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Optimal Information Disclosure and Collusion

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Optimal Information Disclosure and Collusion

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Abstract

This paper studies optimal information disclosure under the threat of collusion. A principal seeks to procure a good from one of two agents who can collude against the principal. The first agent has a publicly known cost of production and the second agent's cost is his private information. The principal decides how much information the first agent receives about the costs of the second agent. In the choice of the optimal disclosure policy, the principal faces a trade-off: More information disclosure makes the elicitation of private information easier but facilitates collusion at the same time. It is optimal for the principal to partially disclose information. Under the optimal information structure, none of the agents receives a positive information rent.

Keywords: Collusion, Optimal information disclosure, Hierarchical delegation

JEL Classification: D82, D83

1 Introduction

Collusion is an important concern in a wide variety of economic institutions such as auctions, procurement relationships, or firms. In auctions, bidders can rig their bids to reduce competition. In procurement relationships, contractors can coordinate their communication with the buyer to keep prices high. In firms, team members can covertly reorganize their interaction to increase rents.

The importance of collusion is empirically well documented. In an influential study, Porter and Zona (1993) document bid rigging in auctions for state highway construction contracts in the United States. More recently, Kawai and Nakabayashi (2015) show that a fifth of contracts for public construction projects in Japan are allocated in auctions where bidding behavior is not competitive.

Given the empirical relevance, a theoretical literature on mechanism design with collusion has been developed to derive implications for the optimal design of institutions. Whereas the early papers in this literature modelled collusion under symmetric information between agents, the more recent part of the literature has analyzed the scope and implications of collusion if agents organize collusion under asymmetric information.¹ In these papers, the principal's optimal choice of allocation rules and transfers is analyzed under the assumption that the information structure of the environment is exogenously given. However, in many institutions, such as auctions or firms, the principal can actively influence which information agents receive. Thus, it arises the question whether and how the principal can curb the adverse effects of collusion by designing the information structure.

In the current paper, I analyze the optimal disclosure of agents' private information in a group of potentially colluding agents. I show that the principal benefits from partially disclosing information. In her choice of the optimal information structure, the principal faces a trade-off between the elicitation of private information and the prevention of collusion. In situations where agents cannot collude, the principal benefits from fully disclosing all information about the agents. This leaves the agents symmetrically informed and allows the principal to elicit all information from the agents without leaving them rents. This can be done by employing a "shoot-the-liar-mechanism" in which agents are punished if their reports contradict each other.

¹Tirole (1986) models collusion under symmetric information, Laffont and Martimort (1997) initiated literature on collusion under asymmetric information.

However, if collusion among agents is possible, it is facilitated by information disclosure. In the case of full disclosure, the agents can perfectly coordinate their behavior to maximize the sum of rents. The principal therefore restricts information disclosure if collusion is possible.

I study the trade-off between information elicitation and collusion prevention in an environment where the principal needs to procure a good that can be provided by each of two agents. The first agent's cost is public information and the second agent is privately informed about his cost advantage over the first agent. Apart from transfer rules and allocations, the principal can commit to disclose information to the first agent about the cost advantage of the second agent. Collusion between the agents is organized by a binding side contract which the first agent proposes to the second agent in a take-it-or-leave-it offer.

I first show that the principal can implement her optimal collusion-proof mechanism by delegating the organization of production to the first agent. On one hand, this finding can explain the prevalence of hierarchical forms of organization in firms or procurement relationships. On the other hand, the result enables me to considerably simplify the principal's intricate problem of designing jointly the mechanism and the information structure.

In the delegated contracting game, I compare the relative performance of different information structures and I identify an optimal information structure. At first I show that the principal prefers to disclose no information over disclosing information which induces the first agent to hold posterior beliefs that can be ordered by *First Order Stochastic Dominance* (FSD). This implies in particular that it is not optimal for the principal to disclose all information to the first agent. Secondly, I solve the principal's problem of finding an optimal information structure and show that it is optimal to partially disclose information.

The first result shows that for information structures with posteriors that satisfy FSD, the costs of preventing collusion outweigh the benefits of easier information revelation. It is simplest to explain the result by comparing the extreme information structures of no disclosure and full disclosure. If the first agent is perfectly informed about the second agent's cost advantage, the principal – who always wants to procure the good – needs to pay a price equal to the first agent's cost, or the highest possible cost realization of the second agent. With no disclosure, the first agent anticipates that he may be able to profitably delegate production to the second agent. Thus, the principal can source the good at a lower price.

The optimality of partial disclosure reflects the trade-off between incentive compatibility and the prevention of collusion. The optimal information structure allows the first agent to perfectly differentiate low values of the second agent's costs. However, any signal which the first agent receives for a low level of cost is also triggered for high levels of the second agent's production costs. Furthermore, the lower the cost indicated by a given signal, the higher the downside risk from high levels of costs. The optimal information structure weighs the positive content (positive probability of low costs) and the negative content (downside risks of high costs) of signals such that the expected costs of the first agent are identical across signal realizations. The principal sets her price equal to this value of expected costs. The first agent therefore does not receive a positive information rent. Moreover, the first agent offers a price equal to the low cost level that is possible after a given signal. The second agent accepts these *aggressive* offers only if his costs are low. Thus, the second agent also receives an information rent of zero.

Even if both agents receive no information rent under the optimal information structure, the equilibrium allocation is distorted. From an efficiency perspective, the price offered by the first agent to the second agent is too high. Thus, the second agent produces the good too rarely relative to the first best.

In the next section, I give an overview of the related literature. In section 3, I introduce the model. In Section 4, I illustrate the paper's main results in a simple example. In Section 5, I show that the optimal collusion-proof mechanism can be implemented in a delegated contracting game. I analyze optimal information structures in section 6 and I conclude in section 7.

2 Related Literature

This paper is most closely related to the literature on mechanism design with collusion under asymmetric information and to the literature on optimal information disclosure. To the best of my knowledge, this paper is the first to analyze the optimal *information design* by a principal who interacts with potentially colluding agents who hold private nonverifiable information.

The literature on mechanism design with collusion under asymmetric information² can be subdivided in papers where the agents can collusively coordinate their participation decisions

²There is an earlier literature on collusion with verifiable information which started from Green and Laffont (1979) and Tirole (1986). The papers by Crémer (1996), McAfee and McMillan (1992), and Caillaud and Jehiel (1998) study collusion under asymmetric information in specific mechanisms.

(Mookherjee and Tsumagari, 2004; Dequiedt, 2007; Pavlov, 2008; Che and Kim, 2009)³ and those papers where they cannot. The current paper belongs to the second part of the literature.

This part includes the papers by Laffont and Martimort (1997, 2000) and Che and Kim (2006). Laffont and Martimort (1997) is the first paper to study collusion under asymmetric information. They derive a collusion-proofness principle and study the optimal collusion-proof and anonymous mechanism in a setting where the agents' private information is independently distributed. Laffont and Martimort (2000) show that collusion imposes no loss on the principal if the private information of agents is uncorrelated but reduces the principal's payoff if types are correlated. Che and Kim (2006) study a very general setup that encompasses the two player-two type models studied by Laffont and Martimort. They generalize the result that a principal incurs no loss from collusion if the agents' types are uncorrelated. Furthermore they show that this result extends to the correlated types case as long as there are more than two agents, and their type distributions satisfy an additional property. Further papers analyze to which extent collusion can explain the phenomenon of delegated contracting. Laffont and Martimort (1998) show that with uncorrelated types, a principal cannot gain from delegating contracting to an agent. Faure-Grimaud et al. (2003) and Celik (2009) study collusion in an organization with a principal, an agent, and a supervisor who has imperfect information about the agent's type. Faure-Grimaud et al. (2003) show that delegated contracting is a possible implementation of the optimal collusion-proof mechanism in a model where the agent has a binary type space and the supervisor observes a binary signal. Celik (2009) shows that delegation is suboptimal in a setting where the agent has more than two types and the supervisor's information is modelled as a partition of the agent's type space. In Faure-Grimaud et al. (2003) and Celik (2009), private information is assumed to be *nested* which implies that the agent knows what the supervisor knows about him. In the current paper, I also model nested private information which is a natural assumption in an environment where one agent is (imperfectly) informed about the other agent's type. Interestingly, Celik (2009) notes that nested private information violates the assumption under which Che and Kim (2006) show that collusion is not harmful to the principal.

This paper is furthermore related to the literature on optimal disclosure of information in

³Mookherjee and Tsumagari (2004) analyze the optimal design of supplier networks. The other papers mainly consider auctions with collusion. In auctions, collusion on participation decisions can take the form of *knockout auctions* which determine the bidders who participate in the auction.

mechanisms. Closest from this literature is Bergemann et al. (2015). They analyze a setting with a seller and a buyer in which information about the buyer’s valuation can be disclosed to the seller. They show that through an appropriate information structure, any pair of (ex-ante) expected profit (for the seller) and consumer surplus (for the buyer) can be achieved as long as the expected profit is higher than the no-information profit, consumers surplus is positive, and the sum of profit and consumer surplus is lower than the maximal social surplus. In the delegated contracting game of this paper, the interaction between the agents is similar to the interaction between seller and buyer in Bergemann et al. (2015). However, unlike in Bergemann et al. (2015), there is a third party – the principal – who seeks to extract surplus from the agents without observing their information.

Bergemann and Pesendorfer (2007) study the joint optimal design of information structure and auction format when the seller can disclose information to bidders. As in Bergemann and Pesendorfer (2007), the current paper analyzes static disclosure of information which occurs before the agents make their participation decision. In this respect, the current paper differs from Esó and Szentes (2007a,b) and Li and Shi (2015), who consider sequential information disclosure, where agents first decide whether to participate and then receive information, or from Bergemann and Wambach (2015) where agents receive new information sequentially.

This paper is furthermore related to the literature on Bayesian Persuasion initiated by Kamenica and Gentzkow (2011). Unlike most of this literature, the current paper allows for commitment to monetary transfers between the principal (respectively, the sender in Bayesian Persuasion) and the agents (resp. the receivers).

3 The Model

Let me now introduce the model. A principal P seeks to procure a good from one of two agents, A_1 and A_2 . P always wants to buy the good.⁴ A_1 can procure the good at cost $z \in \mathbb{R}$. The value of z is common knowledge. A_2 can produce the good at cost c which is drawn from the finite set $C = \{c_1, c_2, \dots, c_n\} \subset \mathbb{R}$. The cost advantage of A_2 over A_1 is given by $\theta = z - c$. It is convenient to denote the type of A_2 by the cost advantage θ , where $\theta \in \Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ such that $\theta_i < \theta_{i+1}$. The prior belief on θ is given by $\lambda \in \Delta\Theta$. To avoid trivial cases, I assume

⁴This could be modelled by assigning a utility of $-\infty$ for P for the case where she does not receive the good.

$\theta_n > 0$.

An allocation determines the agent who procures the good and the transfers that are exchanged between P and the two agents. An allocation is thus given by (x_1, x_2, t_1, t_2) where x_i is the probability that A_i produces the good and t_i is a transfer from P to A_i . As P always procures the good, we consider allocations with $x_1 + x_2 = 1$. A_1 's payoff from the allocation is given by $t_1 - x_1 \cdot z$, and A_2 's payoff amounts to $t_2 - x_2(z - \theta)$. P seeks to minimize the payments that she makes to the agents. Her payoff can be represented as $-t_1 - t_2$. Both agents have an outside option which they value at zero.

Information structures P chooses how much information A_1 receives about the cost advantage of A_2 . I model this by allowing P to freely choose an information structure. An information structure is given by $\sigma = (S, \mu)$ where S is a finite set of signals with generic element $s \in S$ and $\mu \in \Delta(\Theta \times S)$ is a distribution function over the cost advantage and the signal. An information structure has to satisfy the Bayesian consistency requirement that $\sum_{s \in S} \mu(\theta, s) = \lambda(\theta)$. I denote by \mathcal{S} the set of information structures which satisfy these requirements. For a fixed information structure, let $\mu(\theta|s) = \frac{\mu(\theta, s)}{\sum_{s' \in S} \mu(\theta, s')}$ be the conditional probability of θ given the signal realization s . An information structure σ induces the set of posterior beliefs $\{\mu(\cdot|s)\}_{s \in S}$.

I assume that P can observe neither the signal realization s nor the cost advantage θ . A_1 only observes the signal s but not the cost advantage θ . A_2 observes both, the signal s and the cost advantage θ .

Collusion I model collusion as the opportunity for the agents to modify an allocation by changing the identity of the producing agent, by coordinating their reports to P , and by exchanging transfers between each other. Thus, I assume that P can observe neither the identity of the producing agent nor any exchange of transfers or information between the agents. In contrast, the delivery of the good is observable to P and contractible. Starting from an allocation (x_1, x_2, t_1, t_2) , the agents can modify the allocation to $(\chi_1, \chi_2, t_1 + \tau, t_2 - \tau)$ where χ_i is the probability that agent i procures the good with $\chi_1 + \chi_2 = 1$ and τ is a transfer from A_1 to A_2 . Furthermore, the agents can change the transfers t_1 and t_2 directly by coordinating their reports.

The agents organize collusion by signing a binding and enforceable side contract.⁵ In particular, I assume that A_1 can offer the side contract to A_2 in a take-it-or-leave-it offer.

Mechanisms and side contracts I now describe the mechanism which P offers to the agents, and the side contract which A_1 can offer to A_2 . Without loss of generality, I can restrict attention to P offering direct mechanisms of the form

$$\beta = \left(t_1(\hat{s}_1, \hat{s}_2, \hat{\theta}), t_2(\hat{s}_1, \hat{s}_2, \hat{\theta}), x_1(\hat{s}_1, \hat{s}_2, \hat{\theta}), x_2(\hat{s}_1, \hat{s}_2, \hat{\theta}) \right),$$

where t_i is a transfer from P to A_i , x_i is the probability that A_i realizes the project, \hat{s}_i is the report of A_i about $s \in S$, and $\hat{\theta}$ is the report from A_2 to P about $\theta \in \Theta$.

Similarly, there is no loss of generality in focussing on direct side contracts which A_1 offers to A_2 . Such a side contract is given by

$$\gamma = \left(\tau(\hat{\theta}; s), \chi_1(\hat{\theta}; s), \chi_2(\hat{\theta}; s), \rho(\hat{\theta}; s) \right)$$

where τ is a transfer from A_1 to A_2 , χ_i is the probability that A_i realizes the project, $\rho : \Theta \times S \rightarrow S^2 \times \Theta$ is a reporting strategy to the mechanism β , and $\hat{\theta} \in \Theta$ is a report from A_2 to A_1 about $\theta \in \Theta$.

Timing and equilibrium concept The timing of the game is given as follows.

t=0: P chooses the information structure $\sigma \in \mathcal{S}$ and offers a mechanism β to A_1 and A_2 .

t=1: A_1 and A_2 observe σ , β , and the realization of the signal s . A_2 furthermore observes θ .

t=2: A_1 and A_2 each accept or reject P 's offer. If either of them rejects, both agents receive their outside option. Otherwise the game continues.

t=3: A_1 offers a side contract γ to A_2 .

t=4: A_2 accepts or rejects A_1 's offer. If A_2 accepts, the side contract and the mechanism are executed. If A_2 rejects, both agents play the mechanism non-cooperatively.

I study *Bayesian Perfect Equilibria* (BPE) of the game between P , A_1 , and A_2 .

⁵This approach is used by most of the papers in the literature on collusion in mechanism design.

The principal's problem The principal optimally chooses an information structure σ and a direct mechanism β to minimize her expected costs under the constraints that both agents want to participate in the mechanism, that they report their private information truthfully to the mechanism, and that there does not exist a feasible side contract which gives A_1 a strictly higher payoff than to participate non-cooperatively in the mechanism. Formally P 's problem is the following:

$$\begin{aligned} \mathcal{P} : \min_{\sigma, \beta} \mathbf{E} [t_1(s, s, \theta) + t_2(s, s, \theta)] \quad & \text{subject to} \\ \mathbf{E} [t_1(s, s, \theta) - x_1(s, s, \theta)z | s] & \geq 0 \quad \forall s \in S; & (PC_1) \\ \mathbf{E} [t_1(s, s, \theta) - x_1(s, s, \theta)z | s] & \geq \mathbf{E} [t_1(\hat{s}, s, \theta) - x_1(\hat{s}, s, \theta)z | s] \quad \forall \hat{s}, s \in S^2; & (IC_1) \\ t_2(s, s, \theta) - x_2(s, s, \theta)(z - \theta) & \geq 0 \quad \forall s, \theta \in S \times \Theta; & (PC_2) \\ t_2(s, s, \theta) - x_2(s, s, \theta)(z - \theta) & \geq t_2(s, \hat{s}, \hat{\theta}) - x_2(s, \hat{s}, \hat{\theta})(z - \theta) \quad \forall s, \hat{s}, \theta, \hat{\theta} \in S^2 \times \Theta^2; & (IC_2) \\ x_i(s, s, \theta) \in [0, 1], \quad i \in \{1, 2\}, \quad & x_1(s, s, \theta) + x_2(s, s, \theta) = 1. & (F) \end{aligned}$$

and subject to the Collusion-Proofness constraint

$$\begin{aligned} \mathbf{E} [t_1(s, s, \theta) - x_1(s, s, \theta)z | s] & \geq & (CP) \\ \max_{\gamma} \mathbf{E} [t_1(\rho(\theta; s)) + \tau(\theta; s) - \chi_1(\theta; s)z | s] \quad & \text{subject to} \\ t_2(\rho(\theta; s)) - \tau(\theta; s) - \chi_2(\theta; s)(z - \theta) & \geq 0 \quad \forall s, \theta \in S \times \Theta; & (PC_s) \\ t_2(\rho(\theta; s)) - \tau(\theta; s) - \chi_2(\theta; s)(z - \theta) & \geq & (IC_s) \\ t_2(\rho(\hat{\theta}; s)) - \tau(\hat{\theta}; s) - \chi_2(\hat{\theta}; s)(z - \theta) & \quad \forall s, \theta, \hat{\theta} \in S \times \Theta^2; \\ \chi_i(\theta; s) \in [0, 1], \quad i \in \{1, 2\}, \quad & \chi_1(\theta; s) + \chi_2(\theta; s) = 1. & (F_s) \end{aligned}$$

A mechanism β which satisfies the constraints (PC_1) , (IC_1) , (PC_2) , (IC_2) , (F) and (CP) is called *feasible*. A side contract γ which satisfies (PC_s) , (IC_s) , and (F_s) is also called *feasible*.

No collusion benchmark If collusion is not possible, the principal's problem is equivalent to problem \mathcal{P} without the collusion-proofness constraint (CP) . In that case, P can first disclose all information to A_1 and then employ a "shoot-the-liar-mechanism" to implement the first best allocation at no cost. In particular, P could set $x_2(\hat{s}_1) = 1$ if $\hat{s}_1 \leq z$, $x_2(\hat{s}_1) = 0$ if

$\hat{s}_1 > z$ and $x_1(\hat{s}_1) = 1 - x_2(\hat{s}_1)$ where $\hat{s}_1 \in \Theta$ is the report of A_1 and use the transfers $(t_1(\hat{\theta}, \hat{s}_1), t_2(\hat{\theta}, \hat{s}_1)) = (0, \hat{\theta})$ if $\hat{\theta} = \hat{s}_1 \leq z$, $(t_1(\hat{\theta}, \hat{s}_1), t_2(\hat{\theta}, \hat{s}_1)) = (z, 0)$ if $\hat{\theta} = \hat{s}_1 > z$, and $(t_1(\hat{\theta}, \hat{s}_1), t_2(\hat{\theta}, \hat{s}_1)) = -\infty$ otherwise.

4 A simple example

In this section, I illustrate the main results of this paper in an example. Let $z = 4$ and $\Theta = \{1, 2, 3\}$ with the uniform prior λ such that $\lambda(\theta) = \frac{1}{3}$ for all $\theta \in \Theta$.

First, suppose P discloses θ perfectly to A_1 and sets the "shoot-the-liar-mechanism" as described in the previous section. This mechanism always assigns the task of production to A_2 , and pays him – on the equilibrium path – his costs of $4 - \theta$, whereas A_1 never receives a positive transfer. Clearly, this mechanism is not collusion-proof. A_1 can propose a side contract which always assigns production to A_2 , makes the report $(\hat{s}_1, \hat{\theta}) = (1, 1)$ for any value of θ , and demands a payment of $\tau = 3 - (4 - \theta) = \theta - 1$ from A_2 . Participation in this side contract gives A_2 for any type θ the same payoff as playing the mechanism non-cooperatively. A_1 however makes a positive profit for $\theta \in \{2, 3\}$. A_1 profitably exploits that the sum of transfers from P to the agents differs in the reports. Collusion against the mechanism is profitable, because A_1 can design a side-contract which always induces the maximal total transfer to both agents under the mechanism and guarantees A_2 the same payoff as if the mechanism would be played non-cooperatively. In the next section, I show that such a profitable side contract can be found for any mechanism for which the sum of transfers to the agents is not constant over reports. This result holds for all possible information structures. The optimal collusion-proof mechanism therefore features a constant total transfer to the agents. It follows that the optimal collusion-proof mechanism can be implemented as the BPE of a delegated contracting game, where P offers to A_1 a price (equal to the constant total transfer) for delivery of the good, and in turn A_1 offers to A_2 a price for the production of the good.

I compare now the costs of P for the extreme information structures of no disclosure and full disclosure in the delegated contracting game. Suppose at first that P discloses θ perfectly to A_1 . As P always wants to procure the good, she needs to offer a price which is always accepted by A_1 . It follows that P needs to pay a price of 3 to A_1 who then offers a price of $4 - \theta$ to A_2 . Suppose now that P discloses no information to A_1 . Upon acceptance, it is easy to verify that

A_1 optimally offers a price of 2 to A_2 . A_2 accepts this price if $\theta \in \{2, 3\}$. Thus, the expected costs of A_1 are $\Pr(\theta = 1) \cdot 4 + \Pr(\theta \in \{2, 3\}) \cdot 2 = \frac{16}{6}$. A_1 thus accepts if P offers a price of $\frac{16}{6} < 3$. It follows that P prefers no disclosure over full disclosure. As I argue in section 6, no disclosure is preferred by P over any information structure which induces posteriors on Θ that can be ordered by FSD.

Can P improve upon no disclosure? The answer is yes. Consider an information structure with two signals s_1 and s_2 where s_1 is send to A_1 if $\theta \in \{1, 3\}$ and s_2 is send if $\theta = 2$. For this information structure, A_1 offers to A_2 a price of 1 if s_1 has realized and a price of 2 if s_2 has realized. Thus, the expected costs of A_1 is 2 after signal s_2 , and $\Pr(\theta = 3|s_1) \cdot 1 + \Pr(\theta = 1|s_1) \cdot 4 = \frac{15}{6}$ after signal s_1 . This implies that P optimally offers a price of $\frac{15}{6} < \frac{16}{6}$ to A_1 . P therefore prefers this information structure over no disclosure. P can do even better by using an information structure which again uses the signals s_1 and s_2 . s_1 is send to A_1 if $\theta = 3$ and s_2 is send if $\theta = 2$. If $\theta = 1$, s_1 is send with probability α and s_2 is send with probability $1 - \alpha$. The expected costs of A_1 after signal s_1 is then given by $\Pr(\theta = 3|s_1) \cdot 1 + \Pr(\theta = 1|s_1) \cdot 4 = \frac{1+4\alpha}{1+\alpha}$, and the expected costs after s_2 is $\Pr(\theta = 2|s_2) \cdot 2 + \Pr(\theta = 1|s_2) \cdot 4 = \frac{6-4\alpha}{2-\alpha}$. P needs to offer a price at least equal to $\max\left\{\frac{1+4\alpha}{1+\alpha}, \frac{6-4\alpha}{2-\alpha}\right\}$ to A_1 . P sets α optimally such that $\frac{1+4\alpha}{1+\alpha} = \frac{6-4\alpha}{2-\alpha} \Leftrightarrow \alpha = \frac{4}{5}$. A_1 then accepts a price of $\frac{14}{6}$. As I show in section 6, this information structure is optimal for P . Under this optimal information structure, the signal with a higher chance of low costs for A_2 has the larger downside risk of high costs of A_2 . This is a general feature of optimal information structures.

5 Collusion and Delegation

In this section, I show that – for a given information structure – the outcome of the optimal collusion-proof mechanism can be implemented by a delegated contracting game in which P delegates the organization of production to A_1 . While this result is of independent interest, it also simplifies the problem of jointly choosing the optimal information structure and mechanism.

Consider P 's problem \mathcal{P} when some information structure σ is exogenously fixed. P then faces a reduced problem which is given by

$$\mathcal{P}_r : \min_{\beta} \mathbf{E}[t_1(s, s, \theta) + t_2(s, s, \theta)] \quad \text{subject to } (PC_1), (IC_1), (PC_2), (IC_2), (F), (CP).$$

The following Lemma is a crucial step towards the implementation result. It states that a mechanism is only feasible if the sum of transfers to the agents is constant over reports.

Lemma 1. *Any feasible mechanism β satisfies $t_1(s, s, \theta) + t_2(s, s, \theta) = p$ for all $s, \theta \in S \times \Theta$.*

Proof. Toward a contradiction, suppose a mechanism β satisfies all constraints. Let $s^*, \theta^* \in \arg \max_{s, \theta} t_1(s, s, \theta) + t_2(s, s, \theta)$ and suppose there exist $s', \theta' \in S \times \Theta$ such that $t_1(s', s', \theta') + t_2(s', s', \theta') < t_1(s^*, s^*, \theta^*) + t_2(s^*, s^*, \theta^*)$. Consider now the side contract γ which is defined as $\tau(\theta; s) = t_2(s^*, s^*, \theta^*) - t_2(s, s, \theta)$, $\chi_i(\theta; s) = x_i(s, s, \theta)$, and $\rho(\theta; s) = (s^*, s^*, \theta^*)$ for all $s, \theta \in S \times \Theta$ and $i \in \{1, 2\}$. Note that the side contract is feasible, because by construction of the side contract (PC_2) implies (PC_s) , (IC_2) implies (IC_s) , and (F) implies (F_s) . Furthermore

$$\begin{aligned} & \mathbf{E} [t_1(\rho(\theta; s')) + \tau(\theta; s') - \chi_1(\theta; s')z|s'] \\ &= \mathbf{E} [t_1(s^*, s^*, \theta^*) + t_2(s^*, s^*, \theta^*) - t_2(s', s', \theta) - x_1(s', s', \theta)z|s] \\ &> \mathbf{E} [t_1(s', s', \theta) - x_1(s', s', \theta)z|s']. \end{aligned}$$

Thus, (CP) is not satisfied for $s' \in S$, which gives a contradiction. \square

The idea behind the Lemma and its proof is the following: Suppose that for some mechanism, some profile of reports leads to a higher sum of transfers to the agents than all other reports and suppose that the mechanism satisfies participation and incentive constraints for both agents. Then A_1 can design a side contract which always makes the reports that maximize the sum of transfers and implements the same production decision as the mechanism. The transfer between the agents can be used by A_1 to make A_2 as well off as under non-cooperative play in the mechanism. This side contract is strictly profitable for A_1 whenever the sum of transfers to the agents is not maximal.

Lemma 1 implies that P has to make a constant total payment to the agents independently

of their reports. P 's reduced optimization problem can then be restated as

$$\begin{aligned}
\mathcal{P}'_\tau : \min_{p, t_2, x_1, x_2} \quad & p \quad \text{subject to} \\
& \mathbf{E}[p - t_2(s, s, \theta) - x_1(s, s, \theta)z | s] \geq 0 \quad \forall s \in S; & (PC'_1) \\
& \mathbf{E}[p - t_2(s, s, \theta) - x_1(s, s, \theta)z | s] \geq \mathbf{E}[p - t_2(\hat{s}, s, \theta) - x_1(\hat{s}, s, \theta)z | s] \quad \forall \hat{s}, s \in S^2; & (IC'_1) \\
& t_2(s, s, \theta) - x_2(s, s, \theta)(z - \theta) \geq 0 \quad \forall s, \theta \in S \times \Theta; & (PC'_2) \\
& t_2(s, s, \theta) - x_2(s, s, \theta)(z - \theta) \geq t_2(s, \hat{\theta}) - x_2(s, \hat{\theta})(z - \theta) \quad \forall s, \hat{\theta}, \theta \in S^2 \times \Theta^2; & (IC'_2) \\
& x_i(s, s, \theta) \in [0, 1], \quad i \in \{1, 2\}, \quad x_1(s, s, \theta) + x_2(s, s, \theta) = 1. & (F')
\end{aligned}$$

and subject to the rewritten Collusion proofness constraint

$$\begin{aligned}
& \mathbf{E}[p - t_2(s, s, \theta) - x_1(s, s, \theta)z | s] \geq & (CP') \\
& \max_\gamma \mathbf{E}[p - t_2(\rho(\theta; s)) + \tau(\theta; s) - \chi_1(\theta; s)z | s] \quad \text{subject to} \\
& \quad t_2(\rho(\theta; s)) - \tau(\theta; s) - \chi_2(\theta; s)(z - \theta) \geq 0 \quad \forall s, \theta \in S \times \Theta; & (PC'_s) \\
& \quad t_2(\rho(\theta; s)) - \tau(\theta; s) - \chi_2(\theta; s)(z - \theta) \geq & (IC'_s) \\
& \quad \quad t_2(\rho(\hat{\theta}; s)) - \tau(\hat{\theta}; s) - \chi_2(\hat{\theta}; s)(z - \theta) \quad \forall s, \theta, \hat{\theta} \in S \times \Theta^2; \\
& \quad \chi_i(\theta; s) \in [0, 1], \quad i \in \{1, 2\}, \quad \chi_1(\theta; s) + \chi_2(\theta; s) = 1. & (F'_s)
\end{aligned}$$

In this optimization problem, the two agents have opposing preferences. As the total sum of transfers is constant, there is no scope for collusion which increases the total amount of money to be distributed between the agents. For the very same reason, P cannot do better than to delegate the organization of production at a fixed price to A_1 . In particular, define the delegated contracting game δ be defined as follows:

t=0: P offers a payment p to A_1 for delivery of the good.

t=1: A_1 and A_2 observe p and s . A_1 additionally observes θ .

t=2: A_1 accepts or rejects P 's offer.

t=3: A_1 offers a contract $(T(\hat{\theta}), X(\hat{\theta}))$ to A_2 where T is a transfer from A_1 to A_2 , X is the probability that A_2 realizes the project, and $\hat{\theta}$ is A_2 's report about $\theta \in \Theta$.

t=4: A_2 accepts or rejects A_1 's offer.

t=5: Contract is executed.

I study the BPE of this game. Let $(p, t_2(s, s, \theta), x_1(s, s, \theta), x_2(s, s, \theta))$ be a solution to \mathcal{P}' . If there exists a BPE of δ such that P offers p , A_1 always accepts and offers for all $s \in S$ a contract which satisfies $T(\theta) = t_2(s, s, \theta)$ and $X(\theta) = x_2(s, s, \theta)$, and this contract is always accepted by A_2 , then I say that the outcome of the optimal collusion-proof mechanism can be implemented in the delegated contracting game δ . This turns out to be the case.

Proposition 1. *The outcome of the optimal collusion-proof mechanism can be implemented in the delegated contracting game δ .*

Proof. If A_1 accepts P 's offer and the signal is $s \in S$, A_1 optimally offers A_2 the contract $(T(\hat{\theta}), X(\hat{\theta}))$ which satisfies

$$\mathcal{P}_1 : \min_{X, T} \mathbb{E}[T(\theta) + (1 - X(\theta))z|s] \quad \text{subject to}$$

$$T(\theta) - X(\theta)(z - \theta) \geq 0 \quad \forall \theta \in \Theta \quad (\hat{P}C)$$

$$T(\theta) - X(\theta)(z - \theta) \geq T(\hat{\theta}) - X(\hat{\theta})(z - \theta) \quad \forall \theta, \hat{\theta} \in \Theta^2. \quad (\hat{I}C)$$

For the signal realization $s \in S$, set $T(\theta) = t_2(s, s, \theta)$ and $X(\theta) = x_2(s, s, \theta)$. By construction (PC'_2) implies $(\hat{P}C)$ and (IC'_2) implies $(\hat{I}C)$. Moreover (CP') implies that the contract (X, T) is optimal. Finally, (PC_1) implies that it is optimal for A_1 to accept the offered price of p . \square

Proposition 1 is implied by the observation that the agents' interests are opposed under any feasible mechanism. In the optimal collusion-proof mechanism, P makes a constant total transfer to the agents and receives information about A_2 from A_1 . Thus, A_1 only reports information truthfully as long as this does not reduce the share of total transfers that accrue to him. It follows that P cannot do better than to delegate to A_1 the organization of production at the constant total payment.

In the following, I restrict attention to deterministic allocations in the mechanism and the side contract, i.e. $x_i, \chi_i \in \{0, 1\}$. The optimal deterministic collusion-proof mechanism can be implemented as a BPE of the delegated contracting game δ , where at $t = 3$, A_1 offers A_2 to produce the good at a strike price \hat{q} .

6 Optimal Information Structures

In this section, I analyze optimal information structures. Proposition 1 considerably simplifies this task. Instead of having to analyse the problem of jointly finding an optimal mechanism and an optimal information structure in the problem \mathcal{P} , it is sufficient to find the optimal information structure in the delegated contracting game δ . First, I describe equilibrium behavior in the delegated contracting game. I then show that P prefers to disclose no information over using an information structure which induces posteriors that can be ordered by FSD. Finally, I determine an optimal information structure.

In any BPE of the delegated contracting game δ , A_1 accepts the offer from P as long as the offered price p exceeds the expected costs for A_1 , and A_2 accepts the offer from A_1 if the offered price \hat{q} exceeds the cost of A_2 . The expected costs for A_1 after a signal $s \in S$ are thus given by

$$\min_{\hat{q}} \Pr(\hat{q} \geq z - \theta|s)q + \Pr(\hat{q} < z - \theta|s)z = z - \max_{\hat{q}} \Pr(\theta \geq z - \hat{q}|s)(z - \hat{q}).$$

To simplify the notation, define $q \equiv z - \hat{q}$. I denote the expected costs of A_1 for a given information structure σ and a signal realization s by $c(s; \sigma)$ which is given by

$$c(s; \sigma) = z - \max_q q \Pr(\theta \geq q) = z - \max_q q \sum_{\theta \geq q} \mu(\theta|s).$$

For further reference, I denote by σ_\emptyset the information structure which discloses no information to A_1 . The expected costs for A_1 without any further information is given by

$$\bar{c} \equiv c(s; \sigma_\emptyset) = z - \max_q q \sum_{\theta \geq q} \lambda(\theta).$$

P always wants to procure the good. Thus, P needs to make an offer to A_1 which covers A_1 's expected costs for any signal realization. This implies that an information structure is optimal for P if it minimizes the maximal expected costs over all signal realizations. A signal structure σ^* is hence optimal if

$$\sigma^* \in \min_{\sigma \in \mathcal{S}} \max_{s \in S} c(s; \sigma). \quad (1)$$

The first main result in this section shows that the principal prefers to disclose no information over using any information structure which induces posteriors that can be ordered by FSD. Before stating the result, it is helpful to make the following definition.

Definition 1. *The information structure $\sigma = (S, \mu)$ induces posteriors which can be ordered by first order stochastic dominance if for all $s, s' \in S$ either $\sum_{\theta \geq x} \mu(\theta|s) \geq \sum_{\theta \geq x} \mu(\theta|s')$ or $\sum_{\theta \geq x} \mu(\theta|s) \leq \sum_{\theta \geq x} \mu(\theta|s')$ for all $x \in \mathbb{R}$. Denote the set of all information structures which satisfy this property by \mathcal{S}_{FSD} .*

The class of information structures which induce posteriors that can be ordered by FSD includes full disclosure as well as any partition of the set Θ . It turns out that any information structure from this class gives P a weakly lower payoff than disclosing no information at all.

Proposition 2. *P prefers to disclose no information over any information structure with posteriors that are ordered by FSD, i.e. $\max_{s \in S} c(s; \sigma) \geq \bar{c}$ for all $\sigma \in \mathcal{S}_{FSD}$. In particular, full disclosure performs worse than no disclosure.*

Proof. For any information structure σ , it holds that

$$\sum_{\theta \geq x} \lambda(\theta) = \sum_{\theta \geq x} \sum_{s \in S} \mu(\theta, s) = \sum_{s \in S} \Pr(s) \sum_{\theta \geq x} \mu(\theta|s)$$

This implies together with Definition 1 that for any $\sigma \in \mathcal{S}_{FSD}$, there exists $\underline{s} \in S$ such that $\sum_{\theta \geq q} \mu(\theta|\underline{s}) \leq \sum_{\theta \geq q} \lambda(\theta)$. Thus,

$$\max_{s \in S} c(s; \sigma) = \max_{s \in S} z - \max_q q \sum_{\theta \geq q} \mu(\theta|s) = z - \max_q \min_{s \in S} \sum_{\theta \geq q} \mu(\theta|s) \geq z - \max_q q \sum_{\theta \geq q} \lambda(\theta) = \bar{c}$$

□

If an information structure has posteriors that can be ordered by FSD, then the different signal realizations can unambiguously be ranked from *good* to *bad*. As P always seeks to procure the good, she needs to compensate A_1 for his expected costs for all realizations of the signal. The worse signal realizations can be, the higher the payment that P needs to make to A_1 . The strongest ranking from good to bad signals occurs under the full disclosure information structure. Thus, one can even make the statement that full disclosure is the worst information structure for P . A further corollary from Proposition 2 is the following.

Corollary 1. *If $|\Theta| = 2$, then no disclosure is optimal.*

Corollary 1 follows directly from the observation that with a binary type space $\Theta = \{\theta_1, \theta_2\}$, any information structure induces posteriors that can be ranked by *FSD*. Thus, no disclosure is at least as good as any other information structure and therefore optimal.

It arises the question whether P can gain from disclosing information if the type space is richer than binary. Furthermore, it remains open how the optimal information structure avoids to be ranked from good to bad signals.

The following Lemma shows that there exists an optimal information structure which satisfies three properties which help to identify an optimal information structure.

Lemma 2. *There exists an optimal information structure $\sigma = (S, \mu)$ for which*

- i) A_2 realizes the project if and only if $\theta_i \geq \theta_k$ for all $s \in S$ where $\theta_k \in \Theta$ is a cutoff;*
- ii) $\mu(\theta_i, s) > 0 \Rightarrow \mu(\theta_j, s) = 0 \quad \forall s \in S$ for any $\theta_i, \theta_j \in \Theta$ with $i, j \geq k$ given a cutoff $\theta_k \in \Theta$;*
- iii) $|S| = n - k$.*

Proof. The proof is in the Appendix. □

The Lemma states that there exists an optimal information structure for which the identity of the producing agent is determined by a simple threshold, which uses as many signals as types of A_2 above the threshold, and where A_1 can differentiate between those values of the cost advantage for which A_2 produces.

The Lemma implies that partial disclosure is optimal and that the second agent does not receive an information rent. The first point follows from the fact that an optimal information structure exists with less signals than types of the second agent. The second point is implied by property *ii)* of the Lemma which implies that the first agent always offers a price equal to the lowest possible cost of the second agent under a given signal.

The idea behind the Lemma is as follows. Suppose that some optimal information structure does not have a cutoff as described in the Lemma. Then there exists some type $\theta \in \Theta$ for which A_2 produces after some signal s but not after the signal s' . It is then possible to define a new information structure which uses the same set of signals as before but prescribes that signal s' is not send anymore for θ . The probability mass with which s' was send for θ in

the old information structure, is now used to send s in the new information structure. The expected costs of A_1 under the new information structure are weakly smaller than under the old information structure, since after both signals s and s' , the probability of delegation to A_2 has (weakly) increased. Thus, the new information structure is also optimal.

Property *ii*) of Lemma 2 implies that there exists an optimal information structure under which A_2 does not receive an information rent. This follows from the following argument. Consider an information structure which is optimal and has a threshold. Suppose further for some signal s under this information structure, A_1 cannot perfectly tell apart the values θ_i and θ_j which lie above the threshold. Consider now a new information structure, where the signal s is split up in two different signals such that θ_i and θ_j do not send the same signal anymore. If the signal s is split up in the correct proportions, the expected costs of A_1 are weakly lower under the two new signal than under the old signal s .

Finally, property *iii*) of the lemma states that an optimal information structure needs at most (and at least) as many signals as types of A_2 who lie below the threshold. The intuition behind this result is straightforward. Consider an information structure with a threshold which satisfies property *ii*) in the Lemma. The latter property implies that A_1 uses exactly $n - k$ different price offers to A_2 under the optimal information structure. A_1 's expected costs can then not be reduced by using more than $n - k$ signals.

The result in Lemma 2 restricts the set of potentially optimal information structures. It is then possible to specify an optimal information structure and characterize the resulting optimal cutoff. For further reference, I define the variable \hat{k} as follows:

$$\hat{k} \equiv \max \left\{ k \in \{1, \dots, n\} : \sum_{j=k}^n \lambda_j \theta_j > \theta_k \right\}. \quad (2)$$

An optimal information structure is then described in the following proposition.

Proposition 3. $\sigma^* = (S^*, \mu^*)$ is an optimal information structure. Under σ^* , A_2 realizes the project if and only if $\theta_i \geq \theta_{k^*}$ where the optimal cutoff θ_{k^*} is determined by

$$k^* = \begin{cases} \hat{k} & \text{if } \theta_{\hat{k}} \geq \sum_{j=\hat{k}+1}^n \lambda(\theta_j) \theta_j, \\ \hat{k} + 1 & \text{if } \theta_{\hat{k}} < \sum_{j=\hat{k}+1}^n \lambda(\theta_j) \theta_j. \end{cases}$$

σ^* is given by

$$S^* = \{s_{k^*}, s_{k^*+1}, \dots, s_n\} \quad \text{and}$$

$$\mu^*(\theta_i, s_j) = \begin{cases} \lambda(\theta_i) \cdot \mathbf{1}(i = j) & \text{if } i \geq k^*, \\ \lambda(\theta_i) \frac{\lambda(\theta_j)\theta_j}{\sum_{j'=k}^n \lambda(\theta_{j'})\theta_{j'}} & \text{if } i < k^*, j > k^*, \\ \lambda(\theta_i) \frac{\lambda(\theta_j)\theta_j}{\sum_{j'=k}^n \lambda(\theta_{j'})\theta_{j'}} \cdot \mathbf{1}(k^* = \hat{k} + 1) & \text{if } i < k^*, j = k^*. \end{cases}$$

Proof. The proof is in the Appendix. □

The proposition characterizes the optimal threshold θ_k . Furthermore it shows how P pools the positive content and the negative content of signals optimally. Finally, the proposition implies that the principal cannot gain by disclosing information on the types below the threshold to A_1 .

The principal faces the following trade-off in the choice of the optimal cutoff θ_k . For high levels of the optimal cutoff, the principal can set A_1 's expected cost equal to $z - \sum_{i \geq k} \lambda(\theta_i)\theta_i$. This is possible, as the principal can pool for any signal s_i sufficiently many bad types together with the good type θ_i such that A_1 has the same expected costs after any signal realization. As long as there is a sufficient mass of low types, P has an interest to decrease the cutoff θ_k as $z - \sum_{i \geq k} \lambda(\theta_i)\theta_i$ is increasing in k . However, if the threshold becomes smaller, the mass of types for which A_2 does not produce may become so small, that it is not possible anymore to make the expected costs of A_1 identical across signal realizations. In this case, A_1 's expected cost is $z - \theta_k$. This term is decreasing in k . The optimal threshold lies then at the intersection of the functions $z - \sum_{i \geq k} \lambda(\theta_i)\theta_i$ and $z - \theta_k$. Due to the discrete types, the optimal cutoff is as given in the proposition. If $k^* = \hat{k} + 1$, A_1 does not receive an information rent. If $k^* = \hat{k}$, A_1 receives a positive information rent of $\theta_k - \sum_{j=\hat{k}+1}^n \lambda(\theta_j)\theta_j$. Note however, that this rent is converging to zero, as the distances between types in Θ becomes small.

Under the optimal information structure, the relative risk that A_1 needs to produce the good increases as the signal increases from s_j to s_{j+1} . Thus, the more positive content of signal s_{j+1} over signal s_j is offset by a higher downside risk for A_1 . In the optimal information structure, the positive and negative content is balanced across signals such that the expected costs for A_1 are the same. Hereby, it does not matter which values of θ with $\theta < \theta_k$ are used to create this

risk. In particular, the optimal information structure presented in the proposition discloses no information about the relative distribution of values of the cost advantage with $\theta < \theta_k$ to A_1 .

7 Conclusion

This paper is the first to study optimal information disclosure under the threat of collusion. I consider a model with a principal who seeks to procure a good from one of two agents. The agents can collude against the principal. The first agent has a publicly known cost of production and the second agent is privately informed about his cost. The principal decides how much information the first agent receives about the costs of the second agent. I show that the principal faces a trade-off in the choice of an optimal information structure: More information disclosure makes the elicitation of private information from the agents easier but facilitates collusion. I show that the principal cannot do better than to assign the organization of production to the first agent. The principal prefers to disclose no information than to fully disclose information. The principal optimally discloses information partially. Under the optimal information structure, the first agent can differentiate between low levels of the second agent's costs. However, any signal imposes on the first agent the downside risk of high values of the second agent's cost, under which the first agent has to produce himself.

The model studied in this paper is basic, and can be extended in a number of ways. First, in many situations it is natural to think that both agents could hold private information. In other settings, the second agent's private information may naturally influence the first agent's cost of producing the good. Moreover, the restriction to two agents could be relaxed to analyze settings with a higher number of agents.

8 Appendix

Proof of Lemma 2

I start by showing that for any optimal information structure there exists another information structure which is also optimal and satisfies property *i*): Suppose there exists an optimal information structure $\sigma = (S, \mu)$ and two signals $s', s'' \in S$ such that A_2 realizes the project with some type $\theta_i \in \Theta$ for signal s' but not for signal s'' and $\mu(\theta_i, s') > 0$ and $\mu(\theta_i, s'') > 0$. Define the information structure $\tilde{\sigma} = (S, \tilde{\mu})$ such that $\tilde{\mu}(\theta, s) = \mu(\theta, s)$ for $\theta \neq \theta_i$ or $s \in S \setminus \{s', s''\}$, $\tilde{\mu}(\theta_i, s') = \mu(\theta_i, s') + \mu(\theta_i, s'')$, and $\tilde{\mu}(\theta_i, s'') = 0$. It follows that $c(s, \sigma) = c(s, \tilde{\sigma})$ for all $s \in S \setminus \{s', s''\}$, $c(s', \sigma) > c(s', \tilde{\sigma})$, and $c(s'', \sigma) \geq c(s'', \tilde{\sigma})$ as the conditional distribution of θ does not change for signals $s \in S \setminus \{s', s''\}$, whereas A_2 is strictly more likely to realize the project after signals s' and weakly more likely after signal s'' . Thus, the expected costs of A_1 decrease strictly after s' and weakly after s'' . It follows that $\tilde{\sigma}$ is also optimal and satisfies property *i*).

Next, I show that for any optimal information structure that induces a cutoff θ_k , there exists an optimal information structure which also satisfies property *ii*). Suppose that $\sigma = (S, \mu)$ is an optimal information structure with a cutoff θ_k and that there exist $\theta_i, \theta_j \in \Theta$ with $i > j \geq k$ and $s \in S$ with $\mu(\theta_i, s) > 0$ and $\mu(\theta_j, s) > 0$. I suppose without loss of generality that $\theta_j = \min\{\theta \geq \theta_k : \mu(\theta, s) > 0\}$, i.e. that θ_j is the smallest type above the threshold which is in the support of $\mu(\cdot, s)$. It follows that $q(s) = \theta_j$. Define the new information structure $\tilde{\sigma} = (S \cup \{s', s''\} \setminus \{s\}, \tilde{\mu})$ where the signal s is split up into the two new signal realizations s' and s'' such that $\tilde{\mu}(\theta_i, s') = \mu(\theta_i, s)$, $\tilde{\mu}(\theta_j, s') = 0$, $\tilde{\mu}(\theta_j, s'') = \mu(\theta_j, s)$, $\tilde{\mu}(\theta_i, s'') = 0$, and for any $\theta \neq \theta_i, \theta_j$, $\tilde{\mu}(\theta, s') = \alpha\mu(\theta, s)$ and $\tilde{\mu}(\theta, s'') = (1 - \alpha)\mu(\theta, s)$ with $\alpha = \frac{\mu(\theta_i, s)}{\mu(\theta_i, s) + \mu(\theta_j, s)}$. $\tilde{\sigma}$ therefore satisfies $\tilde{\mu}(\theta_i|s') = \mu(\theta_i|s)$, $\tilde{\mu}(\theta_j|s') = 0$, $\tilde{\mu}(\theta_j|s'') = \mu(\theta_j|s)$, $\tilde{\mu}(\theta_i|s'') = 0$, and $\tilde{\mu}(\theta|s) = \tilde{\mu}(\theta|s') = \tilde{\mu}(\theta|s'')$ for all $\theta \neq \theta_i, \theta_j$. It follows that $c(s', \tilde{\sigma}) < c(s, \sigma)$ and $c(s'', \tilde{\sigma}) = c(s, \sigma)$. This implies that $\tilde{\sigma}$ is also optimal and it satisfies property *ii*).

Finally, I show that for any optimal information structure which satisfies *i*) and *ii*), there exists an information structure which is also optimal and satisfies property *iii*). Suppose that the information structure $\sigma = (S, \mu)$ is optimal, induces the cutoff θ_k , and satisfies properties *i*) and *ii*). It follows that in equilibrium $q(s) = \max \text{Supp}\{\mu(\cdot|s)\}$. Define the information structure $\tilde{\sigma} = (\tilde{S}, \tilde{\mu})$ as follows. At first, set $\tilde{S} = \{s_k, s_{k+1}, \dots, s_n\}$. Next, define for any θ_i

the set $S(\theta_i) = \{s \in S : q(s) = \theta_i\}$. Finally, define $\tilde{\mu}$ such that $\tilde{\mu}(\theta, s_i) = \sum_{s \in S(\theta_i)} \mu(\theta, s)$ for all $\theta \in \Theta$ and $i \geq k$. This implies that for any $i \geq k$, $c(s_i; \tilde{\sigma}) \leq \max_{s \in S(\theta_i)} c(s; \sigma)$. Thus, $\tilde{\sigma}$ is also optimal and satisfies the properties *i*), *ii*), and *iii*). \square

Proof of Proposition 3

I solve for the optimal information structure in the class of information structures which satisfy properties *i*), *ii*), and, *iii*) of Lemma 2. An optimal information structure in this class is optimal in the general class of information structures.

For any feasible information structure μ , which induces posteriors $(\mu(\cdot|s))_{s \in S} \in (\Delta^{|\Theta|-1})^{|S|}$, there exists a unique vector $\alpha \in \Delta^{|S|-1}$ such that $\lambda = \sum_{s \in S} \alpha(s) \mu(s)$. Consider now an optimal information structure $\sigma = (S, \mu)$ which satisfies properties *i*), *ii*), and *iii*) of Lemma 2. Property *iii*) implies that σ uses $n - k$ signals s_k, s_{k+1}, \dots, s_n and its characteristic α satisfies

$$\alpha(s_i) \mu(\theta_i | s_i) = \lambda(\theta_i) \quad (3)$$

for any $i \in \{k, \dots, n\}$, because the type θ_i induces signal s_i with probability 1. Together with property *ii*) it follows that $c(s_i; \sigma) = z - \frac{\lambda(\theta_i)}{\alpha(s_i)} \theta_i$. The characterising vector α^* of the optimal information structure σ satisfies

$$\alpha^* \in \arg \max_{\alpha \in \Delta^{n-k-1}} \min_{i \geq k} \frac{\lambda(\theta_i)}{\alpha(s_i)} \theta_i \quad \text{subject to} \quad \alpha(s_i) \geq \lambda(\theta_i) \quad \forall i \geq k.$$

The constraint $\alpha(s_i) \geq \lambda(\theta_i)$ follows from the fact that the signal s_i realizes always for $\theta = \theta_i$. If one neglects the constraint, the optimal α satisfies $\frac{\lambda(\theta_i)}{\alpha(s_i)} \theta_i = \frac{\lambda(\theta_j)}{\alpha(s_j)} \theta_j$ for all $i, j \geq k$. Using $\sum_{s \in S} \alpha(s) = 1$, this would imply

$$\alpha(s_i) = \frac{\lambda(\theta_i) \theta_i}{\sum_{j=k}^n \lambda(\theta_j) \theta_j}. \quad (4)$$

However, it needs to hold that $\mu_i(s_i) = \frac{\lambda(\theta_i)}{\alpha(s_i)} \leq 1$. Given equation (4), this is the case if

$$\sum_{j=k}^n \lambda(\theta_j) \theta_j \leq \theta_i. \quad (5)$$

If the condition of equation (5) is not satisfied, the constraint binds and $\alpha(s_i) = \lambda(\theta_i)$ is optimal.

Note that the condition in equation (5) is violated for some θ_i only if the condition is violated for θ_k . Define \hat{k} as the maximal type for which condition (5) is violated for $\theta_i = \theta_k$:

$$\hat{k} \equiv \max \left\{ k \in \{1, \dots, n\} : \sum_{j=k}^n \lambda(\theta_j)\theta_j > \theta_k \right\}.$$

If P wants to induce a cutoff $\theta_k \leq \theta_{\hat{k}}$, then the maximal expected costs satisfy $\max_{s \in S} c(s; \sigma) = z - \theta_k$. If P wants to induce a cutoff $\theta_k \geq \theta_{\hat{k}+1}$, then the maximal expected costs satisfy $\max_{s \in S} c(s; \sigma) = z - \sum_{j=k}^n \lambda(\theta_j)\theta_j$. It follows that the optimal cutoff k^* satisfies that either $k^* = \hat{k}$ or $k^* = \hat{k} + 1$ is optimal. The first is optimal if $\theta_{\hat{k}} \geq \sum_{j=\hat{k}+1}^n \lambda(\theta_j)\theta_j$, the latter is optimal if the reverse holds true.

Note that there remains some leeway for P to assign types that satisfy $\theta < \theta_{k^*}$ to the signals in S . The distribution μ given in the Proposition is the particular case where the signals in S are not informative about all types $\theta < \theta_{k^*}$. \square

References

- Bergemann, D., Brooks, B., and Morris, S. (2015). The limits of price discrimination. *The American Economic Review*, 105(3):921–957.
- Bergemann, D. and Pesendorfer, M. (2007). Information structures in optimal auctions. *Journal of Economic Theory*, 137(1):580–609.
- Bergemann, D. and Wambach, A. (2015). Sequential information disclosure in auctions. *Journal of Economic Theory*, 159:1074–1095.
- Caillaud, B. and Jehiel, P. (1998). Collusion in auctions with externalities. *Rand Journal of Economics*, 29(4):680–702.
- Celik, G. (2009). Mechanism design with collusive supervision. *Journal of Economic Theory*, 144(1):69–95.
- Che, Y.-K. and Kim, J. (2006). Robustly collusion-proof implementation. *Econometrica*, 74(4):1063–1107.

- Che, Y.-K. and Kim, J. (2009). Optimal collusion-proof auctions. *Journal of Economic Theory*, 144(2):565–603.
- Crémer, J. (1996). Manipulations by coalitions under asymmetric information: The case of groves mechanisms. *Games and Economic Behavior*, 13(1):39–73.
- Dequiedt, V. (2007). Efficient collusion in optimal auctions. *Journal of Economic Theory*, 136(1):302–323.
- Eső, P. and Szentes, B. (2007a). Optimal Information Disclosure in Auctions and the Handicap Auction. *The Review of Economic Studies*, 74(3):705–731.
- Eső, P. and Szentes, B. (2007b). The Price of Advice. *The Rand Journal of Economics*, 38(4):863–880.
- Faure-Grimaud, A., Laffont, J.-J., and Martimort, D. (2003). Collusion, delegation and supervision with soft information. *The Review of Economic Studies*, 70(2):253–279.
- Green, J. and Laffont, J.-J. (1979). On coalition incentive compatibility. *The Review of Economic Studies*, 46(2):243–254.
- Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.
- Kawai, K. and Nakabayashi, J. (2015). Detecting large-scale collusion in procurement auctions. *UC Berkeley, Tohoku University, mimeo*.
- Laffont, J.-J. and Martimort, D. (1997). Collusion under asymmetric information. *Econometrica*, 65(4):875–911.
- Laffont, J.-J. and Martimort, D. (1998). Collusion and delegation. *RAND Journal of Economics*, 29(2):280–305.
- Laffont, J.-J. and Martimort, D. (2000). Mechanism design with collusion and correlation. *Econometrica*, 68(2):309–342.
- Li, H. and Shi, X. (2015). Discriminatory information disclosure. *University of British Columbia, University of Toronto, mimeo*.

- McAfee, R. and McMillan, J. (1992). Bidding rings. *American Economic Review*, 82(3):579–99.
- Mookherjee, D. and Tsumagari, M. (2004). The organization of supplier networks: effects of delegation and intermediation. *Econometrica*, 72(4):1179–1219.
- Pavlov, G. (2008). Auction design in the presence of collusion. *Theoretical Economics*, 3(3):383–429.
- Porter, R. and Zona, J. (1993). Detection of bid rigging in procurement auctions. *Journal of Political Economy*, 101(3):518–38.
- Tirole, J. (1986). Hierarchies and bureaucracies: On the role of collusion in organizations. *Journal of Law, Economics, & Organization*, 2(2):181–214.