2,739	2,739	2,170	2,170	2,993	2,993	2,368	2,368
$\hat{\mathbf{I}}(E(w_i) > \pi_i)$	Subjective Preference: Job Dissatisfaction				Subjective preference: SE by constraint			
$\widehat{\text{Pr}}(w_i > \pi_i)$	0.013 (0.017)		-0.027 (0.018)		0.007 (0.016)		-0.027 (0.018)	
Subjective		0.048** (0.024)		-0.058** (0.025)		0.043* (0.024)		-0.058** (0.026)
	0.071*** (0.018)	0.065*** (0.018)	-0.067*** (0.022)	-0.063*** (0.022)	0.066*** (0.017)	0.061*** (0.017)	-0.051*** (0.017)	-0.047*** (0.018)
N_1	12,880	12,880	12,880	12,880	12,880	12,880	12,880	12,880
N_2	2,990	2,990	2,366	2,366	2,919	2,919	2,309	2,309
$\hat{\mathbf{I}}()$	Wage Controls: City dummies				Wage Controls: city-year dummies			
$\widehat{\text{Pr}}()$	0.019 (0.016)		-0.039** (0.015)		0.019 (0.016)		-0.039** (0.017)	
		0.062*** (0.022)		-0.069*** (0.021)		0.065*** (0.022)		-0.073*** (0.025)
N_1	12,880	12,880	12,880	12,880	12,880	12,880	12,880	12,880
N_2	2,789	2,789	2,210	2,210	2,701	2,701	2,154	2,154

*** p<0.01 ** p<0.05 * p<0.10. Probit Regressions, average marginal effects. Dependent variables in column header. All specifications include a constant, and city-year dummies. Additional controls are included as indicated in the panel headers. SE obtained by bootstrapping OLS, predictions, and probit with 200 replications each. SE clustered at the individual level.

TABLE 12
ROBUSTNESS CHECKS (2)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent Variable	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm	Switch SE to WE	Switch SE to WE	Grow Firm	Grow Firm
$\hat{\mathbf{I}}()$	Wage Controls: education spline function				Wage Controls: alternative spline function			
	0.020 (0.015)		-0.030* (0.018)		0.025 (0.016)		-0.035** (0.018)	
$\widehat{\text{Pr}}()$		0.065*** (0.022)		-0.065*** (0.025)		0.076*** (0.022)		-0.075*** (0.025)
N_1	12,880	12,880	12,880	12,880	12,880	12,880	12,880	12,880
N_2	2,739	2,739	2,170	2,170	2,993	2,993	2,368	2,368
$\hat{\mathbf{I}}()$	Attrition test: Drop 2007 wave				Attrition test: Drop city of Bucamaranga			
	0.021 (0.018)		-0.036* (0.020)		0.022 (0.018)		-0.034** (0.017)	
$\widehat{\text{Pr}}()$		0.068*** (0.026)		-0.073*** (0.028)		0.068*** (0.025)		-0.072*** (0.023)
N_1	10,979	10,979	10,979	10,979	10,517	10,517	10,517	10,517
N_2	2,488	2,488	1,965	1,965	2,543	2,543	2,009	2,009
$\hat{\mathbf{I}}(E(w_i) > \pi_i)$	Attrition test: Invest probability weights							
	0.020 (0.017)		-0.034** (0.017)					
$\widehat{\text{Pr}}(w_i > \pi_i)$		0.047** (0.023)		-0.069*** (0.024)				
N_1			12,880	12,880				
N_2			2,368	2368				

*** p<0.01 ** p<0.05 * p<0.10. Probit Regressions, average marginal effects. Dependent variables in column header. All specifications include a constant, and city-year dummies. SE obtained by bootstrapping OLS, predictions, and probit with 200 replications each. SE clustered at the individual level.