

Müller, Stephan; von Wangenheim, Georg

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Coevolution of Cooperation, Preferences and Cooperative Signals in Social Dilemmas

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Abstract

We study the coevolution of cooperation, preferences and cooperative signals in an environment where individuals engage in a signaling-extended Prisoner's Dilemma. We identify a new type of evolutionary equilibrium – a *transitional equilibrium* – which is constituted and stabilized by the dynamic interaction of multiple Bayesian equilibria. A transitional equilibrium: (1) exists under mild conditions and (2) can stabilize a population that is characterized by the heterogeneity of behavior, preferences, and signaling. We thereby offer an explanation for persistent regularities observed in laboratory and field data on cooperative behavior. Furthermore, this type of equilibria is least demanding with respect to differences in signaling cost between ‘conditional cooperators’ and ‘opportunists’. Indeed and quite surprisingly, a transitional equilibrium is consistent with ‘conditional cooperators’ bearing higher signaling cost in terms of fitness than ‘opportunists’.

JEL Classification numbers: C73, D64, D82.

Keywords: Evolutionary Game Theory, Cooperation, Signaling.

1 Introduction

Several theories have been proposed to explain the evolution of cooperation among humans when cooperation generates a public benefit at a private cost. In this research, the Prisoners dilemma game (henceforth PD) serves commonly as a metaphor for the problem of cooperation. Since natural selection favors defection in this game, any extension that allows for the emergence of cooperation represents a mechanism to promote cooperation. It has been argued, that the essential feature of any mechanism to foster cooperation is that cooperative acts must occur more often between cooperators than expected based on population averages. In other words, the mechanism must induce a positive assortment between cooperative types (Queller, 1985; Fletcher and Zwick 2004).¹

The mechanisms proffered in the literature may vary substantially to how they induce this assortment. Positive assortment can for instance arise because of direct reciprocity in repeated interactions (Trivers, 1971; Axelrod, 1984; Fudenberg and Maskin, 1986), indirect reciprocity based on image scores (Alexander, 1987; Nowak and Sigmund, 1998; Wedekind and Milinski, 2000; Panchanathan and Boyd, 2004), or network reciprocity, where players interact only with their neighbors (Nowak and May, 1992; Hubermann and Glance, 1993; Nowak et al., 1994; Killingback et al., 1999). Another mechanism involves cooperators signaling their type and the play of signal-contingent strategies.

In solving the puzzle of cooperation in social dilemmas the literature so far primarily focused on providing mechanisms that supports the existence of a cooperative equilibrium. We extend this literature by providing an explanation for the following conspicuous regularities of this puzzle. First, there is a persistent pattern, that cooperation is only partial, i.e. only a fraction of the population plays cooperatively when individual rationality calls for defective behavior.² Second, the elicitation of preferences in the laboratory and in the field as well as studies on revealed preferences show that individuals substantially differ in their cooperative attitudes (e.g., Andreoni and Miller, 1993; Cooper et al., 1996; Ockenfels and Weimann, 1999, Fischbacher and Gächter, 2010). Thus, the heterogeneity in behavior seems not to result from mixed strategy play, but appears to be a consequence of differences in preferences. Third, it is rather the rule than the exception that human interactions are accompanied by communication, in particular if the interaction is

¹Indeed, many models of the evolution of altruism share an underlying mathematical structure- that of Hamilton's Price equation formulation of inclusive fitness theory (Hamilton, 1964a,b). Hamilton's relatedness coefficient can be interpreted as the degree of positive assortment of types and need make no reference to common descent (McElreath and Boyd, 2007).

²See Rapaport and Chammah, 1965 and Dawes, 1980 for reviews of these experiments in sociology and psychology. For a survey of some of the studies by economists, see Roth, 1988.

of strategic nature. Humans also differ in this respect and show different ways and intensity of preplay-communication in laboratory and field studies. Importantly, it has been shown that communication influences cooperative behavior (Dawes et al., 1977; Ostrom and Walker, 1991; Brosig, 2002).³

All three phenomena take place at a population level, we therefore take an evolutionary perspective to study these related dimensions of heterogeneity. As a stylized social dilemma, the action set of the PD incorporates the two diametrically opposed behaviors of defection and cooperation. To account for the potential heterogeneity of preferences in equilibrium, we consider an evolutionary model with two types of individuals ‘opportunists’, maximizing individual fitness, and ‘conditional cooperators’, having a preference for joint cooperation.⁴ To emphasize the necessity of communication about preferences, we study the evolution of cooperation in social dilemmas without social information such as reputation in one-shot interactions, which puts other mechanisms like direct or indirect reciprocity out of operation. Any mode of communication comes hardly without any cost, may it be material cost because of effort exerted, resources spent, or forgone opportunities. On the other hand compliance to some code of conduct as a signal for cooperativeness may cause internal cost if it contradicts an individual’s preferences. To account for these aspects, we incorporate communication of types via costly signaling.⁵

We apply the ‘indirect’ evolutionary approach pioneered by Güth and Yaari (1992)⁶ to study the dynamics of the type composition in the population. In that approach players are assumed to be rational, and the evolutionary mechanism shapes the composition of players with different preferences in the population. Recent criticism of this approach (Dekel, Ely, and Yilankaya, 2007) has focused on its assumption that players’ preferences are observable by their opponents, which is not an issue in our signaling framework. We depart from standard applications of this approach in one important manner. Instead of applying the static notion of evolutionary stable strategies (Maynard Smith, 1973) as a stability concept, we explicitly study the dynamic stability of the Bayesian equilibria of the signaling-extended PD. Importantly, considering the full set of Bayesian equilibria and their dynamic stability, puts us in the position to study the transition across different

³It is a stylized fact in experimental research that the opportunity of communication has a robust and strong positive impact on cooperation, for an overview see Sally, 1995.

⁴There is evidence from laboratory and field experiments, that the majority of individuals can be assigned to one of these two classes: Keser and van Winden, 2000; Fischbacher et al., 2001; Frey and Meier, 2004; Fischbacher and Gächter, 2010.

⁵Costly signaling is present in many species, including humans (Zahavi, 1977; Grafen, 1990; Maynard Smith, 1991; Johnstone, 1995; Wright, 1999).

⁶For recent applications see e.g. Wärneryd, 2002; Guttman, 2003; Alger and Weibull, 2013; Hopkins, 2014.

equilibria.

We identify a new type of evolutionary equilibrium which we term transitional equilibrium. In our model under mild conditions on signaling cost and other model parameters a transitional equilibrium exists and is constituted by the dynamic interplay of separating, semi-pooling, and pooling equilibria of the signaling extended PD. This evolutionary equilibrium is characterized by heterogeneity with respect to preferences, behavior, and signaling. Furthermore, it turns out that this type of equilibrium is least demanding with respect to the difference in signaling costs between ‘opportunists’ and ‘conditional cooperators’.

The remainder of the paper is organized as follows. The following section 2 discusses the related theoretical literature in more detail. Our model is presented in section 3. Section 4 presents the set of stable Perfect Bayesian Equilibria (PBE) for a given composition of preferences. This share of cooperative players is endogenized in section 5. Before we conclude in section 7, we discuss our findings in the penultimate section 6.

2 Related Theoretical Literature

In this section we focus on literature which considers the problem of cooperation in social dilemmas under incomplete information regarding the opponent’s type. Most closely related to our approach are the papers of Guttman (2003, 2013), Gintis et al. (2001), and Panchanathan and Boyd (2004). Guttman (2003) is motivated by the seminal paper of Kreps et al. (1982). Therein the authors showed that if one of two players assigns a small probability that the opponent plays the ‘tit-for-tat’ strategy, then, in a finitely repeated Prisoner’s Dilemma (PD), cooperation can be an equilibrium outcome for at least some of the stages. In an ‘indirect’ evolutionary framework Guttman endogenizes the uncertainty assumed by Kreps et al. (1982) regarding the opponent’s preferences. More precisely, the model considers a community consisting of ‘opportunists’ and ‘reciprocators’, who have a preference for mutual cooperation. Furthermore, agents send a costless, random signal that has some informational value for the receiver with respect to the recognition of the opponent’s type. Players are randomly matched to play a finitely repeated PD. In the unique evolutionary equilibrium, both reciprocators and opportunists coexist.⁷ Although the evolutionary equilibrium is characterized by a heteromorphic population, the equilibrium behavior of reciprocators and opportunists differs only in the last round, i.e.

⁷The survival of reciprocators hinges on the assumption that the costless signal emitted by all subjects has some small, but positive correlation with the actual type.

both types show almost identical behavior in equilibrium. Furthermore, if the likelihood of emitting the cooperative signal is independent of the taste parameter measuring the preference for mutual cooperation, then the endogenization of the taste parameter leads to an all-replicator equilibrium, and thereby to full cooperation. Thus, the model is less suitable to explain the regularities of heterogeneous preferences and behavior, in particular in environments with very few repetitions.

Guttman (2013) studies the evolution of an inherited preference to match other agents' contribution to the provision of public goods. Under complete information and randomly matched groups, the unique evolutionary stable matching rate equals one. The model provides a potential explanation for the existence of conditional cooperation, which does not rely on reputation or group selection. However, the informational assumptions are rather strict, which we circumvent by considering a signaling environment, where types are only revealed by equilibrium play. Further, the model predicts a unique preference value and therefore cannot account for the heterogeneity in preferences and behavior, which is the focus of our paper.

Contrary to Guttman (2003), Gintis et al. (2001) consider an environment with no repeated or assortative matching. Furthermore, the signaling in their model is costly. In a multi-player public good game individuals can signal their type by providing a group benefit at a personal cost. These signals may in turn influence a partner's acceptance or rejection of potentially profitable allies. They show, that honest signaling of underlying quality can be evolutionary stable. Necessary conditions for the existence are that signaling is more costly to so called high-quality types and that partners prefer to ally with high-quality types. They show that the payoff difference between high- and low types is positive. As a consequence, the frequency of high types would increase over time. This eventually undermines the separating equilibrium, since it has only limited support.⁸ More precisely, once the share of high types exceeds a certain threshold, high type no longer find it a best response to signal their quality. As a consequence, cooperation could break down. Without an exhaustive search for Nash equilibria and analyzing their dynamic stability which is part of our approach, we just don't know. In the model of Gintis et al. (2001), the monotonic increase in the share of high-quality types is stabilized by the *ad hoc* introduction (see p.112, eq.(12)) of other forces on the population dynamics. Indeed, without the exogenous frequency dependency introduced no heteromorphic population could be stabilized.

The general theme of this strand of literature (see also Lotem et al. 2003, Pan-

⁸That is, the range for the share of high-quality types, such that the conditions for the existence of the honest signaling equilibrium are met, is a open interval with measure less than one.

chanathan and Boyd 2004, Macfarlan et al. 2013) is that costly forms of generosity (like contributions to public goods) serves as a signal to be trustworthy, facilitating the formation of cooperative partnerships in the future. A general problem with the signaling hypotheses is, that it does not explain why quality is signaled by doing good (noted by Gintis et al. 2001 themselves).⁹ Indeed, quality could be signaled by other costly activity like conspicuous consumption.¹⁰ In contrast, in our approach the nature of the signaling technology is not limited to forms of generosity.

Another paper related to our approach is Janssen (2008) which studies the evolution of cooperation in a one-shot PD environment based on the recognition of the opponent’s trustworthiness. Agents costlessly display symbols and they learn which symbols are important to estimate an opponents’ trustworthiness. The simulation based results show both cooperative and defective behavior. In contrast, the evolution of agents’ taste parameters shows the tendency toward homogeneity, since almost all agents in the long run value cooperation over defection (see statistics of parameter α in table 4). Since the result hinges on the assumption that agents can withdraw from playing the game, it does not apply to our idea of random interaction in an unstructured population that cannot be circumvented.

3 Model

The classical Prisoner’s Dilemma is the most prominent and best-studied example of a social dilemma and serves as the basis for our analysis. In this game players can either cooperate (C) or defect (D). The one-shot PD is played recurrently by randomly matched pairs of individuals of a large population. Agents are assumed to process only information on outcomes of their own past interactions. In particular, they process no information on the opponent’s identity or on outcomes in games in which they were not involved.

Similar to Guttman (2003) we focus on the case of a preference for joint cooperation. Players carrying such a preference gain an additional internal payoff m if the outcome of the stage game is mutual cooperation, i.e., (C, C) . In other words, we consider preferences, which induce conditional cooperation.¹¹ We will refer to these players as *high types*. For *low types*, i.e., individuals without this preference utility is identical to material payoffs.

⁹Hopkins (2014) provides a potential solution to the problem. If the ability to reason about others’ mental state, i.e., having a “theory of mind”, is associated with empathy, then humans possessing these attributes can signal their capability by pro-social acts.

¹⁰After all, the prominent example for costly signaling in the context of sexual selection is the peacock’s tail.

¹¹See footnote 4.

Figure 1 presents the payoffs of the PD for the row player. The left table represents material payoffs, which will be decisive for the evolutionary success of different behaviors. The right table represents utility payoffs incorporating the preference for mutual cooperation, which will determine the best response of players. To be of any behavioral significance, we assume $m > \alpha$, such that mutual cooperation becomes a Nash equilibrium if two high types interact.¹²

	C	D
C	1	$-\beta$
D	$1 + \alpha$	0

	C	D
C	$1 + m$	$-\beta$
D	$1 + \alpha$	0

Figure 1: Material (left table) and utility payoff (right table) in the PD, $\alpha, \beta > 0$ and $1 + \beta > \alpha$.

The preference for joint cooperation is assumed to be private information of the agent. In the tradition of Harsanyi (1967, 1968a, 1968b), beliefs about the opponent's type are common knowledge. Like Guttman (2003) and Güth and Ockenfels (2005), we adopt the natural assumption that beliefs correspond to actual frequencies of types.

We employ the indirect evolutionary approach, pioneered by Güth and Yaari (1992)¹³, in which all players are assumed to be rational, and the evolutionary forces determine population's composition of players with different preferences. In other words, preferences determine behavior and behavior in turn determines fitness. Recent criticism of this approach (Dekel et al, 2007) is concerned with the assumption of the observability of agents' preferences. However, in our model preferences are not observable, we only assume that agents have correct beliefs about the distribution of types in the population.

Note that without communication, the impossibility result of Kandori (1992, Proposition 3) applies to such an environment, which states that the unique equilibrium is characterized by full defection, i.e. everybody always defects. We model communication as an additional stage prior to the play of the PD. In that stage, agents can simultaneously send one message concerning their inner motive. As in the standard signaling model (Spence, 1973) we assume the existence of a social technology which enables individuals to signal their positive attitude towards cooperation by incurring some costs. Research on many species including humans (Zahavi, 1977; Grafen, 1990; Maynard Smith, 1991;

¹²As Güth et al. (2000) noted in a different setting, the precise level of m is behaviorally irrelevant. All m -types for whom the same inequality with respect to α holds, form an equivalence class concerning the implied behavior.

¹³The indirect evolutionary approach has also been applied in different strategic settings (ultimatum game, Huck and Oechssler, 1999) or to analyze the evolutionary stability of altruistic preferences (Bester and Güth, 1998), of altruistic and spiteful preferences (Possajennikov, 2000), or of risk preferences (Wärneryd, 2002).

Johnstone, 1995; Wright 1999) supports this assumption. Signaling cost may occur in terms of utility or fitness. Let k_H, k_L denote the signaling cost in terms of utility for high types and low types, respectively. We assume, agents who have a preference for joint cooperation are supposed to bear lower costs for sending the signal, i.e. $k_H < k_L$. Our results suggest that this assumption is not needed. However, it simplifies the analysis of the existence and the stability of the numerous equilibria. Importantly, we make no such assumption on the relationship of the corresponding fitness cost, k_H^f and k_L^f . We refer the reader to our discussion of signaling costs in section 6.

In the current setup, a strategy is a type-contingent and signal-contingent plan. Given the two actions C and D , the two different types, and the two possible signals, there are 64 pure strategies in the signaling extended PD. Since defection is the dominant strategy for low types, we can eliminate all strategies which for the contingency of being a low types specify a cooperative behavior for some signal received. For the remaining 16 pure strategies we will apply the following notation. The first three entries specify the actions and signaling behavior for the high type, the last entry the signaling behavior for the low type. For example CDs, ns denotes the strategy “if high type: cooperate if signal is received, deviate if no signal is received and send signal; if low type: do not signal”. Thus, $p_{CDs, ns}$, $\Pi_L(CDs, ns)$ and $\Pi_H(CDs, ns)$, for example, denote the probability of this strategy being played by a randomly selected individual and her expected payoff if she happens to be of the low or of the high type, respectively. Probabilities and expected payoffs for other strategies are defined accordingly.

To study the stability of the Bayesian equilibria of the signaling-extended PD we explicitly account for the dynamic adjustment of the distribution of types in the population. Both for the set of probabilities by which the strategies are chosen within types and for the shares of types we consider the class of gradual payoff-monotone dynamics (see e.g. Bendor and Swistak, 1998 for definitions), which includes the well known replicator dynamics. Compared to the static approach of evolutionary stable strategies (Maynard Smith and Price, 1973), this dynamic approach allows us to study the dynamic properties not only for one particular equilibrium but also the dynamics across different equilibria. We assume that the dynamic accommodation within the population shares playing the various strategies is fast compared to the dynamics of the population share of high types. This assumption will simplify analysis of the dynamics and is considered adequate since behavior will adapt faster to differences in payoffs than socially and culturally transmitted cooperative preferences.

4 Stable Perfect Bayesian Equilibria with Exogenous Proportions of Conditional Cooperators

In this section we present stable PBE for a given share of high types, which we denote by λ . In our signaling extended PD, there exist one stable separating and three stable pooling equilibria. There are also stable semi-pooling equilibria, however, only one of them is relevant for our subsequent analysis. The others are characterized by strictly negative fitness differentials between high and low types or have a narrow λ -support¹⁴. The following Proposition 1 reports the stable signaling equilibrium, the stable pooling equilibria and the relevant stable semi-pooling equilibrium.

Proposition 1 *In the signaling extended Prisoner's Dilemma exist the following stable Perfect Bayesian Equilibria:*

- (i) *Cooperative Separating Equilibrium (CSE):* $p_{CDs,ns} = 1$
- (ii) *Semi-Pooling Equilibrium (SPE):* $p_{CDs,ns} = 1 - p_{CDs,s} \leq \frac{\lambda(m-a)}{(1-\lambda)\beta}$
- (iii) *Cooperative High Pooling Equilibrium (CHPE):* $p_{CDs,s} = 1 - p_{CCs,s} \geq \frac{k_L}{(1+\alpha)\lambda}$
- (iv) *Cooperative Low Pooling Equilibrium (CLPE):* $p_{CCns,ns} + p_{DCns,ns} = 1$
- (v) *Defective Low Pooling Equilibrium (DLPE):* $p_{DDns,ns} = 1 - p_{CDs,ns} \leq \frac{1}{\lambda} \min \left\{ \frac{k_H + \beta}{1+m+\beta}, \frac{k_H}{1+\alpha} \right\}$

Table 1 presents the conditions for existence and the λ -support of these equilibria.

For a list of the stable semi-pooling equilibria not mentioned in this proposition we refer the reader to Appendix C.

Proof. We leave the derivation and the analysis of stability to the Appendices. See Appendix B for existence and stability of separating and pooling equilibria, and Appendix D for semi-pooling equilibria. \square

¹⁴Here, the λ -support of an equilibrium corresponds to the set of all λ such that the equilibrium under consideration exists.

Strategies	λ -support Condition for Existence	Payoff Differences
Cooperative Separating Equilibrium		
CDs, ns	$\frac{k_H}{1+m} \leq \lambda \leq \frac{k_L}{1+\alpha}$ $k_H < 1 + m$	$\Pi_H - \Pi_L = \lambda(1 + m) - k_H$ $(\Pi_H - \Pi_L)^f = \lambda - k_H^f$
Semi-Pooling Equilibrium		
CDs, ns CDs, s	$\lambda = \frac{k_L}{1+\alpha}$ $k_L < 1 + \alpha$	$\Pi_H - \Pi_L = \lambda(1 + m) - k_H - \beta(1 - \lambda)p_s$ $(\Pi_H - \Pi_L)^f = (\lambda - k_H^f)(1 - p_s) +$ $(k_L^f - k_H^f - (\lambda\alpha + (1 - \lambda)\beta))p_s$
Cooperative High Pooling Equilibrium		
CCs, s CDs, s	$\lambda \geq \max\{\frac{k_L}{1+\alpha}, \frac{\beta}{\beta+m-\alpha}\}$ $k_L < 1 + \alpha$	$\Pi_H - \Pi_L = k_L - k_H - (\lambda(\alpha - m) + (1 - \lambda)\beta)$ $(\Pi_H - \Pi_L)^f = k_L^f - k_H^f - (\lambda\alpha + (1 - \lambda)\beta)$
Cooperative Low Pooling Equilibrium		
$CCns, ns$ $DCns, ns$	$\lambda \geq \frac{\beta}{\beta+m-\alpha}$	$\Pi_H - \Pi_L = -(\lambda(\alpha - m) + (1 - \lambda)\beta)$ $(\Pi_H - \Pi_L)^f = -(\lambda\alpha + (1 - \lambda)\beta) < 0$
Defective High Pooling Equilibrium		
$CDns, ns$ $DDns, ns$	$0 < \lambda < 1$	$\Pi_H - \Pi_L = 0$ $(\Pi_H - \Pi_L)^f = 0$

Table 1: Separating, pooling equilibria, and one semi-pooling equilibrium (see Table 2 in Appendix C for other stable semi-pooling equilibria). Note that equilibria are only stable in the interior of their support. p_s denotes the probability to send the signal conditional on being a low type.

In the *cooperative separating equilibrium*, players apply the strategy CDs, ns . Thus, high types recognize each other and cooperate only among themselves. The intuition behind the fact that the support of this equilibrium has both a lower and an upper is as follows: If there are too few high types, then the cooperative outcome among them cannot compensate for the signaling costs. If on the other hand, there are too many high types, signaling becomes sufficiently profitable for low types. The thresholds for the share of high types have a precise economic interpretation. For high types, the cost-benefit ratio from signaling ($\frac{k_H}{1+m}$) must be smaller than the probability to gain the benefit (λ). The reverse holds true for low types, i.e. their cost-benefit ratio from signaling must exceed ($\frac{k_L}{1+\alpha}$), the likelihood of gaining the benefit.

In the *cooperative low pooling equilibrium*, nobody signals and high types cooperate. This equilibrium exists if there are sufficiently many high types. Only then high types can be compensated for the loss from being cooperative against low types by the cooperative outcome among each other. In other words, if the share of high types falls below a certain threshold, then they will start to prefer defecting when receiving the low signal. Note that this equilibrium is indeed an equilibrium set, since the strategies $CCns, ns$ and $DCns, ns$ are alternative best responses in equilibrium. The share of high types required for this to be an equilibrium decreases in the sucker's payoff, since cooperative behavior becomes more disadvantageous with decreasing sucker's payoffs ($-\beta$). This threshold, too, has an intuitive meaning. Note that $m - \alpha$ (β) measures the incentive to reciprocate cooperative (defective) behavior. In essence, the condition $\frac{\beta}{\beta+m-\alpha} < \lambda$, which can be rewritten as $\lambda(m-\alpha) > (1-\lambda)\beta$, states that the expected gain from reciprocating cooperative behavior must exceed the expected gain from reciprocating defective behavior.

In the *defective low pooling equilibrium*, nobody sends the cooperative signal and everybody defects earning a payoff of zero. Again, because of the lack of distinguishability in equilibrium, this equilibrium is indeed a set where $CDns, ns$ and $DDns, ns$ might be played. This equilibrium set reflects the benchmark solution in the PD without communication and exists for all population compositions between high types and low types.

In the *cooperative high pooling equilibrium*, everybody signals and high types cooperate. This equilibrium exists if there are sufficiently many high types. If the latter's proportion is large enough, they can compensate for the loss from being cooperative against low types by the cooperative outcome among each other. Contrary to the cooperative low pooling equilibrium, an additional restriction with respect to the share of high types arises, reflecting the incentive compatibility for low types to signal. Note that this equilibrium is again an equilibrium set, since the strategies CCs, s and CDs, s are alternative best

responses in equilibrium. In this equilibrium, for low types the reverse logic applies in comparison to the cooperative separating equilibrium, i.e. for low types to find it worthwhile to signal, their cost-benefit ratio ($\frac{k_L}{1+\alpha}$) must be smaller than the likelihood to profit from signaling (λ). The lower bound stemming from the incentive constraint for high types bears the same logic as in the cooperative low pooling equilibrium.

5 Endogenous Proportion of Conditional Cooperators

We now analyze the dynamics of the share of high types (λ) in the population for which we assume that the dynamics have reached a stable equilibrium, as we assumed that inner motives evolve far more slowly than behavioral frequencies. For the sake of a more convenient presentation, we will assume that $k_L, k_H < 1 + \alpha$ which is sufficient for existences of all equilibria presented in Table 1. In other words, we restrict to signaling devices which are less costly than the maximum material payoff in the PD.

The evolution of the proportion of norm bearers is determined by their relative fitness. Fitness is measured by the material payoffs as presented in Figure 1. Analogous to the derivation of the PBE, the differentials in these fitness payoffs among high and low types are the driving force for the evolution of their respective shares. To ease the understanding of the differentials of fitness payoff differentials, we provide some intuition for their size in the relevant PBEs.

In the cooperative separating equilibrium, both types defect in all interactions, except when two individuals of the high type encounter each other. In this case, they cooperate. The low type will thus always earn a fitness payoff of zero, and the high type will earn a fitness payoff of one with probability λ , i.e., the probability that two high types interact. Since high types unconditionally bear the material signaling cost k_H^f , their expected payoff in the cooperative separating equilibrium is $\lambda - k_H^f$, which is also the expected difference of fitness payoffs: $(\Pi_H(CDs, ns) - \Pi_L(CDs, ns))^f = \lambda - k_H^f$. Obviously, this fitness advantage of the high type grows in the share of high types in the population.

In the cooperative low pooling and in the cooperative high pooling, individuals of the high type cooperate in reaction to the signal they send, and all individuals of the low type copy this signal but still defect. Leaving aside signaling costs for a moment, differences in material payoffs then reflect payoffs of unconditional cooperators and defectors in the underlying PD. More precisely, with probability λ , high types meet their own type and realize the cooperative outcome, earning a payoff of one. With the residual probability,

they meet a low type and lose β . Low types always defect and only earn positive payoffs when matched with high types, which happens with probability λ and earns them $1 + \alpha$. A fitness differential to the advantage of the high types thus cannot result from playing the game itself, but only from sufficiently large differences in signaling cost (see Table 1). Obviously, if no signal is sent, as is the case in the cooperative low pooling equilibrium, the fitness payoff of the high type can only be smaller than that of the low type, $(\Pi_H(CCns, ns) - \Pi_L(CCns, ns))^f = -(\lambda\alpha + (1 - \lambda)\beta) < 0$.

Only in the cooperative high pooling equilibrium, a signaling cost disadvantage of the low type may outweigh the disadvantage of the high type from playing cooperatively in the game, so that the high type earns a higher fitness payoff than the low type, $(\Pi_H(CCs, s) - \Pi_L(CCs, s))^f = k_L^f - k_H^f - (\lambda\alpha + (1 - \lambda)\beta)$. Thus, the fitness payoff difference increases (declines) in the share of the high types if defection is more (less) tempting against defection than against cooperation, i.e., if β is larger (smaller) than α . If the proportion of the high type in the population is too small, it is either not worthwhile to mimic the other type, or the chances to meet another high-type individual are so low that cooperation ceases to be the best reaction to the signal sent by all individuals. For these small shares of the high type in the population, the cooperative pooling equilibria break down just like the cooperative separating equilibrium discussed earlier breaks down for shares of the high type that are too large. Finally, in the defective low pooling equilibrium, both types always defect without sending signals and thus all earn the same fitness (and behavioral) payoff of zero.

A stable inner equilibrium, i.e., an equilibrium where both high types and low types coexist may be realized around one stable PBE or by the interplay of several PBEs. We first concentrate on the first case (Proposition 1), and then turn to the second case (Proposition 2). In the first case, the difference in fitness payoffs between high and low types must vanish to constitute a stationary point at this particular value of the share of high types, λ^* . For stability, in the neighborhood of an equilibrium λ^* , high types must earn strictly more than low types for $\lambda < \lambda^*$ and strictly less for $\lambda > \lambda^*$. The only candidate, where a stable heteromorphic population is supported by a single PBE is one associated with the high pooling cooperative equilibrium at $1 - \frac{\alpha - (k_L^f - k_H^f)}{\alpha - \beta}$. This is illustrated in Figure 2. All other equilibria are characterized by either strictly negative or strictly increasing payoff differentials. The high pooling cooperative equilibrium exists and is stable if $1 - \frac{\alpha - (k_L^f - k_H^f)}{\alpha - \beta}$ is inside the λ -support of this equilibrium and the fitness differential decreases in λ , which is the case if $\alpha - \beta > 0$ (see Table 1). Taking these conditions together yields:

Proposition 2 *There is a stable inner equilibrium at $1 - \frac{\alpha - (k_L^f - k_H^f)}{\alpha - \beta}$ if and only if $\max\{\beta + \frac{k_L}{1+\alpha}(\alpha - \beta), \frac{\beta}{\beta+m-\alpha}m\} < k_L^f - k_H^f < \alpha$. In this equilibrium both types send the signal and high types cooperate.*

Proof. All proofs are given in Appendix A. □

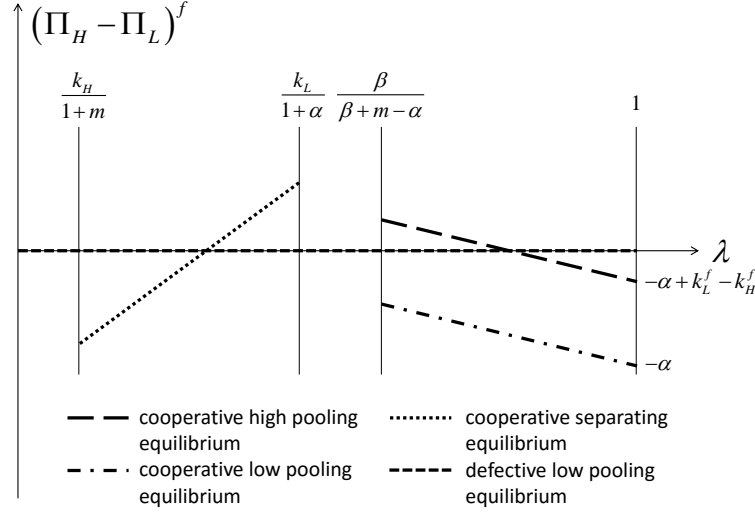


Figure 2: Differences in material payoffs under the conditions of Proposition 2.

The conditions in Proposition 2 reveal that the existence of inner stable equilibria requires that the material signaling costs for high types must exceed the corresponding costs for low types. The spread in signaling cost, however, does not have to compensate for the entire incentive to defect on cooperative behavior (α) for partial cooperation to be supported by the CHPE. Note that the necessary difference in material signaling cost increases in α and β . In other words, the higher the temptation to defect and the higher the loss from being defected against, the higher the required disadvantage in terms of material signaling cost for low types will be. Interestingly, although the precise level of m is not decisive with respect to its behavioural consequence, its level plays a role for partial cooperation induced by the CHPE. The needed spread in signaling cost weakly decreases in the strength of the preference for conditional cooperation m . That is, if high types are more inclined to conditionally cooperate, the signaling device needs to be materially less disadvantageous for low types.

The equilibrium supported by the CHPE is characterized by partial cooperation and the heterogeneity of preferences. However, the equilibrium only exists if we assume that

low types need to bear higher material signaling cost. Moreover, the CHPE as any pooling equilibrium, cannot account for heterogeneity regarding communication. Both limitations will be overcome by the transitional equilibrium, which we will introduce next.

We will now turn to the second case, i.e., whether there is an population equilibrium constituted by the interplay of several stable PBEs. We will refer to such an equilibrium as an *transitional equilibrium*. To give a precise definition we make use of the following notation. Consider a dynamic system $(p(t), \lambda(t))$ with $p \in \Delta$, where Δ denotes the $(n - 1)$ -simplex and $\lambda \in [0, 1]$.

Definition *The triple $(\lambda^*, P_l(\lambda), P_r(\lambda))$ of a scalar λ^* and two distinct equilibrium sets of the fast variable, $P_l(\lambda)$ and $P_r(\lambda)$, is called a transitional equilibrium if there exists an $\epsilon > 0$ such that*

1. *after the system has come close enough to $\lambda^*(t)$ and one of the equilibrium sets at some $t = t_o$, $\lambda(t)$ fluctuates in the interval $(\lambda^* - \epsilon, \lambda^* + \epsilon)$ and $p(t)$ will alternate between the regions of attraction of the two fast-variable equilibria, $P_l(\lambda)$ and $P_r(\lambda)$, for all $t > t_o$ except for a countable number of t_n at which $\lambda(t_n) = \lambda^*$ and $p(t_n)$ is not in the region of attraction of either of the equilibria $P_l(\lambda)$ and $P_r(\lambda)$ if $P_l(\lambda^*)$ and $P_r(\lambda^*)$ do not exist, and*
2. *ϵ may become smaller and eventually approach zero when $p(t)$ becomes faster relative to $\lambda(t)$.*

In other words, λ^* , P_l , and P_r constitute a transitional equilibrium if the dynamic system is always attracted by an equilibrium in the fast variable at which $\text{sign}(\dot{\lambda}) = \text{sign}(\lambda^* - \lambda)$ so that λ is driven from the support of the currently attracting fast-variable equilibrium into the support of the other fast-variable equilibrium. In our case, we may have such an equilibrium only at $\lambda = \frac{k_L}{1+\alpha}$ where two equilibria interplay: the cooperative separating equilibrium and the cooperative high pooling equilibrium (see Figure 3). Note that at $\lambda = \frac{k_L}{1+\alpha}$ a semi-pooling cooperative equilibrium exists.

Proposition 3 states the conditions under which such an stable inner equilibrium may exist as well as its properties.

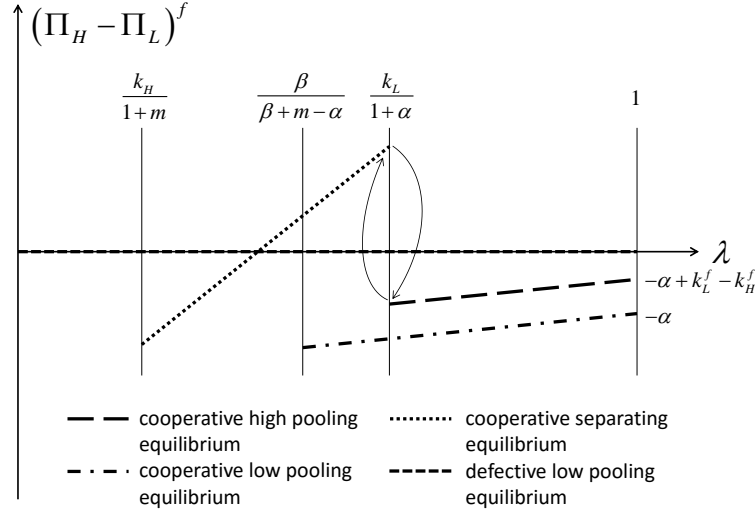


Figure 3: Differences in material payoffs under the conditions of Proposition 3 for $k_L^f > k_H^f$.

Proposition 3 If (1) $\frac{k_L}{1+\alpha} > \max\{\frac{\beta}{\beta+m-\alpha}, k_H^f\}$ and

(2a) $\beta + k_H^f(\alpha - \beta) < k_L^f - k_H^f < \beta + \frac{k_L}{1+\alpha}(\alpha - \beta)$, there is a stable semi-pooling equilibrium at $\lambda^* = \frac{k_L}{1+\alpha}$ with $p_{CDs,s}^* = 1 - p_{CDs,ns}^* = \frac{\lambda^* - k_H^f}{\lambda^* - k_L^f + \lambda^* \alpha + (1 - \lambda^*) \beta}$;

(2b) $k_L^f - k_H^f \leq \beta + k_H^f(\alpha - \beta)$, there is a transitional equilibrium at $\lambda^* = \frac{k_L}{1+\alpha}$, $P_l(\lambda) = \{p | p_{CDs,ns} = 1\}$ (CSE), and $P_r(\lambda) = \{p | p_{CDs,s} = 1 - p_{CCs,s} \geq \frac{k_L}{(1+\alpha)\lambda}\}$ (CHPE).

Before we have a closer look at the conditions of Proposition 3, the following Corollary characterizes the transitional equilibrium in terms of type-contingent behavior and signaling.

Corollary 1 In the transitional equilibrium of Proposition 3 (1) high-type individuals cooperate among each other but also with those low-type individuals who signal to be of the high type, (2) the proportion of low-type individuals who signal to be of the high type fluctuates, and (3) the share of high-type individuals is proportional to the material signaling cost for low types and fluctuates slightly.

Note that the conditions in Proposition 2 and Proposition 3 are mutually exclusive, i.e. there is at most one stable inner equilibrium. The conditions for the transitional equilibrium (see (2b) in Proposition 3) appear less demanding than those in Proposition 2 and those for the semi-pooling equilibrium (see (2a) in Proposition 3), because they do not state a lower bound for the difference between material signaling cost across types, $k_H^f - k_L^f$.

Indeed, quite surprisingly the transitional equilibrium is consistent with high types bearing higher signaling cost in terms of fitness than low types (see discussion for details). Thus, contrary to previous results in the literature, to sustain cooperation in a transitional equilibrium does not hinge on the assumption of some cost advantage for cooperative types. This is because, the material gain in the CSE for high types is determined by their own frequency in the transitional equilibrium $\frac{k_L}{1+\alpha}$, which is independent of k_H and k_H^f . The conditions in Proposition 3 reveal the importance of the strength of the cooperative preference, measured by m also for the existence of a transitional equilibrium. More precisely, the higher m the more likely a transitional equilibrium exists. Furthermore, if the material cost of signaling for both types are small then likely inequalities (1) and (2b) in Proposition 3 will be satisfied. The following corollary summarizes these insights.

Corollary 2 *(1) If the cooperative preference is sufficiently strong and material signaling cost are sufficiently low then a transitional equilibrium exists. (2) If the cooperative preference is sufficiently strong, and the material signaling cost are sufficiently low for high types and do not differ to much across types, then a transitional equilibrium exists.*

These rather mild conditions, in particular if compared to those of Proposition 2, and the property of heterogeneity in all dimensions, i.e., behavior, preferences, and signaling, underline the appeal of the transitional equilibrium. Interestingly, since the long run equilibrium may be characterized by limit cycles, a transitional equilibrium also offers an potential explanation for fluctuations in signaling behavior and therefor the degree of cooperation, without referring to some kind of stochastic shocks.

6 Discussion

Since we place our analysis of the emergence of cooperation in social dilemmas in an environment which can not rely on direct or indirect reciprocity, communication and the implied potential for cooperators to recognize each other are necessary for cooperation to evolve. Thus, we will focus on discussing the nature of signaling cost and their relation across types.¹⁵ We end this section with a brief comment on the endogeneity of the strength of the cooperative preference.

¹⁵The literature also discusses alternative modes of communication: There are models (e.g. Güth, 1995 ; Sethi, 1996) which assume that cooperators can simply recognize each other. There is, however, mixed evidence to what extend humans can unveil incomplete information about cooperative preferences (see Frank et al., 1993; Ockenfels and Selten, 2000; Brosig, 2002). Other models make use of an unsubverted signal like in Arthur Robson's 'secret handshake' model (Robson, 1990). This type of models are prone to what Ken Binmore calls the 'transparent disposition fallacy' (Binmore, 1994).

Let us consider the relation of signaling cost first. The transitional equilibrium is least restrictive regarding the difference in material signaling cost, it even allows for higher fitness cost for high types. Thus, with respect to the relation of signaling cost across types only the assumption $k_L > k_H$ might be considered to be a restriction. If we look at the conditions for existence and the payoff differences in Table 1 of the involved equilibria, we observe that this condition is not needed. That is, the existence of the transitional equilibrium seems to be consistent with both $k_L < k_H$, and $k_L^f < k_H^f$, which we find quite striking. However, to prove the existence of a transitional equilibrium we have to keep track of all stable equilibria at $\frac{k_L}{1+\alpha}$ and their basins of attraction. Since we made use of the assumption $k_L > k_H$ when deriving the set of stable separating, semi-pooling, and pooling equilibria, we can only conjecture, that our results carry over to the case of $k_L < k_H$. Since there are 19 semi-pooling equilibria to be considered in terms of existence and stability we leave this for future research. However, we will try to give the intuition behind this surprising property.

In standard application of signaling theory it is necessary for a separating equilibrium to exist, that types with higher quality bear lower signaling cost. This is not true in our signaling-extended PD. To see why, consider first the case where material cost of signaling and cost in utility terms would coincide. Intuitively, for a separating equilibrium to exist incentive compatibility has to be ensured. In this respect, for high types the cost-benefit ratio of signaling is decisive, which is influenced by the strength of the cooperative preference m . The support of the CSE is given by $\frac{k_H}{1+m} \leq \lambda \leq \frac{k_L}{1+\alpha}$. Thus, the divergence between fitness and utility in the PD governed by m allows signaling cost for low types to fall short of those for high types in a separating equilibrium. However, if $k_L < k_H$, high types would face an evolutionary disadvantage even in the separating equilibrium (see $(\Pi_H - \Pi_L)^f$ for the CSE in Table 1). Hence, for an transitional equilibrium to exist, it is also required that $k_L > k_H$. If we, however, distinguish between material and non-material signaling cost as for the payoffs in the PD, then this implication is not true. This is because, the fitness payoff in the CSE for a high type depends on two things, the frequency of interaction with another high type in the transitional equilibrium $\frac{k_L}{1+\alpha}$, and the material signaling cost k_H^f . The former does not depend on material signaling cost, the latter is independent of the non-material signaling cost. That a distinction between material and non-material signaling cost is highly reasonable will become apparent when we turn to the question regarding the nature of such a signaling device next.

Regarding the nature of a signaling device we will give two illustrative examples. A signal which has no or negligible material cost might consist in sending a smile or some

other positive gesture, or a brief chat at the beginning of a pairwise encounter. According to Frank (1988), cooperators are endowed with an advanced emotional system. This system not only provides the motivation for the cooperative behavior, but also enables them to signal their cooperative attitude.¹⁶ Thus, if it is at all possible for opportunists to send the signal, they would have much higher non-material signaling cost. This would warrant the assumption of $k_L > k_H$.

To give an illustrative example for material signaling cost, consider a situation where individuals elbow their way through a rummage sale. There is a table with one good offered as two variants, goods A and B. There are also two individuals, one preferring good A, the other preferring good B. However getting both goods is the first best outcome for both individuals. They can behave cooperatively, allowing the other to select their preferred good; or they can try to queue-jump and grab both goods, in which case, the other gets none. If both individuals chose not to cooperate, they will grab one of the goods by chance, leaving them in expectation with a lower utility than in the cooperative state. Hence, this example is structurally equivalent to a PD. In this scenario, the signal often used is to make room for the other person. Such a signal is costly in terms of time, which usually has some monetary equivalent. If this gesture is received by both individuals, this might lead to mutual cooperation. Similar to this example, many acts of courtesy may indeed be understood as a signal for a cooperative attitude. Very often, such acts imply foregoing some advantages for the benefit of others. In this example it is *a priori* not clear which type bears higher opportunity cost, leaving the relation of k_L^f and k_H^f ambiguous.

More general, our model can capture any kind of costly behavior prior to the PD, which is socially accepted as the appropriate signaling device. The selection of any particular device appears to be a problem of coordination and is beyond the scope of this paper. However, apparently such devices are used.

Finally, in our model the size of the parameter m measuring the strength of the cooperative preference is not driven by evolutionary forces, since no fitness payoff difference depends on it. However, the size of the parameter does determine the range in which cooperative equilibria exist. Hence, if two separate populations with different levels of m are considered, the one with the higher value is more likely to evolve towards a cooperative state. Thus, the population with the stronger cooperative preference would have

¹⁶In a laboratory experiment Brosig (2002) finds that cooperative individuals are somewhat better at predicting their partner's decisions in one-shot prisoner's dilemma games than are the individualistic ones. This, of course, is also consistent with a better ability to signal. Scharleman et al. (2001), and Eckel and Wilson (2003), for example, explored the reaction of individuals to seeing the faces with whom they were supposedly interacting. Their results support the potential of smiles as a mechanism to allow subjects to read the intentions of others.

an evolutionary edge over the other. Further, if in the course of time, both populations start interacting with each other, a cooperative population might induce cooperation in a defective population and vice versa. To analyze such an environment may be relevant for studying migrational effects on cooperation.

7 Conclusion

This paper tries to shed light on three persistent patterns attributed to cooperative behavior in social dilemmas: (1) heterogeneity in preferences (coexistence of opportunists and conditional cooperators); (2) heterogeneity in behavior (presence of cooperation and defection); and (3) heterogeneity in communication. We study an evolutionary model where individuals are able to signal their preference for joint cooperation before engaging in a one shot Prisoners Dilemma. We derive the full set of Bayesian equilibria in the signaling-extended Prisoner's Dilemma and study their dynamic stability. This exhaustive search puts us in the position to study the transition across different equilibria.

The main insight of our paper is that under mild conditions a new type of evolutionary equilibrium (transitional equilibrium) exists, which is based on the dynamic interplay of separating, semi-pooling, and pooling equilibria. Only this type of equilibrium can stabilize a population state characterized by heterogeneity with respect to all three dimensions: preferences, behavior, and signaling. More precisely, in the transitional equilibrium conditional cooperators cooperate among each other but also with those opportunists who signal to be a cooperator, and the proportion of opportunists who signal to be a cooperator may fluctuate. The transitional equilibrium also provides an explanation for cycles in behavior, preferences and signaling in equilibrium which do not require mutational forces or individual errors in terms of execution of strategies, or perception of cooperative signals. Importantly and quite surprisingly, the transitional equilibrium is consistent with conditional cooperators bearing higher signaling cost in terms of fitness than opportunists. For a transitional equilibrium to exist it suffices that the cooperative preference is sufficiently strong and the material signaling cost for both types of individuals are sufficiently low. As a subsidiary results this solves the problem regarding the instability of separating equilibria on the population level, immanent to costly signaling models of cooperation.

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A Proofs

Proof of Proposition 2. Stability requires a negative slope of the fitness difference function, i.e., $\alpha - \beta > 0$. Let us first consider $\frac{k_L}{1+\alpha} \leq \frac{\beta}{\beta+m-\alpha}$. In this case, the within-support condition amounts to $\frac{\beta}{\beta+m-\alpha} < 1 - \frac{\alpha-(k_L^f-k_H^f)}{\alpha-\beta} < 1$, rearranging yields $\frac{\beta}{\beta+m-\alpha}m < k_L^f - k_H^f < \alpha$. If on the other hand $\frac{k_L}{1+\alpha} > \frac{\beta}{\beta+m-\alpha}$, the within-support condition amounts to $\frac{k_L}{1+\alpha} < 1 - \frac{\alpha-(k_L^f-k_H^f)}{\alpha-\beta} < 1$, rearranging yields $\beta + \frac{k_L}{1+\alpha}(\alpha - \beta) < k_L^f - k_H^f < \alpha$. Note that the first pair of inequalities implies that $\alpha - \beta > 0$, because $\frac{\beta}{\beta+m-\alpha}m < \alpha \iff m(\beta - \alpha) < \alpha(\beta - \alpha) \iff \beta - \alpha < 0$. Thus, the two pairs of inequalities are necessary and sufficient. \square

Proof of Proposition 3. In the proof of proposition 3 we will recurrently make use of the following

Definition
$$\underline{p_{CDs}} \equiv \max \left\{ \frac{k_H(1+\alpha)}{k_L(1+m+\beta)}, \frac{k_H}{k_L} - \left(\frac{m-\alpha+\beta}{1+\alpha} - \frac{\beta}{k_L} \right), \frac{1}{2} \right\}$$

Note that our simplifying assumption $k_H < k_L$ is a sufficient but not a necessary condition for $\underline{p_{CDs}} < 1$.

We observe that for $p_{CDs} = 1 - p_{CCs} > \underline{p_{CDs}}$ and $\lambda(t) = \lambda^* = \frac{k_L}{1+\alpha}$ we have:

$$\begin{aligned} \Pi_H(CDs) &= \frac{k_L}{1+\alpha}(1+m) - (1 - \frac{k_L}{1+\alpha})p_s\beta - k_H \\ &\geq \frac{k_L}{1+\alpha}(1+m) - (1 - \frac{k_L}{1+\alpha})\beta - k_H = \Pi_H(CCs) \end{aligned} \tag{A.1}$$

and

$$\begin{aligned} \Pi_H(CDs) &= \frac{k_L}{1+\alpha}(1+m) - (1 - \frac{k_L}{1+\alpha})p_s\beta - k_H \\ &> \max_x \Pi_H(x), \quad x \in \{CCns, CDns, DCns, DDns, DCs, DDs\}, \end{aligned} \tag{A.2}$$

where the first inequality is strict if $p_s < 1$ and the second inequality follows from $\frac{k_L}{1+\alpha} >$

$\frac{\beta}{\beta+m-\alpha}$, because for $p_{CDs} = 1 - p_{CCs} > \underline{p_{CDs}}$:

$$\begin{aligned}
\Pi_H(DDs) &= \frac{k_L}{1+\alpha}(1+\alpha) - k_H = \frac{k_L(1+m)}{1+\alpha} - \left(\frac{k_L(m-\alpha+\beta)}{\beta(1+\alpha)} - \frac{k_L}{1+\alpha} \right) \beta - k_H < \Pi_H(CCs) \\
\Pi_H(DCs) &= \frac{k_L}{1+\alpha}(1+\alpha) - (1 - \frac{k_L}{1+\alpha})(1-p_s)\beta - k_H < \Pi_H(DDs) \\
\Pi_H(CCs) &= \frac{k_L}{1+\alpha}((1-p_{CDs})(1+m) - p_{CDs}\beta) - (1 - \frac{k_L}{1+\alpha})\beta \\
&< \frac{k_L}{1+\alpha}(1+m) - (1 - \frac{k_L}{1+\alpha})\beta - \frac{k_L}{1+\alpha}(1+m+\beta) \frac{k_H(1+\alpha)}{k_L(1+m+\beta)} = \Pi_H(CCs) \\
\Pi_H(CDns) &= \frac{k_L}{1+\alpha}((1-p_{CDs})(1+m) - p_{CDs}\beta) - (1 - \frac{k_L}{1+\alpha})p_s\beta \\
&< \frac{k_L}{1+\alpha}(1+m) - (1 - \frac{k_L}{1+\alpha})p_s\beta - \frac{k_L}{1+\alpha}(1+m+\beta) \frac{k_H(1+\alpha)}{k_L(1+m+\beta)} = \Pi_H(CDs) \\
\Pi_H(DDns) &= \frac{k_L}{1+\alpha}(1-p_{CDs})(1+\alpha) < k_L \left(1 - \left(\frac{k_H}{k_L} - \left(\frac{m-\alpha+\beta}{1+\alpha} - \frac{\beta}{k_L} \right) \right) \right) = \Pi_H(CCs) \\
\Pi_H(DCns) &= \frac{k_L}{1+\alpha}(1-p_{CDs})(1+\alpha) - (1 - \frac{k_L}{1+\alpha})(1-p_s)\beta < \Pi_H(DDns)
\end{aligned}$$

Hence, continuity of the payoffs and Lipschitz-continuity of the dynamics implies that there exists some $\epsilon \in (0, \lambda^* - k_H^f)$ and some $\mu \in (0, 1 - \underline{p_{CDs}})$ such that for all $\lambda \in (\lambda^* - \epsilon, \lambda^* + \epsilon)$ and all $p_{CDs} > 1 - \mu$ we have $\dot{p}_{CDs} + \dot{p}_{CCs} > 0$ and, due to $p_{CDs} > p_{CCs}$, payoff monotonicity also implies $\dot{p}_{CDs} > 0$. Hence, in every sufficiently small neighborhood of λ^* , p_{CDs} will always grow if it has surpassed some threshold level.

This allows us to prove the proposition under condition (3a): Inserting λ^* , $p_{CDs,s}^*$, and $p_{CDs,ns}^*$ into the second line of Table 1 yields $(\Pi_H - \Pi_L)^f = 0$ and thus $\dot{\lambda} = 0$. Since $p_{CDs,s}^* + p_{CDs,ns}^* = 1$ and thus $\dot{p}_{CDs,s}^* + \dot{p}_{CDs,ns}^* = 0$, the equation $\Pi_L(s) = \lambda^*(1+\alpha) - k_L = 0 = \Pi_L(ns)$ implies $\dot{p} = 0$. λ^* , $p_{CDs,s}^*$, $p_{CDs,ns}^*$ is thus a fix point. To prove stability, we recall that $p_{CDs} = 1$ excludes that any other strategy enters the dynamics. We can thus concentrate our analysis to the variables $\lambda(t)$ and $p_s(t) = p_{CDs,s}(t)$. We rewrite this dynamic system as

$$\begin{aligned}
\dot{\lambda} &= \tilde{h}((\Pi_H - \Pi_L)^f) = \tilde{h}\left((\lambda - k_H^f)(1-p_s) + (k_L^f - k_H^f - (\lambda\alpha + (1-\lambda)\beta))\right), \quad \tilde{h}' > 0 \\
\dot{p}_s &= \tilde{g}(\Pi_L(s) - \Pi_L(ns)) = \tilde{g}(\lambda(1+\alpha) - k_L), \quad \tilde{g}' > 0
\end{aligned}$$

Linearizing the system at the fix point and noting that $\frac{\partial \dot{p}_s}{\partial p_s} = 0$ yields

$$\delta_{1,2} = \frac{1}{2} \frac{\partial \dot{\lambda}}{\partial \lambda} \pm \sqrt{\frac{1}{2} \left(\frac{\partial \dot{\lambda}}{\partial \lambda} \right)^2 + \frac{\partial \dot{p}_s}{\partial \lambda} \frac{\partial \dot{\lambda}}{\partial p_s}} \quad (\text{A.3})$$

as eigenvalues of the characteristic matrix. Since we can write $\frac{\partial \dot{p}_s}{\partial \lambda} \frac{\partial \dot{\lambda}}{\partial p_s} = -(1+\alpha)\tilde{g}'(\cdot) \frac{\lambda - k_H^f}{p_s} \tilde{h}(\cdot) < 0$ and due to the larger velocity of p_s , $\tilde{g}'(\cdot)$ is large enough to turn the term in the

root negative. Thus the sign of the real parts of the eigenvalues is given by the sign of $\frac{\partial \dot{\lambda}}{\partial \lambda} = (1 - p_{CDs,s}^*(1 + \alpha - \beta))\tilde{h}'$ which can easily be shown to be negative if and only if condition (3a) is satisfied. Thus, the fix point is stable if and only if this condition is satisfied.

We now turn to the case of condition (3b), i.e. we assume that the fix point is not stable.

First assume that the system has reached such a state in which $\lambda \in (\lambda^* - \epsilon, \lambda^* + \epsilon)$ and $p_{CDs} > 1 - \mu$ and that $\lambda(t) < \lambda^*$. Then $\Pi_L(s) = \lambda(1 - \alpha)(p_{CCs} + p_{CDs} + p_{CCns} + p_{CDns}) - k_L \approx \lambda(1 - \alpha) - k_L$ will become smaller than $\Pi_L(ns) = (1 + \alpha)(p_{CCs} + p_{DCs} + p_{CCns} + p_{DCns}) \approx 0$ because of the growth of p_{CDs} . Thus, p_s will decline towards zero before $\lambda(t)$ has declined to $\lambda^* - \epsilon$ if the dynamics of p is fast enough relative to the dynamics of λ . As a consequence, $p_{CDs,ns} \rightarrow 1$, i.e. the system is attracted by $P_l(\lambda)$. The fitness payoff difference $(\Pi_H - \Pi_L)^f$ will thus approach $\lambda(t) - k_H^f > 0$ so that $\dot{\lambda}$ will eventually become positive before $\lambda(t) \leq \lambda^* - \epsilon$.

Next, assume that the system has reached a state in which $\lambda \in (\lambda^* - \epsilon, \lambda^* + \epsilon)$ and $p_{CDs} > 1 - \mu$, but, contrary to the previous paragraph, $\lambda(t) > \lambda^*$. Then $\Pi_L(s) = \lambda(1 - \alpha)(p_{CCs} + p_{CDs} + p_{CCns} + p_{CDns}) - k_L \approx \lambda(1 - \alpha) - k_L$ will become larger than $\Pi_L(ns) = (1 + \alpha)(p_{CCs} + p_{DCs} + p_{CCns} + p_{DCns}) \approx 0$ and thus p_s will grow towards one before $\lambda(t)$ has grown to $\lambda^* + \epsilon$ if the dynamics of p is fast enough. As a consequence, $p_{CDs,s} \rightarrow 1$, i.e. the system is attracted by an element of $P_r(\lambda)$. The fitness payoff difference $(\Pi_H - \Pi_L)^f$ will thus approach $k_L^f - k_H^f - (\lambda(t)\alpha + (1 - \lambda(t))\beta) < 0$ so that $\dot{\lambda}$ will eventually become negative before $\lambda(t) \geq \lambda^* + \epsilon$.

Finally, assume that the system has reached a state in which $\lambda = \lambda^*$ and $p_{CDs} > 1 - \mu$. Then either $\dot{\lambda} \neq 0$ so that the system will be driven into one of the states discussed in the two previous paragraphs or $\dot{\lambda} = 0$. But then the fitness payoff difference and thus $\dot{\lambda}$ will quickly become either positive or negative since $\dot{p}_{CDs} > 0$ still holds true. The only exception is the fix point we discussed earlier. Due to condition (3b) it is not stable. As a consequence, the slightest disturbance will drive the system away from the fix point and into the fluctuation described in the two previous paragraphs.

At λ^* , $P_l(\lambda)$, and $P_r(\lambda)$ the system thus satisfies part 1 of the definition of a transitional equilibrium. Part 2 is obvious from the line of argument in the proof of part 1. \square

B Separating and Pooling Equilibria - Existence and Stability

In this section we derive the separating and the pooling equilibria of the signaling-extended PD for a given share of high types $\lambda \in (0, 1)$. Further, we will consider the stability properties of these equilibria. We apply the notion of asymptotical stability as a stability concept. An equilibrium point is a fix point \vec{p}_f of the dynamical system $\dot{p}(t) = F(\vec{p}_f(t))$ and is said to be asymptotically stable if it meets two conditions. First it needs to be Lyapunow-stable, i.e. $\forall \epsilon > 0, \exists \delta > 0 : \|\vec{p}(0) - \vec{p}_f\| < \delta \Rightarrow \|\vec{p}(t) - \vec{p}_f\| < \epsilon, \forall t \geq 0, \forall \vec{p}(t)$ being a trajectory, second it needs to be an attractor, i.e. $\exists \delta > 0 : \text{any trajectory } \vec{p}(t) \text{ with } \|\vec{p}(0) - \vec{p}_f\| < \delta \text{ then } \|\vec{p}(t) - \vec{p}_f\| \xrightarrow{t \rightarrow \infty} 0$. The definitions for an equilibrium set are accordingly (see e.g., Samuelson, 1997).

To proof stability or instability of an equilibrium we will rely on phase diagrams. We will proof instability by arguing that the system cannot be Lyapunow-stable. In case of an equilibrium point in the interior of the support of the equilibrium the involved strategies earn strictly higher payoffs then non-equilibrium strategies. Small perturbation will not alter this property. Payoff monotone dynamics will decrease the share of the non-equilibrium strategies. Hence for analyzing the stability properties in that case it suffices to consider the involved equilibrium strategies and whether the dynamics will reestablish the equilibrium values given a small perturbation. At the boundaries of the support of an equilibrium point a non-equilibrium strategy will earn the same profits as the equilibrium strategies. In that case these strategies needs to be included in the analysis. However with respect to all other strategies the previous argument still applies.

Note that the expected payoff for each type-contingent strategy is additively separable in the payoffs for the two types. We will make use of this property when discussing the stability of equilibria. That is, to proof the (in)stability of a certain equilibrium (set) we will consider contingent-wise changes of behavior. For this purpose, we introduce the following notation. Consider the strategy CDs, ns , then p_{CDs} denotes the probability to play CDs , conditional on being a high type, and p_{ns} the probability not to signal, conditional on being a low type. The definition applies accordingly for any other strategy. Further, we will write expected payoff as linear combination of type-specific payoffs $\Pi_{CDs, ns} = \lambda \Pi_H(CDs) + (1 - \lambda) \Pi_L(ns)$. Finally, in the phase diagrams thick solid lines or points correspond to equilibrium sets or points, respectively. Iso-profit lines are depicted by thick dotted lines.

B.1 Separating Equilibria

B.1.1 High types signal, low types do not signal

Existence

That is, we look for an equilibrium, such that $p_{CCs,ns} + p_{CDs,ns} + p_{DCs,ns} + p_{DDs,ns} = 1$.

$$\Pi_{CCs,ns} = \lambda[\lambda(p_{CCs,ns} + p_{CDs,ns})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.1)$$

$$\Pi_{CDs,ns} = \lambda[\lambda[(p_{CCs,ns} + p_{CDs,ns})(1 + m) - (p_{DCs,ns} + p_{DDs,ns})\beta] - k_H] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.2)$$

$$\Pi_{DCs,ns} = \lambda[\lambda(p_{CCs,ns} + p_{CDs,ns})(1 + \alpha) - (1 - \lambda)\beta - k_H] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.3)$$

$$\Pi_{DDs,ns} = \lambda[\lambda(p_{CCs,ns} + p_{CDs,ns})(1 + \alpha) - k_H] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.4)$$

$$\Pi_{CCns,ns} = \lambda[\lambda[(p_{CCs,ns} + p_{DCs,ns})(1 + m + \beta)] - \beta] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.5)$$

$$\Pi_{CDns,ns} = \lambda[\lambda[(p_{CCs,ns} + p_{DCs,ns})(1 + m) - (p_{CDs,ns} + p_{DDs,ns})\beta]] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.6)$$

$$\Pi_{DCns,ns} = \lambda[\lambda[(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] - (1 - \lambda)\beta] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.7)$$

$$\Pi_{DDns,ns} = \lambda[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] + (1 - \lambda)[\lambda(p_{CCs,ns} + p_{DCs,ns})(1 + \alpha)] \quad (B.8)$$

Note that for $\lambda \in (0, 1)$ and $p_{CCs,ns}, p_{CDs,ns}, p_{DCs,ns}, p_{DDs,ns} > 0$, it follows that $\Pi_{CCs,ns} < \Pi_{CDs,ns}$, $\Pi_{DCs,ns} < \Pi_{DDs,ns}$, $\Pi_{CCns,ns} < \Pi_{CDns,ns}$, and $\Pi_{DCs,ns} < \Pi_{DDs,ns}$. After deletion of these strictly dominated strategies, payoffs of the remaining strategies are given by:

$$\Pi_{CDs,ns} = \lambda[\lambda[p_{CDs,ns}(1 + m) - p_{DDs,ns}\beta] - k_H] , \quad \Pi_{DDs,ns} = \lambda[\lambda(p_{CDs,ns})(1 + \alpha) - k_H] \quad (B.9)$$

$$\Pi_{CDns,ns} = \lambda[-(p_{CDs,ns} + p_{DDs,ns})\beta] , \quad \Pi_{DDns,ns} = 0 \quad (B.10)$$

For a separating equilibrium where high types send the signal and low types don't, only two not dominated strategies are left, CDs,ns and DDs,ns , i.e. $p_{CDs,ns} + p_{DDs,ns} = 1$. Thus, $CDns,ns$ would earn strictly less than $DDns,ns$ (see B.10).

1. Let us first analyze the case $p_{CDs,ns} = 1$. In that case the following three conditions are necessary and sufficient for this to constitute an equilibrium:

- (i) $\Pi_{CDs,ns} > \Pi_{DDs,ns}$, which is always satisfied, because of $m > \alpha$.
- (ii) $\Pi_{CDs,ns} \geq \Pi_{DDns,ns} \Leftrightarrow \lambda \geq \frac{k_H}{1+m}$.
- (iii) $\Pi_{CDs,ns} \geq \Pi_{CDs,s} \Leftrightarrow \lambda \leq \frac{k_L}{1+\alpha}$.

Thus, the three conditions are equivalent to $\frac{k_H}{1+m} \leq \lambda \leq \frac{k_L}{1+\alpha}$. Note that since $k_L > k_H$, and $m > \alpha$ the λ -support for this equilibrium is not empty.

2. Let us now analyze the case $p_{DDs,ns} = 1$. In that case $DDns,ns$ would earn strictly higher payoffs, hence such an equilibrium cannot exist.
3. Finally, let us consider a mixed equilibrium, i.e. $p_{CDs,ns} + p_{DDs,ns} = 1$. In that case the following three conditions are necessary and sufficient for this to constitute an equilibrium:

- (i) $\Pi_{CDs,ns} = \Pi_{DDs,ns} \Leftrightarrow p_{CDs,ns} = \frac{\beta}{\beta+m-\alpha}$.
- (ii) $\Pi_{DDs,ns} \geq \Pi_{DDns,ns} \Leftrightarrow p_{CDs,ns} \geq \frac{\lambda\beta+k_H}{\lambda(1+m+\beta)}$.
- (iii) $\Pi_{CDs,ns} \geq \Pi_{CDs,s} \Leftrightarrow p_{CDs,ns} \leq \frac{k_L}{\lambda(1+\alpha)}$.

At $p_{CDs,ns} = \frac{\beta}{\beta+m-\alpha}$ the last two conditions are equivalent to $\frac{\beta+m-\alpha}{\beta(1+\alpha)}k_H \leq \lambda \leq \frac{\beta+m-\alpha}{\beta(1+\alpha)}k_L$.

Stability

1. Let us first analyze the stability of $p_{CDs,ns} = 1$. This equilibrium is certainly stable in the interior range $\frac{k_H}{1+m} < \lambda < \frac{k_L}{1+\alpha}$ since all payoff inequalities hold strictly. At the upper boundary $\lambda = \frac{k_L}{1+\alpha}$, the strategies CDs,ns and CDs,s earn the same profits, i.e. low types are indifferent between signaling and not sending the signal. Consider a small perturbation such that CDs,s is played with a small positive probability. To reestablish $p_{CDs,ns} = 1$, the share of high types playing

CDs must decrease, because $\Pi_L(s) - \Pi_L(ns) = \lambda p_{CDs}(1 + \alpha) - k_L$. However, for small perturbations CDs is still dominant for high types. Hence, CDs persists as part of the equilibrium strategy and there is no force reestablishing the non-signaling contingency for low types. Thus, the separating equilibrium is not stable at the upper bound. A similar argument establishes, that it is also not stable at the lower bound. At the lower bound $\frac{k_H}{1+m}$, the strategies CDs, ns and $DDns, ns$ earn the same profits, i.e. high types are indifferent between cooperating and incurring the cost of the signal on the one hand, and defecting and no signaling on the other. Consider a random drift, such that $p_{DDns, ns} > 0$, this will lower profits for CDs, ns and leaves profits for $DDns, ns$ unchanged. Hence, the equilibrium will not be restored. In other words, this equilibrium is not stable at $\lambda = \frac{k_H}{1+m}$.

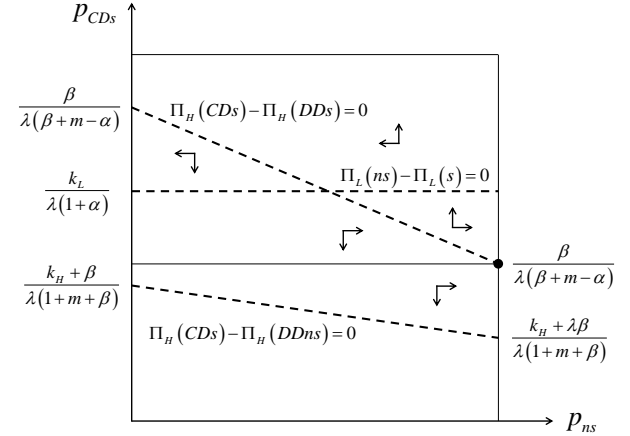
2. Let us next consider the equilibrium with $p_{CDs, ns} = \frac{\beta}{\beta+m-\alpha}$, $p_{DDs, ns} = \frac{m-\alpha}{\beta+m-\alpha}$.

Given the following differences in type-specific payoffs:

$$\Pi_H(CDs) - \Pi_H(DDs) \geq 0 \Leftrightarrow p_{CDs} \geq \frac{\beta - (1 - \lambda)\beta p_{ns}}{\lambda(m - \alpha - \beta)}$$

$$\Pi_L(ns) - \Pi_L(s) \geq 0 \Leftrightarrow p_{CDs} \leq \frac{k_L}{\lambda(1 + \alpha)}$$

, we obtain the following phase diagram. Note that for the support of that equilibrium $\frac{k_L}{\lambda(1+\alpha)} > \frac{k_H + \lambda\beta - (1-\lambda)\beta p_{ns}}{\lambda(m-\alpha-\beta)}$ holds. Note further that the upper bound of the support $\lambda \leq \frac{m-\alpha+\beta}{\beta} \frac{k_L}{1+\alpha}$ implies $\frac{\beta}{m-\alpha+\beta} \leq \frac{k_L}{\lambda(1+\alpha)}$. Additionally, $\frac{k_H + \lambda\beta}{\lambda(1+m+\beta)} \leq \frac{\beta}{m-\alpha+\beta} \Leftrightarrow \frac{k_H}{\lambda(1+\alpha)} \leq \frac{\beta}{m-\alpha+\beta}$. As the diagram clearly indicates, this equilibrium is unstable for all λ in the support.



B.1.2 High types do not signal, low types signal

Existence

That is, we look for an equilibrium, such that $p_{CCns,s} + p_{CDns,s} + p_{DCns,s} + p_{DDns,s} = 1$. Let us again first study the signaling contingency for high types.

$$\Pi_{CCs,s} = \lambda[\lambda(p_{CCns,s} + p_{CDns,s})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.11})$$

$$\Pi_{CDs,s} = \lambda[\lambda[(p_{CCns,s} + p_{CDns,s})(1 + \alpha)] - (1 - \lambda)\beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.12})$$

$$\Pi_{DCs,s} = \lambda[\lambda[(p_{CCns,s} + p_{CDns,s})(1 + m + \beta) - \beta] - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.13})$$

$$\Pi_{DDs,s} = \lambda[\lambda[(p_{CCns,s} + p_{CDns,s})(1 + \alpha)] - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.14})$$

$$\Pi_{CCns,s} = \lambda[\lambda(p_{CCns,s} + p_{DCns,s})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.15})$$

$$\Xi \quad \Pi_{CDns,s} = \lambda[\lambda[(p_{CCns,s} + p_{DCns,s})(1 + \alpha)] - (1 - \lambda)\beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.16})$$

$$\Pi_{DCns,s} = \lambda[\lambda[(p_{CCns,s} + p_{DCns,s})(1 + m + \beta) - \beta] - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.17})$$

$$\Pi_{DDns,s} = \lambda[\lambda[(p_{CCns,s} + p_{DCns,s})(1 + \alpha)] - k_H] + (1 - \lambda)[\lambda(p_{CCns,s} + p_{CDns,s})(1 + \alpha) - k_L] \quad (\text{B.18})$$

Note that for $\lambda \in (0, 1)$ and $p_{CCns,s}, p_{CDns,s}, p_{DCns,s}, p_{DDns,s} > 0$, it follows that $\Pi_{CCs,s} < \Pi_{DCs,s}$, $\Pi_{CDs,s} < \Pi_{DDs,s}$, $\Pi_{CCns,s} < \Pi_{DCns,s}$, and $\Pi_{CDns,s} < \Pi_{DDns,s}$. After deletion of these strictly dominated strategies, payoffs for low types to signal is $-k_L$, whereas signaling yields an expected payoff of $\lambda p_{DCns,s}(1 + \alpha)$. Thus, strategies that imply no signaling for low types generate strictly higher payoffs. Hence, such an equilibrium cannot exist.

B.2 Pooling Equilibria

B.2.1 High types and low types do not signal

Existence

That is, we look for an equilibrium, such that $p_{CCns,ns} + p_{CDns,ns} + p_{DCns,ns} + p_{DDns,ns} = 1$. Let us again first study the signaling contingency for high types.

$$\Pi_{CCs,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)]] \quad (\text{B.19})$$

$$\Pi_{CDs,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + \alpha)] - k_H] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)] \quad (\text{B.20})$$

$$\Pi_{DCs,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)]] \quad (\text{B.21})$$

$$\Pi_{DDs,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + \alpha)] - k_H] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)] \quad (\text{B.22})$$

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$$\Pi_{CCns,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + m + \beta) - \beta] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)]] \quad (\text{B.23})$$

$$\Pi_{CDns,ns} = \lambda[\lambda(p_{CCns,ns} + p_{CDns,ns})(1 + \alpha)] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)] \quad (\text{B.24})$$

$$\Pi_{DCns,ns} = \lambda[\lambda[(p_{CCns,ns} + p_{CDns,ns})(1 + m + \beta) - \beta] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)]] \quad (\text{B.25})$$

$$\Pi_{DDns,ns} = \lambda[\lambda(p_{CCns,ns} + p_{CDns,ns})(1 + \alpha)] + (1 - \lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1 + \alpha)] \quad (\text{B.26})$$

Note that in a pooling equilibrium where nobody sends the signal, $CCns,ns$ and $DCns,ns$ ($CDns,ns$ and $DDns,ns$) will always earn the same profits irrespective of the chosen signal and the particular composition. We will denote profits by $\Pi_{CCns,ns/DCns,ns}$, and $\Pi_{CDns,ns/DDns,ns}$. Since those pairs are indistinguishable we only have to consider the following cases:

1. Consider first the case $p_{CCns,ns} + p_{DCns,ns} = 1$.

- (i) In that case $\Pi_{CCns,ns/DCns,ns} > \Pi_{CCs,ns/DCs,ns}$, $\Pi_{CDns,ns/DDns,ns} > \Pi_{CDs,ns/DDs,ns}$, and $\Pi_{CCns,ns/DCns,ns} > \Pi_{CCns,s/DCns,s}$, because $p_{CDns,ns} = 0$.

(ii) $\Pi_{CCns,ns/DCns,ns} \geq \Pi_{CDns,ns/DDns,ns} \Leftrightarrow \lambda \geq \frac{\beta}{\beta+m-\alpha}$. Because of $\Pi_{CCns,ns/DCns,ns} > \Pi_{CDns,ns/DDns,ns} > \Pi_{CDs,ns/DDs,ns}$, the condition $\lambda \geq \frac{\beta}{\beta+m-\alpha}$ is necessary and sufficient.

2. Consider next the case $p_{CDns,ns} + p_{DDns,ns} = 1$.

(i) In that case $\Pi_{CCns,ns/DCns,ns} < \Pi_{CDns,ns/DDns,ns}$, because $p_{CCns,ns} + p_{CDns,ns} = 0$.

(ii) $\Pi_{CDns,ns/DDns,ns} \geq \Pi_{CDs,ns/DDs,ns} \Leftrightarrow \lambda p_{CDns,ns} \leq \frac{k_H}{1+\alpha}$.

(iii) $\Pi_{CDns,ns/DDns,ns} \geq \Pi_{CCs,ns/DCs,ns} \Leftrightarrow \lambda p_{CDns,ns} \leq \frac{\beta+k_H}{1+m+\beta}$.

(iv) $\Pi_{CDns,ns/DDns,ns} \geq \Pi_{CDns,s/DDns,s} \Leftrightarrow \lambda p_{CDns,ns} \leq \frac{k_L}{1+\alpha}$.

Note that, $\frac{k_L}{1+\alpha} > \frac{k_H}{1+\alpha}$. Thus, (ii) and (iii) are necessary and sufficient.

3. Finally, consider the case $p_{CDns,ns} + p_{DDns,ns} + p_{CCns,ns} + p_{DCns,ns} = 1$.

(i) In that case all no-signaling strategies earn the same payoff: $\lambda[\lambda(p_{CCns,ns} + p_{CDns,ns})(1+m+\beta) - \beta] + (1-\lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1+\alpha)] = \lambda[\lambda(p_{CCns,ns} + p_{CDns,ns})(1+\alpha)] + (1-\lambda)[\lambda(p_{CCns,ns} + p_{DCns,ns})(1+\alpha)] \Leftrightarrow \lambda(p_{CCns,ns} + p_{DCns,ns}) = \frac{\beta}{\beta+m-\alpha}$.

(ii) $\Pi_{CCns,ns/DCns,ns} \geq \Pi_{CCs,ns/DCs,ns} \Leftrightarrow \lambda(p_{CDns,ns} - p_{DCns,ns}) \leq \frac{k_H}{1+m+\beta}$.

(iii) $\Pi_{CDns,ns/DDns,ns} \geq \Pi_{CDs,ns/DDs,ns} \Leftrightarrow \lambda(p_{CDns,ns} - p_{DCns,ns}) \leq \frac{k_H}{1+\alpha}$.

(iv) $\Pi_{CDns,ns/DDns,ns/CCns,ns/DCns,ns} \geq \Pi_{CDns,s/DDns,s/CCns,s/DCns,s} \Leftrightarrow \lambda(p_{CDns,ns} - p_{DCns,ns}) \leq \frac{k_L}{1+\alpha}$.

Note that, because of $k_L > k_H$ and $m > \alpha$, (ii) implies (iii) and (iv). Thus, such an equilibrium exists if and only if $\lambda(p_{CCns,ns} + p_{DCns,ns}) = \frac{\beta}{\beta+m-\alpha}$ and $\lambda(p_{CDns,ns} - p_{DCns,ns}) \leq \frac{k_H}{1+m+\beta}$.

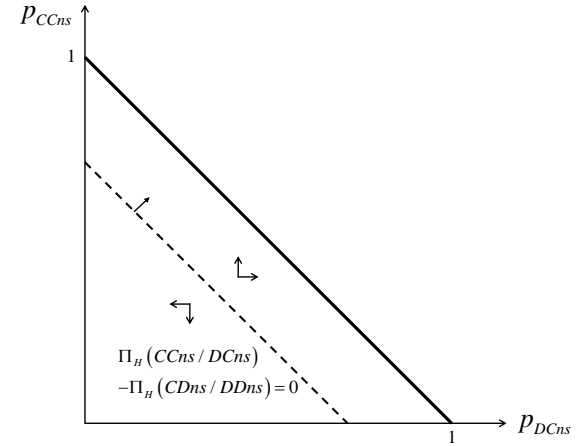
Stability

1. This equilibrium set is stable for $\lambda > \frac{\beta}{m-\alpha+\beta}$ since all inequalities hold strictly, i.e. for any small perturbation the equilibrium strategies earn strictly more than any other strategy. Note that not necessarily the pre-perturbation shares are reestablished, but that the sum of their shares equals unity. At the boundary $\lambda = \frac{\beta}{m-\alpha+\beta}$ there are too few high types and the agents become indifferent between cooperation and defection, i.e. $\Pi_{CCns,ns/DCns,ns} = \Pi_{CDns,ns/DDns,ns}$. Note that it is still a strictly best response not to signal contingent on being a low type. Given the following differences in type-specific payoffs:

$$\Pi_H(CCns) - \Pi_H(CDns) \geq 0 \Leftrightarrow p_{CCns} \geq \frac{\beta}{\lambda(m-\alpha) + \beta} - p_{DCns}$$

, we obtain the following phase diagram.

Note that at $\lambda = \frac{\beta}{m-\alpha+\beta}$ a perturbation from $CCns,ns$ towards $DDns,ns$ decreases the payoffs for the equilibrium strategies strictly more than for $DDns,ns$ and decreases profits for all other strategies weakly more, i.e. those strategies still earn strictly less than $DDns,ns$, and the share of $DDns,ns$ increases. Hence, there is no force reestablishing the equilibrium set. Note that the iso-profit line is shifted towards the boundary as λ approaches the lower limit of the support $\frac{\beta}{m-\alpha+\beta}$. As the diagram clearly indicates, this equilibrium is stable for all $\lambda > \frac{\beta}{m-\alpha+\beta}$ in the support.



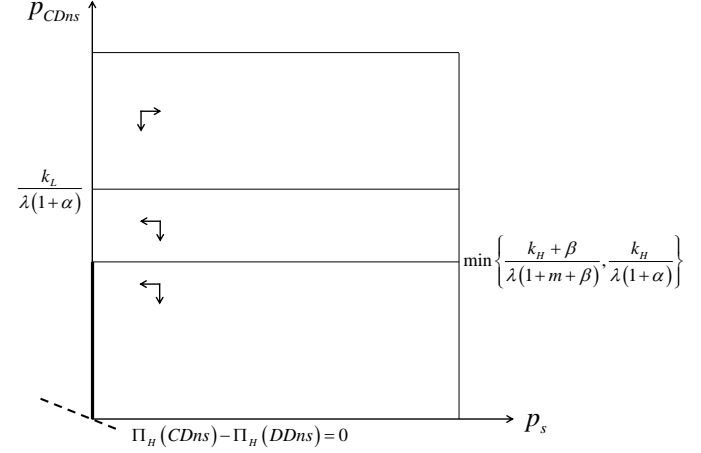
2. This set of equilibria is stable for $p_{CDns,ns} < \frac{1}{\lambda} \min\{\frac{k_H+\beta}{1+m+\beta}, \frac{k_H}{1+\alpha}\}$ since all inequalities hold strictly, i.e. for any small perturbation the equilibrium strategies earn strictly more than any other strategy.

Given the following differences in type-specific payoffs:

$$\Pi_H(CDns) - \Pi_H(DDns) = -\beta(1 - \lambda)p_s \leq 0$$

$$\Pi_L(ns) - \Pi_L(s) = k_L - \lambda(1 + \alpha)p_{CDns} \geq 0 \Leftrightarrow p_{CDns} \leq \frac{k_L}{\lambda(1 + \alpha)}$$

, we obtain the following phase diagram. As the diagram clearly indicates, this equilibrium set is stable for all λ in the support.

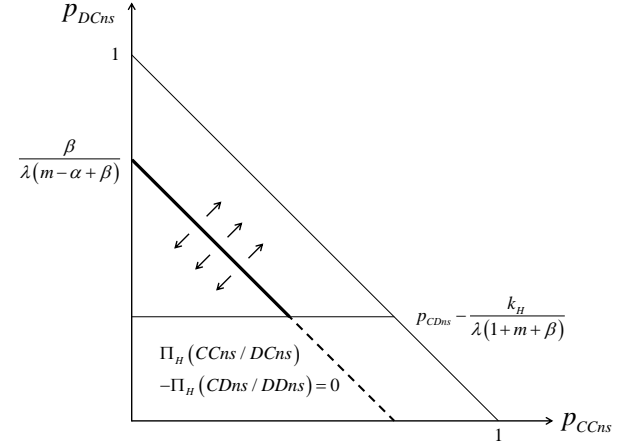


3. Observe that the payoffs for the equilibrium strategies can be written as linear functions in $p_{CCns,ns} + p_{DCns,ns}$.

Given the following differences in type-specific payoffs:

$$\Pi_H(CCns) - \Pi_H(CDns) \geq 0 \Leftrightarrow p_{DCns} \leq \frac{\beta}{\lambda(m - \alpha + \beta)} - p_{CCns}$$

, we obtain the following phase diagram. All other payoff differences of equilibrium strategies vanish. The figure incorporates the two conditions for existence, i.e., $\lambda(p_{CCns,ns} + p_{DCns,ns}) = \frac{\beta}{\beta+m-\alpha}$ and $\lambda(p_{CDns,ns} - p_{DCns,ns}) \leq \frac{k_H}{1+m+\beta}$. As the diagram clearly indicates, this equilibrium set is unstable.



B.2.2 High types and low types signal

Existence

That is, we look for an equilibrium, such that $p_{CCs,s} + p_{CDs,s} + p_{DCs,s} + p_{DDs,s} = 1$. Let us again first study the signaling contingency for high types.

$$\Pi_{CCs,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.27})$$

$$\Pi_{CDs,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + m + \beta) - \beta - k_H] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.28})$$

$$\Pi_{DCs,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_H] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.29})$$

$$\Pi_{DDs,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_H] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.30})$$

$$\Pi_{CCns,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + m + \beta) - \beta] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.31})$$

$$\Pi_{CDns,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + m + \beta) - \beta] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.32})$$

$$\Pi_{DCns,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha)] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.33})$$

$$\Pi_{DDns,s} = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha)] + (1 - \lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1 + \alpha) - k_L] \quad (\text{B.34})$$

Note that in a pooling equilibrium where everybody sends the signal, CCs, s and CDs, s (DCs, s and DDs, s) will always earn the same profits irrespective of the chosen signal and the particular composition. We will denote profits by $\Pi_{CCs,s/CDs,s}$, and $\Pi_{DCs,s/DDs,s}$. Since those pairs are indistinguishable we only have to consider the following cases:

1. Consider first the case $p_{CCs,s} + p_{CDs,s} = 1$.

$$(i) \quad \Pi_{CCs,s/CDs,s} \geq \Pi_{CCns,s/CDns,s} \Leftrightarrow \lambda p_{CDs,s} \geq \frac{k_H}{1+m+\beta}.$$

$$(ii) \quad \Pi_{CCs,s/CDs,s} \geq \Pi_{DCs,s/DDs,s} \Leftrightarrow \lambda \geq \frac{\beta}{\beta+m-\alpha}.$$

$$(iii) \quad \Pi_{CCs,s/CDs,s} \geq \Pi_{DCns,s/DDns,s} \Leftrightarrow \lambda \geq \frac{\beta+k_H}{\beta+m-\alpha+p_{CDs,s}(1+\alpha)}.$$

$$(iv) \Pi_{CCs,s/CDs,s} \geq \Pi_{CCs,ns/CDs,ns} \Leftrightarrow \lambda \geq \frac{k_L}{p_{CDs,s}(1+\alpha)}.$$

Note that (iv) implies (i), for (iv) to be satisfied a strictly positive share needs to play CDs, s . Furthermore, (ii) and (iv) imply (iii). Hence, for $\lambda \geq \max\{\frac{k_L}{p_{CDs,s}(1+\alpha)}, \frac{\beta}{\beta+m-\alpha}\}$ such an equilibrium exists.

2. Consider next the case $p_{DCs,s} + p_{DDs,s} = 1$. This cannot constitute an equilibrium, because not sending the signal contingent on being a low types yields strictly higher payoffs.
3. Finally, consider the case $p_{CDs,s} + p_{DDs,s} + p_{CCs,s} + p_{DCs,s} = 1$.

In that case all signaling strategies earn the same payoff: $\lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1+m+\beta) - \beta - k_H] + (1-\lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1+\alpha) - k_L] = \lambda[\lambda(p_{CCs,s} + p_{CDs,s})(1+m+\beta) - \beta - k_H] + (1-\lambda)[\lambda(p_{CCs,s} + p_{CDs,s})(1+\alpha) - k_L] \Leftrightarrow \lambda(p_{CCs,s} + p_{CDs,s}) = \frac{\beta}{\beta+m-\alpha}$. The following condition are necessary and sufficient for existence.

- (i) $\Pi_{CCs,s/CDs,s} \geq \Pi_{CCns,s/CDns,s} \Leftrightarrow \lambda(p_{CDs,s} - p_{DCs,s}) \geq \frac{k_H}{1+m+\beta}$.
- (ii) $\Pi_{DCs,s/DDs,s} \geq \Pi_{DCns,s/DDns,s} \Leftrightarrow \lambda(p_{CDs,s} - p_{DCs,s}) \geq \frac{k_H}{1+\alpha}$.
- (iii) $\Pi_{CDs,s/DDs,s/CCs,s/DCs,s} \geq \Pi_{CDs,ns/DDs,ns/CCs,ns/DCs,ns} \Leftrightarrow \lambda(p_{CDs,s} - p_{DCs,s}) \geq \frac{k_L}{1+\alpha}$.

Note that (ii) implies (i), and (iii) implies (ii). Hence, such an equilibrium exists if and only if $\lambda(p_{CCs,s} + p_{CDs,s}) = \frac{\beta}{\beta+m-\alpha}$, and $\lambda(p_{CDs,s} - p_{DCs,s}) \geq \frac{k_L}{1+\alpha}$.

Stability

1. Note that at $p_{CDs} = \frac{k_L}{\lambda(1+\alpha)}$ low types are indifferent between signaling and no signaling. As soon as low types start not to signal, CDs earns strictly higher payoffs than CCs such that the incentive for low types to signal will be restored. However, at $\lambda = \frac{k_L}{\lambda(1+\alpha)} p_{CDs}$ equals 1 and therefor can not increase. Thus, this equilibrium is unstable at the upper bound $\frac{k_L}{\lambda(1+\alpha)}$. If $\lambda = \frac{\beta}{m-\alpha+\beta}$, then high types given a received signal are indifferent between cooperative and defective play. For a small increase in the share $p_{DCs,s} + p_{DDs,s}$, the profits for the equilibrium strategies will decline more than

the profits for $DCs, s/DDs, s$. Since the equilibrium strategies and $DCs, s/DDs, s$ will still earn higher profits than any other, there is no force bringing back the system to $p_{CCs, s} + p_{CDs, s} = 1$. Hence, the equilibrium is unstable at $\lambda = \frac{\beta}{m-\alpha+\beta}$.

Given the following differences in type-specific payoffs:

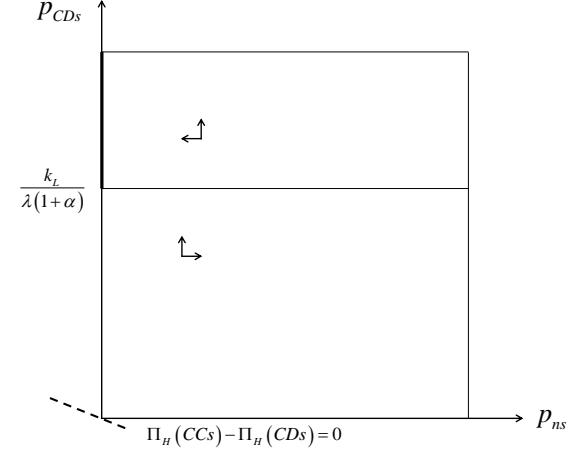
$$\Pi_H(CC_s) - \Pi_H(CD_s) = -\beta(1 - \lambda)p_{ns} \leq 0$$

$$\Pi_L(ns) - \Pi_L(s) \geq 0 \Leftrightarrow p_{CDns} \leq \frac{k_L}{\lambda(1 + \alpha)}$$

, we obtain the following phase diagram.

As the diagram clearly indicates, this equilibrium set is stable for

$$\lambda > \max\left\{\frac{k_L}{p_{CDs, s}(1+\alpha)}, \frac{\beta}{\beta+m-\alpha}\right\}.$$



2. Let $p_{CCs} + p_{CDs} = x$ and $p_{DCs} + p_{DDs} = y$. Note that $y = 1 - x$, because of $p_{CCs} + p_{CDs} + p_{DCs} + p_{DDs} = 1$. Thus, we can write payoffs for high types as: $\Pi_H(CC_s) = \Pi_H(CD_s) = \lambda x(1 + m + \beta) - \beta - k_H$, and $\Pi_H(DC_s) = \Pi_H(DD_s) = \lambda x(1 + \alpha) - k_H$. Given any perturbation that violates the equilibrium condition $\lambda(p_{CCs} + p_{CDs}) = \frac{\beta}{m-\alpha+\beta}$ the equilibrium set will not be restored because $\Pi_H(CC_s/CD_s) - \Pi_H(DC_s/DD_s) = \lambda x(m - \alpha + \beta) - \beta \geq 0 \Leftrightarrow \lambda x \geq \frac{\beta}{m-\alpha+\beta}$. Thus, an increase in x is self enforcing.

C Stable Semi-Pooling Equilibria

strat.	eq. shares	conditions for existence	support	payoff differences
CCs, ns $CDns, ns$	$p_{CCs, ns} = 1 - \frac{k_H + (1-\lambda)\beta}{\lambda(1+m)}$ $p_{CDns, ns} = \frac{k_H + (1-\lambda)\beta}{\lambda(1+m)}$	(1): $\frac{\beta}{\beta+m-\alpha} < \frac{k_H}{1+\alpha} < \frac{1+m}{1+\alpha}$ (2): $\frac{k_H}{1+\alpha} \leq \frac{\beta}{\beta+m-\alpha}$	$\frac{k_H + \beta}{1+m+\beta} \lambda < 1$ $1 - \frac{m-\alpha}{\beta} \frac{k_H}{1+\alpha} < \lambda < 1$	$\Pi_H - \Pi_L =$ $(m - \alpha)\lambda(1 - \frac{k_H + (1-\lambda)\beta}{\lambda(1+m)}) > 0$ $(\Pi_H - \Pi_L)^f =$ $-\alpha\lambda(1 - \frac{k_H^f + (1-\lambda)\beta}{\lambda(1+m)}) < 0$
DCs, ns DCs, s $CDns, ns$ $CDns, s$	$p_{DCs} = \frac{1}{2}(1 - \frac{k_L}{\lambda(1+\alpha)})$ $p_{CDns} = \frac{1}{2}(1 + \frac{k_L}{\lambda(1+\alpha)})$ $p_{ns} = \frac{1}{2}(1 + \frac{1}{(1-\lambda)\beta})(\frac{1+m}{1+\alpha}k_L - k_H)$ $p_s = \frac{1}{2}(1 - \frac{1}{(1-\lambda)\beta})(\frac{1+m}{1+\alpha}k_L - k_H)$	$\beta(1 + \alpha) > \Phi$	$\frac{\beta + k_L - k_H}{\beta + m - \alpha} < \lambda$ $< 1 - \frac{1}{\beta}(\frac{1+m}{1+\alpha}k_L - k_H)$	$\Pi_H - \Pi_L =$ $(m - \alpha)\lambda p_{DCs} - \beta(1 - \lambda)p_s < 0$ $(\Pi_H - \Pi_L)^f =$ $-\alpha\lambda p_{DCs} - \beta(1 - \lambda)p_s < 0$
DCs, ns $CDns, ns$	$p_{DCs, ns} = \frac{1}{2}(1 - \frac{k_H + (1-\lambda)\beta}{\lambda(1+m)})$ $p_{CDns, ns} = \frac{1}{2}(1 + \frac{k_H + (1-\lambda)\beta}{\lambda(1+m)})$		$\lambda > \max\{\frac{k_H + \beta}{1+m+\beta}, \Theta, \Psi\}$	$\Pi_H - \Pi_L = (m - \alpha)\lambda p_{DCs, ns} > 0$ $(\Pi_H - \Pi_L)^f = -\alpha\lambda p_{DCs, ns} < 0$

$$\Phi = \frac{\beta+m-\alpha}{m-\alpha}((1+m)k_L - (1+\alpha)k_H) + \frac{k_L - k_H}{m-\alpha}\beta(1+\alpha), \quad \Theta = 1 - \frac{1}{\beta(1+\alpha)}((1+m)k_L - (1+\alpha)k_H), \quad \Psi = 1 - \frac{(m-\alpha)(1+m+k_H)}{(\beta+m-\alpha)(1+m)+(1+\alpha)\beta}$$

Table 2: Additional Stable Semi-Pooling Equilibria

D Semi-Pooling Equilibria – Existence and Stability

1. Existence

We will turn to the analysis of equilibria where only parts of high types or low types signal.

Before we start we will have a closer look on the payoffs for various strategies and their differences. This will significantly simplify the analysis. The following table gives the payoffs for each strategy:

$$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CC\underline{m}} + p_{CD\underline{m}})(1 + \bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}} + p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta)] - \bar{k}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + (p_{CC\underline{m}} + p_{CD\underline{m}})(1 + \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] - \bar{k}$$

$$\Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1 + \bar{m}) + (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] - \bar{k}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CC\underline{m}} + p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(CC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}} + p_{CC\underline{m}} + p_{DC\underline{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}} + p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta)]$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CC\underline{m}} + p_{DC\underline{m}})(1 + \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)]$$

$$\Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1 + \bar{m}) + (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)]$$

$$\Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}} + p_{CC\underline{m}} + p_{DC\underline{m}})(1 + \alpha) \right]$$

$$\Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}} + p_{CC\underline{m}} + p_{DC\underline{m}})(1 + \alpha) \right]$$

$$\Pi_{\underline{m}}(\bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CC\underline{m}} + p_{CD\underline{m}})(1 + \alpha) \right] - \underline{k}$$

It will be useful to calculate differences among strategies with different behavior but the same signal and among strategies with different signals.

(1) Within-differences:

$$\begin{aligned}
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}} + p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) + p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} - p_{CC\bar{m}} - p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}} - p_{DC\bar{m}} - p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) - p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)]
\end{aligned}$$

$$\begin{aligned}
\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}} + p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}} + p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) + p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}} - p_{CC\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}} - p_{CD\bar{m}} - p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) - p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \\
\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)]
\end{aligned}$$

(2) Cross-differences:

$$\begin{aligned}
\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) &= \lambda \left[(p_{CD\bar{m}} + p_{CD\bar{m}} - p_{DC\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) &= \lambda (1 + \bar{m} + \beta) \left[(p_{CD\bar{m}} + p_{CD\bar{m}} - p_{DC\bar{m}} - p_{DC\bar{m}}) \right] - \bar{k} \\
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \bar{k} \\
\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) + (p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) \right] - \bar{k} \\
\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda (1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\bar{m}} - p_{DC\bar{m}} - p_{DC\bar{m}}) \right] - \bar{k}
\end{aligned}$$

All other differences can be expressed by the within-differences and the four cross differences above.

Observation 1:

$$\begin{aligned}
& \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) = \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\underline{m}} - p_{DC\underline{m}})(\bar{m} - \alpha + \beta) \\
(i) \quad & \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\underline{m}} - p_{DC\underline{m}})(\bar{m} - \alpha + \beta) = \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) \\
& \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) + \lambda(p_{CD\bar{m}} + p_{DC\underline{m}} - p_{CD\underline{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) \\
(ii) \quad & \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \bar{k} - \underline{k}
\end{aligned}$$

A consequence of (ii) of observation 1 is that whenever low types are indifferent in an equilibrium between signaling and not signaling, high types strictly prefer to signal over not to signal given unconditional defective behavior. Put differently, if unconditional defection with and without signal is part of an equilibrium, then low types will prefer not to signal in such an equilibrium.

Observation 2:

$$\begin{aligned}
(i) \quad & \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) \quad \text{note that differences depend only on non-signaling shares} \\
(ii) \quad & \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \\
(iii) \quad & \text{Corollary: } \Pi_{\bar{m}}(CC, \bar{m}) + \Pi_{\bar{m}}(DD, \bar{m}) = \Pi_{\bar{m}}(CD, \bar{m}) + \Pi_{\bar{m}}(DC, \bar{m}) \quad ; \quad \Pi_{\bar{m}}(CC, \underline{m}) + \Pi_{\bar{m}}(DD, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) + \Pi_{\bar{m}}(DC, \underline{m})
\end{aligned}$$

Implication:

If within the 4 signal or 4 non-signal behaviors 3 strategies earn the same profit then all 4 strategies earn the same profit. Hence, as a first consequence, there are for each of the cases signal/ no signal only three possibilities: either all 4 strategies earn the same payoff, 2 equal profitable strategies earn strictly more than 2 others, or a single strategy earns more than all others.

If we look at the corollary of observation 2 that the sum of profits for unconditional strategies must equal the sum of profits for conditional strategies, then both conditional can only earn the same profits in equilibrium if the two unconditional strategies earn the same profits too, i.e. all 4 strategies earn the same, otherwise the two unconditional (conditional) strategies must be dominated by one conditional (unconditional) strategy. Furthermore this dominating strategy dominates the second condition (unconditional) strategy. Hence either all strategies earn the same profits or a conditional an unconditional strategy earn the same (highest) payoffs or a single conditional/unconditional strategy earns the highest payoff. The following Lemma summarizes.

Lemma: For each signaling strategy (signal/ no-signal) the table below gives all possible behavioral combinations that could be part of an equilibrium.

	unconditional versus conditional		
1.	$CC=CD ; DC=DD$	1.1.	$CC=CD=DC=DD$
		1.2.	$CC=CD>DC=DD$
		1.3.	$CC=CD<DC=DD$
2.	$CC>CD ; DC>DD$	2.1.	$CC=DC$
		2.2.	$CC>DC$
		2.3.	$CC<DC$
3.	$CC<CD ; DC<DD$	3.1.	$CD=DD$
		3.2.	$CD>DD$
		3.3.	$CD<DD$

Table Fehler! Kein Text mit angegebener Formatvorlage im Dokument.-1: possible cases for signaling / no signaling

Proof: whenever CC and CD have a strict payoff relation, so do DC and DD, hence either CC/DC and CD/DD have a strict payoff relation or all four strategies earn the same profit. In the former case there are three possible relations among the dominating pair: either the relation is strict, then we have the situation of an unique behavior or they could earn the same payoff. Hence either all behavior earns the same payoff, a pair of conditional and unconditional behavior (CC/DC or CD/DD) earn the highest payoff or any unique behavior earns highest payoff.

If we neglect for a moment that for a given signal all 4 behaviors are part of a semi pooling equilibrium then following the lemma above, the table below gives all possible combinations of strategies in a semipooling equilibrium.

	CC, \underline{m}	DC, \underline{m}	$CC, \underline{m} / DC, \underline{m}$	CD, \underline{m}	DD, \underline{m}	$CD, \underline{m} / DD, \underline{m}$
CC, \bar{m}	N (2.)		N (2.)			
DC, \bar{m}	N (7.)	N (3.)	N (3.)			
$CC, \bar{m} / DC, \bar{m}$	N (2.)	N (3.)	N (3.)		N (6.)	
CD, \bar{m}	N (4.)		N (4.)	N (5.)		N (5.)
DD, \bar{m}	N (4.)		N (4.)	N (5.)	N (1.)	N (5.)
$CD, \bar{m} / DD, \bar{m}$	N (4.)		N (4.)	N (5.)		N (5.)

Table Fehler! Kein Text mit angegebener Formatvorlage im Dokument.-2: N – cannot exist; for colored cells low types don't signal, because either CD, \bar{m} and CD, \underline{m} are not played (blue) or DD, \underline{m} earns highest payoffs (gray) (see 8.-9.)

However, if we have a closer look at the respective differences we can significantly reduce the number of possible combinations.

$$1. \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} = -\bar{k} < 0$$

$$2. \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) = \lambda(1 + \bar{m} + \beta) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} = -\bar{k} < 0$$

$$3. \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} = \lambda \left[(-p_{DC\bar{m}})(1 + \alpha) + (-p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} < 0$$

$$4. \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \stackrel{p_{CC\bar{m}} + p_{DC\bar{m}} = 0}{<} 0$$

$$\Rightarrow: \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Rightarrow p_{CC\bar{m}} + p_{DC\bar{m}} > 0$$

$$5. \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \stackrel{p_{CC\bar{m}} + p_{DC\bar{m}} = 0}{<} 0$$

$$\Rightarrow: \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \geq 0 \Rightarrow p_{CC\bar{m}} + p_{DC\bar{m}} > 0$$

$$6. \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[+ (p_{CD\underline{m}})(1 + \bar{m}) + (-p_{DC\bar{m}})(1 + \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k} \stackrel{p_{CD\underline{m}} = 0}{<} 0$$

$$7. \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) = \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \left[\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) \right] = \lambda(1 + \bar{m} + \beta) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k}$$

$$- \left\{ \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \right\} \stackrel{p_{\bar{m}} = 0}{=} \lambda(1 + \bar{m} + \beta) \left[(-p_{DC\bar{m}}) \right] - \bar{k} - \lambda \left[(p_{DC\bar{m}})(-\beta) \right] = -p_{DC\bar{m}} \lambda(1 + \bar{m}) - \bar{k} < 0$$

$$8. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \bar{k} - \underline{k} \stackrel{\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \leq 0}{<} 0$$

$$9. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k}, \text{ hence if neither } CD, \underline{m} \text{ nor } CD, \bar{m} \text{ is played then low types strictly prefer not to signal, i.e. } p_{\underline{m}} = 1.$$

Observation 3:

$$\Pi_{\bar{m}}(XY, m) - \Pi_{\bar{m}}(XZ, m) = f(p_{CC\bar{m}}, p_{CD\bar{m}}, p_{DC\bar{m}}, p_{DD\bar{m}}) \quad X, Y, Z \in \{C, D\}, m \in \{\underline{m}, \bar{m}\}, Y \neq Z \text{ and}$$

$$\Pi_{\bar{m}}(YX, m) - \Pi_{\bar{m}}(ZX, m) = f(p_{CC\bar{m}}, p_{CD\bar{m}}, p_{DC\bar{m}}, p_{DD\bar{m}}) \quad X, Y, Z \in \{C, D\}, m \in \{\underline{m}, \bar{m}\}, Y \neq Z$$

Before we turn to the 14 remaining cases of table 2, we check for semi-pooling equilibria that contain all 4 behaviors for at least one signal.

1. All 8 strategies are played by high types (4 vs. 4)

Due to $\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k}$ there cannot be an equilibrium such that both equations
 $\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) = \lambda(1 + \bar{m} + \beta) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k}$
are satisfied, required for an equilibrium where all strategies earn the same profits.

2. All four signaling strategies earn same profit, i.e. $\Pi_{\bar{m}}(CC, \bar{m}) = \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) = \Pi_{\bar{m}}(DD, \bar{m})$ (4 versus 2/1)

2.1. $CC, \bar{m} / DC, \bar{m} / CD, \bar{m} / DD, \bar{m}$ vs. $CC, \underline{m} / DC, \underline{m}$, i.e. $\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) > 0$ (*)

2.1.1. $\Pi_{\bar{m}}(CC, \underline{m}) \geq \Pi_{\bar{m}}(DC, \underline{m}) (> \Pi_{\bar{m}}(DD, \underline{m}))$

$$\begin{aligned} \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) &= \lambda(1 + \bar{m} + \beta) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} = 0 \Rightarrow \\ CC, \bar{m} / CC, \underline{m} &\text{ earn same profits, i.e.} \end{aligned} \quad \text{Hence } DD, \bar{m}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} < 0$$

cannot be part of the equilibrium, cannot earn the same profits as CC, \bar{m} . Therefore such an equilibrium cannot exist.

2.1.2. $\Pi_{\bar{m}}(CC, \underline{m}) < \Pi_{\bar{m}}(DC, \underline{m})$, i.e. $p_{CC\underline{m}} = p_{CD\underline{m}} = p_{DD\underline{m}} = 0$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DC\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] = 0 \text{ is violated if } p_{DC\underline{m}} > 0$$

which is necessary for a **semi**-pooling equilibrium.

Therefor such an equilibrium cannot exist.

$$2.2. \quad CC, \bar{m} / DC, \bar{m} / CD, \bar{m} / DD, \bar{m} \quad \text{vs.} \quad CD, \underline{m} / DD, \underline{m}, \text{ i.e. } \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) < 0$$

$$p_{CC\underline{m}} = p_{DC\underline{m}} = 0$$

$$\begin{aligned} \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = 0 \quad (i) \\ \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (ii) \\ \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(\bar{m} - \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (iii) \end{aligned}$$

$$2.2.1. \quad \Pi_{\bar{m}}(DD, \underline{m}) \leq \Pi_{\bar{m}}(CD, \underline{m})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \geq 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda (p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) \quad (v)$$

$$= \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda (p_{CD\underline{m}} - p_{DC\underline{m}})(\bar{m} - \alpha + \beta)$$

$$= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \bar{k} = 0$$

The last equation implies that $(p_{CD\bar{m}} - p_{DC\bar{m}}) \leq 0$ and $(p_{CD\underline{m}} - p_{DC\underline{m}}) > 0$.

$$p_{CC\underline{m}} = p_{DC\underline{m}} = 0 \rightarrow$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] \geq 0 \quad (iv)$$

$$\begin{aligned}
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) \\
&= \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\underline{m}})(\bar{m} - \alpha + \beta) \\
&= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k} = 0
\end{aligned} \tag{v}$$

(iii) is always satisfied in a semi-pooling equilibrium

2.2.1.1. Eq. for $\Pi_{\bar{m}}(DD, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) \xRightarrow{Obs.1} p_{\underline{m}} = 1$:

$$\begin{aligned}
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] = 0 & (i) \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] = & (ii) \\
\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 & (iv) \\
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) & (v) \\
&= \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\underline{m}})(\bar{m} - \alpha + \beta) \\
&= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k} \\
&= \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \bar{k} + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) = 0
\end{aligned}$$

(v) implies $(p_{CD\bar{m}} - p_{DC\bar{m}}) = 0$ then $p_{CD\underline{m}} = \frac{\bar{k}}{\lambda(1 + \alpha)}$ by (v), $p_{DD\underline{m}} = \frac{\bar{k}}{\lambda(1 + \alpha)} \frac{\bar{m} - \alpha}{\beta} - \frac{1 - \lambda}{\lambda}$ by (i) and so (ii), (iv) are remaining:

given that $(p_{CD\bar{m}} - p_{DC\bar{m}}) = 0$ (ii) and (iv) are equivalent., i.e. those two equation amount to one further condition on the shares among high types.

In summary the equilibrium set is given by: $\left\{ \begin{array}{l} (p_{CC\bar{m}}, p_{CD\bar{m}}, p_{DC\bar{m}}, p_{DD\bar{m}}, p_{CD\bar{m}}, p_{DD\bar{m}}, p_{\bar{m}} = 1) \\ p_{CC\bar{m}} = -\frac{\bar{k}}{\lambda(1+\alpha)} + \frac{\beta}{\lambda(\bar{m}-\alpha+\beta)} - p_{DC\bar{m}} \\ p_{CD\bar{m}} = p_{DC\bar{m}}, \\ p_{DD\bar{m}} = -\frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{\beta}{\lambda(\bar{m}-\alpha+\beta)} + \frac{1}{\lambda} - p_{DC\bar{m}} \\ p_{CD\bar{m}} = \frac{\bar{k}}{\lambda(1+\alpha)} \\ p_{DD\bar{m}} = \frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{1-\lambda}{\lambda} \end{array} \right\}$

Note that the condition $p_{DD\bar{m}} \geq 0 \Leftrightarrow \frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{1-\lambda}{\lambda} \geq 0 \Leftrightarrow \lambda \geq 1 - \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta}$. On the other hand in a semi-pooling equilibrium where high apply both types of signals we must have:

$$p_{CD\bar{m}} + p_{DD\bar{m}} < 1 \Leftrightarrow \frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha+\beta}{\beta} - \frac{1-\lambda}{\lambda} < 1 \Leftrightarrow \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha+\beta}{\beta} < 1$$

All conditions of the type

$p \in [0,1], \sum p < 1$ reduce to:

$$(1): p_{DD\bar{m}} \geq 0 \Leftrightarrow \frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{1-\lambda}{\lambda} \geq 0 \Leftrightarrow \lambda \geq 1 - \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta}$$

$$(2): p_{CD\bar{m}} \geq 0 \quad \text{true}$$

$$(3): p_{DD\bar{m}} + p_{CD\bar{m}} < 1 \Leftrightarrow \frac{\bar{k}}{1+\alpha} < \frac{\beta}{\bar{m}-\alpha+\beta}$$

$$(4): p_{CC\bar{m}}, p_{DD\bar{m}}, p_{CD\bar{m}} \geq 0 \Leftrightarrow 0 \leq p_{DC\bar{m}} \leq \frac{1}{\lambda} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right) \min \left\{ \frac{\bar{m}-\alpha}{\beta}, 1 \right\} = \begin{cases} \frac{1}{\lambda} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right) & , \beta < (\bar{m}-\alpha) \\ \frac{1}{\lambda} \frac{\bar{m}-\alpha}{\beta} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right) & , \beta > (\bar{m}-\alpha) \end{cases}$$

Note:

$$a) \frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} - 1 + \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} = -\frac{\bar{m}-\alpha}{\bar{m}-\alpha+\beta} + \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha-\beta}{\beta} > 0 \Leftrightarrow \frac{\bar{k}}{(1+\alpha)} > \frac{\beta(\bar{m}-\alpha)}{(\bar{m}-\alpha+\beta)(\bar{m}-\alpha-\beta)} > \frac{\beta}{\bar{m}-\alpha+\beta}$$

which violates (3), hence $\frac{1}{\lambda} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right)$ is binding, i.e. < 1 .

$$\frac{\beta(\bar{m}-\alpha)}{(\bar{m}-\alpha+\beta)(\bar{m}-\alpha-\beta)} < \frac{\bar{k}}{(1+\alpha)} < \frac{\beta}{\bar{m}-\alpha+\beta} \quad \text{false, since } \frac{(\bar{m}-\alpha)}{(\bar{m}-\alpha-\beta)} > 1, \text{ i.e.}$$

$$b) \frac{\bar{m}-\alpha}{\beta} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right) - 1 + \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} = \frac{\bar{m}-\alpha}{\bar{m}-\alpha+\beta} - 1 < 0 \Rightarrow \frac{1}{\lambda} \frac{\bar{m}-\alpha}{\beta} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right) \text{ is binding, i.e. } < 1.$$

Conditions for existence:

$$1. \frac{\bar{k}}{1+\alpha} < \frac{\beta}{\bar{m}-\alpha+\beta}$$

$$2. \lambda \geq 1 - \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} (>0, \text{ by 1.})$$

3.

$$\beta < (\bar{m} - \alpha): \quad p_{DC\bar{m}} \leq \frac{1}{\lambda} \left(\frac{\beta}{\bar{m} - \alpha + \beta} - \frac{\bar{k}}{1 + \alpha} \right) (> 0, \text{ by 1.})$$

$$\beta \geq (\bar{m} - \alpha): \quad p_{DC\bar{m}} \leq \frac{1}{\lambda} \frac{\bar{m} - \alpha}{\beta} \left(\frac{\beta}{\bar{m} - \alpha + \beta} - \frac{\bar{k}}{1 + \alpha} \right) (> 0, \text{ by 1.})$$

2.2.1.2. Eq. for $\Pi_{\bar{m}}(DD, \underline{m}) < \Pi_{\bar{m}}(CD, \underline{m})$: $p_{DD\underline{m}} = 0$ plugged into

$$\begin{aligned} \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = 0 & (i) \\ \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = & (ii) \\ \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 & (iv) \\ \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda (p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) & (v) \\ &= \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda (p_{CD\underline{m}})(\bar{m} - \alpha + \beta) \\ &= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k} \\ &= \lambda (1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \bar{k} + \lambda (p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) = 0 \end{aligned}$$

(i) gives us $\lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = 0$ which for a semi-pooling equilibrium ($p_{CD\underline{m}} > 0$) requires $p_{\underline{m}} > 0$ hence

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda (1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \underline{k} \leq 0 \text{ Note that with } p_{\underline{m}} > 0 \text{ (ii) will always be satisfied.}$$

$$1.2.1.2.1. \quad \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = 0$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \underline{k} = 0 \quad (*)$$

$$\lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \underline{k} = 0 \Leftrightarrow \lambda(1+\alpha) p_{CD\underline{m}} = \underline{k} - \lambda(1+\alpha)(p_{CD\bar{m}} - p_{DC\bar{m}}) \quad (*)$$

$$\lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] = 0 \quad (i)$$

$$\lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 \quad (ii)$$

$$\lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iv)$$

$$\lambda(1+\alpha) p_{CD\underline{m}} = \bar{k} - \lambda(1+\bar{m} + \beta)(p_{CD\bar{m}} - p_{DC\bar{m}}) \quad (v)$$

Hence $p_{DC\bar{m}} - p_{CD\bar{m}} = \frac{\underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)}$ by (*) and (v) and therefor

$$\lambda(1+\alpha) p_{CD\underline{m}} = \underline{k} - \lambda(1+\alpha)(p_{CD\bar{m}} - p_{DC\bar{m}}) = \underline{k} + \lambda(1+\alpha) \frac{\underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)} \Leftrightarrow p_{CD\underline{m}} = \frac{\underline{k}}{\lambda(1+\alpha)} + \frac{\underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)} \text{ by } (*)$$

$$\text{and by (i) } p_{\underline{m}} = \frac{\lambda}{1-\lambda} \frac{\bar{m} - \alpha}{\beta} \left(\frac{\underline{k}}{\lambda(1+\alpha)} + \frac{\underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)} \right) = \frac{1}{1-\lambda} \frac{\bar{m} - \alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \right).$$

Finally, rearrange (i) to: $\lambda(p_{CD\underline{m}})(\bar{m} - \alpha) - (1-\lambda)\beta = (1-\lambda)[p_{\bar{m}}(-\beta)]$ and plug it into (ii): or equivalently

$$p_{CC\bar{m}} + p_{DC\bar{m}} = \frac{\beta}{\lambda(\bar{m} - \alpha + \beta)} - \frac{\underline{k}}{\lambda(1+\alpha)}, p_{CD\bar{m}} + p_{DD\bar{m}} = 1 - \frac{\beta}{\lambda(\bar{m} - \alpha + \beta)} - \frac{\underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)} = 1 - \frac{\beta - \bar{k} + \underline{k}}{\lambda(\bar{m} - \alpha + \beta)}$$

$$\lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] =$$

Note, that these values imply that (iv) is satisfied : $\lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha + \beta) - \beta + \beta p_{CD\underline{m}} \right] + \lambda p_{CD\underline{m}}(\bar{m} - \alpha) - (1-\lambda)\beta =$

$$\lambda(p_{CC\bar{m}} + p_{DC\bar{m}} + p_{CD\underline{m}})(\bar{m} - \alpha + \beta) - \beta = \underline{k} - \bar{k} > 0$$

In summary equilibrium set is given by

$$\left\{ \begin{array}{l} (p_{CC\bar{m}}, p_{CD\bar{m}}, p_{DC\bar{m}}, p_{DD\bar{m}}, p_{CD\underline{m}}, p_{\underline{m}}) \\ p_{CC\bar{m}} = \frac{(1+\alpha)(\beta+\bar{k}) - (1+\beta+\bar{m})\underline{k}}{\lambda(1+\alpha)(\bar{m}-\alpha+\beta)} - p_{CD\bar{m}} \\ p_{DC\bar{m}} = p_{CD\bar{m}} + \frac{\underline{k} - \bar{k}}{\lambda(\bar{m}-\alpha+\beta)} \\ p_{DD\bar{m}} = 1 - \frac{\beta + \underline{k} - \bar{k}}{\lambda(\bar{m}-\alpha+\beta)} - p_{CD\bar{m}} \\ p_{CD\underline{m}} = \frac{\underline{k}}{\lambda(1+\alpha)} + \frac{\underline{k} - \bar{k}}{\lambda(\bar{m}-\alpha+\beta)} = \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{\lambda(1+\alpha)(\bar{m}-\alpha+\beta)} \\ p_{\underline{m}} = \frac{1}{1-\lambda} \frac{\bar{m}-\alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k} - \bar{k}}{(\bar{m}-\alpha+\beta)} \right) = \frac{1}{1-\lambda} \frac{\bar{m}-\alpha}{\beta} \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(1+\alpha)(\bar{m}-\alpha+\beta)} \end{array} \right\}$$

Note: $p_{CD\bar{m}} + p_{CD\underline{m}} = p_{DC\bar{m}} + \frac{\underline{k}}{\lambda(1+\alpha)}$

$$\Pi_{\underline{m}} = \Pi_{\underline{m}}(\underline{m}) = \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(1+\alpha)] < \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(1+\alpha)] + \underline{k} - \bar{k} = \Pi_{\bar{m}}(DD, \underline{m}) = \Pi_{\bar{m}}$$

Existence:

All conditions of the type

$p \in [0,1], \sum p < 1$ reduce to:

$$(1): p_{CD\underline{m}} > 0 \quad \text{true}$$

$$(2): p_{CD\underline{m}} < 1 \Leftrightarrow \frac{\bar{k}}{1+\alpha} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda \Leftrightarrow \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda$$

$$(3): p_{CC\bar{m}}, p_{DD\bar{m}}, p_{DC\bar{m}} \geq 0 \Leftrightarrow 0 \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{1 - \frac{\beta + \underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)}}_x, \underbrace{\frac{(1+\alpha)(\beta + \bar{k}) - (1+\beta+\bar{m})\underline{k}}{\lambda(1+\alpha)(\bar{m} - \alpha + \beta)}}_y \right\}$$

$$a) x > 0 \Leftrightarrow \lambda > \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \quad x < 1 \quad \text{true}$$

$$b) y > 0 \Leftrightarrow (1+\alpha)(\beta + \bar{k}) - (1+\beta+\bar{m})\underline{k} > 0 \quad y < 1 \Leftrightarrow \frac{(1+\alpha)(\beta + \bar{k}) - (1+\beta+\bar{m})\underline{k}}{(1+\alpha)(\bar{m} - \alpha + \beta)} = \frac{\beta + \bar{k} - \frac{(1+\beta+\bar{m})}{(1+\alpha)}\underline{k}}{(\bar{m} - \alpha + \beta)} < \lambda \quad (\text{by 3a})$$

$$c) x \geq y \Leftrightarrow \lambda \geq \frac{2\beta}{(\bar{m} - \alpha + \beta)} - \frac{\underline{k}}{1+\alpha}$$

$$(4): p_{\underline{m}} \geq 0 \quad \text{true}$$

$$(5): p_{\underline{m}} \leq 1 \Leftrightarrow \lambda \leq 1 - \frac{\bar{m} - \alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \right) = 1 - \frac{\bar{m} - \alpha}{\beta} \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(1+\alpha)(\bar{m} - \alpha + \beta)} (> 0, \text{ by 3b})$$

$$\text{note: } (1+\alpha)\beta - (\bar{m} - \alpha + \beta)\bar{k} > (1+\beta+\bar{m})(\underline{k} - \bar{k}) > 0, \text{ by 3b}) \Rightarrow \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} > \frac{\bar{k}}{1+\alpha} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)}, \text{ i.e. (2) is not binding.}$$

1. (3)b): $(1+\alpha)\beta > (1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}$
2. (3)a) \wedge (2) \wedge (5): $0 < \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda \leq 1 - \frac{\bar{m} - \alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \right) < 1$ ($LHS < RHS$, NOT by 3b) see below)
3. $0 \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{1 - \frac{\beta + \underline{k} - \bar{k}}{\lambda(\bar{m} - \alpha + \beta)}}_x, \underbrace{\frac{\beta + \bar{k} - \frac{1+\beta+\bar{m}}{1+\alpha}\underline{k}}{\lambda(\bar{m} - \alpha + \beta)}}_y \right\}$

$$x \geq y \Leftrightarrow \lambda \geq \frac{2\beta}{(\bar{m} - \alpha + \beta)} - \frac{\underline{k}}{1+\alpha} \left(\stackrel{(3)b)}{>} \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \right)$$

$$\begin{aligned} & \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < 1 - \frac{\bar{m} - \alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \right) \Leftrightarrow \\ & \frac{\bar{m} - \alpha + \beta}{\beta} \frac{\underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} - \frac{\bar{m} - \alpha}{\beta} \frac{\underline{k}}{(1+\alpha)} \Leftrightarrow \\ & \frac{\underline{k} - \bar{k}}{\beta} < \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} - \frac{\bar{m} - \alpha}{(1+\alpha)\beta} \Leftrightarrow \frac{1+\bar{m}}{1+\alpha} \frac{\underline{k}}{\beta} - \frac{\bar{k}}{\beta} < \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} \Leftrightarrow \\ & (1+\bar{m})\underline{k} - (1+\alpha)\bar{k} < \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} (1+\alpha)\beta \Leftrightarrow (1+\alpha)\beta > \frac{\bar{m} - \alpha + \beta}{(\bar{m} - \alpha)} ((1+\bar{m})\underline{k} - (1+\alpha)\bar{k}) \end{aligned}$$

It turns out that this conditions is stronger than 3b), because

$$\frac{\bar{m} - \alpha + \beta}{(\bar{m} - \alpha)} ((1+\bar{m})\underline{k} - (1+\alpha)\bar{k}) - ((1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}) > 0 \Leftrightarrow \underline{k} - \bar{k} > 0$$

Hence we are left with the following conditions for existence:

$$\begin{aligned}
1. & (1+\alpha)\beta > \frac{\bar{m}-\alpha+\beta}{(\bar{m}-\alpha)} \left((1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right) \\
2. & \frac{\beta+\underline{k}-\bar{k}}{(\bar{m}-\alpha+\beta)} < \lambda \leq 1 - \frac{\bar{m}-\alpha}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} + \frac{\underline{k}-\bar{k}}{(\bar{m}-\alpha+\beta)} \right) = 1 - \frac{\bar{m}-\alpha}{\beta} \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(1+\alpha)(\bar{m}-\alpha+\beta)} \\
3. & 0 \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{1 - \frac{\beta+\underline{k}-\bar{k}}{\lambda(\bar{m}-\alpha+\beta)}}_x, \underbrace{\frac{\beta+\bar{k} - \frac{1+\beta+\bar{m}}{1+\alpha}\underline{k}}{\lambda(\bar{m}-\alpha+\beta)}}_y \right\} \\
& x \geq y \Leftrightarrow \lambda \geq \frac{2\beta}{(\bar{m}-\alpha+\beta)} - \frac{\underline{k}}{1+\alpha} \left(\stackrel{(3b)}{>} \frac{\beta+\underline{k}-\bar{k}}{(\bar{m}-\alpha+\beta)} \right)
\end{aligned}$$

$$1.2.1.2.2. \quad \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) < 0 \Rightarrow p_{\underline{m}} = 1$$

$$\begin{aligned}
\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) &= \lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \underline{k} < 0 & (*) \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m}-\alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] = 0 & (i) \\
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) &= \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m}-\alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] = & (ii) \\
\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m}-\alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 & (iv) \\
\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m}-\alpha+\beta) & (v) \\
&= \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\underline{m}})(\bar{m}-\alpha+\beta) \\
&= \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\bar{m}+\beta) + (p_{CD\underline{m}})(1+\alpha) \right] - \bar{k} \\
&= \lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \bar{k} + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m}-\alpha+\beta) = 0
\end{aligned}$$

$$\lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \underline{k} < 0 \quad (*)$$

$$\lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda)(-\beta) = 0 \quad (\text{i})$$

$$\lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] = 0 \quad (\text{ii})$$

$$\lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda)(-\beta) > 0 \quad (\text{iv})$$

$$\lambda(1 + \alpha)p_{CD\bar{m}} = \bar{k} - \lambda(1 + \bar{m} + \beta)(p_{CD\bar{m}} - p_{DC\bar{m}}) \quad (\text{v})$$

Then $p_{CD\bar{m}} = \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)}$ by (i) and

$$p_{CD\bar{m}} - p_{DC\bar{m}} = \frac{\bar{k}}{\lambda(1 + \bar{m} + \beta)} - \frac{(1 + \alpha)}{(1 + \bar{m} + \beta)} p_{CD\bar{m}} = \frac{\bar{k}}{\lambda(1 + \bar{m} + \beta)} - \frac{(1 - \lambda)}{\lambda} \frac{\beta(1 + \alpha)}{(1 + \bar{m} + \beta)(\bar{m} - \alpha)} \text{ by (v).}$$

Furthermore by (ii):

$$\begin{aligned} \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] &= 0 \Leftrightarrow (p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha + \beta) + (1 - p_{CD\bar{m}})(-\beta) = 0 \Leftrightarrow \\ (p_{CC\bar{m}} + p_{CD\bar{m}}) &= (1 - p_{CD\bar{m}}) \frac{\beta}{(\bar{m} - \alpha + \beta)} \Rightarrow p_{DC\bar{m}} + p_{DD\bar{m}} = 1 - (p_{CC\bar{m}} + p_{CD\bar{m}}) - p_{CD\bar{m}} = (1 - p_{CD\bar{m}}) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} \end{aligned}$$

In summary the equilibrium set is given by:

$$\left\{ \begin{array}{l} \left(p_{CC\bar{m}}, p_{CD\bar{m}}, p_{DC\bar{m}}, p_{DD\bar{m}}, p_{CD\underline{m}}, p_{\underline{m}}=1 \right) \\ p_{CC\bar{m}} = \left(1 - p_{CD\underline{m}} \right) \frac{\beta}{(\bar{m} - \alpha + \beta)} - p_{CD\bar{m}} \\ p_{DC\bar{m}} = p_{CD\bar{m}} - \frac{\bar{k}}{\lambda(1 + \bar{m} + \beta)} + \frac{(1 + \alpha)}{(1 + \bar{m} + \beta)} p_{CD\underline{m}} \\ p_{DD\bar{m}} = \left(1 - p_{CD\underline{m}} \right) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} - p_{DC\bar{m}} \\ p_{CD\underline{m}} = \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)} \end{array} \right\}$$

(*) and (iv) remain to be checked:

$$(iv): \bar{k}(\bar{m} - \alpha) - (1 + \alpha)\beta(1 - \lambda) < 0 \Leftrightarrow \lambda < 1 - \frac{\bar{k}}{(1 + \alpha)} \frac{(\bar{m} - \alpha)}{\beta}$$

$$(*): \lambda > 1 - \frac{(\bar{m} - \alpha)}{\beta} \left(\frac{(1 + \beta + \bar{m})\underline{k} - (1 + \alpha)\bar{k}}{(1 + \alpha)(\bar{m} - \alpha + \beta)} \right)$$

note that the lower bound is always smaller than the upper bound due to $\underline{k} > \bar{k}$

All conditions of the type

$p \in [0,1], \sum p < 1$ reduce to:

$$(1): p_{CD\bar{m}} > 0 \quad \text{true}$$

$$(2): p_{CD\bar{m}} < 1 \Leftrightarrow \lambda > \frac{\beta}{(\bar{m} - \alpha + \beta)}$$

$$(3): p_{CC\bar{m}} \geq 0: \frac{\beta}{\lambda(\bar{m} - \alpha + \beta)} \left(1 - \frac{1-\lambda}{\bar{m} - \alpha} (\bar{m} - \alpha + \beta) \right) = \frac{\beta}{(\bar{m} - \alpha + \beta)} \left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) > p_{CD\bar{m}}$$

$$p_{DC\bar{m}} \geq 0: p_{CD\bar{m}} > \frac{\bar{k}(\bar{m} - \alpha) - (1+\alpha)\beta(1-\lambda)}{\lambda(\bar{m} - \alpha)(1+\beta+\bar{m})}$$

$$p_{DD\bar{m}} \geq 0: p_{CD\bar{m}} < \left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} + \frac{\bar{k}}{\lambda(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)}$$

$$\underbrace{\frac{\bar{k}}{\lambda(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)}}_x \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{\left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) \frac{\beta}{(\bar{m} - \alpha + \beta)}}_y, \underbrace{\left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} + \frac{\bar{k}}{\lambda(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)}}_z \right\}$$

$$\text{a) } x > 0 \Leftrightarrow \lambda > 1 - \frac{\bar{k}}{(1+\alpha)} \frac{(\bar{m} - \alpha)}{\beta} \stackrel{(iv)}{\Rightarrow} x < 0$$

$$\text{b) } y > 0 \Leftrightarrow \lambda > \frac{\beta}{(\bar{m} - \alpha + \beta)}$$

$$\text{c) } z > 0 \Leftrightarrow \lambda > \frac{\beta(-\alpha(2+\alpha) + \beta + \bar{m}(2+\bar{m}+\beta)) - \bar{k}(\bar{m} - \alpha)(\bar{m} - \alpha + \beta)}{(1+\bar{m})(\bar{m} - \alpha + \beta)^2}$$

$$\begin{aligned}
z > 0 &\Leftrightarrow \left(\lambda - \frac{(1-\lambda)\beta}{(\bar{m}-\alpha)} \right) \frac{\bar{m}-\alpha}{(\bar{m}-\alpha+\beta)} + \frac{\bar{k}}{(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{(1-\lambda)\beta}{(\bar{m}-\alpha)} > 0 \Leftrightarrow \\
&\left(\lambda - \frac{\beta}{(\bar{m}-\alpha)} + \frac{\lambda\beta}{(\bar{m}-\alpha)} \right) \frac{\bar{m}-\alpha}{(\bar{m}-\alpha+\beta)} + \frac{\bar{k}}{(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{\beta}{(\bar{m}-\alpha)} + \frac{1+\alpha}{1+\bar{m}+\beta} \frac{\lambda\beta}{(\bar{m}-\alpha)} > 0 \Leftrightarrow \\
&\lambda \left(1 + \frac{1+\alpha}{1+\bar{m}+\beta} \frac{\beta}{(\bar{m}-\alpha)} \right) - \frac{\beta}{(\bar{m}-\alpha+\beta)} + \frac{\bar{k}}{(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{\beta}{(\bar{m}-\alpha)} > 0 \Leftrightarrow \\
&\lambda \left(\frac{(1+\bar{m})(\bar{m}-\alpha+\beta)}{(1+\bar{m}+\beta)(\bar{m}-\alpha)} \right) > \frac{\beta}{(\bar{m}-\alpha+\beta)} + \frac{1+\alpha}{1+\bar{m}+\beta} \frac{\beta}{(\bar{m}-\alpha)} - \frac{\bar{k}}{(1+\bar{m}+\beta)}
\end{aligned}$$

Note that for $\frac{\bar{k}}{(1+\alpha)} \geq \frac{\beta}{(\bar{m}-\alpha+\beta)}$ the necessary condition $\frac{\beta}{(\bar{m}-\alpha+\beta)} < \lambda < 1 - \frac{\bar{k}}{(1+\alpha)} \frac{(\bar{m}-\alpha)}{\beta}$ ^{3b)} ^(iv) gives an nonempty interval if

and only if $\frac{\bar{k}}{(1+\alpha)} < \frac{\beta}{(\bar{m}-\alpha+\beta)}$, hence the case $\frac{\bar{k}}{(1+\alpha)} \geq \frac{\beta}{(\bar{m}-\alpha+\beta)}$ can be neglected. It turns out that for the necessary

condition $\frac{\bar{k}}{(1+\alpha)} < \frac{\beta}{(\bar{m}-\alpha+\beta)}$, the condition 3c) is stronger than 3b)

Conditions for existence:

$$1. \frac{\bar{k}}{(1+\alpha)} < \frac{\beta}{(\bar{m}-\alpha+\beta)}$$

$$2. 0 \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{\left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m}-\alpha)}\right) \frac{\beta}{(\bar{m}-\alpha+\beta)}}_y, \underbrace{\left(1 - \frac{(1-\lambda)\beta}{\lambda(\bar{m}-\alpha)}\right) \frac{\bar{m}-\alpha}{(\bar{m}-\alpha+\beta)} + \frac{\bar{k}}{\lambda(1+\bar{m}+\beta)} - \frac{1+\alpha}{1+\bar{m}+\beta} \frac{(1-\lambda)\beta}{\lambda(\bar{m}-\alpha)}}_z \right\}$$

$$3. \max \left\{ \frac{\beta(-\alpha(2+\alpha)+\beta+\bar{m}(2+\bar{m}+\beta)) - \bar{k}(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)}{(1+\bar{m})(\bar{m}-\alpha+\beta)^2}, 1 - \frac{(\bar{m}-\alpha)}{\beta(\bar{m}-\alpha+\beta)} \underbrace{\left((1+\bar{m}+\beta) \frac{\underline{k}}{(1+\alpha)} - \bar{k} \right)}_{(*)} \right\} < \lambda \underset{(iv)}{<} 1 - \frac{\bar{k}}{(1+\alpha)} \frac{(\bar{m}-\alpha)}{\beta}$$

Note that the interval defined by 3. is non-empty due to 1.

$$2.2.2. \Pi_{\bar{m}}(DD, \underline{m}) > \Pi_{\bar{m}}(CD, \underline{m}), \text{ i.e. } p_{CC\underline{m}} = p_{CD\underline{m}} = p_{DC\underline{m}} = 0$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} = 0 \text{ is violated.}$$

Therefore such an equilibrium cannot exist.

3. All four non-signaling strategies earn the same payoffs, i.e. $\Pi_{\bar{m}}(CC, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DD, \underline{m})$

3.1. $CC, \underline{m} / DC, \underline{m} / CD, \underline{m} / DD, \underline{m}$ vs. $CC, \bar{m} / DC, \bar{m}$ i.e. $\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) > 0$

$$p_{CD\bar{m}} = p_{DD\bar{m}} = 0$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \underbrace{\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m})}_{\leq 0} + \bar{k} - \underline{k} < 0 \Rightarrow p_{\bar{m}} = 0$$

$$\text{(i)} \quad \Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] = 0$$

(i) becomes: $\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha)] = 0$ and implies that $(p_{CC\bar{m}} + p_{DC\bar{m}}) = 0$ however a **semi**-pooling equilibrium requires strict positivity for at least one of the shares.

Therefor such an equilibrium cannot exist.

3.2. $CC, \underline{m} / DC, \underline{m} / CD, \underline{m} / DD, \underline{m}$ vs. $CD, \bar{m} / DD, \bar{m}$, i.e. $\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) < 0$

$$p_{CC\bar{m}} = p_{DC\bar{m}} = 0$$

3.2.1. $\Pi_{\bar{m}}(DD, \bar{m}) > \Pi_{\bar{m}}(CD, \bar{m})$, i.e. $p_{CC\bar{m}} = p_{DC\bar{m}} = p_{CD\bar{m}} = 0$

$$\text{Then } \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \underbrace{\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m})}_{=0} + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) = 0 \text{ violates}$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) > \Pi_{\bar{m}}(CD, \bar{m})$$

3.2.2. $\Pi_{\bar{m}}(DD, \bar{m}) \leq \Pi_{\bar{m}}(CD, \bar{m})$, i.e. $p_{CC\bar{m}} = p_{DC\bar{m}} = 0$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \underbrace{\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m})}_{\leq 0} + \bar{k} - \underline{k} < 0 \Rightarrow p_{\bar{m}} = 0$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(\bar{m} - \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = \quad (i)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = \quad (ii)$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} \leq 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] \leq \quad (iv)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) + \lambda(p_{CD\bar{m}} - p_{DC\bar{m}})(\bar{m} - \alpha + \beta) = \quad (v)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CC, \underline{m}) - \lambda(p_{CD\underline{m}} - p_{DC\underline{m}})(\bar{m} - \alpha + \beta) = 0$$

a) If (iii) holds with equality then the last equality implies that $p_{CD\bar{m}} - p_{DC\bar{m}} = 0 \Rightarrow p_{CD\bar{m}} = 0$ then (ii) becomes:

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] = 0 \Rightarrow p_{DD\bar{m}} = 0 \text{ Hence such a **semi**-pooling equilibrium cannot exist.}$$

b) If (iii) holds as a strict inequality then $p_{DD\bar{m}} = 0$ and (ii) becomes $\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(-\beta) \right] = 0$ which holds only for $p_{CD\bar{m}} = 0$, i.e. in a pooling but not **semi**-pooling equilibrium.

4. 1-2 strategies versus 1-2 strategies

In general there are 36 possible matchings: CC, DC, CC and DC, CD, DD, CD and DD, six for each signaling strategy, however as summarized in Table Fehler! Kein Text mit angegebener Formatvorlage im Dokument.-3 we excluded 21 of them; in the following we consider the remaining 14 cases:

	CC, \underline{m}	DC, \underline{m}	$CC, \underline{m} / DC, \underline{m}$	CD, \underline{m}	DD, \underline{m}	$CD, \underline{m} / DD, \underline{m}$
CC, \bar{m}	N (2.)	N (4.1.1)	N (2.)	(4.1.3.)	N (4.1.2.)	N (4.1.2.)
DC, \bar{m}	N (7.)	N (3.)	N (3.)	(4.2.3.2./4.2.3.3.)	N (4.2.1.)	N (4.2.2.)
$CC, \bar{m} / DC, \bar{m}$	N (2.)	N (3.)	N (3.)	(4.3.1.)	N (6.)	N (4.3.2.)
CD, \bar{m}	N (4.)	(4.4.1.)	N (4.)	N (5.)	(4.4.2.)	N (5.)
DD, \bar{m}	N (4.)	N (4.5.)	N (4.)	N (5.)	N (1.)	N (5.)
$CD, \bar{m} / DD, \bar{m}$	N (4.)	(4.6.1.)	N (4.)	N (5.)	(4.6.2.)	N (5.)

Table Fehler! Kein Text mit angegebener Formatvorlage im Dokument.-3: Overview of subcases; N: non-existence of the considered equilibrium; number in parenthesis either refers to the list of payoff differences below Table Fehler! Kein Text mit angegebener Formatvorlage im Dokument.-2 or subsection dealing with the corresponding case.

4.1. CC, \bar{m}

4.1.1. and DC, \underline{m} ($p_{\bar{m}} = 0$)

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CC, \bar{m})] = \quad (i)$$

$$\lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} + \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)]$$

$$= \lambda \left[(p_{CD\bar{m}})(1 + \bar{m}) - p_{DC\bar{m}}(1 + \alpha + \beta) + (p_{CC\bar{m}})(\bar{m} - \alpha) - p_{DD\bar{m}}\beta + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (v)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha) - p_{DC\bar{m}}(1 + \bar{m} + \beta)] - \bar{k} = 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda [p_{DC\bar{m}}(-\beta)] - \beta(1 - \lambda) > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda [p_{DC\bar{m}}(\bar{m} - \alpha)] - \beta(1 - \lambda) > 0 \quad (v)$$

By (iii) such a semi-pooling equilibrium cannot exist.

4.1.2. and DD, \underline{m} or CD, \underline{m} / DD, \underline{m} ($p_{\bar{m}} = 0$)

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda [(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] \leq 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda [(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda [(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda [(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = 0 \quad (v)$$

→

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha)] \leq 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda [p_{CD\bar{m}}(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta)] - \beta(1 - \lambda) > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda [p_{CC\bar{m}}(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta)] - \beta(1 - \lambda) < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = 0 \quad (v)$$

By (i) such a **semi**-pooling equilibrium cannot exist.

4.1.3. and CD, \underline{m}

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \quad (v)$$

$$= \lambda[(p_{CD\bar{m}})(1 + \bar{m}) + (p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) - p_{DC\bar{m}}(1 + \alpha + \beta) - p_{DD\bar{m}}\beta] + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$$

$$= \lambda[(p_{CD\bar{m}})(1 + \bar{m}) + (-p_{DC\bar{m}})(1 + \bar{m} + \beta) - p_{DD\bar{m}}\beta] + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$$

$$= \lambda[(p_{CD\bar{m}})(1 + \bar{m})] + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k} = 0$$

→

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda[(p_{CC\bar{m}})(\bar{m} - \alpha)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (i)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda[(p_{CC\bar{m}})(\bar{m} - \alpha)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda[(p_{CD\bar{m}})(\bar{m} - \alpha)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda[(p_{CD\bar{m}})(-\beta)] + (1 - \lambda)[p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda[(p_{CD\bar{m}})(1 + \bar{m})] + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k} = 0 \quad (v)$$

The sum of (ii) and (iii) implies: $\lambda(\bar{m} - \alpha + \beta) > \beta$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha)] - \underline{k} = \lambda p_{CD\underline{m}}(1 + \alpha) - \underline{k} \quad (vi)$$

$$4.1.3.1. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) > 0 \Leftrightarrow p_{CD\underline{m}} > \frac{\underline{k}}{\lambda(1+\alpha)} (\Rightarrow p_{\underline{m}} = 0)$$

Then $p_{CD\underline{m}} = \frac{\bar{k}}{\lambda(1+\bar{m})}$, $p_{CC\bar{m}} = 1 - \frac{\bar{k}}{\lambda(1+\bar{m})}$ by (v) and (iii) is satisfied and $\frac{(1-\lambda)\beta}{\lambda(\bar{m}-\alpha)} < p_{CC\bar{m}}$ by (ii)

Since $\frac{\bar{k}}{\lambda(1+\bar{m})} < \frac{\underline{k}}{\lambda(1+\alpha)}$ such an equilibrium cannot exist.

$$4.1.3.2. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) < 0 \Leftrightarrow p_{CD\underline{m}} < \frac{\underline{k}}{\lambda(1+\alpha)} (\Rightarrow p_{\underline{m}} = 1)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\underline{m}})(1+\bar{m}) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k} = 0 \quad (\text{v})$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1+\alpha) \right] - \underline{k} = \lambda p_{CD\underline{m}}(1+\alpha) - \underline{k} < 0 \quad (\text{vi})$$

Then $p_{CD\underline{m}} = \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}$, $p_{CC\bar{m}} = 1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}$ by (v) and (ii) is satisfied.

Conditions (iii) and (vi) need to be checked.

$$(ii): \lambda > \frac{\bar{k} + \beta}{(1 + \bar{m} + \beta)}$$

$$(iii): \lambda > 1 - \frac{(\bar{m} - \alpha)}{\beta} \frac{\bar{k}}{(1 + \alpha)}$$

$$(vi): \lambda > 1 - \frac{1}{\beta} \left(\frac{1 + \bar{m}}{(1 + \alpha)} \underline{k} - \bar{k} \right)$$

It turns out, that (iii) implies (vi), hence we are left with:

$$(ii) \wedge (iii): \lambda > \max \left\{ \frac{\bar{k} + \beta}{(1 + \bar{m} + \beta)}, 1 - \frac{(\bar{m} - \alpha)}{\beta} \frac{\bar{k}}{(1 + \alpha)} \right\}$$

$$= \begin{cases} \frac{\bar{k} + \beta}{(1 + \bar{m} + \beta)} & , \frac{\bar{k}}{(1 + \alpha)} > \frac{\beta}{(\bar{m} - \alpha + \beta)} \\ 1 - \frac{(\bar{m} - \alpha)}{\beta} \frac{\bar{k}}{(1 + \alpha)} & , \frac{\bar{k}}{(1 + \alpha)} \leq \frac{\beta}{(\bar{m} - \alpha + \beta)} \end{cases}$$

Finally all condition of the type:

$p \in [0, 1], \sum p < 1$ reduce to:

$$(1): p_{CC\bar{m}} < 1 \quad \text{true}$$

$$(2): p_{CC\bar{m}} > 0 \Leftrightarrow \lambda > \frac{\beta + \bar{k}}{1 + \bar{m} + \beta}$$

$$, \text{ hence } \max \left\{ 1 - \frac{(\bar{m} - \alpha)}{\beta} \frac{\bar{k}}{(1 + \alpha)}, \frac{\beta + \bar{k}}{1 + \bar{m} + \beta} \right\} < \lambda < 1, \text{ hence we need } \frac{\beta + \bar{k}}{1 + \bar{m} + \beta} < 1 \Leftrightarrow \bar{k} < 1 + \bar{m}$$

To summarize: $\left\{ p_{\underline{m}} = 1, p_{CD\underline{m}} = \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}, p_{CC\bar{m}} = 1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right\}$

Conditions for existence:

$$1. (1+\alpha) \frac{\beta}{(\bar{m}-\alpha+\beta)} < \bar{k} < 1+\bar{m}: \quad \frac{\bar{k} + \beta}{(1+\bar{m} + \beta)} < \lambda < 1$$

$$2. \bar{k} \leq (1+\alpha) \frac{\beta}{(\bar{m}-\alpha+\beta)}: \quad 1 - \frac{(\bar{m}-\alpha)}{\beta} \frac{\bar{k}}{(1+\alpha)} < \lambda < 1$$

4.1.3.3. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = 0$

then $p_{CD\underline{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}, p_{CC\bar{m}} = 1 - \frac{\underline{k}}{\lambda(1+\alpha)}$ by (vi) and shares for low types are given by (v): $\frac{\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k}}{(1-\lambda)\beta} = p_{\underline{m}}$

furthermore by (ii) and (iii): $\frac{(1-\lambda)p_{\bar{m}}\beta}{\lambda(\bar{m}-\alpha)} < p_{CC\bar{m}} < 1 - \frac{(1-\lambda)p_{\underline{m}}\beta}{\lambda(\bar{m}-\alpha)}$

$$\begin{aligned}
& \frac{(1-\lambda)\beta - \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k}}{\lambda(\bar{m}-\alpha)} < 1 - \frac{\underline{k}}{\lambda(1+\alpha)} < 1 - \frac{\frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k}}{\lambda(\bar{m}-\alpha)} \Leftrightarrow \\
& \frac{(1-\lambda)\beta - \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k}}{(\bar{m}-\alpha)} < \lambda - \frac{\underline{k}}{(1+\alpha)} < \lambda - \frac{\frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k}}{(\bar{m}-\alpha)} \Leftrightarrow \\
& (1-\lambda)\beta - \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k} - \lambda(\bar{m}-\alpha) < -\frac{(\bar{m}-\alpha)}{(1+\alpha)}\underline{k} < -\frac{(1+\bar{m})}{(1+\alpha)}\underline{k} + \bar{k} \Leftrightarrow \\
& \beta - \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k} - \lambda(\bar{m}-\alpha + \beta) + \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} < \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \frac{(\bar{m}-\alpha)}{(1+\alpha)}\underline{k} < +\bar{k} \Leftrightarrow \\
& \beta - \bar{k} - \lambda(\bar{m}-\alpha + \beta) < \underline{k} < \bar{k}
\end{aligned}$$

The last inequality is violated; hence such an equilibrium cannot exist.

4.2. DC, \bar{m}

4.2.1. and DD, \underline{m} ($p_{\bar{m}} = 0$)

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] < \quad (iii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = 0 \quad (v)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] - \beta(1-\lambda) > 0 \quad (ii)$$

BBy (ii) such a semi-pooling equilibrium cannot exist.

4.2.2. and $CD, \underline{m} / DD, \underline{m}$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(\bar{m} - \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = 0 \quad (v)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda p_{DC\bar{m}}(-\beta) + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda p_{DC\bar{m}}(\bar{m} - \alpha) + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (iv)$$

$$\begin{aligned} \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) &= \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) - [\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m})] = \lambda(1 + \alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} \quad (v) \\ &+ \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] = 0 \end{aligned}$$

(iv) requires $p_{\bar{m}} > 0$, i.e.

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} \geq 0 \quad (*)$$

(i) and (iii) are always satisfied, note that (*) and (v) violate (ii):

$$\lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \alpha) \right] \geq \underline{k} \text{ by } (*) \text{ and } \underbrace{\lambda(1 + \alpha) \left[(p_{CD\underline{m}} - p_{DC\bar{m}}) \right] - \bar{k}}_{>0} + \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] = 0 \text{ by}$$

$$(v), \text{ hence } \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] < 0$$

Hence such a semi-pooling equilibrium cannot exist.

4.2.3. and CD, \underline{m}

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{DC\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\underline{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] < 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) - \left[\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) \right] = 0 \quad (v)$$

(i) and (iii) are always satisfied

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iv)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} - (1 - \lambda) \left[p_{\bar{m}}(-\beta) - p_{\underline{m}}(-\beta) \right] = 0 \quad (v)$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} \quad (*)$$

4.2.3.1. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) > 0$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda)(-\beta) > 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0 \quad (\text{v})$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} > 0 \Rightarrow \textcolor{blue}{p}_{\bar{m}} = 1 \quad (*)$$

(ii) is satisfied; (*) violates (v) because

$$\lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0 \Rightarrow \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} < 0 \Rightarrow \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} < 0$$

Hence such a semi-pooling equilibrium cannot exist.

4.2.3.2. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = 0$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda)p_{\underline{m}} > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda)p_{\bar{m}} > 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} - (1 - \lambda)[p_{\bar{m}}(-\beta) - p_{\underline{m}}(-\beta)] = 0 \quad (\text{v})$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} = 0 \Leftrightarrow p_{CD\bar{m}} - p_{DC\bar{m}} = \frac{\underline{k}}{\lambda(1 + \alpha)} \quad (*)$$

$$p_{CD\bar{m}} - p_{DC\bar{m}} = \frac{\underline{k}}{\lambda(1 + \alpha)} \Rightarrow \textcolor{blue}{p}_{CD\bar{m}} = \frac{1}{2} \left[1 + \frac{\underline{k}}{\lambda(1 + \alpha)} \right], \textcolor{blue}{p}_{DC\bar{m}} = \frac{1}{2} \left[1 - \frac{\underline{k}}{\lambda(1 + \alpha)} \right] \text{ by } (*), \text{ if we plug in these values in (iv) and (ii)}$$

$$p_{\bar{m}} < \frac{\lambda(\bar{m} - \alpha)}{\beta(1 - \lambda)} p_{DC\bar{m}} = \frac{\lambda(\bar{m} - \alpha)}{\beta(1 - \lambda)} \frac{1}{2} \left[1 - \frac{\underline{k}}{\lambda(1 + \alpha)} \right] \quad (\text{iv})$$

$$p_{\underline{m}} < \frac{\lambda(\bar{m} - \alpha)}{\beta(1 - \lambda)} p_{CD\bar{m}} = \frac{\lambda(\bar{m} - \alpha)}{\beta(1 - \lambda)} \frac{1}{2} \left[1 + \frac{\underline{k}}{\lambda(1 + \alpha)} \right] \quad (\text{ii})$$

Furthermore: $\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}=(1-\lambda)\beta[p_{\underline{m}}-p_{\bar{m}}]\Leftrightarrow \textcolor{blue}{p}_{\underline{m}}=\frac{1}{2}\left[1+\frac{1}{(1-\lambda)\beta}\left[\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right]\right], \textcolor{blue}{p}_{\bar{m}}=\frac{1}{2}\left[1-\frac{1}{(1-\lambda)\beta}\left[\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right]\right]$ by (v)

w.r.t. (iv):

$$\begin{aligned} & \frac{1}{2}\left[1-\frac{1}{(1-\lambda)\beta}\left[\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right]\right]<\frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)}\frac{1}{2}\left[1-\frac{\underline{k}}{\lambda(1+\alpha)}\right]\Leftrightarrow \\ & 1-\frac{1}{(1-\lambda)\beta}\left[\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right]<\frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)}\left[1-\frac{\underline{k}}{\lambda(1+\alpha)}\right]\Leftrightarrow \\ & 1-\frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)}<\frac{1}{(1-\lambda)\beta}\left[\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right]+\frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)}\frac{\underline{k}}{\lambda(1+\alpha)}\Leftrightarrow \\ & \beta-\lambda(\bar{m}-\alpha+\beta)<\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}+(\bar{m}-\alpha)\frac{\underline{k}}{(1+\alpha)}\Leftrightarrow \\ & \beta-\lambda(\bar{m}-\alpha+\beta)<\underline{k}-\bar{k} \end{aligned}$$

w.r.t. (ii):

$$\begin{aligned}
& \frac{1}{2} \left[1 + \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right] < \frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)} \frac{1}{2} \left[1 + \frac{\underline{k}}{\lambda(1+\alpha)} \right] \Leftrightarrow \\
& \left[1 + \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right] < \frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)} + \frac{\lambda(\bar{m}-\alpha)}{\beta(1-\lambda)} \frac{\underline{k}}{\lambda(1+\alpha)} \Leftrightarrow \\
& \left[(1-\lambda)\beta + \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right] < \lambda(\bar{m}-\alpha) + (\bar{m}-\alpha) \frac{\underline{k}}{(1+\alpha)} \Leftrightarrow \\
& \beta - \lambda(\bar{m}-\alpha + \beta) < -\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} + \bar{k} + (\bar{m}-\alpha) \frac{\underline{k}}{(1+\alpha)} \Leftrightarrow \\
& \beta - \lambda(\bar{m}-\alpha + \beta) < -(\underline{k} - \bar{k}) \Leftrightarrow \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m}-\alpha + \beta)} < \lambda
\end{aligned}$$

We observe that (ii) is binding.

Finally all condition of the type

$p \in [0,1], \sum p < 1$ reduce to:

$$(1): p_{CD\underline{m}} > 0 \quad \text{true}$$

$$(2): p_{CD\underline{m}} < 1 \Leftrightarrow \frac{\underline{k}}{(1+\alpha)} < \lambda$$

$$(3): p_{\underline{m}} > 0 \quad \text{true}$$

$$(4): p_{\underline{m}} < 1 \Leftrightarrow \lambda < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right]$$

$$\text{i.e. } 0 < \frac{\underline{k}}{(1+\alpha)} < \lambda < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] < 1$$

$$\text{note: } \frac{\underline{k}}{(1+\alpha)} < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \Leftrightarrow \frac{(1+\bar{m}+\beta)}{\beta(1+\alpha)} \underline{k} < 1 + \frac{\bar{k}}{\beta} \Leftrightarrow (1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k},$$

To summarize:

$$\left\{ p_{CD\underline{m}} = \frac{1}{2} \left[1 + \frac{\underline{k}}{\lambda(1+\alpha)} \right], p_{DC\bar{m}} = \frac{1}{2} \left[1 - \frac{\underline{k}}{\lambda(1+\alpha)} \right], \textcolor{blue}{p}_{\underline{m}} = \frac{1}{2} \left[1 + \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right], \textcolor{blue}{p}_{\bar{m}} = \frac{1}{2} \left[1 - \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right] \right\}$$

Conditions:

$$(1): 0 < \max \left\{ \frac{\underline{k}}{(1+a)}, \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - a + b)} \right\} < \lambda < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+a)} \underline{k} - \bar{k} \right] < 1$$

$$\text{note: } a) \frac{\underline{k}}{(1+\alpha)} < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \Leftrightarrow \frac{(1+\bar{m}+\beta)}{\beta(1+\alpha)} \underline{k} < 1 + \frac{\bar{k}}{\beta} \Leftrightarrow (1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}$$

$$\begin{aligned} b) \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] &\Leftrightarrow \frac{(\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} < \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \\ &\Leftrightarrow \underline{k} - \bar{k} < \bar{m} - \alpha - \frac{(\bar{m} - \alpha + \beta)}{\beta(1+\alpha)} [(1+\bar{m})\underline{k} - (1+\alpha)\bar{k}] \\ &\Leftrightarrow \frac{\underline{k} - \bar{k}}{\bar{m} - \alpha} \beta(1+\alpha) < \beta(1+\alpha) - \frac{(\bar{m} - \alpha + \beta)}{(\bar{m} - \alpha)} [(1+\bar{m})\underline{k} - (1+\alpha)\bar{k}] \\ &\Leftrightarrow \beta(1+\alpha) > \frac{(\bar{m} - \alpha + \beta)}{(\bar{m} - \alpha)} [(1+\bar{m})\underline{k} - (1+\alpha)\bar{k}] + \frac{\underline{k} - \bar{k}}{\bar{m} - \alpha} \beta(1+\alpha) \\ &= \frac{(\bar{m} - \alpha + \beta)(1+\bar{m}) - \beta(1+\alpha)}{(\bar{m} - \alpha)} \underline{k} - \frac{(\bar{m} - \alpha)(1+\alpha)}{(\bar{m} - \alpha)} \bar{k} = \frac{(\bar{m} - \alpha)(1+\bar{m} + \beta)}{(\bar{m} - \alpha)} \underline{k} - \frac{(\bar{m} - \alpha)(1+\alpha)}{(\bar{m} - \alpha)} \bar{k} = \bar{k}(1+\bar{m} + \beta)\underline{k} - (1+\alpha)\bar{k} \end{aligned}$$

,i.e. the necessary condition for a non-empty interval a) is weaker than the b)

$$c) \frac{\underline{k}}{(1+\alpha)} < \frac{\beta - (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} \left(< \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} \right) \Leftrightarrow (\bar{m} - \alpha + \beta)\underline{k} < (1+\alpha)\beta - (1+\alpha)(\underline{k} - \bar{k})$$

$$\Leftrightarrow (1+\alpha)\beta > (\bar{m} - \alpha + \beta)\underline{k} + (1+\alpha)(\underline{k} - \bar{k}) = (\bar{m} - \alpha + \beta)\underline{k} + (1+\alpha)(\underline{k} - \bar{k}) = (1+\bar{m} + \beta)\underline{k} - (1+\alpha)\bar{k}$$

$$\text{,i.e. the necessary condition for a non-empty interval a) implies that } \frac{\underline{k}}{(1+\alpha)} < \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)}$$

Conditions for existence:

$$(1): 0 < \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - a + b)} < \lambda < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+a)} \underline{k} - \bar{k} \right] < 1$$

$$(2): \beta(1+\alpha) > \frac{(\bar{m} - \alpha + \beta)}{(\bar{m} - \alpha)} [(1+\bar{m})\underline{k} - (1+\alpha)\bar{k}] + \frac{\underline{k} - \bar{k}}{\bar{m} - \alpha} \beta(1+\alpha)$$

4.2.3.3. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) < 0$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda) > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} - (1 - \lambda)\beta = 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] > 0 \quad (\text{v})$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} < 0 \Rightarrow \textcolor{blue}{p_{\bar{m}}} = 0 \quad (*)$$

(v) is satisfied; $\textcolor{blue}{p_{CD\underline{m}}} = \frac{1}{2} \left[1 + \frac{\bar{k} + (1 - \lambda)\beta}{\lambda(1 + \bar{m})} \right], \textcolor{blue}{p_{DC\bar{m}}} = \frac{1}{2} \left[1 - \frac{\bar{k} + (1 - \lambda)\beta}{\lambda(1 + \bar{m})} \right]$ by (iv),

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda)p_{\underline{m}} > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda)p_{\bar{m}} > 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} - (1 - \lambda) \left[p_{\bar{m}}(-\beta) - p_{\underline{m}}(-\beta) \right] = 0 \quad (\text{v})$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} = 0 \Leftrightarrow p_{CD\underline{m}} - p_{DC\bar{m}} = \frac{\underline{k}}{\lambda(1 + \alpha)} \quad (*)$$

note:

$$1.(iv) - (*): \frac{\bar{k} + \beta}{(1 + \bar{m} + \beta)} - 1 + \frac{1}{\beta(1 + \alpha)} \left((1 + \bar{m})\underline{k} - (1 + \alpha)\bar{k} \right) > 0 \Leftrightarrow \frac{\bar{k}}{(1 + \bar{m} + \beta)} - \frac{1 + \bar{m}}{(1 + \bar{m} + \beta)} + \frac{(1 + \bar{m})\underline{k}}{\beta(1 + \alpha)} - \frac{\bar{k}}{\beta} > 0$$

$$\Leftrightarrow -\frac{(1 + \bar{m})}{(1 + \bar{m} + \beta)\beta} \bar{k} - \frac{1 + \bar{m}}{(1 + \bar{m} + \beta)} + \frac{(1 + \bar{m})}{\beta(1 + \alpha)} \underline{k} > 0 \Leftrightarrow \underline{k} - \frac{(1 + \alpha)}{(1 + \bar{m} + \beta)} \bar{k} > \frac{\beta(1 + \alpha)}{(1 + \bar{m} + \beta)} \Leftrightarrow (1 + \bar{m} + \beta)\underline{k} - (1 + \alpha)\bar{k} > \beta(1 + \alpha)$$

$$2.(iv) - (ii): \frac{\bar{k} + \beta}{(1 + \bar{m} + \beta)} - 1 + \frac{(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta} (1 + \bar{m} + \bar{k}) > 0 \Leftrightarrow$$

$$\frac{\bar{k}}{(1 + \bar{m} + \beta)} - \frac{1 + \bar{m}}{(1 + \bar{m} + \beta)} + \frac{(\bar{m} - \alpha)(1 + \bar{m})}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta} + \frac{(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta} \bar{k} > 0 \Leftrightarrow$$

$$\left(\frac{1}{(1 + \bar{m} + \beta)} + \frac{(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta} \right) \bar{k} > \left(\frac{1}{(1 + \bar{m} + \beta)} - \frac{(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta} \right) (1 + \bar{m}) \Leftrightarrow$$

$$\frac{\bar{k}}{(1 + \bar{m})} > \frac{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta - (1 + \bar{m} + \beta)(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1 + \bar{m}) + (1 + \alpha)\beta + (1 + \bar{m} + \beta)(\bar{m} - \alpha)} = \frac{(1 + \alpha)\beta}{(1 + \bar{m})(\bar{m} - \alpha + \beta)} \Leftrightarrow \frac{\bar{k}}{(1 + \alpha)} > \frac{\beta}{(\bar{m} - \alpha + \beta)} \Leftrightarrow (\bar{m} - \alpha + \beta)\bar{k} > (1 + \alpha)\beta$$

$$\begin{aligned}
& 3.(*)-(ii): 1 - \frac{1}{\beta(1+\alpha)} \left((1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right) - 1 + \frac{(\bar{m}-\alpha)}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} (1+\bar{m}+\bar{k}) > 0 \Leftrightarrow \\
& \frac{(\bar{m}-\alpha)}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} (1+\bar{m}+\bar{k}) > \frac{1}{\beta(1+\alpha)} \left((1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right) \Leftrightarrow \\
& \frac{(\bar{m}-\alpha)(1+\bar{m})}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} > \frac{(1+\bar{m})}{\beta(1+\alpha)} \underline{k} - \frac{\bar{k}}{\beta} - \frac{(\bar{m}-\alpha)}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \bar{k} \Leftrightarrow \\
& 1 > \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{\beta(1+\alpha)(\bar{m}-\alpha)} \underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{\beta(\bar{m}-\alpha)(1+\bar{m})} \bar{k} - \frac{1}{(1+\bar{m})} \bar{k} \Leftrightarrow \\
& 1 > \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{\beta(1+\alpha)(\bar{m}-\alpha)} \underline{k} - \frac{(\bar{m}-\alpha+2\beta)}{\beta(\bar{m}-\alpha)} \bar{k} \Leftrightarrow \beta(1+\alpha) > \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)} \underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)} \bar{k} \\
& \Leftrightarrow \beta(1+\alpha) > \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)} \underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)} \bar{k}
\end{aligned}$$

$$(1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} > (\bar{m}-\alpha+\beta)\bar{k} \Rightarrow ((iv)-(ii) > 0 \Rightarrow (iv)-(*) > 0)$$

$$(1+\alpha)\beta < (\bar{m}-\alpha+\beta)\bar{k} : (iv)$$

$$(\bar{m}-\alpha+\beta)\bar{k} < (1+\alpha)\beta < (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} : (ii)$$

$$(1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} < (1+\alpha)\beta : (*) \text{ oder } (ii)$$

$$\begin{aligned}
& \beta(1+\alpha) > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} \\
& \beta(1+\alpha) > \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)}\bar{k} \\
& (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} - \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} + \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)}\bar{k} \\
& = \frac{(1+\bar{m}+\beta)(\bar{m}-\alpha) - (\bar{m}-\alpha+\beta)(1+\bar{m}) - (1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} + \frac{(\bar{m}-\alpha+\beta)(1+\alpha) + (1+\alpha)\beta - (1+\alpha)(\bar{m}-\alpha)}{(\bar{m}-\alpha)}\bar{k} \\
& = \frac{-2(1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} + \frac{2(1+\alpha)\beta}{(\bar{m}-\alpha)}\bar{k} = \frac{2(1+\alpha)\beta}{(\bar{m}-\alpha)}(\bar{k} - \underline{k}) < 0 \Rightarrow \\
& (1+\alpha)\beta < (\bar{m}-\alpha+\beta)\bar{k} : (iv) \\
& (\bar{m}-\alpha+\beta)\bar{k} < (1+\alpha)\beta < (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} : (ii) \\
& (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} < (1+\alpha)\beta < \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)}\bar{k} : (ii) \\
& \frac{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}{(\bar{m}-\alpha)}\underline{k} - \frac{(\bar{m}-\alpha+\beta)(1+\alpha)+(1+\alpha)\beta}{(\bar{m}-\alpha)}\bar{k} < (1+\alpha)\beta : (*)
\end{aligned}$$

Finally all condition of the type

$p \in [0,1], \sum p < 1$ reduce to:

(1): $p_{CD_m} > 0$ true ,

(2): $p_{CD_m} < 1 \Leftrightarrow \lambda > \frac{\bar{k} + \beta}{(1+\bar{m}+\beta)}$

To summarize:

$$\left\{ \begin{aligned} p_{CD\bar{m}} &= \frac{1}{2} \left[1 + \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right], p_{DC\bar{m}} = \frac{1}{2} \left[1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right], p_{\bar{m}} = 0 \end{aligned} \right\}$$

Conditions for Existence:

$$\lambda > \max \left\{ \frac{\bar{k} + \beta}{(1+\bar{m} + \beta)}, 1 - \frac{1}{\beta(1+\alpha)} \left((1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right), 1 - \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta} (1+\bar{m} + \bar{k}) \right\}$$

$$= \begin{cases} \frac{\bar{k} + \beta}{(1+\bar{m} + \beta)}, & (1+\alpha)\beta < (\bar{m} - \alpha + \beta)\bar{k} \\ 1 - \frac{(\bar{m} - \alpha)}{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta} (1+\bar{m} + \bar{k}), & (\bar{m} - \alpha + \beta)\bar{k} < (1+\alpha)\beta < \frac{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta}{(\bar{m} - \alpha)} \underline{k} - \frac{(\bar{m} - \alpha + \beta)(1+\alpha) + (1+\alpha)\beta}{(\bar{m} - \alpha)} \bar{k} \\ 1 - \frac{1}{\beta(1+\alpha)} \left((1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right), & \frac{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta}{(\bar{m} - \alpha)} \underline{k} - \frac{(\bar{m} - \alpha + \beta)(1+\alpha) + (1+\alpha)\beta}{(\bar{m} - \alpha)} \bar{k} < (1+\alpha)\beta \end{cases}$$

4.3. $CC, \bar{m} / DC, \bar{m}$

4.3.1. CD, \underline{m}

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) - \left[\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CC, \bar{m}) \right] = \quad (v)$$

$$\lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\bar{m} + \beta) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1+\alpha) \right] - \bar{k} + \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] =$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1+\alpha) \right] - \underline{k} \quad (*)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(-p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k} + \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] = \quad (v)$$

$$\lambda \left[(-p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\bar{m}})(1 + \bar{m}) \right] - \bar{k} + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] =$$

$$\lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) - p_{DC\bar{m}}\beta \right] - \bar{k} + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] = 0$$

$$\Pi_{\bar{m}}(\bar{m}) - \Pi_{\bar{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\bar{m}} - p_{DC\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} \quad (*)$$

(iv) is always satisfied in a semi-pooling equilibrium, (i) implies (iii) if DC is played by strictly positive share.

4.3.1.1. $\Pi_{\bar{m}}(\bar{m}) - \Pi_{\bar{m}}(\underline{m}) > 0$

$$\Pi_{\bar{m}}(\bar{m}) - \Pi_{\bar{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} > 0 \Rightarrow p_{\bar{m}} = 1 \quad (*)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}})(-\beta) \right] - (1 - \lambda)\beta = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) \right] - \beta > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) - p_{DC\bar{m}}\beta \right] - \bar{k} = 0 \quad (v)$$

$$(*) \text{ and (v) imply : } \lambda \left[\frac{\underline{k}}{\lambda(1+\alpha)}(1+\bar{m}) - p_{DC\bar{m}}\beta \right] - \bar{k} < 0 \Leftrightarrow \frac{(1+\bar{m})}{(1+\alpha)}\underline{k} - \bar{k} < \lambda\beta p_{DC\bar{m}}$$

(i) and (v) give:

$$p_{DC\bar{m}} = \frac{(1+\bar{m})(\lambda(\bar{m}-\alpha+\beta)-\beta) - \bar{k}(\bar{m}-\alpha)}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta) - \beta(1+\alpha))}$$

$$p_{CC\bar{m}} = \frac{\beta(2(1+\bar{m}) + \beta - \lambda(\bar{m}-\alpha+\beta) - \bar{k})}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta) - \beta(1+\alpha))}$$

$$p_{CD\bar{m}} = \frac{\bar{k}(\bar{m}-\alpha+\beta) + (1+\bar{m}+\beta)(\lambda(\bar{m}-\alpha+\beta) - \beta)}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta) - \beta(1+\alpha))}$$

Given this solution for the respective shares (*) and (iii) need to be checked for.

$$(iii) : -(\bar{m}-\alpha+\beta) \frac{(1+\bar{m})(\lambda(\bar{m}-\alpha+\beta)-\beta) - \bar{k}(\bar{m}-\alpha)}{(2(1+\bar{m})(\bar{m}-\alpha+\beta) - \beta(1+\alpha))} = -\lambda(\bar{m}-\alpha+\beta) p_{DC\bar{m}} < 0$$

$$(*) : \frac{(1+\alpha)(\bar{k}(2\bar{m}-2\alpha+\beta) + \beta(\lambda(\bar{m}-\alpha+\beta) - \beta))}{(2(1+\bar{m})(\bar{m}-\alpha+\beta) - \beta(1+\alpha))} > \underline{k}$$

\Leftrightarrow

$$(iii): \frac{\beta}{(\bar{m}-\alpha+\beta)} + \frac{\bar{k}(\bar{m}-\alpha)}{(1+\bar{m})(\bar{m}-\alpha+\beta)} = \frac{\beta(1+\bar{m})+\bar{k}(\bar{m}-\alpha)}{(1+\bar{m})(\bar{m}-\alpha+\beta)} < \lambda < 1 \quad \wedge \quad \bar{k} < 1+\bar{m}$$

$$(*) : \frac{(1+\alpha)(\bar{k}(2\bar{m}-2\alpha+\beta)+\beta(\lambda(\bar{m}-\alpha+\beta)-\beta))}{(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))} > \underline{k}$$

$$(1+\alpha)(\beta(\lambda(\bar{m}-\alpha+\beta)-\beta)) > \underline{k}(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha)) - \bar{k}(2(1+\alpha)(\bar{m}-\alpha+\beta)-(1+\alpha)\beta)$$

$$(1+\alpha)(\beta(\lambda(\bar{m}-\alpha+\beta)-\beta)) > 2(\bar{m}-\alpha+\beta)(\underline{k}(1+\bar{m})-\bar{k}(1+\alpha)) - (1+\alpha)\beta(\underline{k}-\bar{k})$$

$$\lambda(\bar{m}-\alpha+\beta)-\beta > 2\frac{(\bar{m}-\alpha+\beta)}{\beta}\left(\underline{k}\frac{(1+\bar{m})}{(1+\alpha)}-\bar{k}\right) - (\underline{k}-\bar{k})$$

$$\lambda > \frac{\beta-(\underline{k}-\bar{k})}{(\bar{m}-\alpha+\beta)} + \frac{2}{\beta}\left(\underline{k}\frac{(1+\bar{m})}{(1+\alpha)}-\bar{k}\right)$$

Finally all condition of the type

$$p \in [0,1], \sum p < 1 \text{ reduce to: } \frac{\beta(1+\bar{m})+\bar{k}(\bar{m}-\alpha)}{(1+\bar{m})(\bar{m}-\alpha+\beta)} < \lambda < 1 \quad \wedge \quad \bar{k} < 1+\bar{m} \text{ (derived with mathematica)}$$

$$p_{CD\bar{m}} > 0 : \lambda > \frac{\beta}{(\bar{m}-\alpha+\beta)} - \frac{\bar{k}}{(1+\bar{m}+\beta)}$$

$$p_{CD\bar{m}} < 1 :$$

$$\frac{\bar{k}(\bar{m}-\alpha+\beta)+(1+\bar{m}+\beta)(\lambda(\bar{m}-\alpha+\beta)-\beta)}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))} < 1$$

$$\Leftrightarrow \bar{k}(\bar{m}-\alpha+\beta)-\beta(1+\bar{m}+\beta) < \lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha)) - (1+\bar{m}+\beta)(\bar{m}-\alpha+\beta)$$

$$\Leftrightarrow \bar{k}(\bar{m}-\alpha+\beta)-\beta(1+\bar{m}+\beta) < \lambda((1+\bar{m})(\bar{m}-\alpha)-\beta^2)$$

Note that: $\frac{2}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} \frac{(1+\bar{m})}{(1+\alpha)} - \bar{k} \right) + \frac{\beta - (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} > \frac{\beta + (\bar{m} - \alpha) \frac{\underline{k}}{(1+\alpha)}}{(\bar{m} - \alpha + \beta)}$, since $\frac{\underline{k}}{(1+\alpha)} \left(1 + \bar{m} + \frac{1+\alpha}{2 \frac{(\bar{m} - \alpha + \beta)}{\beta} - 1} \right) > \bar{k}$.

$$p_{DC\bar{m}} = \frac{(1+\bar{m})(\lambda(\bar{m} - \alpha + \beta) - \beta) - \bar{k}(\bar{m} - \alpha)}{\lambda(2(1+\bar{m})(\bar{m} - \alpha + \beta) - \beta(1+\alpha))}$$

In summary the equilibrium is given: $p_{CC\bar{m}} = \frac{\beta(2(1+\bar{m}) + \beta - \lambda(\bar{m} - \alpha + \beta) - \bar{k})}{\lambda(2(1+\bar{m})(\bar{m} - \alpha + \beta) - \beta(1+\alpha))}$

$$p_{CD\bar{m}} = \frac{\bar{k}(\bar{m} - \alpha + \beta) + (1+\bar{m} + \beta)(\lambda(\bar{m} - \alpha + \beta) - \beta)}{\lambda(2(1+\bar{m})(\bar{m} - \alpha + \beta) - \beta(1+\alpha))}$$

Condition for existence: $\frac{2}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} \frac{(1+\bar{m})}{(1+\alpha)} - \bar{k} \right) + \frac{\beta - (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} < \lambda < 1$

4.3.1.2. $\Pi_{\bar{m}}(\bar{m}) - \Pi_{\bar{m}}(\underline{m}) = 0$

$$\Pi_{\bar{m}}(\bar{m}) - \Pi_{\bar{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\alpha) \right] - \underline{k} = 0 \Leftrightarrow 1 - 2p_{DC\bar{m}} - \frac{\underline{k}}{\lambda(1+\alpha)} = p_{CC\bar{m}} \quad (*)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (iii)$$

$$\begin{aligned}
\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(-p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k} + \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] = \\
\lambda \left[(-p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \bar{m}) \right] - \bar{k} + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] &= \\
\lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \bar{m}) - p_{DC\bar{m}}\beta \right] - \bar{k} + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] &= 0
\end{aligned} \tag{v}$$

To summarize:

$$\left\{ \begin{array}{l} (p_{CC\bar{m}}, p_{DC\bar{m}}, p_{CD\underline{m}}, p_{\underline{m}}) \\ p_{DC\bar{m}} = \frac{1}{2} \left(1 - \frac{\underline{k} - \bar{k} - \beta}{\lambda(\bar{m} - \alpha + \beta)} \right) \\ p_{CC\bar{m}} = \frac{\beta + \bar{k} - \frac{1 + \bar{m} + \beta}{1 + \alpha} \underline{k}}{\lambda(\bar{m} - \alpha + \beta)} \\ p_{CD\underline{m}} = \frac{1}{2} \left(1 + \frac{\left(2 \frac{(\bar{m} - \alpha + \beta)}{1 + \alpha} + 1 \right) \underline{k} - \bar{k} - \beta}{\lambda(\bar{m} - \alpha + \beta)} \right) \\ p_{\underline{m}} = \frac{1}{\beta(1 - \lambda)} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) - \frac{1}{2(1 - \lambda)(\bar{m} - \alpha + \beta)} (\underline{k} - \bar{k}) - \frac{\lambda(\bar{m} - \alpha + \beta) - \beta}{2(1 - \lambda)(\bar{m} - \alpha + \beta)} \end{array} \right\}$$

Given this solution for the respective shares (ii) and (iii) need to be checked for.

$$\begin{aligned}
(ii) : \lambda(\bar{m} - \alpha + \beta) &> \beta + \underline{k} - \bar{k} \\
(iii) : \lambda(\bar{m} - \alpha + \beta) &> \beta - (\underline{k} - \bar{k}), \text{ i.e. (ii) implies (iii).}
\end{aligned}$$

Finally all condition of the type

$p \in [0,1], \sum p < 1$ reduce to:

$$\text{(high types): } \lambda > \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} \quad \wedge \quad (1 + \alpha)\beta > (1 + \bar{m} + \beta)\underline{k} - (1 + \alpha)\bar{k} \quad \wedge \quad \underline{k} < 1 + \alpha$$

$$\text{(low types): } \lambda < \min \left\{ \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right), 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right) \right\}$$

note:

$$\frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < 1 \Leftrightarrow \underline{k} - \bar{k} < \bar{m} - \alpha$$

$p_{\bar{m}} > 0$:

$$\frac{1}{\beta(1 - \lambda)} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) > \frac{1}{2(1 - \lambda)(\bar{m} - \alpha + \beta)} (\underline{k} - \bar{k}) + \frac{\lambda(\bar{m} - \alpha + \beta) - \beta}{2(1 - \lambda)(\bar{m} - \alpha + \beta)} \Leftrightarrow$$

$$\frac{2(\bar{m} - \alpha + \beta)}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) > (\underline{k} - \bar{k}) + \lambda(\bar{m} - \alpha + \beta) - \beta \Leftrightarrow$$

$$\lambda < \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right)$$

$$p_{\underline{m}} < 1:$$

$$\frac{1}{\beta(1-\lambda)} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) < \frac{1}{2(1-\lambda)(\bar{m}-\alpha+\beta)} (\underline{k} - \bar{k}) + \frac{\lambda(\bar{m}-\alpha+\beta) - \beta}{2(1-\lambda)(\bar{m}-\alpha+\beta)} + 1 \Leftrightarrow$$

$$\frac{2(\bar{m}-\alpha+\beta)}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) < (\underline{k} - \bar{k}) + \lambda(\bar{m}-\alpha+\beta) - \beta + 2(1-\lambda)(\bar{m}-\alpha+\beta) \Leftrightarrow$$

$$\lambda(\bar{m}-\alpha+\beta) < (\underline{k} - \bar{k}) - \frac{2(\bar{m}-\alpha+\beta)}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) - \beta + 2(\bar{m}-\alpha+\beta) \Leftrightarrow$$

$$\lambda < 2 + \frac{(\underline{k} - \bar{k}) - \beta}{(\bar{m}-\alpha+\beta)} - \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) = 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) \right)$$

LHS < RHS:

$$\frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) - 2 + \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) \right) = 2 \frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + 2 \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) - 2 > 0 \Leftrightarrow$$

$$\frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) > 1 \Leftrightarrow (1+\alpha)\beta \frac{\beta + \bar{k} - \underline{k}}{(\bar{m}-\alpha+\beta)} + 2((1+\bar{m})\underline{k} - (1+\alpha)\bar{k}) > (1+\alpha)\beta$$

from high types:

$$(1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} \Rightarrow$$

$$(1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} < (1+\alpha)\beta < (1+\alpha)\beta \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + 2((1+\bar{m})\underline{k} - (1+\alpha)\bar{k})$$

$$(1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k} < (1+\alpha)\beta \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + 2((1+\bar{m})\underline{k} - (1+\alpha)\bar{k}) \Leftrightarrow$$

$$0 < (1+\alpha)\beta \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + (1+\bar{m} - \beta)\underline{k} - (1+\alpha)\bar{k}$$

$$\Leftrightarrow 0 < (1+\alpha)\beta \frac{\beta}{(\bar{m} - \alpha + \beta)} + (1+\alpha)\beta \frac{\bar{k}}{(\bar{m} - \alpha + \beta)} - (1+\alpha)\beta \frac{\underline{k}}{(\bar{m} - \alpha + \beta)} + (1+\bar{m} - \beta)\underline{k} - (1+\alpha)\bar{k}$$

$$\Leftrightarrow 0 < (1+\alpha)\beta \frac{\beta}{(\bar{m} - \alpha + \beta)} - (1+\alpha) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} \bar{k} + \frac{(1+\bar{m})(\bar{m} - \alpha + \beta) - \beta(1+\bar{m} + \beta)}{(\bar{m} - \alpha + \beta)} \underline{k}$$

$$\Leftrightarrow 0 < (1+\alpha)\beta - (1+\alpha) \frac{\bar{m} - \alpha}{\beta} \bar{k} + \left(\frac{\bar{m} - \alpha}{\beta} - \beta(1+\bar{m}) \right) \underline{k} \Leftrightarrow 0 < (1+\alpha)\beta - (1+\alpha) \frac{\bar{m} - \alpha}{\beta} \bar{k} + \left(\frac{\bar{m} - \alpha}{\beta} - \beta(1+\bar{m}) \right) \underline{k}$$

conditions for existence:

$$1. \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda < \min \left\{ \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right), 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) \right) \right\}$$

$$2. \underline{k} - \bar{k} < \bar{m} - \alpha$$

$$3. \underline{k} < 1 + \alpha$$

$$4. (1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}$$

4.3.1.3. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) < 0$

$$1. \quad \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} < 0 \Rightarrow p_{\bar{m}} = 0 \quad (*)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}})(-\beta) \right] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) \right] - (1 - \lambda)\beta > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\underline{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) - p_{DC\bar{m}}\beta \right] - \bar{k} - (1 - \lambda)\beta = 0 \quad (v)$$

Rewrite (i) and (v) to: $p_{CC\bar{m}} = \frac{p_{DC\bar{m}}\beta}{\lambda(\bar{m} - \alpha)}$, $p_{CD\underline{m}} = \frac{\bar{k} + (1 - \lambda)\beta}{\lambda(1 + \bar{m} + \beta)} + \left(1 + \frac{\beta}{(1 + \bar{m} + \beta)}\right)p_{DC\bar{m}}$ and plug into: $p_{CD\underline{m}} = 1 - p_{CC\bar{m}} - p_{DC\bar{m}}$

$$p_{DC\bar{m}} = \frac{(\bar{m} - \alpha)((1 + \bar{m} + \beta)\lambda - \beta - \bar{k})}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$$

$$\text{EQ: } p_{CC\bar{m}} = \frac{\beta((1 + \bar{m} + \beta)\lambda - \beta - \bar{k})}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$$

$$p_{CD\underline{m}} = \frac{\bar{k}(\bar{m} - \alpha + \beta) + (1 + \bar{m} + \beta)\lambda(\bar{m} - \alpha) + (\bar{m} - \alpha + \beta)(1 - \lambda)\beta}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$$

Finally, check for (*) and (ii) given those values:

w.r.t. (*):

$$\begin{aligned}
& (1+\alpha)\left[\bar{k}(2\bar{m}-2\alpha+\beta)+\beta((2\bar{m}-2\alpha+\beta)-\lambda(\bar{m}-\alpha+\beta))\right]<(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))\underline{k}\Leftrightarrow \\
& \bar{k}\left[2(1+\alpha)(\bar{m}-\alpha+\beta)-\beta(1+\alpha)\right]-\left[2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha)\right]\underline{k}+\beta(1+\alpha)\left[2(\bar{m}-\alpha+\beta)-\beta\right]<\lambda\beta(1+\alpha)(\bar{m}-\alpha+\beta)\Leftrightarrow \\
& \frac{1}{(\bar{m}-\alpha+\beta)}(\underline{k}-\bar{k})-\frac{2}{\beta}\left(\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right)+\frac{2(\bar{m}-\alpha+\beta)-\beta}{(\bar{m}-\alpha+\beta)}=2-\left(\frac{\beta+\bar{k}-\underline{k}}{(\bar{m}-\alpha+\beta)}+\frac{2}{\beta}\left(\frac{(1+\bar{m})}{(1+\alpha)}\underline{k}-\bar{k}\right)\right)<\lambda
\end{aligned}$$

w.r.t. (ii):

$$\lambda > \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)} \wedge \bar{k} < 1 + \bar{m}$$

To summarize:

$$\begin{aligned}
p_{DC\bar{m}} &= \frac{(\bar{m}-\alpha)((1+\bar{m}+\beta)\lambda-\beta-\bar{k})}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))} \\
p_{CC\bar{m}} &= \frac{\beta((1+\bar{m}+\beta)\lambda-\beta-\bar{k})}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))} \\
p_{CD\bar{m}} &= \frac{\bar{k}(\bar{m}-\alpha+\beta)+(1+\bar{m}+\beta)\lambda(\bar{m}-\alpha)+(\bar{m}-\alpha+\beta)(1-\lambda)\beta}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))}
\end{aligned}$$

Finally all condition of the type

$$p \in [0,1], \sum p < 1 \text{ reduce to: } \lambda > \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)} \wedge \bar{k} < 1 + \bar{m}$$

Hence we are left with:

$$\lambda > \max \left\{ \underbrace{\frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)}}_x, \underbrace{2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right)}_y \right\} \wedge \bar{k} < 1 + \bar{m}$$

note:

$$y > x \Leftrightarrow 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right) - \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)} > 0 \Leftrightarrow (1 + \alpha) \beta > (1 + \bar{m} + \beta) \underline{k} - (1 + \alpha) \bar{k} \quad \wedge \quad \underline{k} < 1 + \alpha$$

Conditions for existence:

$$1. \bar{k} < 1 + \bar{m}$$

$$2. \lambda > \max \left\{ \underbrace{\frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)}}_x, \underbrace{2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right)}_y \right\}$$

$$= \begin{cases} 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right), & (1 + \alpha) \beta > (1 + \bar{m} + \beta) \underline{k} - (1 + \alpha) \bar{k} \quad \wedge \quad \underline{k} < 1 + \alpha \\ \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)}, & \text{else} \end{cases}$$

note:

$$y > x \Leftrightarrow 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right) - \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)} > 0 \Leftrightarrow (1 + \alpha) \beta > (1 + \bar{m} + \beta) \underline{k} - (1 + \alpha) \bar{k} \quad \wedge \quad \underline{k} < 1 + \alpha$$

4.3.2. and $CD, \underline{m} / DD, \underline{m} \Rightarrow p_{\underline{m}} = 1$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0$$

(i)

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0$$

(ii)

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0$$

(iii)

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0$$

(iv)

$$\begin{aligned} \Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) &= \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) - [\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(CC, \bar{m})] = \\ \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \bar{k} &+ \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \end{aligned}$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k}$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}})(-\beta) \right] = 0 \quad (i)$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta) \right] - (1 - \lambda)\beta > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) \right] = 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] - (1 - \lambda)\beta < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(-p_{DC\bar{m}})(1 + \bar{m} + \beta) + (p_{CD\underline{m}})(1 + \alpha) \right] - \bar{k} + \lambda \left[(p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DD\underline{m}})(-\beta) \right] - (1 - \lambda)\beta = 0$$

By (iii) such a **semi**-pooling equilibrium cannot exist.

4.4. CD, \bar{m}

4.4.1. and DC, \underline{m}

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m})] = 0 \quad (v)$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k}$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{DC\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (i)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (ii)$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (iv)$$

$$\begin{aligned}
& \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m})] = 0 \\
& \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} \\
& + \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} - p_{CC\underline{m}} - p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}} - p_{DC\underline{m}} - p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) - p_{\underline{m}}(-\beta)] \\
& = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) + (-p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} + \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) + (-p_{DC\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta) - p_{\underline{m}}(-\beta)] \\
& = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \bar{m}) \right] - \bar{k} - \beta(1 - \lambda) [p_{\bar{m}} - p_{\underline{m}}] = 0
\end{aligned} \tag{v}$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} \tag{vi}$$

(i) and (iv) are always satisfied.

4.4.1.1. $\Pi_{\bar{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) > 0$

$$\Pi_{\bar{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} > 0 \Rightarrow \textcolor{blue}{p}_{\bar{m}} = 1 \tag{vi}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \tag{ii}$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\underline{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] > 0 \tag{iii}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \bar{m}) \right] - \bar{k} - \beta(1 - \lambda) = 0 \tag{v}$$

$$\text{EQ: } \textcolor{blue}{p}_{DC\underline{m}} = \frac{1}{2} - \frac{\bar{k} + \beta(1 - \lambda)}{2\lambda(1 + \bar{m})}, \textcolor{blue}{p}_{CD\bar{m}} = \frac{1}{2} + \frac{\bar{k} + \beta(1 - \lambda)}{2\lambda(1 + \bar{m})}$$

(iii) is always satisfied, (ii) and (vi) need to be checked for::

$$(vi): \lambda < 1 - \frac{1}{\beta} \left(\underline{k} \frac{1+\bar{m}}{1+\alpha} - \bar{k} \right)$$

$$(ii): \lambda > \frac{1+\alpha}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \left(\beta \left(1 + \frac{1+\bar{m}}{1+\alpha} \right) - \frac{\bar{k}}{1+\alpha} (\bar{m}-\alpha) \right)$$

$$(vi) \wedge (ii) \wedge \lambda \in (0,1):$$

$$1. \bar{k} < 1+\alpha$$

$$2. \frac{1+\alpha}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \left(\beta \left(1 + \frac{1+\bar{m}}{1+\alpha} \right) - \frac{\bar{k}}{1+\alpha} (\bar{m}-\alpha) \right) < \lambda < 1 - \frac{1}{\beta} \left(\underline{k} \frac{1+\bar{m}}{1+\alpha} - \bar{k} \right)$$

$$3. (1+\alpha)\beta > (\bar{m}-\alpha+\beta)\bar{k} (\Rightarrow 1.)$$

$$4. \underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{-\alpha(1-\beta)+2\beta+\bar{m}(1+\bar{m}-\alpha+\beta)}$$

Finally all conditions of the type

$$p \in [0,1], \sum p < 1 \text{ reduce to: } \frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}$$

$$p_{CD\bar{m}} > 0: \quad \text{true}$$

$$p_{CD\bar{m}} < 1: \Leftrightarrow \lambda > \frac{\beta + \bar{k}}{(1+\bar{m} + \beta)} \wedge \bar{k} < 1 + \bar{m}$$

Conditions:

$$\begin{aligned}
1. \quad & \max \left\{ \underbrace{\frac{1+\alpha}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \left(\beta \left(1 + \frac{1+\bar{m}}{1+\alpha} \right) - \frac{\bar{k}}{1+\alpha} (\bar{m}-\alpha) \right)}_x, \underbrace{\frac{\beta+\bar{k}}{(1+\bar{m}+\beta)}}_y \right\} < \lambda < \underbrace{1 - \frac{1}{\beta} \left(\frac{\underline{k}}{1+\alpha} - \bar{k} \right)}_z \\
2. \quad & (1+\alpha)\beta > (\bar{m}-\alpha+\beta)\bar{k} \\
3. \quad & \underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{-\alpha(1-\beta)+2\beta+\bar{m}(1+\bar{m}-\alpha+\beta)}
\end{aligned}$$

note that 2. implies that $x > y$.

$$x < z : \Leftrightarrow \underline{k} < \frac{(1+\alpha)(\bar{m}-\alpha+2\bar{k})}{(2+\bar{m}+\alpha)} \quad \wedge \quad (1+\alpha)\beta > \left(1+\bar{m} + \frac{(1+\bar{m})\beta+(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \underline{k} - \left(1+\alpha + \frac{2(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \bar{k}$$

note:

$$\begin{aligned}
a) \quad & (\bar{m}-\alpha+\beta)\bar{k} - \left(1+\bar{m} + \frac{(1+\bar{m})\beta+(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \underline{k} + \left(1+\alpha + \frac{2(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \bar{k} = \\
& \left(1+\bar{m} + \frac{(1+\bar{m})\beta+(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \bar{k} - \left(1+\bar{m} + \frac{(1+\bar{m})\beta+(1+\alpha)\beta}{(\bar{m}-\alpha)} \right) \underline{k} < 0 \\
b) \quad & \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{-\alpha(1-\beta)+2\beta+\bar{m}(1+\bar{m}-\alpha+\beta)} - \frac{(1+\alpha)(\bar{m}-\alpha+2\bar{k})}{(2+\bar{m}+\alpha)} = \\
& \frac{(1+\alpha)\beta(\bar{m}-\alpha)}{(1+\bar{m})(\bar{m}-\alpha+\beta)+(1+\alpha)\beta} - \frac{(1+\alpha)(\bar{m}-\alpha)}{(2+\bar{m}+\alpha)} + \frac{(1+\alpha)(\bar{m}-\alpha+2\beta)\bar{k}}{(1+\bar{m})(\bar{m}-\alpha+\beta)+(1+\alpha)\beta} - \frac{(1+\alpha)2\bar{k}}{(2+\bar{m}+\alpha)} < 0, \text{ i.e.} \\
& \underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{-\alpha(1-\beta)+2\beta+\bar{m}(1+\bar{m}-\alpha+\beta)} \text{ is binding.}
\end{aligned}$$

To summarize:

$$\text{EQ: } p_{DC\bar{m}} = \frac{1}{2} - \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}, p_{CD\bar{m}} = \frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}$$

Conditions for existence:

1. $0 < \frac{1+\alpha}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \left(\beta \left(1 + \frac{1+\bar{m}}{1+\alpha} \right) - \frac{\bar{k}}{1+\alpha} (\bar{m}-\alpha) \right) < \lambda < 1 - \frac{1}{\beta} \left(\frac{k}{1+\alpha} - \bar{k} \right) < 1$
2. $(1+\alpha)\beta > (\bar{m}-\alpha+\beta)\bar{k}$
3. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}$

$$4.4.1.2. \Pi_{\bar{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = 0$$

$$\Pi_{\bar{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\alpha) \right] - \underline{k} = 0 \Leftrightarrow (p_{CD\bar{m}} - p_{DC\bar{m}}) = \frac{\underline{k}}{\lambda(1+\alpha)} \quad (\text{vi})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m}-\alpha) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m}-\alpha) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\bar{m}) \right] - \bar{k} - \beta(1-\lambda) \left[p_{\bar{m}} - p_{\underline{m}} \right] = 0 \Leftrightarrow \frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} = \beta(1-\lambda) \left[p_{\bar{m}} - p_{\underline{m}} \right]$$

(v) becomes:

$$\Leftrightarrow \frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} = \beta(1-\lambda) [2p_{\bar{m}} - 1] \Leftrightarrow p_{\bar{m}} = \frac{1}{2} + \frac{1}{2\beta(1-\lambda)} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right]$$

$$p_{CD\bar{m}} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}, p_{DC\underline{m}} = \frac{1}{2} - \frac{\underline{k}}{2\lambda(1+\alpha)} \text{ by (vi)}; \quad p_{\bar{m}} = \frac{1}{2} + \frac{1}{2\beta(1-\lambda)} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \text{ by (v) and}$$

Given this solution for the respective shares (ii) and (iii) need to be checked for.

$$(ii): \lambda(\bar{m} - \alpha + \beta) > \beta + \underline{k} - \bar{k}$$

$$(iii): \lambda(\bar{m} - \alpha + \beta) > \beta - (\underline{k} - \bar{k}), \text{ i.e. (ii) implies (iii).}$$

$$\lambda \in (0,1) \text{ adds } \underline{k} - \bar{k} < \bar{m} - \alpha$$

Finally all conditions of the type

$$p \in [0,1], \sum p < 1 \text{ reduce to:}$$

$$\frac{\underline{k}}{1+\alpha} < \lambda < 1 - \frac{1}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) \left(\Rightarrow \text{nec.: } \frac{\underline{k}}{1+\alpha} < 1 \right)$$

$$(1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}$$

$$1. \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda < 1 - \frac{1}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right)$$

$$2. (1+\alpha)\beta > (\bar{m} - \alpha + \beta)\bar{k}$$

$$3. \underline{k} < \frac{(1+\alpha)(\beta(\bar{m} - \alpha) + \bar{k}(\bar{m} - \alpha + 2\beta))}{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta}$$

$$4. \bar{k} < 1 + \alpha$$

Conditions for existence:

$$4.4.1.3. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) < 0$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) \right] - \underline{k} < 0 \Leftrightarrow \underline{p}_{\underline{m}} = 1 \quad (\text{vi})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda) > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0 \quad (\text{v})$$

$$\text{EQ:} \quad p_{CD\bar{m}} = \frac{1}{2} \left(1 + \frac{\bar{k} - \beta(1 - \lambda)}{\lambda(1 + \bar{m})} \right)$$

$$p_{DC\bar{m}} = \frac{1}{2} \left(1 - \frac{\bar{k} - \beta(1 - \lambda)}{\lambda(1 + \bar{m})} \right)$$

(ii) is satisfied, (iii) and (vi) need to be checked for,

$$(\text{vi}): \lambda < 1 + \frac{1}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right)$$

$$(\text{iii}): \lambda > \frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1 + \bar{m})(\bar{m} - \alpha + \beta) + \beta(1 + \alpha)}$$

$$\text{hence adding } \lambda \in (0, 1): \frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1 + \bar{m})(\bar{m} - \alpha + \beta) + \beta(1 + \alpha)} < \lambda < 1 \quad \wedge \quad \bar{k} < (1 + \bar{m})$$

Finally all conditions of the type:

$p \in [0,1], \sum p < 1$ reduce to:

$$0 < 1 + \frac{\bar{k} - \beta(1 - \lambda)}{\lambda(1 + \bar{m})} < 2 \Leftrightarrow -\lambda(1 + \bar{m} + \beta) < \bar{k} - \beta < \lambda(1 + \bar{m} - \beta) \Leftrightarrow$$

$$\lambda(1 + \bar{m} - \beta) > \bar{k} - \beta \wedge \lambda(1 + \bar{m} + \beta) > \bar{k} - \beta \Leftrightarrow \lambda(1 + \bar{m} - \beta) > \bar{k} - \beta \wedge \lambda > \frac{\bar{k} - \beta}{(1 + \bar{m} + \beta)}$$

$$\text{adding } \lambda \in (0,1) \text{ and } \bar{k} < 1 + \bar{m} \text{ yields } x < \lambda < 1, x = \begin{cases} \frac{\bar{k} - \beta}{1 + \bar{m} - \beta}, \beta \leq \bar{k} \\ \frac{\bar{k} - \beta}{1 + \bar{m} + \beta}, \beta > \bar{k} \end{cases}$$

however, it turns out that $\frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1 + \bar{m})(\bar{m} - \alpha + \beta) + \beta(1 + \alpha)} < \lambda$ is binding.

To summarize:

$$p_{CD\bar{m}} = \frac{1}{2} \left(1 + \frac{\bar{k} - \beta(1 - \lambda)}{\lambda(1 + \bar{m})} \right), \quad p_{DC\bar{m}} = \frac{1}{2} \left(1 - \frac{\bar{k} - \beta(1 - \lambda)}{\lambda(1 + \bar{m})} \right)$$

Conditions for existence:

1. $\frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1 + \bar{m})(\bar{m} - \alpha + \beta) + \beta(1 + \alpha)} < \lambda < 1$
2. $\bar{k} < (1 + \bar{m})$

4.4.2. and $DD, \underline{m} \Rightarrow p_{\underline{m}} = 1$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(\bar{m} - \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) - [\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m})] = 0 \quad (\text{v})$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] - \beta(1 - \lambda) < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] > 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DD\underline{m}})(-\beta) \right] - \beta(1 - \lambda) < 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(-\beta) \right] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) - [\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(CD, \bar{m})] =$$

$$\lambda(1 + \alpha)p_{CD\bar{m}} - \bar{k} + \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) \right] = 0 \Leftrightarrow p_{CD\bar{m}} = \frac{\bar{k}}{\lambda(1 + \bar{m})}$$

$$\text{EQ: } p_{CD\bar{m}} = \frac{\bar{k}}{\lambda(1+\bar{m})}, \quad p_{DD\bar{m}} = 1 - p_{CD\bar{m}}$$

(i),(ii),(iii) and (iv) are satisfied

Finally all conditions of the type $p \in [0,1], \sum p < 1$ reduce to: $\lambda > \frac{\bar{k}}{(1+\bar{m})}$

Condition for existence:

1. $\lambda > \frac{\bar{k}}{(1+\bar{m})}$
2. $\bar{k} < (1+\bar{m})$

however it turns out that this equilibrium is not stable.

4.5. DD, \bar{m} and $DC, \underline{m} \Rightarrow p_{\underline{m}} = 1$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(\bar{m} - \alpha) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1 - \lambda) [p_{\underline{m}}(-\beta)] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m})] = 0 \quad (\text{v})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DC\bar{m}})(-\beta) \right] - \beta(1-\lambda) < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] < 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] - \beta(1-\lambda) > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] < 0 \quad (\text{iv})$$

$$\begin{aligned} \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - \left[\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) \right] = \\ &= -p_{DC\bar{m}}\lambda(1+\bar{m}) - \bar{k} + (1-\lambda)\beta = 0 \Leftrightarrow \frac{(1-\lambda)\beta - \bar{k}}{\lambda(1+\bar{m})} = p_{DC\bar{m}} \end{aligned}$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\alpha) + (p_{CD\bar{m}} - p_{DC\bar{m}})(1+\bar{m} + \beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DC\bar{m}})(-\beta) \right] - \beta(1-\lambda) < 0$$

By (v) $p_{DC\bar{m}} = \frac{(1-\lambda)\beta - \bar{k}}{\lambda(1+\bar{m})}$, however this is incompatible with $p_{DC\bar{m}} > \frac{(1-\lambda)\beta}{\lambda(\bar{m} - \alpha)}$ by (iii)

Hence such a semi-pooling equilibrium cannot exist.

4.6. $CD, \bar{m} / DD, \bar{m}$

4.6.1. and DC, \underline{m}

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m} - \alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m})] = 0 \quad (\text{v})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DC\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m} - \alpha) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CC, \underline{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iv})$$

$$\begin{aligned}
\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &= \Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) - [\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m})] = \\
&\lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1 + \alpha) + (p_{CD\underline{m}} - p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} - \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(\bar{m} - \alpha) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] - (1 - \lambda) [p_{\underline{m}}(-\beta)] = \\
&\lambda \left[(p_{CD\bar{m}})(1 + \alpha) + (-p_{DC\underline{m}})(1 + \bar{m} + \beta) \right] - \bar{k} - \lambda \left[(p_{DC\underline{m}})(-\beta) \right] + \beta(1 - \lambda) p_{\underline{m}} = \\
&\lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) p_{\underline{m}} = 0
\end{aligned}$$

(v)

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k}$$

In a semi-pooling equilibrium (i) and (iv) will always be satisfied.

$$4.6.1.1. \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) > 0$$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} > 0 \Rightarrow \underline{p}_{\underline{m}} = 0 \quad (*)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta) \right] - \beta(1 - \lambda) = 0 \quad (ii)$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\underline{m}})(\bar{m} - \alpha) \right] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} = 0 \quad (v)$$

EQ:

$$p_{DC\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\alpha) - \bar{k}(\bar{m} - \alpha + \beta)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right], p_{CD\bar{m}} = \frac{\beta}{\lambda(\bar{m} - \alpha + \beta)} - p_{DC\bar{m}} \frac{\beta}{\bar{m} - \alpha + \beta}, p_{DD\bar{m}} = 1 - p_{CD\bar{m}} - p_{DC\bar{m}} \Leftrightarrow$$

$$p_{DC\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\alpha) - \bar{k}(\bar{m} - \alpha + \beta)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right], p_{CD\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\bar{m} + \bar{k})}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right], p_{DD\bar{m}} = 1 + \frac{1}{\lambda} \left[\frac{\bar{k}(\bar{m} - \alpha) - \beta(2 + \bar{m} + \alpha)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right]$$

(iii) is satisfied, (*) needs to be checked for, however this equilibrium is not stable.

(*) reduces to:

1. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m} - \alpha) + \bar{k}(\bar{m} - \alpha + 2\beta))}{(1+\bar{m})(\bar{m} - \alpha + \beta) + \beta(1+\alpha)}$
2. $\beta(1+\alpha) > \bar{k}(\bar{m} - \alpha + \beta)$

Finally all conditions of the type

$p \in [0,1], \sum p < 1$ and $0 < \lambda < 1$ reduce to:

1. $\frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1+\bar{m})(\bar{m} - \alpha + \beta) + \beta(1+\alpha)} < \lambda < 1$
2. $\beta(1+\alpha) > \bar{k}(\bar{m} - \alpha + \beta)$

To summarize

$$p_{DC\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\alpha) - \bar{k}(\bar{m} - \alpha + \beta)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right], p_{CD\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\bar{m} + \bar{k})}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right], p_{DD\bar{m}} = 1 + \frac{1}{\lambda} \left[\frac{\bar{k}(\bar{m} - \alpha) - \beta(2 + \bar{m} + \alpha)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right]$$

Conditions for existence:

1. $\frac{\bar{k}(\bar{m}-\alpha)+\beta(2+\alpha+\bar{m})}{(1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha)} < \lambda < 1$
2. $\beta(1+\alpha) > \bar{k}(\bar{m}-\alpha+\beta)$
3. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{(1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha)}$

4.6.1.2. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = 0$

$$\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} - p_{DC\bar{m}})(1+\alpha) \right] - \underline{k} = 0 \quad (0)$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m}-\alpha) + (p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\bar{m}})(\bar{m}-\alpha) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(1+\alpha) - p_{DC\bar{m}}(1+\bar{m}) \right] - \bar{k} + \beta(1-\lambda)p_{\underline{m}} = 0 \quad (\text{v})$$

EQ:

$$\begin{aligned} p_{DC\bar{m}} &= \frac{\beta(1+\alpha) + (1-\bar{m}-\beta+2\alpha)\underline{k} - (1+\alpha)\bar{k}}{2(1+\alpha)(\bar{m}-\alpha+\beta)}, p_{CD\bar{m}} = \frac{\beta(1+\alpha) + (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}}{2(1+\alpha)(\bar{m}-\alpha+\beta)}, p_{DD\bar{m}} = \left[\frac{-\beta + \lambda(\bar{m}-\alpha+\beta) + \bar{k} - \underline{k}}{\lambda(\bar{m}-\alpha+\beta)} \right], \\ p_{\underline{m}} &= -\frac{(1+\alpha)(-\beta(\bar{m}-\alpha) + (\alpha-2\beta-\bar{m})\bar{k}) + ((1+\bar{m})(\bar{m}-\alpha+\beta) + \beta(1+\alpha))\underline{k}}{2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta)}, p_{\bar{m}} = 1 - p_{\underline{m}} \end{aligned}$$

Note, that for the equilibrium values (iii) is satisfied:

Finally all conditions of the type

$p \in [0,1], \sum p < 1$ and $0 < \lambda < 1$ reduce to:

$$-\frac{(1+\alpha)(-\beta(\bar{m}-\alpha)+(\alpha-2\beta-\bar{m})\bar{k})+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}}{2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta)} > 0$$

$p_m > 0$:

$$-\frac{(1+\alpha)(-\beta(\bar{m}-\alpha)+(\alpha-2\beta-\bar{m})\bar{k})+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}}{2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta)} > 0 \Leftrightarrow$$

$$(1+\alpha)(-\beta(\bar{m}-\alpha)+(\alpha-2\beta-\bar{m})\bar{k})+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k} < 0 \Leftrightarrow$$

$$((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-(1+\alpha)(\bar{m}-\alpha+2\beta)\bar{k} < (1+\alpha)\beta(\bar{m}-\alpha) \Leftrightarrow$$

$$\left((1+\bar{m})\left(1+\frac{\beta}{\bar{m}-\alpha}\right)+(1+\alpha)\frac{\beta}{\bar{m}-\alpha}\right)\underline{k}-(1+\alpha)\left(1+2\frac{\beta}{\bar{m}-\alpha}\right)\bar{k} < (1+\alpha)\beta \Leftrightarrow$$

$$\left((1+\bar{m})+(1+\bar{m})\frac{\beta}{\bar{m}-\alpha}+(1+\alpha)\frac{\beta}{\bar{m}-\alpha}\right)\underline{k}-\left((1+\alpha)+2(1+\alpha)\frac{\beta}{\bar{m}-\alpha}\right)\bar{k} < (1+\alpha)\beta$$

$p_m < 1$:

$$(1+\alpha)(-\beta(\bar{m}-\alpha)+(\alpha-2\beta-\bar{m})\bar{k})+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k} > 2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta) \Leftrightarrow$$

$$((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-(1+\alpha)(\bar{m}-\alpha+2\beta)\bar{k} > (1+\alpha)\beta(\bar{m}-\alpha)+2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta) \Leftrightarrow$$

$$4.6.1.3. \Pi_{\bar{m}}(\bar{m})-\Pi_{\underline{m}}(\underline{m}) < 0 \quad (*)$$

$$\Pi_{\bar{m}}(\bar{m})-\Pi_{\underline{m}}(\underline{m})=\lambda[(p_{CD\bar{m}})(1+\alpha)]-\underline{k} < 0 \Rightarrow p_{\underline{m}}=1$$

$$\Pi_{\bar{m}}(CD,\bar{m})-\Pi_{\bar{m}}(DD,\bar{m})=\lambda[(p_{CD\bar{m}})(\bar{m}-\alpha)+(p_{DD\bar{m}})(-\beta)]+(1-\lambda)[p_{\bar{m}}(-\beta)]=0 \quad (ii)$$

$$\Pi_{\bar{m}}(DC,\underline{m})-\Pi_{\bar{m}}(DD,\underline{m})=\lambda[(p_{DC\bar{m}})(\bar{m}-\alpha)]+(1-\lambda)[p_{\underline{m}}(-\beta)] > 0 \quad (iii)$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda)p_{\underline{m}} = 0 \quad (\text{v})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha) + (p_{DD\bar{m}})(-\beta) \right] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DC\underline{m}})(\bar{m} - \alpha) \right] - \beta(1 - \lambda) > 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0 \quad (\text{v})$$

(ii) and (v):

$$\lambda \left[(p_{CD\bar{m}})(\bar{m} - \alpha + \beta) + p_{DC\underline{m}}\beta - \beta \right] = 0$$

$$\lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0$$

$$p_{CD\bar{m}} = \frac{\beta - p_{DC\underline{m}}\beta}{\lambda(\bar{m} - \alpha + \beta)}$$

$$\lambda \left[(p_{CD\bar{m}})(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = \lambda \left[\frac{\beta - p_{DC\underline{m}}\beta}{\lambda(\bar{m} - \alpha + \beta)}(1 + \alpha) - p_{DC\underline{m}}(1 + \bar{m}) \right] - \bar{k} + \beta(1 - \lambda) = 0$$

$$\Rightarrow p_{DC\underline{m}} = \frac{1}{\frac{\beta(1 + \alpha)}{(\bar{m} - \alpha + \beta)} + \lambda(1 + \bar{m})} \left[\frac{\beta(1 + \alpha)}{\lambda(\bar{m} - \alpha + \beta)} - \bar{k} + \beta(1 - \lambda) \right]$$

$$\text{EQ: } \begin{aligned} p_{CD\bar{m}} &= \frac{\beta(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}, p_{DC\bar{m}} = \frac{\beta((1-\lambda)(\bar{m}-\alpha+\beta)+\lambda(1+\alpha))-(\bar{m}-\alpha+\beta)\bar{k}}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))} \\ p_{DD\bar{m}} &= \frac{(\bar{m}-\alpha)(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))} \end{aligned}$$

We need to check for (*) and (iii):

$$\begin{aligned} (*) : & \frac{(1+\alpha)((\bar{m}-\alpha+2\beta)\bar{k}+\beta(2\lambda(\bar{m}-\alpha+\beta)-(\bar{m}-\alpha+2\beta)))-((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}}{(1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha)} < 0 \Leftrightarrow \\ & ((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-(1+\alpha)(\bar{m}-\alpha+2\beta)\bar{k} > (1+\alpha)\beta(\bar{m}-\alpha)+2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta) \Leftrightarrow \\ & \lambda < \frac{(1+\alpha)(\beta-\bar{k})(\bar{m}-\alpha+2\beta)+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}}{2(1+\alpha)\beta(\bar{m}-\alpha+\beta)} \\ & = \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k}}{2(1+\alpha)\beta(\bar{m}-\alpha+\beta)} \end{aligned}$$

Finally all conditions of the type

$p \in [0,1], \sum p < 1$ and $0 < \lambda < 1$ and (iii) reduce to:

1. $0 < \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} < \lambda < 1$
2. $\beta(1+\alpha) > (\bar{m}-\alpha+\beta)\bar{k}$

adding the upper bound due to (*) we end up with:

$$1. \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} < \lambda < \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)}$$

$$2. \beta(1+\alpha) > (\bar{m}-\alpha+\beta)\bar{k}$$

note that LHS < RHS due to $\bar{k} < \underline{k}$

$$\text{RHS} < 1: \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} < 1 \Leftrightarrow$$

$$((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k} < \beta(1+\alpha)(\bar{m}-\alpha) \Leftrightarrow$$

$$((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k} < \beta(1+\alpha)(\bar{m}-\alpha) \Leftrightarrow$$

$$\frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\underline{k}-\frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\bar{k} < \beta(1+\alpha) \Leftrightarrow$$

note:

$$\frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\underline{k}-\frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\bar{k} > (\bar{m}-\alpha+\beta)\bar{k} \Leftrightarrow$$

$$\frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\underline{k}-\frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))+(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)}{(\bar{m}-\alpha)}\bar{k} > 0 \Leftrightarrow$$

$$\underline{k}-\bar{k} > 0, \text{ i.e. } \frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\underline{k}-\frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\bar{k} < \beta(1+\alpha) \text{ is binding.}$$

To summarize:

$$p_{CD\bar{m}} = \frac{\beta(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}, p_{DC\bar{m}} = \frac{\beta((1-\lambda)(\bar{m}-\alpha+\beta)+\lambda(1+\alpha))-(\bar{m}-\alpha+\beta)\bar{k}}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}$$

$$p_{DD\bar{m}} = \frac{(\bar{m}-\alpha)(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}$$

Conditions for existence:

$$\begin{aligned}
1. \quad & \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} < \lambda < \frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\underline{k}-((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} \\
2. \quad & \beta(1+\alpha) > \frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\underline{k} - \frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\bar{k}
\end{aligned}$$

4.6.2. and $DD, \underline{m} \Rightarrow p_{\underline{m}} = 1$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m}-\alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m}-\alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) [p_{\bar{m}}(-\beta)] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m}-\alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(\bar{m}-\alpha) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1-\lambda) [p_{\bar{m}}(-\beta)] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1+\alpha) \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}}) \right] - \bar{k} = 0 \quad (\text{v})$$

$$\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{DD\bar{m}})(-\beta) \right] - \beta(1-\lambda) < 0 \quad (\text{i})$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(\bar{m}-\alpha) + (p_{DD\bar{m}})(-\beta) \right] = 0 \quad (\text{ii})$$

$$\Pi_{\bar{m}}(DC, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{DD\underline{m}})(-\beta) \right] - \beta(1-\lambda) < 0 \quad (\text{iii})$$

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] < 0 \quad (\text{iv})$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) = \lambda(1+\alpha) \left[(p_{CD\bar{m}}) \right] - \bar{k} = 0 \quad (\text{v})$$

$$\text{EQ: } p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}, p_{DD\bar{m}} = \frac{(\bar{m}-\alpha)}{\beta} \frac{\underline{k}}{\lambda(1+\alpha)}, p_{DD\underline{m}} = 1 - \frac{(\bar{m}-\alpha+\beta)}{\beta} \frac{\underline{k}}{\lambda(1+\alpha)}$$

(i), (iii) and (iv) are satisfied.

Finally all conditions of the type

$p \in [0,1], \sum p < 1$ and $0 < \lambda < 1$ reduce to:

$$1. 0 < \frac{\bar{k}(\bar{m}-\alpha+\beta)}{(1+\alpha)\beta} < \lambda < 1$$

$$2. \beta(1+\alpha) > (\bar{m}-\alpha+\beta)\bar{k}$$

To summarize:

$$p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}, p_{DD\bar{m}} = \frac{(\bar{m}-\alpha)}{\beta} \frac{\underline{k}}{\lambda(1+\alpha)}, p_{DD\underline{m}} = 1 - \frac{(\bar{m}-\alpha+\beta)}{\beta} \frac{\underline{k}}{\lambda(1+\alpha)}$$

Conditions for existence:

$$1. \frac{\bar{k}(\bar{m}-\alpha+\beta)}{(1+\alpha)\beta} < \lambda < 1$$

$$2. \beta(1+\alpha) > (\bar{m}-\alpha+\beta)\bar{k}$$

Finally we consider semi-pooling equilibria with only low types pooling, i.e. $\Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} = 0$. For this equality to hold we necessarily need $p_{CD\bar{m}} > 0$ or $p_{CD\underline{m}} > 0$ but not both since this would correspond to a pooling among high types.

$$4.7. \quad \Pi_{\underline{m}}(\bar{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{CD\underline{m}} - p_{DC\bar{m}} - p_{DC\underline{m}})(1 + \alpha) \right] - \underline{k} = 0$$

$$4.7.1. \quad p_{CD\bar{m}} > 0, \text{ i.e. } p_{CC\bar{m}} = p_{CD\underline{m}} = p_{DC\bar{m}} = p_{DD\underline{m}} = 0$$

$$p_{\underline{m}} \in (0, 1):$$

$$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k} <$$

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k} <$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(CC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] <$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right]$$

$$\Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] <$$

$$\Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) \right]$$

$$\Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) \right]$$

$$\Pi_{\underline{m}}(\bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \underline{k}$$

\Rightarrow

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \bar{m}) + (p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] < 0$$

$$\Pi_{\bar{m}}(DD, \underline{m}) = 0$$

$$\Pi_{\underline{m}}(\underline{m}) = 0 = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) \right] - \underline{k} = \Pi_{\underline{m}}(\bar{m}) \Leftrightarrow p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1 + \alpha)} \Rightarrow DD, \underline{m} \prec DD, \bar{m}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \bar{m}) + (p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k}$$

$$\Pi_{\underline{m}}(\underline{m}) = 0$$

$$\Pi_{\underline{m}}(\bar{m}) = \lambda \left[(p_{CD\bar{m}})(1 + \alpha) \right] - \underline{k}$$

$$4.7.1.1. \quad \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) > 0 \Rightarrow p_{CD\bar{m}} = 1$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) > 0 \Leftrightarrow$$

$$\lambda(\bar{m} - \alpha + \beta) > \lambda\beta + (1 - \lambda)p_{\bar{m}}\beta \Leftrightarrow p_{\bar{m}} < \frac{\lambda(\bar{m} - \alpha)}{(1 - \lambda)\beta}$$

$$\Pi_{\underline{m}}(\underline{m}) = 0 = \lambda(1 + \alpha) - \underline{k} = \Pi_{\underline{m}}(\bar{m}) \Leftrightarrow \lambda = \frac{\underline{k}}{(1 + \alpha)}$$

To summarize:

$$p_{CD\bar{m}} = 1$$

Conditions for existence:

1. $\lambda = \frac{\underline{k}}{(1+\alpha)}$
2. $\underline{k} < (1+\alpha)$
3. $p_{\bar{m}} < \frac{\lambda(\bar{m}-\alpha)}{(1-\lambda)\beta}$

note that 3. is only binding if:

$$\frac{\lambda(\bar{m}-\alpha)}{(1-\lambda)\beta} < 1 \Leftrightarrow \lambda < \frac{\beta}{(\bar{m}-\alpha+\beta)} \Leftrightarrow \frac{\underline{k}}{(1+\alpha)} < \frac{\beta}{(\bar{m}-\alpha+\beta)}$$

$$4.7.1.2. \quad \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) = 0$$

$$\Pi_{\bar{m}}(CD, \bar{m}) = \Pi_{\bar{m}}(DD, \bar{m}) \Leftrightarrow$$

$$\lambda[(p_{CD\bar{m}})(\bar{m}-\alpha+\beta)] = \lambda\beta + (1-\lambda)p_{\bar{m}}\beta \Leftrightarrow p_{CD\bar{m}} = \frac{\lambda\beta + (1-\lambda)p_{\bar{m}}\beta}{\lambda(\bar{m}-\alpha+\beta)}$$

$$\Pi_{\underline{m}}(\underline{m}) = 0 = \lambda[(p_{CD\bar{m}})(1+\alpha)] - \underline{k} = \Pi_{\underline{m}}(\bar{m}) \Leftrightarrow p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}$$

\Rightarrow

$$\frac{\lambda\beta + (1-\lambda)p_{\bar{m}}\beta}{\lambda(\bar{m}-\alpha+\beta)} = \frac{\underline{k}}{\lambda(1+\alpha)} \Leftrightarrow p_{\bar{m}} = \frac{(\bar{m}-\alpha+\beta)\underline{k} - \lambda(1+\alpha)\beta}{(1-\lambda)(1+\alpha)\beta}$$

$$\text{EQ: } p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}, p_{DD\bar{m}} = 1 - \frac{\underline{k}}{\lambda(1+\alpha)}, p_{\bar{m}} = \frac{(\bar{m}-\alpha+\beta)\underline{k} - \lambda(1+\alpha)\beta}{(1-\lambda)(1+\alpha)\beta}, p_{\underline{m}} = \frac{(1+\alpha)\beta - (\bar{m}-\alpha+\beta)\underline{k}}{(1-\lambda)(1+\alpha)\beta}$$

Conditions for existence:

1. $\frac{\underline{k}}{(1+\alpha)} < \lambda < \frac{(\bar{m}-\alpha+\beta)}{\beta} \frac{\underline{k}}{(1+\alpha)}$
2. $(1+\alpha)\beta > (\bar{m}-\alpha+\beta)\underline{k}$
3. $\underline{k} < (1+\alpha)$

4.7.2. $p_{CD\underline{m}} > 0$, i.e. $p_{CC\bar{m}} = p_{CD\bar{m}} = p_{DC\bar{m}} = p_{DD\bar{m}} = 0$

$p_{\underline{m}} \in (0,1)$:

$$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1+\bar{m}) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k} <$$

$$\Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1+\bar{m}) + (p_{DC\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1+\alpha) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k} <$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1+\alpha) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(CC, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1+\bar{m}) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] <$$

$$\Pi_{\bar{m}}(DC, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1+\bar{m}) + (p_{CD\underline{m}} + p_{DD\underline{m}})(-\beta) \right] + (1-\lambda) \left[p_{\underline{m}}(-\beta) \right]$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1+\alpha) \right] + (1-\lambda) \left[p_{\bar{m}}(-\beta) \right] <$$

$$\Pi_{\bar{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1+\alpha) \right]$$

$$\Pi_{\underline{m}}(\underline{m}) = \lambda \left[(p_{CC\underline{m}} + p_{DC\underline{m}})(1+\alpha) \right]$$

$$\Pi_{\underline{m}}(\bar{m}) = \lambda \left[(p_{CC\underline{m}} + p_{CD\underline{m}})(1+\alpha) \right] - \underline{k}$$

Since CD, \underline{m} is strictly dominated by unconditional defection such an equilibrium cannot exist.

2. Stability (The denotation of the following equilibria in the first column refers to the corresponding subsection in App. A.)

2.2.1.1.	$CC\bar{m}$ $CD\bar{m}$ $DC\bar{m}$ $DD\bar{m}$ $CD\underline{m}$ $DD\underline{m}$ \underline{m}	$p_{CC\bar{m}} = -\frac{\bar{k}}{\lambda(1+\alpha)} + \frac{\beta}{\lambda(\bar{m}-\alpha+\beta)} - p_{DC\bar{m}}$ $p_{CD\bar{m}} = p_{DC\bar{m}},$ $p_{DD\bar{m}} = -\frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{\beta}{\lambda(\bar{m}-\alpha+\beta)} + \frac{1}{\lambda} - p_{DC\bar{m}}$ $p_{CD\underline{m}} = \frac{\bar{k}}{\lambda(1+\alpha)}$ $p_{DD\underline{m}} = \frac{\bar{k}}{\lambda(1+\alpha)} \frac{\bar{m}-\alpha}{\beta} - \frac{1-\lambda}{\lambda}$	$\lambda \geq 1 - \frac{\bar{k}}{(1+\alpha)} \frac{\bar{m}-\alpha}{\beta}$	1. $\frac{\bar{k}}{1+\alpha} < \frac{\beta}{\bar{m}-\alpha+\beta}$ 2. $\beta < (\bar{m}-\alpha): \quad p_{DC\bar{m}} \leq \frac{1}{\lambda} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right)$ $\beta \geq (\bar{m}-\alpha): \quad p_{DC\bar{m}} \leq \frac{1}{\lambda} \frac{\bar{m}-\alpha}{\beta} \left(\frac{\beta}{\bar{m}-\alpha+\beta} - \frac{\bar{k}}{1+\alpha} \right)$
	<p>Note that the payoffs for non-signaling high types is independent of their own share. However payoffs for all other behavioral strategies strictly increase in the share $p_{CD\underline{m}}$ and weakly decrease in $p_{DD\underline{m}}$. Hence if the set is perturbed such that the equilibrium level for $p_{CD\underline{m}}$ is exceeded than there is no force bringing it back to that level. Hence this set of equilibria is unstable.</p>			$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\underline{m}})(1+\bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}} + p_{DD\underline{m}})(-\beta) \right] - \beta(1-\lambda) - \bar{k}$ $\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1+\bar{m}) + (p_{CD\underline{m}})(1+\alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] - \bar{k}$ $\Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[(p_{CD\underline{m}})(1+\bar{m}) + (p_{CC\bar{m}} + p_{CD\bar{m}})(1+\alpha) + (p_{DD\bar{m}})(-\beta) \right] - \beta(1-\lambda) - \bar{k}$ $\Pi_{\bar{m}}(DD, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\underline{m}})(1+\alpha) \right] - \bar{k}$ $\Pi_{\underline{m}}(CD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1+\bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right]$ $\Pi_{\underline{m}}(DD, \underline{m}) = \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1+\alpha) \right]$

2.2.1.2.1.	$CC\bar{m}$ $CD\bar{m}$ $DC\bar{m}$ $DD\bar{m}$ $CD\underline{m}$ \underline{m} \bar{m}	$p_{CC\bar{m}} = \frac{(1+\alpha)(\beta+\bar{k}) - (1+\beta+\bar{m})\underline{k}}{\lambda(1+\alpha)(\bar{m}-\alpha+\beta)} - p_{CD\bar{m}}$ $p_{DC\bar{m}} = p_{CD\bar{m}} + \frac{\underline{k}-\bar{k}}{\lambda(\bar{m}-\alpha+\beta)}$ $p_{DD\bar{m}} = 1 - \frac{\beta+\underline{k}-\bar{k}}{\lambda(\bar{m}-\alpha+\beta)} - p_{CD\bar{m}}$ $p_{CD\underline{m}} = \frac{\underline{k}}{\lambda(1+\alpha)} + \frac{\underline{k}-\bar{k}}{\lambda(\bar{m}-\alpha+\beta)} = \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{\lambda(1+\alpha)(\bar{m}-\alpha+\beta)}$ $p_{\underline{m}} = \frac{1}{1-\lambda} \frac{\bar{m}-\alpha}{\beta} \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(1+\alpha)(\bar{m}-\alpha+\beta)}$	$\frac{\beta+\underline{k}-\bar{k}}{(\bar{m}-\alpha+\beta)} < \lambda$ $\leq 1 - \frac{\bar{m}-\alpha}{\beta} \frac{(1+\beta+\bar{m})\underline{k} - (1+\alpha)\bar{k}}{(1+\alpha)(\bar{m}-\alpha+\beta)}$	1. $(1+\alpha)\beta > \frac{\bar{m}-\alpha+\beta}{(\bar{m}-\alpha)} ((1+\bar{m})\underline{k} - (1+\alpha)\bar{k})$ 2. $0 \leq p_{CD\bar{m}} \leq \min \left\{ \underbrace{1 - \frac{\beta+\underline{k}-\bar{k}}{\lambda(\bar{m}-\alpha+\beta)}}_x, \underbrace{\frac{\beta+\bar{k} - \frac{1+\beta+\bar{m}}{1+\alpha}\underline{k}}{\lambda(\bar{m}-\alpha+\beta)}}_y \right\}$ $x \geq y \Leftrightarrow \lambda \geq \frac{2\beta}{(\bar{m}-\alpha+\beta)} - \frac{\underline{k}}{1+\alpha} \left(\stackrel{(3)(b)}{>} \frac{\beta+\underline{k}-\bar{k}}{(\bar{m}-\alpha+\beta)} \right)$
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	<p>Note that the payoff for non-signaling high types is independent of their own share. However payoffs for all other behavioral strategies strictly increase in the share $p_{CD\bar{m}}$ and weakly decrease in $p_{DD\bar{m}}$. Consider a perturbation such that the equilibrium level for $p_{CD\bar{m}}$ is exceeded and $p_{DD\bar{m}}$ decreases. Payoffs for signaling high types strictly increase and there is no force bringing $p_{CD\bar{m}}$ back to that level. Hence this set of equilibria is unstable.</p>	$\begin{aligned}\Pi_{\bar{m}}(CC, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k} \\ \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + p_{CD\bar{m}}(1 + \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k} \\ \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k} \\ \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k} \\ \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \\ \Pi_{\underline{m}}(\underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) \right] \\ \Pi_{\underline{m}}(\bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \underline{k}\end{aligned}$
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2.2.1.2.2.

$CC\bar{m}$ $CD\bar{m}$ $DC\bar{m}$ $DD\bar{m}$ $CD\underline{m}$ \underline{m}	$p_{CC\bar{m}} = (1 - p_{CD\bar{m}}) \frac{\beta}{(\bar{m} - \alpha + \beta)} - p_{CD\bar{m}}$ $p_{DC\bar{m}} = p_{CD\bar{m}} - \frac{\bar{k}}{\lambda(1 + \bar{m} + \beta)} + \frac{(1 + \alpha)}{(1 + \bar{m} + \beta)} p_{CD\bar{m}}$ $p_{DD\bar{m}} = (1 - p_{CD\bar{m}}) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} - p_{DC\bar{m}}$ $p_{CD\underline{m}} = \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)}$	$\max \left\{ \frac{\beta(-\alpha(2 + \alpha) + \beta + \bar{m}(2 + \bar{m} + \beta)) - \bar{k}(\bar{m} - \alpha)(\bar{m} - \alpha + \beta)}{(1 + \bar{m})(\bar{m} - \alpha + \beta)^2}, \right.$ $\left. 1 - \frac{(\bar{m} - \alpha)}{\beta(\bar{m} - \alpha + \beta)} \left((1 + \bar{m} + \beta) \frac{\bar{k}}{(1 + \alpha)} - \bar{k} \right) \right\}$ $< \lambda < 1 - \frac{\bar{k}}{(1 + \alpha)} \frac{(\bar{m} - \alpha)}{\beta}$	$1. \frac{\bar{k}}{(1 + \alpha)} < \frac{\beta}{(\bar{m} - \alpha + \beta)}$ $2. 0 \leq p_{CD\bar{m}} \leq$ $\min \left\{ \frac{\left(1 - \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) \frac{\beta}{(\bar{m} - \alpha + \beta)}}{\left(1 - \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)} \right) \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)} + \frac{\bar{k}}{\lambda(1 + \bar{m} + \beta)} - \frac{1 + \alpha}{1 + \bar{m} + \beta} \frac{(1 - \lambda)\beta}{\lambda(\bar{m} - \alpha)}} \right\}$
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Note that the payoffs for non-signaling high types is independent of their own share.

However payoffs for all other behavioral strategies strictly increase in the share $p_{CD\bar{m}}$.

Hence if the set is perturbed such that the equilibrium level for $p_{CD\bar{m}}$ is exceeded than

$$\begin{aligned}\Pi_{\bar{m}}(CC, \bar{m}) &= \lambda \left[(1 + \bar{m}) - (1 + \bar{m} + \beta)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (1 + \alpha) - (1 + \alpha + \beta)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \bar{k} \\ \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[(1 + \bar{m}) - (p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) - (1 + \bar{m})(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(1 + \alpha) - (1 + \alpha)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \bar{k} \\ \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right]\end{aligned}$$

there is no force bringing it back to that level. Hence this set of equilibria is unstable.

$$\begin{aligned}\Pi_{\bar{m}}(CC, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + p_{CD\bar{m}}(1 + \alpha) + (p_{DC\bar{m}} + p_{DD\bar{m}})(-\beta) \right] - \bar{k} \\ \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}} + p_{CD\bar{m}})(1 + \alpha) \right] - \bar{k} \\ \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right]\end{aligned}$$

$$\begin{aligned}\Pi_{\bar{m}}(CC, \bar{m}) &= \lambda \left[(1 + \bar{m}) - (1 + \bar{m} + \beta)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) + (1 + \alpha) - (1 + \alpha + \beta)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \bar{k} \\ \Pi_{\bar{m}}(DC, \bar{m}) &= \lambda \left[(1 + \bar{m}) - (p_{CC\bar{m}} + p_{CD\bar{m}})(\bar{m} - \alpha) - (1 + \bar{m})(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \beta(1 - \lambda) - \bar{k} \\ \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda \left[(1 + \alpha) - (1 + \alpha)(p_{DC\bar{m}} + p_{DD\bar{m}}) \right] - \bar{k} \\ \Pi_{\bar{m}}(CD, \underline{m}) &= \lambda \left[(p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (p_{CD\bar{m}} + p_{DD\bar{m}})(-\beta) \right]\end{aligned}$$

4.1.3.2.

$CC\bar{m}$
 $CD\bar{m}$
 \underline{m}

$$p_{CD\bar{m}} = \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}$$

$$p_{CC\bar{m}} = 1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}$$

$$p_{\underline{m}} = 1$$

$$1. \frac{\bar{k} + \beta}{(1+\bar{m} + \beta)} < \lambda < 1: \quad \frac{\beta}{(\bar{m} - \alpha + \beta)} < \frac{\bar{k}}{(1+\alpha)} \left(< \frac{1+\bar{m}}{(1+\alpha)} \right)$$

$$2. 1 - \frac{(\bar{m} - \alpha)}{\beta} \frac{\bar{k}}{(1+\alpha)} < \lambda < 1: \quad \frac{\bar{k}}{(1+\alpha)} \leq \frac{\beta}{(\bar{m} - \alpha + \beta)}$$

$$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda(p_{CC\bar{m}} + p_{CD\bar{m}})(1+\bar{m}) - \beta(1-\lambda) - \bar{k}$$

$$\Pi_{\bar{m}}(CD, \underline{m}) = \lambda(p_{CC\bar{m}})(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)]$$

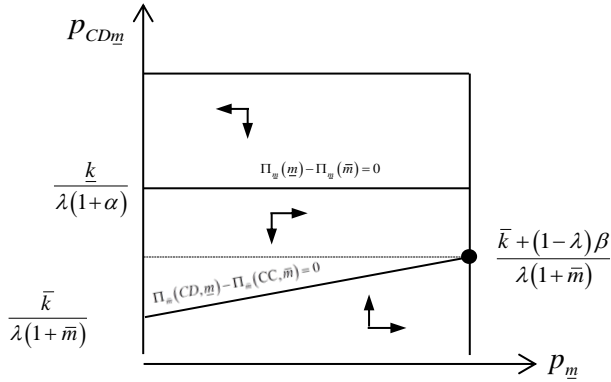
$$\Pi_{\underline{m}}(\underline{m}) = \lambda p_{CC\bar{m}}(1+\alpha)$$

$$\Pi_{\underline{m}}(\bar{m}) = \lambda(p_{CC\bar{m}} + p_{CD\bar{m}})(1+\alpha) - \underline{k}$$

$$\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) - \beta(1-\lambda) - \bar{k} - (1-\lambda)[p_{\bar{m}}(-\beta)] \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\beta(1-\lambda) + \bar{k} + (1-\lambda)[p_{\bar{m}}(-\beta)]}{\lambda(1+\bar{m})} = \frac{\bar{k} + \beta(1-\lambda)p_{\underline{m}}}{\lambda(1+\bar{m})}$$

$$\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{CC\bar{m}}(1+\alpha) - \lambda(p_{CC\bar{m}} + p_{CD\bar{m}})(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\underline{k}}{\lambda(1+\alpha)}, \text{ note that } \frac{\underline{k}}{\lambda(1+\alpha)} > \frac{\bar{k} + \beta(1-\lambda)}{(1+\bar{m})} \text{ due to the condition (see support) } \lambda > 1 - \frac{(\bar{m} - \alpha)\bar{k}}{\beta(1+\alpha)}.$$

Hence we obtain the following phase diagram:



As the diagram clearly indicates this equilibrium is stable.

Note that the lower bound in the support condition implies

$$\left(\frac{\underline{k}}{\lambda(1+\alpha)} > \right) \frac{\bar{k}}{\lambda(1+\alpha)} > \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})}$$

Differences in fitness payoffs:

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda(p_{CC\bar{m}})(1+\bar{m}) - \lambda p_{CC\bar{m}}(1+\alpha) = \lambda(\bar{m} - \alpha)p_{CC\bar{m}} = \lambda(\bar{m} - \alpha) \left(1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right) > 0$$

$$(\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}))' = -\alpha\lambda \left(1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right) < 0$$

$p_{CD\bar{m}} = \frac{1}{2} \left[1 + \frac{\underline{k}}{\lambda(1+\alpha)} \right]$ $p_{DC\bar{m}} = \frac{1}{2} \left[1 - \frac{\underline{k}}{\lambda(1+\alpha)} \right]$ $p_{\underline{m}} = \frac{1}{2} \left[1 + \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right]$ $p_{\bar{m}} = \frac{1}{2} \left[1 - \frac{1}{(1-\lambda)\beta} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right] \right]$	$0 < \frac{\beta + (\underline{k} - \bar{k})}{(\bar{m} - a + b)} < \lambda < 1 - \frac{1}{\beta} \left[\frac{(1+\bar{m})}{(1+a)} \underline{k} - \bar{k} \right] < 1$	$\beta(1+\alpha) > \frac{(\bar{m} - \alpha + \beta)}{(\bar{m} - \alpha)} \left[(1+\bar{m})\underline{k} - (1+\alpha)\bar{k} \right] + \frac{\underline{k} - \bar{k}}{\bar{m} - \alpha} \beta(1+\alpha)$
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$\Pi_{\bar{m}}(DC, \bar{m}) = \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$
 $\Pi_{\bar{m}}(CD, \underline{m}) = \lambda p_{DC\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)]$
 $\Pi_{\underline{m}}(\underline{m}) = \lambda p_{DC\bar{m}}(1+\alpha)$
 $\Pi_{\underline{m}}(\bar{m}) = \lambda p_{CD\bar{m}}(1+\alpha) - \underline{k}$
 $\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\bar{m}}(1+\bar{m}) - (1-\lambda)[p_{\bar{m}}(-\beta)] \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) - \beta(1-\lambda)p_{\bar{m}} - \bar{k} - \lambda(1-p_{CD\bar{m}})(1+\bar{m}) + \beta(1-\lambda)(1-p_{\bar{m}}) \geq 0 \Leftrightarrow$
 $2\lambda p_{CD\bar{m}}(1+\bar{m}) - 2\beta(1-\lambda)p_{\bar{m}} - \bar{k} - \lambda(1+\bar{m}) + \beta(1-\lambda) \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{2\beta(1-\lambda)p_{\bar{m}} + \bar{k} + \lambda(1+\bar{m}) - \beta(1-\lambda)}{2\lambda(1+\bar{m})} = \frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda) + 2\beta(1-\lambda)p_{\bar{m}}}{2\lambda(1+\bar{m})}$
 $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{DC\bar{m}}(1+\alpha) - \lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow \lambda(1+\alpha) - 2\lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\lambda(1+\alpha) + \underline{k}}{2\lambda(1+\alpha)} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$

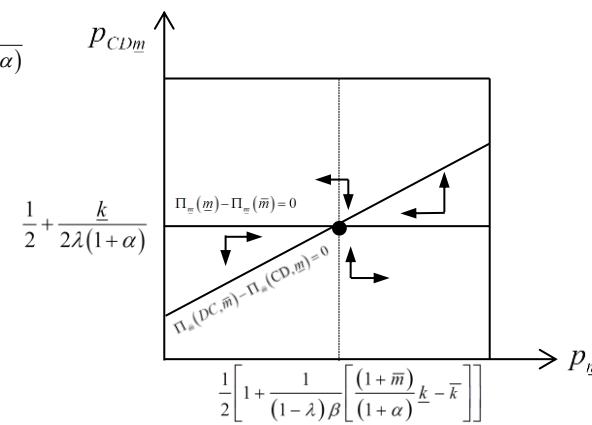
Hence we obtain the following phase diagram:

Differences in fitness payoffs:

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda p_{DC\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \lambda p_{DC\bar{m}}(1+\alpha) = \lambda(\bar{m} - \alpha)p_{DC\bar{m}} - \beta(1-\lambda)p_{\bar{m}}$$

$$(\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}))' = -\alpha\lambda - \beta(1-\lambda)p_{\bar{m}} < 0$$

As the diagram clearly indicates this equilibrium is stable.



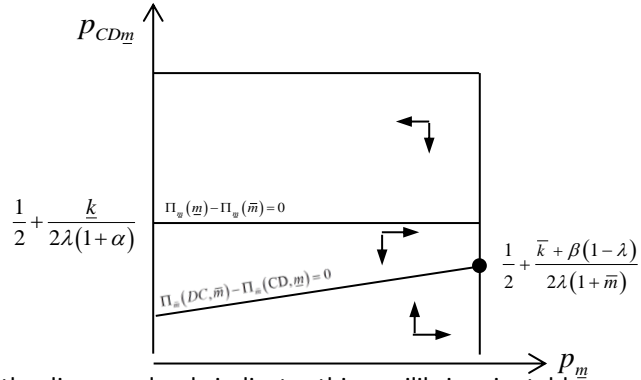
$DC\bar{m}$	$p_{CD\bar{m}} = \frac{1}{2} \left[1 + \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right]$	$\lambda > \max \left\{ \frac{\bar{k} + \beta}{(1+\bar{m} + \beta)}, 1 - \frac{1}{\beta(1+\alpha)} ((1+\bar{m})\underline{k} - (1+\alpha)\bar{k}), 1 - \frac{\bar{m} - \alpha}{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta} (1+\bar{m} + \bar{k}) \right\}$
$CD\underline{m}$	$p_{DC\bar{m}} = \frac{1}{2} \left[1 - \frac{\bar{k} + (1-\lambda)\beta}{\lambda(1+\bar{m})} \right]$	
\underline{m}	$p_{\bar{m}} = 0$	

as above: $\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda) + 2\beta(1-\lambda)p_{\underline{m}}}{2\lambda(1+\bar{m})}$

$$\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\lambda(1+\alpha) + \underline{k}}{2\lambda(1+\alpha)} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$$

Note that $\frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)} > \frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}$ since this is equivalent to the second argument in the support condition.

Hence we obtain the following phase diagram:



As the diagram clearly indicates this equilibrium is stable.

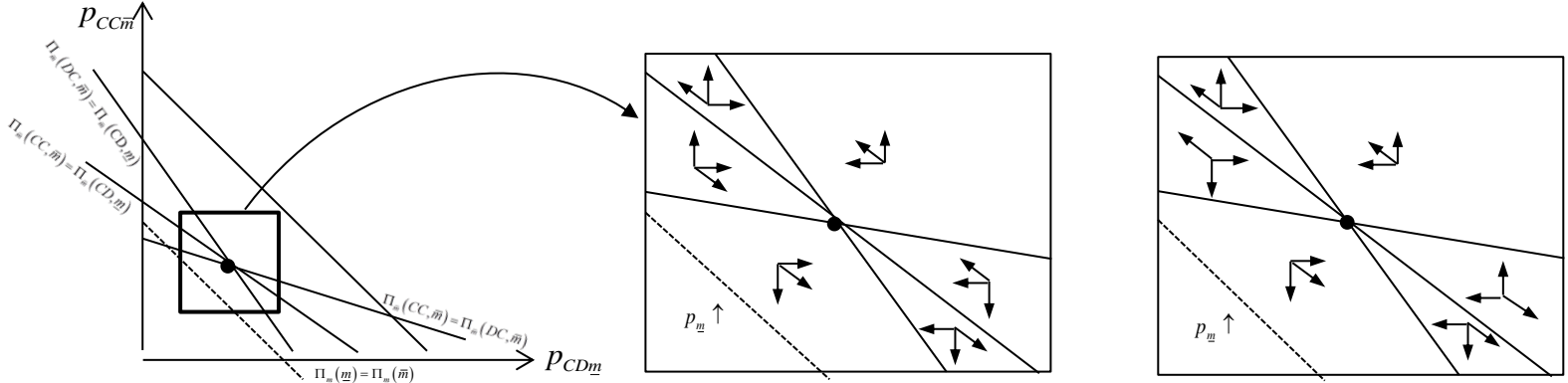
$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda p_{DC\bar{m}}(1+\bar{m}) - \lambda p_{DC\bar{m}}(1+\alpha) = \lambda(\bar{m} - \alpha)p_{DC\bar{m}} > 0$$

$$(\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}))^f = -\alpha\lambda < 0$$

Differences in fitness payoffs:

$$\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}) = \lambda p_{DC\bar{m}}(1+\bar{m}) - \lambda p_{DC\bar{m}}(1+\alpha) = \lambda(\bar{m} - \alpha)p_{DC\bar{m}} > 0$$

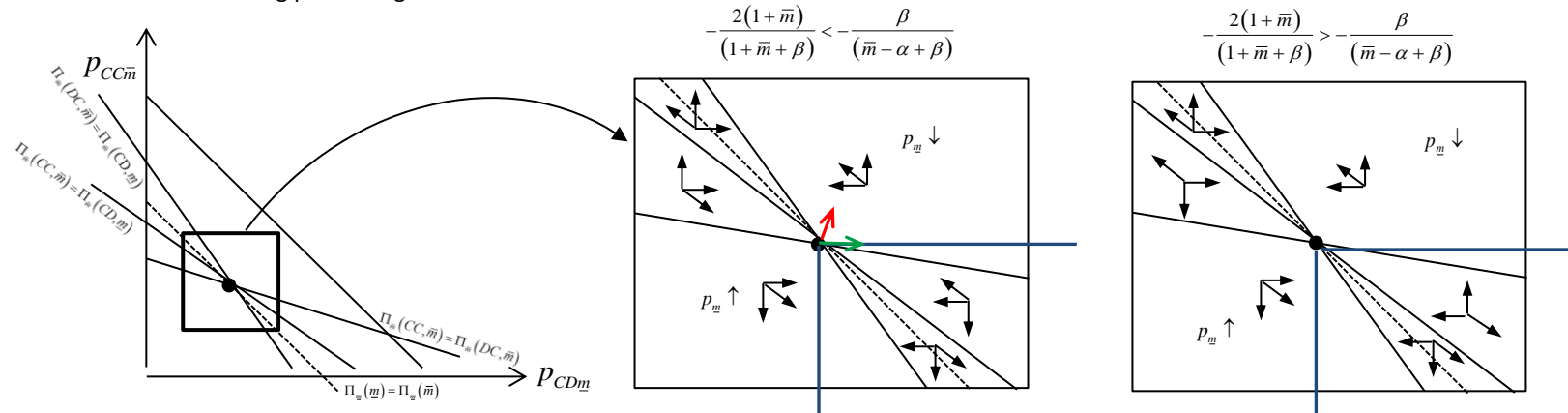
$$(\Pi_{\bar{m}}(CD, \underline{m}) - \Pi_{\underline{m}}(\underline{m}))^f = -\alpha\lambda < 0$$

<div>4.3.1.1.</div> <div> $DC\bar{m}$ $CC\bar{m}$ $CD\bar{m}$ \bar{m} </div>	$p_{DC\bar{m}} = \frac{(1+\bar{m})(\lambda(\bar{m}-\alpha+\beta)-\beta)-\bar{k}(\bar{m}-\alpha)}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))}$ $p_{CC\bar{m}} = \frac{\beta(2(1+\bar{m})+\beta-\lambda(\bar{m}-\alpha+\beta)-\bar{k})}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))}$ $p_{CD\bar{m}} = \frac{\bar{k}(\bar{m}-\alpha+\beta)+(1+\bar{m}+\beta)(\lambda(\bar{m}-\alpha+\beta)-\beta)}{\lambda(2(1+\bar{m})(\bar{m}-\alpha+\beta)-\beta(1+\alpha))}$	$\frac{2}{\beta} \left(\frac{\underline{k}}{(1+\alpha)} - \bar{k} \right) + \frac{\beta - (\underline{k} - \bar{k})}{(\bar{m} - \alpha + \beta)} < \lambda < 1$
	<p>We obtain the following phase diagram (derivation below):</p>  <p>As the diagram clearly indicates this equilibrium is unstable.</p>	

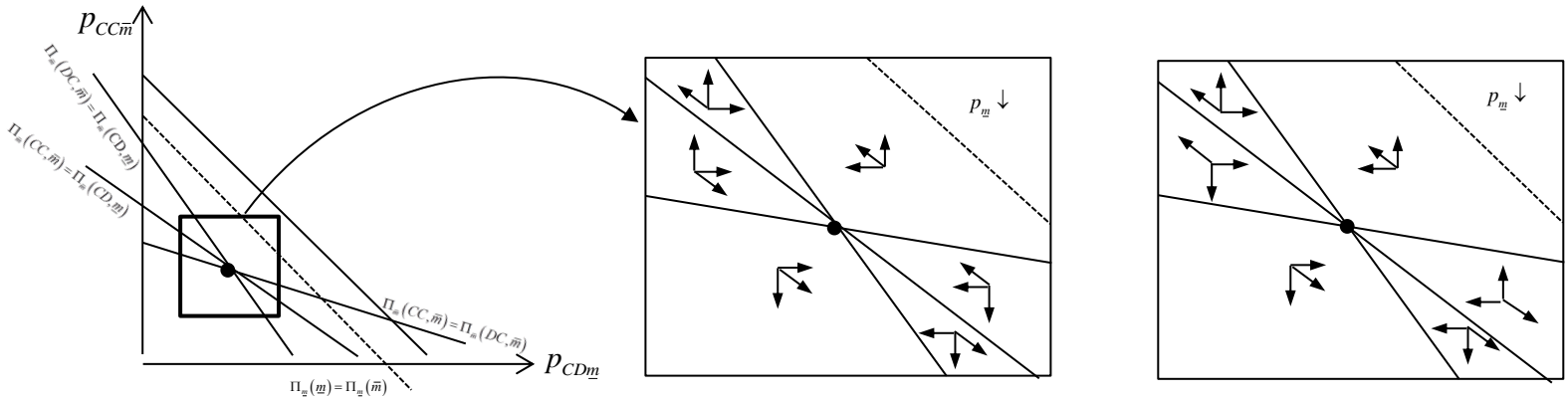
<div>4.3.1.2.</div> <div> $DC\bar{m}$ $CC\bar{m}$ $CD\bar{m}$ \bar{m} \underline{m} </div>	$\left\{ \begin{aligned} p_{DC\bar{m}} &= \frac{1}{2} \left(1 - \frac{\underline{k} - \bar{k} - \beta}{\lambda(\bar{m} - \alpha + \beta)} \right) \\ p_{CC\bar{m}} &= \frac{\beta + \bar{k} - \frac{1+\bar{m}+\beta}{1+\alpha} \underline{k}}{\lambda(\bar{m} - \alpha + \beta)} \\ p_{CD\bar{m}} &= \frac{1}{2} \left(1 + \frac{\left(\frac{2(\bar{m} - \alpha + \beta)}{1+\alpha} + 1 \right) \underline{k} - \bar{k} - \beta}{\lambda(\bar{m} - \alpha + \beta)} \right) \\ p_{\underline{m}} &= \frac{1}{\beta(1-\lambda)} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) - \frac{1}{2(1-\lambda)(\bar{m} - \alpha + \beta)} (\underline{k} - \bar{k}) - \frac{\lambda(\bar{m} - \alpha + \beta) - \beta}{2(1-\lambda)(\bar{m} - \alpha + \beta)} \end{aligned} \right\}$	$1. \frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda < \min \left\{ \frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right), 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right) \right) \right\}$ $2. \underline{k} - \bar{k} < \bar{m} - \alpha$ $3. \underline{k} < 1 + \alpha$ $4. (1+\alpha)\beta > (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}$
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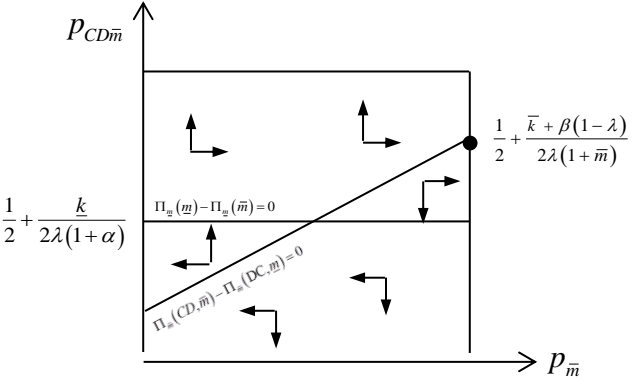
	$\Pi_{\bar{m}}(CC, \bar{m}) = \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + p_{DC\bar{m}}(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k}$ $\Pi_{\bar{m}}(DC, \bar{m}) = \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + p_{CC\bar{m}}(1 + \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k}$ $\Pi_{\bar{m}}(CD, \underline{m}) = \lambda (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right]$ $\Pi_{\underline{m}}(\underline{m}) = \lambda (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha)$ $\Pi_{\underline{m}}(\bar{m}) = \lambda (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) - \underline{k}$ $\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(DC, \bar{m}) \geq 0 \Leftrightarrow \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + p_{DC\bar{m}}(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k} - \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + p_{CC\bar{m}}(1 + \alpha) \right] - (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] + \bar{k} \geq 0 \Leftrightarrow$ $\lambda \left[(p_{CC\bar{m}})(\bar{m} - \alpha) + p_{DC\bar{m}}(-\beta) \right] \geq (1 - \lambda) \beta p_{\bar{m}} \Leftrightarrow p_{CC\bar{m}} \geq \frac{(1 - \lambda) \beta p_{\bar{m}} + \lambda \beta}{\lambda(\bar{m} - \alpha + \beta)} - \frac{\beta}{(\bar{m} - \alpha + \beta)} p_{CD\bar{m}}$ $\Pi_{\bar{m}}(CC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \geq 0 \Leftrightarrow \lambda \left[(p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \bar{m}) + p_{DC\bar{m}}(-\beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) + p_{\bar{m}}(-\beta) \right] - \bar{k} - \lambda (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) - (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \geq 0 \Leftrightarrow$ $\lambda \left[(p_{CD\bar{m}})(1 + \bar{m}) - (1 - p_{CD\bar{m}} - p_{CC\bar{m}})(1 + \bar{m} + \beta) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k} \geq 0 \Leftrightarrow \lambda \left[2(p_{CD\bar{m}})(1 + \bar{m}) + p_{CC\bar{m}}(1 + \bar{m} + \beta) \right] \geq (1 - \lambda) \beta p_{\underline{m}} + \bar{k} + (1 + \bar{m} + \beta) \lambda \Leftrightarrow p_{CC\bar{m}} \geq 1 + \frac{(1 - \lambda) \beta p_{\underline{m}} + \bar{k}}{\lambda(1 + \bar{m} + \beta)} - \frac{2(1 + \bar{m})}{(1 + \bar{m} + \beta)} p_{CD\bar{m}}$ $\Pi_{\bar{m}}(DC, \bar{m}) - \Pi_{\bar{m}}(CD, \underline{m}) \geq 0 \Leftrightarrow \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + p_{CC\bar{m}}(1 + \alpha) \right] + (1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] - \bar{k} - \lambda (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \bar{m}) - (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] \geq 0 \Leftrightarrow$ $\lambda \left[2p_{CD\bar{m}}(1 + \bar{m}) + p_{CC\bar{m}}(1 + \alpha) \right] - \lambda(1 + \bar{m}) - \bar{k} + 2(1 - \lambda) \left[p_{\underline{m}}(-\beta) \right] + (1 - \lambda) \beta \geq 0 \Leftrightarrow p_{CC\bar{m}} \geq \frac{\lambda(1 + \bar{m}) + \bar{k} + 2(1 - \lambda) \beta p_{\underline{m}} - (1 - \lambda) \beta}{\lambda(1 + \alpha)} - \frac{2(1 + \bar{m})}{(1 + \alpha)} p_{CD\bar{m}}$ $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda (p_{CC\bar{m}} + p_{DC\bar{m}})(1 + \alpha) - \lambda (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) + \underline{k} \geq 0 \Leftrightarrow \lambda (1 - p_{CD\bar{m}})(1 + \alpha) - \lambda (p_{CC\bar{m}} + p_{CD\bar{m}})(1 + \alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CC\bar{m}} \leq 1 + \frac{\underline{k}}{\lambda(1 + \alpha)} - 2p_{CD\bar{m}}$ <p>Note: $-\frac{2(1 + \bar{m})}{(1 + \alpha)} < -2 < -\frac{2(1 + \bar{m})}{(1 + \bar{m} + \beta)} < ? > -\frac{\beta}{(\bar{m} - \alpha + \beta)}$</p>
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Hence we obtain the following phase diagram:

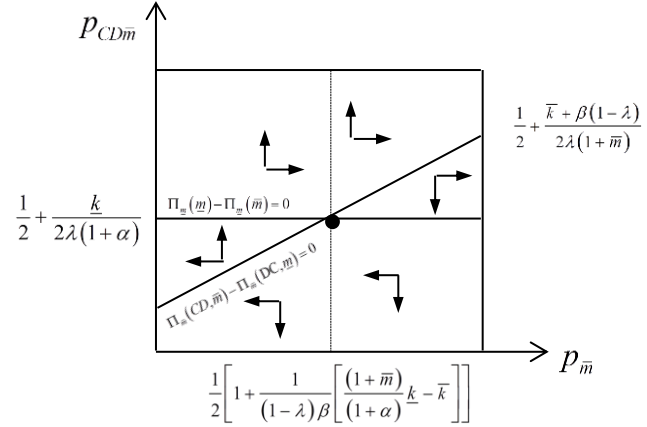


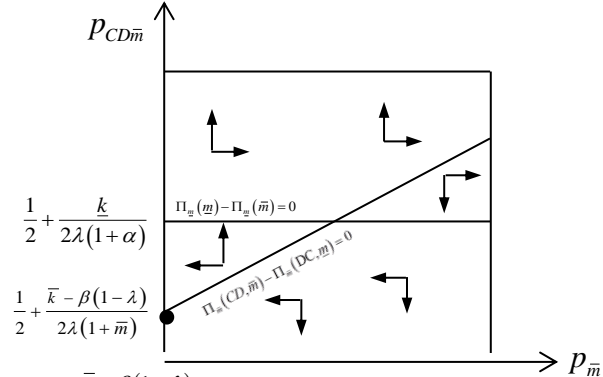
We saw for the previously analyzed equilibrium with strict inequality with respect to the signaling decision among low types that this equilibrium is unstable. The question here is whether the indeterminacy of the low types in equilibrium could have a stabilizing effect. It turns out that it doesn't. The reason is that if $p_{\bar{m}}$ decreases the new intersection of the iso-profit lines (not an equilibrium) lies in the fourth quadrant relative to the equilibrium point. We will argue for the most favorite scenario that an adjustment in $p_{\bar{m}}$ will not stabilize the equilibrium. Consider therefore a perturbation of the type indicated by the red arrow. For such an perturbation equilibrium will not be restored in the absence of an adjusting $p_{\bar{m}}$. What kind of adjustment is most favorable with respect to stabilization? The instability can only be circumvented if the induced shift of the intersection point of iso-profit lines and thereby a shift of regions with the depicted dynamics would bring the perturbed point into a region with dynamics point at the equilibrium. Most favorable is a strong and fast movement to right at the boundary of the fourth quadrant (indicated by the green arrow). It is important to note that even this most favorable movement cannot induce the perturbed point to be pushed into a region to the left of the dotted line (unaltered by changes in $p_{\bar{m}}$), because then $p_{\bar{m}}$ would start to increase again. And if this is assumed to be fast and strong, then the iso-profit lines will be shifted back towards its equilibrium locations. In other words the only thing that can happen is that the population state pointed at by the red arrow is find itself in the area between the $\Pi_{\bar{m}}(DC, \bar{m}) = \Pi_{\bar{m}}(CD, \bar{m})$ -isoline and the dotted line. However this will not lead to a reestablishment of equilibrium but to further movement away. The same argument applies to the second diagram.

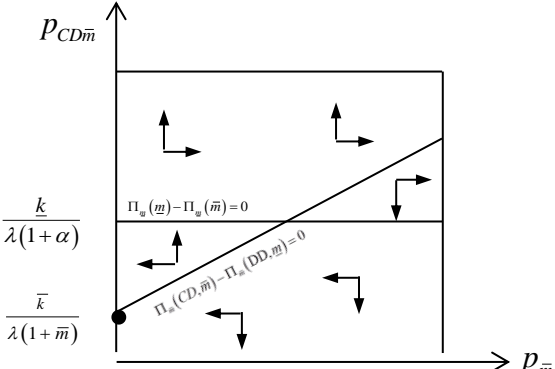
4.3.1.3.	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="margin-bottom: 10px;"> $DC\bar{m}$ $p_{DC\bar{m}} = \frac{(\bar{m} - \alpha)((1 + \bar{m} + \beta)\lambda - \beta - \bar{k})}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$ </div> <div style="margin-bottom: 10px;"> $CC\bar{m}$ $p_{CC\bar{m}} = \frac{\beta((1 + \bar{m} + \beta)\lambda - \beta - \bar{k})}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$ </div> <div> $CD\underline{m}$ $p_{CD\underline{m}} = \frac{\bar{k}(\bar{m} - \alpha + \beta) + (1 + \bar{m} + \beta)\lambda(\bar{m} - \alpha) + (\bar{m} - \alpha + \beta)(1 - \lambda)\beta}{\lambda(2(1 + \bar{m})(\bar{m} - \alpha + \beta) - \beta(1 + \alpha))}$ </div> </div>	$\lambda > \max \left\{ \underbrace{\frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)}}_x, 2 - \underbrace{\left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right)}_y \right\}$ <p>note:</p> $y > x \Leftrightarrow 2 - \left(\frac{\beta + \bar{k} - \underline{k}}{(\bar{m} - \alpha + \beta)} + \frac{2}{\beta} \left(\frac{(1 + \bar{m})}{(1 + \alpha)} \underline{k} - \bar{k} \right) \right) - \frac{\beta + \bar{k}}{(1 + \bar{m} + \beta)} > 0$ $\Leftrightarrow (1 + \alpha)\beta > (1 + \bar{m} + \beta)\underline{k} - (1 + \alpha)\bar{k} \quad \wedge \quad \underline{k} < 1 + \alpha$	$\bar{k} < 1 + \bar{m}$
	<p>We obtain the following phase diagram (derivation above):</p>  <p>As the diagram clearly indicates this equilibrium is unstable.</p>		

4.4.1.1.	$CD\bar{m}$ $DC\underline{m}$ \bar{m}	$p_{DC\underline{m}} = \frac{1}{2} - \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}$ $p_{CD\bar{m}} = \frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})}$	$0 < \frac{1+\alpha}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta} \left(\beta \left(1 + \frac{1+\bar{m}}{1+\alpha} \right) - \frac{\bar{k}}{1+\alpha} (\bar{m}-\alpha) \right) < \lambda$ $< 1 - \frac{1}{\beta} \left(\frac{k}{1+\alpha} - \bar{k} \right) < 1$	<ol style="list-style-type: none"> 1. $(1+\alpha)\beta > (\bar{m}-\alpha+\beta)\bar{k}$ 2. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m}-\alpha)+\bar{k}(\bar{m}-\alpha+2\beta))}{(\bar{m}-\alpha+\beta)(1+\bar{m})+(1+\alpha)\beta}$
	<p>as below:</p> $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\underline{m}}(1+\bar{m}) - (1-\lambda)[p_{\underline{m}}(-\beta)] \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{2\beta(1-\lambda)p_{\bar{m}} + \bar{k} + \lambda(1+\bar{m}) - \beta(1-\lambda)}{2\lambda(1+\bar{m})} = \frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda) + 2\beta(1-\lambda)p_{\bar{m}}}{2\lambda(1+\bar{m})}$ $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{DC\underline{m}}(1+\alpha) - \lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\lambda(1+\alpha) + \underline{k}}{2\lambda(1+\alpha)} = \frac{1}{2} + \frac{k}{2\lambda(1+\alpha)}$ <p>Hence we obtain the following phase diagram:</p>  <p>As the diagram clearly indicates this equilibrium is unstable. Note $\frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})} > \frac{1}{2} + \frac{k}{2\lambda(1+\alpha)} \Leftrightarrow \frac{\bar{k} + \beta(1-\lambda)}{(1+\bar{m})} > \frac{k}{(1+\alpha)} \Leftrightarrow \lambda < 1 - \frac{1}{\beta} \left(\frac{k}{1+\alpha} - \bar{k} \right)$, what is equivalent to the upper bound of the support.</p>			

4.4.1.2.	$CD\bar{m}$ $DC\underline{m}$ \bar{m} \underline{m}	$p_{CD\bar{m}} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$ $p_{DC\underline{m}} = \frac{1}{2} - \frac{\underline{k}}{2\lambda(1+\alpha)}$ $p_{\bar{m}} = \frac{1}{2} + \frac{1}{2\beta(1-\lambda)} \left[\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right]$	$\frac{\beta + \underline{k} - \bar{k}}{(\bar{m} - \alpha + \beta)} < \lambda < 1 - \frac{1}{\beta} \left(\frac{(1+\bar{m})}{(1+\alpha)} \underline{k} - \bar{k} \right)$	<ol style="list-style-type: none"> 1. $(1+\alpha)\beta > (\bar{m} - \alpha + \beta)\bar{k}$ 2. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m} - \alpha) + \bar{k}(\bar{m} - \alpha + 2\beta))}{(\bar{m} - \alpha + \beta)(1+\bar{m}) + (1+\alpha)\beta}$ 3. $\bar{k} < 1 + \alpha$
	<p> $\Pi_{\bar{m}}(CD, \bar{m}) = \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$ $\Pi_{\bar{m}}(DC, \underline{m}) = \lambda p_{DC\underline{m}}(1+\bar{m}) + (1-\lambda)[p_{\underline{m}}(-\beta)]$ $\Pi_{\underline{m}}(\underline{m}) = \lambda p_{DC\underline{m}}(1+\alpha)$ $\Pi_{\underline{m}}(\bar{m}) = \lambda p_{CD\bar{m}}(1+\alpha) - \underline{k}$ </p> <p> $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\underline{m}}(1+\bar{m}) - (1-\lambda)[p_{\underline{m}}(-\beta)] \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{2\beta(1-\lambda)p_{\bar{m}} + \bar{k} + \lambda(1+\bar{m}) - \beta(1-\lambda)}{2\lambda(1+\bar{m})} = \frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda) + 2\beta(1-\lambda)p_{\bar{m}}}{2\lambda(1+\bar{m})}$ </p> <p> $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{DC\underline{m}}(1+\alpha) - \lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\lambda(1+\alpha) + \underline{k}}{2\lambda(1+\alpha)} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$ </p> <p>Hence we obtain the following phase diagram:</p> <p>Note:</p> $\frac{1}{2} + \frac{\bar{k} + \beta(1-\lambda)}{2\lambda(1+\bar{m})} > \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$ $\Leftrightarrow \frac{\bar{k} + \beta(1-\lambda)}{(1+\bar{m})} > \frac{\underline{k}}{(1+\alpha)} \Leftrightarrow \lambda < 1 - \frac{1}{\beta} \left(\frac{\underline{k}}{1+\alpha} - \bar{k} \right)$ <p>, what is equivalent to the upper bound of the support. As the diagram clearly indicates this equilibrium is unstable.</p>			



<div style="text-align: center;"> $CD\bar{m}$ $DC\underline{m}$ \underline{m} </div>	$p_{CD\bar{m}} = \frac{1}{2} \left(1 + \frac{\bar{k} - \beta(1-\lambda)}{\lambda(1+\bar{m})} \right)$ $p_{DC\underline{m}} = \frac{1}{2} \left(1 - \frac{\bar{k} - \beta(1-\lambda)}{\lambda(1+\bar{m})} \right)$	$\frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1 + \bar{m})(\bar{m} - \alpha + \beta) + \beta(1 + \alpha)} < \lambda < 1$	1. $\bar{k} < (1 + \bar{m})$
4.4.1.3.	<p>as above:</p> $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1 + \bar{m}) + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\underline{m}}(1 + \bar{m}) - (1 - \lambda)[p_{\underline{m}}(-\beta)] \geq 0$ $\Leftrightarrow p_{CD\bar{m}} \geq \frac{2\beta(1-\lambda)p_{\bar{m}} + \bar{k} + \lambda(1+\bar{m}) - \beta(1-\lambda)}{2\lambda(1+\bar{m})} = \frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda) + 2\beta(1-\lambda)p_{\bar{m}}}{2\lambda(1+\bar{m})}$ $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{DC\underline{m}}(1 + \alpha) - \lambda p_{CD\bar{m}}(1 + \alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\lambda(1+\alpha) + \underline{k}}{2\lambda(1+\alpha)} = \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$ <p>Hence we obtain the following phase diagram:</p>  <p>Note: $\frac{1}{2} + \frac{\bar{k} - \beta(1-\lambda)}{2\lambda(1+\bar{m})} < \frac{1}{2} + \frac{\underline{k}}{2\lambda(1+\alpha)}$</p> <p>As the diagram clearly indicates this equilibrium is unstable.</p>		

4.4.2.	$CD\bar{m}$ $DD\underline{m}$ \underline{m}	$p_{CD\bar{m}} = \frac{\bar{k}}{\lambda(1+\bar{m})}$ $p_{DD\underline{m}} = 1 - \frac{\bar{k}}{\lambda(1+\bar{m})}$	$\lambda > \frac{\bar{k}}{(1+\bar{m})}$	$1. \bar{k} < (1+\bar{m})$
		$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$ $\Pi_{\bar{m}}(DD, \underline{m}) = 0$ $\Pi_{\underline{m}}(\underline{m}) = 0$ $\Pi_{\underline{m}}(\bar{m}) = \lambda p_{CD\bar{m}}(1+\alpha) - \underline{k}$ $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\bar{k} + \beta(1-\lambda)p_{\bar{m}}}{\lambda(1+\bar{m})}$ $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow -\lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\underline{k}}{\lambda(1+\alpha)}$ <p>Hence we obtain the following phase diagram:</p>  <p>As the diagram clearly indicates this equilibrium is unstable.</p>		

$CD\bar{m}$	$p_{DC\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\alpha) - \bar{k}(\bar{m} - \alpha + \beta)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right]$		
$DD\bar{m}$	$p_{CD\bar{m}} = \frac{1}{\lambda} \left[\frac{\beta(1+\bar{m} + \bar{k})}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right]$		
$DC\bar{m}$		$\frac{\bar{k}(\bar{m} - \alpha) + \beta(2 + \alpha + \bar{m})}{(1+\bar{m})(\bar{m} - \alpha + \beta) + \beta(1+\alpha)} < \lambda < 1$	1. $\beta(1+\alpha) > \bar{k}(\bar{m} - \alpha + \beta)$
\bar{m}	$p_{DD\bar{m}} = 1 + \frac{1}{\lambda} \left[\frac{\bar{k}(\bar{m} - \alpha) - \beta(2 + \bar{m} + \alpha)}{\beta(1+\alpha) + (1+\bar{m})(\bar{m} - \alpha + \beta)} \right]$		2. $\underline{k} < \frac{(1+\alpha)(\beta(\bar{m} - \alpha) + \bar{k}(\bar{m} - \alpha + 2\beta))}{(1+\bar{m})(\bar{m} - \alpha + \beta) + \beta(1+\alpha)}$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) \geq 0 \Leftrightarrow \lambda [p_{CD\bar{m}}(1+\bar{m}) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \lambda p_{CD\bar{m}}(1+\alpha) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(\bar{m} - \alpha) + \lambda(1 - p_{CD\bar{m}} - p_{DC\bar{m}})(-\beta) + (1-\lambda)[p_{\bar{m}}(-\beta)] \geq 0 \Leftrightarrow$$

$$\lambda p_{CD\bar{m}}(\bar{m} - \alpha + \beta) - \lambda\beta + \lambda\beta p_{DC\bar{m}} + (1-\lambda)[p_{\bar{m}}(-\beta)] \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\lambda\beta - \lambda\beta p_{DC\bar{m}} + (1-\lambda)\beta p_{\bar{m}}}{\lambda(\bar{m} - \alpha + \beta)} = \frac{\lambda\beta + (1-\lambda)\beta p_{\bar{m}}}{\lambda(\bar{m} - \alpha + \beta)} - \frac{\beta}{(\bar{m} - \alpha + \beta)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=1}{=} \frac{\beta}{\lambda(\bar{m} - \alpha + \beta)} - \frac{\beta}{(\bar{m} - \alpha + \beta)} p_{DC\bar{m}}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Leftrightarrow \lambda [p_{CD\bar{m}}(1+\bar{m}) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\bar{m}}(1+\bar{m}) - (1-\lambda)[p_{\underline{m}}(-\beta)] \geq 0 \Leftrightarrow$$

$$\lambda [p_{CD\bar{m}}(1+\bar{m}) + (1 - p_{CD\bar{m}} - p_{DC\bar{m}})(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{DC\bar{m}}(1+\bar{m}) - (1-\lambda)[p_{\underline{m}}(-\beta)] \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\bar{m} + \beta) - \lambda p_{DC\bar{m}}(1+\bar{m} - \beta) - \lambda\beta - \beta(1-\lambda)p_{\underline{m}} - \bar{k} + \beta(1-\lambda)p_{\underline{m}} \geq 0 \Leftrightarrow$$

$$\Leftrightarrow p_{CD\bar{m}} \geq \frac{\lambda p_{DC\bar{m}}(1+\bar{m} - \beta) + 2\lambda\beta + 2\beta(1-\lambda)p_{\underline{m}} + \bar{k} - \beta}{\lambda(1+\bar{m} + \beta)} = \frac{2\lambda\beta + 2\beta(1-\lambda)p_{\underline{m}} + \bar{k} - \beta}{\lambda(1+\bar{m} + \beta)} + \frac{(1+\bar{m} - \beta)}{(1+\bar{m} + \beta)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=1}{=} \frac{\beta + \bar{k}}{\lambda(1+\bar{m} + \beta)} + \frac{(1+\bar{m} - \beta)}{(1+\bar{m} + \beta)} p_{DC\bar{m}}$$

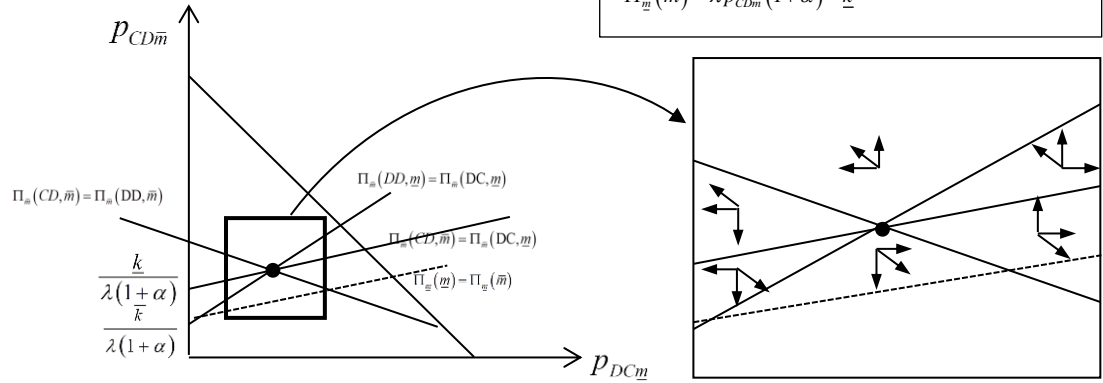
$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1+\alpha) - \bar{k} - \lambda p_{DC\bar{m}}(1+\bar{m}) - (1-\lambda)[p_{\underline{m}}(-\beta)] \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\bar{k} - \beta(1-\lambda)p_{\underline{m}}}{\lambda(1+\alpha)} + \frac{(1+\bar{m})}{(1+\alpha)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=1}{=} \frac{\bar{k}}{\lambda(1+\alpha)} + \frac{(1+\bar{m})}{(1+\alpha)} p_{DC\bar{m}}$$

$$\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow \lambda p_{DC\bar{m}}(1+\alpha) - \lambda p_{CD\bar{m}}(1+\alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\underline{k}}{\lambda(1+\alpha)} + p_{DC\bar{m}}$$

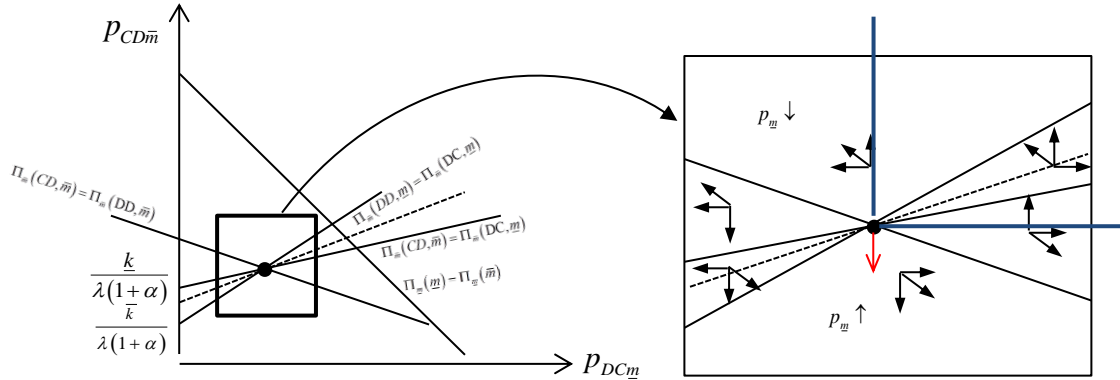
Note that $-\frac{\beta}{(\bar{m} - \alpha + \beta)} < \frac{(1+\bar{m} - \beta)}{(1+\bar{m} + \beta)} < 1 < \frac{(1+\bar{m})}{(1+\alpha)}$.

Hence we obtain the following phase diagram:

As the diagram clearly indicates this equilibrium is unstable. If in the course of the dynamics the dotted line is crossed, low types start to prefer not to signal since signaling is not often enough rewarded by $CD\bar{m}$ -player and too often punished by $DC\bar{m}$ -player. The induced decline in the share of signaling low types will shift all three constraints downwards.



$$\begin{aligned} \Pi_{\bar{m}}(CD, \bar{m}) &= \lambda [p_{CD\bar{m}}(1+\bar{m}) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} \\ \Pi_{\bar{m}}(DD, \bar{m}) &= \lambda p_{CD\bar{m}}(1+\alpha) - \bar{k} \\ \Pi_{\bar{m}}(DC, \underline{m}) &= \lambda p_{DC\bar{m}}(1+\bar{m}) + (1-\lambda)[p_{\underline{m}}(-\beta)] \\ \Pi_{\underline{m}}(\underline{m}) &= \lambda p_{DC\bar{m}}(1+\alpha) \\ \Pi_{\underline{m}}(\bar{m}) &= \lambda p_{CD\bar{m}}(1+\alpha) - \underline{k} \end{aligned}$$

4.6.1.2.	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div>$CD\bar{m}$</div> <div>$DD\bar{m}$</div> <div>$DC\bar{m}$</div> <div>\bar{m}</div> <div>\underline{m}</div> </div> <div style="margin-top: 10px;"> $p_{DC\bar{m}} = \frac{\beta(1+\alpha) + (1-\bar{m}-\beta+2\alpha)\underline{k} - (1+\alpha)\bar{k}}{2(1+\alpha)(\bar{m}-\alpha+\beta)}$ $p_{CD\bar{m}} = \frac{\beta(1+\alpha) + (1+\bar{m}+\beta)\underline{k} - (1+\alpha)\bar{k}}{2(1+\alpha)(\bar{m}-\alpha+\beta)}$ $p_{DD\bar{m}} = \left[\frac{-\beta + \lambda(\bar{m}-\alpha+\beta) + \bar{k} - \underline{k}}{\lambda(\bar{m}-\alpha+\beta)} \right],$ $p_{\bar{m}} = -\frac{(1+\alpha)(-\beta(\bar{m}-\alpha) + (\alpha-2\beta-\bar{m})\bar{k}) + ((1+\bar{m})(\bar{m}-\alpha+\beta) + \beta(1+\alpha))\underline{k}}{2(1+\alpha)(1-\lambda)(\bar{m}-\alpha+\beta)}$ $p_{\bar{m}} = 1 - p_{\underline{m}}$ </div>	See Appendix Fehler! Verweisquelle konnte nicht gefunden werden.	
	<p>Hence we obtain the following phase diagram:</p>  <p>We will apply here the same logic as in the case for the equilibrium 4.3.1.2. Consider a perturbation that pushes the population state in the lower triangular region (red arrow). Given that low types will strictly prefer not to signal, which in turn shifts the intersection point of the iso-profit lines into the first quadrant relative to the equilibrium. As the picture clearly indicates this will not help to stabilize the equilibrium.</p>		

$CD\bar{m}$	$p_{CD\bar{m}} = \frac{\beta(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}$	$\frac{(1+\alpha)\beta(\bar{m}-\alpha+2\beta)+(\bar{m}-\alpha)(\bar{m}-\alpha+\beta)\bar{k}}{2\beta(1+\alpha)(\bar{m}-\alpha+\beta)} < \lambda < \frac{\beta(1+\alpha) > \frac{((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))\bar{k}}{(\bar{m}-\alpha)} - \frac{((1+\alpha)(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}{(\bar{m}-\alpha)}\bar{k}}$
$DD\bar{m}$		
$DC\bar{m}$	$p_{DC\bar{m}} = \frac{\beta((1-\lambda)(\bar{m}-\alpha+\beta)+\lambda(1+\alpha))-(\bar{m}-\alpha+\beta)\bar{k}}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}$	
\underline{m}	$p_{DD\bar{m}} = \frac{(\bar{m}-\alpha)(\lambda(1+\bar{m}+\beta)+\bar{k}-\beta)}{\lambda((1+\bar{m})(\bar{m}-\alpha+\beta)+\beta(1+\alpha))}$	

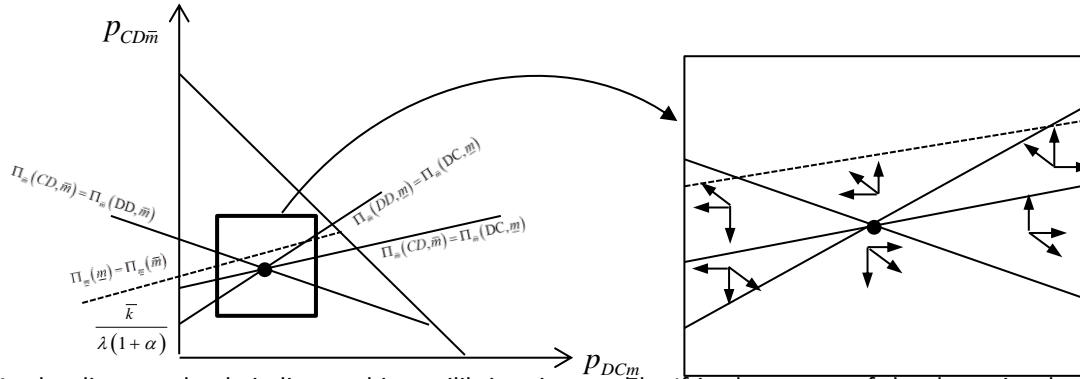
as above

$$\begin{aligned} \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) &\geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\lambda\beta - \lambda\beta p_{DC\bar{m}} + (1-\lambda)\beta p_{\bar{m}}}{\lambda(\bar{m}-\alpha+\beta)} = \frac{\lambda\beta + (1-\lambda)\beta p_{\bar{m}}}{\lambda(\bar{m}-\alpha+\beta)} - \frac{\beta}{(\bar{m}-\alpha+\beta)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=0}{=} \frac{\lambda\beta}{\lambda(\bar{m}-\alpha+\beta)} - \frac{\beta}{(\bar{m}-\alpha+\beta)} p_{DC\bar{m}} \\ \Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &\geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\lambda p_{DC\bar{m}}(1+\bar{m}-\beta) + 2\lambda\beta + 2\beta(1-\lambda)p_{\bar{m}} + \bar{k} - \beta}{\lambda(1+\bar{m}+\beta)} = \frac{2\lambda\beta + 2\beta(1-\lambda)p_{\bar{m}} + \bar{k} - \beta}{\lambda(1+\bar{m}+\beta)} + \frac{(1+\bar{m}-\beta)}{(1+\bar{m}+\beta)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=0}{=} \frac{2\lambda\beta - \beta + \bar{k}}{\lambda(1+\bar{m}+\beta)} + \frac{(1+\bar{m}-\beta)}{(1+\bar{m}+\beta)} p_{DC\bar{m}} \\ \Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DC, \underline{m}) &\geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\bar{k} - \beta(1-\lambda)p_{\bar{m}}}{\lambda(1+\alpha)} + \frac{(1+\bar{m})}{(1+\alpha)} p_{DC\bar{m}} \stackrel{p_{\bar{m}}=0}{=} \frac{\bar{k} - \beta(1-\lambda)}{\lambda(1+\alpha)} + \frac{(1+\bar{m})}{(1+\alpha)} p_{DC\bar{m}} \\ \Pi_{\underline{m}}(\underline{m}) - \Pi_{\bar{m}}(\bar{m}) &\geq 0 \Leftrightarrow \lambda p_{DC\bar{m}}(1+\alpha) - \lambda p_{CD\bar{m}}(1+\alpha) + \bar{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\bar{k}}{\lambda(1+\alpha)} + p_{DC\bar{m}} \end{aligned}$$

Note that $-\frac{\beta}{(\bar{m}-\alpha+\beta)} < \frac{(1+\bar{m}-\beta)}{(1+\bar{m}+\beta)} < 1 < \frac{(1+\bar{m})}{(1+\alpha)}$.

In comparison to 4.6.1.1. the three lines corresponding to equal profits among the equilibrium strategies shifts such that the equilibrium lies below the dotted line, which is constant with respect to changes in the share of signaling low types.

Hence we obtain the following phase diagram:



As the diagram clearly indicates this equilibrium is unstable. If in the course of the dynamics the dotted line is crossed, low types start to prefer to signal since signaling is often enough rewarded by $CD\bar{m}$ -player and not too often punished by $DC\bar{m}$ -player. The induced incline in the share of signaling low types will shift all three constraints upwards.

4.6.2.

$CD\bar{m}$ $DD\bar{m}$ $DD\underline{m}$ \underline{m}	$p_{CD\bar{m}} = \frac{k}{\lambda(1+\alpha)}$ $p_{DD\bar{m}} = \frac{(\bar{m}-\alpha)}{\beta} \frac{k}{\lambda(1+\alpha)}$ $p_{DD\underline{m}} = 1 - \frac{(\bar{m}-\alpha+\beta)}{\beta} \frac{k}{\lambda(1+\alpha)}$	$\frac{\bar{k}(\bar{m}-\alpha+\beta)}{(1+\alpha)\beta} < \lambda < 1$	$\beta(1+\alpha) > (\bar{m}-\alpha+\beta)\bar{k}$
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$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) \geq 0 \Leftrightarrow \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + p_{DD\bar{m}}(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k} - \lambda p_{CD\bar{m}}(1 + \alpha) + \bar{k} \geq 0 \Leftrightarrow \lambda \left[p_{CD\bar{m}}(\bar{m} - \alpha) + (1 - p_{CD\bar{m}} - p_{DD\bar{m}})(-\beta) \right] \geq (1 - \lambda) \beta p_{\bar{m}} \Leftrightarrow$$

$$p_{CD\bar{m}} \geq \frac{(1-\lambda)\beta p_{\bar{m}} + \lambda(1-p_{DD\bar{m}})\beta}{\lambda(\bar{m}-\alpha+\beta)} = \frac{(1-\lambda)\beta p_{\bar{m}} + \lambda\beta}{\lambda(\bar{m}-\alpha+\beta)} - \frac{\beta}{(\bar{m}-\alpha+\beta)} p_{DD\bar{m}} \stackrel{p_{\bar{m}}=0}{=} \frac{\beta}{(\bar{m}-\alpha+\beta)} - \frac{\beta}{(\bar{m}-\alpha+\beta)} p_{DD\bar{m}}$$

$$\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \geq 0 \Leftrightarrow \lambda [p_{CD\bar{m}}(1 + \bar{m}) + p_{DD\bar{m}}(-\beta)] + (1 - \lambda) [p_{\bar{m}}(-\beta)] - \bar{k} \geq 0 \Leftrightarrow \lambda [p_{CD\bar{m}}(1 + \bar{m}) + (1 - p_{CD\bar{m}} - p_{DD\bar{m}})(-\beta)] \geq \bar{k} + (1 - \lambda)\beta p_{\bar{m}} \Leftrightarrow$$

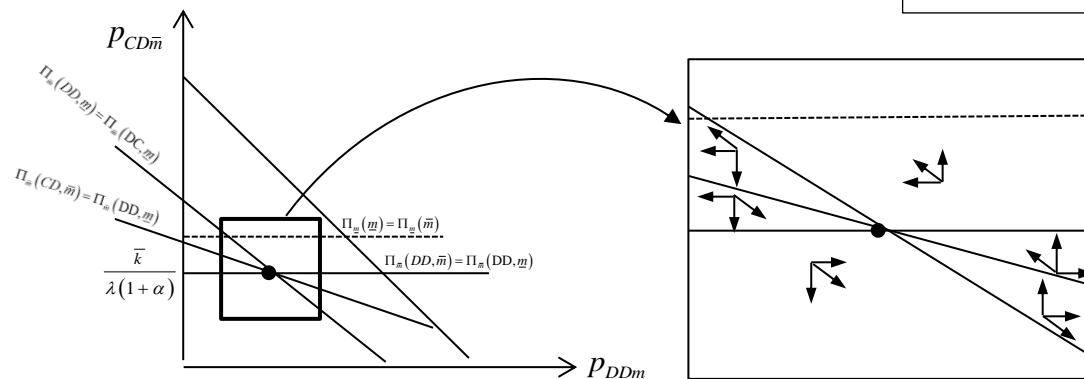
$$p_{CD\bar{m}} \geq \frac{\bar{k} + (1-\lambda)\beta p_{\bar{m}} + \lambda\beta}{\lambda(1+\bar{m}+\beta)} - \frac{\beta}{(1+\bar{m}+\beta)} p_{DD\bar{m}} \stackrel{p_{\bar{m}}=0}{=} \frac{\bar{k} + \lambda\beta}{\lambda(1+\bar{m}+\beta)} - \frac{\beta}{(1+\bar{m}+\beta)} p_{DD\bar{m}}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) - \Pi_{\bar{m}}(DD, \underline{m}) \geq 0 \Leftrightarrow \lambda p_{CD\bar{m}}(1 + \alpha) - \bar{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \geq \frac{\bar{k}}{\lambda(1 + \alpha)}$$

$$\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow -\lambda p_{CD\bar{m}}(1 + \alpha) + \underline{k} \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\underline{k}}{\lambda(1 + \alpha)}$$

Note: $-\frac{\beta}{(1+\bar{m}+\beta)} > -\frac{\beta}{(\bar{m}-\alpha+\beta)}$

Hence we obtain the following phase diagram:



Since the phase diagram is ambiguous with respect to stability we will study the Eigenvalues of the linearized system.

$$\Pi_{\bar{m}}(CD, \bar{m}) = \lambda \left[p_{CD\bar{m}}(1 + \bar{m}) + p_{DD\bar{m}}(-\beta) \right] + (1 - \lambda) \left[p_{\bar{m}}(-\beta) \right] - \bar{k}$$

$$\Pi_{\bar{m}}(DD, \bar{m}) = \lambda p_{CD\bar{m}}(1 + \alpha) - \bar{k}$$

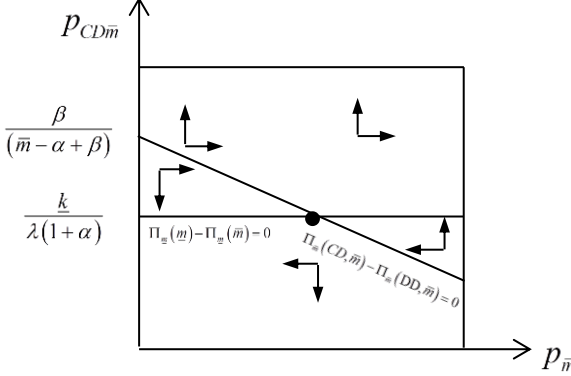
$$\Pi_{\bar{m}}(DD, \underline{m}) = 0$$

$$\Pi_m(\underline{m}) = 0$$

$$\Pi_m(\bar{m}) = \lambda p_{CD\bar{m}}(1 + \alpha) - \underline{k}$$

$ \begin{array}{ccc} \begin{array}{c} p \\ CD, \bar{m} \\ \varepsilon \end{array} & \begin{array}{c} q \\ DD, \bar{m} \\ -\varepsilon \end{array} & \begin{array}{c} DD, \underline{m} \\ -\varepsilon \end{array} \\ : & & : \\ \Delta \Pi_{\bar{m}}(CD, \bar{m}) = \lambda(1 + \bar{m})\varepsilon & & \Delta \Pi_{\bar{m}}(CD, \bar{m}) = -\lambda(\beta)\varepsilon \\ \Delta \Pi_{\bar{m}}(DD, \bar{m}) = \lambda(1 + \alpha)\varepsilon & & \Delta \Pi_{\bar{m}}(DD, \bar{m}) = 0 \\ \Delta \Pi_{\bar{m}}(DD, \underline{m}) = 0 & & \Delta \Pi_{\bar{m}}(DD, \underline{m}) = 0 \\ \text{hence } p_{CD, \bar{m}} \uparrow \text{ and } p_{DD, \underline{m}} \downarrow \Rightarrow f_p^p > 0 & \text{hence } p_{CD, \bar{m}} \downarrow \text{ and } p_{DD, \underline{m}} \uparrow \Rightarrow f_q^q > 0 & \text{payoff monotonicity} \\ & & \Pi_{\bar{m}}(DD, \bar{m}) = \Pi_{\bar{m}}(DD, \underline{m}) \Leftrightarrow p_{DD, \bar{m}} = p_{DD, \underline{m}} \end{array} $	
$ \text{hence } \begin{pmatrix} f_p^p & f_q^p \\ f_p^q & f_q^q \end{pmatrix} = \begin{pmatrix} & \\ & \\ & \\ & \end{pmatrix} > 0 $	
Hence at least one of the Eigenvalues is strictly positive and therefore this equilibrium is unstable.	

4.7.1.1	$CD\bar{m}$ \bar{m} \underline{m}	$p_{CD\bar{m}} = 1$	$\lambda = \frac{\underline{k}}{(1 + \alpha)}$	1. $\underline{k} < (1 + \alpha)$ 2. $p_{\bar{m}} < \frac{\lambda(\bar{m} - \alpha)}{(1 - \lambda)\beta}$, note that 2. is only binding if: $\frac{\lambda(\bar{m} - \alpha)}{(1 - \lambda)\beta} < 1 \Leftrightarrow \lambda < \frac{\beta}{(\bar{m} - \alpha + \beta)} \Leftrightarrow \frac{\underline{k}}{(1 + \alpha)} < \frac{\beta}{(\bar{m} - \alpha + \beta)}$
	<p> $\Pi_{\bar{m}}(CD, \bar{m}) = \lambda p_{CD\bar{m}}(1 + \bar{m}) + (1 - \lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$ $\Pi_{\bar{m}}(\underline{m}) = 0$ $\Pi_{\bar{m}}(\bar{m}) = \lambda p_{CD\bar{m}}(1 + \alpha) - \underline{k}$ </p> <p>Hence we obtain the following phase diagram:</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(\underline{m}) = \frac{\underline{k}}{(1 + \alpha)}(1 + \bar{m}) - \bar{k} + (1 - \lambda)[p_{\bar{m}}(-\beta)]$ $(\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(\underline{m}))' = \frac{\underline{k}}{(1 + \alpha)} - \bar{k} + (1 - \lambda)[p_{\bar{m}}(-\beta)]$ </div> <p>As the diagram clearly indicates this equilibrium set is stable.</p> <p>Any perturbation induces a drift towards the separating equilibrium $p_{CD\bar{m}} = 1, p_{\bar{m}} = 0$.</p>			

4.7.1.2.	$CD\bar{m}$ $DD\bar{m}$ \bar{m} \underline{m}	$p_{CD\bar{m}} = \frac{\underline{k}}{\lambda(1+\alpha)}$ $p_{DD\bar{m}} = 1 - \frac{\underline{k}}{\lambda(1+\alpha)}$ $p_{\bar{m}} = \frac{(\bar{m} - \alpha + \beta)\underline{k} - \lambda(1+\alpha)\beta}{(1-\lambda)(1+\alpha)\beta}$ $p_{\underline{m}} = \frac{(1+\alpha)\beta - (\bar{m} - \alpha + \beta)\underline{k}}{(1-\lambda)(1+\alpha)\beta}$	$\frac{\underline{k}}{(1+\alpha)} < \lambda < \frac{(\bar{m} - \alpha + \beta)}{\beta} \frac{\underline{k}}{(1+\alpha)}$	$1. (1+\alpha)\beta > (\bar{m} - \alpha + \beta)\underline{k}$ $2. \underline{k} < (1+\alpha)$
	<p> $\Pi_{\bar{m}}(CD, \bar{m}) = \lambda[p_{CD\bar{m}}(1+\bar{m}) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k}$ $\Pi_{\bar{m}}(DD, \bar{m}) = \lambda p_{CD\bar{m}}(1+\alpha) - \bar{k}$ $\Pi_{\underline{m}}(\underline{m}) = 0$ $\Pi_{\underline{m}}(\bar{m}) = \lambda p_{CD\bar{m}}(1+\alpha) - \underline{k}$ $\Pi_{\bar{m}}(CD, \bar{m}) - \Pi_{\bar{m}}(DD, \bar{m}) \geq 0 \Leftrightarrow \lambda[p_{CD\bar{m}}(1+\bar{m}) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] - \bar{k} - \lambda p_{CD\bar{m}}(1+\alpha) + \bar{k} \geq 0 \Leftrightarrow \lambda[p_{CD\bar{m}}(\bar{m} - \alpha) + p_{DD\bar{m}}(-\beta)] + (1-\lambda)[p_{\bar{m}}(-\beta)] \geq 0 \Leftrightarrow$ $p_{CD\bar{m}} \geq \frac{\lambda\beta - (1-\lambda)\beta p_{\bar{m}}}{\lambda(\bar{m} - \alpha + \beta)}$ $\Pi_{\underline{m}}(\underline{m}) - \Pi_{\underline{m}}(\bar{m}) \geq 0 \Leftrightarrow p_{CD\bar{m}} \leq \frac{\underline{k}}{\lambda(1+\alpha)}$ </p> <p>Hence we obtain the following phase diagram:</p>  <p>As the diagram clearly indicates this equilibrium is unstable.</p>			