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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel -  
Session: Collusion and Exclusionary Practices in Oligopoly, No. E02-V2

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Rasch, Alexander; Gössl, Florian (2016) : The scope for collusion under different pricing schemes, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2016: Demographischer Wandel - Session: Collusion and Exclusionary Practices in Oligopoly, No. E02-V2, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

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# The scope for collusion under different pricing schemes

## Abstract

We analyze and compare the incentives to collude under different pricing schemes in a differentiated-products market where customers have elastic demand. We show that allowing firms to set two-part tariffs as opposed to linear prices facilitates collusion at maximum prices independent of the degree of differentiation. However, compared to a situation where firms can only set fixed fees that are independent of the quantity purchased, collusion at maximum prices is less sustainable with two-part tariffs. The results have important implications for competition policy where the perspective—static or dynamic—may be crucial.

*JEL classification:* D43; L13; L41.

*Keywords:* Collusion; Fixed fees; Linear pricing; Price discrimination; Two-part tariffs; Non-linear pricing.

# 1 Introduction

In this paper, we analyze the impact of different pricing structures on firms' ability to collude. In particular, we are interested in nonlinear pricing, a common business practice in many industries. Examples include mobile telecommunications, media markets, amusement parks, gas, and electricity. In these industries, the pricing structures consist of at least two components: a fixed (entry) fee which is independent of the quantity demanded and a linear per-unit price.

Another prominent example of an industry where prices consist of multiple fixed and variable components is air cargo or air freight. In this market, shipping or freight rates can be considered flat fees within a certain weight segment and/or type of commodity. Different from that, (per-kilo and/or per-km) surcharges typically depend on the exact chargeable weight and/or distance and can be considered linear. Collusive agreements on a global scale have been revealed in the air cargo industry in recent years. In June 2008, the United States Department of Justice announced that major airlines have agreed to plead guilty and pay fines exceeding \$500 million for fixing one or more components of total air cargo rates (U.S. Department of Justice, 2008). Similar judgements were made in other jurisdictions such as the European Union where fines amounted to a total of €799 million (European Commission, 2010), Canada (over \$24 million in fines, see Canadian Competition Bureau, 2013), and New Zealand (NZ\$42.5 million in fines, see Commerce Commission New Zealand, 2013).<sup>1</sup> Many major airlines were involved in the cartel in one or more jurisdictions including Air Canada, Air France-KLM, British Airways, Cathay Pacific, Cargolux, Emirates, Japan Airlines, LAN Chile, Martinair, SAS Cargo Group, Singapore Airlines, and Qantas. Lufthansa—the largest German carrier—was also involved in the cartel in the European Union but did not have to pay a fine because it reported the cartel to the European Commission (European Commission, 2010). However, rail operator Deutsche Bahn announced in December 2014 that it is suing Lufthansa, amongst other airlines, seeking damages amounting to €1.76 billion for the airlines' role in the cartel (Reuters, 2014). Furthermore, an investigation by the Swiss competition authority (COMCO) concluded that “airlines had agreed on freight rates, fuel surcharges, war risk surcharges, customs clearance surcharges for the U.S. and the commissioning of surcharges” and fined several airlines a total of CHF 11 million (Swiss Competition Commission, 2014, p. 1).<sup>2</sup>

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<sup>1</sup>In contrast to the US, the European Commission did not find sufficient evidence that airlines coordinated on freight rates but based their decision on the coordination of fuel surcharges only (European Commission, 2010).

<sup>2</sup>The sum of fines is relatively small in Switzerland because, as COMCO reports, Lufthansa and its subsidiary Swiss Air received full immunity because they reported the cartel.

We take the firm conduct observed in the air cargo price fixing case as a motivating example to address the question as to whether and how the possibility to coordinate on multiple fixed and linear price components instead of agreeing upon one (linear or fixed) collusive price only influences firms' ability to collude. To that end, building on Yin (2004) who models two-part tariff competition in duopoly, we analyze the incentives to collude under different pricing schemes in a differentiated-products setup à la Hotelling (1929) with elastic demand. Total demand is elastic as local demand—the demand of a single customer—decreases in price and in the distance to the respective firm (i.e., transport cost). We start by analyzing collusive incentives in a baseline setting where firms can set and coordinate on a single linear or fixed price only. Turning to two-part tariffs (i.e., setting a fixed fee in addition to a linear price), we find that the comparison regarding the sustainability of collusion crucially depends on the type of the single price (linear or fixed). The scope for collusion is largest for all values of the transport-cost parameter when firms are allowed to use fixed fees only whereas setting linear prices only results in the lowest incentives to collude. When setting two-part tariffs, collusion is easier to sustain compared to linear pricing but harder to sustain compared to fixed fees.

The main effect which renders collusion more attractive under nonlinear pricing compared to linear pricing are the relatively large gains from deviation under linear pricing. When a firm deviates in the linear-pricing scenario to increase its market share, lowering its price has an additional positive effect on profits because it increases local demand. This effect is absent under nonlinear pricing because firms use the fixed part of the tariff to compete for market shares which results in relatively low gains from deviation.

Collusion is easiest to sustain in the fixed-fee scenario. This is also caused by relatively lower gains from deviation under fixed fees relative to two-part tariffs. Under two-part tariffs, the deviating firm is able to fine-tune local demand using the linear part of the tariff especially when differentiation is large and optimal deviation does not entail covering the whole market, giving rise to larger gains from deviation and a lower incentive to collude.

Our result of intermediate incentives to collude under two-part tariffs is also relevant for the ongoing discussion of whether the simplification of tariff structures benefits customers. For example, the British Office of Gas and Electricity Markets (OFGEM) recently prohibited “complex multi-tier tariffs, where, for example, customers are initially charged a higher rate, which only falls if their consumption increases above certain levels” (Office of Gas and Electricity Markets, 2014, p. 1) in the markets for electricity and gas in the UK.

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In addition, several other airlines received significant fine reductions because of their cooperation during the investigation.

The argument brought forward by regulators is that increasing transparency and enhancing comparability of prices through simplification of tariffs leads to an increase in customer surplus. Consistent with this argument, when interpreting two-part tariffs as complex and fixed fees as well as linear prices as simplified tariffs, our model predicts an increase in customer surplus in a static, competitive environment when firms are not allowed to offer two-part tariffs.

When considering dynamic effects of simplified tariff structures, however, our analysis points out that the effects of simplification on customer surplus are not clear-cut at all. Prohibiting firms to set two-part tariffs but allowing them to charge simpler flat fees harms customers as it fosters collusion. However, the incentives to collude are reduced when firms may only set linear prices instead of two-part tariffs. As a policy implication, we stress that a careful approach to simplifying tariff structures is necessary to prevent pro-collusive effects even if short-term customer surplus increases.

Two-part tariffs or nonlinear pricing can be considered a form of second-degree price discrimination (see Varian, 1989) in the sense that all customers are offered the same schedule of price-quantity combinations. When customers are heterogeneous in their demand, they self-select different quantities and hence end up paying different per-unit prices. A typical example of two-part tariffs are quantity discounts which can take a large number of different forms (e.g., loyalty discounts, rebates).

Although price discrimination and nonlinear tariffs are important features of antitrust concerns, the literature on the impact of different pricing schemes on collusion is sparse.<sup>3</sup> To the best of our knowledge, this is the first study analyzing the incentives to collude when firms compete in two-part tariffs as opposed to linear prices. Concerning the relationship between third-degree price discrimination and collusion, Liu and Serfes (2007) investigate the impact of the availability of customer-specific information for market segmentation in a linear-city model on the feasibility to collude. A higher degree of market segmentation accompanied by a more diversified pricing structure is possible as the quality of customer information increases. Better information has opposing effects regarding the sustainability of collusion: on the one hand, it implies higher collusive profits and harsher punishment; on the other hand, deviation becomes more profitable. The authors show that the latter effect dominates,

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<sup>3</sup>There is a large body of literature on the use of two-part tariffs in monopoly starting with Oi (1971). As far as the formal treatment of competition with nonlinear prices is concerned, Armstrong and Vickers (2001) as well as Rochet and Stole (2002) analyze nonlinear pricing in a setting with both horizontal and vertical heterogeneity whereas Laffont et al. (1998) focus on competition in two-part tariffs in the context of access pricing. We build our analysis on Yin (2004), who focuses on a duopoly model with horizontal product differentiation, in order to isolate the effect of different pricing schemes.

i.e., collusion is harder to sustain as the firms' ability to segment customers improves.

A related study to Liu and Serfes (2007) is Colombo (2010): the author allows for different degrees of product differentiation (i.e., firms are not located at the extremes of the linear city and hence are not maximally differentiated) and analyzes perfect (or first-degree) price discrimination. With perfect price discrimination, firms may set prices based on the exact location of a customer (so-called delivered pricing).<sup>4</sup> The author shows that collusion is easier to sustain the lower transport costs are<sup>5</sup> and that colluding on discriminatory prices is harder than on a uniform price.<sup>6</sup>

Both Liu and Serfes (2007) and Colombo (2010) find that third-degree and perfect price discrimination tend to reduce firms' incentives to collude. In contrast, our study suggests increased incentives to collude under second-degree price discrimination.

In their contribution, Fong and Liu (2011) analyze intertemporal price discrimination in an overlapping-generations model and show that, in comparison to uniform pricing, loyalty rewards of different forms facilitate collusion. Loyalty rewards are a form of quantity discounts and can also be interpreted as second-degree price discrimination. Customers live for two periods and demand at most one unit in each period. Firms can then price discriminate by allowing for a discount for repeat customers. Fong and Liu (2011) show that with loyalty discounts, deviating firms are unable to steal the industry profit for one period and hence, collusion is more likely to occur. This effect is further strengthened when firms can commit to offering discounts because the commitment limits firms' options when deciding upon the optimal deviation strategy. In contrast to our model, firms in Fong and Liu (2011) can set linear prices only. If they can discriminate, they may set two different linear prices in a given period for first-time and repeat customers. In our model, firm can set both a linear and a fixed tariff component at the same point in time.

The rest of this paper is organized as follows. In section 2, we present the setup and derive demand functions. In section 3, we derive profits in the competitive, collusion, and deviation cases for linear prices (subsection 3.1), fixed fees (subsection 3.2), and two-part tariffs (subsection 3.3). The result-

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<sup>4</sup>This is a special case of the analysis in Liu and Serfes (2007) with maximally differentiated firms. Further contributions investigating the implications of delivered pricing on collusion are, among others, Jorge and Pires (2008) and Miklós-Thal (2008).

<sup>5</sup>In the standard setting with unit demand and without the ability of firms to discriminate between customers, lower transport costs make it harder for firms to sustain collusion at maximum prices (Chang, 1991).

<sup>6</sup>Note that in his setup, firms always punish and deviate using discriminatory prices which is different from the present setup where punishment and deviation profits depend on the pricing instruments available to the firms.

ing critical discount factors are compared in subsection 4. The last section concludes.

## 2 The model

We consider a model of horizontal product differentiation à la Hotelling (1929) with two symmetric firms 1 and 2 which are located at the extremes of the linear city of unit length, i.e., at  $L_1 = 0$  and  $L_2 = 1$ . Fixed and marginal costs are equal to zero.<sup>7</sup> Firms discount future profits by the common discount factor  $\delta$  per period. We compare the incentives to collude in three different pricing scenarios in the following section:

- (i) linear-price scenario (denoted by subscript  $L$ ): firms compete in prices  $p_{i,L}$  per unit purchased (see subsection 3.1);
- (ii) fixed-fee scenario (denoted by subscript  $F$ ): firms compete in fixed fees  $f_{i,F}$ , i.e., customers pay a flat (subscription) fee independent of actual usage (with  $i \in \{1, 2\}$ ) (see subsection 3.2); and
- (iii) two-part tariffs (denoted by subscript  $T$ ): firms compete in tariffs which are made up of a fixed component  $f_{i,T}$  and a variable part  $p_{i,T}$  charged per unit sold (see subsection 3.3).

Customers of mass one are uniformly distributed along the line. Each customer buys either from firm 1 or from firm 2. Building on Yin (2004) who models two-part tariff competition in duopoly, we allow individual demands to be elastic.<sup>8</sup> A customer who is located at  $x$  and purchases  $q$  receives the following utility when buying from firm  $i$ :

$$U(x, q, p_i, f_i) = q - \frac{q^2}{2} - q(p_i + \tau|L_i - x|) - f_i \quad (1)$$

where  $\tau$  is the transport-cost parameter. We note that the quantity demanded depends on transport cost. In the product differentiation interpretation of the model, this would mean that mismatch costs occur for each unit purchased. Then,  $q\tau|L_i - x|$  represents the total disutility suffered by a customer with preferred product characteristics of  $x$  when consuming a product that is not ideal (and thus not located at  $x$  but at  $L_i$ ). Note that the larger are  $q$  and/or  $|L_i - x|$ , the greater the disutility.

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<sup>7</sup>We will relax the assumption of zero marginal costs below.

<sup>8</sup>More precisely, we use the shipping model with linear demand which is discussed in section 3.2 in Yin (2004).

Customers maximize their utility when deciding on the quantity they want to purchase. This implies that a customer has the following local demand at firm  $i$

$$\begin{aligned} \max_q U(x, q, p_i, f_i) &\Rightarrow \frac{\partial U}{\partial q} = 1 - q - p_i - \tau|L_i - x| = 0 \\ \Rightarrow q(x, p_i, f_i) &= \begin{cases} 1 - p_i - \tau|L_i - x| & \text{if } U(x, q, p_i, f_i) \geq 0 \\ 0 & \text{else.} \end{cases} \end{aligned} \quad (2)$$

Customer heterogeneity with respect to product preferences is also reflected in the individual demand which decreases as the difference in preferences and actual product characteristics grows.

Before analyzing the three different pricing regimes and their impact on collusion, a note on the measure for collusive stability seems in order. We will derive the critical discount factor for the different scenarios. Using grim-trigger strategies (see Friedman, 1971), we can compute critical discount factors according to the well-known formula

$$\delta \geq \bar{\delta} := \frac{\pi^d - \pi^c}{\pi^d - \pi^*} \quad (3)$$

where  $\pi^c$ ,  $\pi^d$ , and  $\pi^*$  denote collusive profits, deviation profits and competitive (punishment) profits, respectively. All things equal, a lower (higher) punishment or deviation profit leads to a stabilization (destabilization) of the collusive agreement whereas the opposite is true for a change in the collusive profit.

Our focus here is on the situation where the market is covered, i.e., all customers along the line buy which is why the following is assumed to hold for customers' transport costs:

**Assumption 1.** *Transport costs are not too high:  $0 < \tau \leq 2/5 =: \bar{\tau}$ .*

The assumption guarantees that the whole market is served under any of the pricing scenarios to be considered. For larger transport costs, firms prefer not to serve the customers located around  $1/2$ . As a result, firms are local monopolists and the notion of collusion has no bite. Note that the assumption regarding transport costs is standard in the literature (see Yin, 2004).

We proceed with the derivation of the profits and the critical discount factors in the three scenarios.



## 3 Analysis

### 3.1 Linear pricing

We first consider firms' incentives to collude in a situation where they set linear prices. The results presented in this section are due to Rothschild (1997). We start by analyzing the customers' purchasing decision. Plugging the local demand specified in expression (2) into the utility expression in (1) implies that the indifferent customer located at  $\tilde{x}$  is given by

$$U(\tilde{x}, p_i) = U(\tilde{x}, p_j) \Leftrightarrow \tilde{x}(p_i, p_j) = \frac{1}{2} - \frac{p_i - p_j}{2\tau}.$$

Consider the case where the indifferent customer  $\tilde{x}$  is located in between both firms, i.e.,  $0 \leq \tilde{x} \leq 1$ . Then, aggregate demand of firm  $i$  is given by

$$Q_i(p_i, p_j) = \int_0^{\tilde{x}(p_i, p_j)} (1 - \tau x - p_i) dx.$$

In the punishment stage, firms compete by simultaneously setting linear prices. The profit function of firm  $i$  is given by

$$\pi_i(p_i, p_j) = p_i Q_i(p_i, p_j).$$

We can derive the competitive equilibrium prices and profits in the standard way. Dropping subscripts, they are given by

$$p_L^* = \frac{2 + 3\tau - \sqrt{4 - 4\tau + 13\tau^2}}{4}$$

and

$$\pi_L^* = \frac{(2 + 3\tau - \sqrt{4 - 4\tau + 13\tau^2})(2 - 4\tau + \sqrt{4 - 4\tau + 13\tau^2})}{32},$$

respectively. As in the Hotelling model with linear transport costs and unit demand, the model converges to Bertrand competition as differentiation vanishes ( $\tau \rightarrow 0$ ). Equilibrium prices increase in  $\tau$  in the relevant range of  $\tau$  but the mark-up is smaller than in the model with unit demand. Larger differentiation dampens competition, giving firms an incentive to raise prices with both elastic and unit demand. Besides this competition effect, with elastic demand, mark-up is lower because firms have an additional incentive to lower prices to counter the reduction of individual demand caused by an increase in

$\tau$  (elasticity effect).<sup>9</sup> An equivalent argument holds for the equilibrium profits.

If firms collude (at maximum prices), they share the market equally and jointly set the optimal linear price in order to maximize industry profit which equals

$$\bar{p}_L^c = \frac{1}{2} - \frac{\tau}{8}.$$

The resulting collusive profit of each firm is given by

$$\bar{\pi}_L^c = \frac{(4 - \tau)^2}{128}.$$

Note that the results for the collusive scenario are derived under the assumption that collusion can be maintained, i.e., the incentive-compatibility constraint is not binding here. In the next section, we will shortly analyze the case where such full collusion is not sustainable.

Given that the competitor sticks to collusion and sets the optimal collusive price  $p_L^c$ , we can derive the price set by a deviating firm  $i$ . When deriving this price, we have to distinguish between the cases where firm  $i$  serves (i) the whole market and (ii) shares the market with the other firm. For relatively large values of the transport-cost parameter, it is optimal to leave some market share to the competitor when deviating because covering the whole market would require a steep downward adjustment of the price. Below a cut-off value of  $\tau$ ,  $\tau'$ , the optimal deviation leads to a market share of 1 of the deviating firm.

Define  $A := \sqrt{592 - 392\tau + 637\tau^2}$  and  $\tau' := (4\sqrt{249} - 16)/233 \approx 0.2022$ . The optimal deviating price and the resulting profit are then given by

$$p_L^d = \begin{cases} \frac{1}{2} - \frac{9\tau}{8} & \text{if } 0 < \tau \leq \tau' \\ \frac{40+14\tau-A}{72} & \text{if } \tau' < \tau \leq \bar{\tau} \end{cases}$$

and

$$\pi_L^d = \begin{cases} \frac{1}{4} - \frac{\tau}{4} - \frac{45\tau^2}{64} & \text{if } 0 < \tau \leq \tau' \\ \frac{(40+14\tau-A)(208+952\tau+20A+7A\tau-539\tau^2)}{497664\tau} & \text{if } \tau' < \tau \leq \bar{\tau}. \end{cases}$$

Using the profits in the collusive, deviating, and punishment phases and (3), the critical discount factor under linear pricing is given by

$$\bar{\delta}_L = \begin{cases} \frac{91\tau^2+24\tau-16}{28\tau\sqrt{4-4\tau+13\tau^2-16+20\tau-5\tau^2}} & \text{if } 0 < \tau \leq \tau' \\ \frac{637\tau^2 A - 15893\tau^3 - 392\tau A + 12876\tau^2 + 592A - 17520\tau - 3520}{637\tau^2 A - 108864\tau^2\sqrt{13\tau^2-4\tau+4} + 376795\tau^3 + 392\tau A - 49332\tau^2 + 592A + 44688\tau - 3520} & \text{if } \tau' < \tau \leq \bar{\tau} \end{cases}$$

<sup>9</sup>The separation of effects due to a change in product differentiation into competition and elasticity effects was first discussed by Mérel and Sexton (2010).

As first shown by Rothschild (1997), the critical discount factor first increases in the transport-cost parameter and then decreases.<sup>10</sup> We discuss the effects in more detail in the context of Proposition 4 below. We next turn to the case of fixed fees.

### 3.2 Fixed fees

Fixed fees can be interpreted as a special case of two-part tariffs where the linear part  $p_i$  is set equal to zero. Then, local demand at firm  $i$  depends on the location of the customer only:  $q(x) = 1 - \tau|L_i - x|$ . In this case, the marginal customer is given by

$$U(\tilde{x}, f_i) = U(\tilde{x}, f_j) \Leftrightarrow \tilde{x}(f_i, f_j) = \frac{1}{2} - \frac{f_i - f_j}{\tau(2 - \tau)}.$$

In the punishment stage, firms compete by simultaneously setting fixed fees. Firm  $i$  maximizes

$$\pi_{i,F}(f_i, f_j) = f_i \tilde{x}(f_i, f_j) \tag{4}$$

with respect to  $f_i$ . We have the following result:

**Lemma 1.** *In the punishment scenario with fixed fees, firms set an equilibrium fixed fee of*

$$f_F^* = \tau - \frac{\tau^2}{2}$$

and make a profit of

$$\pi_F^* = \frac{\tau}{2} - \frac{\tau^2}{4}. \tag{5}$$

*Proof.* Differentiating  $\pi_{1,F}(f_1, f_2)$  with respect to  $f_1$  yields

$$\frac{\partial \pi_{1,F}(f_1, f_2)}{\partial f_1} = \frac{2f_2 - 4f_1 + 2\tau - \tau^2}{2\tau(2 - \tau)}.$$

The second-order condition is given by

$$\frac{\partial^2 \pi_{1,F}(f_1, f_2)}{\partial f_1^2} = -\frac{2}{\tau(2 - \tau)} < 0.$$

Setting  $\partial \pi_{1,F}(f_1, f_2) / \partial f_1 = 0$ , using symmetry, solving for  $f_1$ , and re-substituting into  $\pi_{1,F}(f_1, f_2)$  immediately leads to the lemma.  $\square$

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<sup>10</sup>For an in-depth analysis and explanation of the results, see Mérel and Sexton (2010) or Rasch and Herre (2013).

Fixed prices and profits increase in differentiation for  $\tau$  as defined by Assumption 1. It is well-known that differentiation softens competition, resulting in an incentive to increase fees in order to extract more surplus from each customer because switching suppliers becomes more costly as differentiation increases. Profits are always larger under fixed fees compared to linear pricing. When a fixed fee is available, firms can use it to directly target consumer surplus, resulting in larger profits. Similar to linear pricing, as differentiation vanishes ( $\tau \rightarrow 0$ ), competition becomes tougher and profits converge to zero.

Under full collusion, each firm serves exactly half of the market and sets the fixed fee such that all surplus of the customer located at  $1/2$  is extracted. We arrive at the following result:

**Lemma 2.** *With fixed fees, collusive prices and profits are given by*

$$\bar{f}_F^c = \frac{(2 - \tau)^2}{8}$$

and

$$\bar{\pi}_F^c = \frac{(2 - \tau)^2}{16}. \quad (6)$$

*Proof.* The customer at position  $x = 1/2$  when buying from firm 1 has a utility of  $1/2 - \tau/2 + \tau^2/8 - f_1$ . The optimal  $f_1$  follows immediately. The collusive profit is then given by  $\bar{\pi}_{i,F}^C = \bar{f}_{i,F}^C/2$ . Due to symmetry, the values for firm 2 are identical.  $\square$

In the fixed-fee scenario, firms can fully extract the surplus of the indifferent consumer in order to maximize joint profits. Extracting all surplus of the indifferent consumer is impossible under linear pricing, resulting in a larger collusive profit with fixed fees. Turning to the optimal deviating strategy given that the other firms sets the collusive fixed fee, we again have to distinguish between the cases where the deviator finds it profitable to serve the whole market or leaves some market share to the other firm. Define  $\tau'' := 2/13$ . We then find the following:

**Lemma 3.** *With fixed fees, deviation prices and profits are given by*

$$f_F^d = \begin{cases} \frac{1}{2} - \frac{3\tau}{2} + \frac{5\tau^2}{8} & \text{if } 0 < \tau \leq \tau'' \\ \frac{(2-\tau)(2+3\tau)}{16} & \text{if } \tau'' < \tau \leq \bar{\tau} \end{cases}$$

and

$$\pi_F^d = \begin{cases} \frac{1}{2} - \frac{3\tau}{2} + \frac{5\tau^2}{8} & \text{if } 0 < \tau \leq \tau'' \\ \frac{(2-\tau)(2+3\tau)^2}{256\tau} & \text{if } \tau'' < \tau \leq \bar{\tau}. \end{cases} \quad (7)$$

*Proof.* The optimal deviation fixed fee set by firm 1 when the other firm sets pricing according the collusive agreement is determined by maximizing the profit function  $\pi_1(f_1, \bar{f}_F^c)$  over  $f_1$ . The partial derivative is given by

$$\frac{\partial \pi_1(f_1, \bar{f}_F^c)}{\partial f_1} = \frac{-16f_1 - 3\tau^2 + 4\tau + 4}{8\tau(2 - \tau)}.$$

Observe that the second-order condition is satisfied due to Assumption 1:

$$\frac{\partial^2 \pi_{1,F}(f_1, \bar{f}_{2,F}^C; \tau)}{\partial f_1^2} = -\frac{2}{\tau(2 - \tau)} < 0$$

Solving the first-order condition for  $f_1$  leads to  $f_F^{d,l} = (2 - \tau)(2 + 3\tau)/16$ . Given  $f_F^{d,l}$ , we have to ensure that the market share of the deviating firm does not exceed 1, i.e.,  $\tilde{x}(f_F^{d,l}, \bar{f}_F^c) \leq 1 \Leftrightarrow \tau \geq 2/13$ . Thus, for  $\tau \geq 2/13$ , the optimal deviation fixed fee is  $f_F^{d,l}$ . For  $\tau \leq 2/13$ , the optimal fixed fee is set such that the indifferent customer is at location  $x = 1$ :  $1 = \tilde{x}(f_1, \bar{f}_F^c) \Leftrightarrow f_1 = 1/2 - 3t/2 + 5t^2/8 =: f_F^{d,s}$ . Plugging the respective fixed fees into the profit function (4) leads to the deviation profits.  $\square$

Using (3) and the respective profits we just derived, we can calculate the critical discount factor and analyze its slope.

**Proposition 1.** *When firms can set fixed fees only, the critical discount factor is given by*

$$\bar{\delta}_F = \begin{cases} \frac{9\tau-2}{14\tau-4} & \text{if } 0 < \tau \leq \tau'' \\ \frac{2-5\tau}{11\tau+2} & \text{if } \tau'' < \tau \leq \bar{\tau}, \end{cases}$$

where  $\tau'' := 2/13$ .

Furthermore,  $\bar{\delta}_F$  is decreasing in  $\tau$ .

*Proof.* The first part of the lemma follows from substituting expressions (5), (6), and (7) into inequality (3) and simplifying. For the second part, the derivative of  $\bar{\delta}_F$  with respect to  $\tau$  is given by

$$\frac{\partial \bar{\delta}_F}{\partial \tau} = \begin{cases} \frac{-2}{(7\tau-2)^2} & \text{if } 0 < \tau \leq \tau'' \\ \frac{-32}{(11\tau+2)^2} & \text{if } \tau'' < \tau \leq \bar{\tau}. \end{cases}$$

$\partial \bar{\delta}_F / \partial \tau < 0$  follows immediately.  $\square$

The observation that the critical discount factor decreases in the scope of product differentiation is similar to the case analyzed in Chang (1991) and Häckner (1996). In those contributions, it is shown that with unit demand, the

effect of an increased level of product differentiation on deviation—which is less profitable as customers incur higher transport costs—outweighs the opposing, sustainability-decreasing effect on competitive profits which increase as firms enjoy a greater degree of market power. As the linear price is zero in the present setup, local demand is independent of any linear pricing component; hence, a similar result concerning the impact of product differentiation on the critical discount factor is obtained.

We next analyze the case where firms choose two-part tariffs.

### 3.3 Two-part tariffs

In a situation where firms set both linear prices and fixed fees, the marginal customer is located at

$$U(\tilde{x}, p_i, f_i) = U(\tilde{x}, p_j, f_j) \quad (8)$$

$$\Leftrightarrow \tilde{x}(p_i, f_i, p_j, f_j) = \frac{1}{2} - \frac{p_i - p_j}{2\tau} - \frac{f_i - f_j}{2\tau(2 - p_i - p_j - \tau)}. \quad (9)$$

If firms compete in the punishment stage, each of them maximizes

$$\pi_i(p_i, f_i, p_j, f_j) = p_i Q_i(p_i, f_i, p_j, f_j) + f_i \tilde{x}(p_i, f_i, p_j, f_j)$$

with respect to  $p_i$  and  $f_i$ . The first term of the profit function is the revenue generated by charging a variable price per unit purchased while the second term is the market share or the number of customers multiplied with the fixed fee. The results in the competitive stage are due to Yin (2004) for the situation with a fully covered market where the following prices and profits result:

$$p_T^* = \frac{\tau}{4},$$

$$f_T^* = \frac{3\tau}{4} - \frac{9\tau^2}{16},$$

and

$$\pi_T^* = \frac{\tau}{2} - \frac{11\tau^2}{32}. \quad (10)$$

In two-part-tariff competition, the main instrument used to compete for the indifferent customer is the fixed fee. The use of the linear price can be best thought of as a sequential procedure. Firms determine the optimal linear price as a function of market share and then compete for the indifferent customer and market share by setting fixed fees.

When setting the linear price for given market shares, firms have to balance two opposing effects caused by the fact that individual demands depend on

both price *and* location. As a consequence, local demand decreases for each customer as the distance between the customer and the respective firm increases. Lowering the linear price leads to a larger surplus for all customers. In particular, it leads to a larger surplus of the marginal customer, i.e., the customer with the lowest demand at one firm, which can potentially be extracted via the fixed fee. On the other hand, lowering the linear price leads to a larger rent given to customers located closer to the firms. This gives firms an incentive to increase the linear price in order to extract additional surplus from these inframarginal customers. In the resulting compromise, firms set a linear price above marginal cost.

While the competitive profit in both the two-part-tariff and the fixed-fee scenario is larger compared to the profits under linear prices because of the ability of the firms to directly extract surplus via a fixed fee, it is lower under two-part tariffs compared to the fixed-fee scenario. With two-part tariffs, firms can gain additional profits for a given market share by setting a positive linear price. This option is not present in the fixed-fee scenario. Anticipating this source of additional income, firms behave more aggressively when competing for market share using the fixed fee. The result is a lower competitive profit under two-part tariff competition.

Under full collusion, firms again share the market equally, and set a linear price to maximize overall customer surplus which they partly extract via the fixed fee. We then have the following result:

**Lemma 4.** *In the two-part-tariff scenario, the collusive prices and profits are given by*

$$\begin{aligned}\bar{p}_T^c &= p_T^* = \frac{\tau}{4}, \\ \bar{f}_T^c &= \frac{(4 - 3\tau)^2}{32},\end{aligned}$$

and

$$\bar{\pi}_T^c = \frac{1}{4} - \frac{\tau}{4} + \frac{5\tau^2}{64}. \quad (11)$$

*Proof.* Writing the profit function of firm 1 (which is w.l.o.g.) as a function of  $p_1$  yields

$$\pi_{1,T} = \frac{1}{4} \left( 1 - 2p_1 - \tau + p_1^2 + p_1\tau + \frac{\tau^2}{4} \right) + p_1 \int_0^{\frac{1}{2}} (1 - p_1 - \tau x) dx.$$

The first-order condition is given by

$$\frac{\partial \pi_{1,T}}{\partial p_1} = -\frac{p_1}{2} + \frac{\tau}{8}.$$

Observe that the second-order condition is satisfied. Setting  $\partial\pi_{1,T}/\partial p_1 = 0$  and solving for  $p_1$  yields  $\bar{p}_{i,T}^C$ . The optimal fixed fee is derived by substituting  $\bar{p}_{i,T}^C$  into  $f_{1,T}$ . The collusive profit follows immediately by substituting  $\bar{p}_{i,T}^C$  and  $f_{1,T}$  into the profit function.  $\square$

Observe that the linear prices under competition and collusion are identical. This is because the firms' market shares are identical in both scenarios. Because the linear price can be thought of as a function of market share only, this results in the same linear price. The optimal fixed fee is larger without competition as firms are now able to extract all surplus of the indifferent consumer.

The largest collusive profits can be obtained with two-part tariffs while they are lowest under linear pricing. In contrast to linear pricing, the ability to use fixed fees allows firms to fully extract the indifferent consumer's surplus. The availability of a linear price to extract additional surplus of inframarginal consumers ranks the two-part-tariff scenario above the fixed-fee scenario.

Next assume that firm  $i$ 's competitor follows the collusive agreement by setting  $\bar{p}_T^c$  and  $\bar{f}_T^c$  and define  $B := \sqrt{61 - 83\tau + 28\tau^2}$  and  $\tau''' := 14/19 - 2\sqrt{30}/19 \approx 0.1603$ . The optimal deviation strategy of firm  $i$  and the resulting deviation profit are characterized as follows:

**Lemma 5.** *Under two-part tariffs, the optimal deviation from the collusive agreement yields the following prices and profits:*

$$p_T^d = \begin{cases} \frac{\tau}{2} & \text{if } \tau \leq \tau''' \\ \frac{8-5\tau-B}{3} & \text{else,} \end{cases}$$

$$f_T^d = \begin{cases} \frac{1}{2} - 2\tau + \frac{11\tau^2}{8} & \text{if } \tau \leq \tau''' \\ \frac{197}{9} - \frac{268\tau}{9} + \frac{181\tau^2}{18} - \frac{25B}{9} + \frac{35\tau B}{18} & \text{else,} \end{cases}$$

and

$$\pi_T^d = \begin{cases} \frac{1}{2} - \frac{3\tau}{2} + \frac{7\tau^2}{8} & \text{if } \tau \leq \tau''' \\ \frac{1}{27\tau} (5\tau - 8 + B) (58 - 86\tau + 31\tau^2 - 8B + 5\tau B) & \text{if } \tau''' < \tau \leq \bar{\tau}. \end{cases} \quad (12)$$

*Proof.* See the appendix.  $\square$

For low levels of differentiation, the deviating firm covers the whole market. This becomes costly as differentiation increases because customers close to the firm sticking to the collusive agreement are expensive to attract. If transport costs are sufficiently large, this becomes too costly such that some market share is covered by the non-deviating firm.



Given firms' profits in all scenarios under two-part-tariff pricing, we are now able to calculate the critical discount factor according to condition (3). We summarize our findings in the following lemma.

**Proposition 2.** *The discount factor under two-part tariffs for which collusion with grim-trigger strategies can be sustained is given by*

$$\bar{\delta}_T = \begin{cases} \frac{17\tau-4}{26\tau-8} & \text{if } \tau \leq \tau''' \\ \frac{1}{2} \frac{3584\tau^2 B + 18745\tau^3 - 10624\tau B - 83856\tau^2 + 7808B + 124176\tau - 60928}{1792\tau^2 + 9737\tau^3 - 5312\tau B - 42576\tau^2 + 3904B + 62304\tau - 30464} & \text{if } \tau''' < \tau \leq \bar{\tau}, \end{cases}$$

where  $\tau''' := 14/19 - 2\sqrt{30}/19$  and  $B := \sqrt{61 - 83\tau + 28\tau^2}$ .

Furthermore,  $\bar{\delta}_T$  is decreasing in  $\tau$ .

*Proof.* The first part of the lemma follows from substituting expressions (10), (11), and (12) into inequality (3) and simplifying. For part two of the lemma, the derivative of  $\bar{\delta}_T$  with respect to  $\tau$  is given by

$$\frac{\partial \bar{\delta}_T}{\partial \tau} = \begin{cases} \frac{-8}{(13\tau-4)^2} & \text{if } \tau \leq \tau''' \\ \frac{1}{B(1792B+9737\tau^3-5312\tau B-42576\tau^2+3904B+62304\tau-30464)^2} (432(112980\tau^5 + 21318\tau^4 B - 672977\tau^4 - 95401\tau^3 B + 1539029\tau^3 + 149280\tau^2 B - 1653042\tau^2 - 91392\tau B + 795440\tau + 15232B - 119072)) & \text{if } \tau''' < \tau \leq \bar{\tau}, \end{cases}$$

For  $\tau \leq \tau'''$ , it follows immediately that  $\partial \bar{\delta}_T / \partial \tau < 0$ . For  $\tau''' < \tau < \bar{\tau}$ , first note that because  $B > 0$  in the relevant range of  $\tau$ , the first part of the expression is positive. Define the second part of the expression as

$$G(\tau) = (432(112980\tau^5 + 21318\tau^4 B - 672977\tau^4 - 95401\tau^3 B + 1539029\tau^3 + 149280\tau^2 B - 1653042\tau^2 - 91392\tau B + 795440\tau + 15232B - 119072)).$$

$G(\tau)$  has one root at  $\tau = 4/9$  which contradicts  $\tau''' < \tau < \bar{\tau}$ . Plugging in a smaller, positive value of  $\tau$ , e.g.,  $\tau = 1/3$  gives  $G(1/3) \approx -1231.62$ . Because the function is continuous, it follows that  $\partial \bar{\delta}_T / \partial \tau < 0$  if  $\tau''' < \tau < \bar{\tau}$ .  $\square$

Consider the impact of the level of product differentiation on the critical discount factor for two-part tariffs. As mentioned before, firms use the fixed part of the two-part tariff to compete for the indifferent customer. As a consequence, the impact of the product differentiation is similar to the case where firms only charge a fixed fee, i.e., an increase in differentiation increases the sustainability of collusion.

We next compare profits and analyze the outcomes in the different scenarios.

## 4 Comparisons

### 4.1 Profits

We start with the comparison of firm profits. Regarding competitive profits, Yin (2004) points out that the competitive profits with two-part tariffs are always higher than those under linear pricing. We complement the analysis by comparing these profits to the one with fixed fees and by comparing the collusive profits in the three pricing regimes. We can state the following result:

**Proposition 3.** *For any value  $\tau \in (0, \bar{\tau}]$  it holds that*

$$(i) \pi_L^* < \pi_T^* < \pi_F^* \text{ and}$$

$$(ii) \bar{\pi}_L^c < \bar{\pi}_F^c < \bar{\pi}_T^c.$$

Punishment profits are larger with two-part tariffs compared to linear pricing although intuition would first point in the opposite direction: typically, competitive pressure is higher if firms have more instruments available, which should lead to lower profits. In the present case, however, firms use the fixed fee to compete for the indifferent customer (both under a fixed fee and a two-part tariff) without having to pay attention to (inframarginal) local demand. Once market shares are set, firms are monopolists when it comes to optimizing local demand (or customer surplus) through the linear price. Consumer surplus can then partly be appropriated through the fixed fee (see also Yin, 2004). Moreover, in the linear-pricing case, firms cannot directly target consumer surplus but have to use the linear component to compete for the indifferent customer. As a result, firms make a lower profit under linear pricing which means that in this scenario, punishment is harshest.

Competitive profits are lower with two-part tariffs compared to fixed fees. Under two-part tariffs, when competing for market shares via the fixed fee, firms anticipate that they can earn additional profits by adjusting the linear price once market shares are set. This additional source of income induces firms to compete more intensely for market shares by lowering the fixed fees, resulting in relatively lower profits under two-part tariffs.

When firms collude, the (strictly) largest (industry) profit is obtained when two-part tariffs are used because in this scenario firms have most instruments available to extract customer surplus. However, the comparison between the fixed-price scenario and linear pricing is less clearcut a priori. Under linear pricing, firms are unable to extract all surplus from the indifferent customer and hence from any customer located closer to them because they can only

set a linear price.<sup>11</sup> When firms are able to charge a fixed fee only, they can fully extract the indifferent customer's surplus but leave positive surplus to the other customers located closer to the firm(s). Because local demand is decreasing in the distance to the firms, these customers have a positive surplus even after paying the fixed fee. With two-part tariffs, the additional instrument of a linear price allows firms to further increase their profits by extracting additional surplus off those customers located in closer proximity to their locations. In conclusion, collusion is most profitable if firms can set two-part tariffs.

## 4.2 Critical discount factors

In the following proposition, we present our main result which sheds light on how the different profits affect the discount factors in the three pricing scenarios considered. Interestingly, we find that the ranking of critical discount factors is the same for all values of  $\tau$ .

**Proposition 4.** *For any value  $\tau \in (0, \bar{\tau}]$  it holds that  $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$ .*

*Proof.* We start by showing that  $\bar{\delta}_T < \bar{\delta}_L$  holds for  $0 < \tau \leq \bar{\tau}$ . At  $\tau = 0$ , all three critical discount factors are equal to  $1/2$ . Since  $\bar{\delta}_L$  is increasing at first and both  $\bar{\delta}_F$  and  $\bar{\delta}_T$  are decreasing,  $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$  holds for values of  $\tau$  close to 0. For  $\tau > 0$ , we look for a solution to  $\bar{\delta}_T = \bar{\delta}_L$  with respect to  $\tau$  in the regions of  $\tau$  defined by the respective deviation profits. For  $0 < \tau < \tau'''$  we find that  $\bar{\delta}_T = \bar{\delta}_L$  only holds for  $\tau = 0$  which contradicts that  $0 < \tau < \tau'''$ . For the area of  $\tau''' < \tau < \tau'$ ,  $\bar{\delta}_T = \bar{\delta}_L$  holds for  $\tau \approx 0.0829$  which violates  $\tau''' < \tau$ . For  $\tau > \tau'$ , the only solution to  $\bar{\delta}_T = \bar{\delta}_L$  is  $\tau = 0$ . Since the discount factors are continuous functions,  $\bar{\delta}_T < \bar{\delta}_L$  follows.

In the second step, we show in the same way that  $\bar{\delta}_F < \bar{\delta}_T$  holds for  $0 < \tau \leq \bar{\tau}$ . In the region where  $0 < \tau < \tau''$ ,  $\bar{\delta}_F = \bar{\delta}_T$  only holds for  $\tau = 0$  which contradicts  $\tau > 0$ . For  $\tau'' < \tau < \tau'''$ , the only solution to  $\bar{\delta}_F = \bar{\delta}_T$  is at  $\tau \approx 0.1355$  which is not in the range of  $\tau'' < \tau < \tau'''$ . For  $\tau > \tau'''$ ,  $\bar{\delta}_F = \bar{\delta}_T$  only holds for  $\tau = 0$ . Since the discount factors are continuous functions,  $\bar{\delta}_F < \bar{\delta}_T$  follows.

As  $\bar{\delta}_T < \bar{\delta}_L$  and  $\bar{\delta}_F < \bar{\delta}_T$ , it follows that  $\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$ . □

A graphical illustration of the critical discount factors is given in *Figure 1*.

<sup>11</sup>Under linear pricing, even the indifferent customer is left with a positive surplus. This is because the firms optimize against a linear demand function. Furthermore, it can be shown that optimal collusion involves serving the customer located at  $1/2$  only if  $\tau \leq 2/3$  which holds under Assumption 1. For larger values of  $\tau$ , firms do not have an incentive to serve all customers in the market, leading to a number of customers close to  $1/2$  being left out.

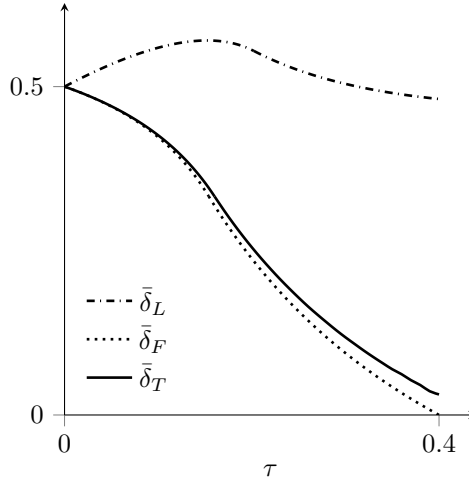
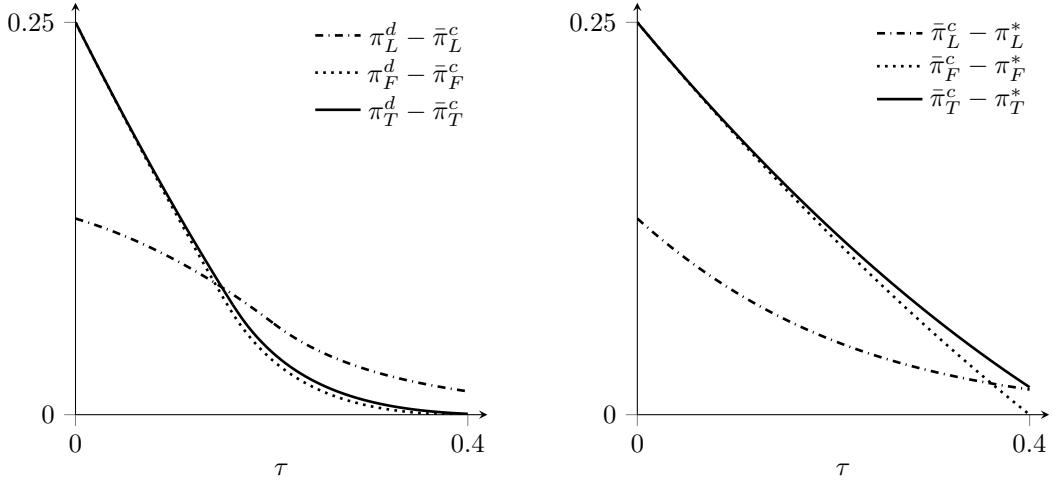


Figure 1: Comparison of the critical discount factors in the three pricing scenarios.

In order to understand the intuition behind the result, we have a closer look at the gains from deviation and the losses from punishment. They are illustrated in *Figure 2* for the three pricing scenarios.



(a) Comparison of losses from punishment. (b) Comparison of losses from punishment.

Figure 2: Comparison of deviation incentives in the three pricing scenarios.

Now consider an increase in the degree of product differentiation which affects these two main effects regarding the deviation incentives in the following

standard way. On the one hand, there are decreasing gains from deviation. As a matter of fact, deviation becomes more difficult (or less profitable) because it is harder for firms to extend their market share by attracting those customers who are located closer to the competitor. On the other hand, the losses from punishment decrease as switching between firms becomes more costly for customers and hence firms enjoy greater market power. Whereas the first effect makes collusion easier to sustain, the second effect leads to the opposite outcome.

Next consider the relative importance of these two effects. Under linear pricing, the second effect dominates as the differentiation parameter moderately increases. This is due to the fact that with elastic demand, firms lower their linear prices to gain market shares which drives up the demand by the inframarginal customers. As a result, the decrease in the losses from punishment is slowed down and collusion is harder to sustain. As the differentiation parameter increases even further and becomes sufficiently large, the effect on the inframarginal demand is reduced (as demand is already rather low) such that the first effect dominates. As a result, collusion becomes easier to sustain.

Under fixed pricing and two-part tariffs, we find the standard positive relationship between the degree of differentiation and the sustainability of collusion. In these cases, we know from the derivation of the profits that the fixed fee is used as the main instrument to compete for the indifferent customer in the punishment case and as the main means to undercut the collusive prices when deviating. As extending market shares becomes more difficult when firms becomes more differentiated, the effect on the gains from deviation dominates and collusion becomes easier to sustain. At the same time, however, firms competing in fixed prices only cannot use the linear component to fine-tune their deviation strategy but must solely rely on the market-extension effect. As a consequence, collusion is relatively easier to sustain under fixed pricing compared to two-part tariffs as deviation is less profitable in the former regime.

### 4.3 Customer surplus and social welfare

In this subsection, we comment on the implications of the above outcomes for customers and total welfare.

We start by analyzing customer surplus which we denote by  $\Lambda$ . Given firm symmetry, it amounts to

$$\Lambda = 2 \int_0^{\frac{1}{2}} U(x, p_i, f_i) dx.$$

Customer surplus in the three different pricing scenarios under competition

amounts to

$$\Lambda_L^* = \frac{11\tau^2}{12} - \frac{5\tau}{8} + \frac{1}{4} + \frac{(2\tau + 1)(\sqrt{4 - 4\tau + 13\tau^2})}{8},$$

$$\Lambda_F^* = \frac{13\tau^2}{24} - \frac{5\tau}{4} + \frac{1}{2},$$

and

$$\Lambda_T^* = \frac{67\tau^2}{69} - \frac{5\tau}{4} + \frac{1}{2}.$$

Under full collusion, we have

$$\bar{\Lambda}_L^c = \frac{7\tau^2}{384} - \frac{\tau}{16} + \frac{1}{8},$$

$$\bar{\Lambda}_F^c = -\frac{\tau^2}{12} + \frac{\tau}{4},$$

and

$$\bar{\Lambda}_T^c = -\frac{7\tau^2}{48} + \frac{\tau}{4}.$$

A straightforward comparison of the customer surpluses in the three different pricing scenarios under competition and collusion yields the following proposition:<sup>12</sup>

**Proposition 5.** *For any value  $\tau \in (0, \bar{\tau}]$  it holds that*

(i)  $\Lambda_F^* < \Lambda_T^* < \Lambda_L^*$  and

(ii)  $\bar{\Lambda}_T^c < \bar{\Lambda}_F^c < \bar{\Lambda}_L^c$ .

Turning to total welfare, we assume that total welfare denoted by  $\Upsilon$  puts equal weights on customer surplus and firm profits. Thus, it is given by

$$\Upsilon = \Lambda + 2\pi_i.$$

Under competition, we thus have

$$\Upsilon_L^* = -\frac{31\tau^2}{48} - \frac{\tau}{2} + \frac{1}{4} + \frac{(3\tau + 2)(\sqrt{4 - 4\tau + 13\tau^2})}{16},$$

$$\Upsilon_F^* = \frac{\tau^2}{24} - \frac{\tau}{4} + \frac{1}{2},$$

---

<sup>12</sup>Note that the comparison of collusive outcomes here and for social welfare (see below) is valid as long as collusion is stable in all three pricing scenarios.

and

$$\Upsilon_T^* = \frac{\tau^2}{96} - \frac{\tau}{4} + \frac{1}{2}.$$

Under full collusion, we have

$$\bar{\Upsilon}_L^c = \frac{13\tau^2}{384} - \frac{\tau}{4} + \frac{1}{2},$$

$$\bar{\Upsilon}_F^c = \frac{\tau^2}{24} - \frac{\tau}{4} + \frac{1}{2},$$

and

$$\bar{\Upsilon}_T^c = \frac{\tau^2}{96} - \frac{\tau}{4} + \frac{1}{2}.$$

Note that the differences in social welfare are solely driven by the differences in the linear prices, where  $0 = p_F^* < p_T^* < p_L^*$  and  $0 = \bar{p}_F^c < \bar{p}_T^c < \bar{p}_L^c$ . We arrive at the following result:

**Proposition 6.** *For any value  $\tau \in (0, \bar{\tau}]$  it holds that*

(i)  $\Upsilon_L^* < \Upsilon_T^* < \Upsilon_F^*$  and

(ii)  $\bar{\Upsilon}_L^c < \bar{\Upsilon}_T^c < \bar{\Upsilon}_F^c$ .

As by definition the linear price is equal to zero and hence equal to marginal costs under fixed prices in the competition and the collusion phase, there is no distortion from a social welfare point of view. As a consequence, social welfare is higher than in the other two pricing regimes. By the same argument, social welfare is lower when firm set linear prices compared to the situation where they set two-part tariffs. In the latter scenario, the linear price is below the one under linear pricing which is why there is less distortion.

*Table 1* summarizes the results from this section.

Table 1: Comparison of the three regimes under competition and full collusion.

	Competition	Full collusion
Profits	$\pi_L^* < \pi_T^* < \pi_F^*$	$\bar{\pi}_L^c < \bar{\pi}_F^c < \bar{\pi}_T^c$
Critical discount factors	–	$\bar{\delta}_F < \bar{\delta}_T < \bar{\delta}_L$
Customer surplus	$\Lambda_F^* < \Lambda_T^* < \Lambda_L^*$	$\bar{\Lambda}_T^c < \bar{\Lambda}_F^c < \bar{\Lambda}_L^c$
Social welfare	$\Upsilon_L^* < \Upsilon_T^* < \Upsilon_F^*$	$\bar{\Upsilon}_L^c < \bar{\Upsilon}_T^c < \bar{\Upsilon}_F^c$

## 5 Further aspects

### 5.1 Partial collusion

Note that we have only considered the possibility of collusion at maximum prices so far. However, firms may set a lower collusive price in order to reduce firms' incentive to deviate from collusion for those cases where the actual discount factor is below the critical discount factor. In the standard Hotelling setup, Chang (1991) shows that a price above the competitive price can be sustained as long as firms are (slightly) differentiated. In the present setup with elastic demand, we discuss the derivation of the optimal collusive price under partial collusion for the case where firms set fixed costs. The other cases can be analyzed in an analogous fashion but are less tractable. Below we will report the results in a numerical example for all three cases (see *Figure 3*).<sup>13</sup>

We start by deriving the optimal deviation under partial collusion. To this end, suppose that the competitor sticks to collusion and sets a fixed fee of  $f_F^c$ .

Consider the case where  $\tau > \tau''$  which means that a deviating firm does not want to serve the whole market under collusion at maximum prices. Now if the collusive price has to be adjusted downwards to sustain collusion, a deviating firm is even less interested in capturing the whole market. Then, maximizing the deviating firm's profit function yields the following optimal deviation price:

$$\begin{aligned} \max_{f_F^d} \pi_F^d &= f_F^d \left( \frac{1}{2} - \frac{f - f_F^c}{\tau(2 - \tau)} \right) \\ \Rightarrow f_F^d(f_F^c) &= \frac{\tau(2 - \tau)}{4} + \frac{f_F^c}{2}. \end{aligned}$$

Given this price, the profit amounts to

$$\pi_F^d(f_F^c) = \frac{(\tau(2 - \tau) + 2f_F^c)^2}{16\tau(2 - \tau)}.$$

In order for collusion to be maintained when firms play grim-trigger strategies, it must hold that

$$\begin{aligned} \frac{\pi_F^c(f_F^c)}{2(1 - \delta)} &= \pi_F^d(f_F^c) + \frac{\delta\pi_F^c}{1 - \delta} \\ \Leftrightarrow \frac{f_F^c}{2(1 - \delta)} &= \frac{(\tau(2 - \tau) + 2f_F^c)^2}{16\tau(2 - \tau)} + \frac{\delta\pi_F^c}{1 - \delta} \end{aligned}$$

---

<sup>13</sup>The analysis below follows Rasch and Wambach (2009). See the derivation and discussion of their *Proposition 3*.



Solving this equality for  $f_F^c$  gives the following  $\delta$ -adjusted collusive price such that collusion can be (just) maintained:

$$f_F^c(\delta) = \frac{\tau(2-\tau)(3\delta-1)}{2(1-\delta)}$$

where  $\partial f_F^c(\delta)/\partial\delta > 0$ ,  $f_F^c(0) = f_F^*$ , and  $f_F^c(\bar{\delta}_F) = \bar{f}_F^c$ .<sup>14</sup> The resulting profit is then given as

$$\pi_F^c(\delta) = \frac{\tau(2-\tau)(3\delta-1)}{4(1-\delta)}.$$

Note that it holds that  $\partial\pi_F^c(\delta)/\partial\delta > 0$ ,  $\pi_F^c(0) = \pi_F^*$ , and  $\pi_F^c(\bar{\delta}_F) = \bar{\pi}_F^c$  (see also *Figure 3*).

**Proposition 7.** *Under partial collusion, it holds that for any  $\tau \in (0, \bar{\tau}]$ ,*

- (i) *there exist  $\delta'$  and  $\delta''$  with  $0 < \delta' \leq \delta'' < \bar{\delta}_T$  such that if  $\delta \in [0, \delta'] \Rightarrow \pi_F^c \geq \pi_T^c$  and  $\delta \in [\delta'', \bar{\delta}_T] \Rightarrow \pi_F^c \leq \pi_T^c$  and*
- (ii) *for any  $\tau \in (0, \bar{\tau}]$ , there exist  $\delta'''$  and  $\delta''''$  with  $0 < \delta''' \leq \delta'''' < \bar{\delta}_T$  such that if  $\delta \in [0, \delta'''] \Rightarrow \Lambda_F^c \leq \Lambda_T^c$  and  $\delta \in [\delta'''', \bar{\delta}_T] \Rightarrow \Lambda_T^c \leq \Lambda_F^c$ .*

*Proof.* Ad (i): as  $\pi_F^* > \pi_T^*$ ,  $\delta'$  exists. Moreover, as  $\bar{\pi}_F^c < \bar{\pi}_T^c$ ,  $\delta''$  exists.

Ad (ii): similar argument as for (i). □

The proposition has an important implication for competition policy. On the one hand, two-part tariffs lead to higher maximum profits which is bad news for customers. On the other hand, compared to the scenario with fixed fees, collusion at maximum collusive prices can only be sustained for a smaller range of the discount factor and if firms are forced to reduce their collusive prices, collusive profits may actually be *lower* which benefits customers. *Figure 3* illustrates the findings for the case where  $\tau = 1/4$ .

## 5.2 Positive marginal costs

Our results continue to hold qualitatively when introducing a common positive marginal cost for the firms despite the inefficiency this causes in case of fixed fees because firms then make a loss with each unit sold. As marginal costs increase, collusion becomes easier to sustain under fixed fees and two-part tariffs for any given value of the transport-cost parameter. The reason is a

<sup>14</sup>Note that given a lower collusive price, a deviating may abstain from serving the whole market even for  $\tau < \tau''$ . For this to be true, it can be shown that the discount factor must be sufficiently small ( $\delta \leq 1/3$ ) such that the  $\delta$ -adjusted price is low enough. Contact authors for details.

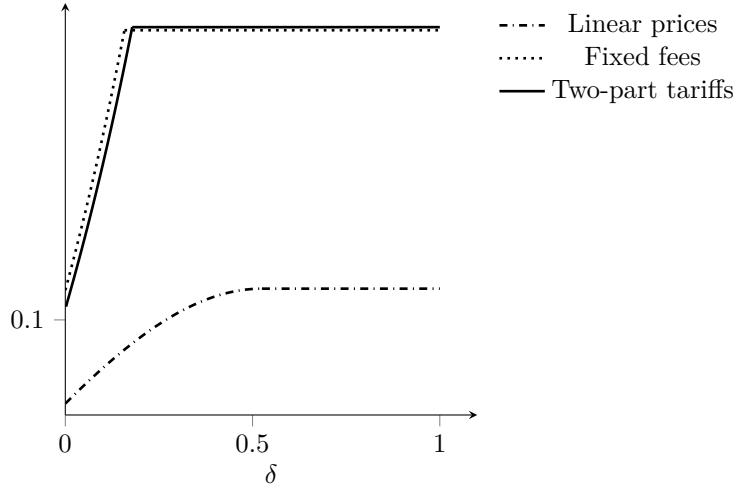


Figure 3: Comparison of the collusive profits in the three pricing scenarios (for  $\tau = 1/4$ ).

lower utility for consumers located in the middle of the unit interval, leading to an increased incentive by firms to serve only consumers located in their proximity under non-linear pricing, i.e., to behave like local monopolists. The cases of  $c = 1/10$  and  $c = 1/5$  are displayed in *Figure 4*.

### 5.3 Constant collusive profits

In the main analysis, we determine the optimal collusive profit for each pricing scheme. In order to reduce the multiplicity of effects and, additionally, as a robustness check, we now fix the level of the collusive profit across all scenarios. We choose the optimal collusive profit under linear pricing,  $\pi_L^C$ , as the highest level of collusive profits feasible in all three pricing scenarios we analyse. In order to calculate the critical discount factors, we determine the optimal deviation profits under fixed fees and two-part tariffs given the common collusive profit. Observe that the linear-pricing scenario as well as competitive profits remain at the same level as in the main analysis. The resulting critical discount factors are displayed in *Figure 5*. Qualitatively, the findings of our main analysis are unaffected by assuming a constant collusive profit. Collusion is easiest to sustain in the fixed-fee scenario and hardest to sustain under linear pricing while incentives to collude are intermediate in the two-part-tariff scenario. More interestingly, holding the collusive profit constant allows us to isolate the influence of the different incentives to deviate on the sustainability of collusion. *Figure 6 (a)* shows the corresponding deviation profits. The incentive to deviate is largest under linear pricing. The reason is that firms

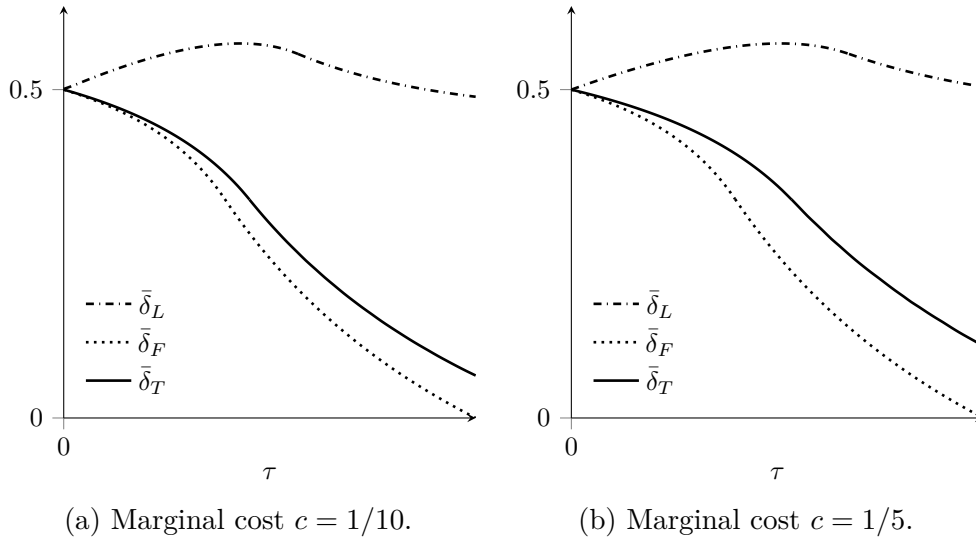


Figure 4: Critical discount factors with positive marginal costs.

use the fixed fee as a main means of increasing market share when deviating. In contrast, under linear pricing, lowering the price from the collusive price level leads to an increase in market share *and* increase in local demand. This demand increase dampens the loss of profits as differentiation increases. Given the critical discount factors in *Figure 5* and because punishment profits are unchanged (that is, punishment is harshest under linear pricing, see also *Figure 6 (b)*) and the collusive profit is constant, the smaller deviation incentive under non-linear pricing is the main effect causing a higher sustainability of collusion in these pricing scenarios.

## 6 Conclusion

Motivated by the recent world-wide cartel case in the air cargo industry where firms coordinated on multiple price components, we analyze the influence of three different pricing regimes on firms' ability to collude. It is shown that full collusion is easiest to sustain with fixed fees. Compared to two-part tariffs, where incentives to collude are intermediate, this result is driven by smaller gains from deviation under fixed fees. We show that this pro-collusive effect dominates the opposing effect of a smaller loss from punishment under fixed fees. Collusion at maximum prices can be sustained for the smallest range of the discount factor under linear pricing when compared to both nonlinear-pricing scenarios. This result is driven by the relatively large gains from deviation under linear pricing which is caused by an increase in local demand due to undercutting, an effect that is a lot weaker under nonlinear pricing.

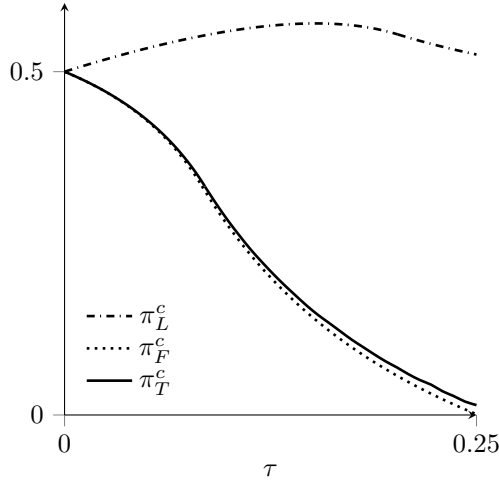


Figure 5: Critical discount factors with a constant collusive profit.

The present analysis not only helps to get a better understanding of recent anti-trust cases but also has implications for competition policy in a more general context. Customer protection agencies as well as policymakers often criticize firms' complex pricing schedules designed to price-discriminate between customers. They demand that firms reduce the complexity of their pricing schemes in order to make decisions for customers easier and more transparent. This paper highlights that the implications of such changes are not clearcut. It is true that a smaller number of available contracts, i.e., less instruments to price-discriminate among customers, may reduce prices customers have to pay in a static context. Indeed, when moving from two-part tariffs to linear prices or fixed fees, we find that customer rents increase. However, there may be the undesired anti-competitive consequence that collusion turns out to be easier to sustain and firms end up generating higher supra-competitive profits in a dynamic setting when moving to a simpler pricing regime. As a consequence, it seems of importance for competition authorities to carefully assess how the simplification of pricing structures—through fixed or linear prices—is achieved.

An aspect which we have not analyzed is the strategic choice of the pricing schedule employed by firms. If firms decide on the tariff before they collude, they may use other pricing techniques when they choose to deviate. Furthermore, it may be interesting to investigate the effect of the number of firms on the incentives to collude under nonlinear pricing. We leave this as an open question for future research.

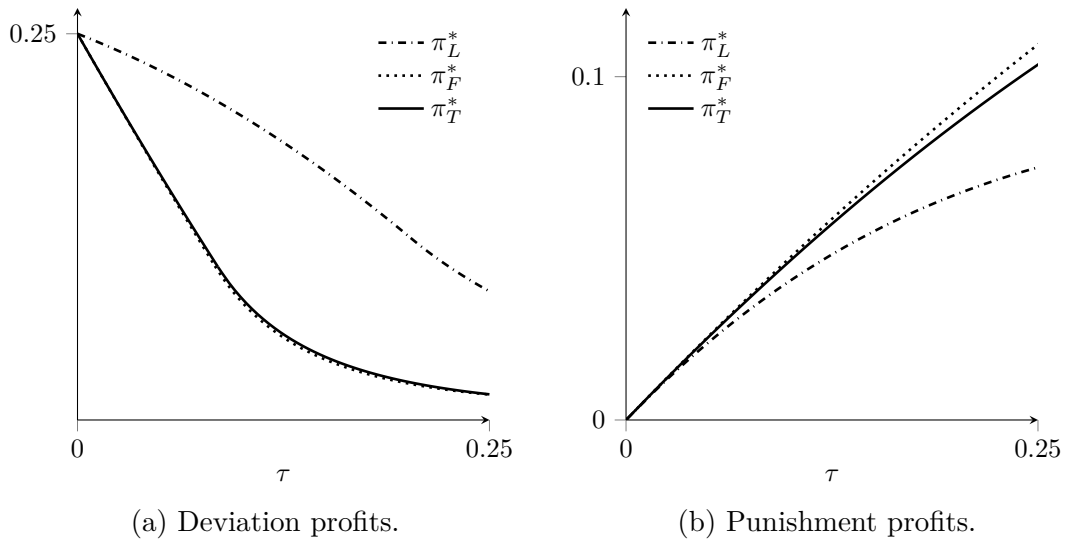


Figure 6: Deviation and punishment profits with a constant collusive profit.

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## Appendix

### Proof of *Lemma 5*

*Proof.* For the proof it is helpful to rewrite the profit function of the deviating firm as a function of the linear prices of both firms, the fixed fee of the firm that sticks to collusion as well as the marginal customer. Maximization is then over  $p_i$  and  $\tilde{x}$ . The idea is that the deviating firm can choose the optimal market size and, given the market size, the optimal linear price. The profit function of firm  $i$  is then given by

$$\pi_T^d = \tilde{x} \left( f_j + \frac{(1 - p_i - \tau\tilde{x})^2}{2} - \frac{(1 - p_j - \tau(1 - \tilde{x}))^2}{2} \right) + p_i \int_0^{\tilde{x}} (1 - p_i - \tau y) dy$$

where the condition for the indifferent customer was solved for firm  $i$ 's fixed fee and substituted. Substituting the collusive values of firm  $j$  leads to

$$\pi_T^d = \tilde{x} \left( \frac{(4-3\tau)^2}{32} + \frac{(1-p_i-\tau\tilde{x})^2}{2} - \frac{(1-\frac{\tau}{4}-\tau(1-\tilde{x}))^2}{2} \right) + p_i \int_0^{\tilde{x}} (1-p_i-\tau y) dy$$

Differentiating with respect to  $p_i$  gives

$$\frac{\partial \pi_T^d}{\partial p_i} = \frac{\tau \tilde{x}^2}{2} - p_i \tilde{x},$$

differentiating the profit function w.r.t. to  $\tilde{x}$  leads to

$$\frac{\partial \pi_T^d}{\partial \tilde{x}} = \frac{1}{2} + \frac{\tau}{2} - \frac{\tau^2}{2} - \frac{p_i^2}{2} + \tilde{x} p_i \tau - 4\tau \tilde{x} + \frac{5\tau^2 \tilde{x}}{2}.$$

Solving for  $p_i$  and  $\tilde{x}$  leads to

$$p_i^{D,l} = \frac{8}{3} - \frac{5\tau}{3} - \frac{B}{3}$$

and

$$\tilde{x}^D = \frac{1}{\tau} \left( \frac{16}{3} - \frac{10\tau}{3} - \frac{2B}{3} \right),$$

where  $B := \sqrt{61 - 83\tau + 28\tau^2}$ . Given these two critical values, we now check whether second order conditions are satisfied, i.e., if they constitute a maximum of the profit function. The Hessian of the profit function is given by

$$H(p_i, \tilde{x}) = \begin{bmatrix} -\tilde{x} & \tau \tilde{x} - p_i \\ \tau \tilde{x} - p_i & \tau p_i - 4\tau + \frac{5\tau^2}{2} \end{bmatrix}.$$

The determinant of the Hessian is given by

$$D(p_i, \tilde{x}) = \tau p_i \tilde{x} + 4\tau x - \frac{5\tau^2 \tilde{x}}{2} - \tau^2 \tilde{x}^2 - p_i^2.$$

It follows that the pair  $p_i^{D,l}, \tilde{x}^D$  is a local maximum if  $D(p_i^{D,l}, \tilde{x}^D) > 0$  and  $\partial^2 \pi_T^d / \partial p_i^2 < 0$ . Substituting the critical values into the determinant and simplifying gives

$$D(p_i^{D,l}, \tilde{x}^D) = -\frac{B(5\tau - 8 + B)}{3}.$$

Observe that  $D(p_i^{D,l}, \tilde{x}^D)$  is a function of  $\tau$  only, define it  $D(p_i^{D,l}, \tilde{x}^D) := K(\tau)$ .  $K(\tau)$  has three roots, one at  $\tau = 1/2 - \sqrt{5}/2 \approx -0.618$ , a second one at  $\tau = 83/56 + \sqrt{57}/56 \approx 1.617$  and a third one at  $\tau = 83/56 - \sqrt{57}/56 \approx 1.347$ .



As none of these roots in the range of  $\tau$  as defined by Assumption 1, it follows that  $K(\tau)$  does not have any roots in the relevant range. Substituting a valid value into  $K(\tau)$  leads to  $K(0.1) \approx 0.537$ . It follows that  $D(p_i^{D,l}, \tilde{x}^D) > 0$ . The second condition follows immediately,  $\partial^2 \pi_T^d / \partial p_i^2 = -\tilde{x} < 0$  as  $\tilde{x} \in [0, 1]$ .

The resulting optimal fixed fee  $f_i^{D,l}$  follows from substituting the collusive values of firm  $j$ ,  $p_i^{D,l}$ , and  $\tilde{x}^D$  into equation (9) and solving for  $f_i$ . Re-substituting the optimal values into the profit function leads to the deviation profit for large  $\tau$ . Clearly,  $\tilde{x}^D$  cannot exceed 1. Solving  $\tilde{x}^D \leq 1$  for  $\tau$  leads to

$$\tau \leq \frac{14}{19} - \frac{2\sqrt{30}}{19} =: \tau'''.$$

For  $\tau > \tau'''$ , the optimal deviating values are given by  $p_i^{D,l}$  and  $f_i^{D,l}$ . For  $\tau \leq \tau'''$ , the optimal linear price is derived by substituting  $\tilde{x} = 1$  into the first-order condition and solving for  $p_i$ , leading to

$$p_i^{D,s} = \frac{\tau}{2}.$$

The optimal fixed fee is calculated by setting  $\tilde{x} = 1$ , substituting firm  $j$ 's collusive values and the optimal linear price of firm  $i$  into equation (9) and solving for  $f_i$  which gives

$$f_i^{D,s} = \frac{1}{2} - 2\tau + \frac{11\tau^2}{8}.$$

The respective profit follows immediately. □